Transition form factors of C-even quarkonia in photon-photon processes

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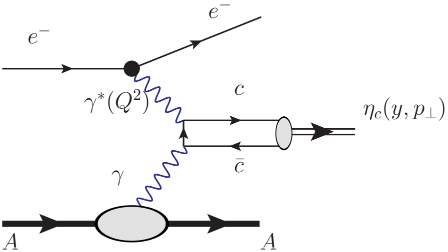
Electron-ion collisions



the nuclear radius: $R_A = r_0 A^{1/3}$, with $r_0 = 1.1$ fm

$$\sigma(eA \to e\eta_c A) = \int d\omega_e dQ^2 \frac{d^2 N_e}{d\omega_e dQ^2} \times \sigma(\gamma^* A \to \eta_c A)$$

$$\sigma(\gamma^* A \to \eta_c A) = \int d\omega_A \frac{dN_A}{d\omega_A} \times \sigma_{\rm TT}(\gamma^*\gamma \to \eta_c; W_{\gamma\gamma}, Q^2, 0)$$

$$W_{\gamma\gamma} = \sqrt{4\omega_e\omega_A - p_\perp^2}$$

$$\omega_e = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{+y}$$

$$\omega_A = \frac{\sqrt{M^2 + p_\perp^2}}{2} e^{-y}$$

$$\frac{dN_A}{d\omega_A} = \frac{2Z^2\alpha_{em}}{\pi\omega_A}\left[\xi K_0(\xi)K_1(\xi) - \frac{\xi^2}{2}(K_1^2(\xi) - K_0^2(\xi))\right]$$

$\xi = R_A\omega_A/\gamma_L$, K_0 and K_1 -modified Bessel functions

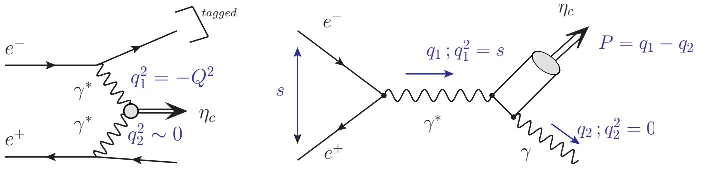
i.e.: *Ann.Rev.Nucl.Part.Sci*.55, 271(2005)

$$p_\perp^2 = \left(1 - \frac{\omega_e}{E_e}\right)Q^2$$

$$\frac{d^2 N_e}{d\omega_e dQ^2} = \frac{\alpha_{em}}{\pi\omega_e Q^2}\left[\left(1 - \frac{\omega_e}{E_e}\right)\left(1 - \frac{Q^2_{min}}{Q^2}\right) + \frac{\omega_e^2}{2E_e^2}\right]$$

$Q^2_{min} = m_e^2\omega_e^2/[E_e(E_e - \omega_e)]$ and $Q^2_{max} = 4E_e(E_e - \omega_e)$

$\gamma^*\gamma^*$ transition form factor



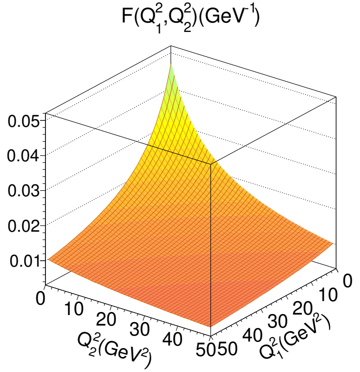
- We are interested in the coupling of η_Q to two (off-shell) photons

$$\mathcal{M}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_Q) = i\,4\pi\alpha_{\rm em}\,\varepsilon_{\mu\nu\alpha\beta}\epsilon_1^\mu\epsilon_2^\nu q_1^\alpha q_2^\beta \underbrace{\mathcal{F}_{\eta_Q}(t_1,t_2)}_{\text{transition FF}} \qquad t_1 = \frac{q_1^2}{m_Q^2}, \quad t_2 = \frac{q_2^2}{m_Q^2}\,.$$

- the $\gamma^*\gamma^*$ transition form factor describes the following observables:

1. **on-shell**: $t_1 = t_2 = 0$: the decay width $\eta_Q \to \gamma\gamma$.
2. **space-like region**: $t_1 < 0, t_2 = 0$: exclusive production of η_Q in *single-tagged* e^+e^- collisions. $t_1 < 0, t_2 < 0 \to$ *double tagged* e^+e^- collisions.
3. **time–like region**: $t_1 > 0, t_2 = 0$: exclusive production of $\eta_Q\gamma$ in e^+e^- annihilation; Dalitz decay $\eta_Q \to \gamma\ell^+\ell^-$ or $\eta_Q \to 4\ell$.

