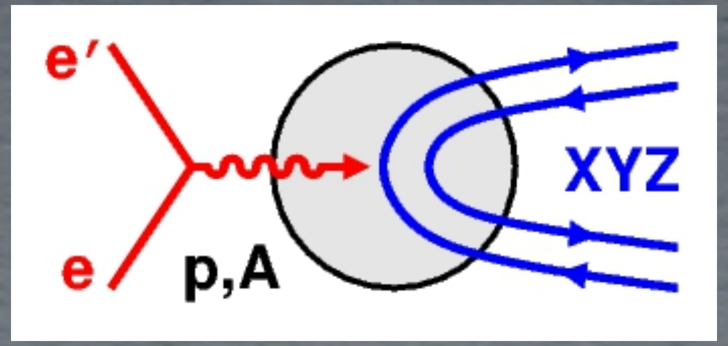


CFNS, Stony Brook

Apr, 2025

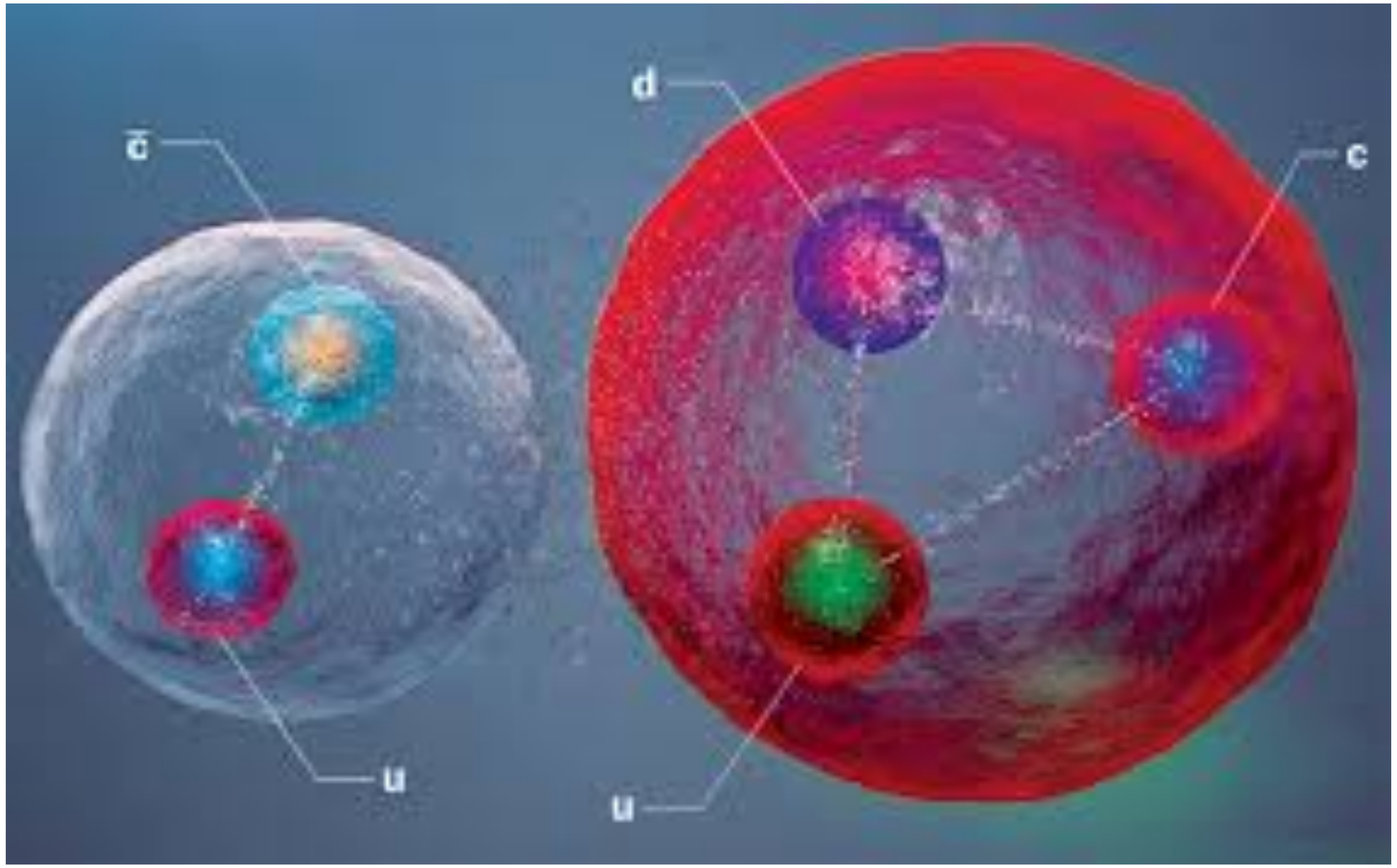
Heavy Exotic States & K-Matrix Analysis at the EIC

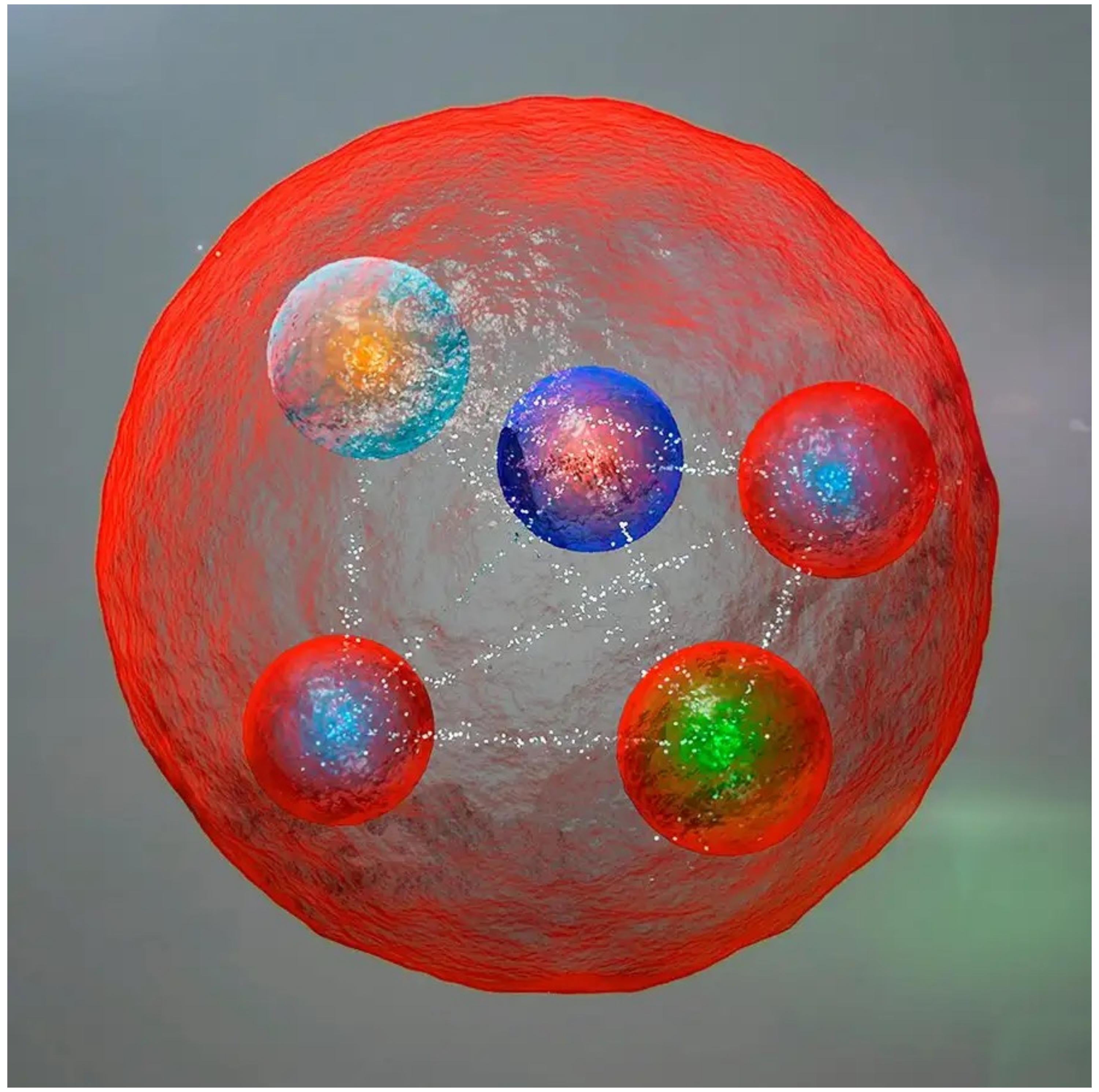
Eric Swanson

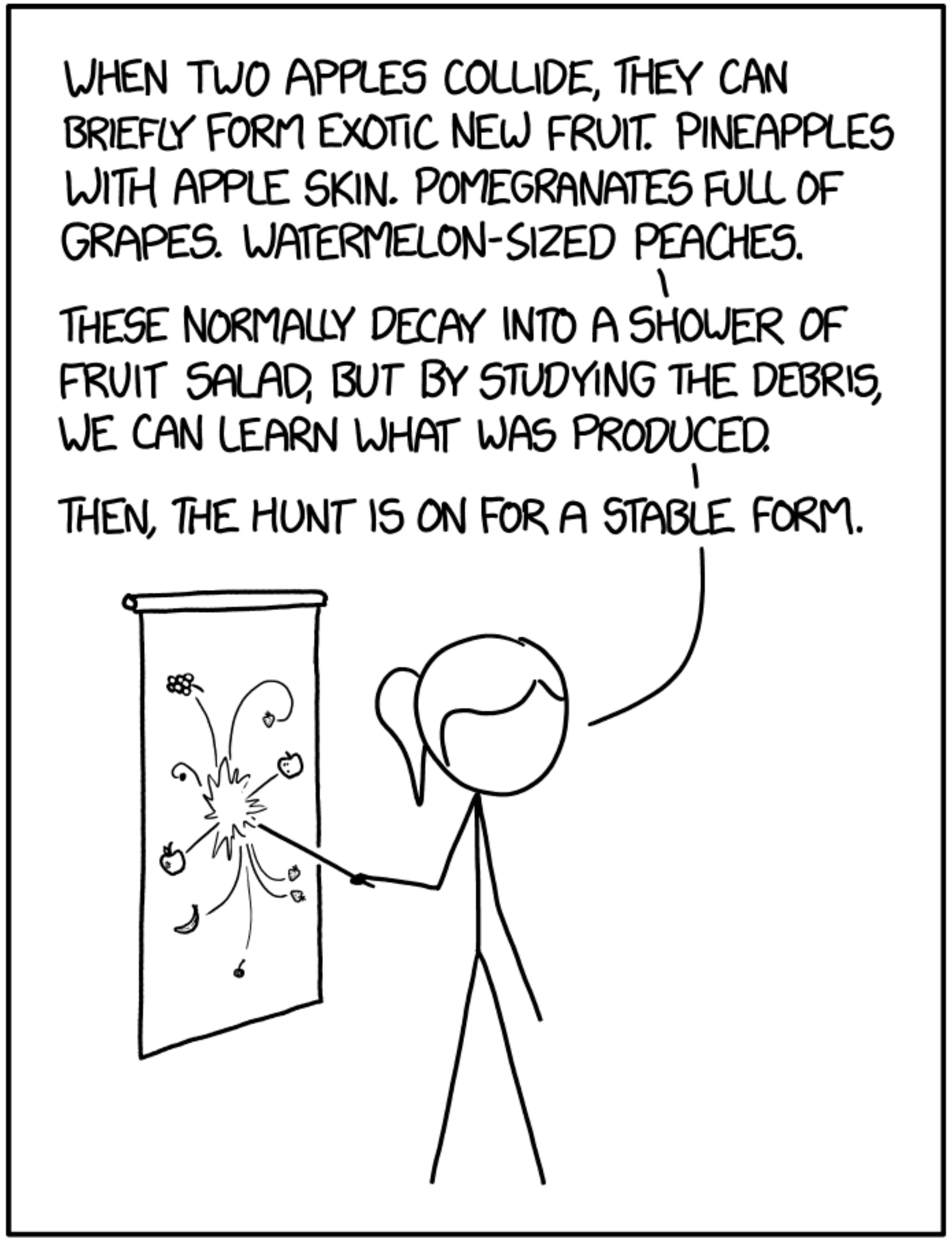
ExoHad
Exotic Hadrons Topical Collaboration

University of Pittsburgh

Heavy Exotic States



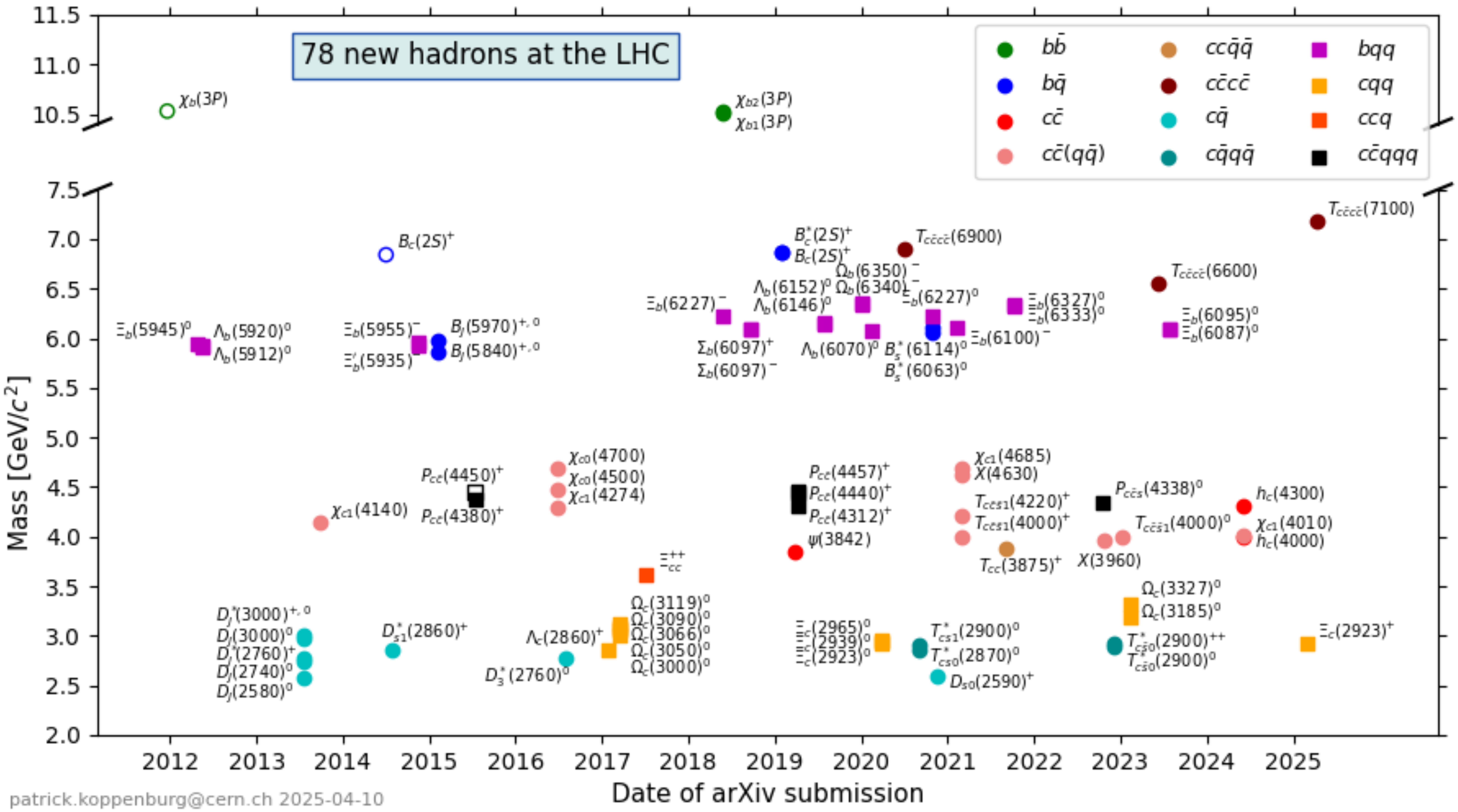




HOW NEW TYPES OF FRUIT ARE DEVELOPED

h/t Randall Munroe

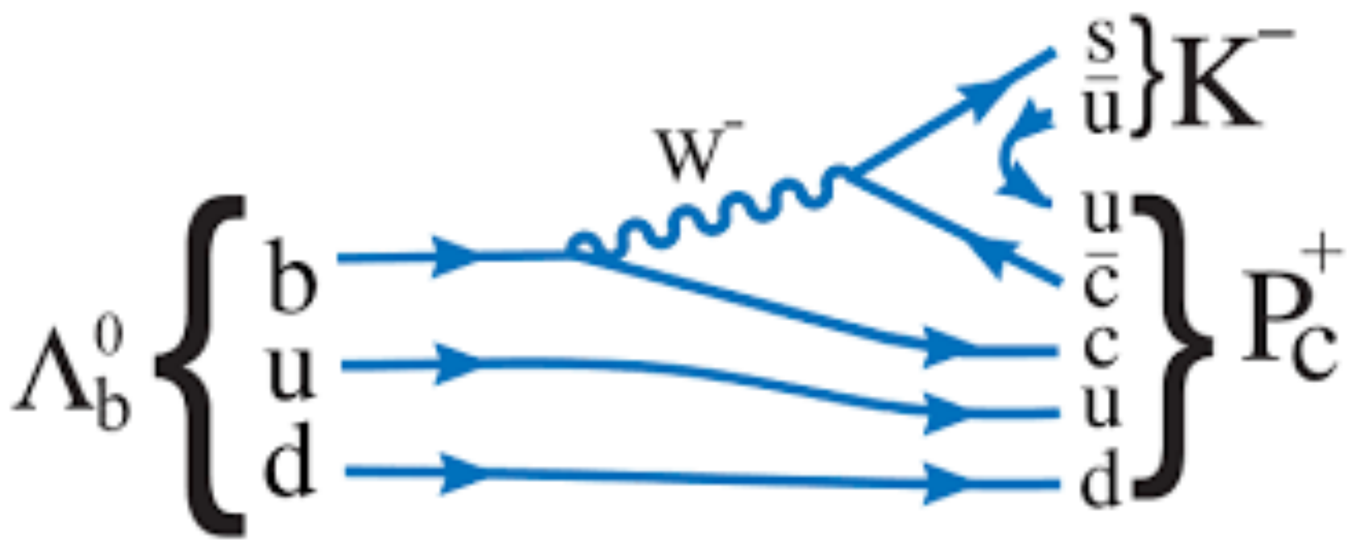
Exotics States Abound!



