One Born-Oppenheimer effective theory for all multiquark states

Exotic heavy meson spectroscopy and structure with EIC
April 15, 2025

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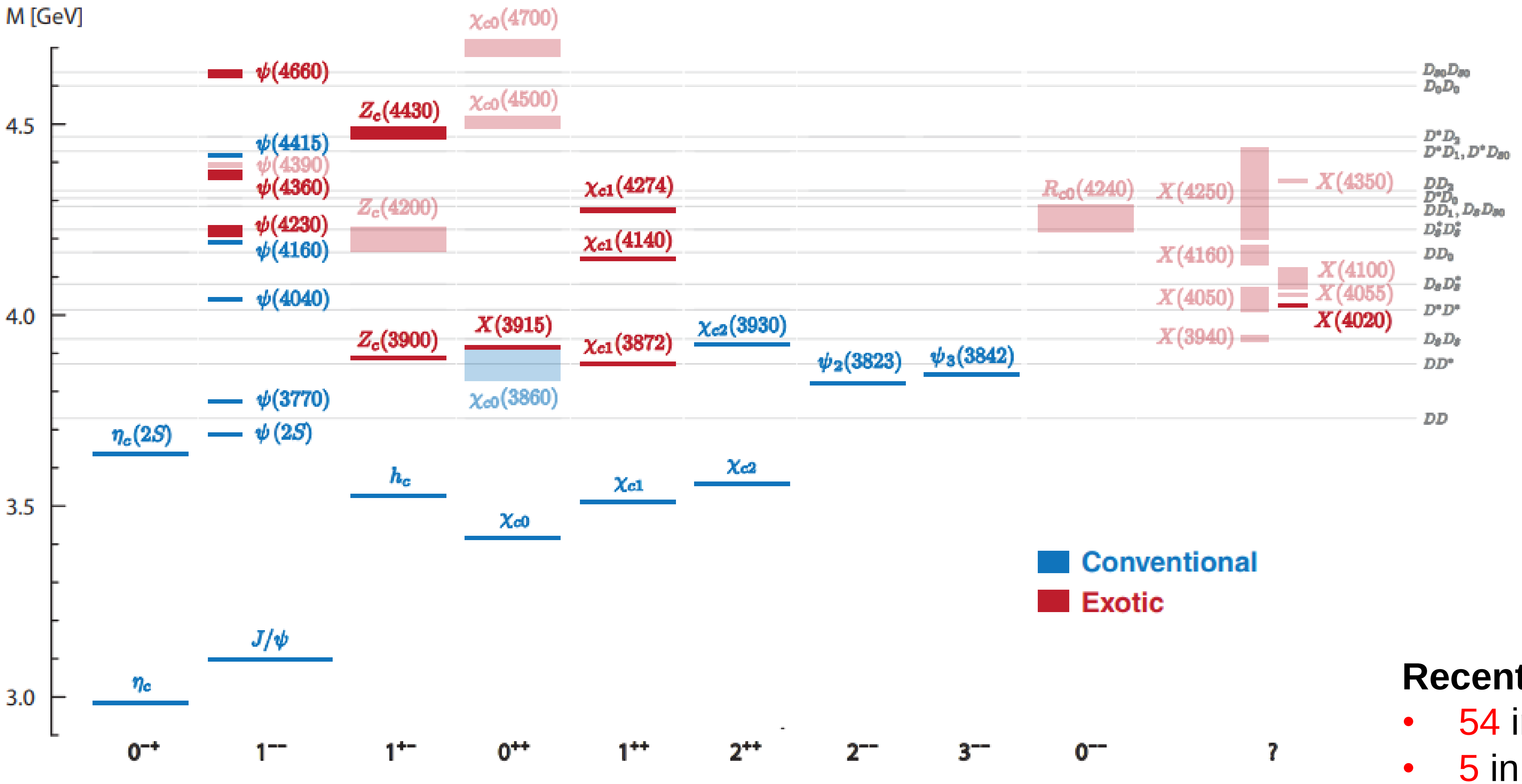
Phys. Rev. D 110, 094040 (2024) **(Editors Suggestion)**
&
arXiv 2411.14306 **(Under review at PRL)**

Technical University of Munich

Deutsche Forschungsgemeinschaft

Exotic mesons



Recent count:

- 54 in $c\bar{c}$ sector
- 5 in $b\bar{b}$ sector
- 4 with all c and $\bar{c}$
- 1 with two charm quarks.

Lebed talk on Monday (April 14)

Eichmann, Fischer, Heupel, Santowski, Few Body Syst. 61 (2020)

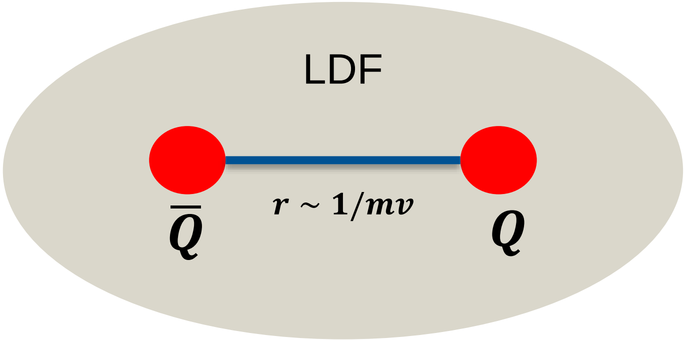
Born-Oppenheimer EFT

BOEFT: Exotic Hadron

- Exotic hadron ($Q\bar{Q}X, QQX, \ldots\ldots$), X: any combination of light quark and gluons (LDF) for color singlet.

- Hierarchy of scales in hybrids:

$$m \gg mv \gtrsim \Lambda_{\mathrm{QCD}} \gg mv^2$$



  - ❖ Mass of heavy quark: $\boldsymbol{m}$
  - ❖ Energy scale for LDF: $\boldsymbol{\Lambda_{\mathrm{QCD}}}$
  - ❖ Relative momentum between heavy quarks: $mv \sim 1/\mathrm{r}$
  - ❖ Heavy Quark kinetic energy scale: mv^2

- Time-scale for dynamics of $Q\bar{Q}$: $\sim \frac{1}{mv2} \gg \frac{1}{\Lambda_{\mathrm{QCD}}}$

Heavy quarks **static** with respect to light quarks or gluons

Born-Oppenheimer (BO) Approximation

Braaten, Langmack, Smith Phys. Rev. D. 90, 014044 (2014)
Juge, Kutti, Morningstar, Phys. Rev. Lett. 90, 161601 (2003)

BOEFT: Quantum #’s

- **BO-quantum number** (**r ≠ 0**): heavy quarks static, Cylindrical symmetry group $D_{\infty h}$

Labelling LDF static energies:

✓ Absolute value of component of LDF angular momentum $\boldsymbol{K}$

$$|\boldsymbol{r}\cdot\boldsymbol{K}| \equiv \boldsymbol{\Lambda} = \mathbf{0}, \mathbf{1}, \mathbf{2}, \dots\dots\dots (\text{or } \boldsymbol{\Sigma}, \boldsymbol{\Pi}, \boldsymbol{\Delta}, \boldsymbol{\Phi}, \dots\dots)$$

✓ Product of charge conjugation and parity ($\boldsymbol{CP}$):

$$\boldsymbol{\eta} = +\mathbf{1}\ (\mathbf{g}),\ -\mathbf{1}\ (\boldsymbol{u})$$

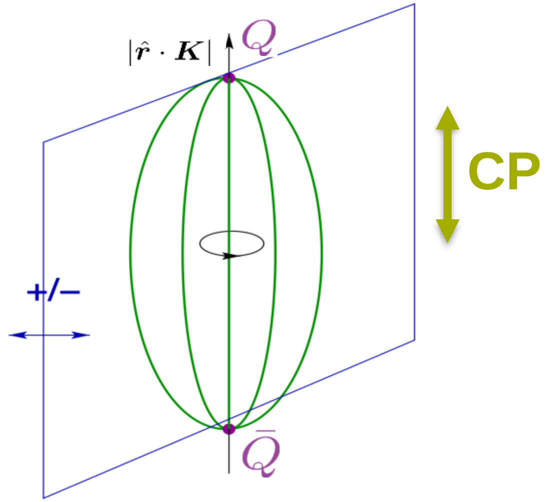
✓ $\boldsymbol{\sigma}$: Eigenvalue of reflection about a plane containing static sources.

$$\sigma = P\,(-1)^{K_{\text{light}}} = \pm 1$$

Born, Oppenheimer, Annalen der Physik 389 (1927)

Landau, Lifshitz & Pitaevskii, QM book

$$\Lambda_\eta^\sigma$$



Examples:

$\boldsymbol{K}^{\boldsymbol{PC}}$	$\boldsymbol{\Lambda}_{\boldsymbol{\eta}}^{\boldsymbol{\sigma}}$
0^{++}	Σ_g^+
0^{+-}	Σ_u^+
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$
2^{--}	$\{\Sigma_g^-, \Pi_g, \Delta_g\}$

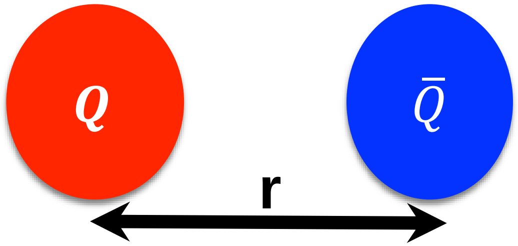
- **Spherical symmetry** restored in **r → 0** limit:
  Labelled by LDF quantum #’s:

$$\kappa = \{K^{PC}, f\}$$

Exotic Hadron

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

$Q\bar{Q}X$



color: $3 \otimes \bar{3} = 1 \oplus 8$

X = gluon → **Hybrid**

$X = (q\bar{q})_8$ → **Tetraquark (Molecule, compact..)**

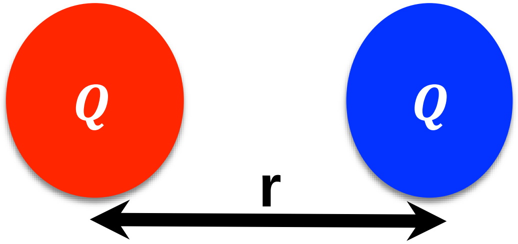
$X = (qqq)_8$ → **Pentaquark (Molecule, compact..)**

and so on

Total angular momentum of **$Q\bar{Q}X$** or **QQX** :

$$J = L + S_Q$$

QQX



color: $3 \otimes \bar{3} = \bar{3} \oplus 6$

$X = q_3$ → **Double heavy baryon**

$X = (\bar{q}\bar{q})_3$ / $(\bar{q}\bar{q})_{\bar{6}}$ → **Tetraquark**

$X = (qq\bar{q})_3$ / $(qq\bar{q})_6$ → **Pentaquark**

and so on

X_8 : **Adjoint hadrons** (gluelump, adjoint meson, adjoint baryon….)

$X_{3/6}$: **triplet or sextet hadrons** (meson, baryon….)

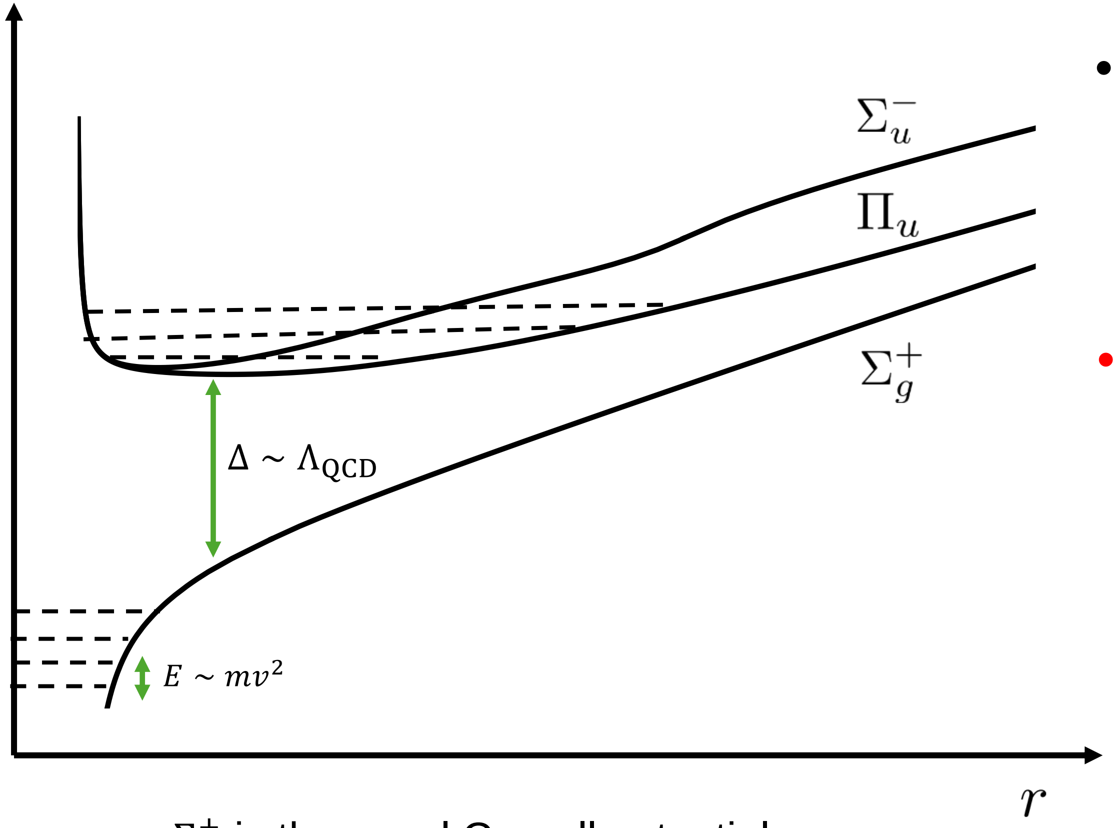
BOEFT can address all these states with inputs from Lattice QCD

BOEFT

- BOEFT Lagrangian: $L_{\text{BOEFT}} = L_{Q\bar{Q}} + L_{Q\bar{Q}g} + L_{Q\bar{Q}q\bar{q}} + L_{\text{mixing}} + \cdots$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040 Castellà , Soto Phys. Rev. D. 102, 014012 (2020)

Brambilla, Krein, Castellà , Vairo Phys. Rev. D. 97, (2018)



- Gap of order Λ_{QCD} allows us to focus individually on low-lying states corresponding to quarkonium, hybrid, tetraquark etc.

- L_{mixing}: Mixing between different states with similar masses and same quantum-numbers.

Ex: Hybrid-quarkonium mixing, Tetraquark-hybrid & Tetraquark-quarkonium mixing etc.

R. Oncala, J. Soto, Phys. Rev. D96 014004 (2017)

Ajeli, Brambilla, AM, Vairo, In preparation

Σ_g^+ is the usual Cornell potential

Σ_u^-: similar to Cornell potential except repulsive octet behavior at small distances.

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- BOEFT Lagrangian:

$$L_{\text{BOEFT}} = \int d^3\boldsymbol{R} \int d^3\boldsymbol{r} \sum_{\kappa\lambda\lambda'} \text{Tr}\Big\{ \Psi^{\dagger}_{\kappa\lambda}(\mathbf{r},\, \mathbf{R},\, t) \Big[i\partial_t\, \delta_{\lambda\lambda'} - V_{\kappa\lambda\lambda'}(r) + P^{i\dagger}_{\kappa\lambda}(\theta,\phi) \frac{\boldsymbol{\nabla}^2_r}{m_Q} P^i_{\kappa\lambda'}(\theta,\phi) \Big] \Psi_{\kappa\lambda'}(\mathbf{r},\, \mathbf{R},\, t) \Big\}$$

LDF-quantum #: $\kappa = \{K^{PC}, f\}$ BO-quantum #: Λ^{σ}_{η} $\lambda = \pm\Lambda$

Projection vectors for $D_{\infty h}$: $P^i_{K\lambda}(\theta,\varphi) = D^{\lambda *}_{K i}(0,\theta,\varphi)$

- **BO potentials: Potential between Q & $\bar{Q}$** due to LDF (light quarks, gluons).