$\gamma^*\gamma^* \to \eta_c(1S)$ Transition Form Factor



- In a Drell-Yan like frame, where $q_1 = q_1^+ n^+ + q_{1\perp}, q_2 = q_2^- n^- + q_{2\perp}$, and $q_i^2 = -\boldsymbol{q}_i^2$, the transition factor has a light-front wave function representation. Brodsky & Huang '99.

$$n^{+\mu} n^{-\nu} M_{\mu,\nu}(\gamma^*(q_1)\gamma^*(q_2) \to \eta_c)$$
$$= -i4\pi\alpha_{em}(q_1^x q_2^y - q_1^y q_2^x) F(Q_1^2, Q_2^2)$$

$$F(Q_1^2, Q_2^2) = e_c^2 \sqrt{N_c} 4m_c \cdot \int \frac{dz d^2\boldsymbol{k} \psi_S(z, \boldsymbol{k})}{z(1-z)16\pi^3} \left\{ \frac{(1-z)}{(\boldsymbol{k} - (1-z)\boldsymbol{q}_2)^2 + \varepsilon^2} + \frac{z}{(\boldsymbol{k} + z\boldsymbol{q}_2)^2 + \varepsilon^2} \right\},$$
$$Q_i^2 = \vec{q}_{i\perp}, \varepsilon^2 = z(1-z)\boldsymbol{q}_1^2 + m_c^2$$

- $z, 1-z$ are LF-"+" momentum fractions of quark/antiquark, $\boldsymbol{k}$ is relative transverse momentum. NR-limit: $\boldsymbol{k} \to 0$, $z \to 1/2$.

Two approaches to quarkonium light front wave functions

Terentev substitution - LFWF from potential model

- Quark three-momentum in bound state rest frame

$$\vec{k} = (\boldsymbol{k}, k_z), \qquad k_z = \Big(z - \frac{1}{2}\Big) M_{c\bar{c}} \qquad M^2_{c\bar{c}} = \frac{\boldsymbol{k}^2 + m_c^2}{z(1-z)}$$

- radial WF $u_{nP}(k)$ becomes (with appropriate Jacobian) radial LFWF $\psi(z, \boldsymbol{k})$
- canonical spin is substituted by LF helicity via **Melosh transform**

$$\xi_Q = R(z, \boldsymbol{k}) \chi_Q \,, \qquad \xi^*_{\bar{Q}} = R^*(1-z, -\boldsymbol{k}) \chi^*_{\bar{Q}} \qquad R(z, \boldsymbol{k}) = \frac{m_c + z M_{c\bar{c}} - i\vec{\sigma} \cdot (\vec{n} \times \boldsymbol{k})}{\sqrt{(m_c + z M_{c\bar{c}})^2 + \boldsymbol{k}^2}}$$

- We use a variety of interquark potentials summarized in Eur. Phys. J. C **79** (2019) no.6, 495

Basis light front quantization (BLFQ)

- bound state WFs from effective LF-Hamiltonian

$$H_{\rm eff} |\chi_c; \lambda_A, P_+, \boldsymbol{P}\rangle = M^2_\chi |\chi_c; \lambda_A, P_+, \boldsymbol{P}\rangle \,,$$

- we use LFWFs from Y. Li, P. Maris and J. P. Vary, Phys. Rev. D **96** (2017), 016022
- effective Hamiltonian which contains a term motivated by a “soft-wall” confinement from LF-holography, as well as a longitudinal confinement potential supplemented by one gluon exchange including the full spin-structure.

Light-Front Wave Functions from rest-frame: $J = 0$

$$\psi_{NR}(\vec{p},\lambda_1,\lambda_2) = \sum_{m_l,\,m_s} \underbrace{Y_{l\,m_l}(\hat{p})\langle \frac{1}{2}\frac{1}{2},\lambda_1\lambda_2|S,m_s\rangle\langle L,S,m_l m_s|JJ_z\rangle}_{\text{spin-orbit}} \underbrace{\varphi(|\vec{p}|)}_{\text{radial}}$$

$$= \underbrace{\frac{1}{\sqrt{2}}\xi_Q^{\tau\dagger}\,\hat{\mathcal{O}} i\sigma_2\,\xi_{\bar{Q}}^{\bar{\tau}*}}_{\text{spin-orbit}} \underbrace{\frac{u_{nl}(p)}{p}}_{\text{radial}} \frac{1}{\sqrt{4\pi}};$$

$$\vec{p} = (\vec{p}_\perp, p_z) = \left(\boldsymbol{k}, \frac{1}{2}(2z-1)M_{Q\bar{Q}}\right).$$

$$\hat{\mathcal{O}} = \begin{cases} \mathbb{I} & S=0, L=0. \\ \frac{\vec{\sigma}\cdot\vec{p}}{p}, & S=1, L=1. \end{cases}$$