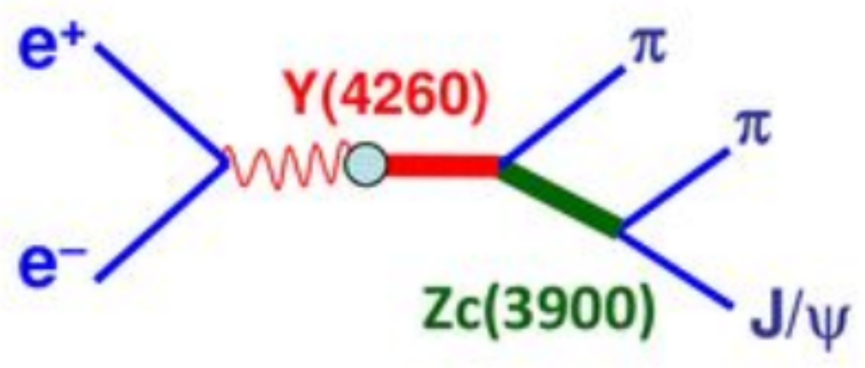
h/t Patrick Koppenburg

Exotic Production Modes

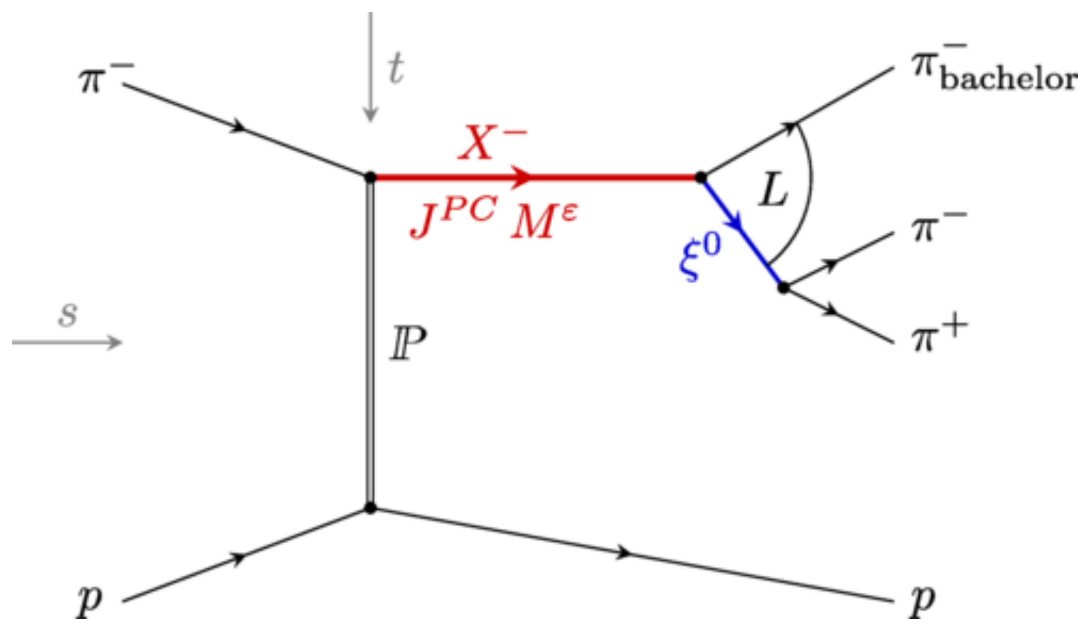
$B/\Lambda_b \to HX$



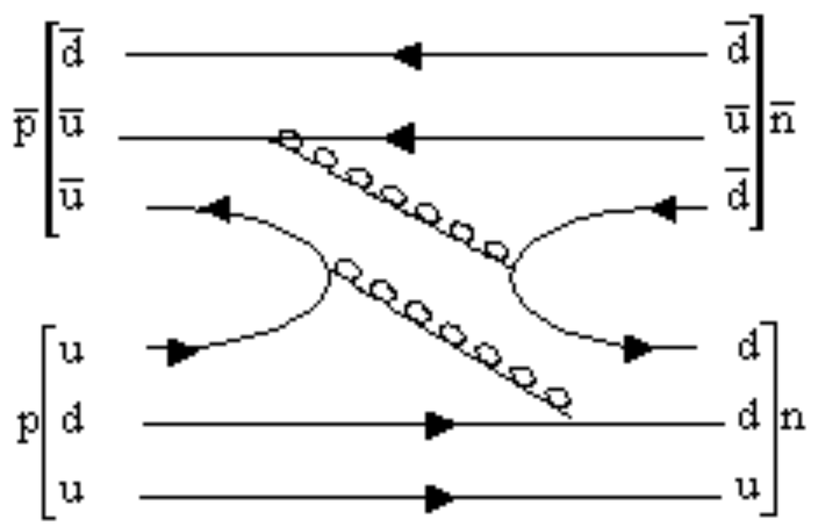
$e^+e^- \to X$



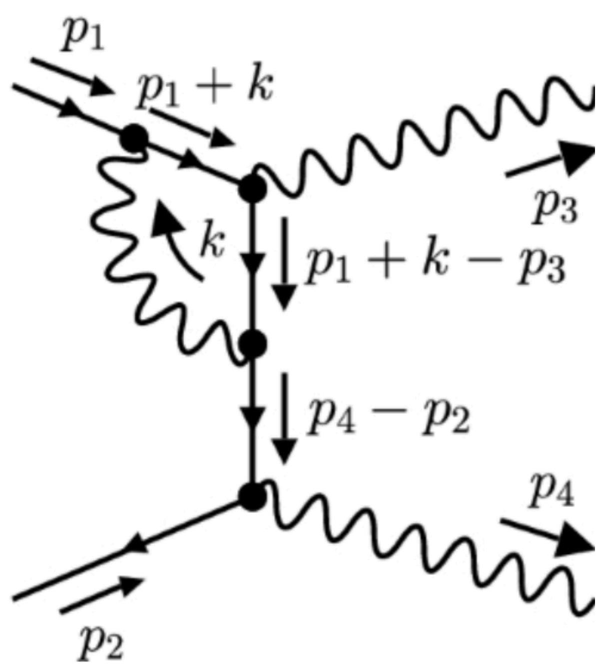
$\pi N \to BX$



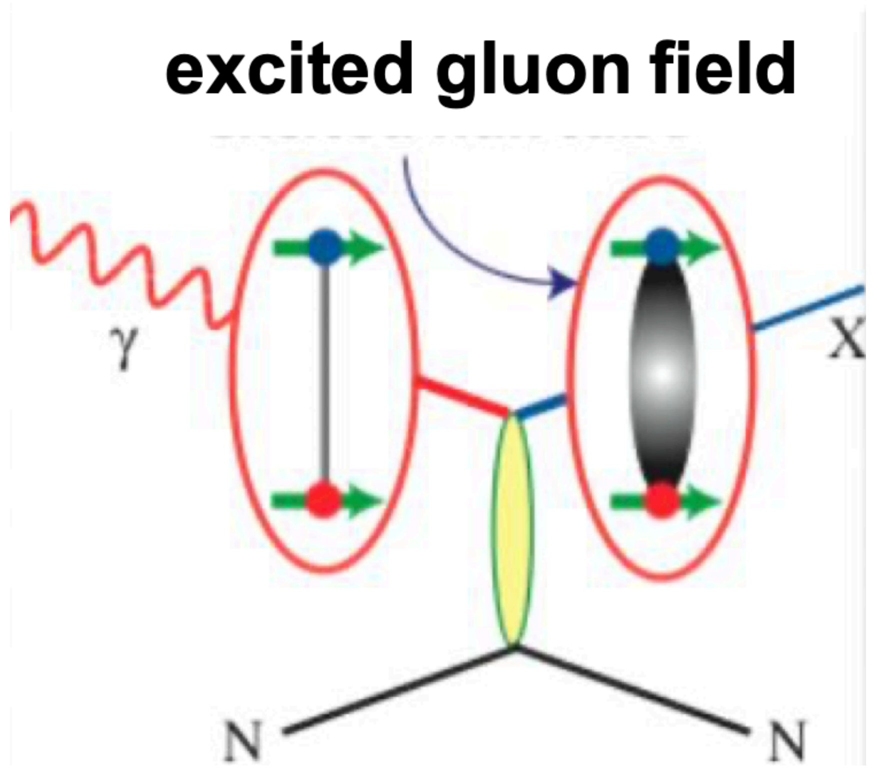
$p\bar{p}/pp \to X$



$\gamma\gamma \to X$



$\gamma p \to BX$



(for reference)

Table 1: Exotica organized by the way they are produced. References are given in the decay column.

Process	Production	Decay	Particle
B and Λ_b Decays	$B \to K + X$	$X \to \pi^+\pi^- J/\psi$ [4, 109, 110, 111, 112, 113, 114] $X \to D^{*0}\bar{D}^0$ [115, 116, 117] $X \to \gamma J/\psi$ [118, 119, 120, 121] $X \to \gamma\psi(2S)$ [118, 120]	$X(3872)$
		$X \to \omega J/\psi$ [106, 122, 123]	$X(3872)$ $Y(3940)$
		$X \to \gamma\chi_{c1}$ [124]	$X(3823)$
		$X \to \phi J/\psi$ [125, 126, 127, 128, 129, 130, 131, 132]	$Y(4140)$ $Y(4274)$ $X(4500)$ $X(4700)$
	$B \to K + Z$	$Z \to \pi^\pm\chi_{c1}$ [133, 134]	$Z_1(4050)$ $Z_2(4250)$
		$Z \to \pi^\pm J/\psi$ [46, 135]	$Z_c(4200)$ $Z_c(4430)$
		$Z \to \pi^\pm\psi(2S)$ [30, 135, 136, 137, 138, 139]	$Z_c(4240)$ $Z_c(4430)$
	$B \to K\pi + X$	$X \to \pi^+\pi^- J/\psi$ [140]	$X(3872)$
	$\Lambda_b \to K + P_c$	$P_c \to pJ/\psi$ [35]	$P_c(4380)$ $P_c(4450)$
e^+e^- Annihilation	$e^+e^- \to Y$	$Y \to \pi\pi J/\psi$ [23, 29, 141, 142, 143, 144, 145]	$Y(4008)$ $Y(4260)$
		$Y \to \pi\pi\psi(2S)$ [108, 146, 147, 148]	$Y(4360)$ $Y(4660)$
		$Y \to \omega\chi_{c0}$ [149]	$Y(4230)$
		$Y \to \Lambda_c\bar{\Lambda}_c$ [150]	$X(4630)$
		$Y \to \pi\pi\Upsilon(1S, 2S, 3S)$ [151, 152] $Y \to \pi\pi h_b(1P, 2P)$ [153]	$Y_b(10888)$
	$e^+e^- \to \pi + Z$	$Z \to \pi J/\psi$ [22, 23, 31, 32] $Z \to D^*\bar{D}$ [33, 154, 155]	$Z_c(3900)$
		$Z \to \pi h_c$ [156, 157] $Z \to D^*\bar{D}^*$ [158, 159]	$Z_c(4020)$
		$Z \to \pi^\pm\psi(2S)$ [148]	$Z_c(4055)$
		$Z \to \pi\Upsilon(1S, 2S, 3S)$ [160, 161, 162] $Z \to \pi h_b(1P, 2P)$ [160]	$Z_b(10610)$ $Z_b(10650)$
		$Z \to B\bar{B}^*$ [163]	$Z_b(10610)$
		$Z \to B^*\bar{B}^*$ [163]	$Z_b(10650)$
	$e^+e^- \to \gamma + X$	$X \to \pi^+\pi^- J/\psi$ [52]	$X(3872)$
	$e^+e^- \to \pi^+\pi^- + X$	$X \to \gamma\chi_{c1}$ [164]	$X(3823)$
	$e^+e^- \to J/\psi + X$	$X \to D\bar{D}^*$ [41, 165]	$X(3940)$
		$X \to D^*\bar{D}^*$ [41]	$X(4160)$
$\gamma\gamma$ Collisions	$\gamma\gamma \to X$	$X \to \omega J/\psi$ [166, 167]	$X(3915)$
		$X \to D\bar{D}$ [168, 169]	$Z(3930)$
		$X \to \phi J/\psi$ [170]	$X(4350)$
Hadron Collisions	pp or $p\bar{p} \to X+$ anything	$X \to \pi^+\pi^- J/\psi$ [27, 171, 172, 173]	$X(3872)$
		$X \to \phi J/\psi$ [174]	$Y(4140)$
		$X \to B_s\pi^\pm$ [175]	$X(5568)$

h/t Lebed, Mitchell, Swanson 1610.04528

(for reference)

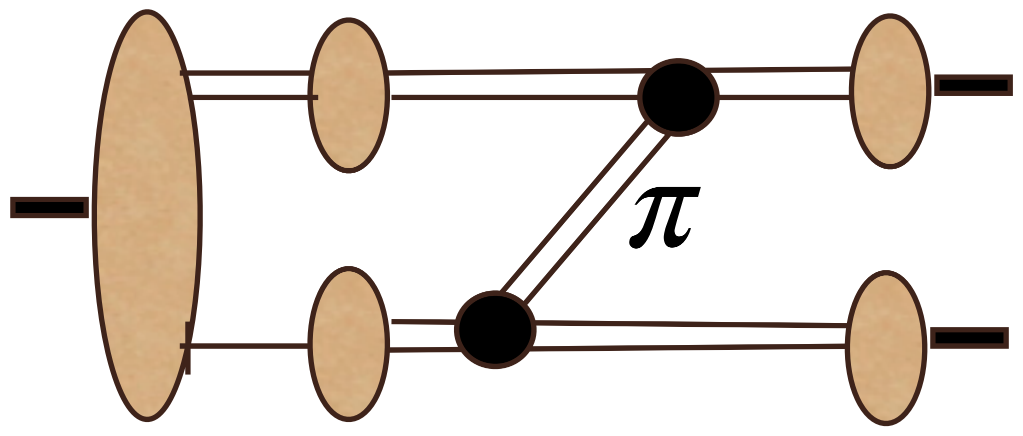
Table 3: Major experiments in the past, present, and future of heavy-quark exotics studies.

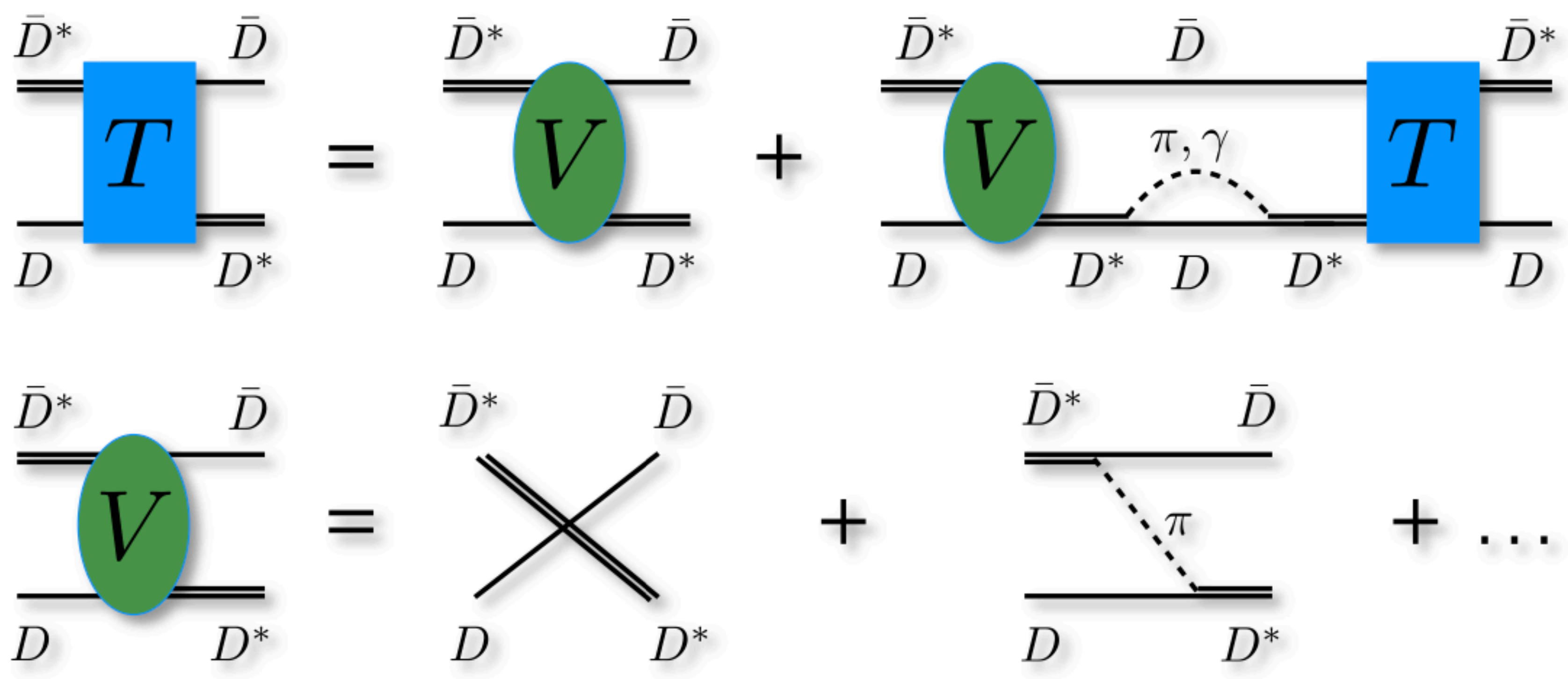
Experiment	Highlights	Accelerator	Years	Institute	Production
BaBar	$Y(4260)$ [29] $Y(4360)$ [108]	PEP-II	1999–2008	SLAC (Menlo Park, California, USA)	e^+e^- annihilation ($E_{\rm CM} \approx 10$ GeV): $e^+e^- \to B\bar{B};\ B \to KX$ $e^+e^- \to Y_b$ $e^+e^- \to \pi Z_b$ $e^+e^-(\gamma_{\rm ISR}) \to Y$ $e^+e^-(\gamma_{\rm ISR}) \to \pi Z_c$ $e^+e^- \to J/\psi + X$ $\gamma\gamma \to X$
Belle	$X(3872)$ [4] $Y(3940)$ [106] $X(3915)$ [166] $Z_c(4430)$ [30, 136, 137] $Z_b(10610)$, $Z_b(10650)$ [160, 162, 163] $Y_b(10888)$ [151, 152]	KEKB	1998–2010	KEK (Tsukuba, Japan)	
Belle II	Upcoming continuation of Belle	SuperKEKB	2018–		
CLEO-c	$Y(4260)$ [142] $\pi^+\pi^-h_c$ [177]	CESR-c	2003–2008	Cornell U. (Ithaca, New York, USA)	e^+e^- annihilation ($E_{\rm CM} \approx 4$ GeV): $e^+e^- \to Y$ $e^+e^- \to \pi Z$ $e^+e^- \to \gamma X$
BESIII	$Z_c(3900)$ [22, 154] $Z_c(4020)$ [156, 158] $Y(4230)$ [149] $X(3872)$ [52]	BEPCII	2008–	IHEP (Beijing, China)	
CDF	$Y(4140)$ [126] $Y(4274)$ [132] $X(3872)$ [178, 179, 172]	Tevatron	1985–2011	Fermilab (Batavia, Illinois, USA)	$p\bar{p}$ collisions ($E_{\rm CM} \approx 2$ TeV): $p\bar{p} \to X$ + any $p\bar{p} \to B$ + any; $B \to KX$
D0	$X(3872)$ [171] $Y(4140)$ [174] $X(5568)$ [175]				
ATLAS	$\chi_b(3P)$ [180]	LHC	2010–	CERN (Geneva, Switzerland)	pp collisions ($E_{\rm CM} = 7, 8, 13$ TeV): $pp \to X$ + any $pp \to B$ + any; $B \to KX$ $pp \to \Lambda_b$ + any; $\Lambda_b \to KP_c$
CMS	$X(3872)$ [28] $Y(4140)$, $Y(4274)$ [130]				
LHCb	$Z_c(4430)$ [138, 139] $X(3872)$ [109] $P_c(4380)$, $P_c(4450)$ [35] $Y(4140)$, $Y(4274)$ [125, 131]				
COMPASS	photoproduction [181] $a_1(1420)$ [182]	SPS	2002-2011		μ/π beam on N target ($p_{beam} \approx 160, 200$ GeV) $\pi N \to XN$ $\gamma N \to XN$
$\bar{\rm P}$ANDA	Upcoming	HESR		GSI (Darmstadt, Germany)	$\bar{p}$ beam on p target ($p_{beam} \approx 1.5$–15 GeV): $p\bar{p} \to X$ $p\bar{p} \to X$ + any
GlueX	Beginning (searches for light quark hybrid mesons)	CEBAF	2016–	Jefferson Lab (Newport News, Virginia, USA)	γ beam on p target ($E_{beam} \le 11$ GeV): $\gamma p \to Xp$
CLAS12					

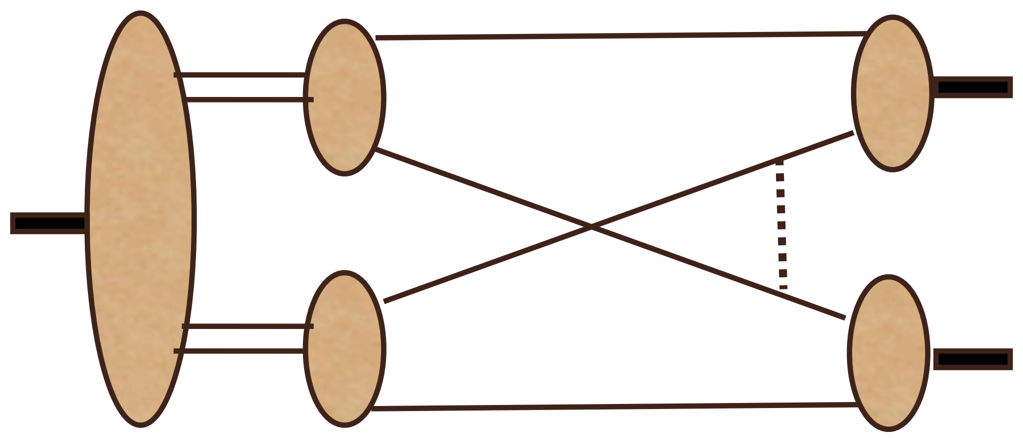
h/t Lebed, Mitchell, Swanson 1610.04528

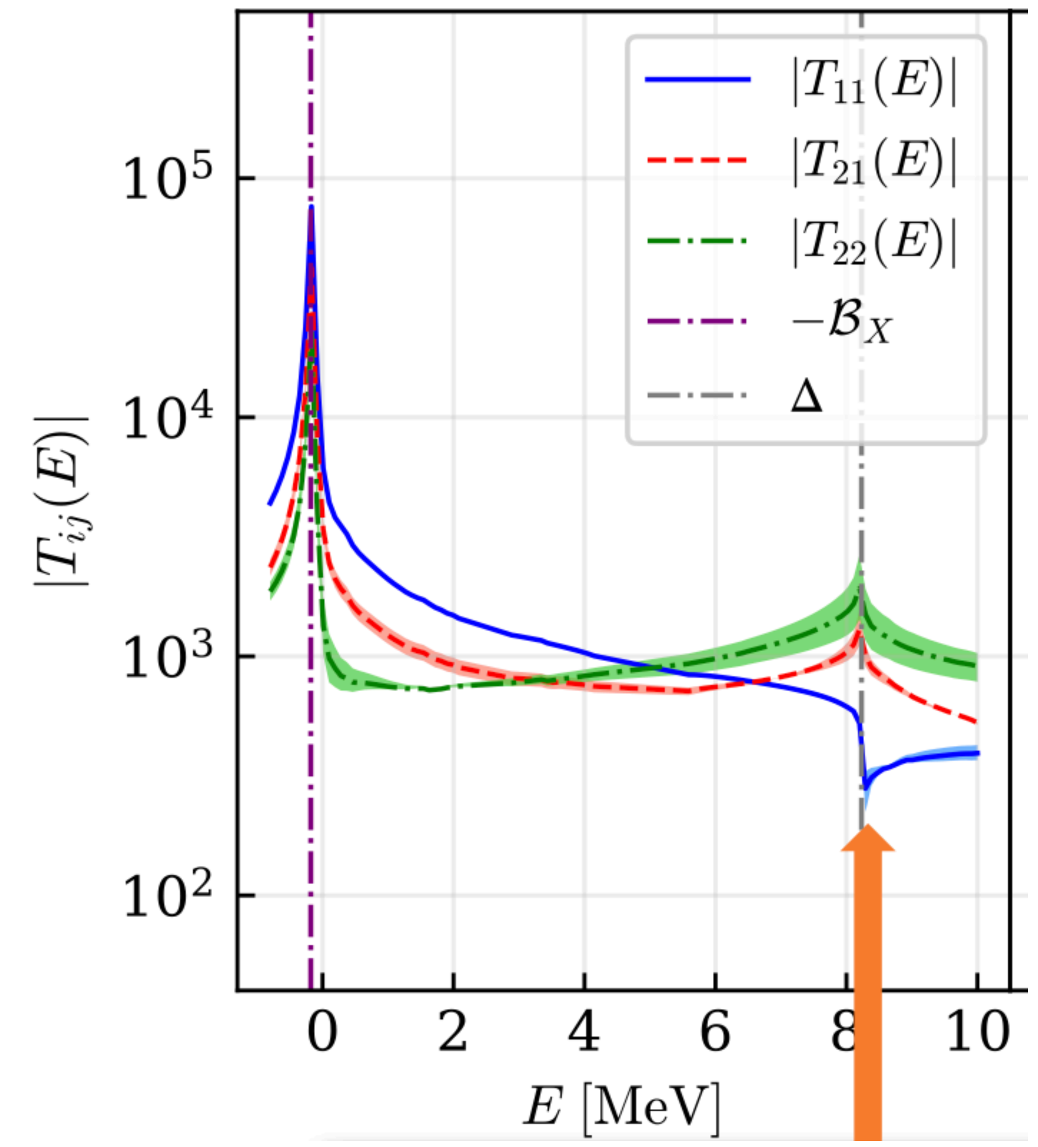
Exotic Interpretations -- molecules

X(3872)





