The Role of Diquark Dynamics in the Structure of Exotic Hadrons

Richard Lebed

Arizona State University

Exotic Heavy Meson Spectroscopy and Structure with EIC

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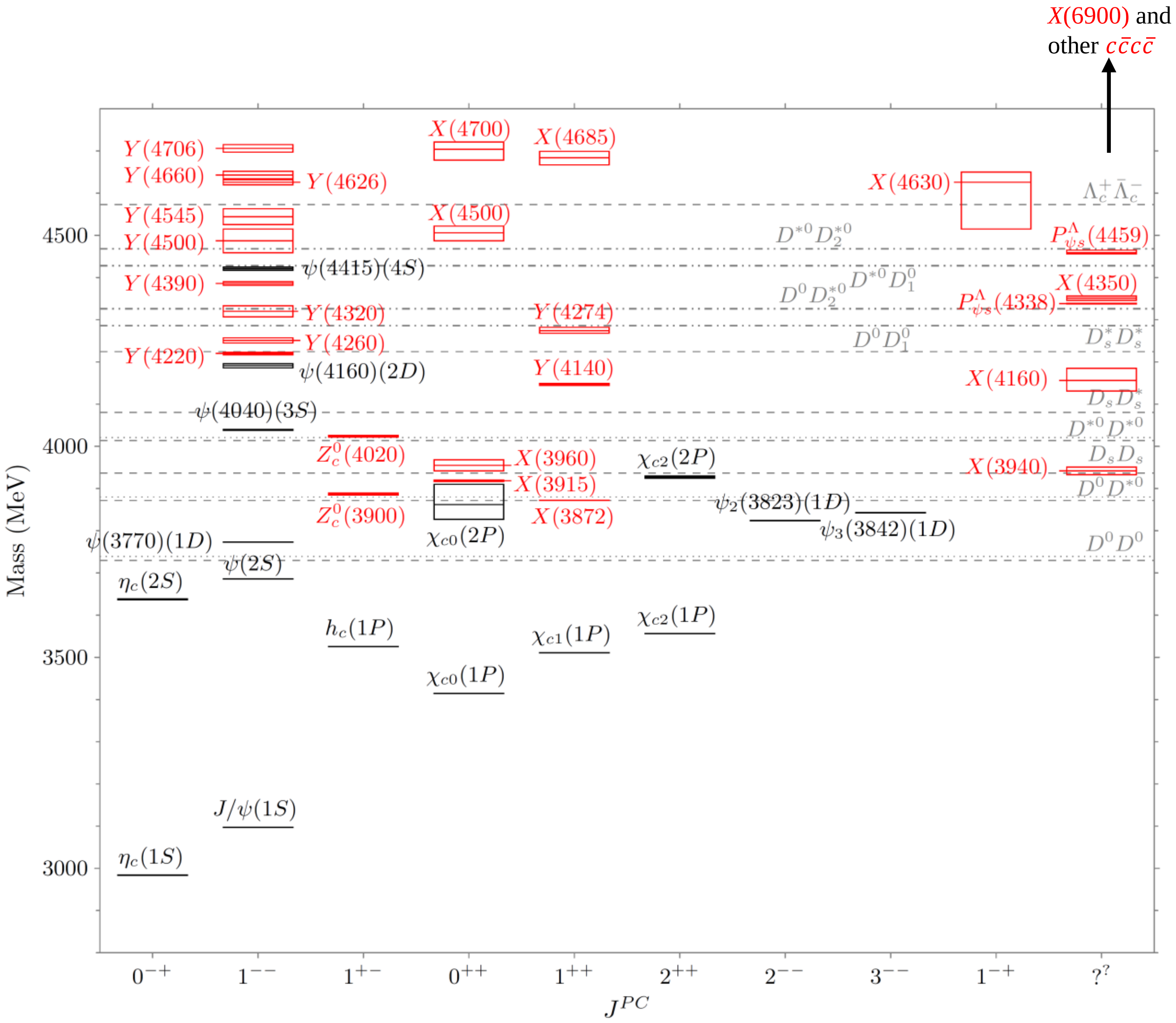
What EIC can do with exotics

- "The EIC will have sufficient energy to produce essentially all known $XYZP$ candidates, including both the charmonium and bottomonium energy ranges."
- "The ability to vary photon virtuality in photoproduction processes permits measurements of the spatial extent of the states, which provides crucial information on their substructure."
  - *Summary of Topical Group on Hadron Spectroscopy*, Snowmass 2021 [arXiv:2207.14594]
- In addition, multiquark states can have large spin eigenvalues. Clever use of L & T photons can allow access to multiple spin states

Neutral hidden-charm system, April 2025



Several of the states are quite close to di-hadron thresholds

Most prominent example:

$$m_{X(3872)} - m_{D^0} - m_{D^{*0}}$$
$$= -50 \pm 90 \text{ keV}$$

cf. the deuteron:

$$m_d - m_p - m_n$$
$$= -2.2452(2) \text{ MeV}$$

Heavy-quark exotics census: April 2025

- **72** observed exotics, both tetraquarks and pentaquarks
  - 54 in the charmonium sector (including open-strange)
  - 5 in the (much less explored) bottomonium sector
  - 7 with a single c quark (and an s, a u, and a d)
  - 1 with a single b quark (and an s, a u, and a d)
  - 4 with all c and $\bar{c}$ quarks
  - 1 with two c quarks
- A naïve count estimates well over 100 more exotics are waiting to be discovered