Spherical harmonic for S-wave (l =0):

$$Y_{00} = \sqrt{\frac{1}{4\pi}}$$

Spherical harmonic for P-wave (l =1):

$$\begin{aligned} Y_{10} &= \sqrt{\frac{3}{4\pi}}\cos(\theta)\,, \\ Y_{11} &= -\sqrt{\frac{3}{8\pi}}\sin(\theta)e^{i\phi}\,, \\ Y_{1-1} &= \sqrt{\frac{3}{8\pi}}\sin(\theta)e^{-i\phi}\,. \end{aligned}$$

Clebsch-Gordan coefficients

$$\langle L=1, S=1; m_l, m_s|J=0, J_z=0\rangle\,,$$

$$\begin{aligned} \langle 1,1;+1,-1|00\rangle &= \sqrt{\frac{1}{3}}\,, \\ \langle 1,1;-1,+1|00\rangle &= \sqrt{\frac{1}{3}}\,, \\ \langle 1,1;0,0|00\rangle &= -\sqrt{\frac{1}{3}}\,, \end{aligned}$$

Light-Front Wave Functions from rest-frame: $J = 0$ - Melosh-transf.

Melosh-transformation of spin-orbit part: $\xi_Q = R(z, \boldsymbol{k})\chi_Q, \quad \xi^*_{\bar{Q}} = R^*(1-z, -\boldsymbol{k})\chi^*_{\bar{Q}}$,

$$R(z, \boldsymbol{k}) = \frac{m_Q + zM - i\vec{\sigma}\cdot(\vec{n}\times\boldsymbol{k})}{\sqrt{(m_Q + zM)^2 + \boldsymbol{k}^2}}$$

$\hat{\mathcal{O}}' = R^\dagger(z, \boldsymbol{k})\mathcal{O}\, i\sigma_2 R^*(1-z, -\boldsymbol{k})(i\sigma_2)^{-1}$ from Pauli matrices properties: $i\sigma_2\vec{\sigma}^*(i\sigma_2)^{-1} = -\vec{\sigma}$

$$\hat{\mathcal{O}}' = R^\dagger(z, \boldsymbol{k})\hat{\mathcal{O}}\, R(1-z, -\boldsymbol{k})\,.$$

Pseudoscalar (S-wave)

$$\begin{pmatrix} \Psi_{++}(z,\boldsymbol{k}) & \Psi_{+-}(z,\boldsymbol{k}) \\ \Psi_{-+}(z,\boldsymbol{k}) & \Psi_{--}(z,\boldsymbol{k}) \end{pmatrix} = \frac{\psi_S(z,\boldsymbol{k})}{\sqrt{z(1-z)}} \begin{pmatrix} -k_x + ik_y & m_Q \\ -m_Q & -k_x - ik_y \end{pmatrix}$$

$$\psi_S(z,\boldsymbol{k}) = \frac{\pi}{\sqrt{2M}}\frac{u_{n0}(p)}{p}$$

Scalar (P-wave)

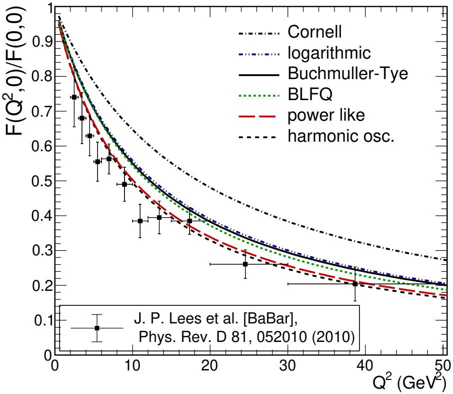
$$\begin{pmatrix} \Psi_{++}(z,\boldsymbol{k}) & \Psi_{+-}(z,\boldsymbol{k}) \\ \Psi_{-+}(z,\boldsymbol{k}) & \Psi_{--}(z,\boldsymbol{k}) \end{pmatrix} = \frac{-\psi_P(z,\boldsymbol{k})}{\sqrt{z(1-z)}} \begin{pmatrix} k_x - ik_y & m_Q(1-2z) \\ m_Q(1-2z) & -k_x - ik_y \end{pmatrix}$$

$$\psi_P(z,\boldsymbol{k}) = \frac{\pi\sqrt{M}}{\sqrt{2}\sqrt{M^2 - 4m_Q^2}}\frac{u_{n1}(p)}{p}$$

Normalisation

$$1 = \int_0^1 \frac{\mathrm{d}z}{z(1-z)}\int\frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}\sum_{\lambda\bar{\lambda}}|\Psi_{\lambda\bar{\lambda}}(z,\boldsymbol{k})|^2 = \int_0^1 \frac{\mathrm{d}z}{z(1-z)}\int\frac{\mathrm{d}^2\vec{k}_\perp}{16\pi^3}2M_{c\bar{c}}\psi(z,\boldsymbol{k})$$