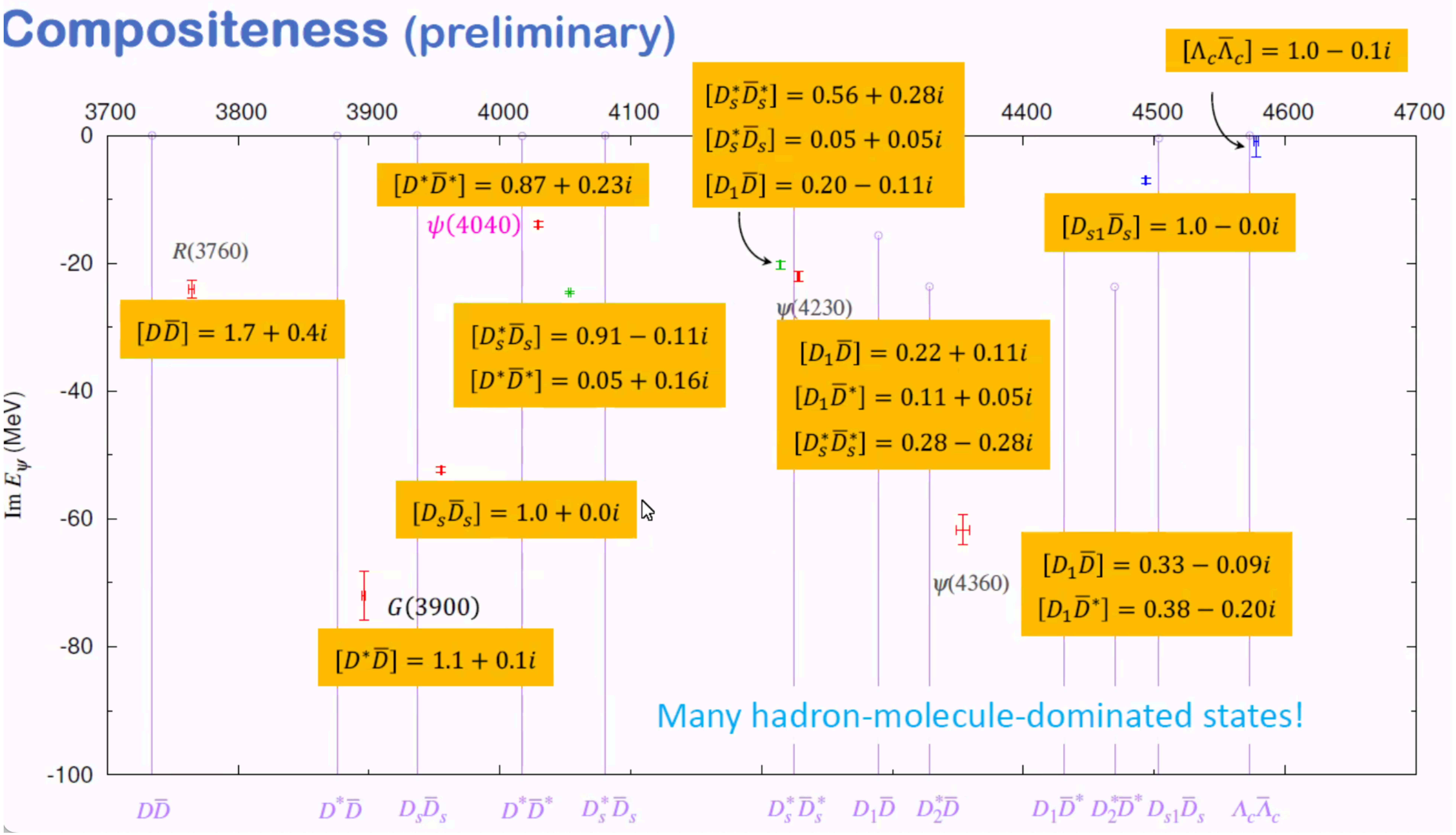




arXiv:hep-ph/0311229

h/t F-K Guo

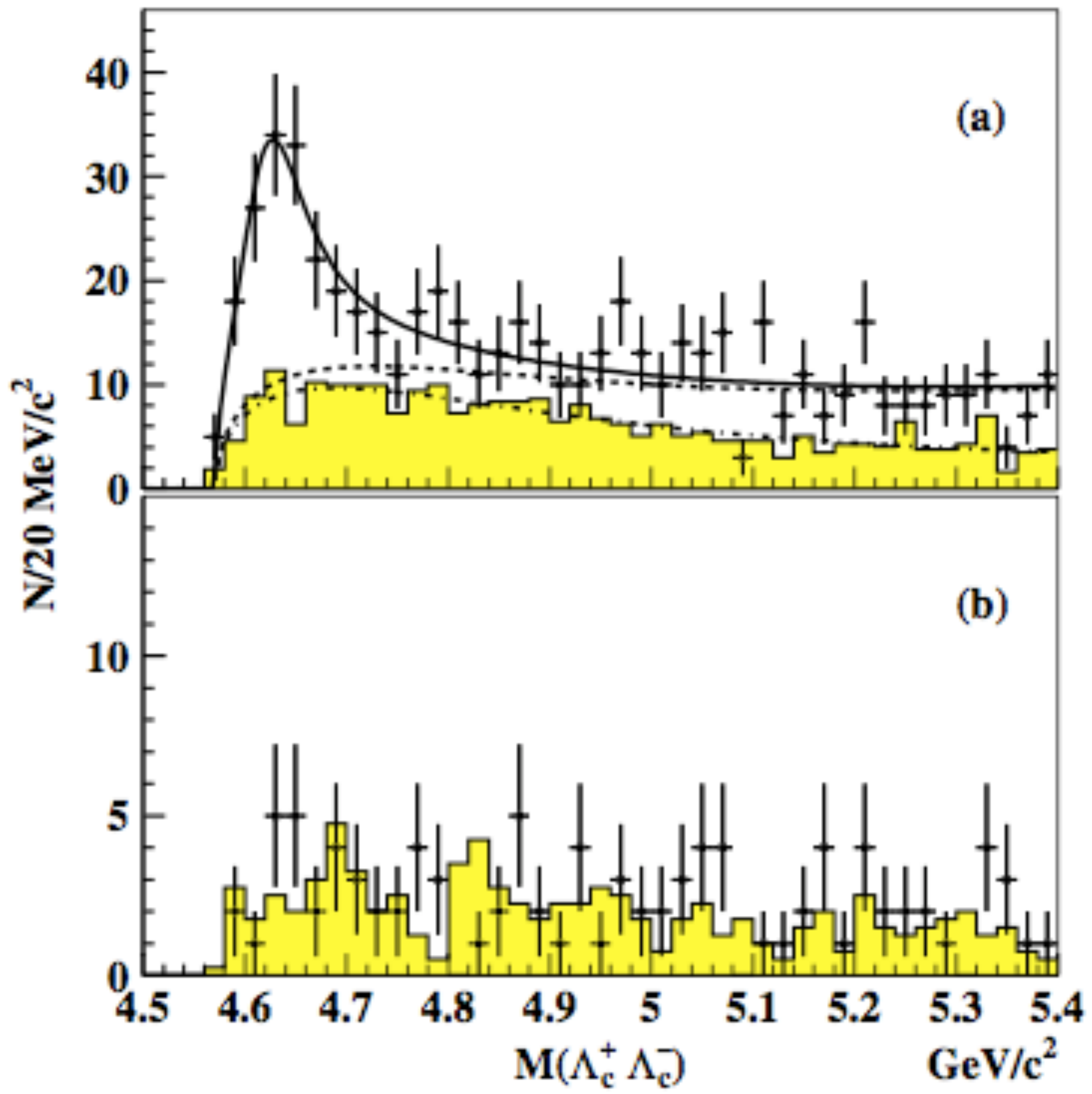
Exotic Interpretations -- molecules



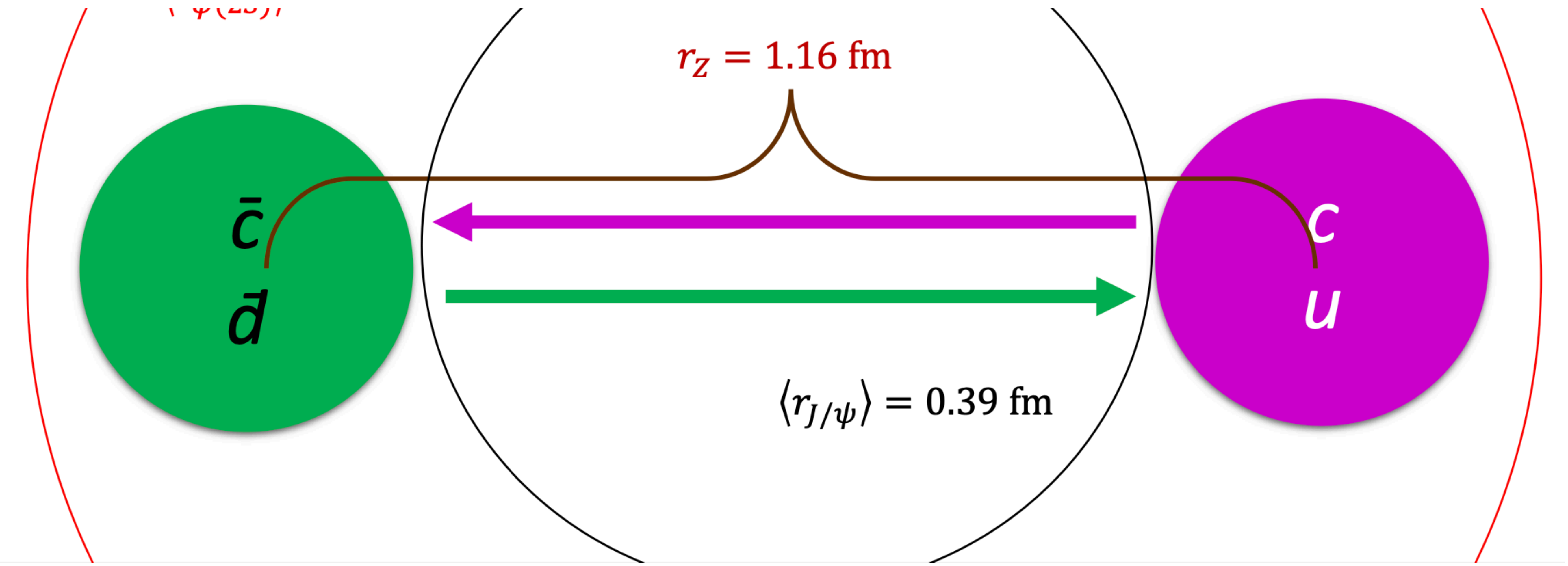
h/t Christian Hanhart

Exotic Interpretations -- threshold effects

X(4630)



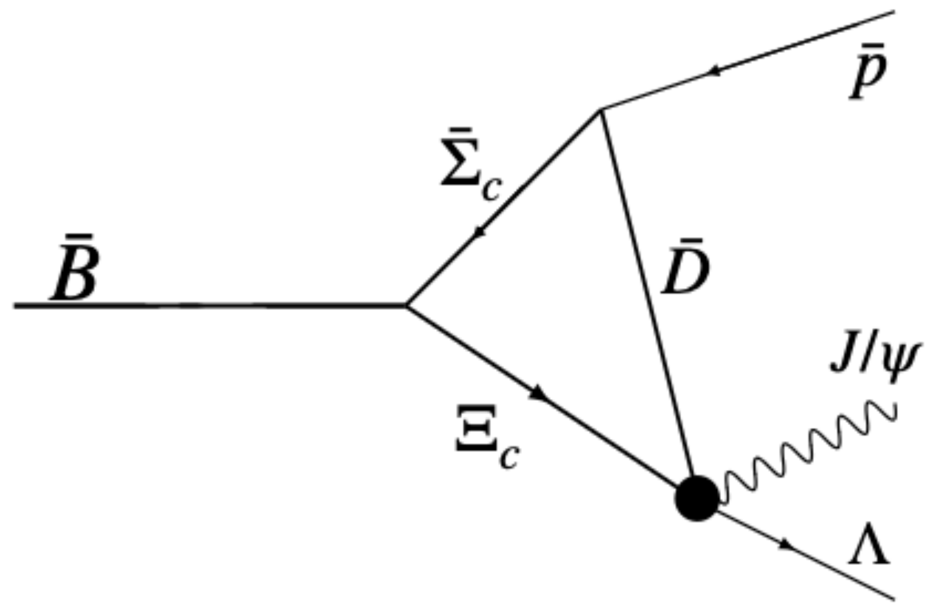
Exotic Interpretations -- dynamical dyquarks

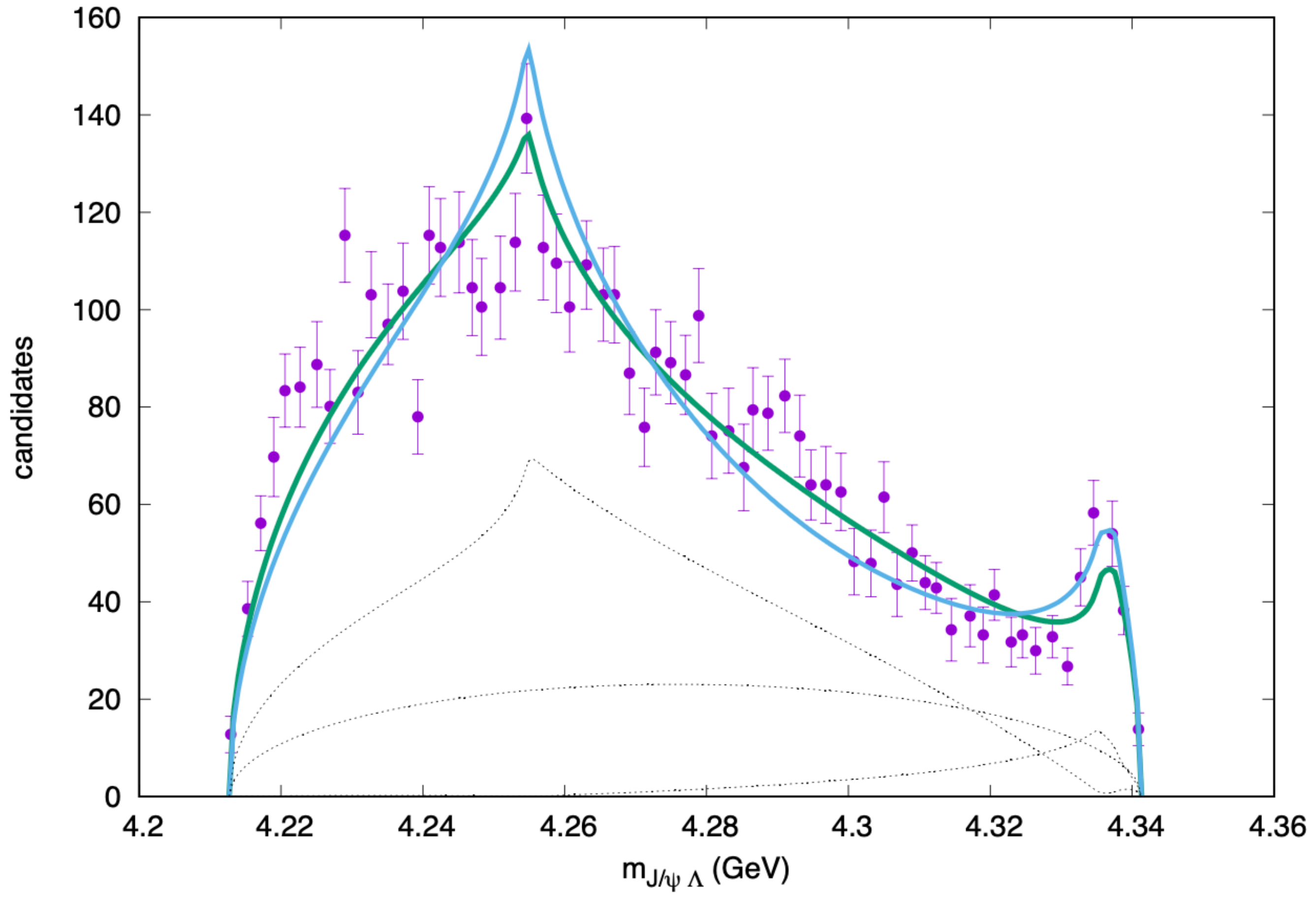


h/t R. Lebed

Exotic Interpretations -- loop singularities

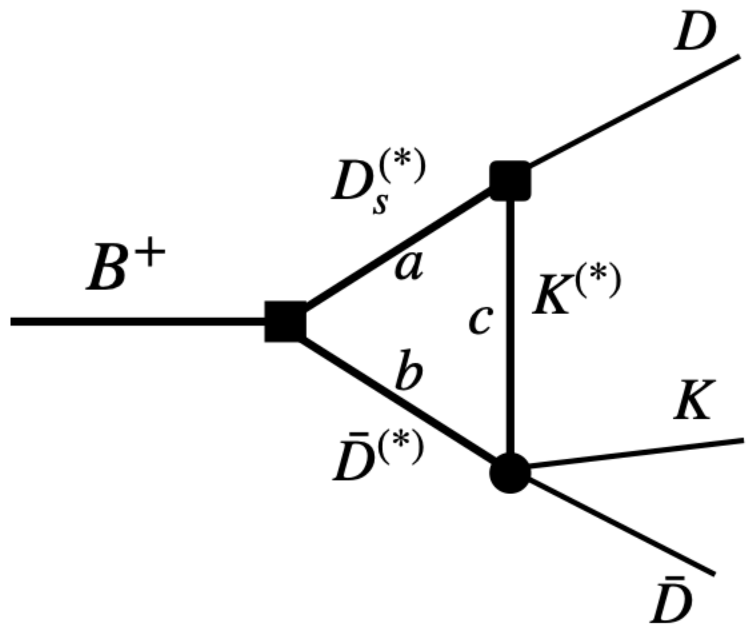
$P_{\psi s}^{\Lambda}(4338)$

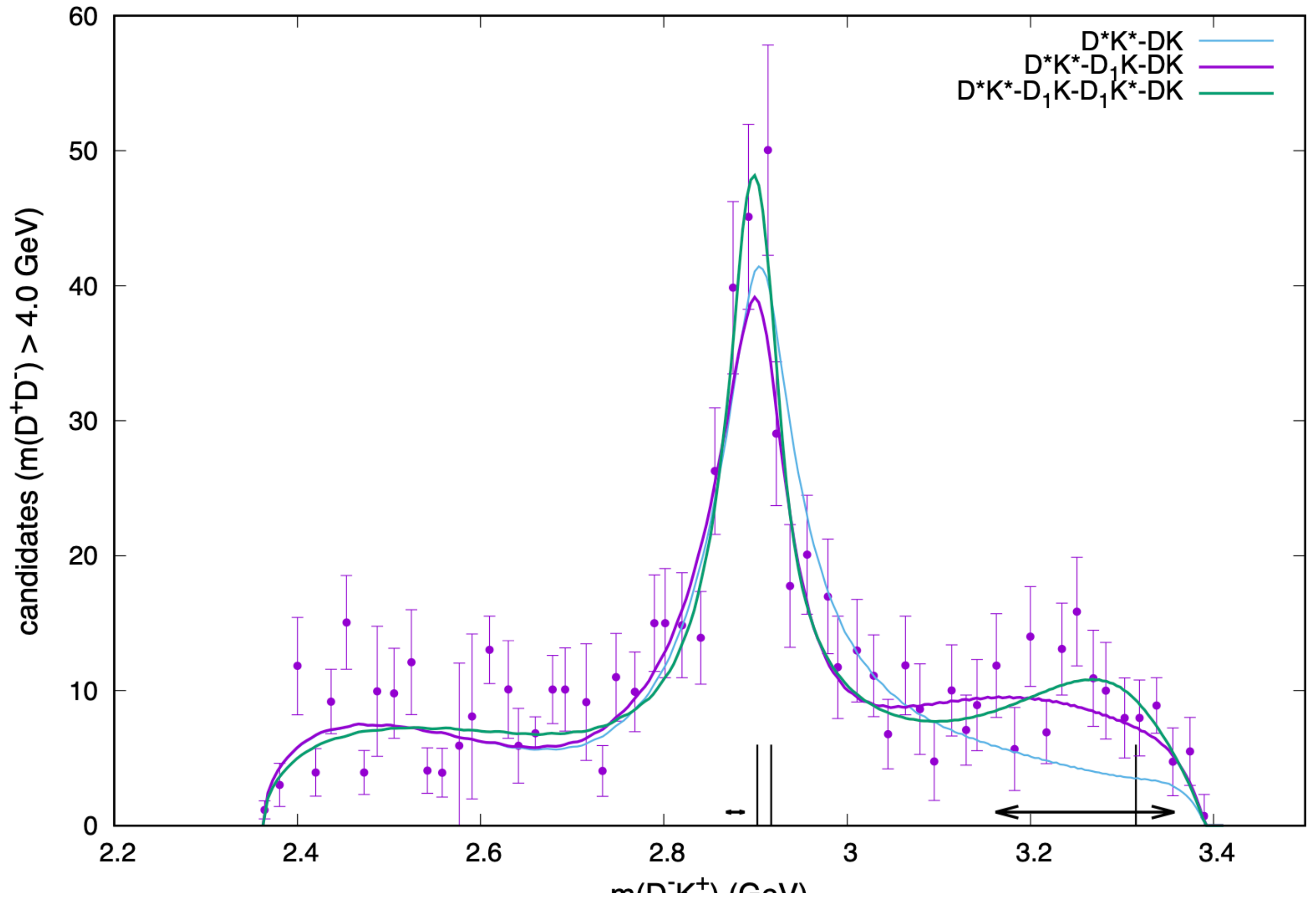




arXiv:2208.05106

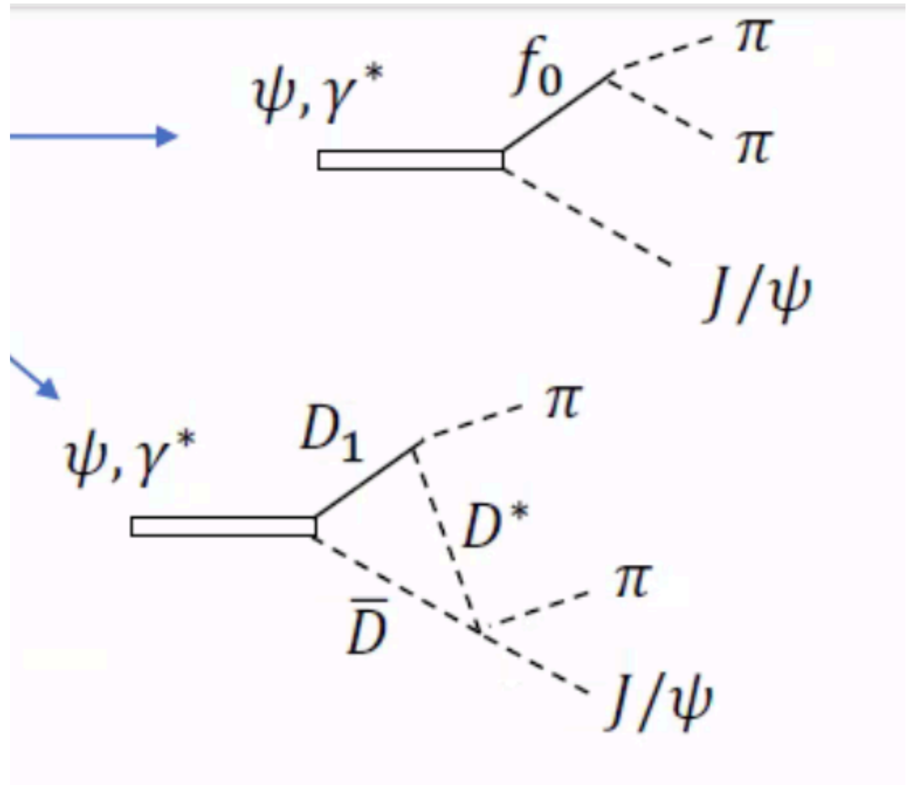
X(2900)