Born-Oppenheimer (BO) potential:

$$V_{\kappa\lambda\lambda'}(r) = \boxed{E^{(0)}_{\kappa,|\lambda|}(r)}\delta_{\lambda\lambda'} + \boxed{\frac{V^{(1)}_{\kappa\lambda\lambda'}(r)}{m_Q}} + \dots ,$$

Static Energy — Spin-dependent potentials

Brambilla, Lai, Segovia, Castellà, Phys. Rev. D. 101, (2020)

Brambilla, Lai, Segovia, Castellà, Vairo Phys. Rev. D. 99, (2019)

Soto, Valls, Phys. Rev. D 108 (2023)

Castellà , Soto Phys. Rev. D. 102, (2020)

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Good quantum numbers:

- BO-orbital momentum: $\boldsymbol{L} = \boldsymbol{L}_Q + \boldsymbol{K}$
- Heavy quark Spin: $\boldsymbol{S}_Q$ (HQSS limit)
- Total angular momentum: $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}_Q$

$\boldsymbol{K}$: LDF angular-momentum or spin

$\boldsymbol{L_Q}$: orbital-angular momentum of QQ or $Q\bar{Q}$ pair.

Coupled Equations (**spin-averaged**) for lowest Hybrids (Q$\bar{\text{Q}}$g) and Tetraquarks (QQ$\overline{\text{qq}}$ or Q$\bar{\text{Q}}$q$\bar{\text{q}}$):

LDF quantum # K=1

parity σ_P:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} \\ -2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma & 0 \\ 0 & E_\Pi \end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix}$$

Opposite parity $-\sigma_P$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\, \partial_r + \frac{l(l+1)}{m_Q r^2} + E_\Pi\right]\psi^{(N)}_{\Pi,-\sigma_P} = \mathcal{E}_N\, \psi^{(N)}_{\Pi,-\sigma_P}$$

Berwein, Brambilla, Castellà , Vairo Phys. Rev. D. 92 (2015)

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Coupled Equations (**spin-averaged**) for Doubly Heavy Baryons (QQq) & Pentaquarks (QQq̄qq or QQ̄qqq):

LDF quantum # K=1/2

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + E_{K_\eta}\right]\psi^{(N)}_{K_\eta,\sigma_P} = \mathcal{E}_N\,\psi^{(N)}_{K_\eta,\sigma_P}$$

LDF quantum # K=3/2

parity σ_P:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2\partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} E_{(1/2)_u} & 0 \\ 0 & E_{(3/2)_u}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2,\sigma_P} \\ \psi^{(N)}_{3/2,\sigma_P}\end{pmatrix} = \mathcal{E}_n\begin{pmatrix}\psi^{(N)}_{1/2,\sigma_P} \\ \psi^{(N)}_{3/2,\sigma_P}\end{pmatrix}$$

Opposite parity $-\sigma_P$:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2\partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+3)+\frac{17}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} E_{(1/2)_u} & 0 \\ 0 & E_{(3/2)_u}(r)\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2,-\sigma_P} \\ \psi^{(N)}_{3/2,-\sigma_P}\end{pmatrix} = \mathcal{E}_n\begin{pmatrix}\psi^{(N)}_{1/2,-\sigma_P} \\ \psi^{(N)}_{3/2,-\sigma_P}\end{pmatrix}$$

Castellà , Soto Phys. Rev. D. 102, 014013 (2020) Castellà , Soto Phys. Rev. D. 104, 074027 (2021)

$\boldsymbol{X(3872)}$ & $\boldsymbol{T_{cc}^{+}(3875)}$

$\chi_{c1}(3872)$

First XYZ exotic state seen by Belle

Phys. Rev. Lett. 91, 262001 (2003)

- Quark content $c\,\bar{c}$ + light quarks

- Quantum numbers: **$J^{PC}=1^{++}$ (Isospin=0)**

LHCb, Phys. Rev. Lett. 110, 222001 (2013)   LHCb, Phys. Rev. D. 92, 011102 (2015)

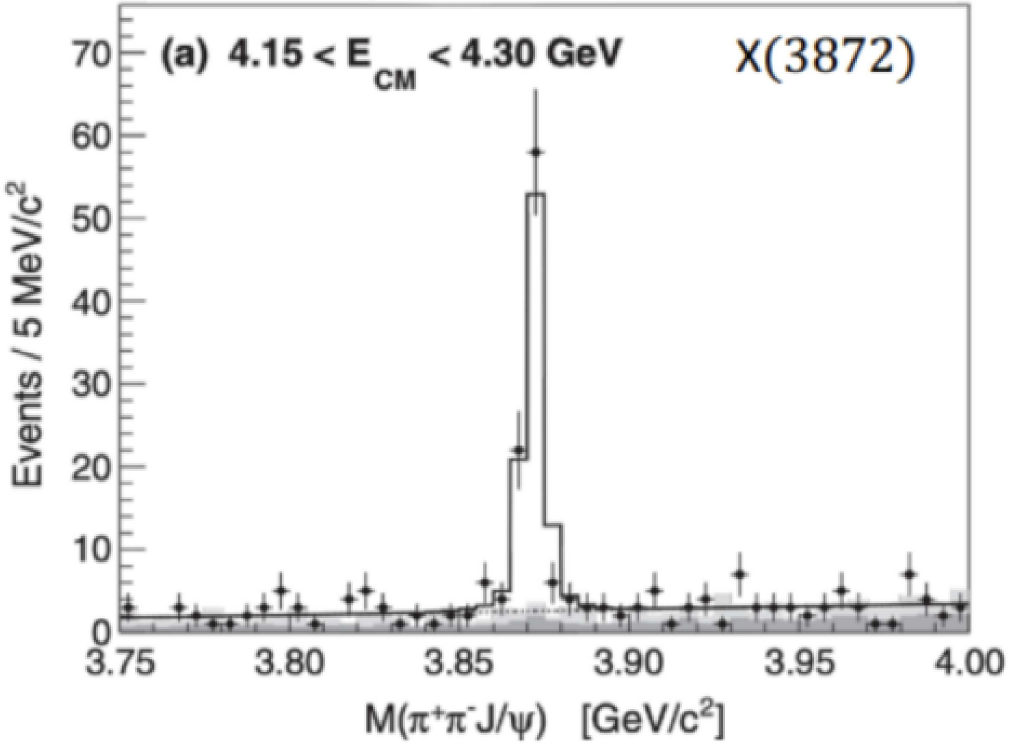
Mass extremely close to $D^{*0}D^0$ threshold (within 100 keV)

$$m_{\chi_{c1}(3872)} - (m_{D^{*0}} + m_{\bar{D}^0}) = -0.07 \pm 0.12 \text{ MeV}.$$

LHCb, JHEP 08 (2020) 123

$e^+e^- \to \gamma X(3872);\quad X(3872) \to \pi^+\pi^- J/\psi$

BesIII coll. PRL 122 (2019) 202001



BOEFT: $Q\bar{Q}q\bar{q}$ multiplets

$Q\bar{Q}$ color state	$q\bar{q}$ spin K^{PC}	Static energies	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
Octet	0^{-+}	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	T_1^0
			1	$\{1^{--}, (0,1,2)^{-+}\}$	T_2^0
			2	$\{2^{++}, (1,2,3)^{+-}\}$	T_3^0
	1^{--}	$\{\Sigma_g^{+\prime}, \Pi_g\}$	1	$\{1^{+-}, (0,1,2)^{++}\}$	T_1^1
		$\{\Sigma_g^{+\prime}\}$	0	$\{0^{-+}, 1^{--}\}$	T_2^1
		$\{\Pi_g\}$	1	$\{1^{-+}, (0,1,2)^{--}\}$	T_3^1
		$\{\Sigma_g^{+\prime}, \Pi_g\}$	2	$\{2^{-+}, (1,2,3)^{--}\}$	T_4^1

Isospin-1 channel:
$Z_c(3900), Z_c(4200), Z_b(10610)$, $Z_b(10610)$ **states:**

Mixing between $K^{PC} = 0^{-+}$ and $K^{PC} = 1^{--}$

Light-quark spin-symmetry !!

Voloshin, Phys. Rev. D. 93, 074011 (2016)

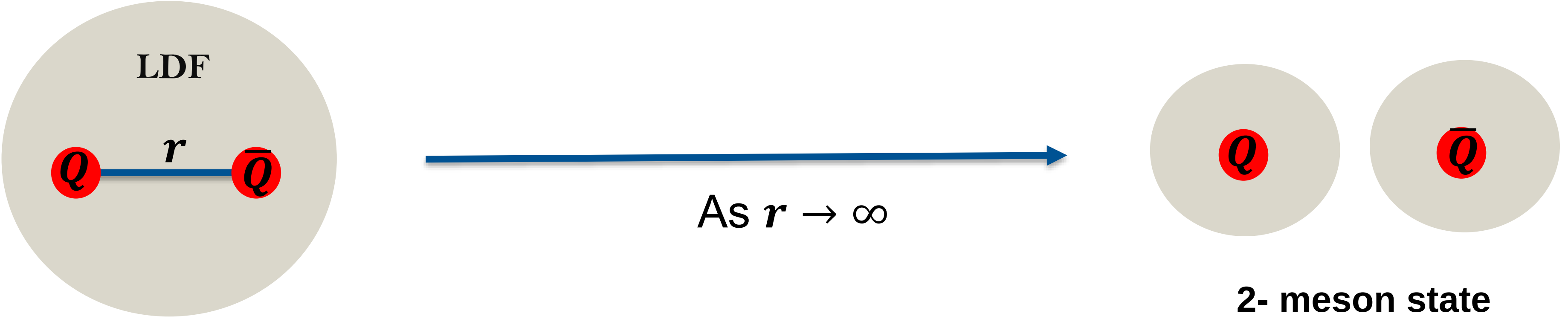
Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Isospin-0 channel:
$\chi_{c1}(3872), X_b$

Brambilla, AM, Scirpa, Vairo 2411.14306

BO potentials: Tetraquarks



Consider $Q\bar{Q}q\bar{q}$ system:

BO-quantum # Λ_η^σ as $r \to 0$:

$Q\bar{Q}$ (color)	Light Spin K^{PC}	Λ_η^σ $(D_{\infty h})$
Octet	0^{-+}	Σ_u^-
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

BO-quantum # Λ_η^σ for meson-antimeson as $r \to \infty$

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

s-wave+s-wave
Ex. $\mathbf{D\bar{D}}$ threshold

Meson-antimeson have same BO-quantum # Λ_η^σ !!!

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

Quarkonium and Tetraquarks

String breaking: quarkonium & meson-antimeson

Bulava, Hoerz, Knechtli, Koch, Moir, Morningstar, Peardon, Phys. Lett. B. 793 (2019)

Bulava, Knechtli, Koch, Morningstar, Peardon, Phys. Lett. B. 854 (2024)

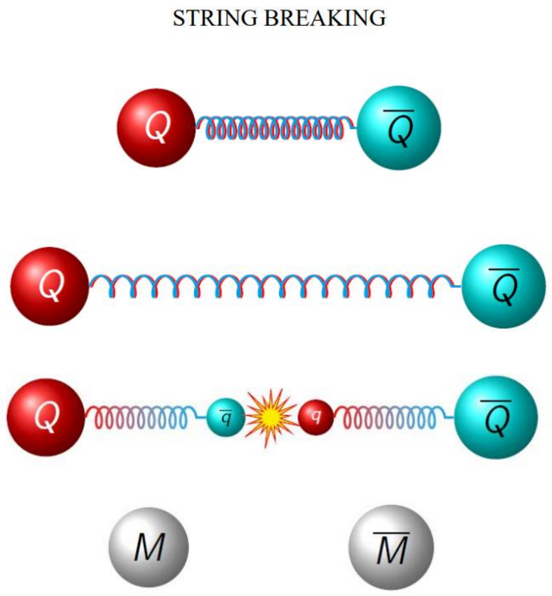
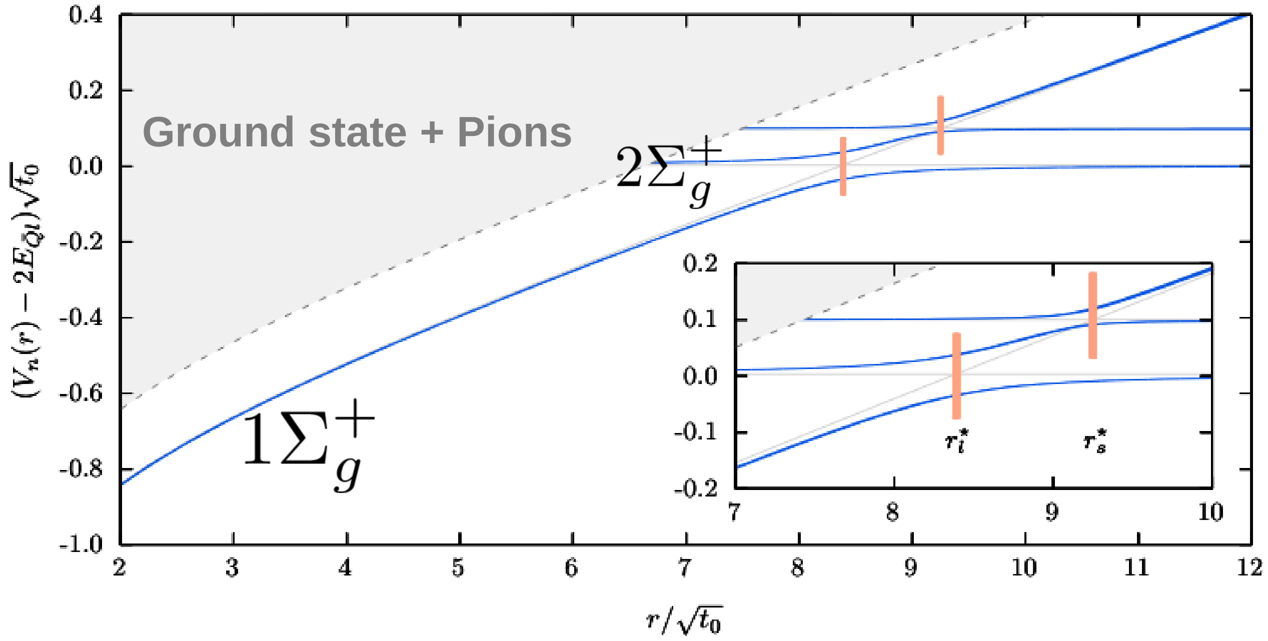


Figure from R. Bruschini talk

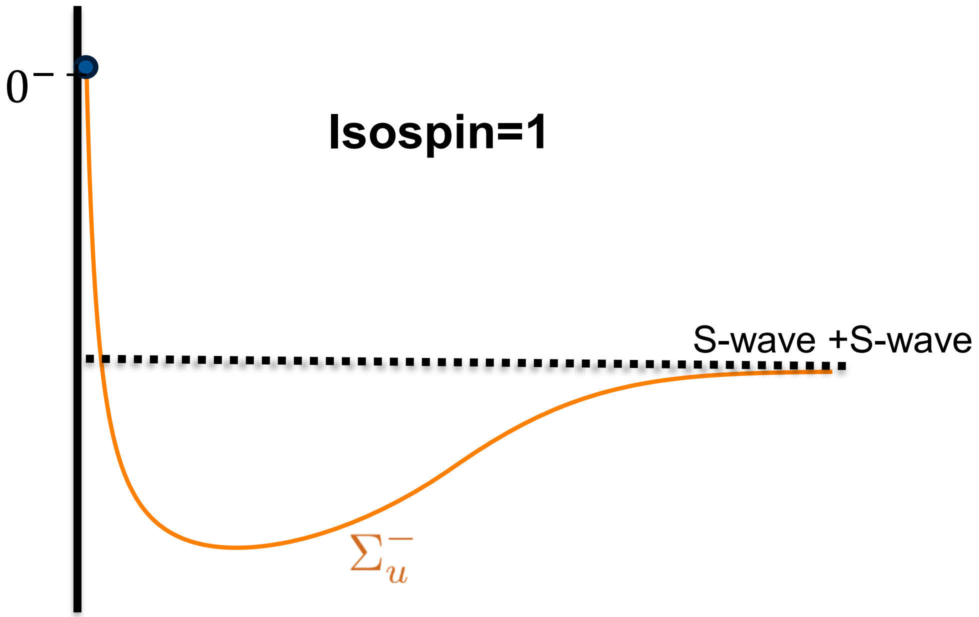
$$m_\pi \approx 200 - 340 \text{ MeV} \quad m_K \approx 440 - 480 \text{ MeV}$$



