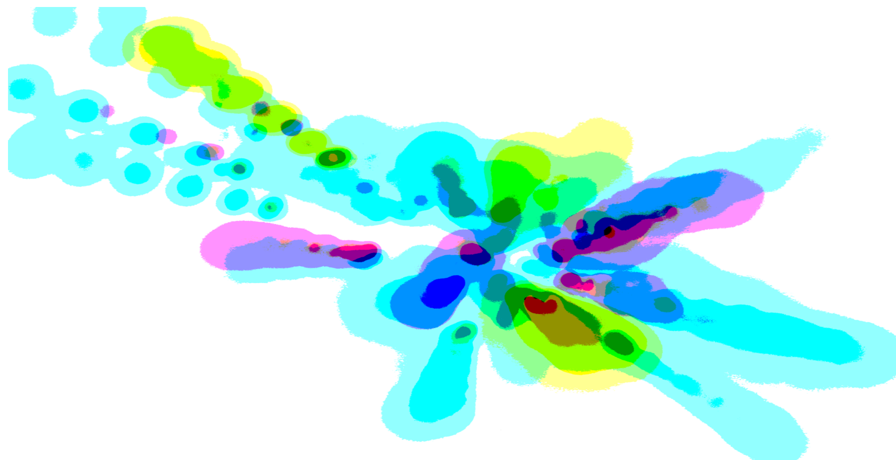
The chiral anomaly in polarized deeply inelastic scattering: Topological screening and sphaleron transitions

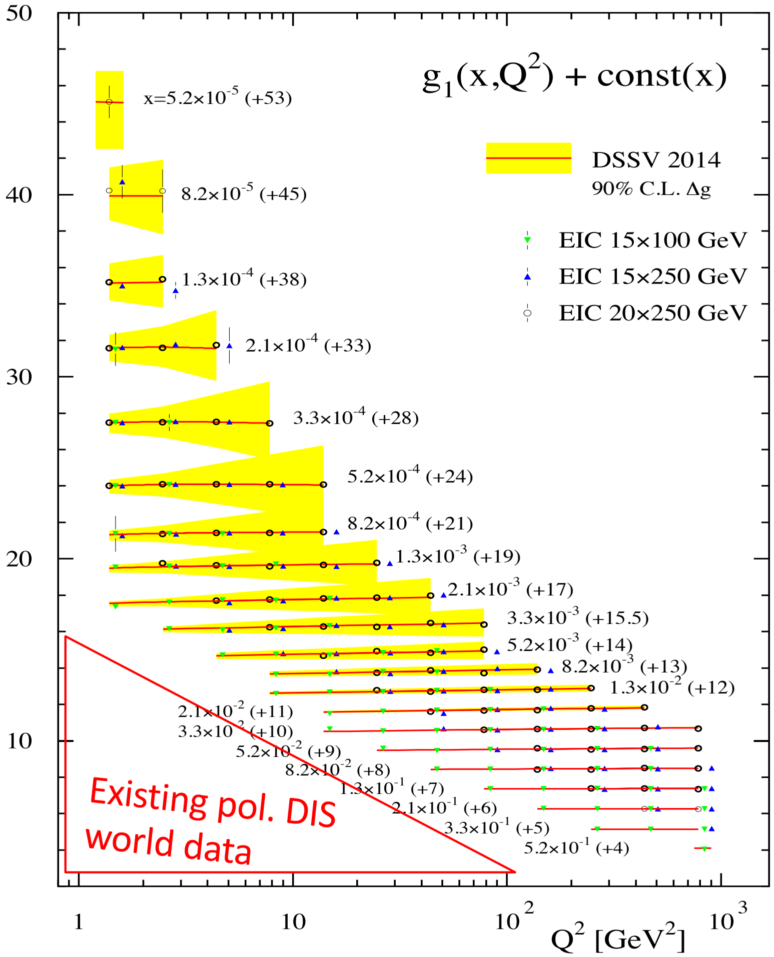


Raju Venugopalan

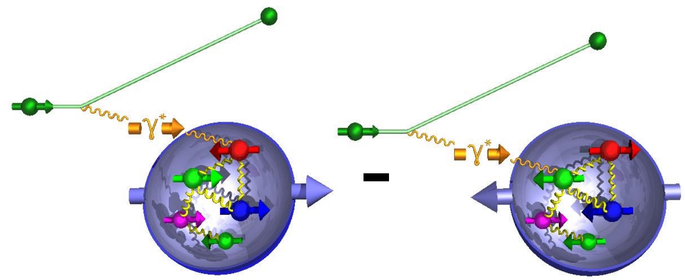
Brookhaven National Laboratory and Stony Brook University