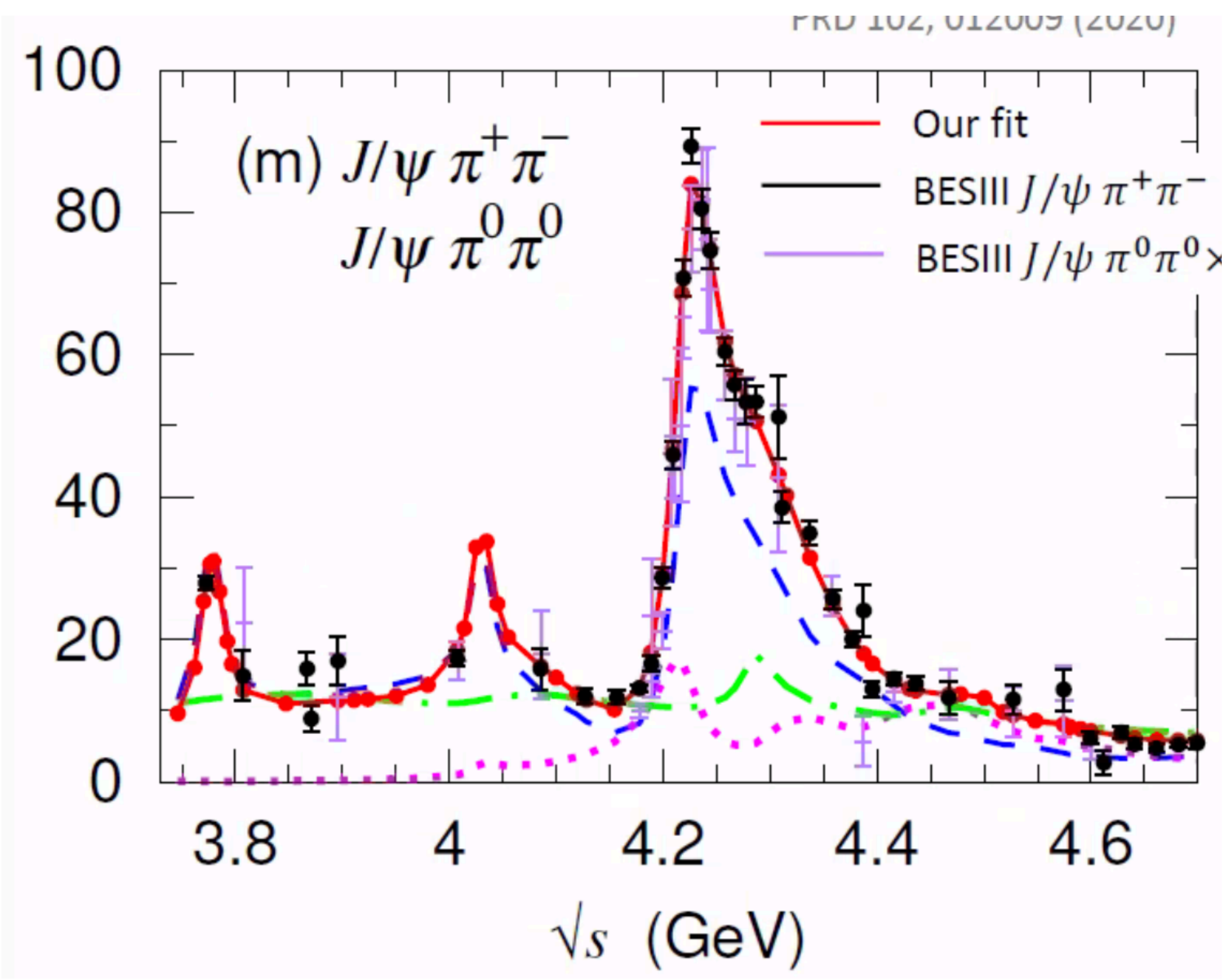




arXiv:2008.12838

Y(4230)





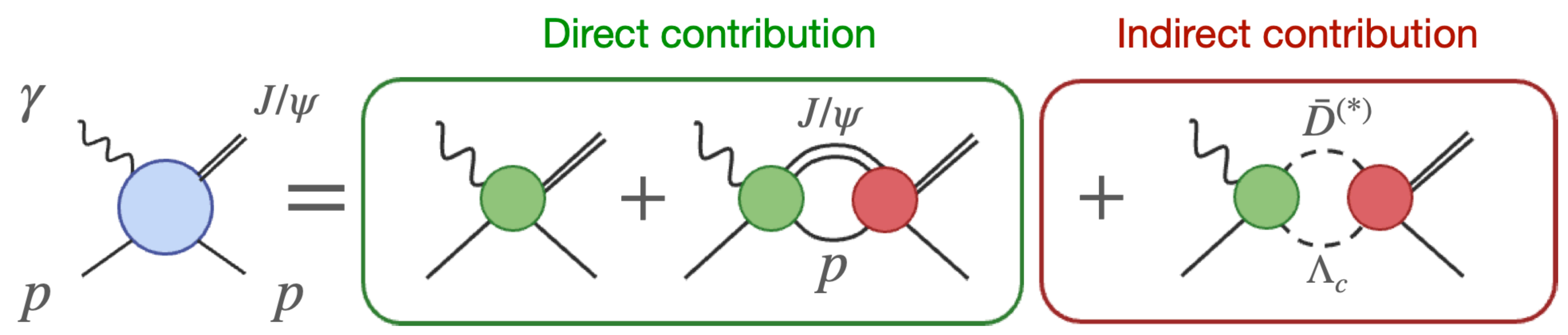
Exotics @ EIC

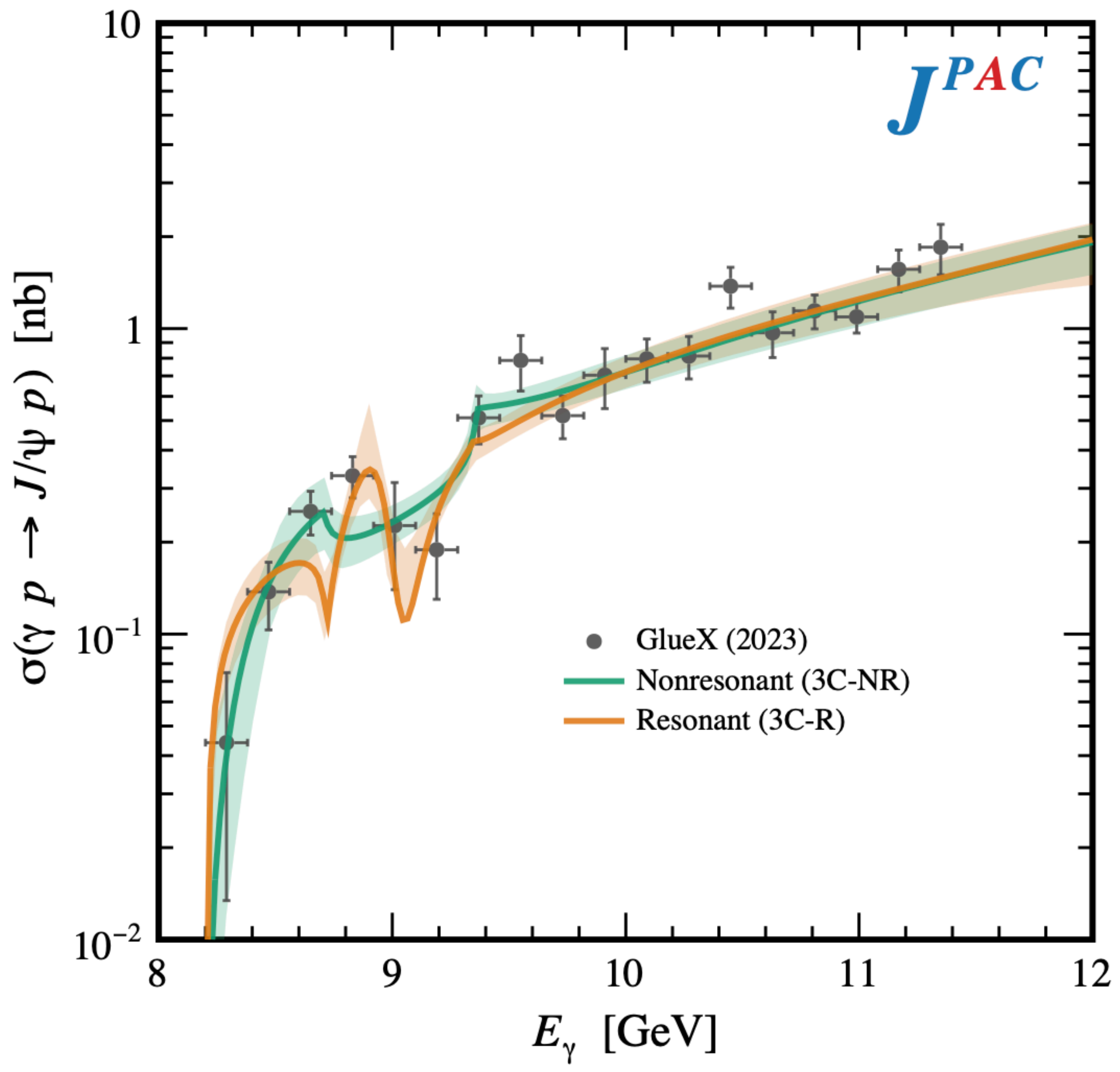
👍

- large luminosity
- high energy
- access to quantum numbers
- polarized photons and protons
- orthogonal to other production methods

Exotics @ EIC

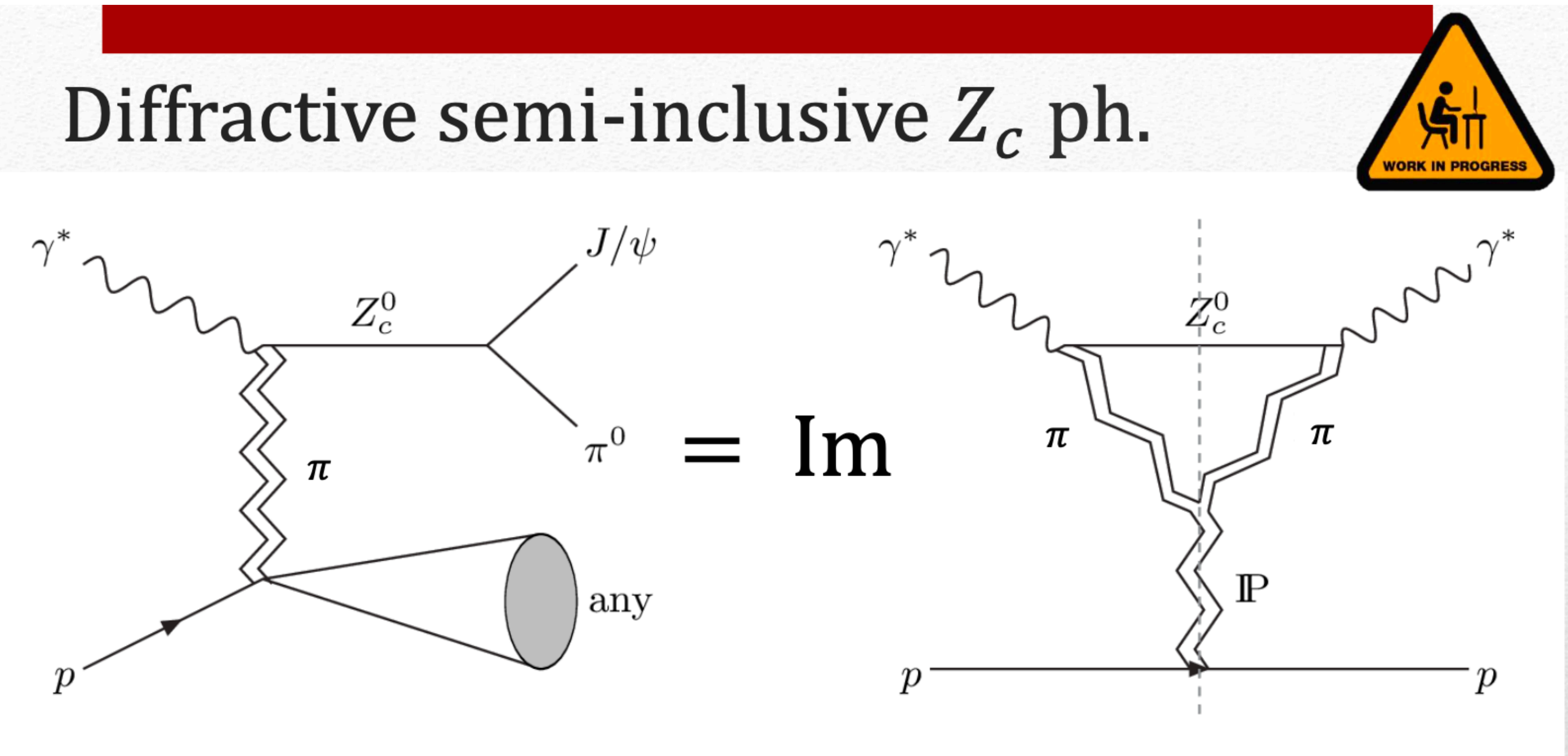
- orthogonal to other production methods





arXiv:2305.01449

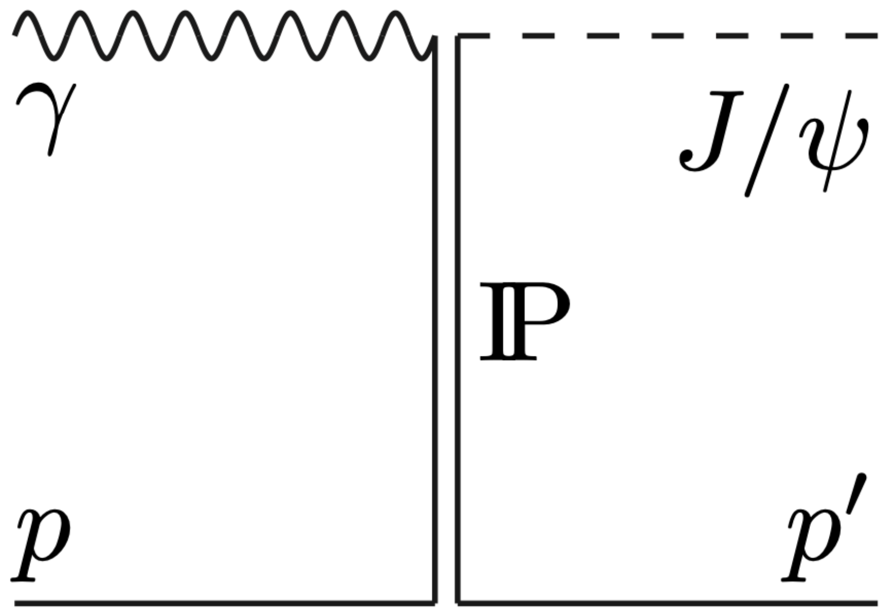
Diffractive semi-inclusive Z_c ph.

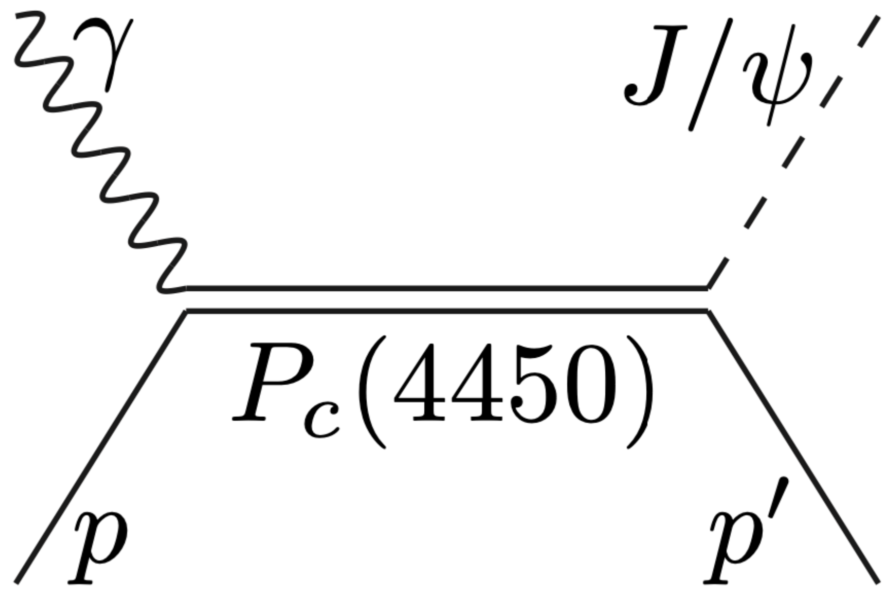


If the target fragments are separated from the beam ones, one can invoke Regge factorization

Quark-Regge duality allows to replace the intermediate hadrons with Pomeron, prediction reliable for $x_B \sim 1$, $t \ll W_{\gamma p}^2$

h/t Alessandro Pilloni (2020)





arXiv:1801.10211

Exotics @ EIC

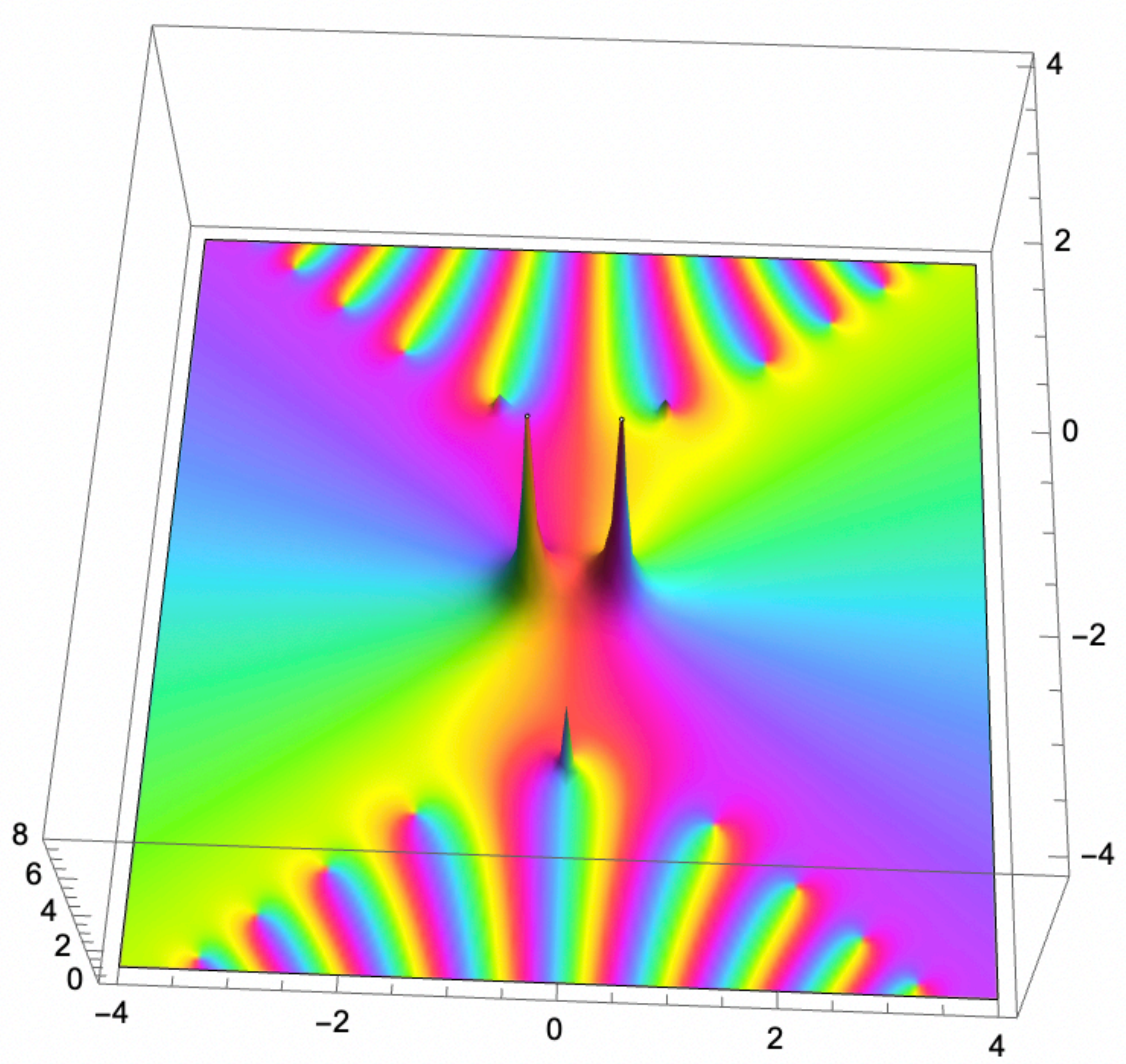
👍

- large luminosity
- high energy
- access to quantum numbers
- polarized photons and protons
- orthogonal to other production methods

👎

- is the orthogonality useful?
- reactions will be complicated!

