New Developments in Heavy Quark Production Theory

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In collaboration with: J. Cammarota, F. Feng, Y. Huang, Y. Jia, J.-W. Qiu, W.-L. Sang, K. Watanabe, X. Xiong, D.-S. Yang, ...

Jia-Yue Zhang

Jefferson Lab

HEFTY COLLABORATION

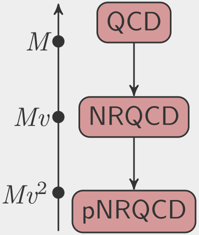
Center for Frontiers in Nuclear Science

1. **QCD Factorization for Heavy Quarkonium Production**

2. Production of Fully-Heavy Tetraquark

3. Summary

NRQCD Factorization *Bodwin, Braaten, Lepage, PRD1995*



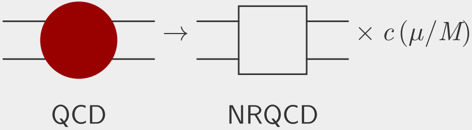
Vairo, Hadron 2011

See Vitev's talk for more details

- Quarkonium energy scale *Braaten, 1997*

	$c\bar{c}$	$b\bar{b}$	$t\bar{t}$
M	1.5 GeV	4.7 GeV	180 GeV
Mv	0.9 GeV	1.5 GeV	16 GeV
Mv^2	0.5 GeV	0.5 GeV	1.5 GeV

- Integrate out the heavy($\sim M$) degrees of freedom



Universality of LDMEs

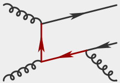
	$\langle\mathcal{O}(^3S_1^{[1]})\rangle$ [GeV^3]	$\langle\mathcal{O}(^1S_0^{[8]})\rangle$ [10^{-2} GeV^3]	$\langle\mathcal{O}(^3P_0^{[8]})\rangle/m_c^2$ [10^{-3} GeV^3]	$\langle\mathcal{O}(^3S_1^{[8]})\rangle$ [10^{-3} GeV^3]
Butenschoen, Kniehl, PRD2011	1.32	3.04	-4.04	1.68
Chao, et al., PRL2012	1.16	8.9	0.30	0.56
Gong, et al., PRL2013	1.16	9.7	-9.5	-4.6
Bodwin, et al., PRL2014		9.9	1.1	1.1
Zhang, et al., PRL2015	0.65	0.78	17	10
Feng, et al., PRD2019	1.16	5.66	3.42	1.77

Selection of J/ψ LDMEs from NLO fits using Tevatron (and LHC) data. ***Lansberg, 2020***

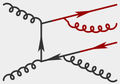
- Both size and sign of universal LDMEs are different.

Heavy Quarkonium Production with Multiple Hard Scales

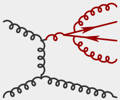
E.g. Perturbative expansion of color-singlet $Q\bar{Q}$ production: ***Kang, Qiu, Sterman, PRL2012***



$$\hat{\sigma}^{\mathrm{LO}} \rightarrow \frac{\alpha_s^3(p_{\mathrm{T}})}{p_{\mathrm{T}}^8}$$



$$\hat{\sigma}^{\mathrm{NLO}} \rightarrow \frac{\alpha_s^3(p_{\mathrm{T}})}{p_{\mathrm{T}}^6} \otimes \alpha_s(\mu) \ln \frac{\mu^2}{\mu_0^2}$$



$$\hat{\sigma}^{\mathrm{NNLO}} \rightarrow \frac{\alpha_s^2(p_{\mathrm{T}})}{p_{\mathrm{T}}^4} \otimes \alpha_s^3(\mu) \ln^m \frac{\mu^2}{\mu_0^2}$$

- Hierarchy between p_{T} and m_Q provides another way of expansion.
- When $p_{\mathrm{T}} \gg 2m_Q$, NNLO becomes dominant.