Normalized transition form factor: $\gamma^*\gamma \to \eta_c$



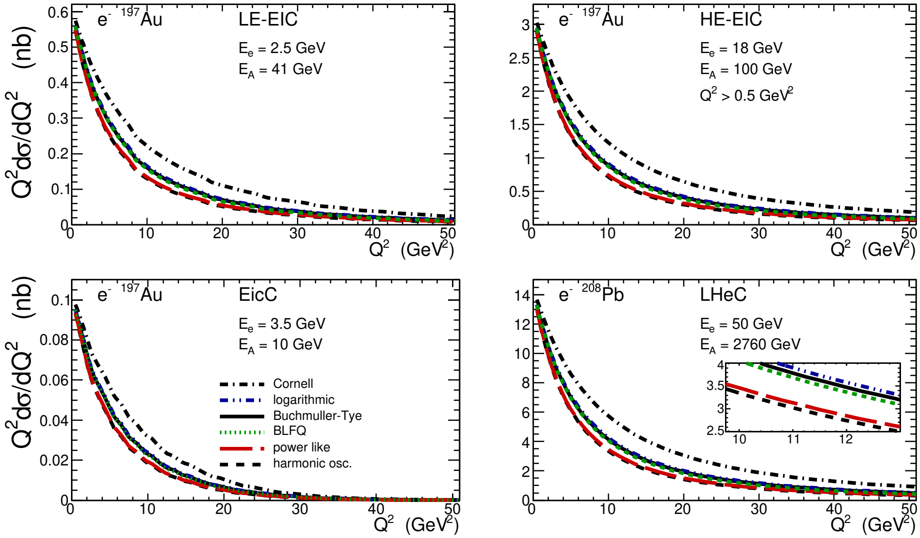
In the Melosh spin-rotation formulation $\tilde{\psi}_{\uparrow\downarrow}(z, k_\perp), \tilde{\psi}_{\uparrow\uparrow}(z, k_\perp)$, are related to the same radial wave function $\psi(z, k_\perp)$ as:

$$\tilde{\psi}_{\uparrow\downarrow}(z, k_\perp) \to \frac{m_c}{\sqrt{z(1-z)}} \psi(z, k_\perp)\,,$$

$$\tilde{\psi}_{\uparrow\uparrow}(z, k_\perp) \to \frac{-|\vec{k}_\perp|}{\sqrt{z(1-z)}} \psi(z, k_\perp)\,,$$

$$F(Q^2, 0) = e_c^2 \sqrt{N_c}\, 4 \int \frac{dz d^2\vec{k}_\perp}{\sqrt{z(1-z)}16\pi^3} \left\{ \frac{1}{\vec{k}_\perp{}^2 + \varepsilon^2} \tilde{\psi}_{\uparrow\downarrow}(z, k_\perp) + \frac{\vec{k}_\perp{}^2}{[\vec{k}_\perp{}^2 + \varepsilon^2]^2} \left(\tilde{\psi}_{\uparrow\downarrow}(z, k_\perp) + \frac{m_c}{k_\perp} \tilde{\psi}_{\uparrow\uparrow}(z, k_\perp) \right) \right\},$$

Differential distribution in photon virtuality



$Q^2 > 0.5\,\mathrm{GeV}^2$

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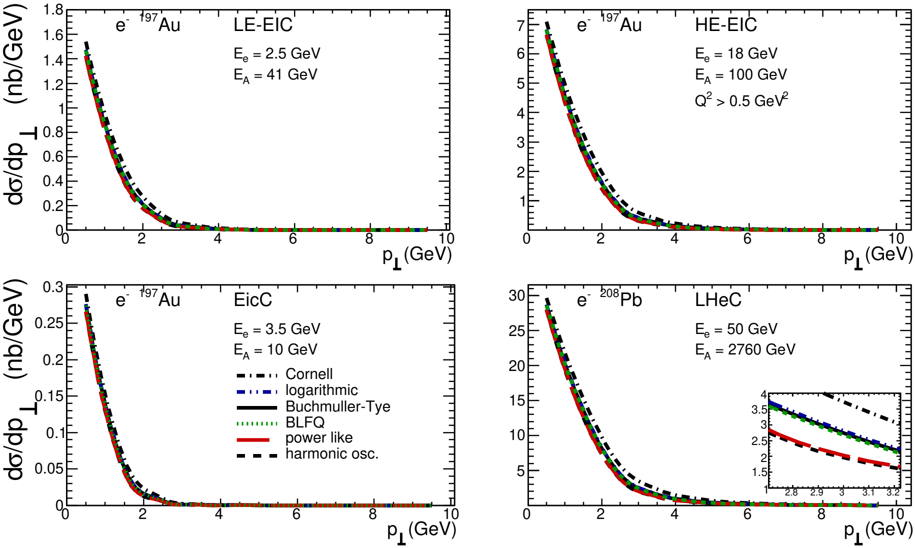
Differential distribution in transverse momentum of η_c