K-matrix Analysis

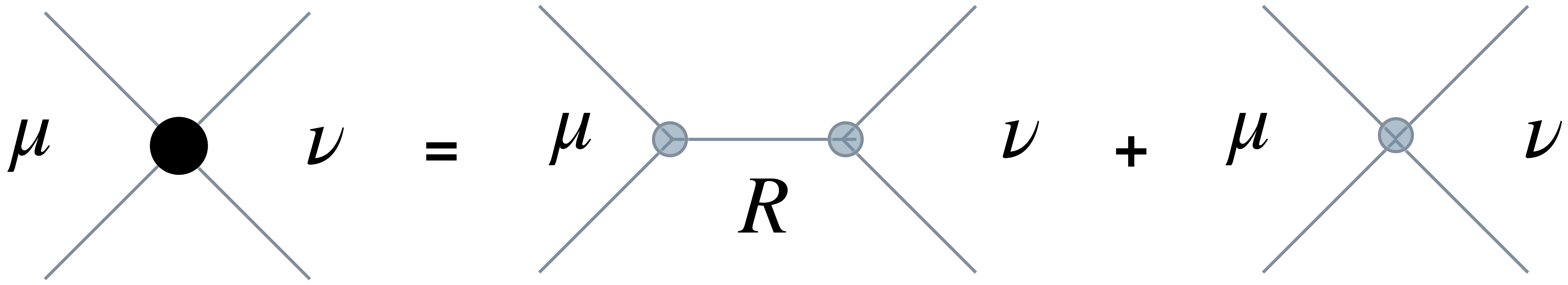


{a real example!}

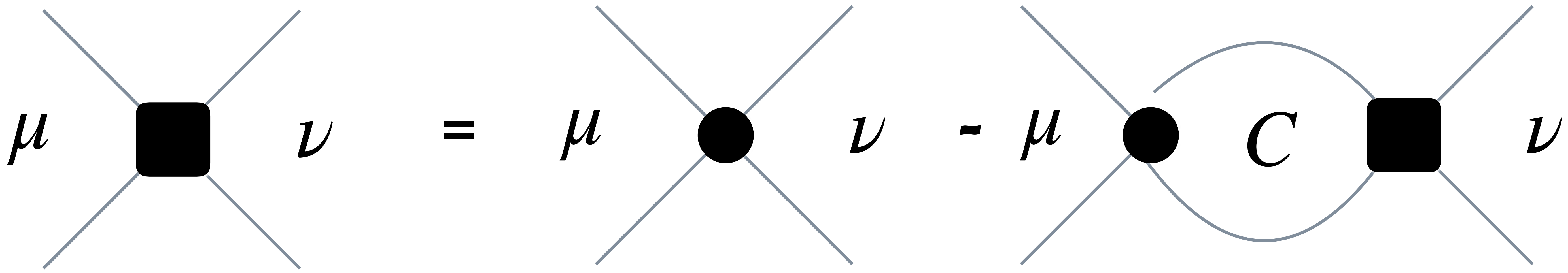
h/t Randall Munroe

K-matrix Formalism

K



$\mathscr{M}$

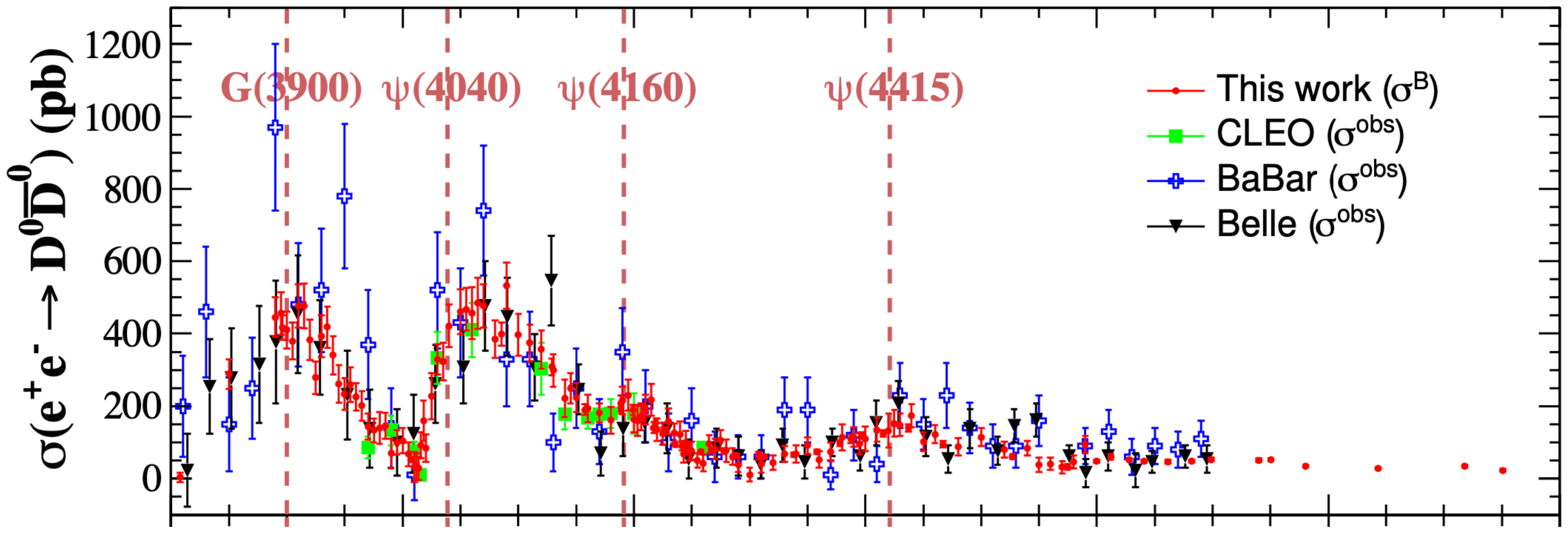


K-matrix Formalism

h/t chat-gpt

K-matrix Formalism 👍

- (sometimes) computationally simple
- respects two-body unitarity (sums of Breit-Wigners are not encouraged!)
- more reliable extraction of couplings (from pole residues); avoids assumptions in extracting branching fractions



h/t BESIII

- employing a Chew-Mandelstam representation permits avoiding kinematic singularities at $s = 0$ and pseudothreshold (and in formfactor models)

$$\mathscr{C}(s + i\epsilon) = \mathscr{C}(s_1) - \frac{s - s_1}{\pi} \int_{s_0} \frac{ds'}{s' - s_1} \frac{\rho(s')g^2(s')}{s' - s - i\epsilon}$$

K-matrix Formalism 👎

- Three-body unitarity?

- "Analyticity" -- model for the (meromorphic) structure; unintended structure (form factors); continuing to the complex plane

- left hand cuts? (➛ N/D)

- ghost poles can befuddle

- extracted couplings tend to show model-dependence

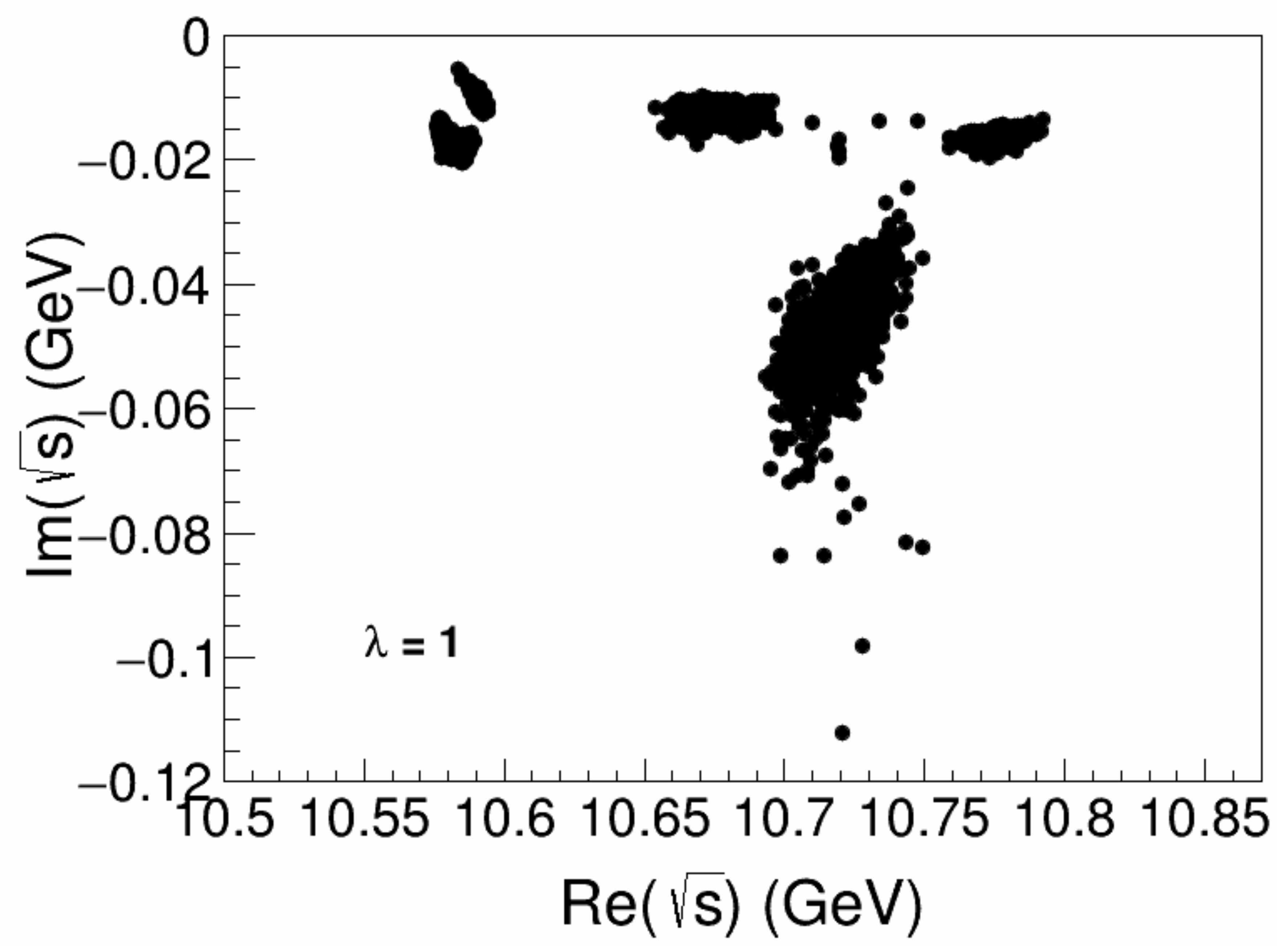
- overfitting is all too easy

- "reality" is often ascribed to the fit parameters/ overfitting is a problem

Can arise due to spurious non-invertibility of (1+CK).

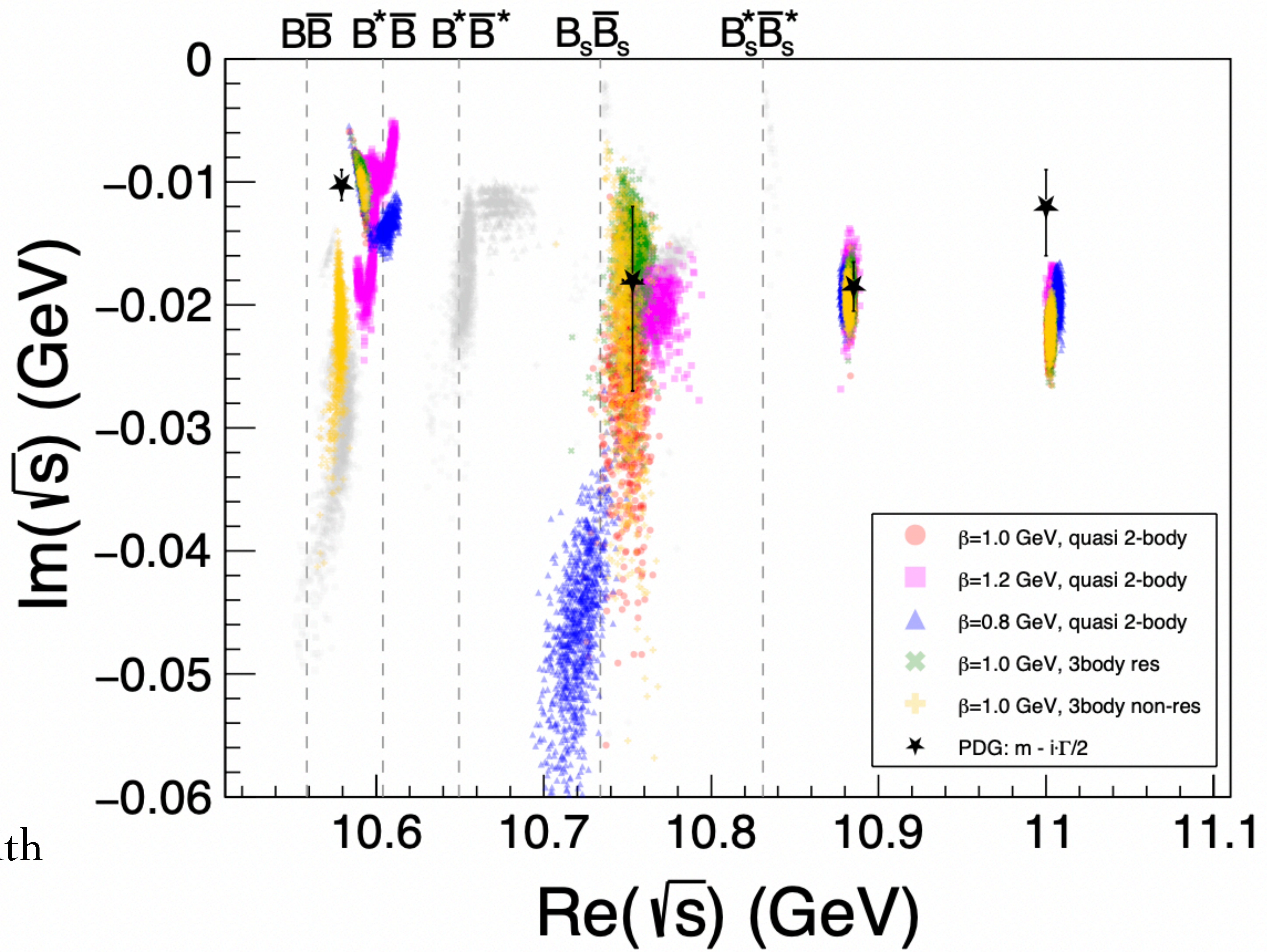
Identify by anomalous residues and behaviour under rescaling the couplings $\hat{g} \to \lambda\hat{g}$.

h/t Nils Hüsken



K-matrix Applications

pole positions (+ 1000 bootstrap datasets)