String breaking radius ≈ 1.22 fm

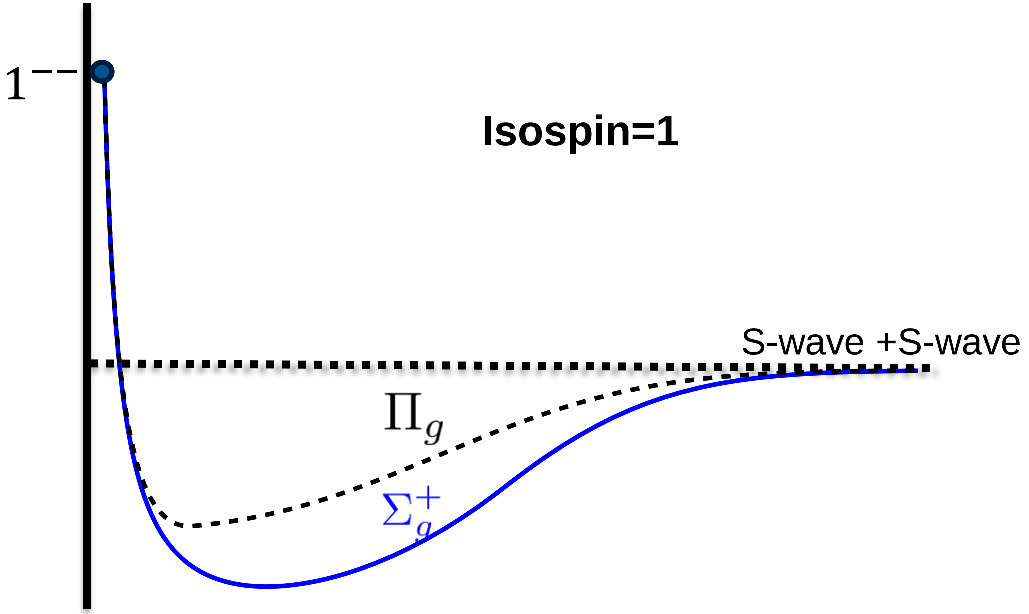
Avoided crossing between static energies **with same** BO-quantum # Σ_g^+

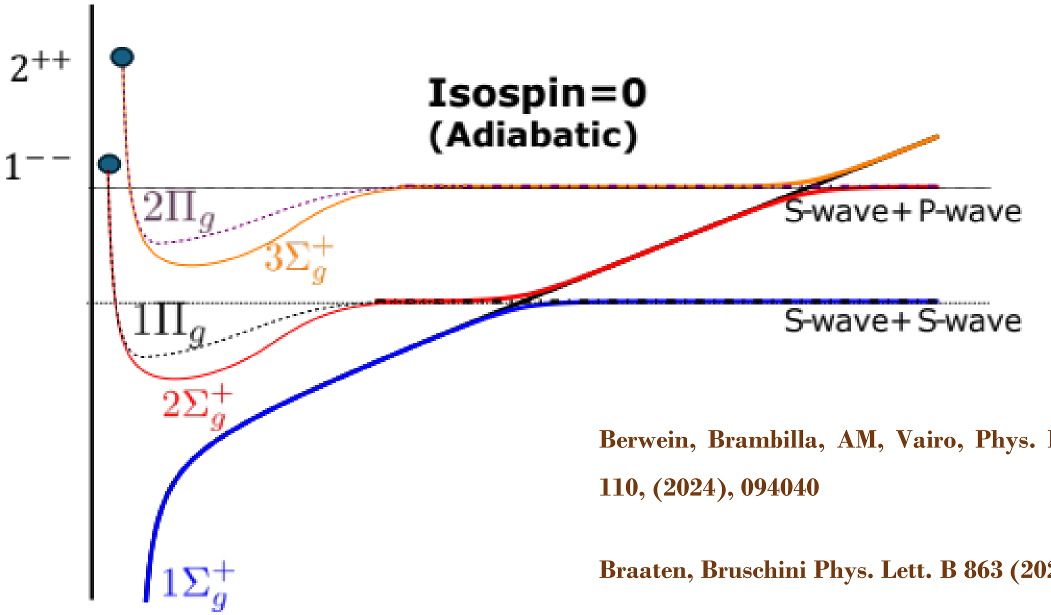
Tetraquarks



Key Takeaways: Tetraquark static energy behavior

- Repulsive behavior at small r ($r \to 0$)
- Heavy meson pair threshold at large r ($r \to \infty$)
- Avoided crossing with quarkonium static energy (Isopsin=0)





Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

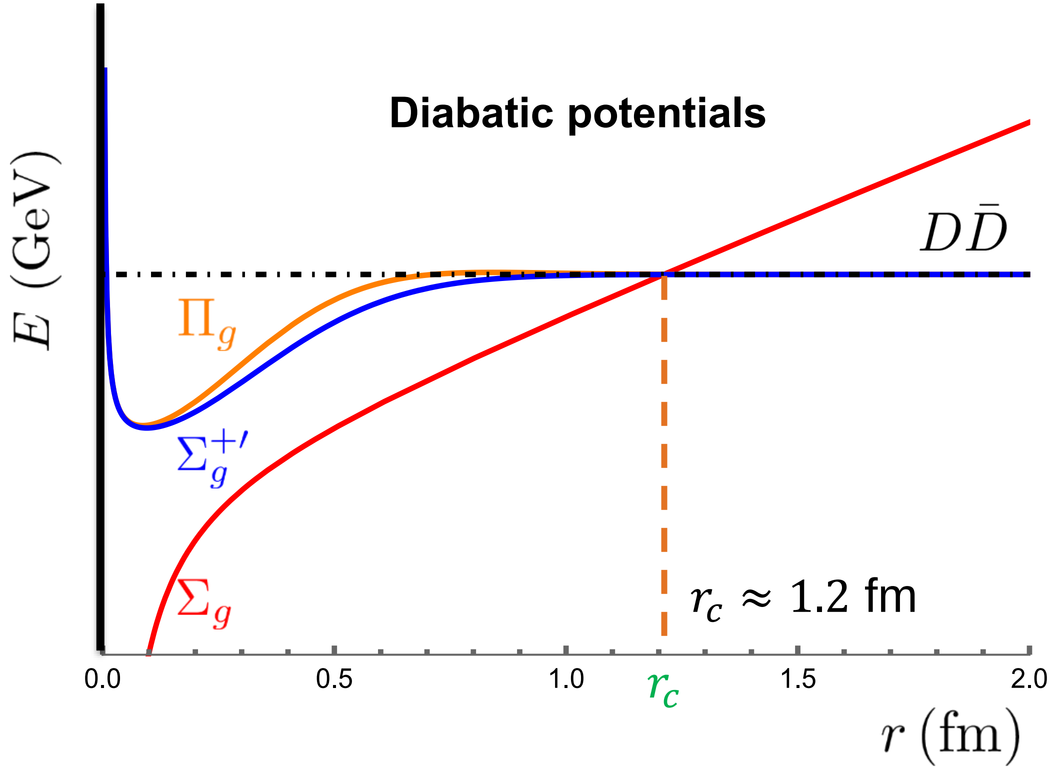
Braaten, Bruschini Phys. Lett. B 863 (2025) 139386

$\chi_{c1}(3872)$

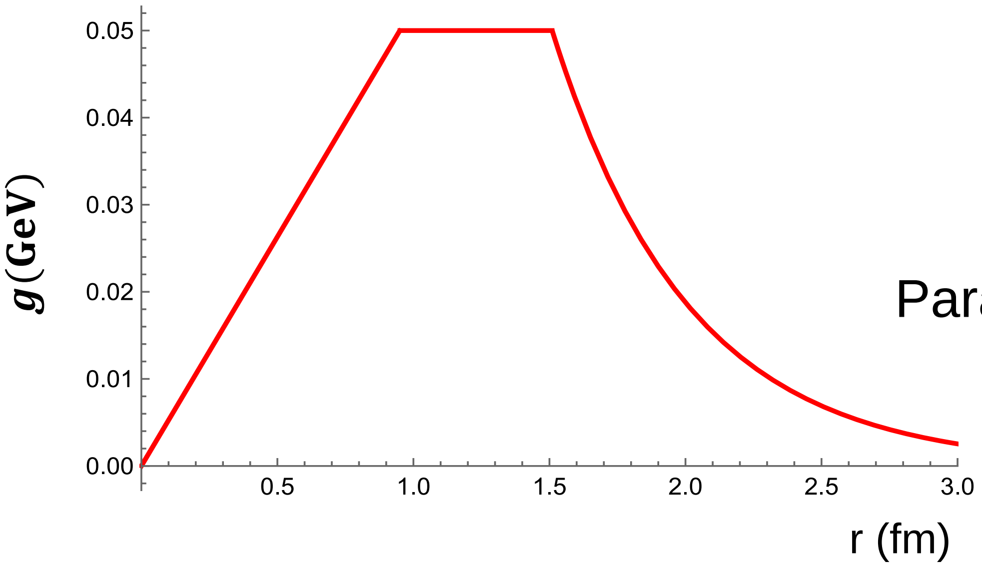
Coupled-channel Equations: spin-averaged

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1) \end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r) \end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix}$$

$l = 1$



Mixing potential g(r)



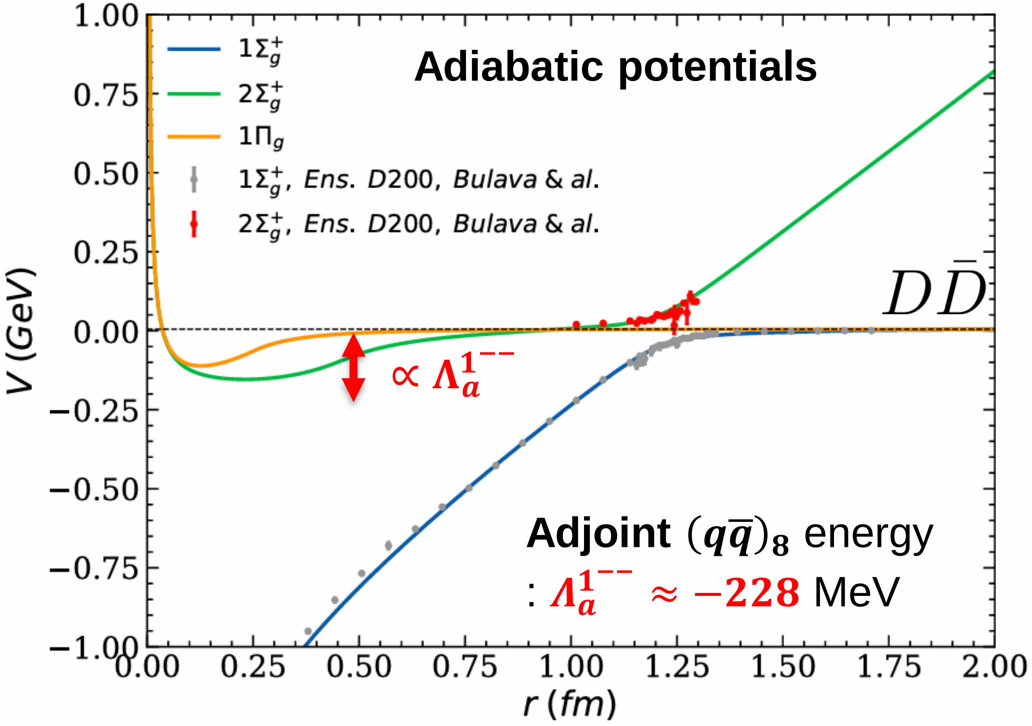
Parametrization of Mixing potential **g(r)**

- Short-distance $r \to 0$ behavior fixed by pNRQCD

Castellà, Phys. Rev. D. 106, (2022), 094020

- Around r_c: fixed to lattice QCD value

$\chi_{c1}\,(3872)$



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Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Alasiri, Braaten, AM, Phys. Rev. D 110, 054029 (2024)

Coupled-channel Equations:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l\,(l+1) & 0 & 0 \\ 0 & l(l+1)+2 & -2\sqrt{l(l+1)} \\ 0 & -2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_{\Sigma_g^+}(r) & g(r) & 0 \\ g(r) & E_{\Sigma_g^{+\prime}}(r) & 0 \\ 0 & 0 & E_{\Pi_g}(r)\end{pmatrix}\right]\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix} = \mathcal{E}\begin{pmatrix}\psi_\Sigma \\ \psi_{\Sigma'} \\ \psi_\Pi\end{pmatrix}$$

$l = 1$

Lattice inputs on string breaking from

Bulava et al Phys. Lett. B. 854 (2024)

Results:

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8-13\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius $\sim$ 15 fm.
4) Deeper state in bottom sector: 15 MeV below spin-isospin averaged $B\bar{B}$ threshold.

$\chi_{c1}(3872)$

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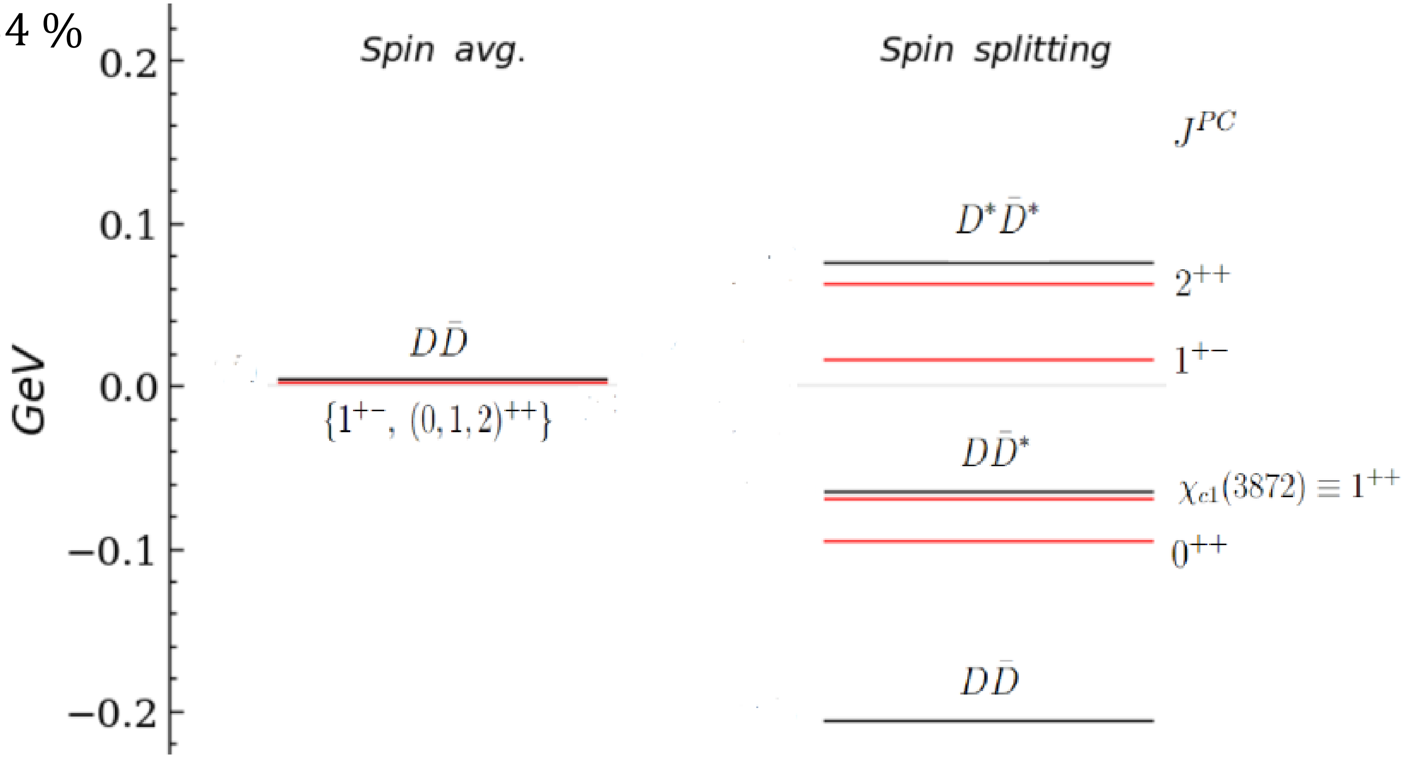
Results:

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8 - 13\ \%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\ \%$, $|\psi_\Pi|^2 \approx 54\ \%$
3) Radius ~ 15 fm.
4) **Adjoint** $(q\bar{q})_8$ energy: $\Lambda_a^{1^{--}} \approx -228$ MeV:
   No bound states in higher multiplets T_2^1, T_3^1, T_4^1 ….

Using lattice QCD spin-splitting results for hybrids ($Q\bar{Q}g$)



Multiplet T_1^1: $\{1^{+-}, (0, 1, 2)^{++}\}$

1^{++} state: Identified with $\chi_{c1}(3872)$

1^{+-} state: Mass around 3.957 (11) GeV. Identified with X(3940) ?