CFNS workshop, January 14, 2025

Interplay of perturbative and non-perturbative dynamics in polarized DIS



g_1 extracted from longitudinal spin asymmetry

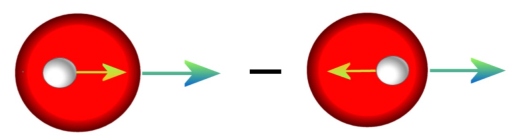


First moment $\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x,Q^2)$

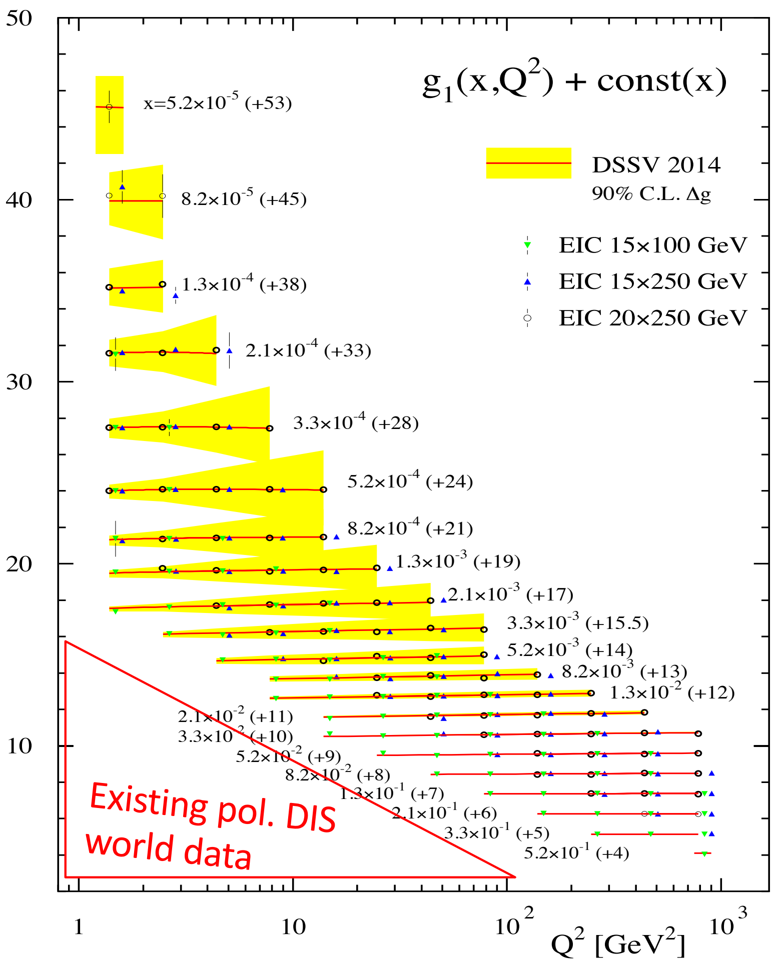
In the parton model, it is the net quark helicity

$$\Sigma(Q^2) = \sum_f \int_0^1 dx_B\, \left(\Delta q_f(x_B,Q^2) + \Delta\bar{q}_f(x_B,Q^2)\right)$$

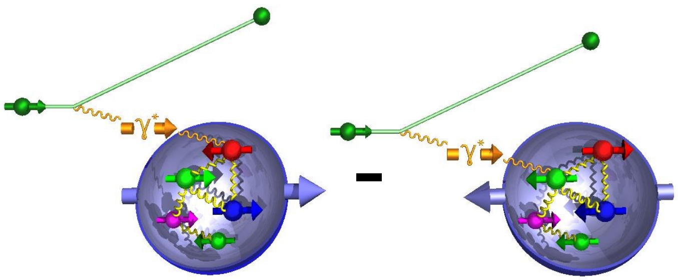
Δq_f = Diff. in parton densities of left and right handed quarks of flavor f
$\Delta\bar{q}_f$ = Ditto for anti-quarks



Interplay of perturbative and non-perturbative dynamics in polarized DIS



g_1 extracted from longitudinal spin asymmetry



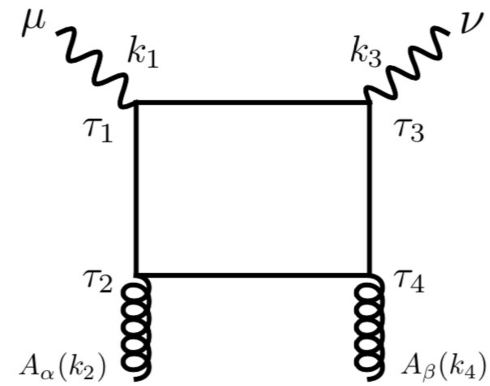
First moment $\Delta\Sigma(Q^2) \propto \int_0^1 dx\, g_1(x,Q^2)$

In QCD, the physics is far more subtle and rich – elements include,

- The chiral anomaly and validity of QCD factorization theorems
- Anomalous Ward identities, large N, topological screening of ΔΣ
- Novel axion-like dynamics, and sphaleron-like transitions at small x

Worldline approach to polarized DIS: box diagram

Anti-symmetric piece of hadron tensor



$$\tilde{W}_{\mu\nu}(q,P,S) = \frac{2M_N}{P\cdot q}\epsilon_{\mu\nu\alpha\beta}q^\alpha\Big\{S^\beta g_1(x_B,Q^2) + \Big[S^\beta - \frac{(S\cdot q)P^\beta}{P\cdot q}\Big]g_2(x_B,Q^2)\Big\}$$

$$i\tilde{W}^{\mu\nu}(q,P,S) = \frac{1}{2\pi e^2}\,\mathrm{Im}\int d^4x\, e^{-iqx}\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_3}{(2\pi)^4}e^{-ik_1\frac{x}{2}}e^{ik_3\frac{x}{2}}\langle P,S|\tilde{\Gamma}_A^{\mu\nu}[k_1,k_3]|P,S\rangle$$

with the polarization tensor $\Gamma_A^{\mu\nu}[k_1,k_3] = \int\frac{d^4k_2}{(2\pi)^4}\int\frac{d^4k_4}{(2\pi)^4}\,\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4]\;\mathrm{Tr_c}(\tilde{A}_\alpha(k_2)\tilde{A}_\beta(k_4))$

Antisymmetric piece of box diagram

$$\Gamma_A^{\mu\nu\alpha\beta}[k_1,k_3,k_2,k_4] \equiv -\frac{g^2e^2e_f^2}{2}\int_0^\infty\frac{dT}{T}\int\mathcal{D}x\int\mathcal{D}\psi\;\exp\Big\{-\int_0^T d\tau\Big(\frac{1}{4}\dot{x}^2+\frac{1}{2}\psi\cdot\dot{\psi}\Big)\Big\}$$
$$\times\Big[V_1^\mu(k_1)V_3^\nu(k_3)V_2^\alpha(k_2)V_4^\beta(k_4) - (\mu\leftrightarrow\nu)\Big]$$