The internal structure of exotics is unresolved

Mesons depicted here, but each model has a baryonic analogue

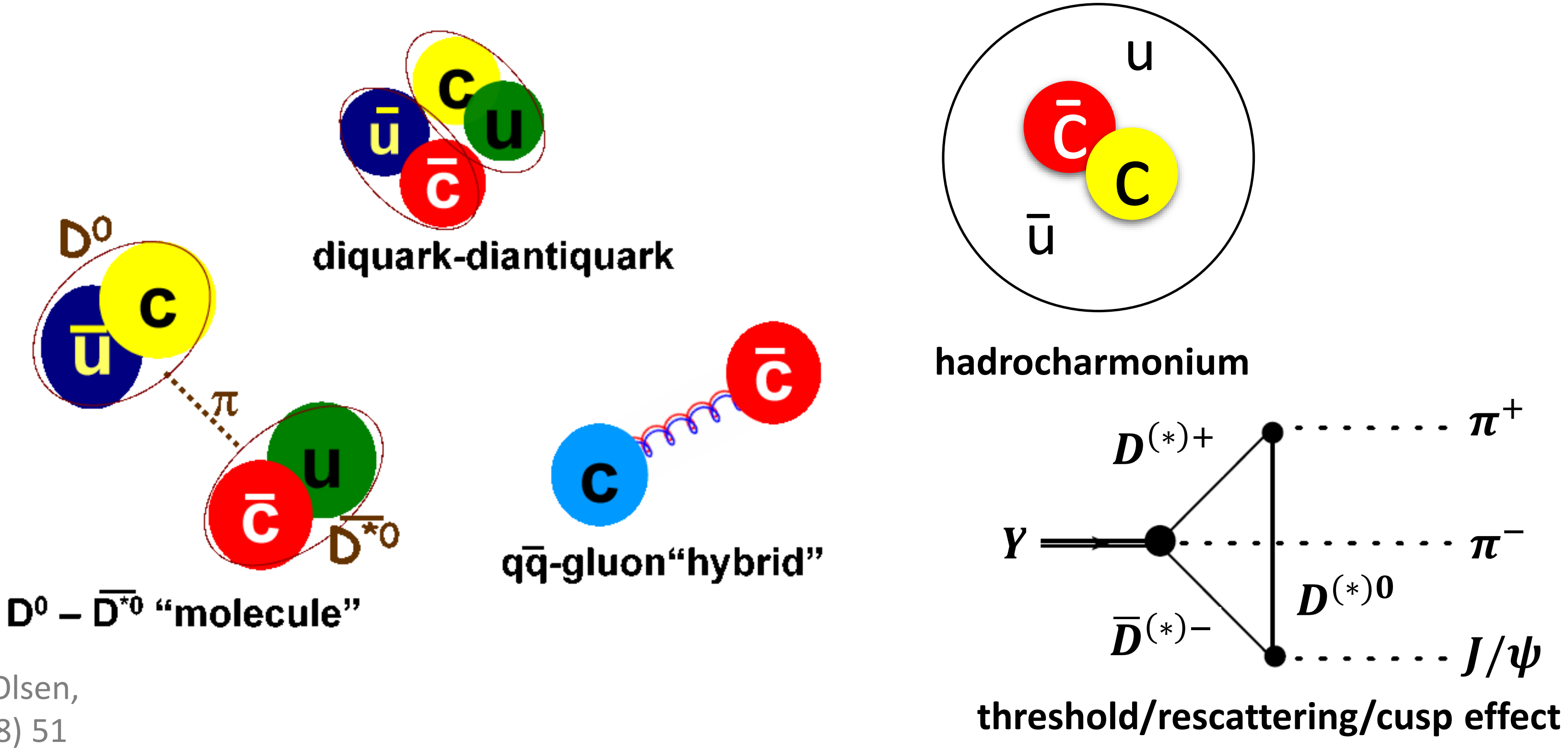


Image adapted from Godfrey & Olsen, Ann. Rev. Nucl. Part. Sci. **58** (2008) 51

"Each of the interpretations provides a natural explanation of parts of the data, but neither explains all of the data. It is quite possible that both kinds of structures appear in Nature. It may also be the case that certain states are superpositions of the compact and molecular configurations."
—Karliner, Santopinto, *et al.*, 2203.16583

The plan:

1) Develop a model that predicts a full spectrum for the expected exotics

2) Determine both mass spectrum and decay patterns

3) Check whether yet-unobserved states are missing for a good reason

4) Try to understand why some lie quite close to di-hadron thresholds

Why diquarks?

- Short-distance attraction of two color-$\mathbf{3}$ quarks into a color-$\overline{\mathbf{3}}$ diquark is *fully half as strong* as combining $\mathbf{3}$ and $\overline{\mathbf{3}}$ into **color-neutral** singlet (*i.e.*, **diquark attraction** nearly as strong as the **confining attraction**)

- The $SU(2)$ analogue: Just as one computes a spin-spin coupling,
$$\vec{s}_1 \cdot \vec{s}_2 = \frac{1}{2}\left[(\vec{s}_1 + \vec{s}_2)^2 - \vec{s}_1{}^2 - \vec{s}_2{}^2\right],$$
from two particles in representations 1 and 2 combined into representation 1+2:

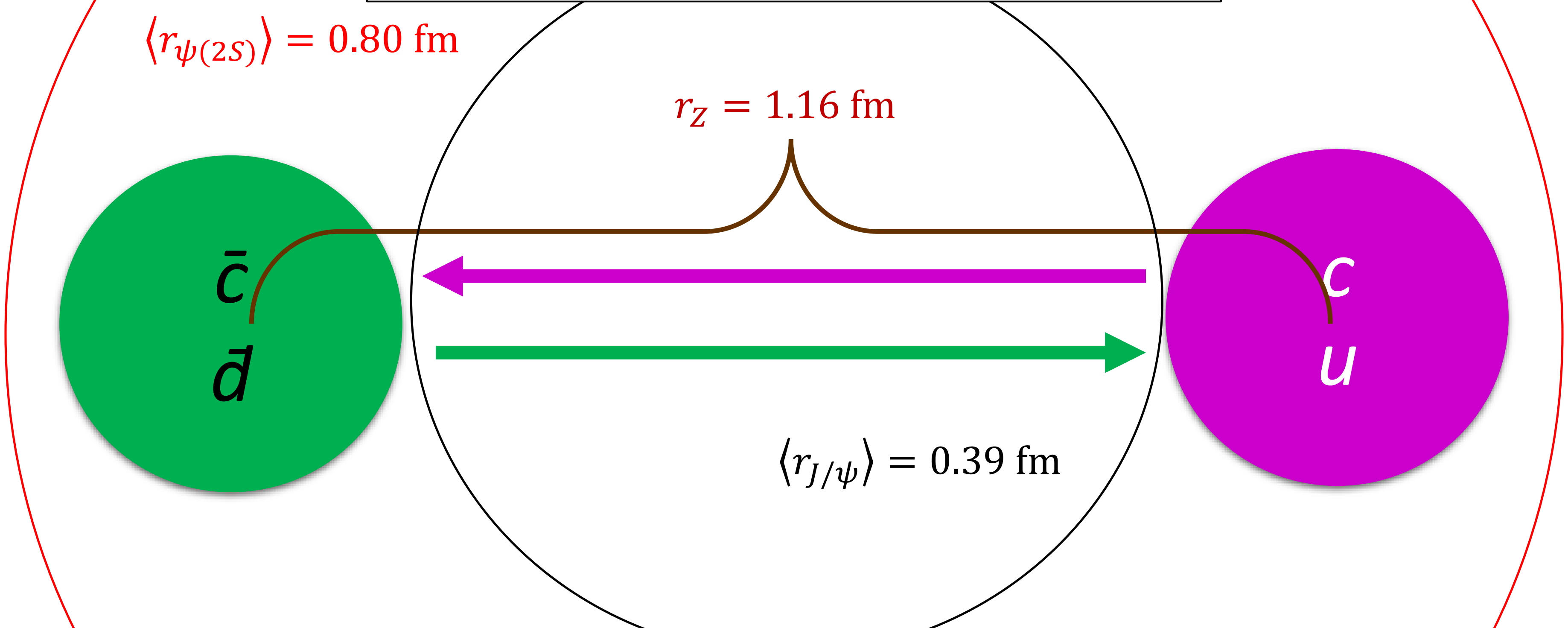
- If s_1, s_2 = spin $\frac{1}{2}$, and $\vec{s}_1 + \vec{s}_2$ = spin 0, get $-\frac{3}{4}$;
if spin 1, get $+\frac{1}{4}$

Diquarks allow large, but still strongly bound, states

Belle [K. Chilikin *et al.*, PRD **90**, 112009 (2014)] finds:

$$\frac{\mathrm{B.R.}\,[Z_c^-\,(4430) \to \psi(2S)\pi^-]}{\mathrm{B.R.}\,[Z_c^-\,(4430) \to J/\psi\pi^-]} > \mathbf{10}$$

and LHCb [R. Aaij *et al.*, PRL **112**, 222002 (2014)] has not yet reported seeing the J/ψ $(1S)$ mode



The dynamical diquark *picture*:

Brodsky, Hwang, RFL [PRL **113**, 112001 (2014)]

- Heavy quarks provide nucleation points for diquark formation
- Separation of heavy quarks during production process (in heavy-hadron decays or high-energy collisions) leaves diquarks as identifiable constituent components of multiquark hadrons
- Diquark-antidiquark pair remain strongly connected by color flux tube → tetraquark $(Qq)(\bar{Q}\bar{q})$
- Same color-triplet mechanism supports pentaquark formation, using a triquark: $[Q(\bar{q}_1\bar{q}_2)](\bar{Q}\bar{q})$ [RFL, PLB **749**, 454 (2015)]

The dynamical diquark *model*:

RFL [PRD **96**, 116003 (2017)]

- Exotic eigenstate: the configuration once kinetic energy of the heavy di-(tri-)quarks converted into potential energy of the color flux tube
- Two heavy, slow sources connected by light degrees of freedom? That's the adiabatic approximation → ordinary Schrödinger equation
- In energy regions where only one potential-energy function important (away from level crossings), can use the single-channel approximation Together, these form the Born-Oppenheimer (BO) approximation
- BO potentials are the same ones in lattice simulations of heavy-quark hybrids, labeled by axial quantum numbers such as Σ_g^+, Π_u^-, *etc.*

Dynamical diquark model, first numerical results

Giron, RFL, Peterson [JHEP **05**, 061 (2019)]

- When the heavy sources coincide, BO potentials become degenerate (called *parity doubling* in atomic physics)
  → requires development of a coupled Schrödinger equation solver
- Our detailed simulations showed that all exotics known at that time fit into the ground-state Σ_g^+ BO potential, in the $1S$, $1P$, $2S$, $2P$ orbitals
- Using precisely the lattice-simulated BO potentials, the result for the $(cq)(\bar{c}\bar{q})$ $\Sigma_g^+(1S)$ multiplet-average mass naturally matches $m_{X(3872)}$, and those for $\Sigma_g^+(1P)$, $\Sigma_g^+(2S)$ beautifully match $m_{Y(4220)}$, $m_{Z_c(4430)}$, respectively
- But these are multiplet-average masses—Need to include fine structure