2^{++} state: Mass around 4.004 (14) GeV.

0^{++} state: Mass around 3.846 (11) GeV. Also indicated in the lattice calculations: Prelovsek et al JHEP 06 (2021) 035.

$\chi_{c1}(3872)$

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1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 8-13\,\%$
2) Tetraquark percentage: $|\psi_{\Sigma'}|^2 \approx 38\,\%$, $|\psi_\Pi|^2 \approx 54\,\%$
3) Radius > 15 fm.

We **naturally** get $8-13\,\%$ quarkonium component in $\chi_{c1}(3872)$ due to **avoided level crossing**

Radiative decays:

$$\mathcal{R}_{\gamma\psi} = \frac{\Gamma_{\chi_{c1}(3872)\to\gamma\psi(2s)}}{\Gamma_{\chi_{c1}(3872)\to\gamma J/\psi}},$$

Our estimate: $\boldsymbol{R_{\gamma\psi} = 2.99 \pm 2.36}$ (assuming only through $\chi_{c1}(2P)$ **component**)

LHCb: $\boldsymbol{R_{\gamma\psi} = 1.67 \pm 0.25}$ Aaij et al arXiv: 2406.17006

Compositeness:

BES III: $\boldsymbol{Z = 0.18^{+0.20}_{-0.23}}$

Ablikim et al. Phys. Rev. Lett 132, 151903 (2024)

EMPPR: $0.052 < Z < 0.14$

Esposito, Maiani, Pilloni, Polosa, Riquer Phys. Rev. D 105, L031503 (2022)

Agreement with our $8-13\,\%$ quarkonium (compact) component

Lattice QCD:

$c\bar{c}$ operator along with $D\bar{D}^*$ relevant for $\chi_{c1}(3872)$ signal

Padmanath, Lang, Prelovsek Phys. Rev. D 92, 034501 (2015) Prelovsek and Leskovec Phys. Rev. Lett 111, 192001 (2013)

X_b

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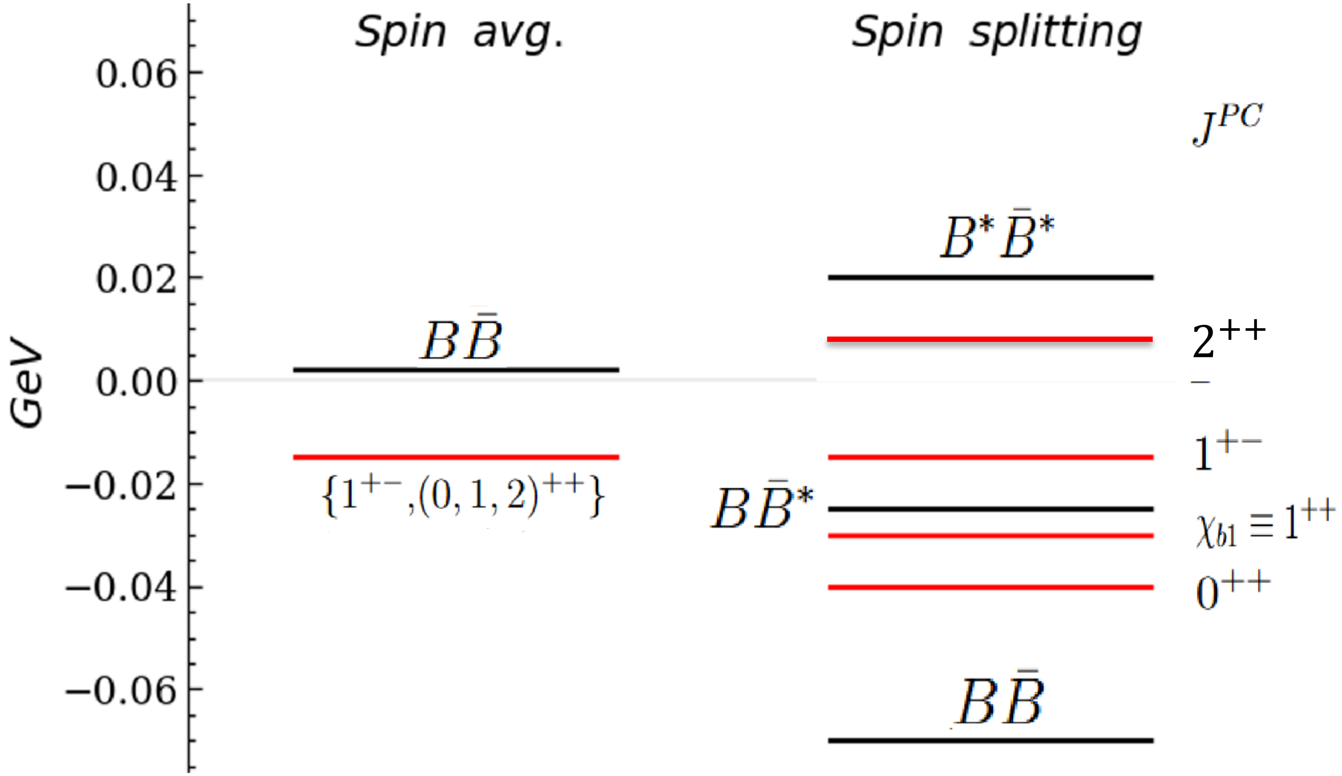
Using lattice QCD spin-splitting results for hybrids ($Q\bar{Q}g$)

Results with
adjoint meson energy $\approx$ $\mathbf{-228}$ MeV

1) Quarkonium percentage: $|\psi_\Sigma|^2 \approx 1.5$ %
2) Tetraquark percentage:

$$|\psi_{\Sigma'}|^2 \approx 45.4\ \%,\ |\psi_\Pi|^2 \approx\ 53.1\ \%$$



Multiplet T_1^1: $\{1^{+-},(0,1,2)^{++}\}$

1^{++}: identified with X_b: Mass around 10.595 GeV

1^{+-} state: Mass around 10.612 GeV.

2^{++} state: Mass around 10.635 GeV.

0^{++} state: Mass around 10.576 GeV.

$T_{cc}^{+}(3875)$

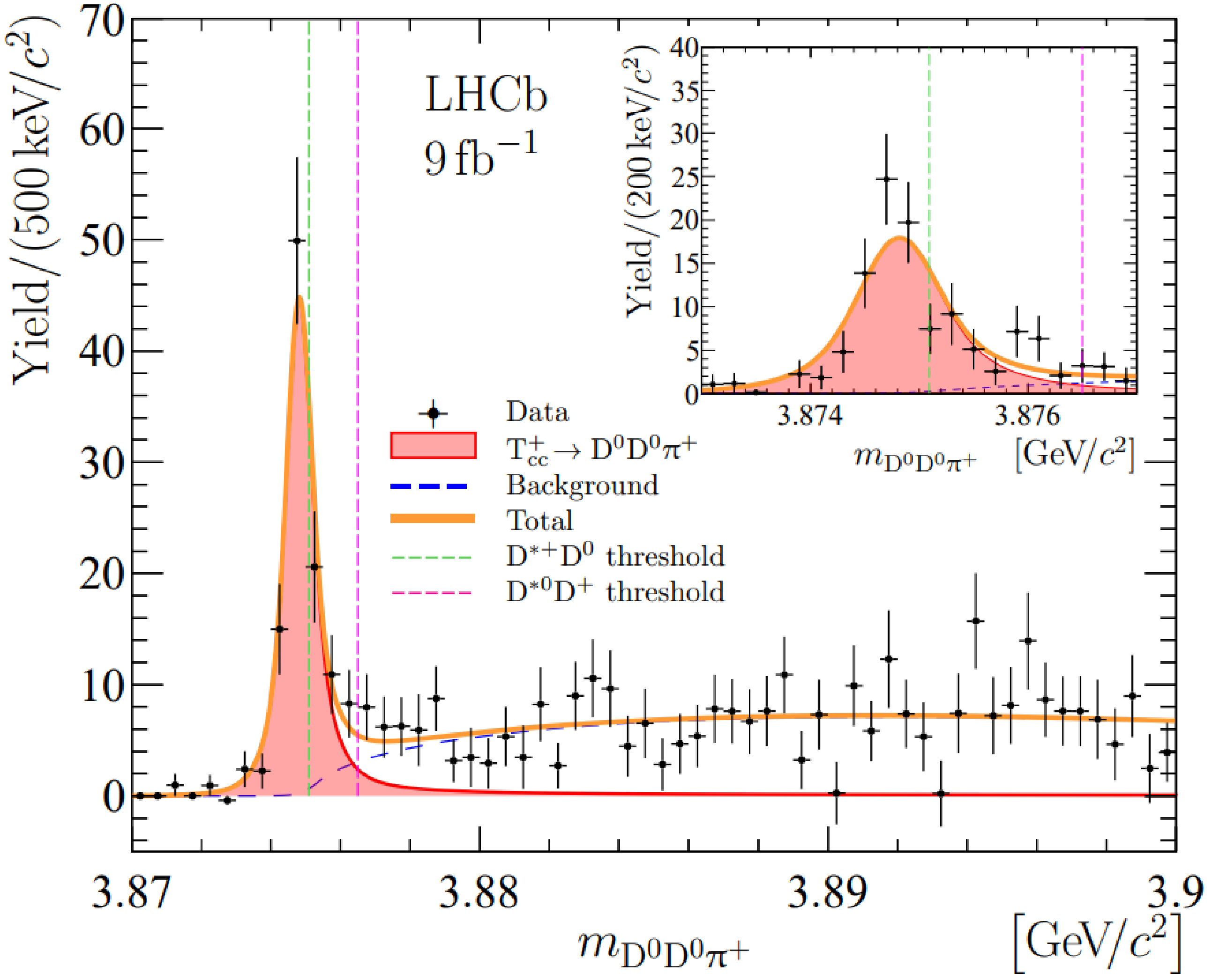
First doubly charmed tetraquark seen by LHCb

$$T_{cc}^{+}(3875) \to D^0 D^0 \pi^+$$

- Exotic quark content $cc\bar{u}\bar{d}$
- Consistent with **isoscalar** with $\mathbf{J^P=1^+}$
- **Longest lived** Exotic particle: $\Gamma \sim 50$ keV

Mass below $D^{*+}D^0$ threshold and very narrow

$$m_{T_{cc}^+} - (m_{D^{*+}} + m_{D^0}) = -0.27 \pm 0.06 \text{ MeV}.$$



LHCb (Nature Phys. 18 (2022) 7, 751; Nature Comm. 13 (2022) 3351)

BOEFT: $QQ\bar{q}\bar{q}$ multiplets

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

light antiquarks

{qq'}, 1^+ [qq'], 0^+

"Bad diquark" **"Good diquark"**

Defines the Born-Oppenheimer static potentials $\Sigma_g^+, \{\Sigma_g^-, \Pi_g\}$

Bad diquark – Good diquark
$\approx$ 200 MeV

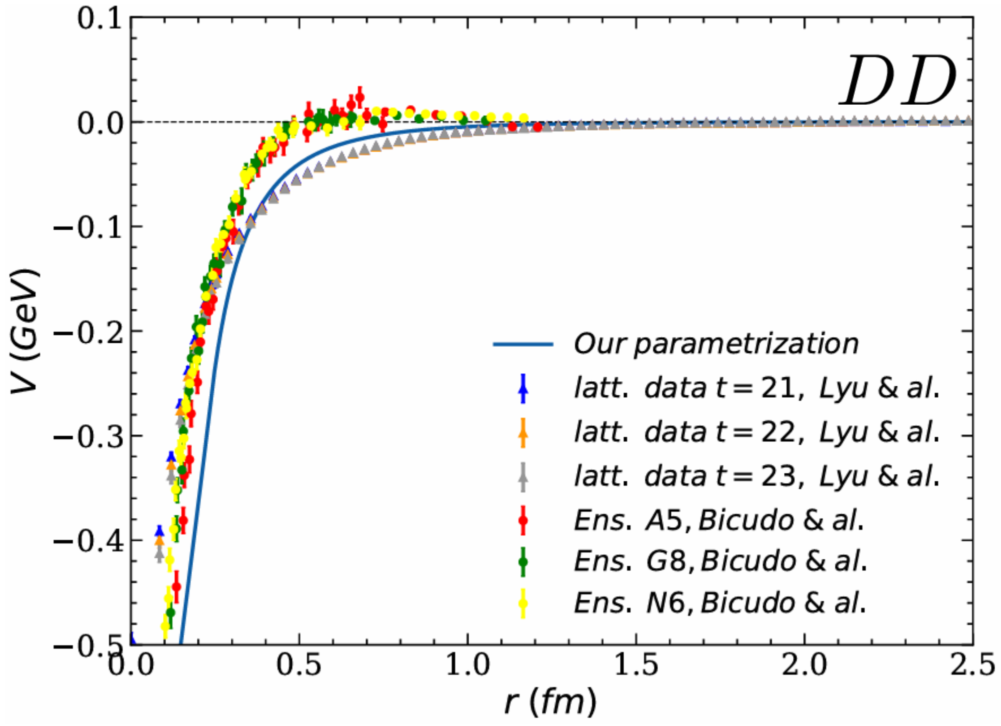
QQ color state	Light spin K^{PC}	Static energies	Isospin I	l	J^{PC}	
					$S_Q = 0$	$S_Q = 1$
anti-triplet $\bar{3}$	0^+	$\{\Sigma_g^+\}$	0	0	—	1^+
				1	1^-	—
	1^+	$\{\Sigma_g^-, \Pi_g\}$	1	0	0^-	—
				1	1^-	$(0,1,2)^+$

J^P for T_{cc}^+

$T_{cc}^{+}(3875)$

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Brambilla, AM, Scirpa, Vairo 2411.14306



Critical good diquark energy: $\Lambda_t^{0^+} \approx -478$ MeV

Lyu, Aoki, Doi, Hatsuda, Ikeda, Meng, Phys. Rev. Lett. 131, 161901 (2023)

Bicudo, Marinkovic, Mueller, Wagner, arXiv 2409.10786

Schrödinger Equation:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + V_{\Sigma_g^+}\right]\psi_{\Sigma_g^+} = \mathcal{E}_N\,\psi_{\Sigma_g^+}\,.$$

$l = 0$

Results:

1) T_{cc} state : 320 keV below DD threshold
2) Radius ~ 8 fm or larger.
3) Deeper bound state in bb sector: T_{bb} 116 MeV below BB threshold.
4) Deeper bound state in bc sector: T_{bc} 25 MeV below DB threshold.