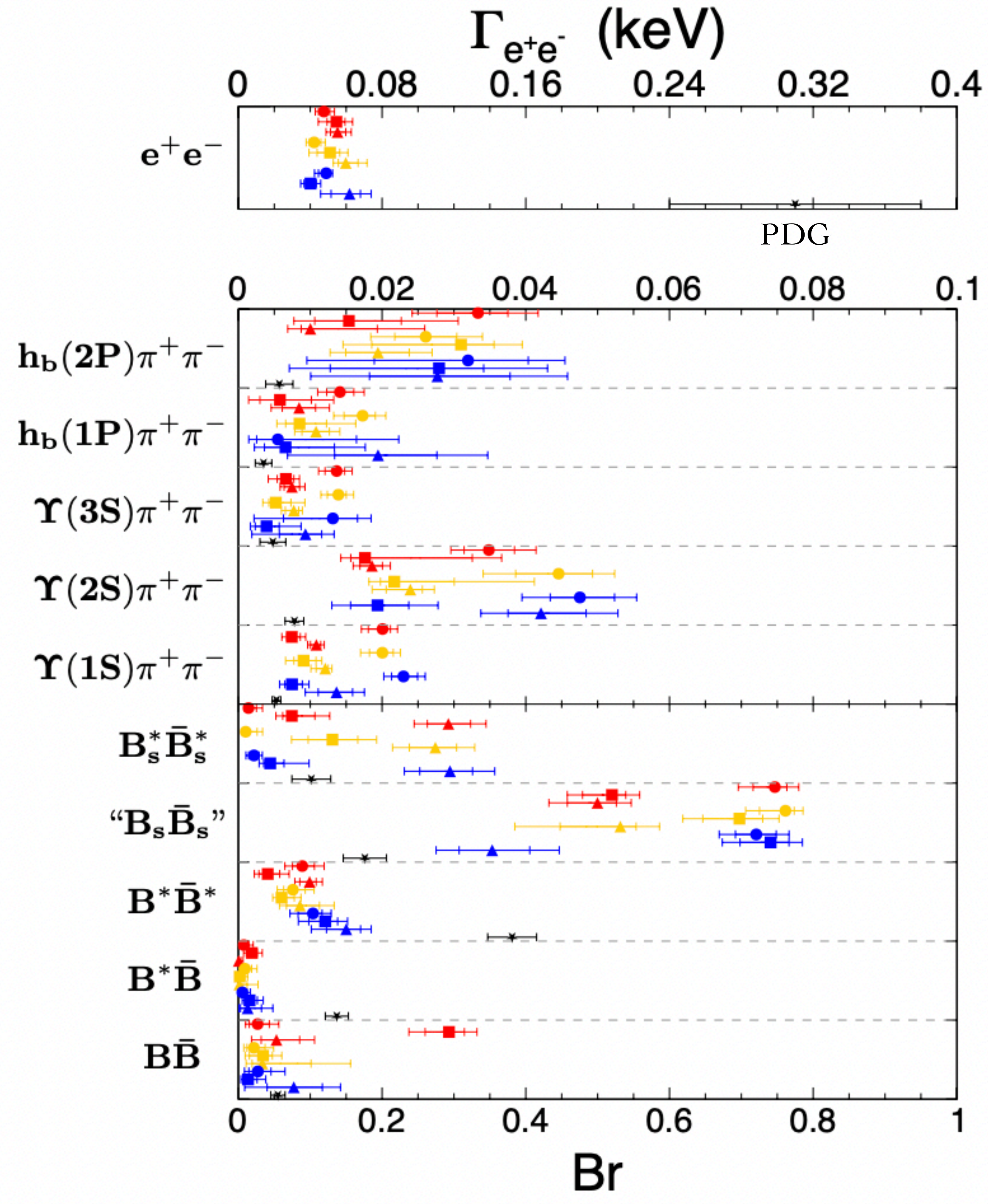


grey=ghost poles with sizable residues

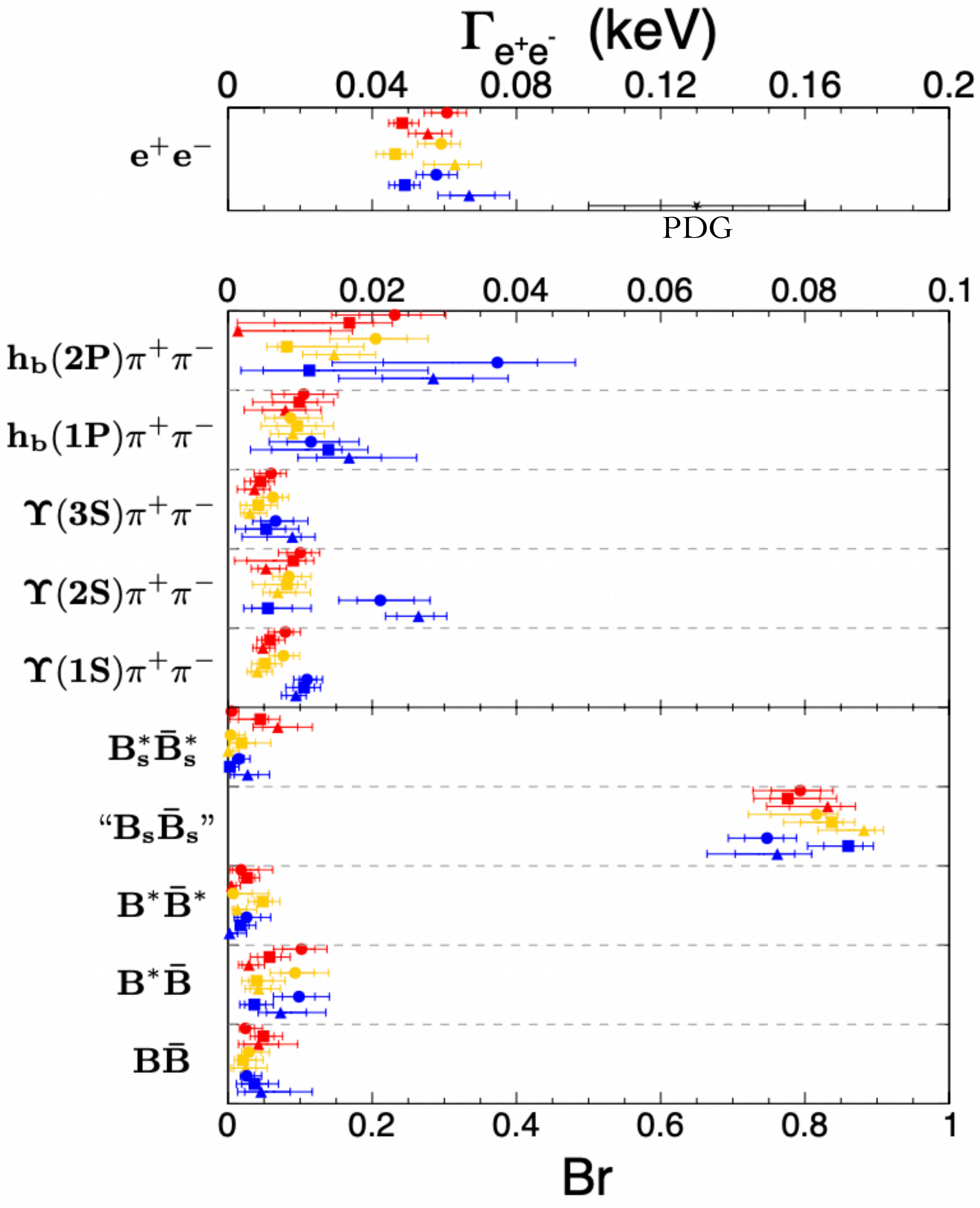
arXiv: 2204.11915

K-matrix Applications

couplings and branching fractions



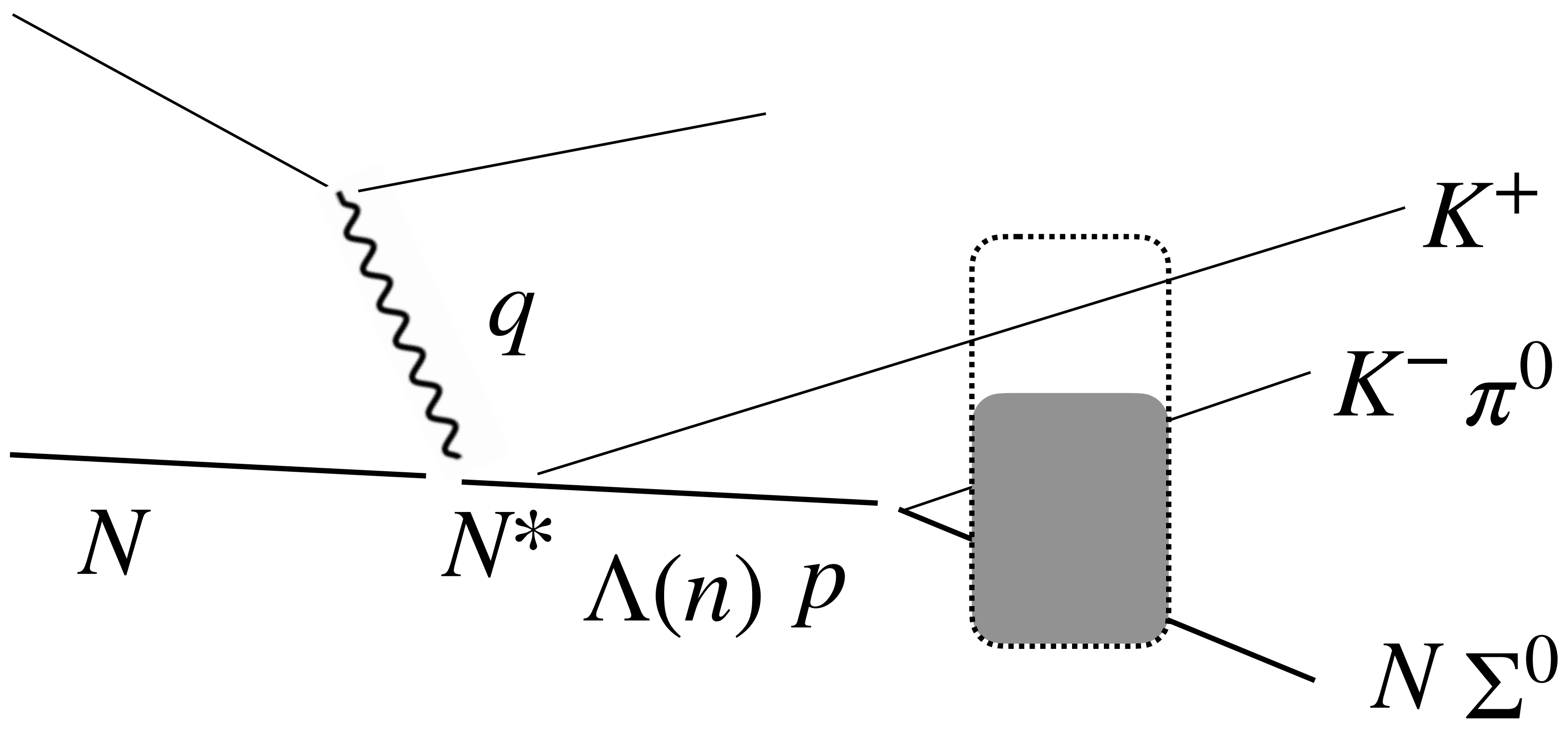
Υ(5S)



Υ(6S)

arXiv: 2204.11915

Ex. Electroproduction of $K(KN, \pi\Sigma)$ @ CLAS/JLab



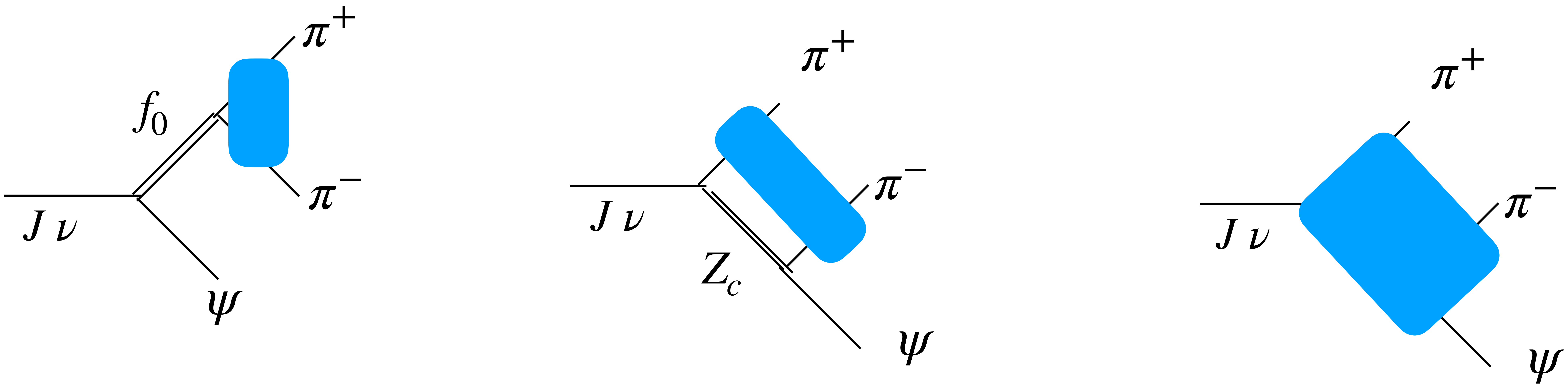
$$\mathcal{M} \sim \sum_{nm} \bar{u}\gamma_\mu u\, D^{\mu\nu}(q)\, \bar{N}^*_m \gamma_\nu N \cdot BW(N^*_m) \cdot g_{N^*\Lambda_n K} \cdot BW(p, \Lambda_n) \cdot g_{KN}$$

Model as $\gamma^* N \to K(KN, \Sigma\pi)$ scattering

$$K_{\alpha\beta} = \sum_{R=\Lambda(1405),\Lambda(1520)} \frac{g_{\alpha R} g_{\beta R}}{m_R^2 - s} + f_{\alpha\beta}$$

Ex. $e^+e^- \to \pi\pi J/\psi$

Merge K-matrix and isobar formalisms



27.7.24

Ex. $e^+e^- \to \pi\pi J/\psi$

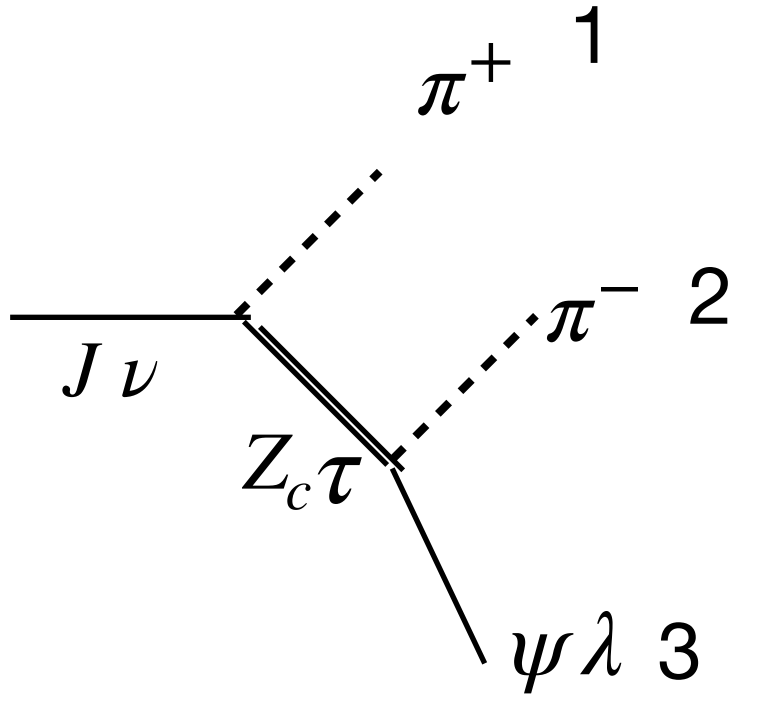
$$\mathscr{M} \sim D^{1}_{\Lambda,\nu-\nu'}(\hat{k}_e)\, h^{V_J:ee}_{\nu,\nu'} \cdot BW(s) \cdot D^{J*}_{\Lambda\rho}(\alpha,\beta,\gamma) \cdot O^{\rho}_{\lambda_1\lambda_2\lambda_3}$$

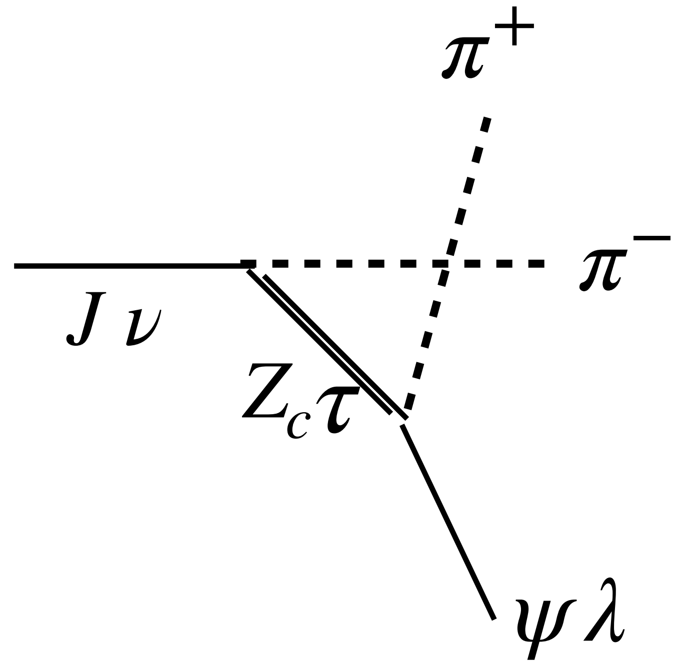
arXiv:1910.04566

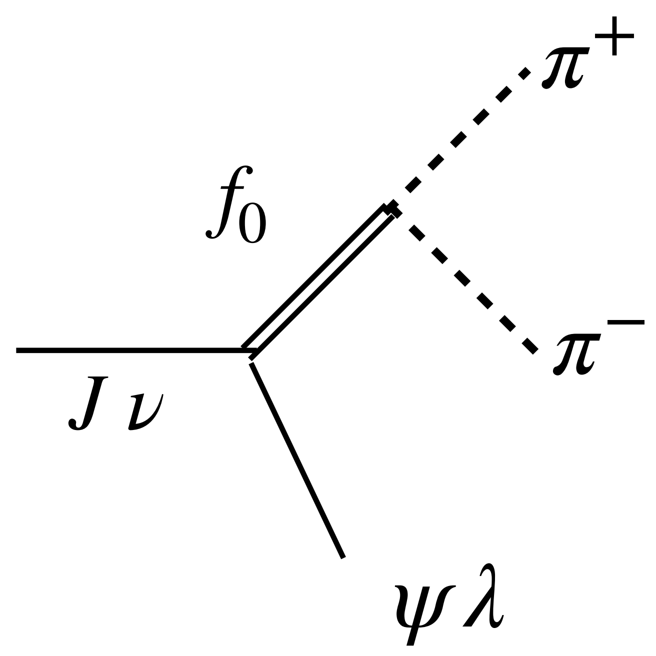
$$\mathrm{d}\sigma/\mathrm{d}\Phi_3 = N \sum_{\Lambda,\Lambda'} \rho_{\Lambda\Lambda'} \sum_{\nu,\nu'} D^{J*}_{\Lambda,\nu}(\alpha,\beta,\gamma) D^{J}_{\Lambda',\nu'}(\alpha,\beta,\gamma) \sum_{\{\lambda\}} O^{\nu}_{\{\lambda\}} O^{\nu'*}_{\{\lambda\}},$$

$$d\sigma = \frac{|h^{Y:ee}|^2 \cdot |BW(s;Y)|^2 \cdot \sum_{\nu\{\lambda\}} O^{\nu}_{\{\lambda\}} O^{\nu*}_{\{\lambda\}}}{(2\pi)^3\, 64 p_{1CM}\, s^{3/2}}\, dm^2_{12} dm^2_{23}$$

Ex. $e^+e^- \to \pi\pi J/\psi$







spectator: k=1 $\hat{\theta}_{1(1)} = 0$ θ_{23} $\zeta^3_{1(0)} = \zeta^3_{1(3)}$

k=2 $\hat{\theta}_{2(1)}$ θ_{13} $\zeta^3_{2(0)} = \zeta^3_{2(3)}$

k=3 $\hat{\theta}_{3(1)}$ θ_{12} $\zeta^3_{3(0)} = \zeta^3_{3(3)} = 0$

$$\cos(\hat{\theta}_{3(1)}) = \frac{(m_0^2+m_3^2-\sigma_3)(m_0^2+m_1^2-\sigma_1) - 2m_0^2(\sigma_2 - m_3^2 - m_1^2)}{\lambda^{1/2}(m_0^2, m_1^2, \sigma_1)\lambda^{1/2}(m_0^2, \sigma_3, m_3^2)},$$

$$\cos(\hat{\theta}_{1(2)}) = \frac{(m_0^2+m_1^2-\sigma_1)(m_0^2+m_2^2-\sigma_2) - 2m_0^2(\sigma_3 - m_1^2 - m_2^2)}{\lambda^{1/2}(m_0^2, m_2^2, \sigma_2)\lambda^{1/2}(m_0^2, \sigma_1, m_1^2)},$$

$$d^j_{\lambda\lambda'}(\hat{\theta}_{2(1)}) = (-1)^{\lambda-\lambda'} d^j_{\lambda\lambda'}(\hat{\theta}_{1(2)}),$$

$$\cos(\zeta^3_{1(3)}) = \frac{2m_3^2(\sigma_2 - m_0^2 - m_2^2) + (m_0^2 + m_3^2 - \sigma_3)(\sigma_1 - m_3^2 - m_2^2)}{\lambda^{1/2}(m_0^2, m_3^2, \sigma_3)\lambda^{1/2}(\sigma_1, m_3^2, m_2^2)},$$

$$\cos(\zeta^3_{3(2)}) = \frac{2m_3^2(\sigma_1 - m_0^2 - m_1^2) + (m_0^2 + m_3^2 - \sigma_3)(\sigma_2 - m_3^2 - m_1^2)}{\lambda^{1/2}(m_0^2, m_3^2, \sigma_3)\lambda^{1/2}(\sigma_2, m_3^2, m_1^2)},$$

$$\cos(\theta_{12}) = \frac{2\sigma_3(\sigma_2 - m_3^2 - m_1^2) - (\sigma_3 + m_1^2 - m_2^2)(m_0^2 - \sigma_3 - m_3^2)}{\lambda^{1/2}(m_0^2, m_3^2, \sigma_3)\lambda^{1/2}(\sigma_3, m_1^2, m_2^2)},$$

$$\cos(\theta_{23}) = \frac{2\sigma_1(\sigma_3 - m_1^2 - m_2^2) - (\sigma_1 + m_2^2 - m_3^2)(m_0^2 - \sigma_1 - m_1^2)}{\lambda^{1/2}(m_0^2, m_1^2, \sigma_1)\lambda^{1/2}(\sigma_1, m_2^2, m_3^2)},$$

$$\cos(\theta_{31}) = \frac{2\sigma_2(\sigma_1 - m_2^2 - m_3^2) - (\sigma_2 + m_3^2 - m_1^2)(m_0^2 - \sigma_2 - m_2^2)}{\lambda^{1/2}(m_0^2, m_2^2, \sigma_2)\lambda^{1/2}(\sigma_2, m_3^2, m_1^2)}.$$

(123) (321)

$\delta_{\nu,\tau}$

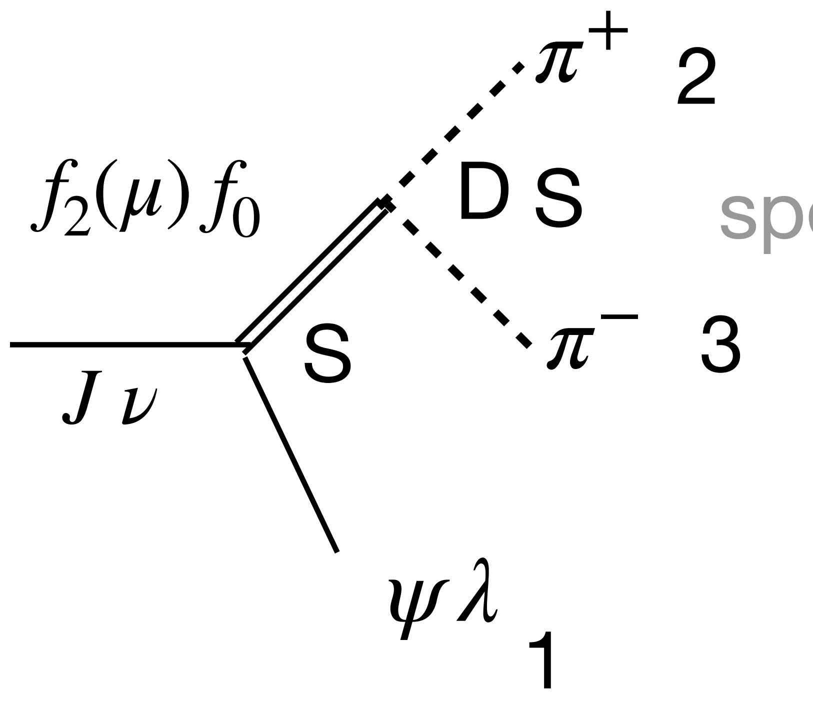
k=1: $$O^\nu_\lambda = \sum_{\tau\lambda'} BW(s;Y)\, d^1_{\nu,\tau}(\hat{\theta}_{1(1)})\, h^{Y:\pi Z}_\tau\, BW(\sigma_1;Z)\, d^1_{\tau,-\lambda'}(\theta_{23})\, h^{Z:\pi\psi}_{\lambda'}\, d^1_{\lambda',\lambda}(\zeta^3_{1(0)})$$

k=2: $$O^\nu_\lambda = \sum_{\tau\lambda'} BW(s;Y)\, d^1_{\nu,\tau}(\hat{\theta}_{2(1)})\, h^{Y:\pi Z}_\tau\, BW(\sigma_2;Z)\, d^1_{\tau,-\lambda'}(\theta_{13})\, h^{Z:\pi\psi}_{\lambda'}\, d^1_{\lambda',\lambda}(\zeta^3_{2(0)})$$

1 $\delta_{\lambda',\lambda}$

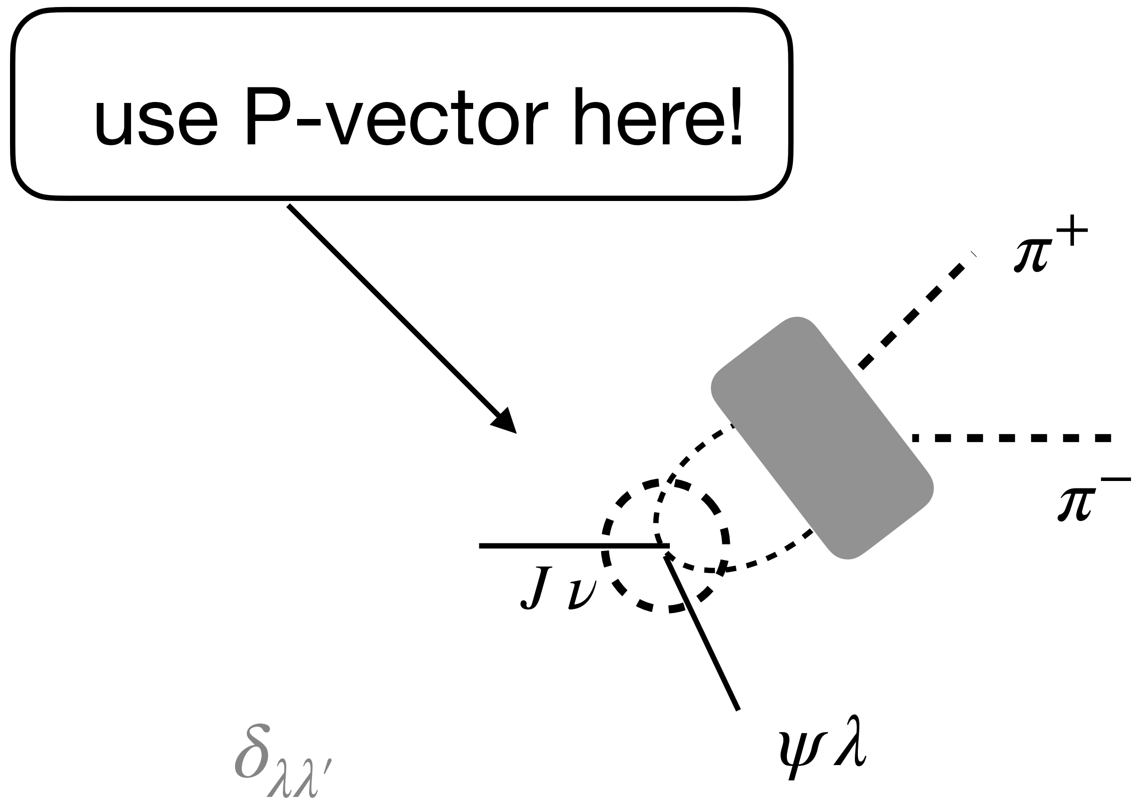
k=3: $$O^\nu_\lambda = \sum_{\lambda'} BW(s;Y)\, d^1_{\nu,-\lambda'}(\hat{\theta}_{3(1)})\, h^{Y:f\psi}_{\lambda'}\, BW(\sigma_3;f)\, d^0_{0,0}(\theta_{12})\, h^{f:\pi\pi}\, d^1_{\lambda',\lambda}(\zeta^3_{3(0)})$$