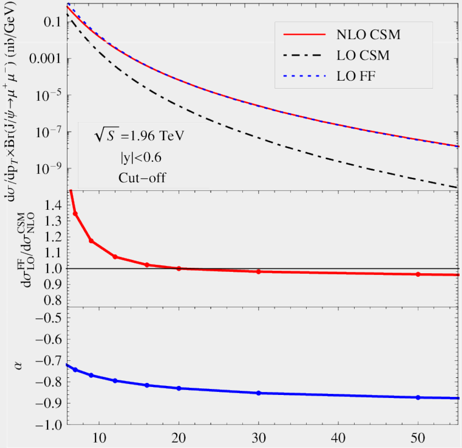
→ Expansion in powers of α_s might not be ideal when $p_{\mathrm{T}} \gg 2m_Q$.

Fixed-Order NRQCD v.s. Fragmentation

- QCD factorization

$$d\sigma^{(\mathrm{NLO})}_{\mathrm{NRQCD}} \propto \Big[d\hat{\sigma}^{A(\mathrm{LO})}_{ab\to[Q\bar{Q}(v8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(v8)]\to J/\psi} + d\hat{\sigma}^{S(\mathrm{LO})}_{ab\to[Q\bar{Q}(a8)]} \otimes \mathcal{D}^{(\mathrm{LO})}_{[Q\bar{Q}(a8)]\to J/\psi} \Big]$$

- LO fragmentation/NLP gives a nice description of NLO fixed-order results when p_{T} is sufficiently large.
- Fragmentation is much easier to calculate.



Kang, Ma, Qiu, Sterman, PRD2015

QCD Factorization for Heavy Quarkonium Production

pQCD+NRQCD Factorization

$$\mathrm{d}\sigma_{hh'\to J/\psi X} = \sum_n \left\langle O_n^{J/\psi} \right\rangle \otimes \sum_{a,b} \int \mathrm{d}x_a \mathrm{d}x_b f_{a/h}(x_a,\mu) f_{b/h'}(x_b,\mu) \times \left[\mathrm{d}\tilde{\sigma}^{\mathrm{Resum}}_{ab\to c\bar{c}[n]X} + \mathrm{d}\tilde{\sigma}^{\mathrm{NRQCD}}_{ab\to c\bar{c}[n]X} - \mathrm{d}\tilde{\sigma}^{\mathrm{Asym}}_{ab\to c\bar{c}[n]X} \right]$$

Kang, Ma, Qiu, Sterman, PRD2015; Lee, Qiu, Sterman, Watanabe, 2022

- pQCD factorization + FFs: $\mathrm{d}\tilde{\sigma}^{\mathrm{Resum}}_{ab\to c\bar{c}[n]X} = \mathrm{LP} + \mathrm{NLP} + \ldots$
- pQCD fixed-order: $\mathrm{d}\tilde{\sigma}^{\mathrm{NRQCD}}_{ab\to c\bar{c}[n]X}$
- pQCD asymptotic contribution:

$$\mathrm{d}\tilde{\sigma}^{\mathrm{Asym}}_{ab\to c\bar{c}[n](P)X} = \mathrm{d}\tilde{\sigma}^{\mathrm{Resum}}_{ab\to c\bar{c}[n](P)X}\Big|_{\text{fixed order}} \approx \begin{cases} \mathrm{d}\tilde{\sigma}^{\mathrm{NRQCD}}_{ab\to c\bar{c}[n](P)X} & p_{\mathrm{T}} \gg m_c \\ \mathrm{d}\tilde{\sigma}^{\mathrm{Resum}}_{ab\to c\bar{c}[n](P)X} & p_{\mathrm{T}} \gtrsim m_c \end{cases}$$

Modified DGLAP *Kang, Ma, Qiu, Sterman, PRD2014*

- Physical cross sections does NOT depend on the factorization scale.

$$\frac{\mathrm{d}}{\mathrm{d}\ln\mu_f^2}\left(E\frac{\mathrm{d}\tilde{\sigma}^{\text{Resum}}_{hh'\to c\bar{c}[n](P)X}}{\mathrm{d}^3P}\right)=0$$