Lüscher Method: pion mass 280 MeV: virtual state $9.9^{+3.6}_{-7.1}$ MeV below DD^* threshold

Padmanath & Prelovsek , Phys. Rev. Lett. 129, 032002 (2022)

HALQCD collaboration: pion mass 146 MeV: virtual state $59^{+53}_{-99}{}^{+2}_{-67}$ keV below DD^* threshold

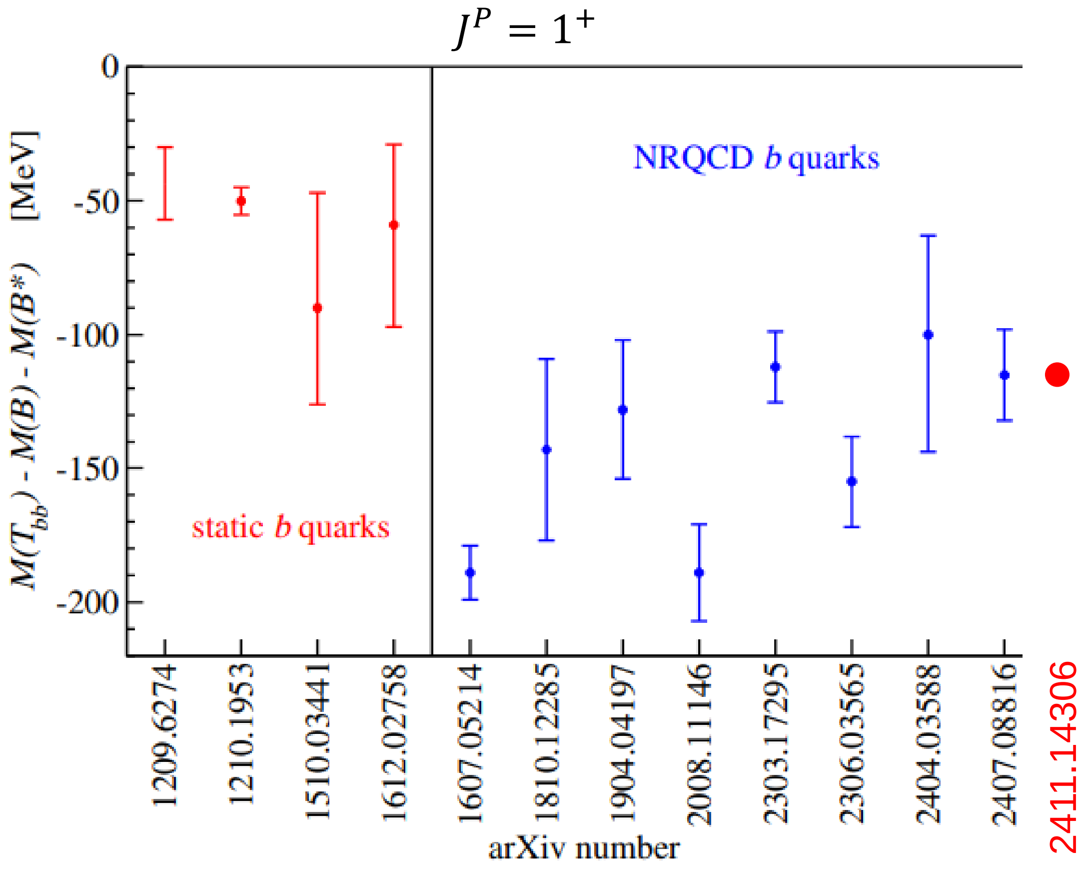
physical pion mass 135 MeV: bound state

Lyu et al, Phys. Rev. Lett. 131, 161901 (2023)

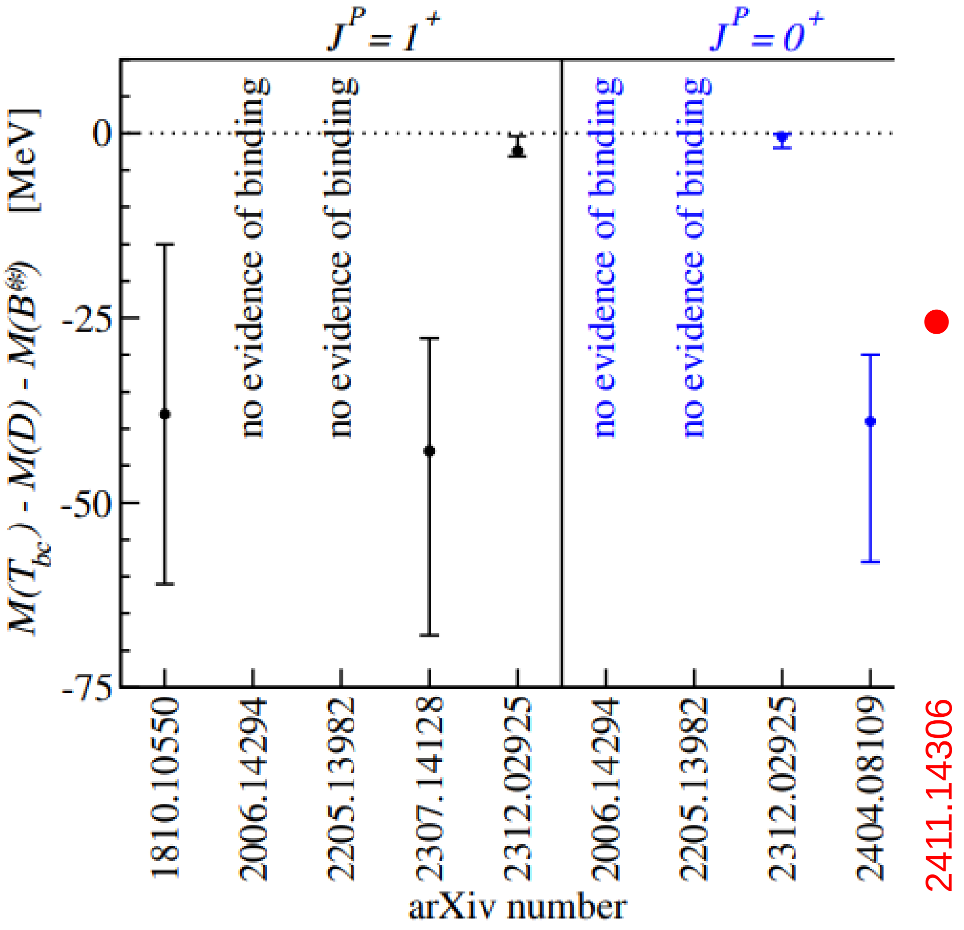
T_{bb} & T_{bc}

Brambilla, AM, Scirpa, Vairo 2411.14306

T_{bb} binding energy comparison:



T_{bc} binding energy comparison:



EFT and heavy-quark-diquark symmetry prediction for T_{bb}: 133 ± 25 MeV

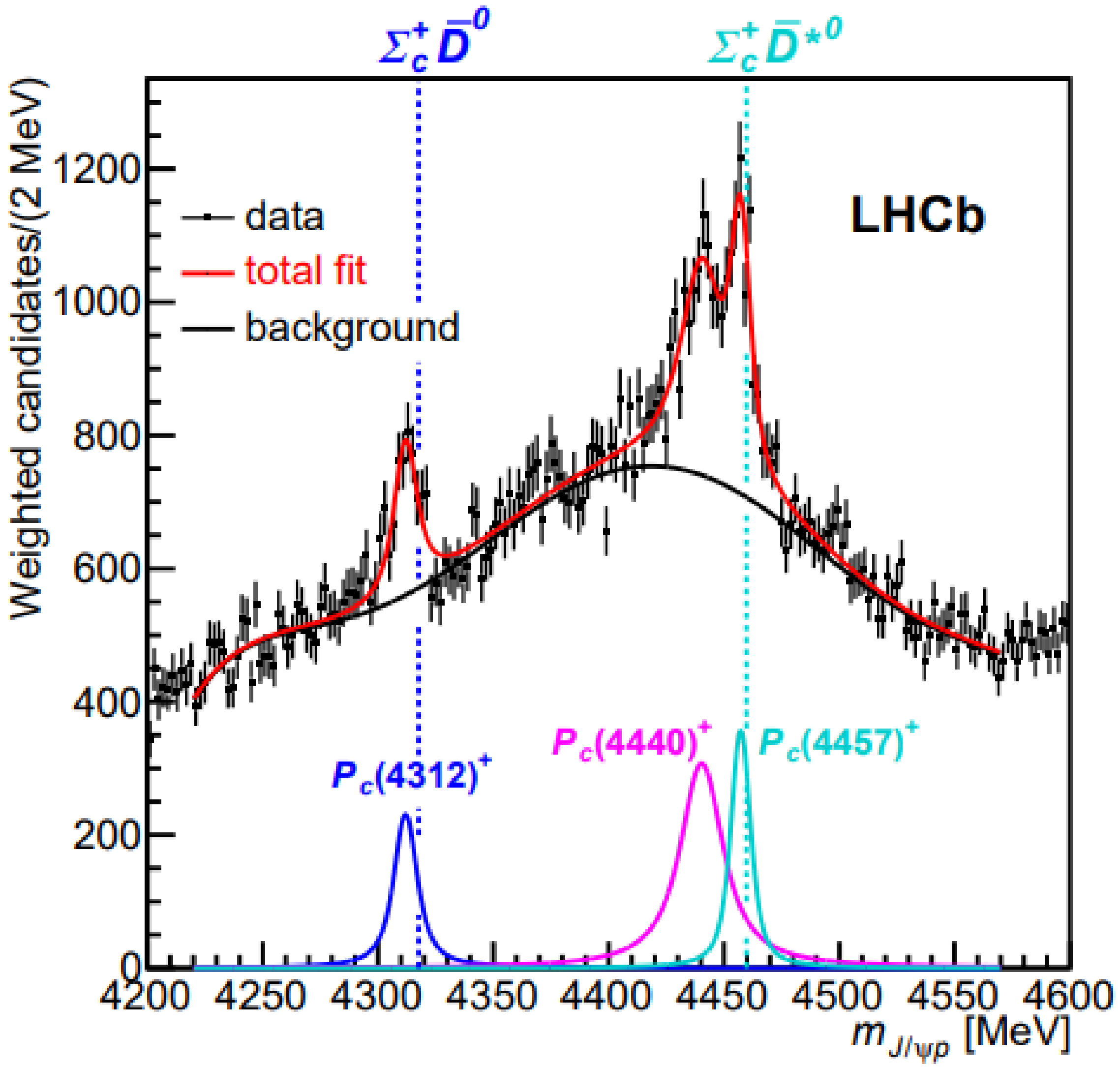
Braaten, He, AM, Phys. Rev. D. 103, 016001 (2021)

Our result 25 MeV for both $J^P = \{0^+, 1^+\}$

Figures from Randy Lewis talk: LepageFest 2024

Pentaquarks

Pentaquark



LHCb, Phys. Rev. Lett. 122, (2019), 222001

Observed states

- **4** states with isospin $\boldsymbol{I = 1/2}$:

$P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$

- **2** states with isospin $\boldsymbol{I = 0}$: $P_{c\bar{c}s}(4338)^0$
  $P_{c\bar{c}s}(4459)^0$

J^P quantum numbers not established except $P_{c\bar{c}s}(4338)^0$

PDG 2025

Brambilla, AM, Vairo, in preparation

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

Pentaquark

TUM

Lowest pentaquark multiplets:

$Q\bar{Q}$ color state	Light spin k^P	BO quantum # $D_{\infty h}$	l	J^P $\{S_Q = 0, S_Q = 1\}$
Octet	$(1/2)^+$	$(1/2)_g$	$1/2$	$\{1/2^-, (1/2, 3/2)^-\}$
$\mathbf{8}$	$(3/2)^+$	$\{(1/2)'_g, (3/2)_g\}$	$3/2$	$\{3/2^-, (1/2, 3/2, 5/2)^-\}$

No lattice QCD results on adjoint baryon mass: $\Lambda_{(1/2)^+}$ and $\Lambda_{(3/2)^+}$

Treat them as free parameter to reproduce $P_{c\bar{c}}$ spectrum.

Coupled-channel Equations $k^P = (1/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{(l-1/2)(l+1/2)}{m_Q r^2} + V_{(1/2)_g}\right]\psi^{(N)}_{(1/2)^+} = \mathcal{E}_{1/2}\,\psi^{(N)}_{(1/2)^+}$$

$l = 1/2$

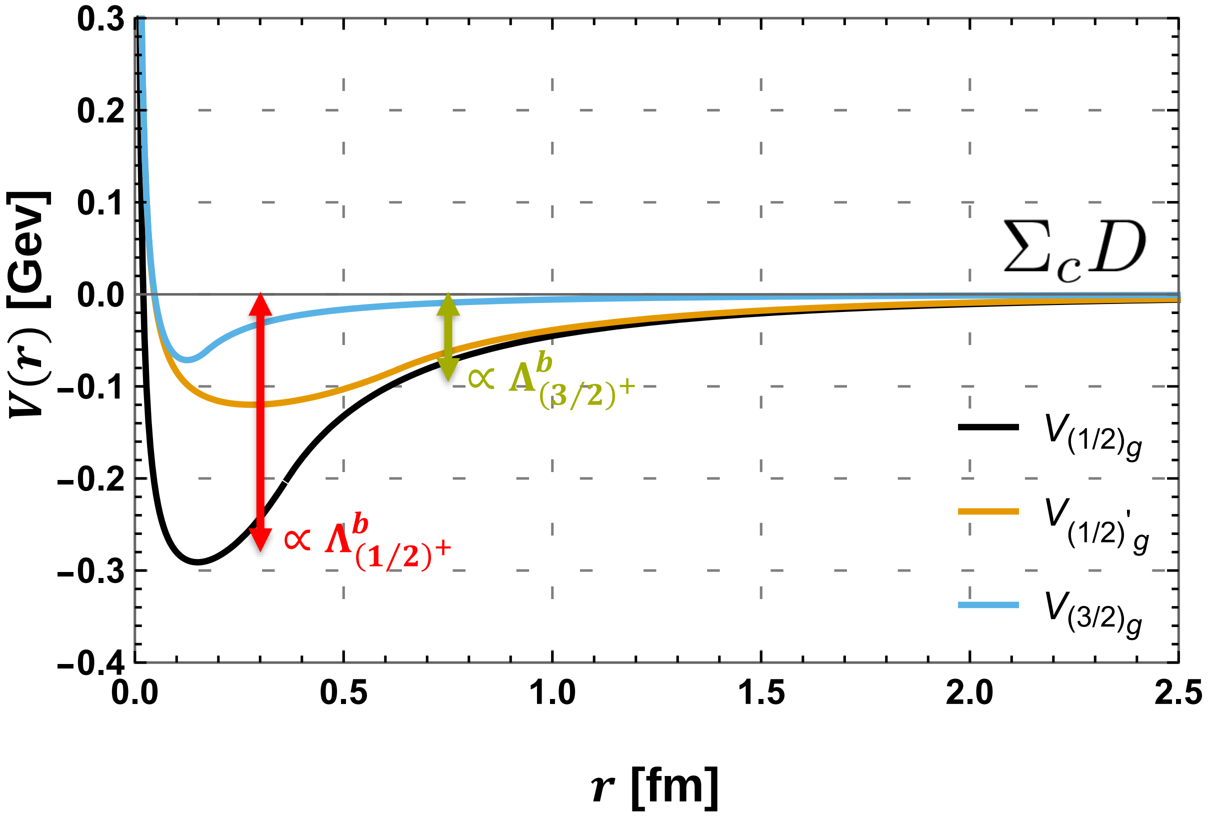
Coupled-channel Equations $k^P = (3/2)^+$:

$$\left[-\frac{1}{m_Q r^2}\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l-1)-\frac{9}{4} & -\sqrt{3l(l+1)-\frac{9}{4}} \\ -\sqrt{3l(l+1)-\frac{9}{4}} & l(l+1)-\frac{3}{4}\end{pmatrix} + \begin{pmatrix} V_{(1/2)'_g} & 0 \\ 0 & V_{(3/2)_g}\end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{1/2} \\ \psi^{(N)}_{3/2}\end{pmatrix} = \mathcal{E}_{3/2}\begin{pmatrix}\psi^{(N)}_{1/2} \\ \psi^{(N)}_{3/2}\end{pmatrix}$$

$l = 3/2$

Pentaquark

Brambilla, AM, Vairo, in preparation

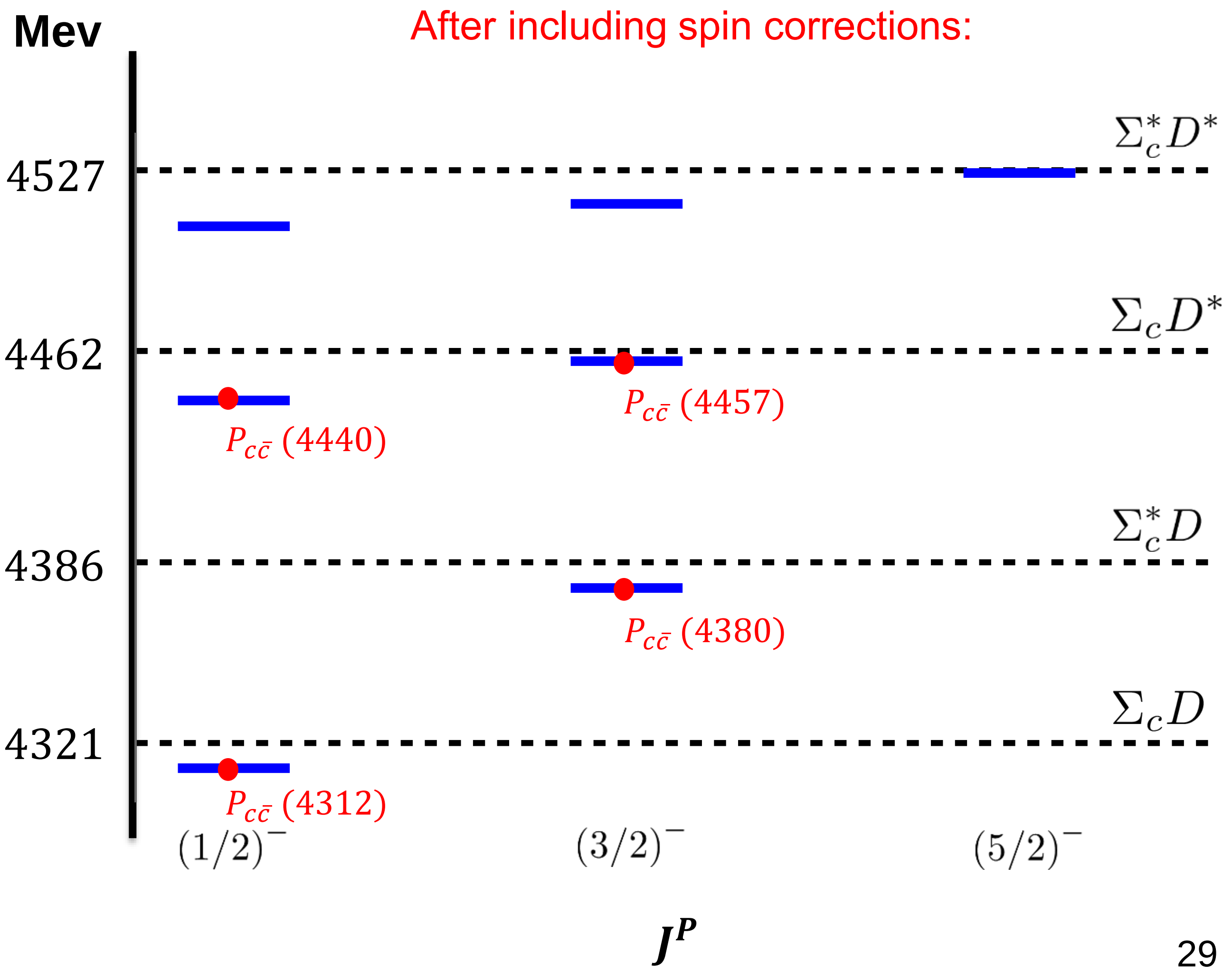


$(qqq)_8$ energy:

$\Lambda^b_{(1/2)^+} \approx -364$ MeV, $\Lambda^b_{(3/2)^+} \approx -159$ MeV

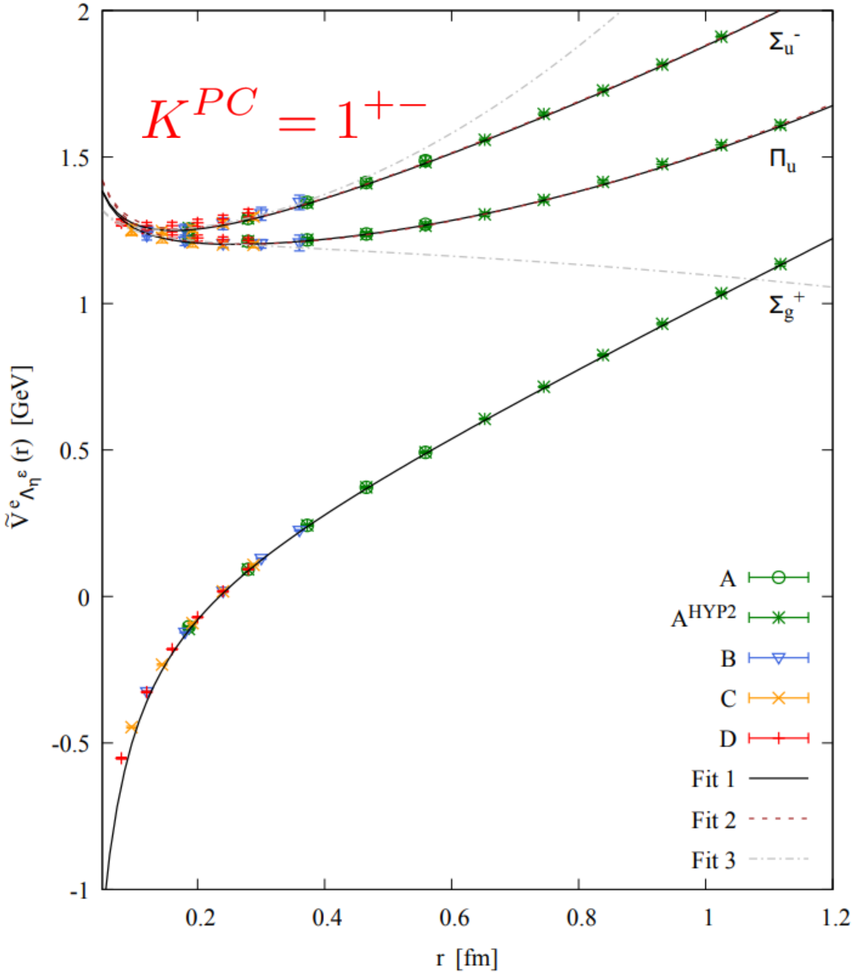
Decay studies to $\Lambda_c - \mathrm{D}^{(*)}$ and $J/\psi + X$ (X: light hadrons) under progress.

After including spin corrections:



Hybrids

BOEFT: Hybrids



Schlosser and Wagner Phys. Rev. D. 105, (2022)

- Coupled Schrödinger Eq:

$$\left[-\frac{1}{m_Q r^2}\,\partial_r r^2 \partial_r + \frac{1}{m_Q r^2}\begin{pmatrix} l(l+1)+2 & -2\sqrt{l(l+1)} \\ -2\sqrt{l(l+1)} & l(l+1)\end{pmatrix} + \begin{pmatrix} E_\Sigma & 0 \\ 0 & E_\Pi \end{pmatrix}\right]\begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix} = \mathcal{E}_N \begin{pmatrix}\psi^{(N)}_{\Sigma,\sigma_P} \\ \psi^{(N)}_{\Pi,\sigma_P}\end{pmatrix}$$

$$\left[-\frac{1}{m_Q r^2}\,\partial_r\, r^2\,\partial_r + \frac{l(l+1)}{m_Q r^2} + E_\Pi\right]\psi^{(N)}_{\Pi,-\sigma_P} = \mathcal{E}_N\,\psi^{(N)}_{\Pi,-\sigma_P}$$

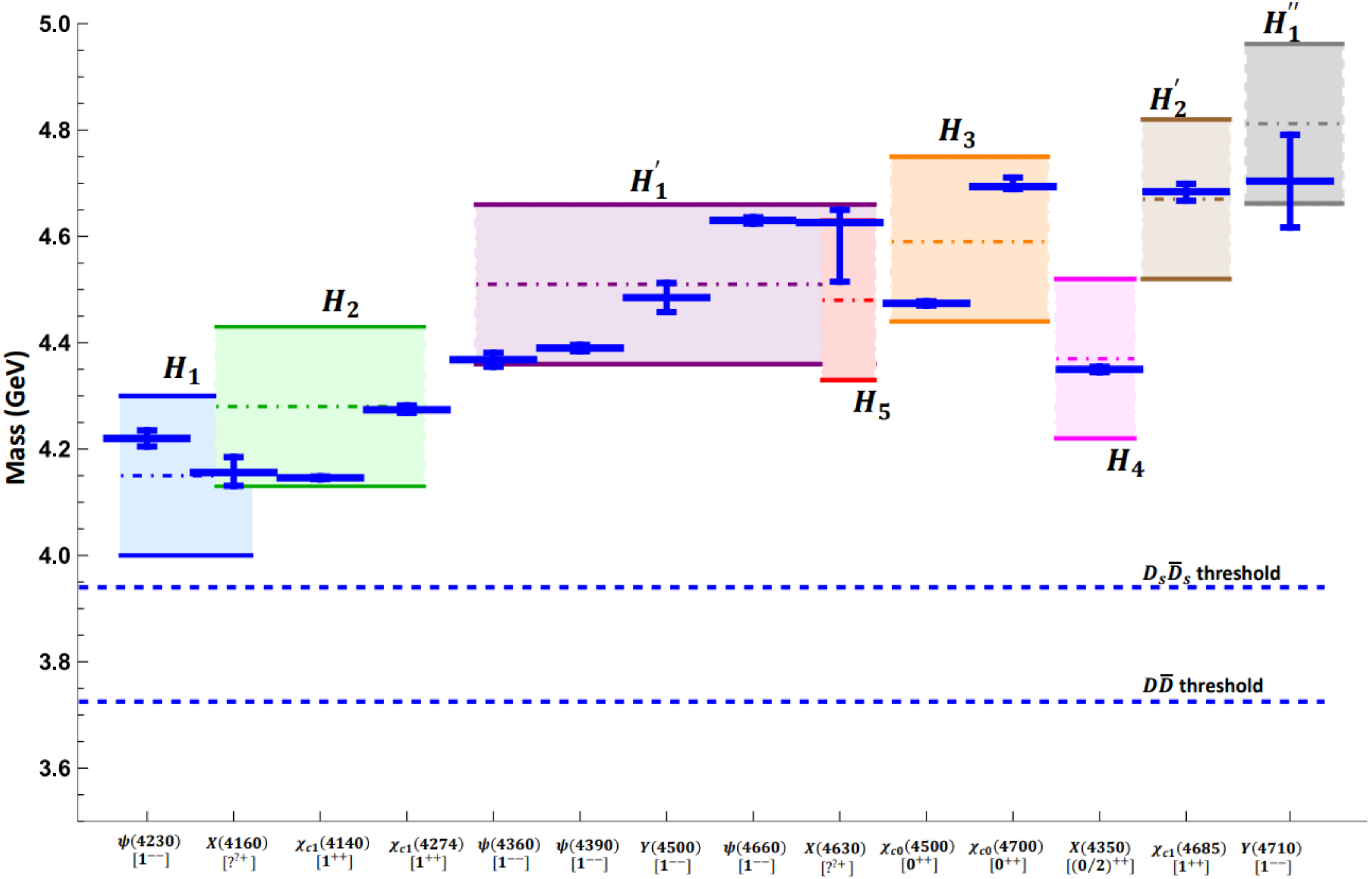
Multiplet	J^{PC}	$M_{c\bar{c}g}$	$M_{b\bar{b}g}$
H_1	$\{1^{--}, (0,1,2)^{-+}\}$	4155	10786
H_1'		4507	10976
H_1''		4812	11172
H_2	$\{1^{++}, (0,1,2)^{+-}\}$	4286	10846
H_2'		4667	11060
H_2''		5035	11270
H_3	$\{0^{++}, 1^{+-}\}$	4590	11065
H_3'		5054	11352
H_3''		5473	11616
H_4	$\{2^{++}, (1,2,3)^{+-}\}$	4367	10897
H_5	$\{2^{--}, (1,2,3)^{-+}\}$	4476	10948

Λ- doubling: opposite parity states non-degenerate.

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

BOEFT: Hybrids

- **Charmonium hybrids**: comparison with experimental results:



	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

PDG 2022

Brambilla, Lai, AM, Vairo

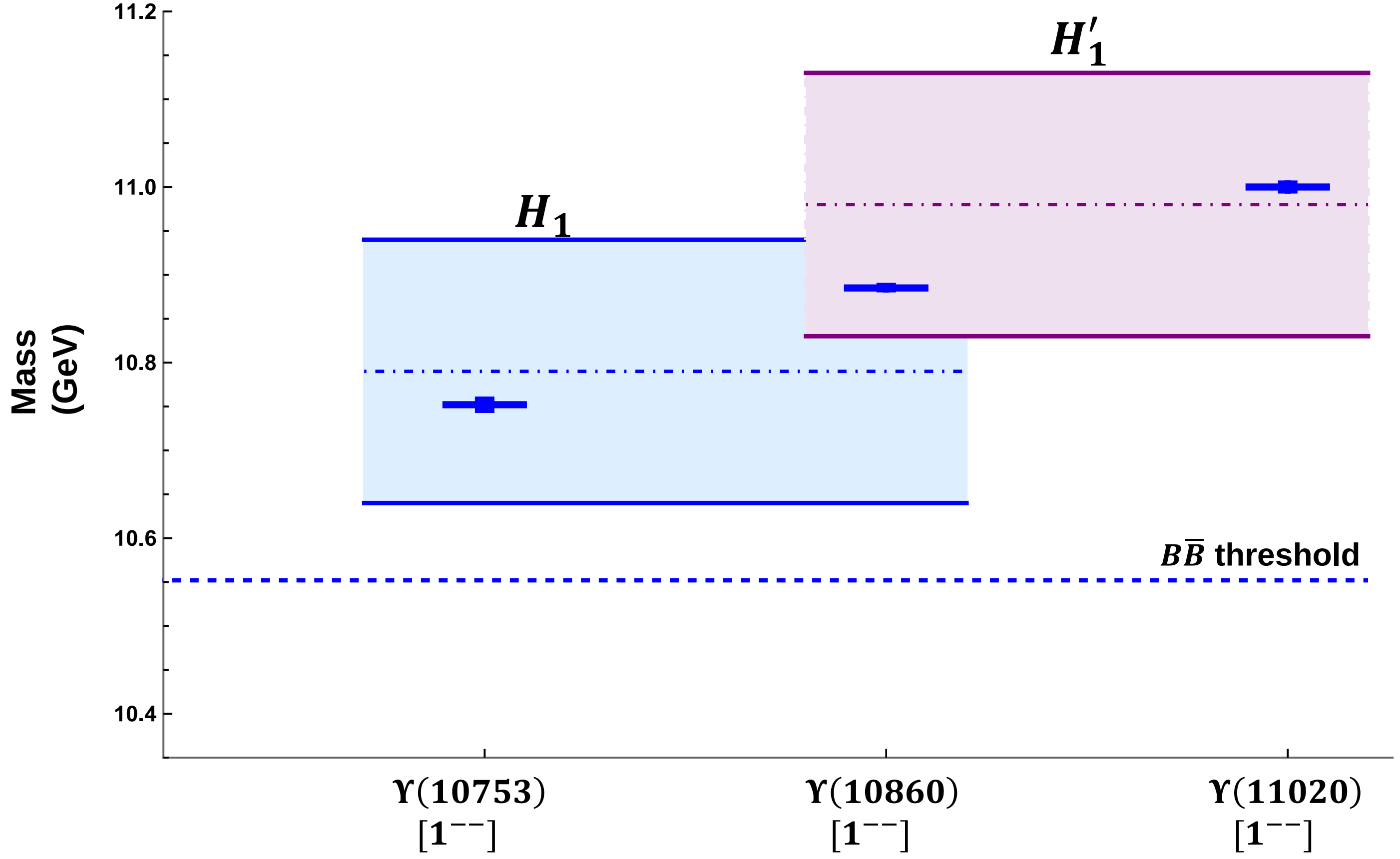
Phys. Rev. D 107, 054034 (2023)

Berwein, Brambilla, Castellà , Vairo

Phys. Rev. D. 92, 114019 (2015)

BOEFT: Hybrids

- **Bottomonium hybrids**: comparison with experimental results:



PDG 2022

	l	$J^{PC}\{s=0, s=1\}$	$E_n^{(0)}$
H_1	1	$\{1^{--}, (0,1,2)^{-+}\}$	Σ_u^-, Π_u
H_2	1	$\{1^{++}, (0,1,2)^{+-}\}$	Π_u
H_3	0	$\{0^{++}, 1^{+-}\}$	Σ_u^-
H_4	2	$\{2^{++}, (1,2,3)^{+-}\}$	Σ_u^-, Π_u
H_5	2	$\{2^{--}, (1,2,3)^{-+}\}$	Π_u

Brambilla, Lai, AM, Vairo
Phys. Rev. D 107, 054034 (2023)

Berwein, Brambilla, Castellà , Vairo
Phys. Rev. D. 92, 114019 (2015)

Hybrid Decays

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

- Spin-conserving decay due to $\boldsymbol{r}\cdot\boldsymbol{E}$ term :

$|S_H = 1> \dashrightarrow |S_Q = 1>$
$|S_H = 0> \dashrightarrow |S_Q = 0>$

$$\Gamma(H_m \to Q_n) = \frac{4\alpha_s(\Delta E)\,T_F}{3N_c}\, T^{ij}\,(T^{ij})^\dagger\,\Delta E^3$$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.

$$T^{ij} \equiv \langle H_m | r^j | Q_n\rangle = \int d^3\mathbf{r}\, \Psi^{i\dagger}_{(m)}(\mathbf{r})\, r^j\, \Phi^{Q\bar{Q}}_{(n)}(\mathbf{r})$$

$$\langle H_m|\boldsymbol{r}|Q_n\rangle = \sqrt{T^{ij}(T^{ij})^\dagger}$$

$\Psi^i_{(m)}$: Hybrid wf
Φ^Q_n : Quarkonium wf

R. Oncala, J. Soto, Phys. Rev. D96, 014004 (2017).

J. Castellà, E. Passemar, Phys. Rev. D104, 034019 (2021)

- Spin-flipping decay due to $\boldsymbol{S}.\boldsymbol{B}$ term:

$|\chi_H\rangle$: Hybrid spin wf
$|\chi_Q\rangle$: Quarkonium spin wf

$|S_H = 1> \dashrightarrow |S_Q = 0>$
$|S_H = 0> \dashrightarrow |S_Q = 1>$

$$T^{ij} \equiv \langle H_m|\left(S_1^j - S_2^j\right)|Q_n\rangle = \left[\int d^3\mathbf{r}\,\Psi^{i\dagger}_{(m)}(\mathbf{r})\,\Phi^Q_{(n)}(\mathbf{r})\right]\langle\chi_H|\left(S_1^j - S_2^j\right)|\chi_Q\rangle$$

Depends on overlap of quarkonium and hybrid wavefunctions.