$Q^2 > 0.5\,\mathrm{GeV}^2$

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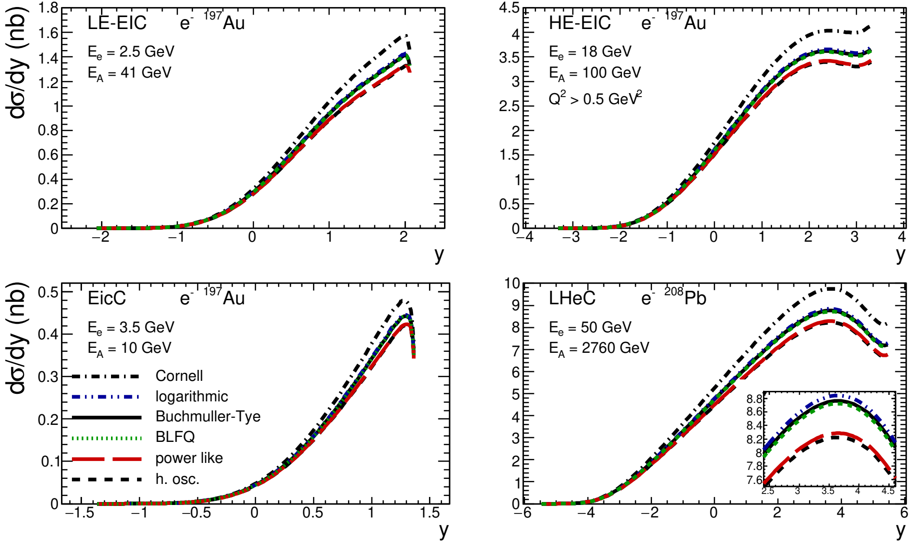
Differential distribution in rapidity of η_c



$Q^2 > 0.5\,\mathrm{GeV}^2$

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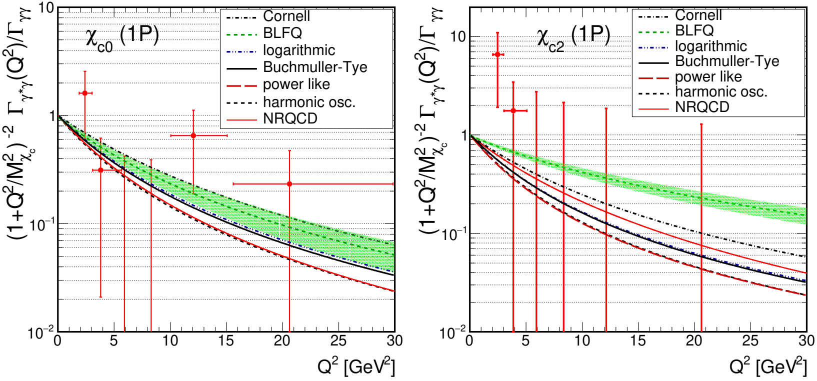
$\gamma^*\gamma$ cross-section and off-shell width



Off-shell decay width $\Gamma^*(Q^2)$ for χ_{c0} (on the l.h.s.) and χ_{c2} (on the r.h.s.) compared to the Belle data Phys.Rev.D 97 (2018) 5, 052003.

$$\Gamma_{\gamma^*\gamma}(Q^2) = \Gamma^*_{\mathrm{TT}}(Q^2) + \epsilon_0 2\Gamma^*_{\mathrm{LT}}(Q^2)$$

$$\Gamma^*(Q^2) = \frac{(4\pi\alpha_{\mathrm{em}})^2}{16\pi M} F^2_{\mathrm{TT}}(Q^2)\,.$$

$$\Gamma^*_{\mathrm{TT}}(Q^2) = (4\pi\alpha_{\mathrm{em}})^2 \left\{ \frac{F^2_{\mathrm{TT2}}(Q^2)}{80\pi M} + \frac{M^3 F^2_{\mathrm{TT0}}(Q^2)}{120\pi}\left(1+\frac{Q^2}{M^2}\right)^4 \right\}.$$

$$\Gamma^*_{\mathrm{LT}}(Q^2) = (4\pi\alpha_{\mathrm{em}})^2 \frac{1}{160\pi}\left(1+\frac{Q^2}{M^2}\right)^2 M Q^2 F^2_{\mathrm{LT}}(Q^2)\,.$$