Dynamical diquark model, fine structure & isospin

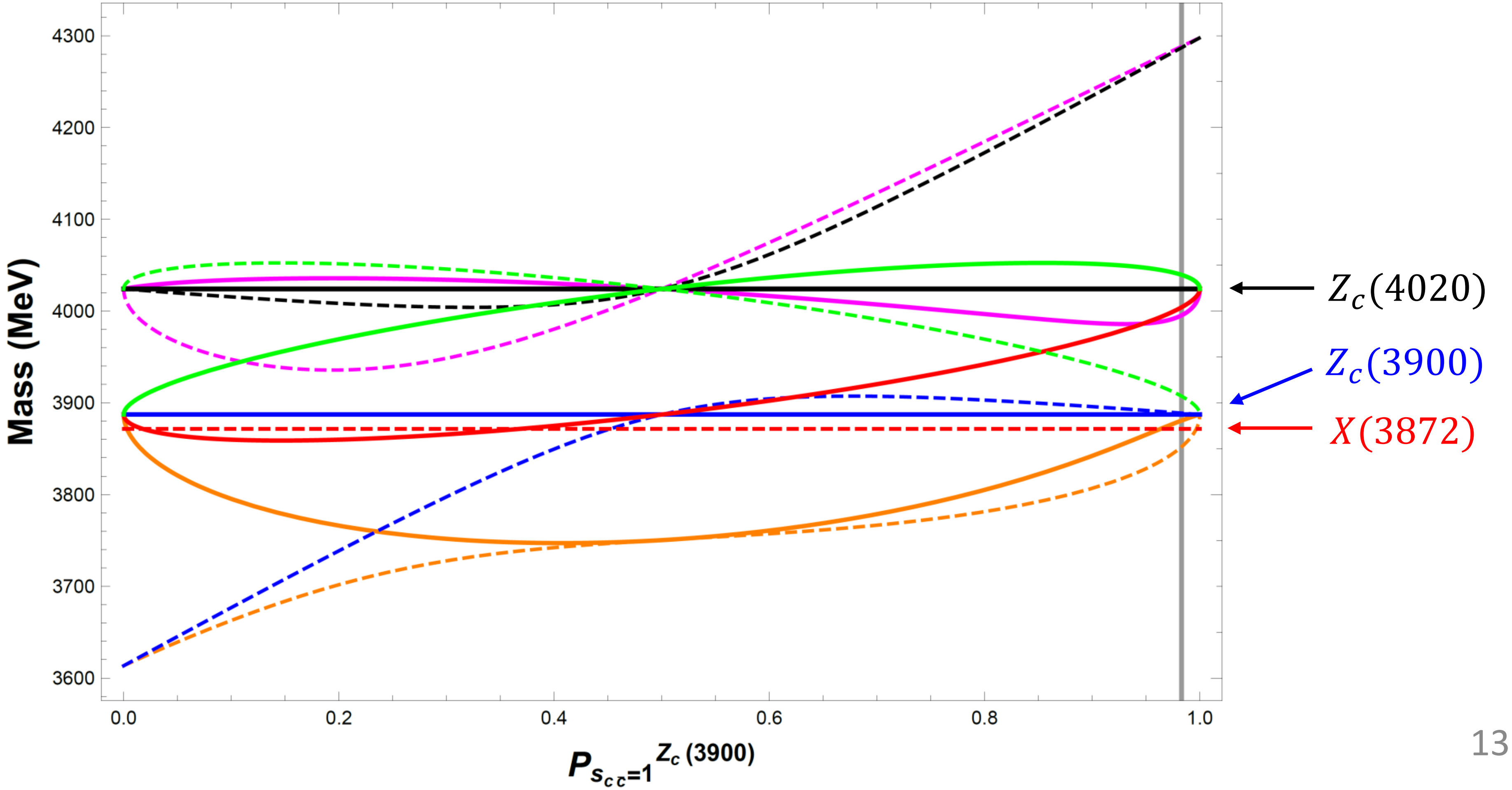
Giron, RFL, Peterson [JHEP **01**, 124 (2020)]

- Only a few known exotics in each multiplet $\Longrightarrow$ Need to identify most physically important perturbation Hamiltonian operators
- *e.g.*, the multiplet $(cq)(\bar{c}\bar{q})\ \Sigma_g^+(1S)$ contains $6\ I = 0$ and $6\ I = 1$ states, and we know only $X(3872)$ [$I = 0$], $Z_c(3900)$ & $Z_c(4020)$ [$I = 1$]
- Fixes 2 operators, taken to be
  (1) quark spin-spin coupling within each diquark, and
  (2) isospin-spin exchange between diquarks (analogous to π exchange)
- Naturally predicts $X(3872)$ to be lightest narrow state in multiplet
- Naturally predicts $Z_c(3900)$ to decay preferentially to J/ψ ($s_{c\bar{c}} = 1$) and $Z_c(4020)$ to h_c ($s_{c\bar{c}} = 0$), as is observed

Dynamical diquark model, $(cq)(\bar{c}\bar{q})$ ground states

Figure from J. Giron, PhD dissertation (2021)

- The model also predicts masses for the other 9 states in $\Sigma_g^+(1S)$:



If the model is any good, it must also apply to other flavor/spin sectors

- Using the same Hamiltonian operators, apply to the sectors:
- $(cq)(\bar{c}\bar{q})$ $P=-1$ (*e.g.*, Y), $\Sigma_g^+(1P)$: Giron, RFL [PRD **101**, 074032 (2020)]
- $(bq)(\bar{b}\bar{q})$: Giron, RFL [PRD **102**, 014036 (2020)]
- $(cs)(\bar{c}\bar{s})$: Giron, RFL [PRD **102**, 014036 (2020)]
- $(cc)(\bar{c}\bar{c})$: Giron, RFL [PRD **102**, 074003 (2020)]
- $(cq)(\bar{c}\bar{s})$: Giron, Martinez, RFL [PRD **104**, 054001 (2021)]
- $(cu)(\bar{c}ud)$, $(cs)(\bar{c}ud)$ pentaquarks: Giron, RFL [PRD **104**, 114028 (2021)]

But what about the closeness of some exotics to di-hadron thresholds?