→ Modified inhomogeneous evolution equations for FFs up to NLP:

$$\frac{\partial}{\partial\ln\mu^2}D_{[Q\bar{Q}(\kappa)]\to H}(z,\mu^2)=\frac{\alpha_s(\mu)}{2\pi}\sum_n\int_z^1\frac{dz'}{z'}P_{[Q\bar{Q}(n)]\to[Q\bar{Q}(\kappa)]}\left(\frac{z}{z'}\right)D_{[Q\bar{Q}(n)]\to H}(z',\mu^2),$$

$$\frac{\partial}{\partial\ln\mu^2}D_{f\to H}(z,\mu^2)=\frac{\alpha_s(\mu)}{2\pi}\sum_{f'}\int_z^1\frac{dz'}{z'}P_{f\to f'}\left(\frac{z}{z'}\right)D_{f'\to H}(z',\mu^2)$$
$$+\frac{\alpha_s^2(\mu)}{\mu^2}\sum_\kappa\int_z^1\frac{dz'}{z'}P_{f\to[Q\bar{Q}(\kappa)]}\left(\frac{z}{z'}\right)D_{[Q\bar{Q}(\kappa)]\to H}(z',\mu^2)$$

Comparison with Tevatron Data

- LP dominant:

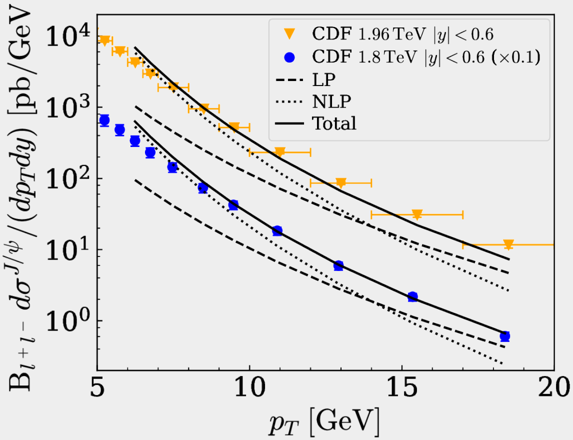
$$p_{\mathrm{T}} \gtrsim 5(2m_c) \sim 15\mathrm{GeV}$$

- NLP important:

$$5(2m_c) \gtrsim p_{\mathrm{T}} \gtrsim (2m_c)$$

- Matching to fixed-order NRQCD

$$p_{\mathrm{T}} \sim 2m_c$$



Lee, Qiu, Sterman, Watanabe, 2022

Heavy Quarkonium Production at EIC

- Leptoproduction of quarkonium is suppressed by the large momentum transfer:

$$\frac{\mathrm{d}\sigma^{\mathrm{SIDIS}}}{\mathrm{d}x_B \mathrm{d}Q^2 \mathrm{d}p_{\mathrm{T}}^2 \mathrm{d}y} \propto \frac{1}{Q^4}$$

- Additional scales are introduced for measurement of photoproduction of quarkonium at HERA.
- Collinear factorization allows systematic treatment of photoproduction and QED radiation.
- Joint QED and QCD factorization for the single inclusive hadron production:

Liu, Melnitchouk, Qiu, Sato, PRD2021; JHEP2021

$$\frac{\mathrm{d}\sigma_{ep\to HX}}{\mathrm{d}p_{\mathrm{T}}} \approx f_{i/e} \otimes f_{j/p} \otimes \left(D_k^H \otimes C_{ij\to k} + D_{c\bar{c}}^H \otimes C_{ij\to c\bar{c}} \right)$$

- $f_{i/e,p}$: **universal** lepton/parton distribution function (LDF/PDF)
- D_*^H: **universal** hadron fragmentation function
- $C_{ij\to *}$: perturbative calculable **IR&CO-safe** hard scattering coefficient

Lepton Distribution Function

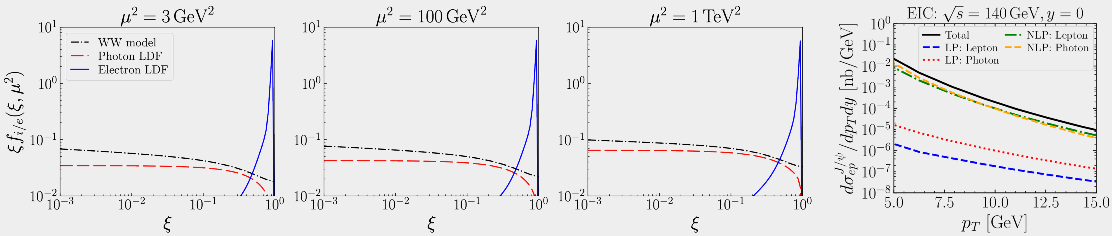
- Combined factorization leads to mixed evolution of LDFs.