$\gamma^*\gamma^*$-transition form factors for $J^{PC} = 1^{++}$ axial mesons

$$\frac{1}{4\pi\alpha_{\rm em}}\mathcal{M}_{\mu\nu\rho} = i\Big(q_1 - q_2 + \frac{Q_1^2 - Q_2^2}{(q_1+q_2)^2}(q_1+q_2)\Big)_\rho \tilde{G}_{\mu\nu}\,\frac{M}{2X}F_{\rm TT}(Q_1^2,Q_2^2) + ie^L_\mu(q_1)\tilde{G}_{\nu\rho}\,\frac{1}{\sqrt{X}}F_{\rm LT}(Q_1^2,Q_2^2) + ie^L_\nu(q_2)\tilde{G}_{\mu\rho}\,\frac{1}{\sqrt{X}}F_{\rm TL}(Q_1^2,Q_2^2)\,.$$

- Above we introduced

$$\tilde{G}_{\mu\nu} = \varepsilon_{\mu\nu\alpha\beta}q_1^\alpha q_2^\beta\,,\; X = (q_1\cdot q_2)^2 - q_1^2 q_2^2$$

and the polarization vectors of longitudinal photons

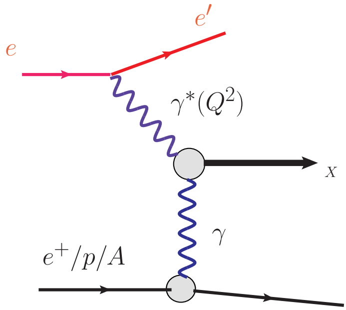
$$e^L_\mu(q_1) = \sqrt{\frac{-q_1^2}{X}}\Big(q_{2\mu} - \frac{q_1\cdot q_2}{q_1^2}q_{1\mu}\Big), \qquad e^L_\nu(q_2) = \sqrt{\frac{-q_2^2}{X}}\Big(q_{1\nu} - \frac{q_1\cdot q_2}{q_2^2}q_{2\nu}\Big).$$

- $F_{\rm TT}(0,0) = 0$, there is **no decay to two photons** (Landau-Yang).
- $F_{\rm LT}(Q^2,0) \propto Q$ (absence of kinematical singularities).

$$f_{\rm LT}(Q^2) = \frac{F_{\rm LT}(Q^2,0)}{Q}$$

- $f_{\rm LT}(0)$ gives rise to so-called “reduced width” $\tilde{\Gamma}$.

Accessing transition form factors

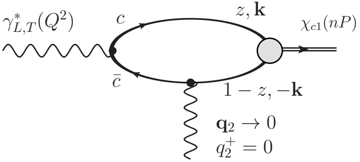


- We want to have at least **one virtual photon** to study the Q^2–dependence of transition FFs. For 1^{++} states it is stricly necessary to obtain finite cross section. This excludes ultraperipheral heavy ion collisions, where photons are quasi-real.
- Electron scattering gives us access to finite Q^2 and a whole polarization density matrix of virtual photons.
- Feasible options are:
  1. single tag e^+e^- collisions. Here the tagged lepton couples to the virtual photon, while photons from the lepton "lost in the beampipe" are quasireal.
  2. electron-proton or electron-ion scattering. Here especially heavy ions which have large charge give rise to a large quasireal photon flux enhanced by Z^2.

Ion	Cu	Ru	Au	Pb
Z	29	44	79	82

- Ion is scattered at very small angle, not measured. Photon exchange is associated with rapidity between produced syatem and ion.

Transition amplitude in the Drell-Yan frame



LF-Fock state expansion

$$|\chi_{c1}; \lambda, P_+, \boldsymbol{P}\rangle = \sum_{i,j,\sigma,\bar\sigma} \frac{\delta^i_j}{\sqrt{N_c}} \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \Psi^\lambda_{\sigma\bar\sigma}(z,\boldsymbol{k}) \times \Big| Q_{i\sigma}(zP_+, \mathbf{p}_Q)\bar{Q}^j_{\bar\sigma}((1-z)P_+, \mathbf{p}_{\bar Q})\Big\rangle + \dots$$

$$\boldsymbol{k} = (1-z)\boldsymbol{p}_Q - z\boldsymbol{p}_{\bar Q} \qquad \boldsymbol{P} = \boldsymbol{p}_Q + \boldsymbol{p}_{\bar Q}$$

