Jet definition and TMD factorisation in SIDIS

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with E. Iancu, A. H. Mueller, and F. Yuan (2408.03129, submitted to PRL)

Goals and outline

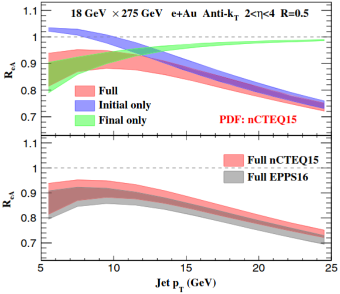
This talk, in a nutshell:

A small-x perspective on TMD factorisation for single-inclusive **jet** production in DIS.

- First calculation of Sudakov logs in SIDIS with jets and their dependence on the jet algorithm.
- New asymmetric jet distance measure which ensures TMD factorisation.
- Emergence of the DGLAP and CSS evolutions from the small x approach.
  $\Rightarrow$ combined high-energy, CSS and DGLAP evolution within the TMD formalism.

Why is it potentially interesting for this workshop?

- Jets at moderate x are complementary probes of CNM effects. Li & Vitev, PRL (126), 2021



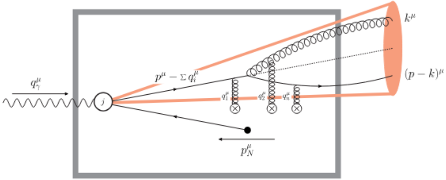


Fig. from Sievert, Vitev, PRD 98 (2018)

- At moderate/low $p_T \ll Q$, important to disentangle purely initial state effects ("intrinsic k_T") from CNM effects on the final state jet evolution.
- Using suitable jet definitions should help not mixing the effects together.

Single inclusive jet production in DIS

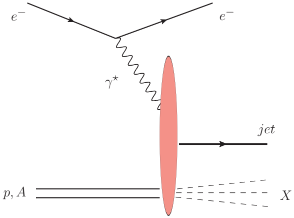
[PC, Iancu, Mueller, Yuan, 2408.03129]

⇒ Measure **jets** in DIS events and bin in terms of $P_\perp$ measured in Breit (or dipole) frame:

$$\frac{\mathrm{d}\sigma^{eA\to \mathrm{e'+jet}+X}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2\mathrm{d}P_\perp}$$

⇒ In the case of a hadron measurement, see Altinoluk, Jalilian-Marian, Marquet, 2406.08277. See also talk by J. Jalilian-Marian yesterday.

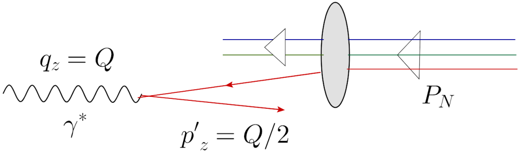
⇒ Also accesses the sea quark TMD at small x in the limit $Q^2 \gg \boldsymbol{P}_\perp^2$.



Breit frame and target picture at LO

- Breit frame: head-on $\gamma^* + A$ collision.
  $q^\mu = (0, 0, 0, q_z = Q)$
  See also talk by Felix Ringer on Thursday.

- Photon absorbed by the struck quark.



- Quark produced with $\boldsymbol{P}_\perp = \boldsymbol{0}_\perp$:

$$\left.\frac{\mathrm{d}\sigma^{\gamma^\star_{\mathrm{T}}+A\to q+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{4\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\delta^{(2)}(\boldsymbol{P}_\perp)xf_q(x)$$

- Dominated by aligned jet configurations $z = \frac{k_{\mathrm{jet}}\cdot P}{P\cdot q} \sim 1$.

Breit frame and target picture at LO and small x

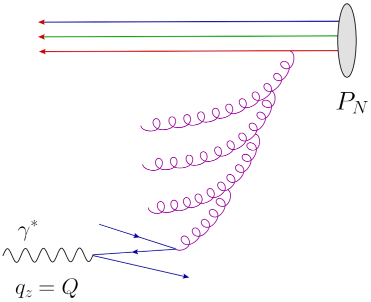
- At small x, rise of the gluon distribution ($\lambda \sim 0.2 - 0.3$)

$$Q_s^2(x) \sim \alpha_s \frac{xG(x, Q_s^2)}{\pi A^{2/3}} \sim \frac{A^{1/3}}{x^\lambda}$$

See also talk by Farid Salazar yesterday.