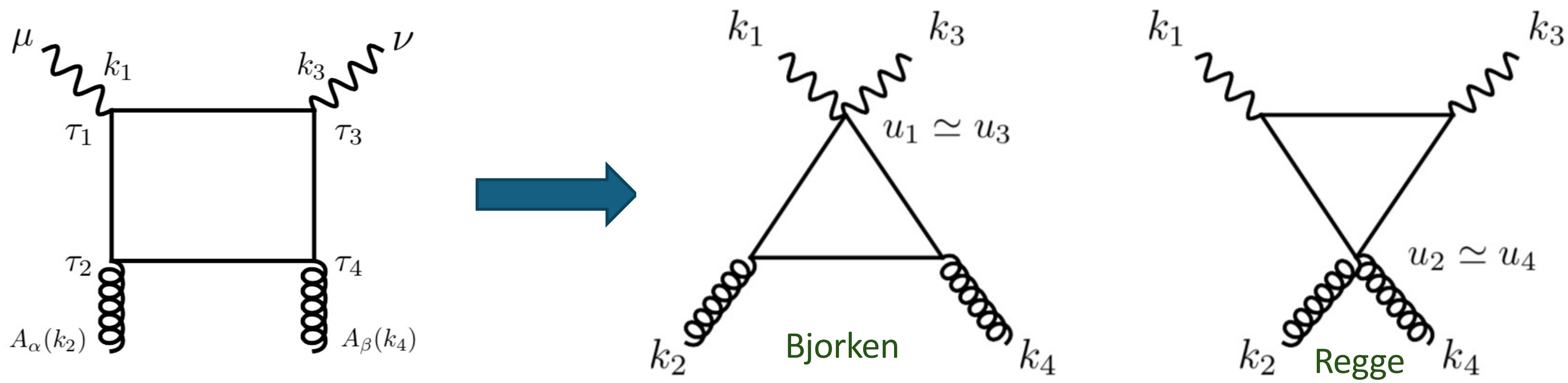
Product of boson and Grassmann worldline currents $V_i^\mu(k_i) \equiv \int_0^T d\tau_i(\dot{x}_i^\mu + 2i\psi_i^\mu k_j\cdot\psi_j)e^{ik_i\cdot x_i}$

“Pert. Theory without Feynman diagrams”, M. Strassler, NPB 385 (1992) 145
Review: C. Schubert, Phys. Repts. (2001)

A. Tarasov, RV, PRD (2021, 2022)

Finding triangles in boxes in Bjorken and Regge asymptotics

A. Tarasov, RV, PRD (2021, 2022)



$$S^\mu g_1(x_B, Q^2)\Big|_{Q^2\to\infty} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x}\left(1-\frac{x_B}{x}\right)\int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \frac{l^\mu}{l^2} \langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$ + non-pole

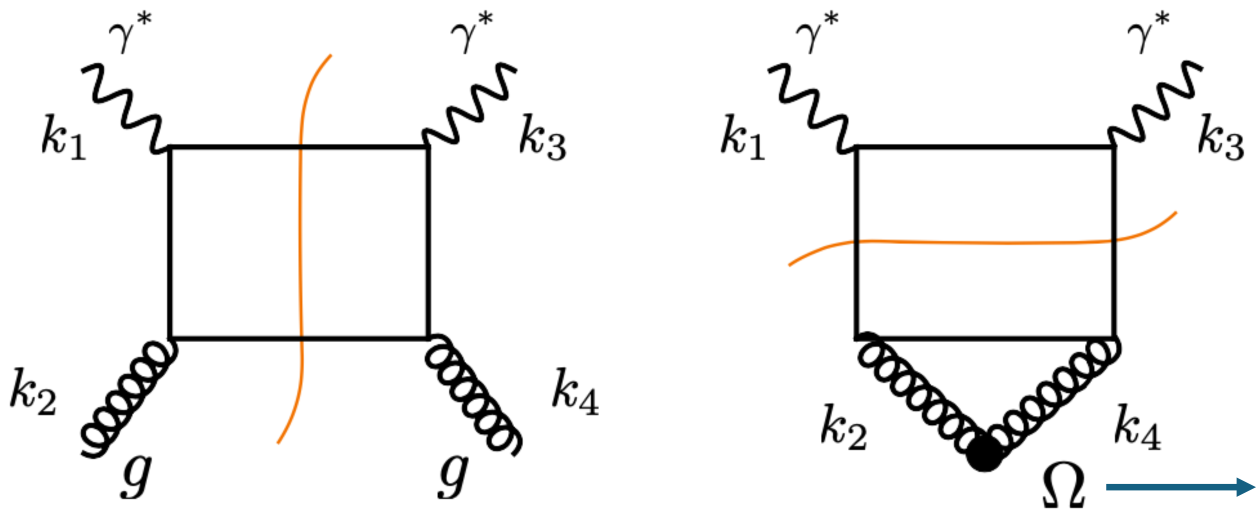
$$S^\mu\, g_1(x_B, Q^2)|_{x_B\to 0} = \sum_f e_f^2 \frac{\alpha_s}{i\pi M_N} \int_{x_B}^1 \frac{dx}{x} \int \frac{d\xi}{2\pi} e^{-i\xi x} \lim_{l_\mu\to 0} \frac{l^\mu}{l^2} \langle P', S|\mathrm{Tr}_c F_{\alpha\beta}(\xi n)\tilde{F}^{\alpha\beta}(0)|P, S\rangle$$ + non-pole

We discovered the “anomaly pole not just in its first moment but in $g_1(x,Q^2)$ itself

Very important to impose exact kinematics without making collinear or high energy approximations...

Reproduced in a Feynman diagram analysis
S. Bhattacharya, Y. Hatta, W. Vogelsang, PRD (2023)

Finding triangles in boxes in Bjorken and Regge asymptotics



Topological charge density: $F\tilde{F}$

How does one regulate this pole? Such a pole also exists in QED but the story in QCD is much more subtle

Anomalies in the worldline formulation of QFT

Fermion action in background of scalar, pseudoscalar, vector and axial vector fields:

$$S_{\text{fermion}}[\bar{\Psi}, \Phi, \Pi, A, B, \Psi] = \int d^4x \, \bar{\Psi}^I \left[i\partial\!\!\!/ - \Phi + i\gamma^5 \Pi + A\!\!\!/ + \gamma^5 B\!\!\!/ \right]^{IJ} \Psi^J$$

Effective action: $-\mathcal{W}[A, B, \Phi, \Pi] = \mathrm{Ln}\,\mathrm{Det}\,[\mathcal{D}]$ with $\mathcal{D} = p\!\!\!/ - i\Phi(x) - \gamma_5 \Pi - A\!\!\!/ - \gamma_5 B\!\!\!/$

Split into real and imaginary parts: $\mathcal{W}_R = -\frac{1}{2}\mathrm{Ln}\left(\mathcal{D}^\dagger \mathcal{D}\right)$; $\mathcal{W}_I = \frac{1}{2}\mathrm{Arg}\,\mathrm{Det}\left(\mathcal{D}^2\right)$