- We evaluate the $\gamma^*\gamma^* \to \chi_{c1}$ amplitude in the Drell-Yan frame where $q_{1\mu} = q_{1+}n^+_\mu + q_{1-}n^-_\mu$ and $q_{2\mu} = q_{2-}n^-_\mu + q^\perp_{2\mu}$, using the light front plus-component of the current:

$$\langle\chi_{c1}(\lambda)|J_+(0)|\gamma^*_L(Q^2)\rangle = 2q_{1+}\sqrt{N_c}e^2e_f^2 \int \frac{dz d^2\boldsymbol{k}}{z(1-z)16\pi^3} \times \sum_{\sigma,\bar\sigma} \Psi^{\lambda *}_{\sigma\bar\sigma}(z,\boldsymbol{k})\,(\boldsymbol{q}_2\cdot\nabla_{\boldsymbol{k}})\Psi^{\gamma_L}_{\sigma\bar\sigma}(z,\boldsymbol{k},Q^2) \propto i\boldsymbol{q}_2\langle\chi_{c1}(\lambda)|\boldsymbol{r}|\gamma^*_L(Q^2)\rangle.$$

- for *spacelike* photons, the plus component of the current is free from parton number changing or instantaneous fermion exchange contributions.
- "Transition dipole moment" with dipole sizes controlled by photon WF $r \sim 1/\sqrt{m_c^2 + Q^2/4} < 0.15\,\mathrm{fm}$.

$\gamma^*\gamma^*$ cross sections and the reduced width

- photon-photon cross sections:

$$\sigma_{ij} = \frac{32\pi(2J+1)}{N_i N_j} \frac{\hat{s}}{2M\sqrt{X}} \frac{M\Gamma}{(\hat{s}-M^2)^2 + M^2\Gamma^2} \Gamma^{ij}_{\gamma^*\gamma^*}(Q_1^2, Q_2^2, \hat{s}),$$

where $\{i,j\} \in \{\mathrm{T}, \mathrm{L}\}$, and $N_\mathrm{T} = 2$, $N_\mathrm{L} = 1$ are the numbers of polarization states of photons. In terms of our helicity form factor, we obtain for the LT configuration, putting at the resonance pole $\hat{s} \to M^2$, and $J = 1$ for the axial-vector meson:

reduced width

$$\tilde{\Gamma}(A) = \lim_{Q^2 \to 0} \frac{M^2}{Q^2} \Gamma^{\mathrm{LT}}_{\gamma^*\gamma^*}(Q^2, 0, M^2) = \frac{\pi\alpha_{\mathrm{em}}^2 M}{3} f_{\mathrm{LT}}^2(0),$$

- provides a useful measure of size of the relevant e^+e^- cross section in the $\gamma\gamma$ mode. For a $c\bar{c}$ state:

$$f_{\mathrm{LT}} = -e_f^2 M^2 \frac{\sqrt{3N_c}}{8\pi} \int_0^\infty \frac{dk\, k^2 u(k)}{(k^2+m_c^2)^2} \frac{1}{\sqrt{M_{c\bar{c}}}} \left\{ \frac{2}{\beta^2} - \frac{1-\beta^2}{\beta^3} \log\left(\frac{1+\beta}{1-\beta}\right) \right\}, \quad \beta = \frac{k}{\sqrt{k^2+m_c^2}}$$

- nonrelativistic limit:

$$\tilde{\Gamma}(A) = \frac{2^5 \alpha_{\mathrm{em}}^2 e_c^2 N_c}{M_\chi^4} |R'(0)|^2$$

Reduced width of $\chi_{c1}(1P)$

Table: Reduced width of $\chi_{c1}(1P)$

potential model	m_c (GeV)	$\lvert R'(0)\rvert$ (GeV$^{5/2}$)	$\tilde{\Gamma}(\chi_{c1})_{\rm NR}$ (keV)	$\tilde{\Gamma}(\chi_{c1})$ (keV)
power-law	1.33	0.22	0.97	0.50
Buchmüller-Tye	1.48	0.25	0.82	0.30
Cornell	1.84	0.32	0.56	0.09
harmonic oscillator	1.4	0.27	1.20	0.53
logarithmic	1.5	0.24	0.72	0.27

- Considerably larger values of $\tilde{\Gamma}(\chi_{c1})$ are quoted in the literature. For example Danilkin & Vanderhaeghen (2017) report a value of $\tilde{\Gamma}(\chi_{c1}) \approx 1.6\,\mathrm{keV}$ from a sum rule analysis. Li et al. (2022) obtain $\tilde{\Gamma}(\chi_{c1}) \approx 3\,\mathrm{keV}$ from a LFWF approach.
- A measurement of the reduced width would therefore be very valuable.