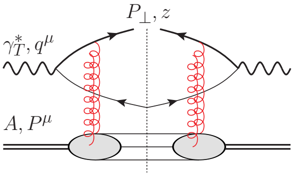
- **Sea** quark comes from a $g \to q\bar{q}$ splitting.
- For $Q^2 \gg P_\perp^2 \gg Q_s^2$, this splitting is DGLAP-like:

$$\left.\frac{\mathrm{d}\sigma^{\gamma_\mathrm{T}^\star + A \to q + X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{8\pi^2 \alpha_{\mathrm{em}} e_f^2}{Q^2} \frac{\alpha_s}{2\pi^2} \frac{1}{P_\perp^2} \int_x^1 \mathrm{d}\xi P_{qg}(\xi) \frac{x}{\xi} G\left(\frac{x}{\xi}, P_\perp^2\right)$$



- NB: a LO small x calculation achieves partial NLO accuracy in standard collinear factorization.

TMD factorisation in SIDIS at LO from the dipole picture



- Longitudinal boost to the dipole frame with $q^0 \sim q_z \gg Q$: $\gamma^* \to q\bar{q}$ splitting+interaction with the "shockwave" (CGC EFT).
  [Mueller (1990), Nikolaev and Zakharov (1991)]

- For $Q^2 \gg P_\perp^2, Q_s^2$, the CGC result factorises in terms of the (sea) quark TMD $x\mathcal{F}_q(x, P_\perp^2)$ [Marquet, Xiao, Yuan, PLB 682, 207 (2009)]

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^\star + A \to \mathrm{jet}+X}}{\mathrm{d}^2 \boldsymbol{P}_\perp}\right|_{\mathrm{LO}} = \frac{8\pi^2 \alpha_{\mathrm{em}} e_f^2}{Q^2} \times \underbrace{\frac{N_c}{\pi^2} \int_{\boldsymbol{b}_\perp} \int \frac{\mathrm{d}^2 \boldsymbol{q}_\perp}{(2\pi)^2} \mathcal{D}(x, \boldsymbol{q}_\perp) \left[1 - \frac{\boldsymbol{P}_\perp \cdot (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)}{(P_\perp^2 - (\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2)} \ln \frac{P_\perp^2}{(\boldsymbol{P}_\perp - \boldsymbol{q}_\perp)^2}\right]}_{\text{sea quark TMD}}$$

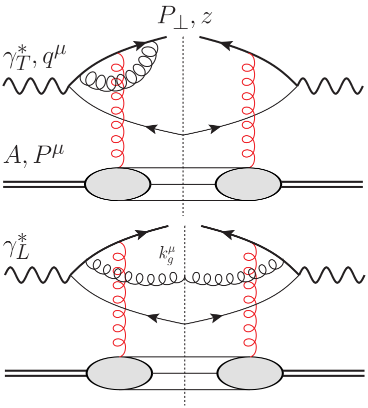
$$\underset{P_\perp \gg Q_s}{\longrightarrow} \frac{8\pi^2 \alpha_{\mathrm{em}} e_f^2}{Q^2} \frac{\alpha_s}{2\pi^2} \frac{1}{P_\perp^2} \int_x^1 \mathrm{d}\xi P_{qg}(\xi) \frac{x}{\xi} G\left(\frac{x}{\xi}, P_\perp^2\right)$$

Outline of the NLO computation in the dipole picture

- NLO calculation at small x for general **jet** kinematics performed in [PC, Ferrand, Salazar, JHEP 05 (2024) 110].
  For single hadron, see [Bergabo, Jalilian-Marian, JHEP 01 (2023) 095 (inclusive), Fucilla, Grabovsky, Li, Szymanowski, Wallon, JHEP 02 (2024) 165 (diffractive)]
  For dijet, see PC, Salazar, Venugopalan, JHEP 11 (2021) 222
- High energy factorisation with collinearly improved BK/BFKL. [Altinoluk, Jalilian-Marian, Marquet, 2406.08277]
- Consider the limit $Q^2 \gg P_\perp^2 \gg Q_s^2$ in the NLO impact factor

$$\left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star+A\to q+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma_{\mathrm{CGC}}^{\gamma_{\mathrm{T}}^\star+A\to q+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \left[1+\alpha_s\mathcal{I}(\boldsymbol{P}_\perp, Q, R, x_\star)\right]$$

- NLO impact factor depends on the jet definition.