Entire dynamics of the anomaly comes from W_I - the phase of the Dirac determinant

Heat kernel regularization of the phase as a worldline path integral

W_I can also be expressed as a worldline Lagrangian of 0+1- bosonic (coordinate) and Grassmann fields

Jacobian for zero modes multiplied by G-parity factor

$$W_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_0^\infty dT\,\mathcal{N}\int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi\ \mathrm{tr}\ \chi\,\bar{\omega}(0)\exp\left[-\int_0^T d\tau\,\mathcal{L}_{(\alpha)}(\tau)\right]$$

Worldline Lagrangian with chiral symmetry breaking interpolating parameter α

$$\mathcal{L}_{(\alpha)}(\tau) = \mathcal{L}(\tau)\Big|_{\Phi\to\alpha\Phi, B\to\alpha B} \quad \text{with} \quad \mathcal{L}(\tau) = \frac{\dot{x}^2}{2\mathcal{E}} + \frac{1}{2}\psi\dot{\psi} - i\dot{x}^\mu\mathcal{A}_\mu + \frac{\mathcal{E}}{2}\mathcal{H}^2 + i\mathcal{E}\psi^\mu\psi_5\mathcal{D}_\mu\mathcal{H} + \frac{i\mathcal{E}}{2}\psi^\mu\psi^\nu\mathcal{F}_{\mu\nu}$$

in the chiral basis $\mathcal{A}_\mu \equiv \begin{pmatrix} A^L_\mu & 0 \\ 0 & A^R_\mu \end{pmatrix} = \begin{pmatrix} A_\mu + B_\mu & 0 \\ 0 & A_\mu - B_\mu \end{pmatrix}$ and $\mathcal{H} \equiv \begin{pmatrix} 0 & iH \\ -iH^\dagger & 0 \end{pmatrix} = \begin{pmatrix} 0 & i\Phi + \Pi \\ -i\Phi + \Pi & 0 \end{pmatrix}$

Interesting titbit: Very analogous to problem of regularizing a chiral gauge theory – “standard model on the lattice”

D’Hoker, Gagne, hep-th/9508131, hep-th/9512080

Heat kernel regularization of the phase as a worldline path integral

W_I can also be expressed as a worldline Lagrangian of 0+1- bosonic (coordinate) and Grassmann fields

Jacobian for zero modes multiplied by G-parity factor

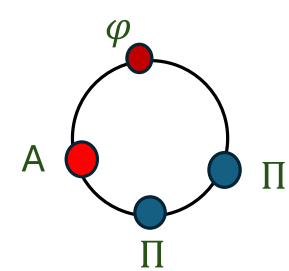
$$W_{\mathcal{I}} = -\frac{i}{32}\int_{-1}^{1} d\alpha \int_{0}^{\infty} dT\,\mathcal{N}\int_{\mathrm{PBC}} \mathcal{D}x\,\mathcal{D}\psi \;\mathrm{tr}\;\chi\,\bar{\omega}(0)\,\exp\left[-\int_{0}^{T} d\tau\,\mathcal{L}_{(\alpha)}(\tau)\right]$$

Worldline Lagrangian with chiral symmetry breaking interpolating parameter α

Can combine real and imaginary parts in a " perturbative" expansion

$$W = \sum_{n=1}^{\infty}\frac{1}{n}\int\frac{d^Dp_1}{(2\pi)^D}\cdots\frac{d^Dp_n}{(2\pi)^D}(2\pi)^D\delta^{(D)}(p_1+\cdots+p_n)\int\frac{d^Dq}{(2\pi)^D}\;\mathrm{tr}\frac{1}{\not{q}-im}$$
$$\times\,(i\tilde{\varphi}_1+\gamma_5\tilde{\Pi}_1+\tilde{\not{A}}_1)\cdots\frac{1}{\not{q}-\not{p}_1-\cdots-\not{p}_{n-1}-im}(i\tilde{\varphi}_n+\gamma_5\tilde{\Pi}_n+\tilde{\not{A}}_n)$$

Quark loop with external sources

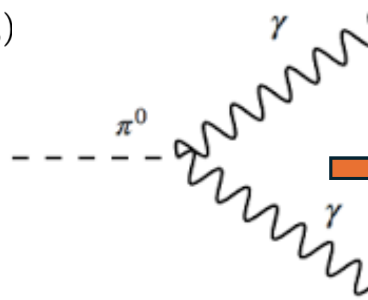


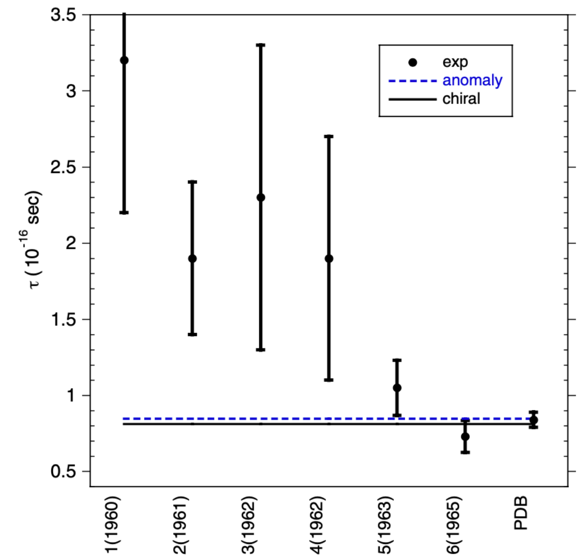
Even # of insertions belong to W_R and odd to W_I

Alvarez-Gaume, della Pietra^2, PLB (1986)
Ball, Osborn, PLB (1986)
Alvarez-Gaume, Ginsparg, Ann. Phys. (1986)

What's in a phase? WZW terms and "QCD axion" from worldline action

$$W_{\Im}[\Pi^5] = -\frac{i}{5} \int_{p^1,\ldots,p^5} (2\pi)^4 \delta^{(4)}(p^1 + \cdots + p^5)\,(-4im)\,\mathrm{Tr_c}(\tilde{\Pi}_1 \cdots \tilde{\Pi}_5)$$
$$\times\ \varepsilon_{\mu_1\cdots\mu_4} p^1_{\mu_1} \cdots p^4_{\mu_4} I'(p^i)$$





A.Bernstein,B.Holstein, RMP (2013)

Most precise prediction in QCD to date…

Likewise, expanding to O($\Phi\,\Pi\,A^2$),

$$S^{\bar\eta}_{\mathrm{WZW}} = -i\frac{\sqrt{2\,n_f}}{F_{\bar\eta}} \int d^4x\,\bar\eta\,\Omega$$

$\bar\eta$ is the primordial η', Ω the topological charge density, F_η the $\bar\eta$ decay const.