Semi-inclusive Hybrid-to-Quarkonium transition decay rate
= spin-conserving + spin-flipping decay rates.

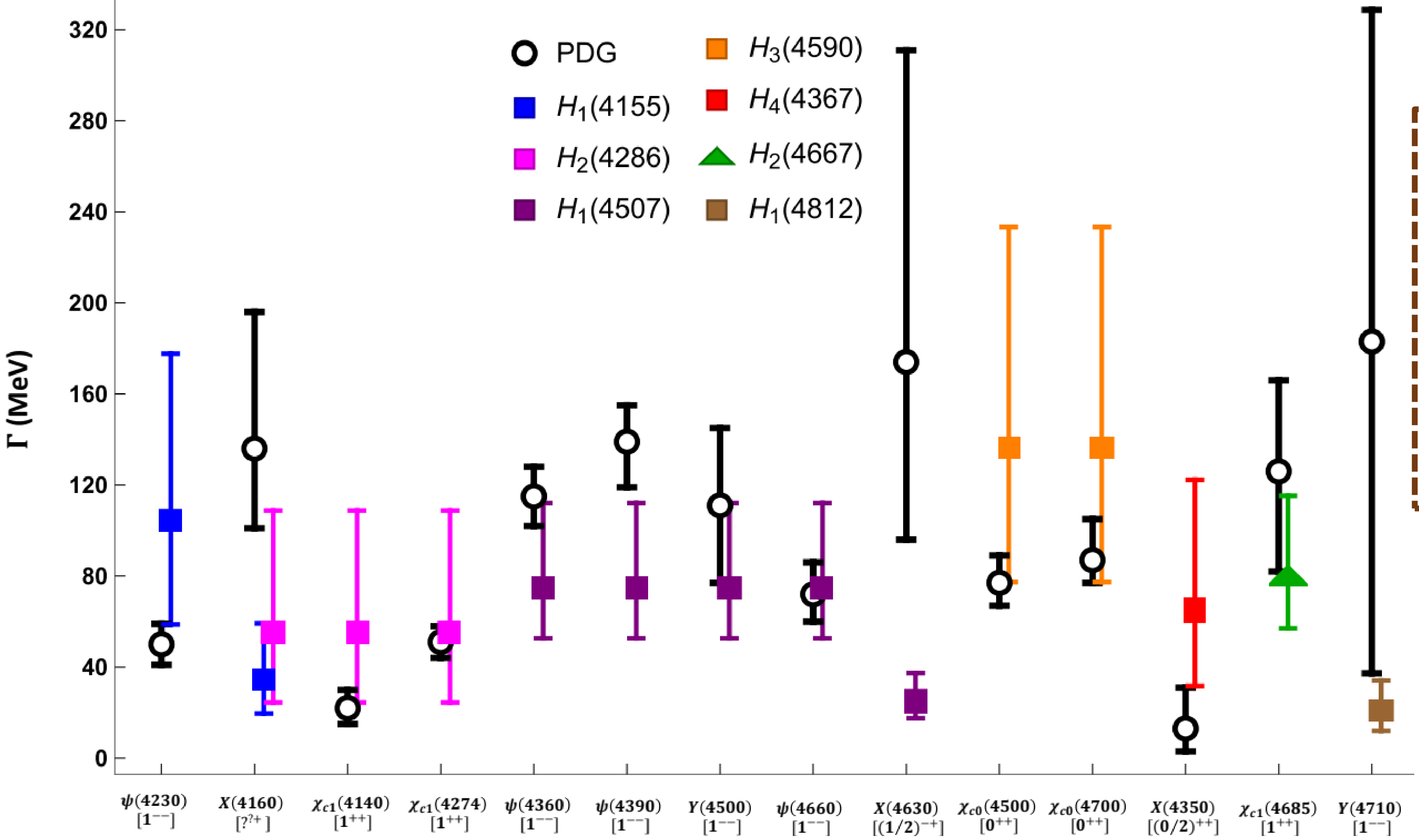
Our estimate of decay rate are lower-bounds for the **total width of hybrids**

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

Semi-inclusive process:

$H_m \to Q_n + X$; Q_n: low-lying quarkonium (states below threshold) & X: light hadrons.



$H_m \to Q_n + X$

spin-conserving + spin-flipping decays

=

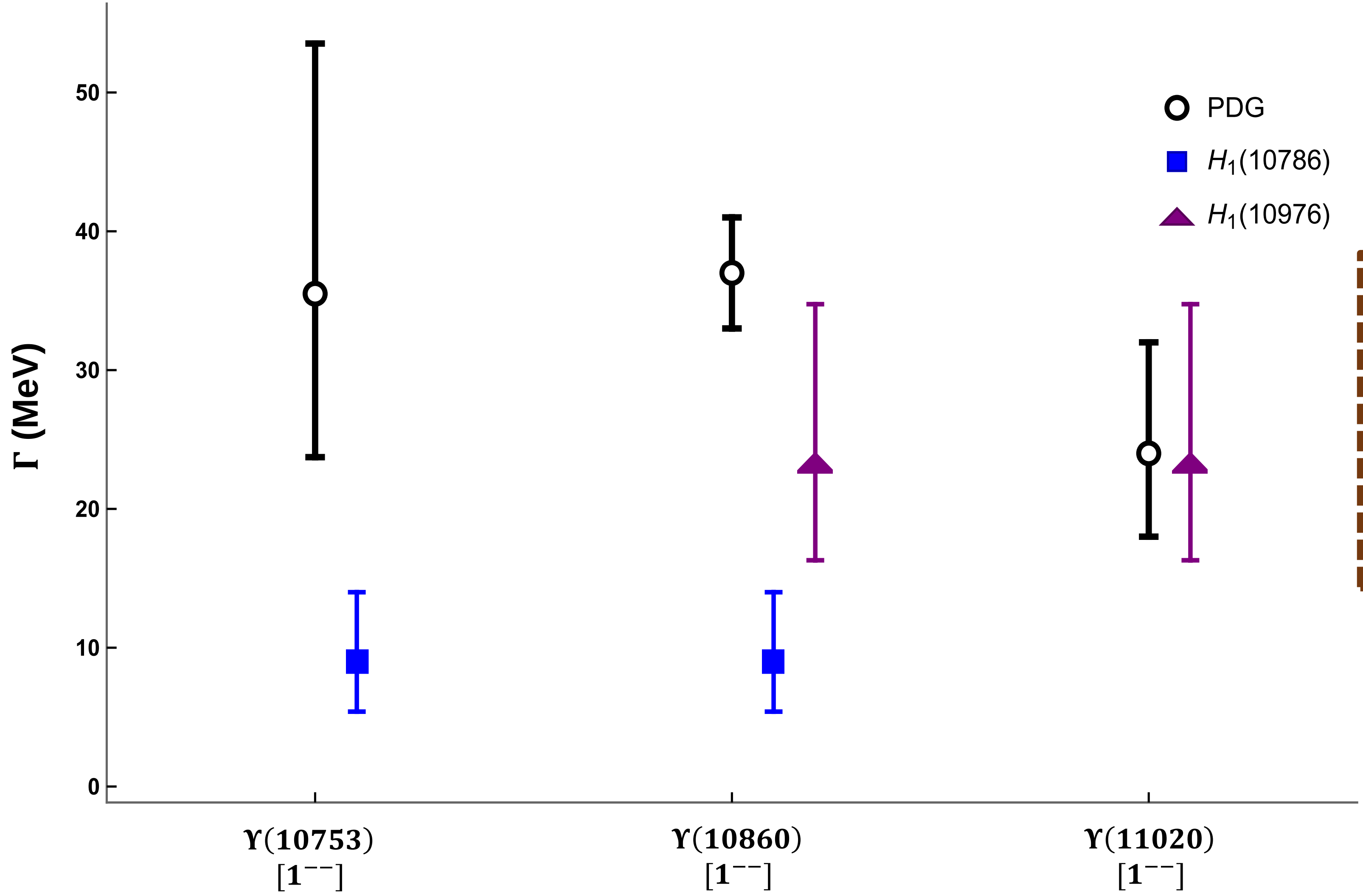
lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Results

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

Semi-inclusive process:

$\boldsymbol{H_m \rightarrow Q_n + X}$; Q_n: low-lying quarkonium (states below threshold) & X: light hadrons.



$\boldsymbol{H_m \rightarrow Q_n + X}$

spin-conserving + **spin-flipping decays**

=

lower bound on the total decay widths of hybrids which is compared with inclusive rate of physical states in PDG.

Disclaimer

$\Upsilon(nS)$+Dipion transition rules out pure hybrid interpretation for $\Upsilon(10753)$

Belle II, JHEP 07, 116 (2024) (2023)

Hybrid: Mixing with heavy-light

- **Hybrid decays to s-wave + s-wave meson pairs:**

Conventional Wisdom: Hybrid decays to two S-wave mesons forbidden!

Kou & Pene, Phys Lett B 631 (2005) Page, Phys Lett B 407 (1997) Farina, Tecocoatzi, Giachino, Santopinto & Swanson, Phys Rev D 102 (2020)

Decay allowed based on BO-quantum #

Bruschini Phys. Rev. D 109 L031501 (2024)
J. Castella JHEP 06, 107 (2024)

Hybrid

Light spin K^{PC}	Static energies $D_{\infty h}$	l	J^{PC} $\{S_Q=0, S_Q=1\}$	Multiplets
1^{+-}	$\{\Sigma_u^-, \Pi_u\}$	1	$\{1^{--}, (0,1,2)^{-+}\}$	H_1
	$\{\Pi_u\}$	1	$\{1^{++}, (0,1,2)^{+-}\}$	H_2
	$\{\Sigma_u^-\}$	0	$\{0^{++}, 1^{+-}\}$	H_3
	$\{\Sigma_u^-, \Pi_u\}$	2	$\{2^{++}, (1,2,3)^{+-}\}$	H_4
	$\{\Pi_u\}$	2	$\{2^{--}, (1,2,3)^{-+}\}$	H_5

BO-quantum # Λ_η^σ for threshold

$K_{\bar{q}}^P \otimes K_q^P$	K^{PC}	Static energies $D_{\infty h}$
$(1/2)^- \otimes (1/2)^+$	0^{-+}	$\{\Sigma_u^-\}$
	1^{--}	$\{\Sigma_g^+, \Pi_g\}$

s-wave+s-wave
Ex. $\mathbf{D\bar{D}}$ threshold

Σ_u^- component in hybrids couple with Σ_u^- component in s-wave+s-wave !!!!

Hybrid: Decays to heavy-light

Multiplet	J^{PC}		Potential	
H_1	1^{--}	$(0,1,2)^{-+}$	$\Pi_u \& \Sigma_u^-$	allowed
H_2	1^{++}	$(0,1,2)^{+-}$	Π_u	forbidden
H_3	0^{++}	1^{+-}	Σ_u^-	allowed
H_4	2^{++}	$(1,2,3)^{+-}$	$\Pi_u \& \Sigma_u^-$	allowed
H_5	2^{--}	$(1,2,3)^{-+}$	Π_u	forbidden

Taken from R. Bruschini talk: Exotic hadron spectroscopy 2024, Swansea

Bruschini Phys. Rev. D 109 L031501 (2024)

Recent lattice computation for $c\bar{c}$ hybrid 1^{-+} **decay** to

$$D_1\bar{D} : 258(133)\ \text{MeV}$$

$$D^*\bar{D} : 88(18)\ \text{MeV}$$

$$D^*\bar{D}^* : 150(118)\ \text{MeV}$$

Shi et al. Phys. Rev. D 109, 094513 (2024)

Summary/Outlook

- Born-Oppenheimer EFT: Tool based on QCD and Born-Oppenheimer approximation to study Exotic states.

- Behavior of tetraquark / pentaquark static energies (Lattice needs to confirm the small ***r*** behavior!):

  - $Q\bar{Q}$ **systems: Repulsive** behavior at small r $(\boldsymbol{r \to 0})$**.** Avoided crossing quarkonium static energy (isospin=0).
  - QQ **systems: Attractive (**Triplet) or repulsive (sextet) behavior at small r $(\boldsymbol{r \to 0})$**.**
  - Heavy meson pair or heavy meson baryon threshold at large r $(\boldsymbol{r \to \infty})$**.**
  - $Q\bar{Q}$ systems: Avoided crossing between tetraquark and quarkonium static energy (Isospin=0).

- New results regarding $\chi_{c1}(3872)$, $T_{cc}^{+}(3875)$ and $P_{c\bar{c}}$ pentaquark states.

- Extend BOEFT approach for production studies:
  1) understanding hadro-production of $\chi_{c1}(3872)$, and $T_{cc}^{+}(3875)$.
  2) photo-production studies relevant for EIC collider, Glue X. More difficult than hadro-production studies !!!

What is an XYZ Meson ???

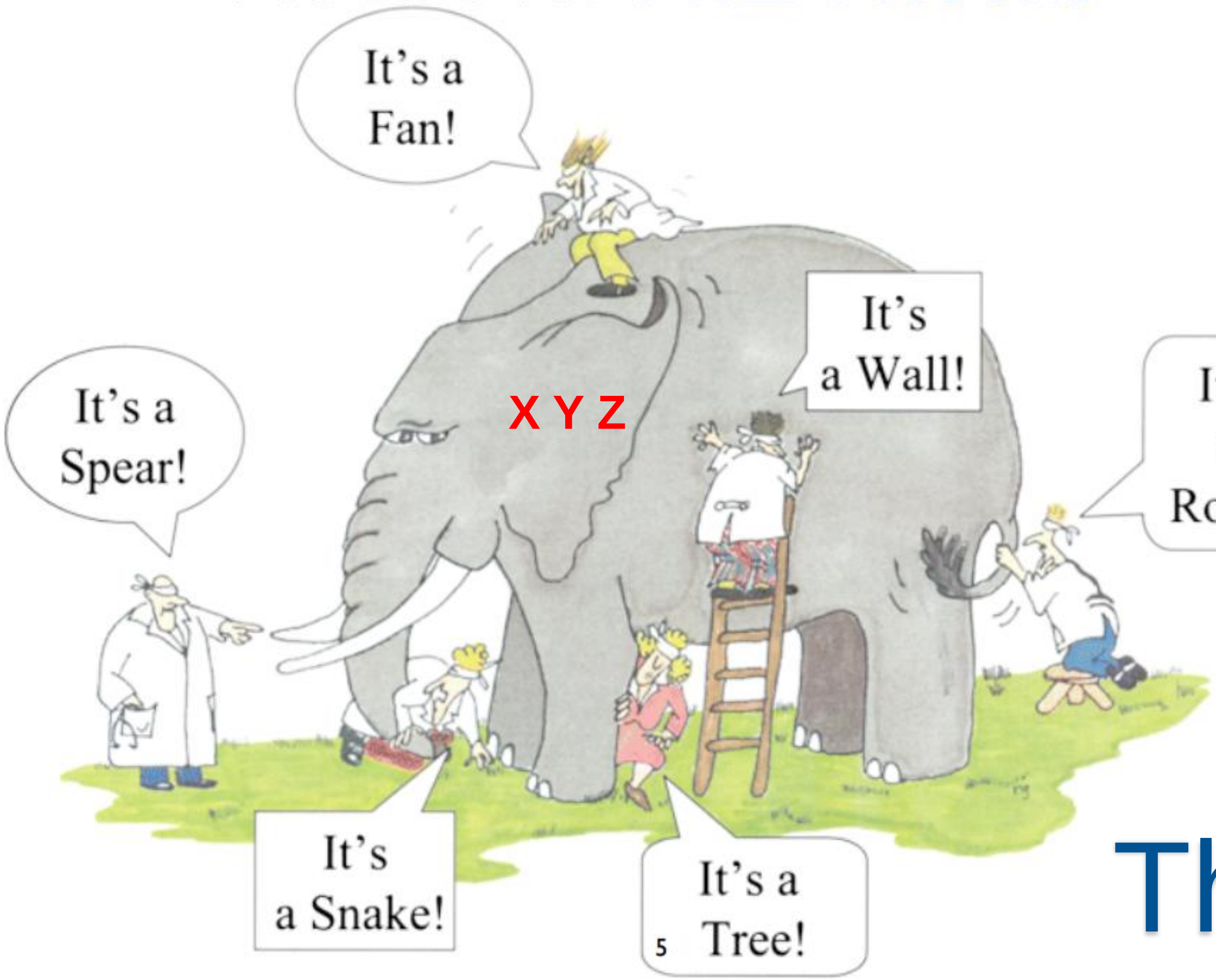


Figure from Eric Braaten talk:

Charm 2020 conference

Thank you!!