- Since the constituents are the same, *e.g.*, $(cq)(\bar{c}\bar{q})$ *vs.* $(c\bar{q})(\bar{c}q)$, some exotics should naturally lie close ($\sim 10\text{ MeV}$) to thresholds

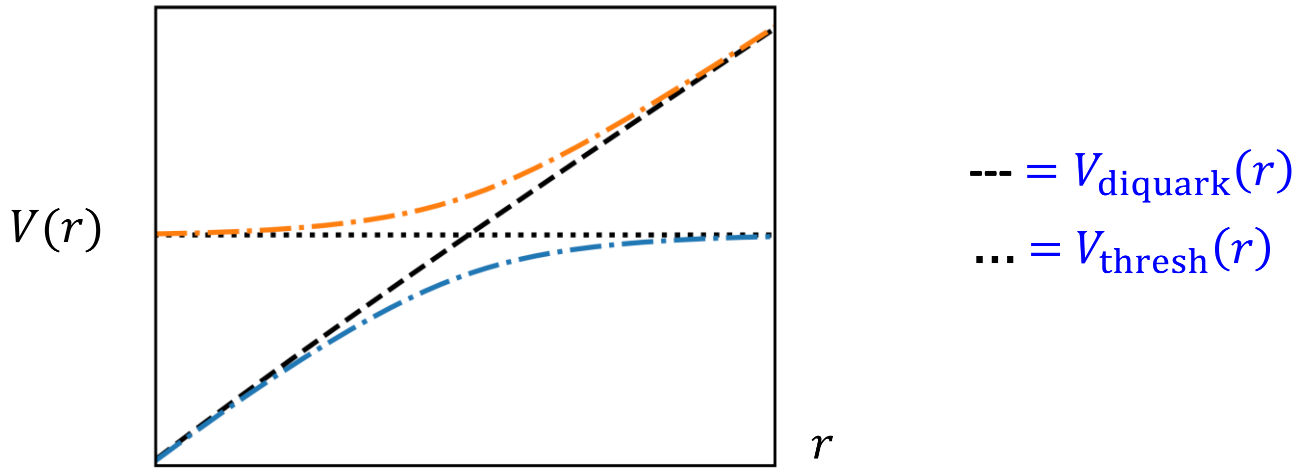
- But $m_{X(3872)} - m_{D^0} - m_{D^{*0}} = -50 \pm 90\text{ keV}$ cannot be an accident!

- This binding energy is much smaller than expected for a "conventional" hadron molecule*—More likely a threshold rescattering effect [coupling to near-on-shell particle pair leads to enhanced amplitude]
  *one primarily bound by light-meson exchange

- Lots of work done to explain some exotics as purely threshold effects, but not every threshold seems to have a prominent associated state

Diabatic formalism

- But what if both types of potentials are present (diquark-antidiquark and di-hadron threshold)?
- This is a well-known problem in atomic physics: Must perform coupled-channel calculation to find mixed-configuration eigenstates near level crossing, where adiabatic approximation fails
- Rigorous method to incorporate these effects: **diabatic formalism**



Diabatic formalism

- Choose heavy-source separation $\boldsymbol{r}_0$ where configuration mixing small
- Solve Schrödinger equation for eigenstates $|\xi_i(\boldsymbol{r}_0)\rangle$, with i labeling unmixed diquark/di-hadron components
- Given interaction Hamiltonian for light degrees of freedom H_{light}, compute diabatic potential matrix $V_{ji}(\boldsymbol{r}, \boldsymbol{r}_0) \equiv \langle \xi_j(\boldsymbol{r}_0) | H_{\text{light}} | \xi_i(\boldsymbol{r}_0) \rangle$
- The rest of the Hamiltonian is the heavy-source kinetic-energy operator, $\mathrm{K} = \mathrm{diag}\{-\hbar^2 \nabla^2 / 2\mu_i\}$
- Solve the coupled Schrödinger equation $[\mathrm{K} + V(\boldsymbol{r})]\Psi(\boldsymbol{r}) = E\Psi(\boldsymbol{r})$ for eigenstates $|\Psi(\boldsymbol{r})\rangle$ as linear combinations of $|\xi_i(\boldsymbol{r})\rangle$

Diabatic formalism for heavy-quark states

- Only missing ingredient: What is the full mixing potential V (specifically, off-diagonal elements of the diabatic potential matrix)?
- Lattice simulations will be capable of calculating these (*e.g.*, string-breaking potential static energies), but in the meantime, model them as Gaussians that rapidly transition at the level crossing
- Diabatic approach first applied to mixing of hadron thresholds with conventional quarkonium: Bruschini & Gonzalez [PRD **102**, 074002 (2020)]
- We use the same techniques, but with diquark states
  The coupled-channel Schrödinger solver from prior work comes in very handy!
  RFL, Martinez [PRD **106**, 074007 (2022)]

Diabatic formalism results: Bound states

- It is not at all unnatural for a diquark state near a threshold to acquire a very large di-hadron component, while others do not:

J^{PC}	E (MeV)	$\delta\bar{\delta}$	$D\bar{D}^*$	$D_s\bar{D}_s$	$D^*\bar{D}^*$	$D_s^*\bar{D}_s^*$	$\langle r\rangle$(fm)	$\langle r^2\rangle^{1/2}$(fm)
0^{++}	3905.4	63.0%		27.4%	8.4%	1.2%	0.596	0.605
1^{++}	3871.5	8.6%	91.4%				4.974	5.459
2^{++}	3922.3	83.1%		1.5%	13.9%	1.5%	0.443	0.497

- Knowing explicit diquark (short-distance) as well as di-hadron (long-distance) components allows one to probe effects sensitive to short-distance structure, such as radiative decays: *e.g.*, B.R.[$X(3872) \longrightarrow \gamma\psi(2S)$]$= (1.3 \pm 0.5)\%$

Diabatic formalism: mass shifts and widths

RFL, Martinez [PRD **110**, 074002 (2024)]