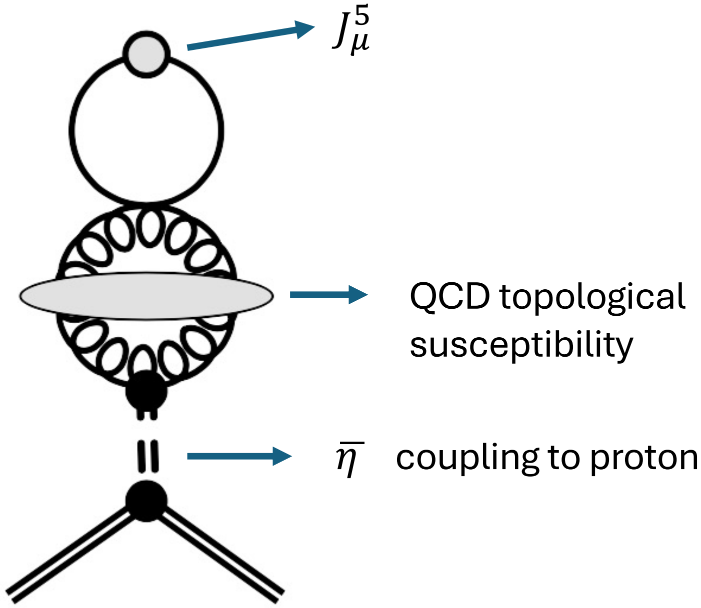
In the chiral limit, for $N_c \to \infty$, the $\bar\eta$ is the QCD axion

The proton's quark helicity from the topology of the QCD vacuum

A remarkable result by Shore and Veneziano :

$$\Delta\Sigma = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\sqrt{\chi'(0)}$$

Proton's quark helicity proportional to slope of QCD topological susceptibility in the forward limit



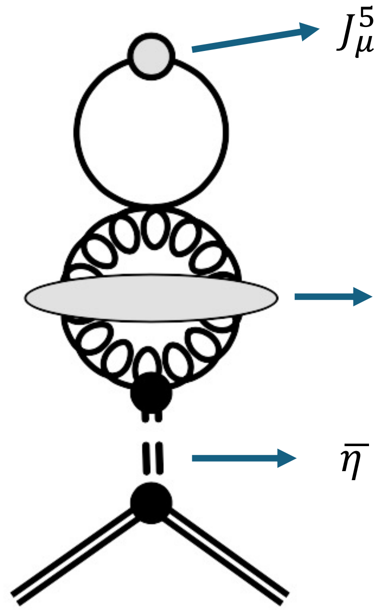
A key element in this analysis is a Goldberger-Treiman relation between axial vector and pseudoscalar form factors

Veneziano (1989)
Shore, Veneziano (1990, 1992)
Shore, Narison, Veneziano (1998)
Also, Jaffe, Manohar (1990)

The proton's quark helicity from the topology of the QCD vacuum

A remarkable result by Shore and Veneziano:

$$\Delta\Sigma = \sqrt{\frac{2}{3}}\frac{2n_f}{M_N}\, g_{\eta_0 NN}\sqrt{\chi'(0)}$$

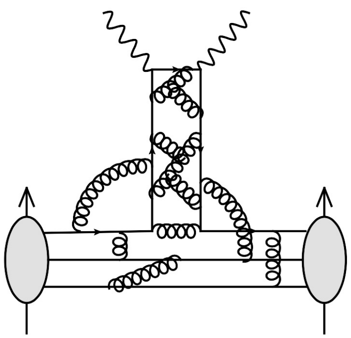


Can be understood as a primordial $\bar{\eta}$ "axion" propagating from the polarized proton, acquiring mass (Witten-Veneziano) from topological bubbles of the QCD vacuum, and coupling to J_μ^5 through the topological charge density

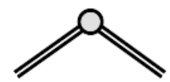
Bottom line: shift anomaly pole $l^2 \rightarrow l^2 - m_{\eta'}^2$

Tarasov, RV 2109.10370, and in preparation

Back to DIS, quark helicity and Kogut-Susskind pole



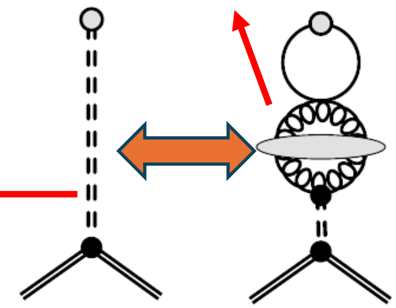
$$\langle P', S|J_5^\mu|P, S\rangle = \bar{u}(P', S)\left[\gamma^\mu\gamma_5 G_A(l^2) + l^\mu\gamma_5 G_P(l^2)\right]u(P, S)$$



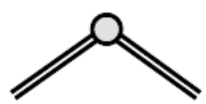
Direct axial vector coupling of J_5^μ to polarized proton

Massless "prodigal Goldstone" $\bar{\eta}$ field

Triangle coupling to top. susceptibility



I) Consider first the direct axial vector coupling:

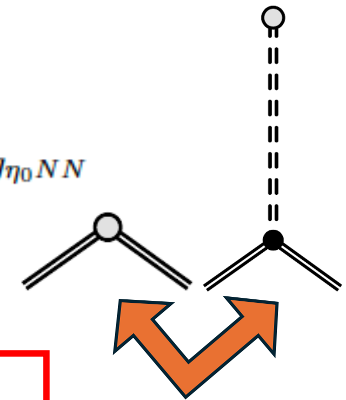


Since $G_P(l^2)$ cannot have a pole for $l^2 \to 0$, $\langle P, S|J_5^\mu|P, S\rangle = 2M_N\, G_A(0)\, S^\mu$