Perhaps Born-Oppenheimer can address whole pattern !!! .

Backup Slides

BOEFT

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

- Radial Schrödinger equation:

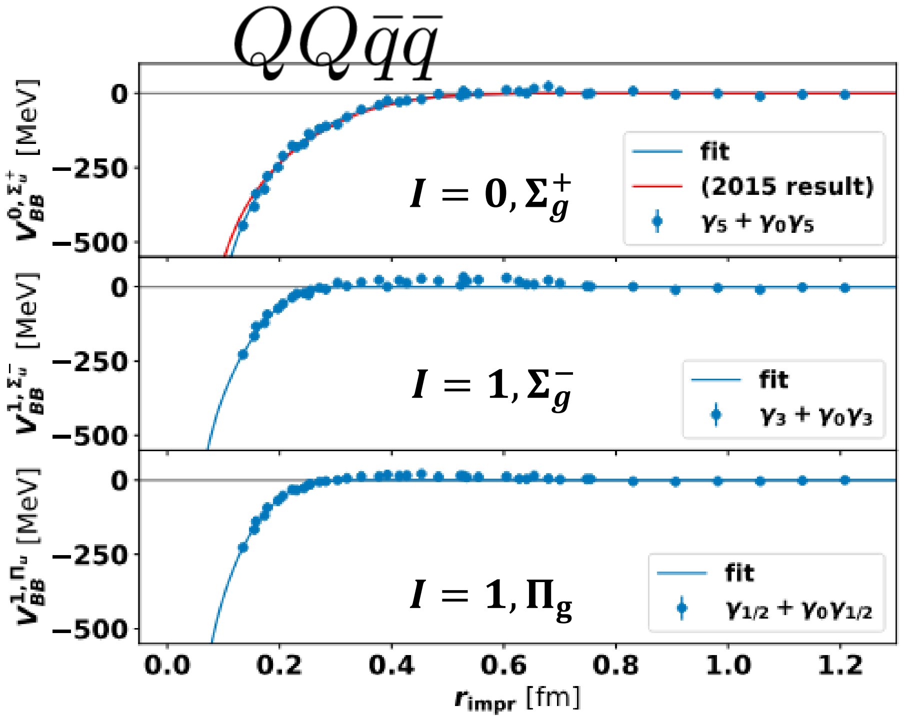
  Mixing different static energies with same LDF-quantum #: $\kappa = \{K^{PC}, f\}$

$$\sum_{\lambda}\left[-\frac{1}{m_Q r^2}\partial_r\, r^2\partial_r + \frac{1}{m_Q r^2}\boxed{M_{\lambda'\lambda}} + E^{(0)}_{\kappa,|\lambda|}(r)\delta_{\lambda\lambda'}\right]\psi^{(N)}_{\kappa\lambda}(r) = \mathcal{E}_N\,\psi^{(N)}_{\kappa\lambda'}(r)$$

Mixing term $M_{\lambda'\lambda}$: from angular momentum piece:
Coupling static states with different BO-quantum numbers Λ^{σ}_{η}

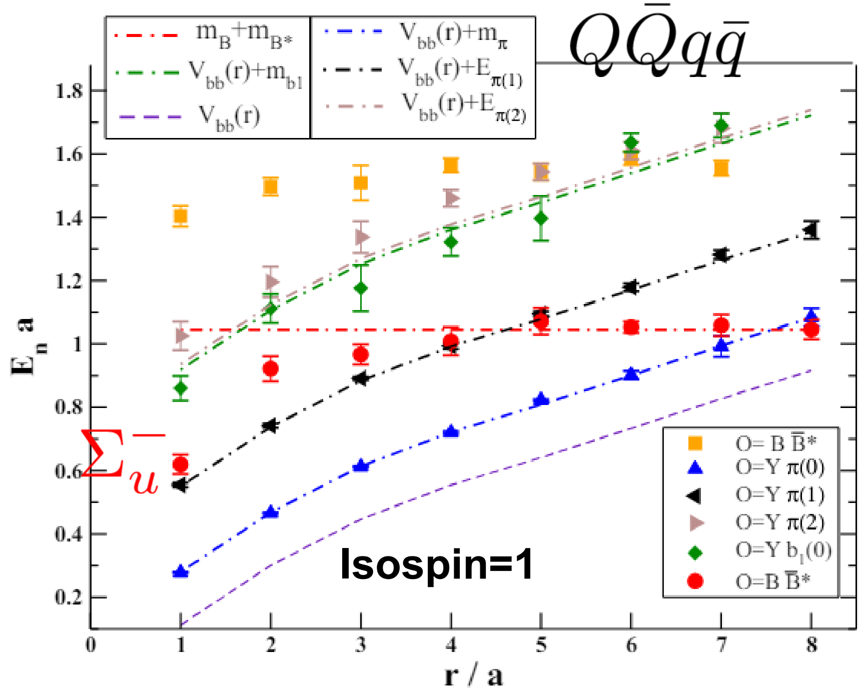
Mixing term $M_{\lambda'\lambda}$: Angular momentum L^2 spherically symmetric
but static states have cylindrical symmetry

BO potentials



Mueller et al, PoS LATTICE2023, 64 (2024)

Bicudo, Cichy, Peters, Wagner, Phys. Rev. D. 93, (2016)



Prelovsek, Bahtiyar, Petkovic, Phys. Lett. B. 805, (2020)

Tetraquark / pentaquark BO potentials:

Quark configurations can be rearranged to have two hadron state.

Similarity with molecular physics. Molecules going to constituent atoms when internuclear separation very large.

$T_{cc}^{+}(3875)$

For illustration purpose, we model $V_{\Sigma_g^+}$ considering short-distance behavior from [75] and long-distance behavior with a two-pion exchange potential [76]

$$V_{\Sigma_g^+} = \begin{cases} \frac{\kappa_3}{r} + E_{0^+} + A_{\Sigma_g^+} r^2 & r < R_{\Sigma_g^+} \\ F_{\Sigma_g^+} e^{-r/d}/r^2 & r > R_{\Sigma_g^+} . \end{cases} \quad (7)$$

where $\kappa_3 = -0.120$ and $A_{\Sigma_g^+} = 0.197\,\mathrm{GeV}^3$ [75], the parameters $F_{\Sigma_g^+}$ and $R_{\Sigma_g^+}$ are determined by imposing continuity up to first derivatives. We treat 0^+ triplet meson energy E_{0^+} as free parameter to obtain $T_{cc}^{+}(3875)$ state.

Brambilla, AM, Scirpa, Vairo 2411.xxxxx

χ_{c1} (3872)

We use the lattice parametrization (where energy levels are normalized with respect to twice the energy of the static heavy-light pair $E_{Q\bar{l}}$) in [71] for $V_{\Sigma_g^+}$ across all r. For $V_{\Sigma_g^{+\prime}}$ and V_{Π_g}, we model the short-distance behavior using the quenched BO-potential parametrization from [75] due to lack of lattice computation, long-distance behavior with a two-pion exchange potential [76], and the asymptotic limit ($r \to \infty$) with a constant $E_1 = 0.005$ GeV as in [71]:

$$V_{\Sigma_g^+}(r) = V_0 + \frac{\gamma}{r} + \sigma r, \tag{2}$$

$$V_\Lambda(r) = \begin{cases} \frac{\kappa_8}{r} + E_1 + A_\Lambda r^2 + B_\Lambda r^4 & r < R_\Lambda \\ F_\Lambda e^{-r/d}/r^2 + E_1 & r > R_\Lambda . \end{cases} \tag{3}$$

where $\Lambda \equiv \{\Sigma_g^{+\prime}, \Pi_g\}$, $\gamma = -0.434$, $\sigma = 0.198\,\mathrm{GeV}^2$, $\kappa_8 = 0.037$, $A_{\Sigma_g^{+\prime}} = 0.0065\,\mathrm{GeV}^3$, $B_{\Sigma_g^{+\prime}} = 0.0018\,\mathrm{GeV}^5$, $A_{\Pi_g} = 0.0726\,\mathrm{GeV}^3$, $B_{\Pi_g} = -0.0051\,\mathrm{GeV}^5$, $d \sim 1/(2m_\pi) \sim 1/0.3\,\mathrm{GeV}^{-1} \sim 0.65\,\mathrm{fm}$ and parameters F_Λ and R_Λ are determined by imposing continuity up to first derivatives. The constant $V_0 = -1.142$ GeV is interpreted as $-2E_{Q\bar{l}}$. For $V_{\Sigma_g^+-\Sigma_g^{+\prime}}$, it must vanish as $r \to 0$ based on pNRQCD [77], and approach zero asymptotically as $r \to \infty$, with a peak near the string-breaking region[4]. Hence, we parametrize $V_{\Sigma_g^+-\Sigma_g^{+\prime}}$ as

$$V_{\Sigma_g^+-\Sigma_g^{+\prime}} = \begin{cases} \frac{g}{r_1} r & r < r_1 \\ g & r_1 \le r \le r_2 \\ A \exp(-r/r_0) & r > r_2 , \end{cases} \tag{4}$$

where the parameters $g = 0.05$ GeV, $r_1 = 0.95$ fm, and $r_2 = 1.51$ fm are fixed considering the lattice data in [71], $r_0 = 0.5$ fm is the Sommer scale, $A = 1.02$ GeV has been fixed by demanding the continuity of the potential at r_2.

Brambilla, AM, Scirpa, Vairo 2411.xxxxx

BOEFT: Lattice Operators

Berwein, Brambilla, AM, Vairo, Phys. Rev. D. 110, (2024), 094040

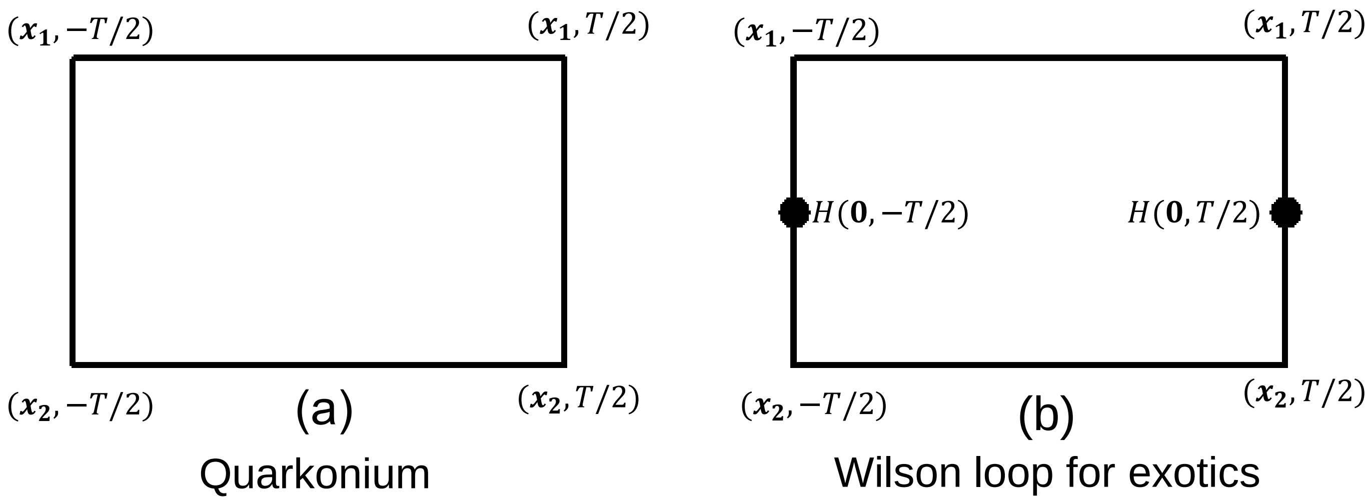
NRQCD operator (gauge invariant) for exotic hadron $Q\bar{Q}X$ or QQX :

$$\mathcal{O}_{\kappa,\lambda}(t,\boldsymbol{r}) = \chi^\dagger(t,\boldsymbol{r}/2)\,\phi(t;\boldsymbol{r}/2,\boldsymbol{0})\,P^{\alpha\dagger}_{\kappa,\lambda}H^\alpha_\kappa(t,\boldsymbol{0})\,\phi(t;\boldsymbol{0},-\boldsymbol{r}/2)\,\psi(t,-\boldsymbol{r}/2)$$

H^α_κ : LDF (gluon or light-quarks) operator characterizing X based on quantum # κ (isospin, color etc..)

$P^\alpha_{\kappa,\lambda}$: Projection vectors for projecting onto cyclindrical symmetry $D_{\infty h}$ representations.

$$E^{(0)}_{\kappa,|\lambda|}(r) = \lim_{T\to\infty}\frac{i}{T}\log\left[\langle \text{vac}|\mathcal{O}_{\kappa,\lambda}(T/2,\,\boldsymbol{r},\,\boldsymbol{R})\,\mathcal{O}^\dagger_{\kappa,\lambda}(-T/2,\,\boldsymbol{r},\,\boldsymbol{R})|\text{vac}\rangle\right]$$

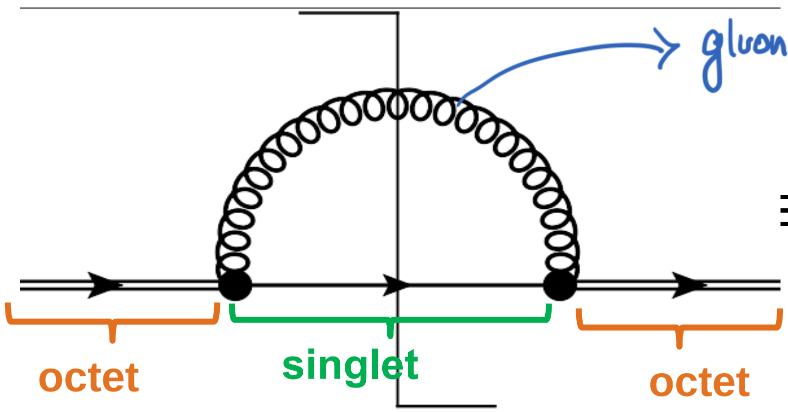


Hybrid Decays

Brambilla, Lai, AM, Vairo Phys. Rev. D 107, 054034 (2023)

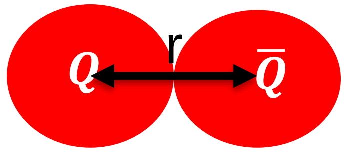
- BOEFT can describe decays of hybrids to quarkonium.
- Semi-inclusive process: $H_m \to Q_n + X$; Q_n: low-lying quarkonium (states below threshold) & X: light hadrons.
  - ✓ ΔE: Large energy difference ⇒ $\Delta E \equiv E_{H_m} - E_{Q_n} \gtrsim 1$ GeV.
  - ✓ Constituent gluon of the hybrid is a spectator.
  - ✓ Hierarchy of scales: $\Delta E \gg \Lambda_{QCD} \gg mv^2$ (ΔE ≫ Λ_QCD: Perturbative computation)

matching **pNRQCD** and **BOEFT:**



≡ Im

Virtual gluon resolves color structure of $Q\bar{Q}$ pair ($\mathbf{r} \to \mathbf{0}$) in quarkonium and hybrid in short-distance limit



Quarkonium ---> Singlet

Hybrid ---> Octet

- Decays are computed from local imaginary terms in the hybrid potential (BOEFT potential).

Optical theorem: $\sum_n \Gamma(H_m \to Q_n) = -2\,\mathrm{Im}\,\langle H_m|V|H_m\rangle$

DISCLAIMER!!!
Decay to open-flavor threshold states not accounted here.