$\chi_{c1}(3872)$ – the $[c\bar{c}]\, 2^3P_1$ component

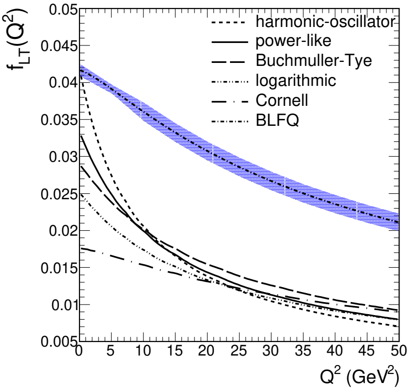


Figure: The dimensionless $\gamma_L^* \gamma \to \chi_{c1}(2P)$ transition form factor $f_{\rm LT}(Q^2)$.

- We use LFWFs for $n = 1$ radial excitation of the p-wave charmonium.
- We trace the different Q^2–dependences to differences of the z–dependence and constituent c-quark mass used in different models.
- error band for BLFQ reflects dependence on basis-size as proposed by its authors.

Reduced $\gamma_L^*\gamma$ width for $\chi_{c1}(3872)$

Table: The reduced width of the $\chi_{c1}(2P)$ state for several models of the charmonium wave functions with specific c-quark mass.

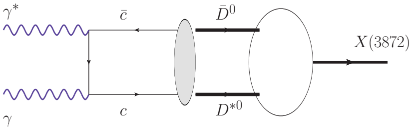
$c\bar{c}$ potential	m_c (GeV)	$f_{\rm LT}(0)$	$\tilde{\Gamma}_{\gamma\gamma}$ (keV)
harmonic oscillator	1.4	0.041	0.36
power-law	1.334	0.033	0.24
Buchmüller-Tye	1.48	0.029	0.18
logarithmic	1.5	0.025	0.14
Cornell	1.84	0.018	0.07
BLFQ	1.6	0.044	0.42

- First evidence for the production of $\chi_{c1}(3872)$ in single-tag e^+e^- collisions was reported by Belle Phys. Rev. Lett. **126** (2021) no.12, 122001 From three measured events, they provided a range for its reduced width, $0.02\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma} < 0.5\,\mathrm{keV}$. Recent update by Achasov et al. Phys. Rev. D **106** (2022) no.9, 093012 using a corrected value for the branching ratio $\mathrm{Br}(\chi_{c1}(3872) \to \pi^+\pi^- J/\psi)$ and reads

$$0.024\,\mathrm{keV} < \tilde{\Gamma}_{\gamma\gamma}(\chi_{c1}(3872)) < 0.615\,\mathrm{keV}$$

- all our results, including the BLFQ approach, lie **well within the experimentally allowed range**. Therefore, $\gamma\gamma$ data do not exclude the $c\bar{c}$ option, although there is certainly some room for a contribution from an additional meson-meson component.

Possible molecule contribution to $\tilde{\Gamma}$?



- apparently nothing (?) is known about “hadronic molecule” contribution to the reduced width.
- Spins of heavy quarks in $\chi_{c1}(3872)$ are entangled to be in the spin-triplet state (M. Voloshin, 2004). But near threshold the $c\bar{c}$ state produced via $\gamma\gamma$-fusion is in the $^1S_0$ state. (It's different for gluons, where color octet populates $^3S_1$!)
- $\rightarrow$ “handbag mechanism” suppressed in heavy quark limit.
- What about purely hadronic models?

Summary

- The LFWF representation of $\gamma^*\gamma^*$ transition form factors includes relativistic corrections.
- Ver sparse experimental information even for ground–state charmonia. Babar have measured $\gamma^*\gamma \to \eta_c$. For χ_{c2} and χ_{c0} off-shell widths from single tag cross-section we compared to Belle data (2017).
- We propose to investigate photon-photon fusion mechanisms also in future electron-Ion colliders. Differential distributions for photon virtuality for η_c production have been shown for LE-EIC, HE-EIC, EicC and LHeC. The estimated cross-section is in the range: $(0.1 - 60)$ nb.
- More detailed simulation of distributions and hadronic background necessary. There also is background from diffractive J/ψ production via $J/\psi \to \eta_c\gamma$.
- The reduced width of the ground state $\chi_{c1}(1P)$, for one longitudinal and one real photon $\tilde{\Gamma}$ is obtained in the ballpark of ~ 0.5 keV.
- In the case of $\chi_{c1}(3872)$, the values obtained for a 2^3P_1 charmonium are well within the range of the first Belle data. This suggests an important role of the $c\bar{c}$ Fock state for production in the $\gamma^*\gamma$ mode. (Of course there is still room for additional contributions.)
- Electroproduction of $\chi_{c1}(1P), \chi_{c1}(3872)$ in the Coulomb field of a heavy nucleus may give access to form factor $f_{\rm LT}(Q^2)$. This is additional information on the structure. We know how to estimate it for $c\bar{c}$ states.