Ex. $e^+e^- \to \pi\pi J/\psi$



spectator: k=1 $\hat{\theta}_{1(1)} = 0$ θ_{23}

$\zeta^1_{1(0)} = 0$



Notice that this gives predictions for $K\bar{K}J/\psi$ and $\eta\eta J/\psi$

$\delta_{\nu,-\lambda'}$ $\quad 1 \quad$ $\delta_{\lambda\lambda'}$

$$^{(1)}O^{\nu}_{\lambda} = \sum_{R\lambda'} BW(s;Y)\, d^1_{\nu,0-\lambda'}(\hat{\theta}_{1(1)})\, h^{Y:f_R\psi}_{0\lambda'}\, BW(\sigma_1; f_R)\, d^0_{00}(\theta_{23})\, h^{f_R:\pi\pi}_{00}\, d^1_{\lambda',\lambda}(\zeta^1_{1(0)})$$

$$\to \delta_{\nu,-\lambda}\, BW(s;Y) \sum_{\alpha\beta} h^{Y:(\alpha\beta)_S\psi}_{0\lambda}\, T_{\alpha\beta:\pi\pi}(\sigma_1)$$

$$\approx \delta_{\nu,-\lambda}\, BW(s;Y)\; h^{Y:(\pi\pi)_S\psi}_{0\lambda}\, T_{\pi\pi:\pi\pi}(\sigma_1)$$

with $\quad T = (1+KC)^{-1}K = K(1+CK)^{-1},\; C = R - i\rho$

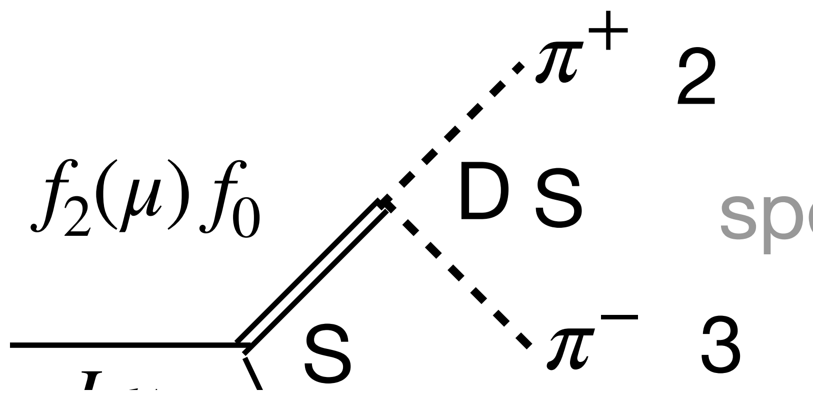
So we have $\delta_{\nu,-\lambda}\, BW(s;Y) \displaystyle\sum_{\gamma} (1+KC)^{-1}_{\pi\pi:\gamma} \left(\frac{g_{R:\gamma}\, g_{R:\pi\pi}}{m_R^2 - s} + f_{\gamma:\pi\pi} \right) h^{Y:(\pi\pi)\psi}$

Alternatively, we can bend the ψ line back and think of this as production from a $J\psi$ P-vector

$$\delta_{\nu,-\lambda}\, BW(s;Y) \sum_{\gamma} (1+KC)^{-1}_{\pi\pi:\gamma} \left(\frac{g_{R:Y\psi}\, g_{R:\gamma}}{m_R^2 - s} + f_{Y\psi:\gamma} \right)$$

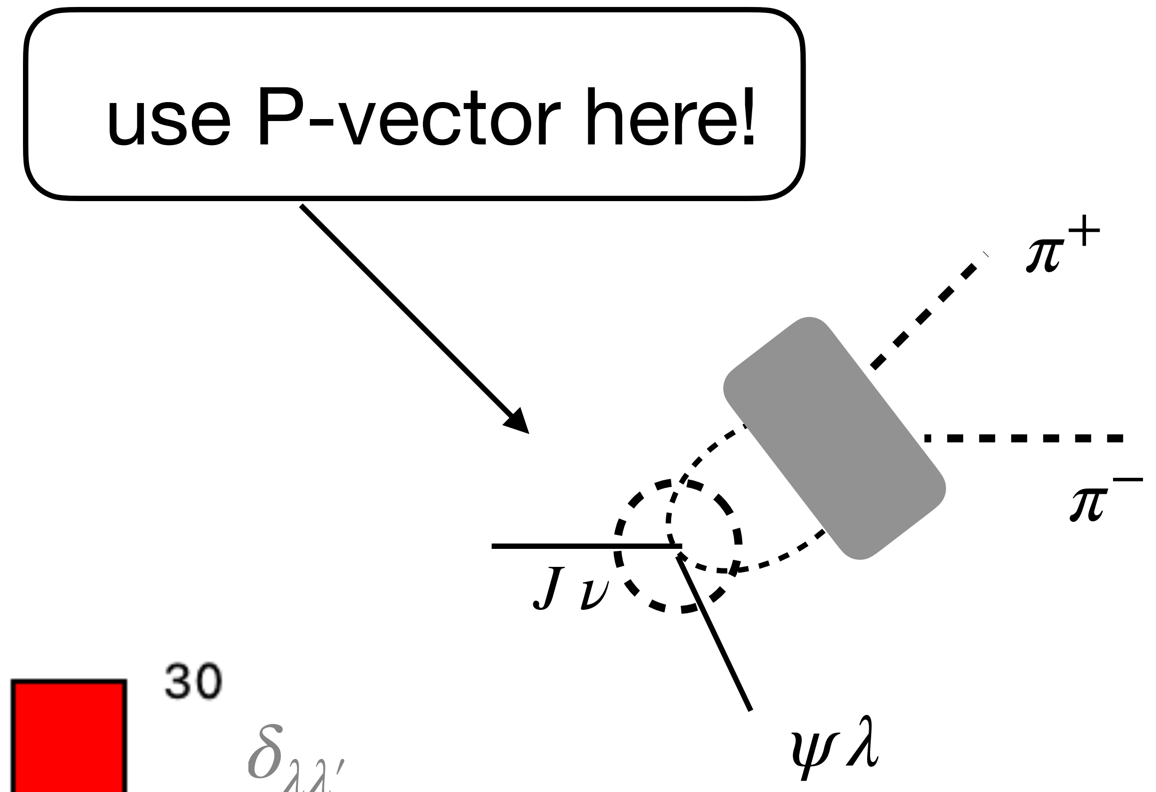
The formulations are equivalent under $f_{\gamma:Y\psi} = f_{\gamma:\pi\pi} h^{Y:(\pi\pi)\psi}$ and $g_{R:Y\psi} = g_{R:\pi\pi} h^{Y:(\pi\pi)\psi}$.

Ex. $e^+e^- \to \pi\pi J/\psi$

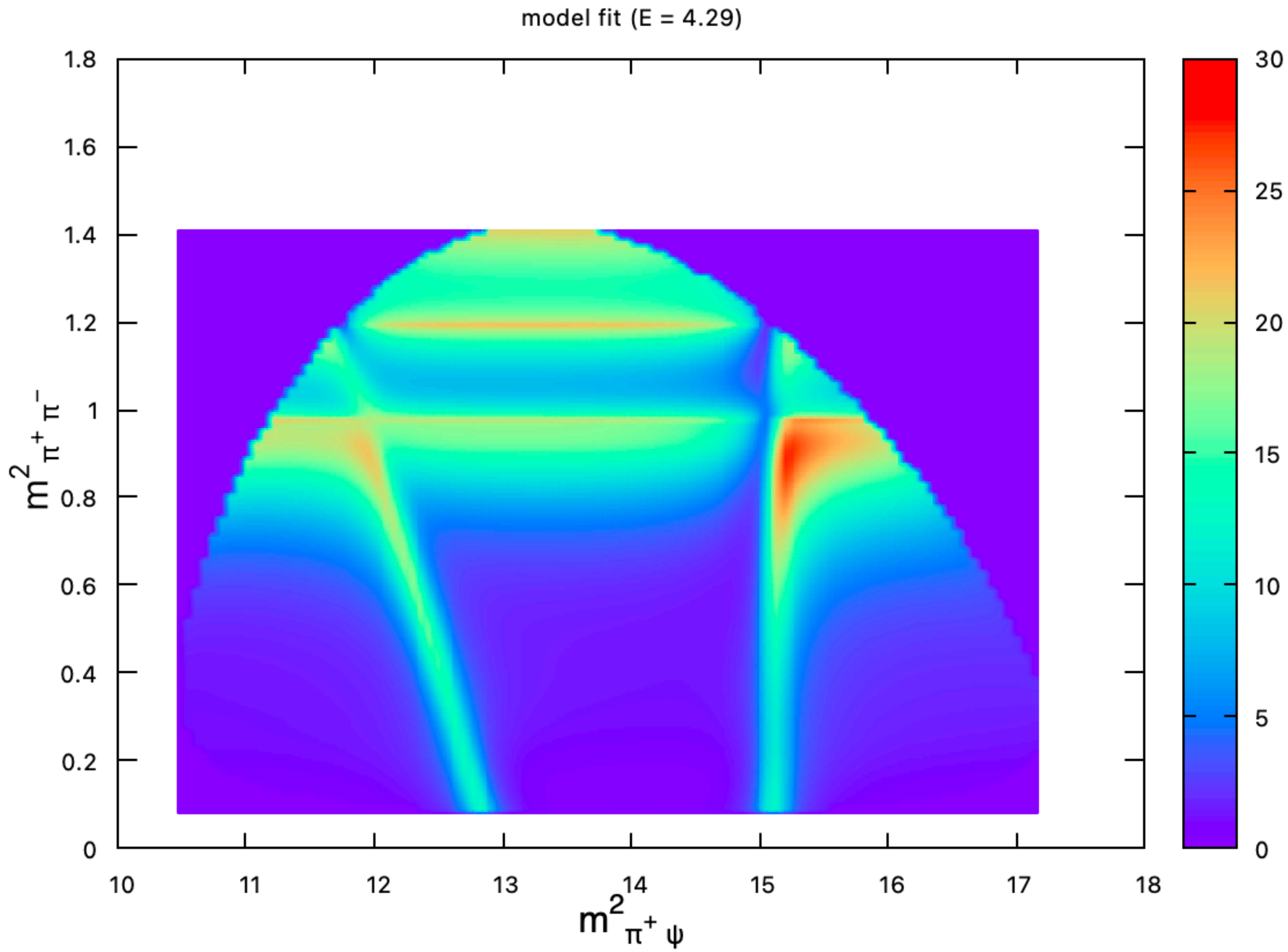


spectator: k=1 $\hat{\theta}_{1(1)} = 0$ θ_{23}

$\zeta^1_{1(0)} = 0$



Notice that this gives predictions for $K\bar{K}J/\psi$ and $\eta\eta J/\psi$



$\delta_{\lambda\lambda'}$

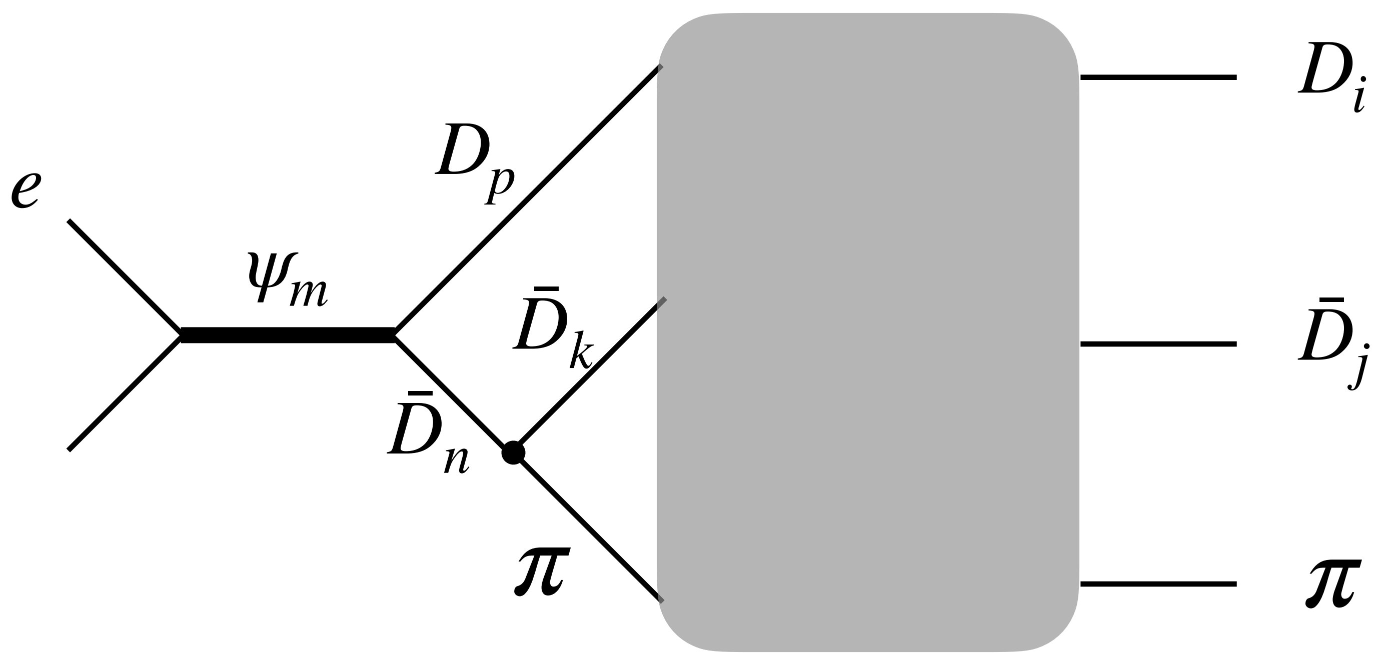
$K = K(1 + CK)^{-1}$, $C = R - i\rho$

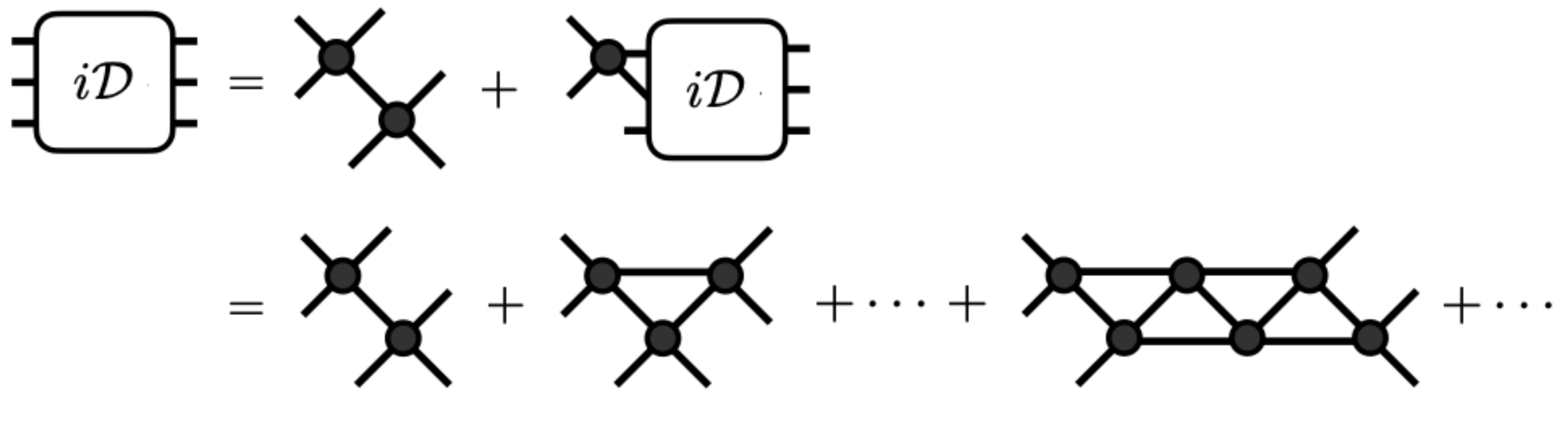
production from a $J\psi$ P-vector

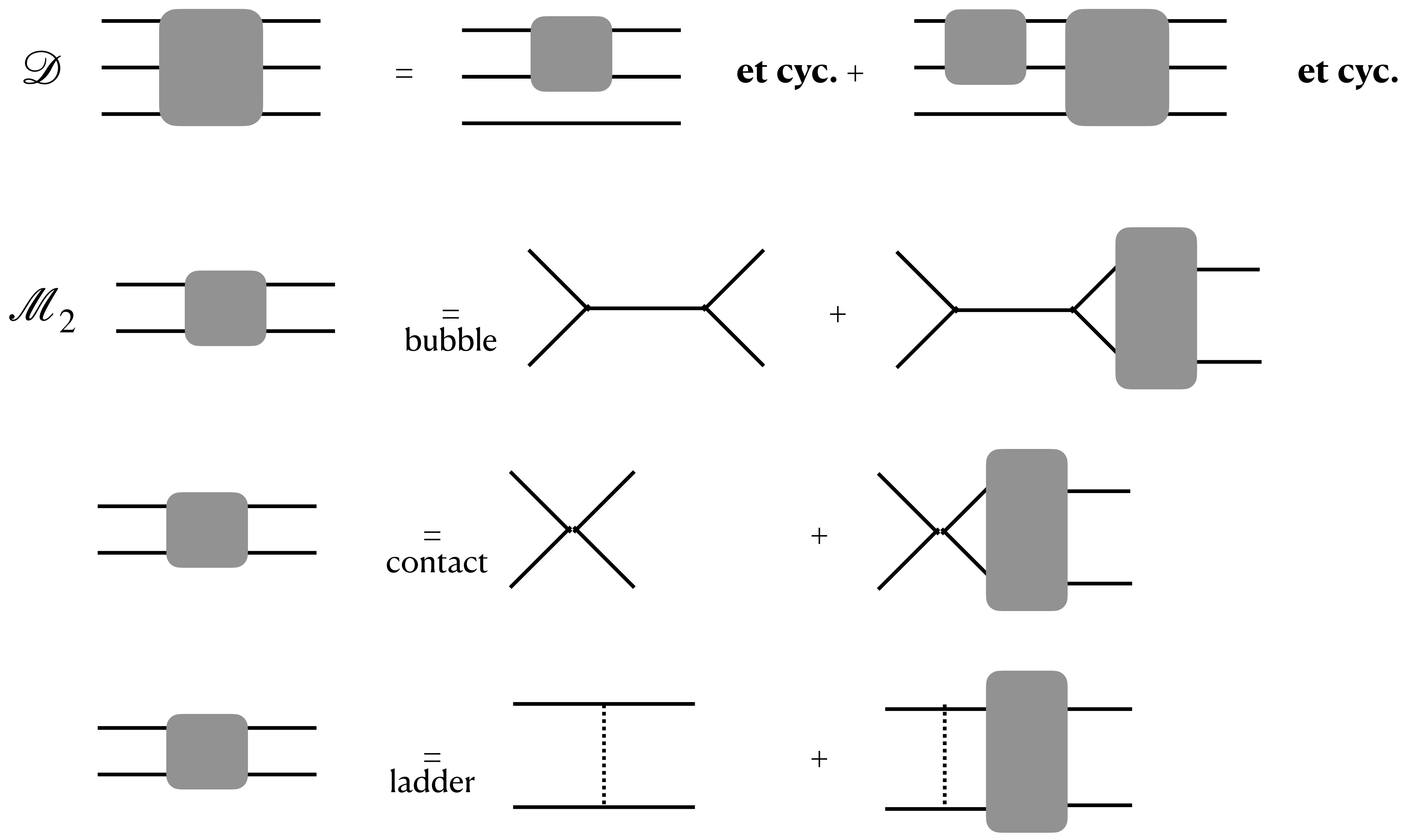
ulations are equivalent under $f_{\gamma:Y\psi} = f_{\gamma:\pi\pi} h^{Y:(\pi\pi)\psi}$ and

$_{R:\pi\pi} h^{Y:(\pi\pi)\psi}$.

Ex. incorporate three-body unitarity







Sebastian Dawid, Nils Hüsken

Conclusions

- EIC will be a complementary & powerful probe of exotic hadrons

- these will almost certainly appear in complicated reactions

- which will require sophisticated modelling

- expertise developed at COMPASS, JLab/JPAC (LHCb, BEPC) will go a long way in helping!