Examples of NLO diagrams

Jet sequential recombination algorithms

- Popular jet definitions nowadays use sequential recombination algorithms.
  (Unlike cone-based jet definitions)

- Example with jets in e^+e^-: JADE, k_t algorithms,...
  JADE, Z.Phys.C 33 (1986), Catani, Dokshitzer, Olsson, Turnock, Webber, PLB 269, 432 (1991)

- Distance measure d_{ij} between particles i, j.
  Ex: $d_{ij} = M_{ij}^2/Q^2$ for JADE def.

- Sequential clustering of particles.

  $\rightarrow$ For each pair of particles (i, j), work out the distance d_{ik}.
  $\rightarrow$ Find the minimum of all d_{ij}.
  $\rightarrow$ If the min is $< d_{\mathrm{cut}}$, recombine i and j and repeat from step 1.
  Otherwise, terminate the iteration.

Jet definitions in DIS

- Jet definitions designed to ensure factorisation of inclusive jet cross sections in terms of universal pdf. Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)
- Longitudinally invariant "generalized-k_t" algorithms.

$$d_{ij} = \min(p_{t,i}^{2k}, p_{t,j}^{2k})\frac{\Delta R_{ij}^2}{R^2}\,, \quad d_{iB} = p_{t,i}^{2k}$$

Catani, Dokshitzer, Seymour, Webber, NPB, 406 (1993), Cacciari, Salam, Soyez, JHEP 0804:063,2008

$\Rightarrow$ Many jet analysis at HERA chose longitudinally invariant k_t algorithm in the Breit frame. Ex: α_s extraction from jet cross-sections, ZEUS, PLB 547 (2002), H1 PLB 653, 134 (2007), ...

- e^+e^- spherically invariant jet definitions in the Breit frame.

$$d_{ij} = \min(E_i^{2k}, E_j^{2k})\frac{1-\cos(\theta_{ij})}{1-\cos(R)}\,, \quad d_{iB} = E_i^{2k}$$

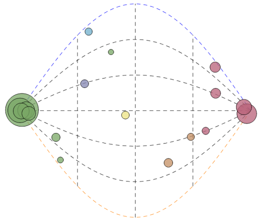
$\Rightarrow$ Recent studies on TMD factorisation with jets in DIS at moderate x use this definition with WTA scheme.
Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi, PRL 121, (2018), JHEP 10, 031 (2019)

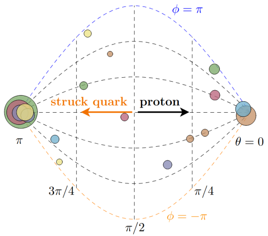
Known issues with previous options

- Spherically invariant jet definitions in the Breit frame are not boost invariant.
  $\Rightarrow$ Hard to distinguish beam remnant from backward jets.

- Longitudinally invariant jet definitions in Breit frame fail to cluster hadrons in the forward region.

Fig. from Arratia, Makris, Neill, Ringer, Sato, PRD 104 (2021)





Jet definition in DIS: summary

- Not all jet definitions ensure factorisation of the fully inclusive **jet** cross section.

  [Catani, Dokshitzer, Webber, PLB 285, 291 (1992), Webber, J. Phys. G 19, 1567 (1993)]

- Does the same phenomenon arise for TMD factorisation?

Jet def	distance measure	dipole frame NLO clustering condition ($R \ll 1$)
LI C/A	$d_{ij} = \frac{\Delta R_{ij}^2}{R^2}$	$\frac{M_{ij}^2 z^2}{\boldsymbol{P}_\perp^2 R^2} \leq 1$
SI C/A	$d_{ij} = \frac{1-\cos(\theta_{ij})}{1-\cos(R)}$ **in Breit frame**	$\frac{M_{ij}^2 z^2}{Q^2 R^2} \leq 1$
new LI jet def	$d_{ij} = M_{ij}^2/(z_i z_j Q^2 R^2)$	$\frac{M_{ij}^2}{z_i z_j Q^2 R^2} \leq 1$

See also Centauro algorithm, Arratia, Makris, Neill, Ringer, Sato, PRD 104 (2021)

- $M_{ij}^2 = (k_i + k_j)^2$, $z_i = (k_i \cdot P)/(P \cdot q) = k_i^+/q^+$.
- Goal: find a jet definition which ensures TMD factorisation of the single inclusive jet cross-section.