The helicity of the proton equals its axial vector charge

$$\Delta\Sigma = G_A(0)$$

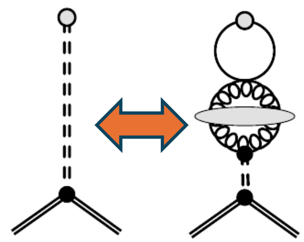
Goldberger-Treiman relation and anomaly cancelation



II) The anomaly equation + the Dirac equation
link axial vector and pseudoscalar channels: Goldberger-Treiman relation $G_A(0) = \frac{\sqrt{2\tilde{n}_f}}{2M_N} F_{\bar{\eta}}\, g_{\eta_0 NN}$

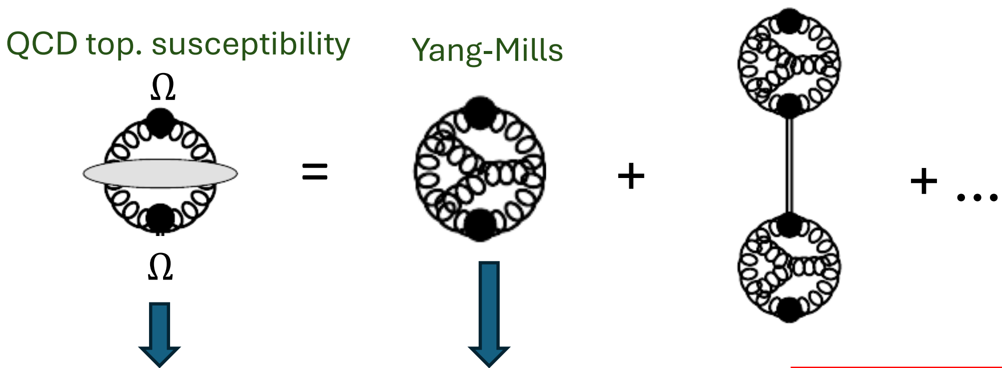
$g_{\eta_0 \mathrm{NN}}$ is coupling of isosinglet projection η_0 to proton

III) Absence of pseudoscalar pole implies



$$\sqrt{2\tilde{n}_f}\, F_{\bar{\eta}} = 2n_f \lim_{l\to 0} i\, \langle 0|T\, \Omega \eta_0|0\rangle$$

Topological mass generation and quark helicity



1/N corrections to YM top. susceptibility induced by WZW coupling

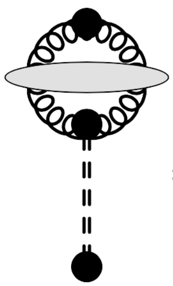
$$\chi(l^2) = l^2 \frac{1}{l^2 - m_{\eta'}^2} \chi_{\mathrm{YM}}(l^2) \quad \text{with} \quad \boxed{m_{\eta'}^2 \equiv -\frac{2\,n_f}{F_{\bar{\eta}}^2} \chi_{\mathrm{YM}}(0)}$$

Witten-Veneziano formula

Immediate consequence is screening of topological charge by the vacuum in the chiral limit: $\chi(l^2) \rightarrow 0 \; for \; l^2 \rightarrow 0$

Topological mass generation and quark helicity

Similarly, $\langle 0|T\Omega\eta_0|0\rangle \equiv$



$$= -i\frac{1}{l^2}\frac{\sqrt{2\tilde{n}_f}}{F_{\bar{\eta}}}\chi(l^2)$$

From previous slide,

$$\sqrt{2\tilde{n}_f}\,F_{\bar{\eta}} = 2n_f \lim_{l\to 0} i\,\langle 0|T\,\Omega\eta_0|0\rangle\,.$$
$$= -i\frac{1}{l^2}\frac{\sqrt{2\tilde{n}_f}}{F_{\bar{\eta}}}\chi(l^2)$$

$$\propto\ \chi' \text{ for } l^2 \to 0$$

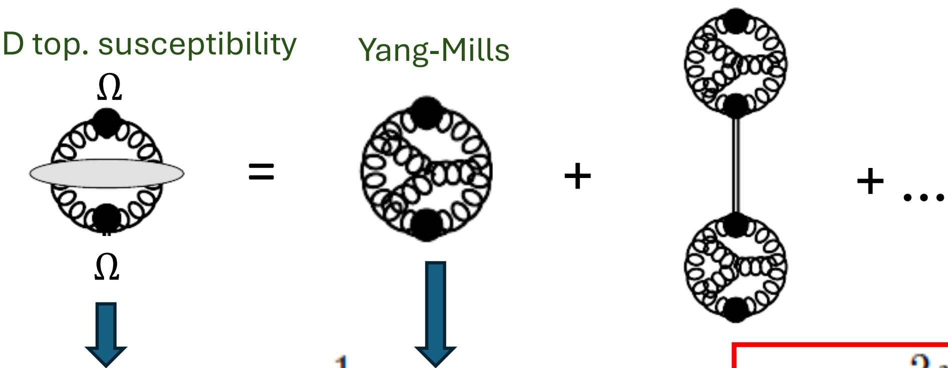
Combining with the Goldberger-Treiman relation, obtain result of Shore and Veneziano

$$\Delta\Sigma(Q^2) = \sqrt{\frac{2}{3}}\,\frac{2n_f}{M_N}\,g_{\eta_0 NN}\,\sqrt{\chi'(0)}$$