~thank you~

The X(3872) in 1992

190

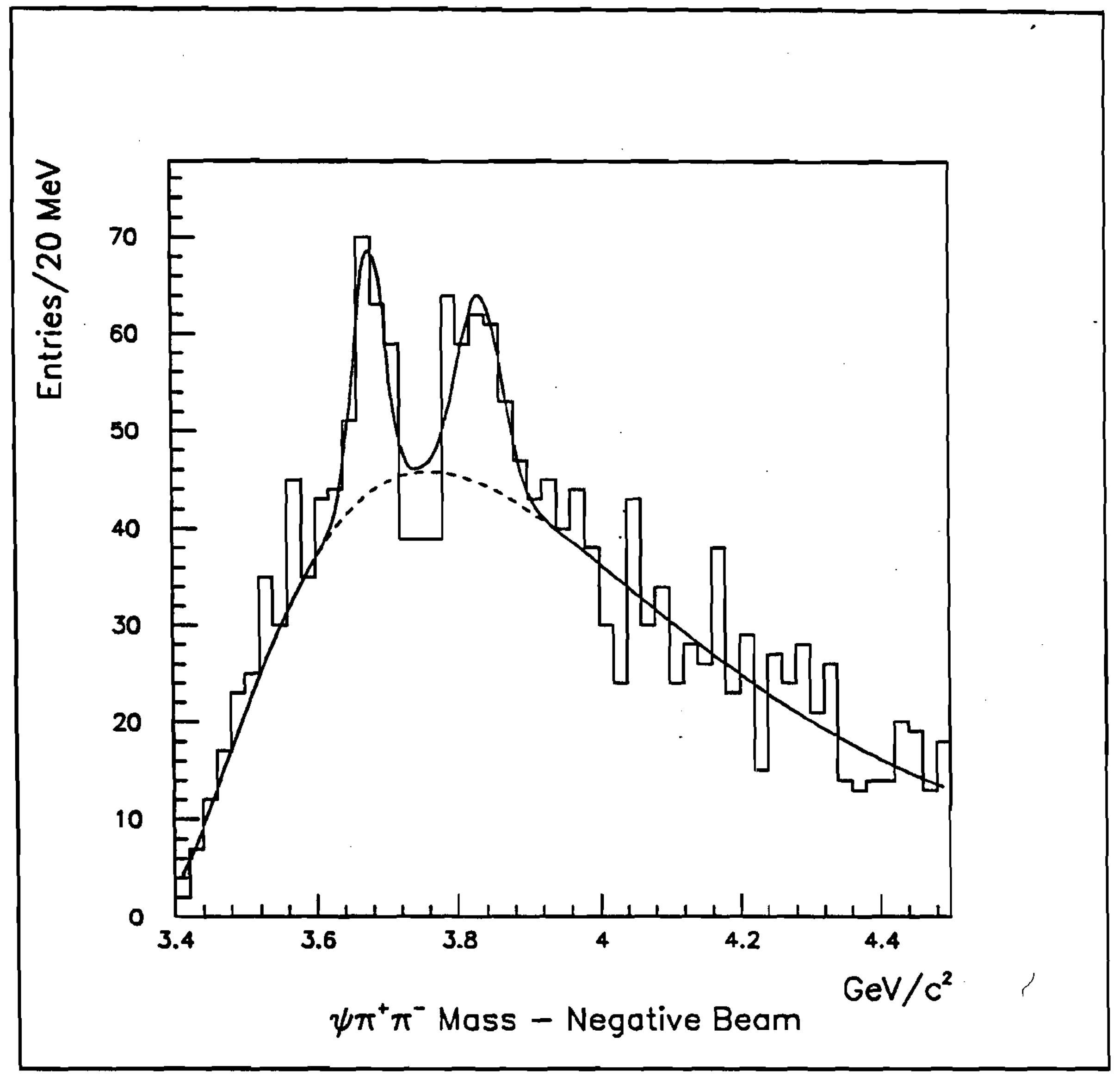
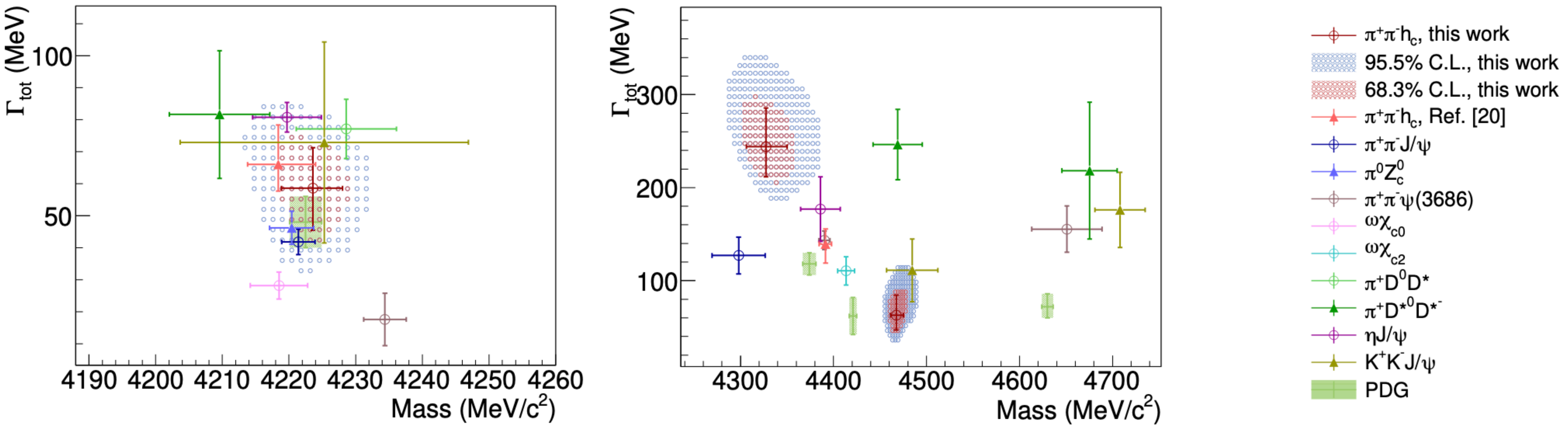


Figure 6.12 $\psi\pi\pi$ mass spectrum, standard cuts, negative beam.

A single peak above background does not fit the observed signal well. A second peak above the ψ' was added to the fit to improve this. The fit parameters are shown on the following page:

E-705

Tom LeCompte,
Northwestern thesis
E705 at FNAL

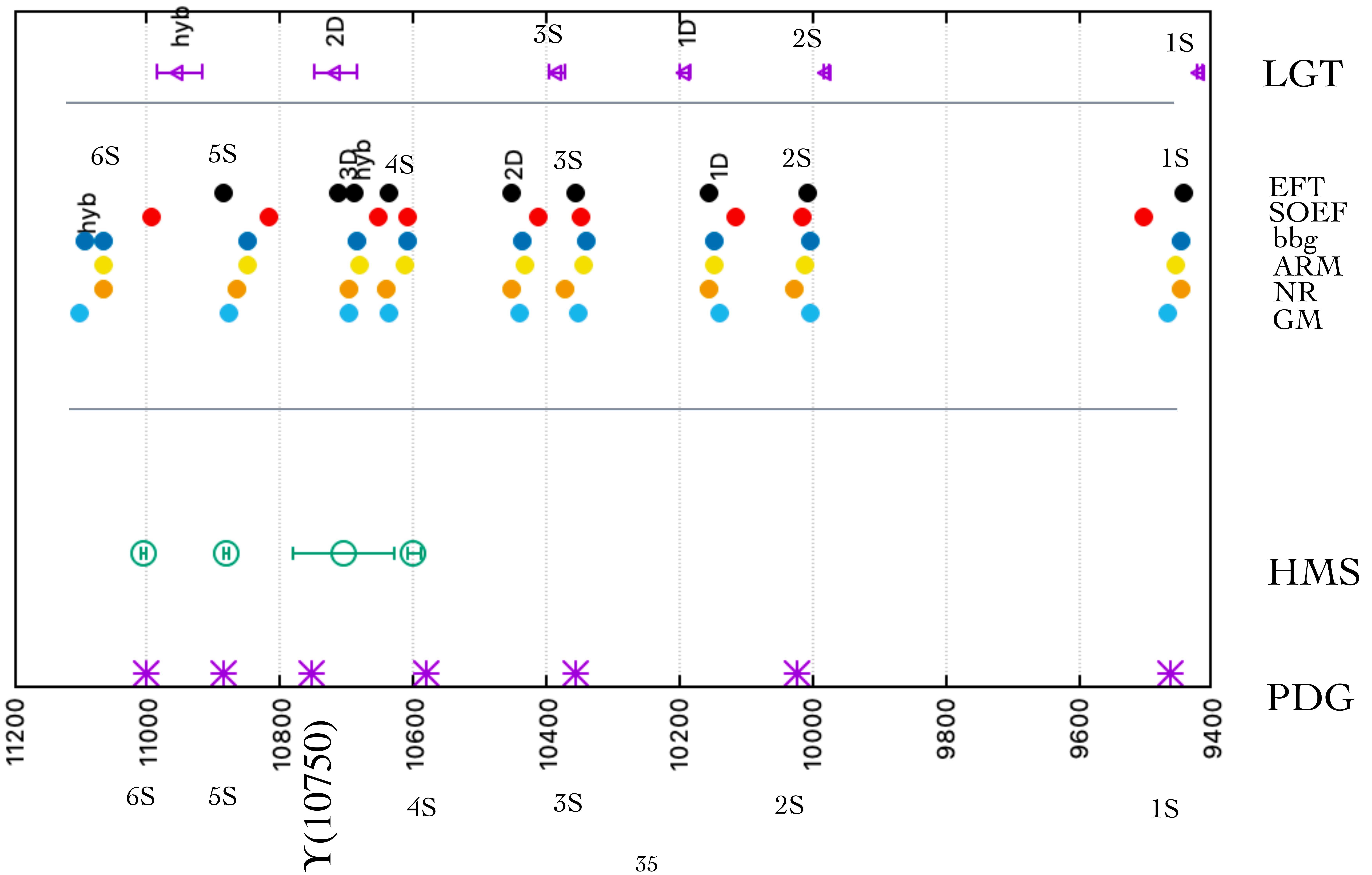


https://arxiv.org/pdf/2504.04096

vector bottomonium

masses

[BW params, poles, quark model eigenvalues, LGT plateaus]



Discussion

$\Gamma(ee)$ (keV)

state	RPP	our estimate	LS	GM	SOEF
$\Upsilon(4S)$	0.272	(0.003 - 0.62)	0.31	0.39	0.21
$\Upsilon(5S)$	0.31	(0.037 - 0.068)	0.28	0.33	0.18
$\Upsilon(6S)$	0.13	(0.043 - 0.074)	0.26	0.27	0.15
$\Upsilon(10750)$	(0.01 - 0.40)[a]	(0.004 - 0.10)		2.38 eV [b]	

[a] from ambiguous solutions in Ref. [5]

[b] assuming a $3D$ state

JPAC Helicity Formalism Summary

DalitzPlotDecomposition.pdf

20.5.23

$$M^{\Lambda}_{\{\lambda\}} = \sum_{\nu} D^{J*}_{\Lambda,\nu}(\alpha,\beta,\gamma) O^{\nu}_{\{\lambda\}},$$

$$\mathrm{d}\sigma/\mathrm{d}\Phi_3 = N \sum_{\Lambda,\Lambda'} \rho_{\Lambda\Lambda'} \sum_{\nu,\nu'} D^{J*}_{\Lambda,\nu}(\alpha,\beta,\gamma) D^{J}_{\Lambda',\nu'}(\alpha,\beta,\gamma) \sum_{\{\lambda\}} O^{\nu}_{\{\lambda\}} O^{\nu'*}_{\{\lambda\}}, \quad (3)$$

where N is an overall normalization factor, and ρ is the spin-density matrix of the decaying particle. It is clear that in the unpolarized case, when $\rho_{\Lambda\Lambda'} \sim \delta_{\Lambda\Lambda'}$, the dependence on α, β, and γ drops out. Conversely, when one integrates over the Euler angles, the remaining distribution is not sensitive to the polarization.

The amplitude $M^{\Lambda}_{\{\lambda\}}$ can be written as a sum of three terms, each one defining its own aligned configuration,

$$O^{\nu}_{\{\lambda\}}(\{\sigma\}) = \sum_{(ij)k} \sum_{s}^{(ij)\to i,j} \sum_{\tau} \sum_{\{\lambda'\}} n_J n_s \, d^{J}_{\nu,\tau-\lambda'_k}(\hat{\theta}_{k(1)}) H^{0\to(ij),k}_{\tau,\lambda'_k} X_s(\sigma_k) \, d^{s}_{\tau,\lambda'_i-\lambda'_j}(\theta_{ij}) H^{(ij)\to i,j}_{\lambda'_i,\lambda'_j}$$
$$\times \, d^{j_1}_{\lambda'_1,\lambda_1}(\zeta^{1}_{k(0)}) \, d^{j_2}_{\lambda'_2,\lambda_2}(\zeta^{2}_{k(0)}) \, d^{j_3}_{\lambda'_3,\lambda_3}(\zeta^{3}_{k(0)}),$$

helicity amps

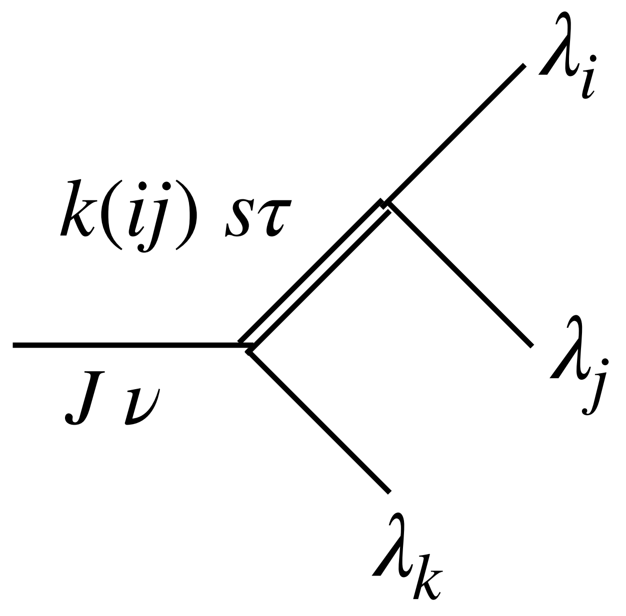
3 topologies

possible isobars

isobar helicities

X_s = isobar BW model

final state Wigner spin rotations



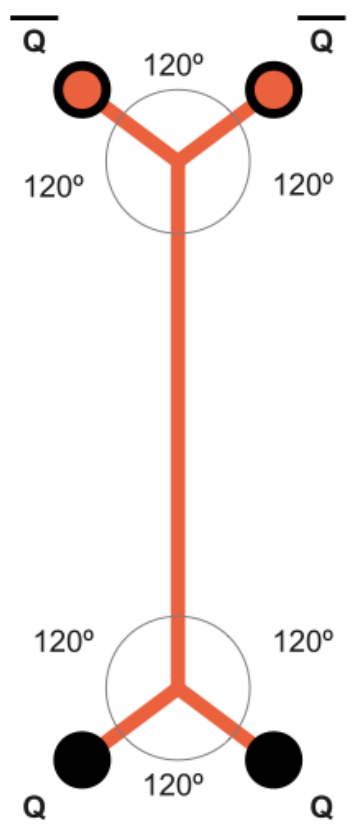
k(ij) refers to the (ij)-isobar /or/ the k spectator

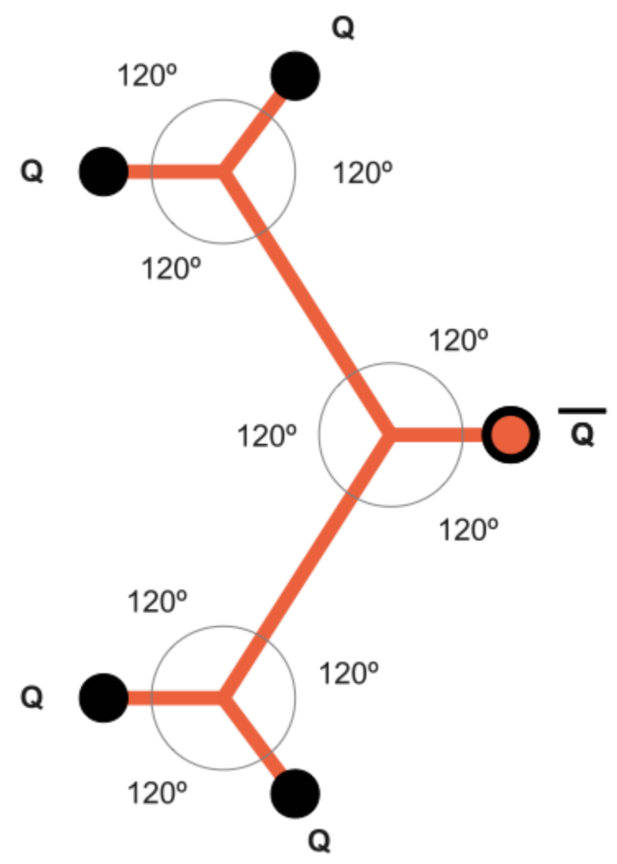
$\sigma_k = m^2_{ij}$ is the (ij) isobar invariant mass

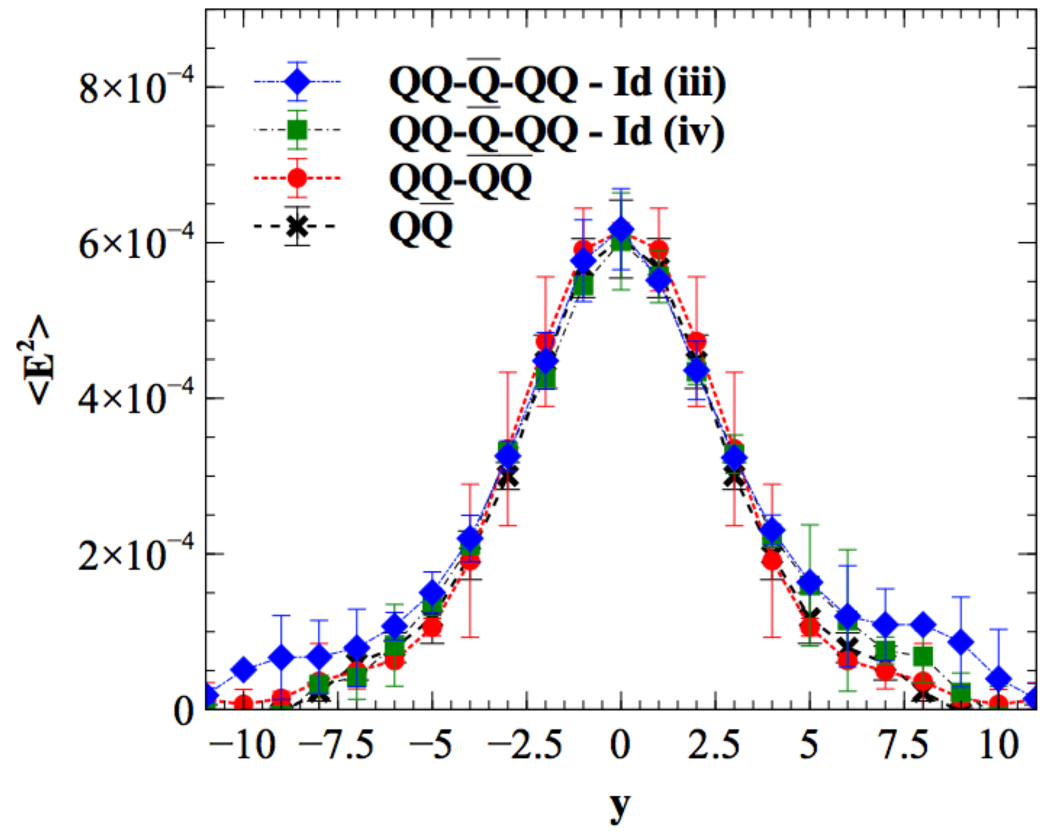
$\{\lambda_i\}$ are the final state polarisations

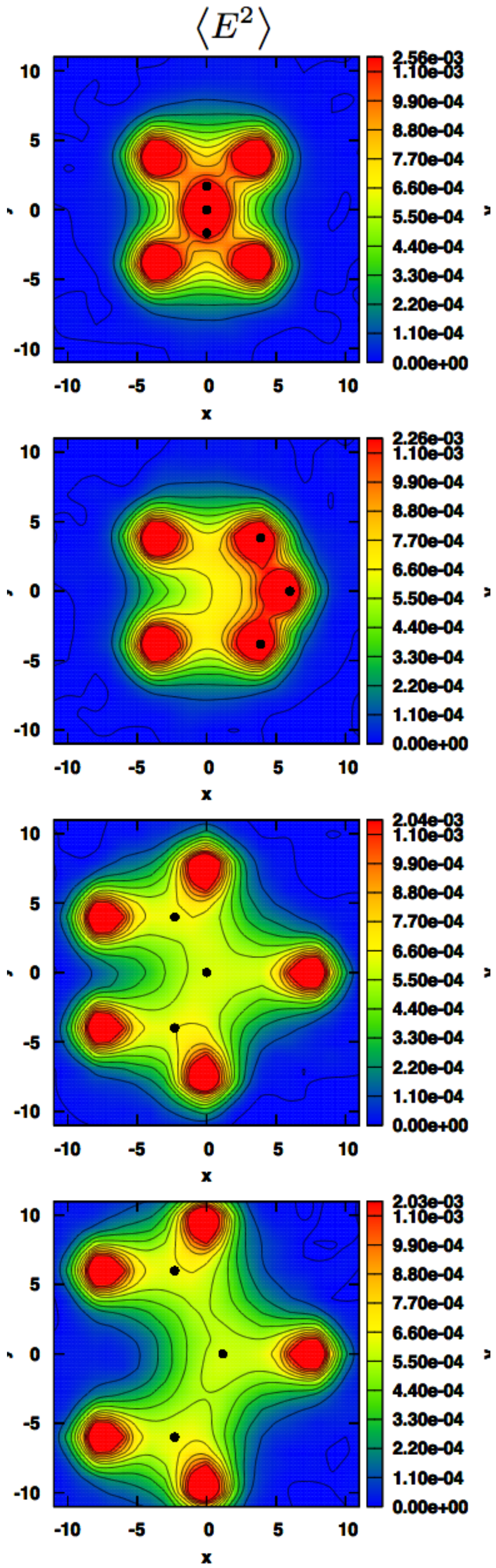
$\hat{\theta}_{k(1)}$, θ_{ij}, $\zeta^{\ell}_{k(0)}$ are fixed angular functions of invariants w/ $\hat{\theta}_{1(1)} = 0$ (thus the reference frame topology simplifies)

What is the interaction?









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