- Results of previous slide treats exotics as ordinary bound states with no lower open-heavy-flavor thresholds to which they can decay
- The atomic physics method to include them: Fano [PR **124**, 1866 (1961)]
- Used in Cornell model $c\bar{c}$ calculations: [PRD **17**, 3090 (1978)]
- Mixing of $c\bar{c}$ with exotics: Bruschini & Gonzalez [PRD **103**, 074009 (2021)]
- Diquark model [RFL, Martinez] sample results:

J^{PC}	$M - M_A$ [MeV]	M [GeV]	M_R [GeV]
0^{++}	-5.07	3.89876	3.89470
1^{++}			
2^{++}	-0.36	3.91708	3.90260
1^{--}	-5.88	4.26370	4.21240

Diabatic formalism: scattering amplitudes

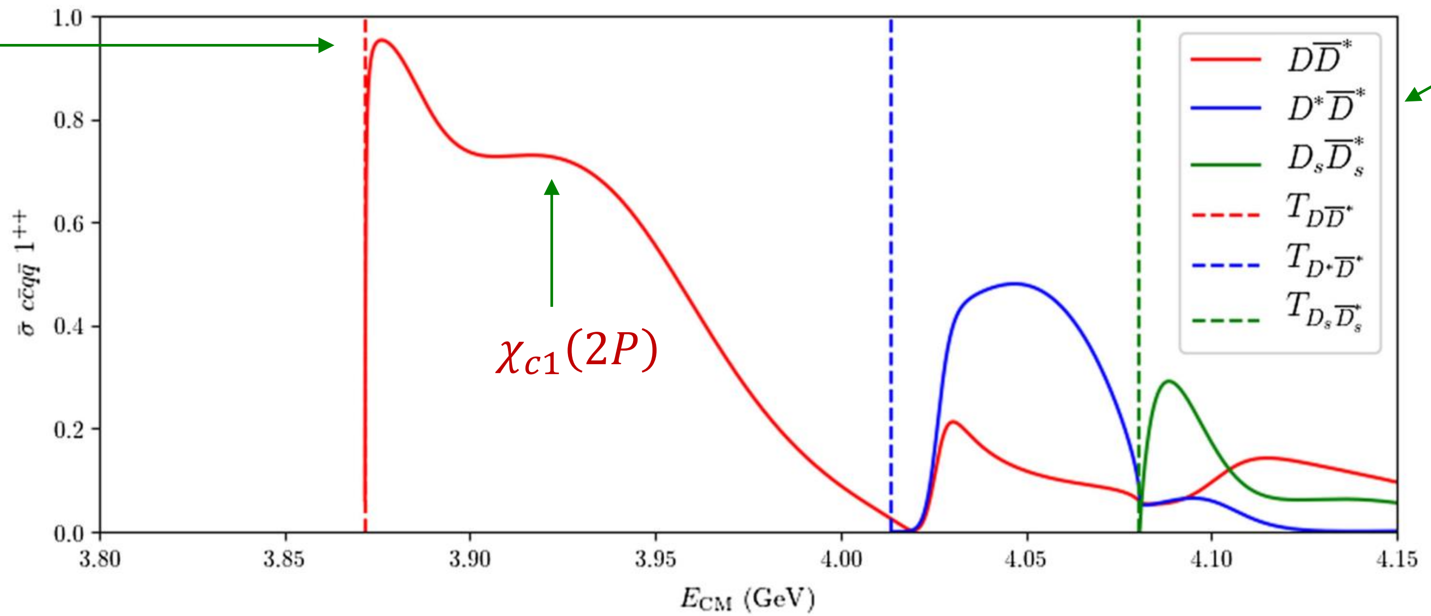
RFL, Martinez [PRD **108**, 014013 (2023)]

- Physical resonances are poles in S-matrix (or K-matrix) amplitudes
- Elastic 2-meson scattering wave functions: spherical Bessel functions
- Diabatic V_{mix} with diquarks introduces scattering phase shifts δ_J, which can be used to produce scattering cross sections σ_J:

$\chi_{c1}(3872)$

Scaled to remove phase space

Elastic scattering cross sections



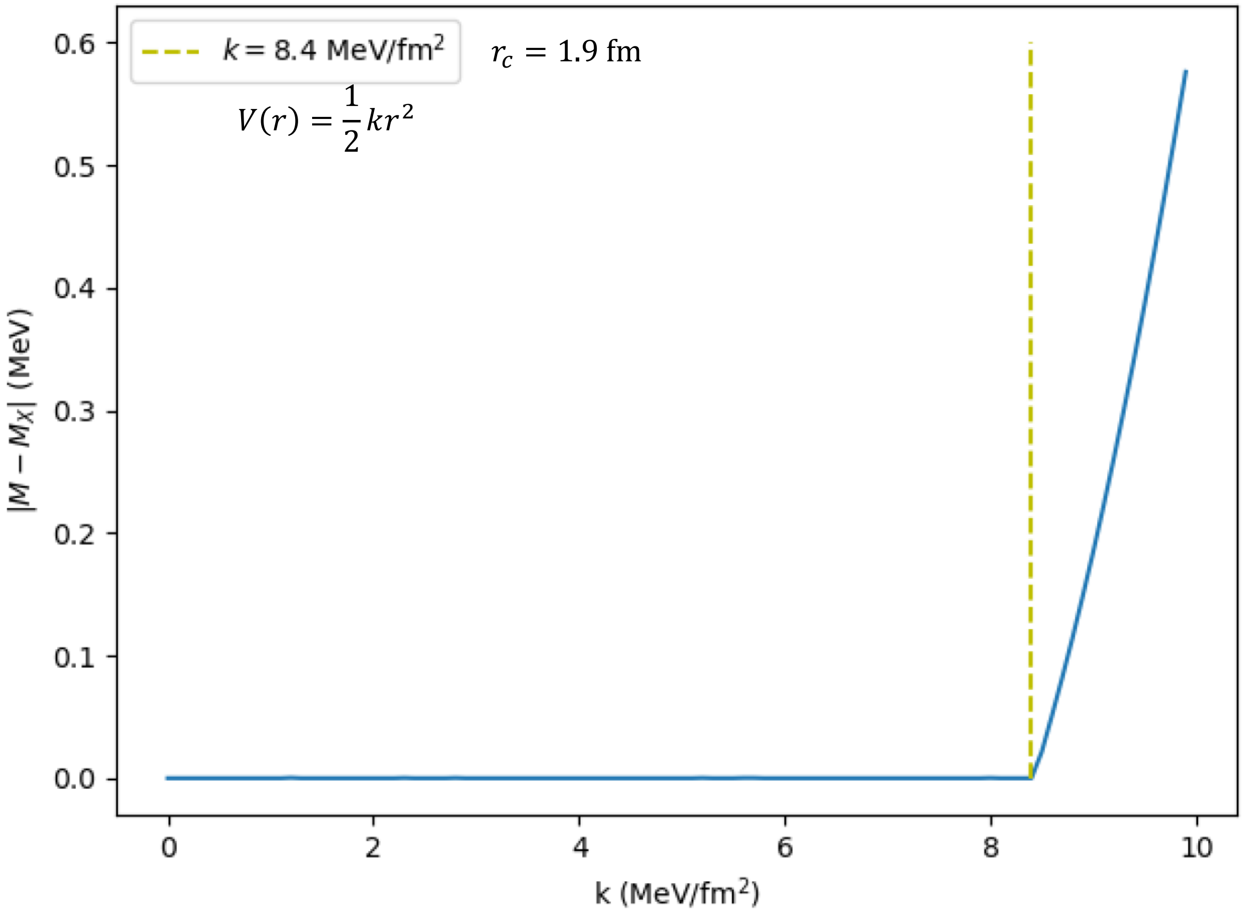
True diquark-molecular unification

RFL, Martinez [2502.20567, just accepted by PRD]

- A trivial modification in the diabatic formalism! Just replace free di-meson energy (sum of masses) with an attractive potential $V(r)$
- Exploratory study: Take $\chi_{c1}(3872)$ and $\chi_{c0}(3915)$ measured masses as fixed, and ask how diquark mass, and depth and range of $V(r)$ must scale to maintain these constraints
- (We used square-well and simple harmonic oscillator $V(r)$ examples)
- A result to surprise no one: If $V(r)$ becomes too strong, no choice of diquark parameters preserves the fit, and the bound state becomes pure di-meson molecular (illustration next slide):

Breaking the diquark model with a molecule

How close to the $\chi_{c1}(3872)$ mass?



Is any of this approach valid for lighter quarks?

- Flavor universality of QCD says that the diquark mechanism still occurs for lighter flavors
- But the original dynamical diquark model relies on heavy quarks to nucleate distinguishable diquarks
- Perhaps some hint persists for, say, $s\bar{s}s\bar{s}$ tetraquarks? Candidates like $\phi(2170)$ with weird properties exist…
- My new postdoc, Shahriyar Jafarzade, is investigating this possibility

ExoHad
Exotic Hadrons Topical Collaboration

https://www.exohad.org/

Diabatic framework: The future

- If the adiabatic calculation of light states like $s\bar{s}s\bar{s}$ is fruitful, then apply the diabatic formalism to them
- The current diabatic calculations treat all $\Sigma_g^+(1S)$ states as degenerate, but fine structure is easy to incorporate into H_{light}
- The current diabatic calculations treat V_{mix} parameters for all di-meson thresholds as degenerate; can incorporate proper heavy-quark spin and flavor dependence

Summary & Conclusions

1) So many heavy-quark exotics have now been observed
   that a theory to predict their complete spectrum has become **imperative**
2) Molecules alone are not enough: Many exotics lie far from constituent thresholds
3) Models based upon diquarks hold promise:
   Fully predicted spectrum, whole state bound by strong QCD forces,
   many phenomenological successes (especially in the dynamical diquark model)
4) But many exotics *are* very close to thresholds ⟶ adiabatic nature of
   Born-Oppenheimer approximation can be generalized to **diabatic formalism**
   when di-hadron thresholds are nearby, unifying diquark & molecular pictures
5) Initial calculations of dynamical diquark model using diabatic framework
   now complete, research in multiple future directions now underway

Backup Slides

The orbitally excited $\Sigma_g^+(1P)$ multiplet

Giron, RFL [PRD **101**, 074032 (2020)]

- The lightest negative-parity states (like Y) live here:
  14 with $I = 0$, 14 with $I = 1$
- Multiplet $\Sigma_g^+(1P)$ contains precisely $4\ J^{PC} = 1^{--}, I = 0$ (Y) states
- Analysis requires more Hamiltonian operators: spin-orbit and tensor
- But which states are experimentally confirmed?
  BESIII data is rapidly improving but still presents ambiguities
- Our analysis predicts full multiplet under several possible assignments:
  e.g., using $Y(4220), Y(4320), Y(4390)$ predicts the $J^{PC} = 0^{--}, I = 1$ $Z_c(4240)$ seen by LHCb [PRL **122**, 22202 (2014)]

If the model is any good, it must also apply to other flavor sectors

- Using the same Hamiltonian operators, apply to:
- the $(bq)(\bar{b}\bar{q})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
  Here, just the masses of $Z_b(10610)$, $Z_b(10650)$, and their B.R.'s to $\Upsilon(nS)$, $h_b(nP)$ are enough to predict full $\Sigma_g^+(1S)$ multiplet
- the $(cs)(\bar{c}\bar{s})$ sector {Giron, RFL [PRD **102**, 014036 (2020)]}
  Here, $X(3915)$ (peculiar: no open-charm decay) is the lowest state, $X(4140)$ is analogue of $X(3872)$, and all other $\Sigma_g^+(1S)$ are predicted because the Hamiltonian has one less operator (zero isospin!)