Topological mass generation and quark helicity

QCD top. susceptibility

Yang-Mills

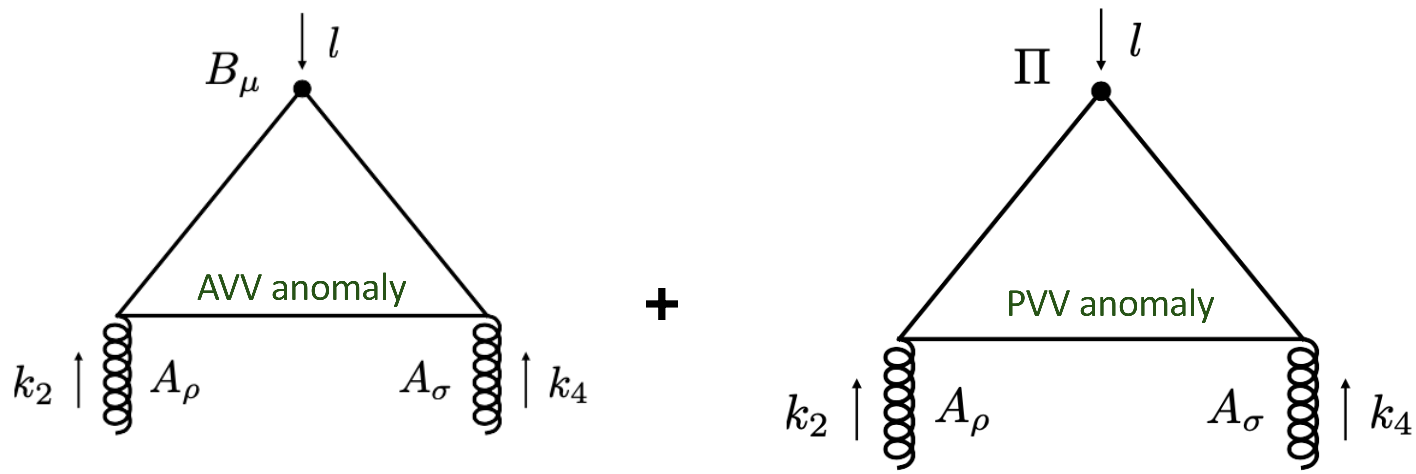


1/N corrections to YM top. susceptibility induced by WZW coupling

$$\chi(l^2) = l^2 \frac{1}{l^2 - m_{\eta'}^2} \chi_{\rm YM}(l^2)$$ with $$m_{\eta'}^2 \equiv -\frac{2\,n_f}{F_{\bar{\eta}}^2} \chi_{\rm YM}(0)$$ Witten-Veneziano formula

Immediate consequence is topological screening: $\chi(l^2) \rightarrow 0 \; for \; l^2 \rightarrow 0$

What happens for finite quark masses?



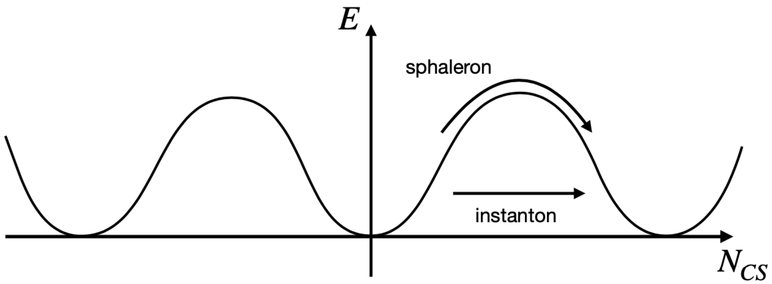
A remarkable result by Adler (+Bardeen) is that the PVV anomaly (present for $m \neq 0$) in QED exactly cancels the AVV anomaly when $l^2 \to 0$. *Caveat emptor*, same story in pQCD – Castelli et al. (2024)

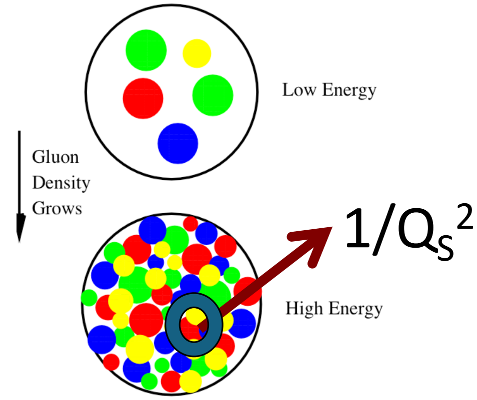
A corollary is that massless QED is not a well-defined theory

However chiral QCD is a well-defined theory and pQCD arguments are untenable. We showed that the anomaly pole is canceled by the $\bar{\eta}$ - the same physics that governs topological generation of the η' mass

What happens in QCD for finite quark mass? Quantitatively not much – but qualitatively very interesting Tarasov+RV-to appear. Spoiler alert: the chiral condensate plays a key role…

What about g_1 at small x_{Bj} ?





Saturation can induce over the barrier sphaleron-like transitions: $\bar{\eta}$ "*axion*" propagates in this "medium"

The medium does not affect the η' mass but can induce a drag

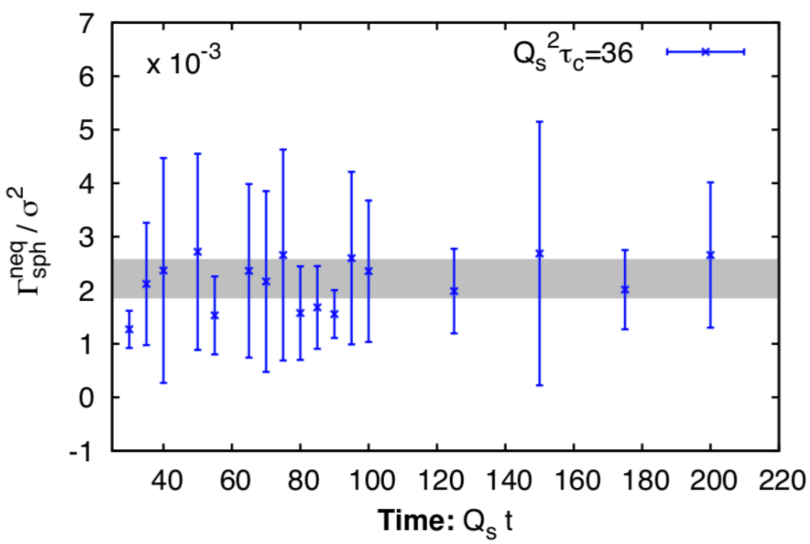
drag coefficient is proportional to the sphaleron transition rate…