Sudakov logarithms in the NLO impact factor

- NLO Sudakov logs $L = \ln(Q^2/\boldsymbol{P}_\perp^2)$ depend on the jet definition!

For LI C/A (or anti-k_t):

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{3}{4}L^2 + \left(\frac{3}{4} - \ln(R)\right)L + \mathcal{O}(1)\right]$$

while for SI C/A ($\beta = 2$) and our new jet definition ($\beta = 0$)

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2 + \left(\frac{3(1-\beta/2)}{4} + \ln(R)\right)L + \mathcal{O}(1)\right]$$

- From CSS evolution of the quark TMD alone, we expect the log structure

$$\frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}L^2 + \frac{3}{4}L\right]$$

$\Rightarrow$ TMD factorisation implies $\beta = 0$.
New LI jet definition in DIS suitable for TMD factorisation with jets.

- Sudakov DL for a jet measurement is half the DL for hadron measurement.

Heuristic derivation of the Sudakov double log

- To DLA, Sudakov comes the virtual gluon emissions in the phase space forbidden to the real ones:

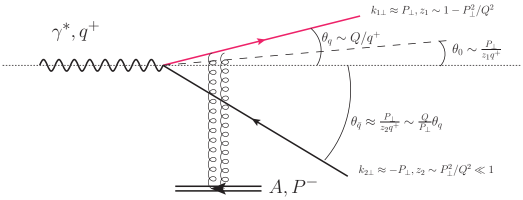
$$S_{\mathrm{DL}} = -\frac{\alpha_s C_F}{2\pi} \int_{P_\perp^2}^{Q^2} \frac{\mathrm{d}k_{g\perp}^2}{k_{g\perp}^2} \int_{k_{g\perp}^2/Q^2}^{z_{\max}} \frac{\mathrm{d}z_g}{z_g} ,$$

- $k_{g\perp} \gg P_\perp$ forbidden due to the constraint $P_\perp^2 \gg Q^2$.
- Virtual emissions are effectively cut in the UV by the hard scale Q^2.
- Lower limit on $z_g \Leftrightarrow \tau_g \sim 1/k_g^- \gg \tau_\gamma \sim 1/q^-$: excludes gluons contributing to (collinearly improved) high energy evolution.
- $z_{\max}$ depends on the jet constraint $\Rightarrow$ a real gluon is forbidden only if it is **not** in the jet.

For a hadron measurement, $z_{\max} = 1$ and $S_{\mathrm{DL}} = -\frac{\alpha_s C_F}{2\pi} \ln(Q^2/P_\perp^2)$. Altinoluk, Jalilian-Marian, Marquet, 2406.08277

Physical interpretation of the jet constraint

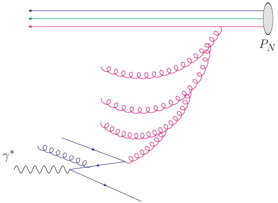
- Our new clustering condition equivalent to $\theta_{ij} \leq R\theta_{\rm jet}$ with $\theta_{\rm jet} \sim Q/q^+$.
- Angle of the jet set by its **virtuality** rather by its transverse momentum.
  (Naively, $\theta_{\rm jet} \sim \frac{P_\perp}{zq^+}$.)
- Soft gluons contributing to Sudakov must have
  $\theta_g \gg \theta_{\rm jet} \Leftrightarrow z_g \leq k_{g\perp}/Q$.
  Stronger constraint than $\theta_g \gg \frac{P_\perp}{zq^+} \Leftrightarrow z_g \leq k_{g\perp}/P_\perp$!
- Using $z_{\rm max} = k_{g\perp}/Q$ gives $S_{\rm DL} = -\frac{\alpha_s C_F}{4\pi} \ln(Q^2/P_\perp^2)$.



$\Uparrow$

Aligned jet configuration in dipole frame.