If the model is any good, it must also apply to other flavor sectors

- Using the same Hamiltonian operators, apply to:
- the $(cq)(\bar{c}\bar{s})$ sector {Giron, Martinez, RFL [PRD **104**, 054001 (2021)]}
  Here, LHCb's recently observed $Z_{cs}(4000)$, $Z_{cs}(4220)$ [PRL **127**, 082001 (2021)] belong to $SU(3)_{\rm flavor}$ multiplets of $J^{PC} = 1^{++}$ and 1^{+-}, but their strange states can mix, like K_{1A}, K_{1B}
- the $(cu)(\bar{c}ud)$ and $(cs)(\bar{c}ud)$ pentaquarks {Giron, RFL [PRD **104**, 114028 (2021)]}
  Here, all the known nonstrange states: $P_c(4312)$, $P_c(4337)$, $P_c(4450)$, $P_c(4457)$, easily accommodated

If the model is any good, it must also apply to other flavor sectors

- Using the same Hamiltonian operators, apply to:
- the $(cc)(\bar{c}\bar{c})$ sector {Giron, RFL [PRD **102**, 074003 (2020)]}
  Here, identical particle constraints limit the number of allowed states
- Not easy to describe $(cc)(\bar{c}\bar{c})$ states using "conventional" di-hadron molecule picture
- Our calculations indicate $X(6900)$ [Sci. Bull. **65**, 1983 (2020)] seen by LHCb is almost certainly in an excited orbital, most likely $\Sigma_g^+(2S)$
- So where are the lower $(cc)(\bar{c}\bar{c})$ states?
  Preliminary results from CMS and ATLAS indicate seeing them!

Even the Naming Scheme Has to Be Updated

LHCb Collaboration, 2206.15233

- "I found a hadron with valence-quark content c, s, u, d"

- "Which one? There are three kinds, not counting antiparticles:"
- $c\bar{s}u\bar{d}$: $T^{X}_{c\bar{s}\,J}(\mathrm{mass})^{++}$
- $c\bar{s}d\bar{u}$: $T^{X}_{c\bar{s}\,J}(\mathrm{mass})^{0}$
- $cs\bar{u}\bar{d}$: $T^{X}_{cs\,J}(\mathrm{mass})^{0}$

J = total spin,
X = symbol for parity & isospin
(*e.g.*, a for $P = +, I = 1$)

- Examples of all three of these types have already been observed!

Not all exotic candidates have heavy quarks

- $\pi_1(1600)$ (discovered 1998) is believed to be a hybrid meson because its $J^{PC} = 1^{-+}$ is not accessible to $q\bar{q}$ states
- $f_0(1710)$ is believed to have a sizeable glueball component because the quark model predicts one fewer 0^{++} states than are seen, and of them $f_0(1710)$ shows up most prominently in J/ψ decays (a glue-rich environment)
- $\phi(2170)$ has a peculiar decay pattern and may be an $s\bar{s}g$ hybrid or the $s\bar{s}q\bar{q}$ tetraquark analogue to the $c\bar{c}q\bar{q}$ state $Y(4230)$
- **Lesson 2**: Exotics studies require high- and low-energy intercommunity dialogue

What kind of “state” is it?

- Not every “bump” in the data is a Breit-Wigner resonance, *i.e.*, corresponds to a pole in one particular region of the complex scattering amplitude
- Distinguishing “resonances” from “virtual states” and “bound states”, not to mention (supposedly) simple “threshold rescattering effects”, requires:

1) Careful determination of amplitude dependence on energy/mass across the resonant region to measure the lineshape in different decay modes
2) Algorithms to model amplitudes in such a way as to obey bedrock quantum-field theory principles like unitarity

Lesson 4: Modern hadron spectroscopy analysis will require collaborations featuring frequent interactions between experimentalists and theorists, and the collective work of multiple theory researchers

Key scattering formulas

$$\frac{1}{p^{(i)}r}\sin\left(p^{(i)}r-\ell\frac{\pi}{2}\right)\to e^{i\delta_\ell^{(i)}}\frac{1}{p^{(i)}r}\sin\left(p^{(i)}r-\ell\frac{\pi}{2}+\delta_\ell^{(i)}\right)$$

$$\psi^{(i)}_{J^{PC},m_J}(\mathbf{r})=\frac{1}{r}\sqrt{\frac{2\mu^{(i)}}{\pi p^{(i)}}}\sum_k i^{\ell_k^{(i)}}a^{(i)}_{J^{PC};k}$$

$$\times\frac{1}{p^{(i)}r}\sin\left(p^{(i)}r-\ell_k\frac{\pi}{2}+\delta^{(i)}_{J^{PC};k}\right)\mathrm{Y}^{J,m_J}_{\ell_k^{(i)}s_k^{(i)}}(\hat{\mathbf{r}})$$

$$\psi^{i\leftarrow i'}_{J^{PC},m_J;k'}(\mathbf{r})=\frac{1}{r}\sqrt{\frac{2\mu^{(i)}}{\pi p^{(i)}}}\sum_k i^{\ell_k^{(i)}}\left[\delta_{ii'}\delta_{kk'}\sin\left(p^{(i)}r-\ell_k^{(i)}\frac{\pi}{2}\right)+p^{(i)}f^{i\leftarrow i'}_{J^{PC};k,k'}e^{i(p^{(i)}r-\ell_k^{(i)}\frac{\pi}{2})}\right]\mathrm{Y}^{J,m_J}_k(\hat{\mathbf{r}})$$

$$\sigma^{i\leftarrow i'}_{J^{PC}}=\frac{4\pi(2J+1)}{(2s_{M_1^{(i')}}+1)(2s_{\bar{M}_2^{(i')}}+1)}\sum_{k,k'}|f^{i\leftarrow i'}_{J^{PC};k,k'}|^2$$