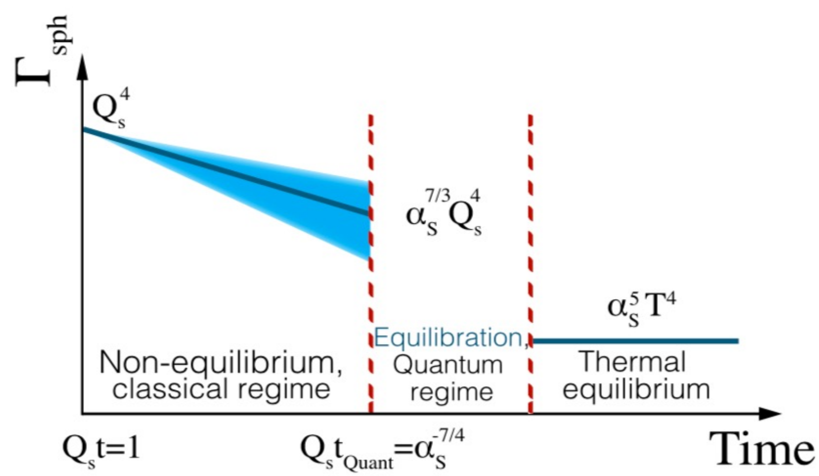
McLerran, Mottola, Shaposhnikov, PRD(1991)

Semi-classical EFT computation: $g_1^{\rm Regge}(x_B, Q^2) \propto \dfrac{Q_S^2 m_{\eta'}^2}{F_{\tilde{\eta}}^3 M_N} \exp\left(-4\, n_f C\, \dfrac{Q_S^2}{F_{\tilde{\eta}}^2}\right)$ ⇨ rapid quenching proton's spin

Tarasov, RV (2022)

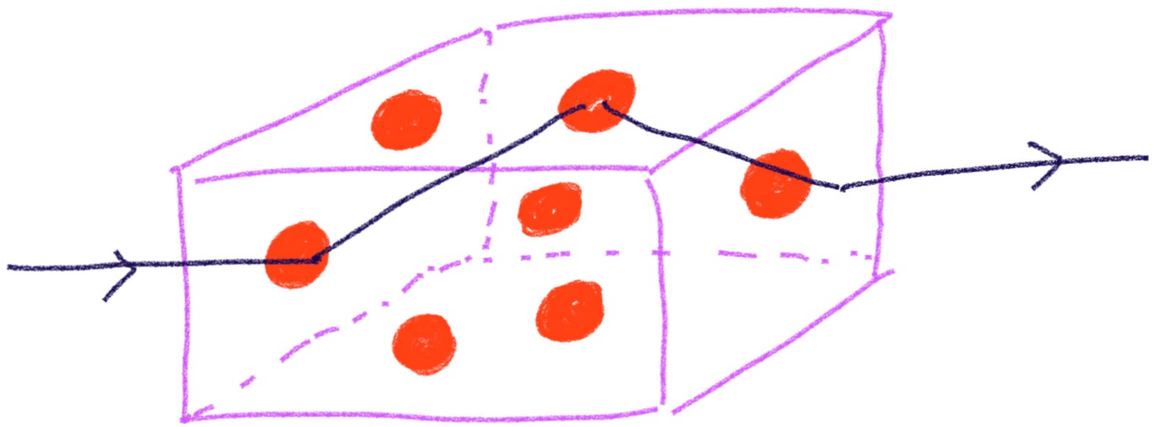
Sphaleron transition rate in overoccupied gauge fields





Mace,Schlichting,RV (2016)

Spin diffusion in topologically disordered medium

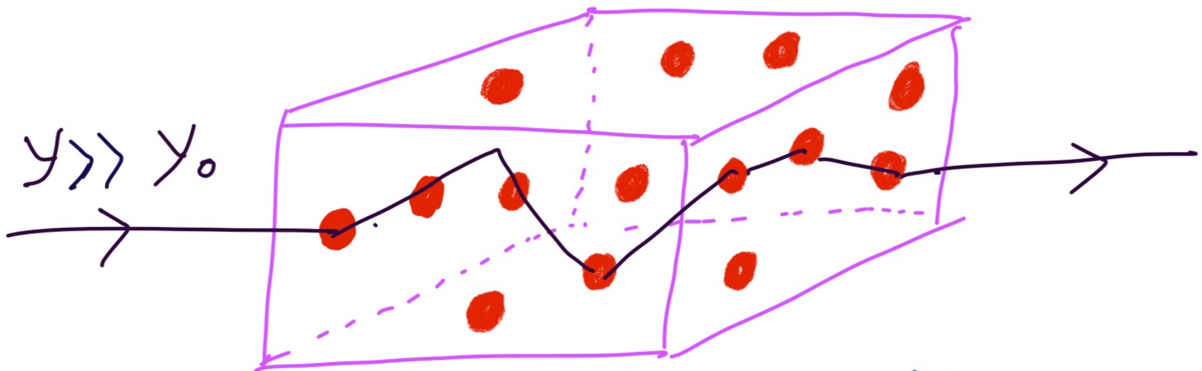


Dense gluon blob of size $1/\sqrt{\sigma}$ given by $\Gamma^{Y}_{\text{sphaleron}} = \#\,\sigma^2$ & carrying topological charge

---

Helicity flip for massless quarks given by $N_L - N_R = n_f \nu$, where ν is the topological charge and $\Gamma^{Y}_{\text{sphaleron}} \propto \langle \nu^2 \rangle$

Spin diffusion in topologically disordered medium



As x decreases ($y >> y_0$), the blobs become smaller ($Q_s(y) > Q_s(y_0)$) and denser with more topological charge

Helicity flip for massless quarks given by $N_L - N_R = n_f \nu$,
where ν is the topological charge and $\Gamma^{Y}_{\text{sphaleron}} \propto \langle \nu^2 \rangle$

Outlook

Measurements in polarized DIS sensitive to the anomaly can provide fresh insight into non-perturbative dynamics in QCD - exciting prospect of measuring sphaleron-like topological transitions

Can constrain non-perturbative models – for instance holographic models

These considerations also apply to observables sensitive to the conformal anomaly

-- In particular, so-called generalized parton distributions measured in exclusive reactions

For e.g., Bhattacharya,Hatta,Vogelsang, PRD (2023)
Also, Bhattacharya, Hatta, Schoeleber (2024)