Complete NLO result for $Q^2 \gg P_\perp^2 \gg Q_s^2$



- The complete NLO corrections are organized as

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{NLO}} = \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{R} + \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{V}$$

- Term where $P_\perp$ is given by a $q \to qg$ hard collinear splitting

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{R} = \frac{8\pi^2\alpha_{\mathrm{em}}e_f^2}{Q^2}\frac{\alpha_s}{2\pi^2}\frac{1}{P_\perp^2}\int_x^{1-\frac{RP_\perp}{Q}} \mathrm{d}\xi P_{qq}(\xi)\frac{x}{\xi}f_q\left(\frac{x}{\xi}, P_\perp^2\right)$$

- Sudakov logs and finite pieces, for our asymmetric jet clustering definition.

$$\left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{V} = \left.\frac{\mathrm{d}\sigma^{\gamma_{\mathrm{T}}^*+A\to j+X}}{\mathrm{d}^2\boldsymbol{P}_\perp}\right|_{\mathrm{LO}} \times \frac{\alpha_s C_F}{\pi}\left[-\frac{1}{4}\ln^2\left(\frac{Q^2}{P_\perp^2}\right) + \left(\frac{3}{4} + \ln(R)\right)\ln\left(\frac{Q^2}{P_\perp^2}\right) - \frac{3}{2}\ln(R) + \frac{11}{4} - \frac{3\pi^2}{4} + \frac{3}{4}\ln^2(x_\star) + \frac{3}{8}\ln(x_\star) + \mathcal{O}(R^2)\right]$$

- $x_\star$ factorisation scale: gluons with $z_g \leq x_\star P_\perp^2/Q^2$ resummed with high energy evolution.

DGLAP+CSS resummation

- Beyond LO, the quark TMD also depends upon the photon virtuality Q^2.
- $x\mathcal{F}_q(x, P_\perp^2, Q^2)$ = number of quarks in the target with given x and $P_\perp$, as probed with a longitudinal resolution fixed by Q^2.
- Taking derivative w.r.t. $\ln(Q^2)$ and assuming Markovian evolution:

$$\frac{\partial \mathcal{F}_q(x, P_\perp^2, Q^2)}{\partial \ln Q^2} = \frac{C_F}{2\pi}\left\{\frac{\alpha_s(P_\perp^2)}{P_\perp^2}\int_{\Lambda^2}^{P_\perp^2} \mathrm{d}\ell_\perp^2\, \mathcal{F}_q(x, \ell_\perp^2, Q^2) - \int_{P_\perp^2}^{Q^2} \frac{\mathrm{d}\ell_\perp^2}{\ell_\perp^2}\alpha_s(\ell_\perp^2)\mathcal{F}_q(x, P_\perp^2, Q^2)\right\} + \frac{3}{2}\frac{\alpha_s(P_\perp^2)C_F}{\pi}\mathcal{F}_q(x, P_\perp^2, Q^2)$$

$\Rightarrow$ "diagonal" version of the CSS equation for the quark TMD from the dipole picture.

- Integrating our 1-loop result up to Q^2 yields:

$$xf_q(x, Q^2) = xf_q^{(0)}(x, Q^2) + \int_{\Lambda^2}^{Q^2} \frac{\mathrm{d}P_\perp^2}{P_\perp^2}\frac{\alpha_s(P_\perp^2)}{2\pi}\int_x^1 \mathrm{d}\xi\, \mathcal{P}_{qq}(\xi)\,\frac{x}{\xi} f_q\left(\frac{x}{\xi}, P_\perp^2\right)$$

$\Rightarrow$ DGLAP evolution of the quark pdf from the dipole picture.

Similar results obtained for the WW gluon TMD in PC, Iancu, 2406.04238

Conclusion

- Clarifying the jet definition (clustering algorithm) for jet production in SIDIS in the TMD limit $Q^2 \gg P_\perp^2$.
- Calculation of the Sudakov effect for jet production in SIDIS.
- NLO calculation in the high-energy formalism (dipole picture, CGC). Conclusions remain valid at moderate x.
- Emergence of the DGLAP and CSS evolutions of the quark TMD from the small-x approach.
- Sudakov suppression affects the jet p_t spectrum at low p_T, where it competes with CNM effects on final state evolution.

THANK YOU!