→ LDFs are **non-perturbative**.

- Weizäcker-Williams distribution at LO

$$f_{\gamma/l}^{\rm WW}(\xi,\mu^2)=\frac{\alpha}{2\pi}P_{\gamma l}(\xi)\left(\ln\frac{\mu^2}{\xi m_l^2}-1\right)$$

Hinderer, Schlegel, Vogelsang, PRD2015



Qiu, Watanabe, in preparation

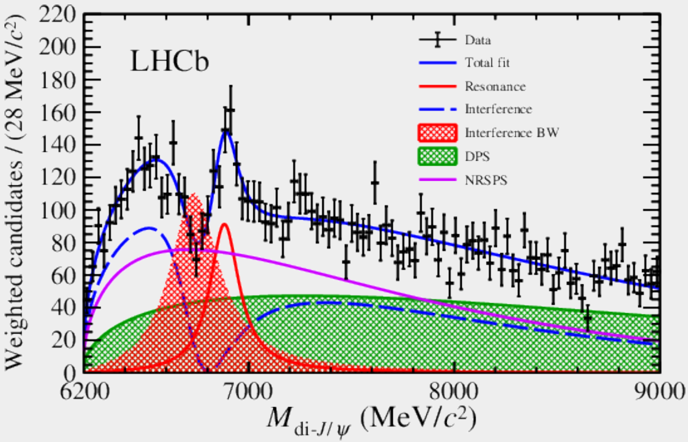
Boer, et al., 2025

1. QCD Factorization for Heavy Quarkonium Production

2. Production of Fully-Heavy Tetraquark

3. Summary

Discovery of $X(6900)$



Invariant mass spectrum of J/ψ-pair candidates (*LHCb, 2020*)

Production

- Duality relations: *Berezhnoy, et al., 2011, 2012; Kaliner, et al., 2017*
- Color evaporation model: *Carvalho, et al., 2016; Maciuła, et al., 2020*
- NRQCD-inspired: *Ma, Zhang, 2020; Feng, et al., 2020*
  - Gluon fragmentation: *Feng, et al., PRD2022; Zhu, 2020*
  - LO at LHC: *Ma, Zhang, 2020; Feng, et al.,PRD2023*
  - LO at B factories: *Feng, et al., PLB2021, CPC2021*
  - LO at electron-ion colliders: *Feng, et al., PRD2024*
- $\gamma\gamma$ interaction: *Gonçalves, Moreira, 2021*

NRQCD Factorization

- To produce T_{4Q}, one needs to produce two charm quarks and two anti-charm quarks at short distances $\sim 1/m_Q$ before the hadronization.
- NRQCD factorization formula

$$\sigma(T_{4Q}) = \sum_n \frac{F_n(\mu_\Lambda)}{m_Q^{d_n-4}} \left\langle 0 \left| \mathcal{O}_n^{T_{4Q}}(\mu_\Lambda) \right| 0 \right\rangle ,$$

- NRQCD production operators

$$\mathcal{O}_n^{T_{4Q}} = O_n \left(\sum_X \sum_{m_J} |T_{4Q} + X\rangle \langle T_{4Q} + X| \right) O_{n'}^\dagger .$$

NRQCD Operators

We construct all the NRQCD local operators at leading order of velocity expansion for the S-wave tetraquark with $J^{PC}=0^{++},1^{+-},2^{++}$

$$
\begin{aligned}
O^{(0)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} &= -\frac{1}{\sqrt{3}}[\psi_a^t(\mathrm{i}\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(\mathrm{i}\sigma^2)\chi_d^*]\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \\
O^{(0)}_{\mathbf{6}\otimes\bar{\mathbf{6}}} &= [\psi_a^t(\mathrm{i}\sigma^2)\psi_b][\chi_c^\dagger(\mathrm{i}\sigma^2)\chi_d^*]\,\mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}}, \\
O^{i;(1)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} &= \frac{\mathrm{i}}{\sqrt{2}}\epsilon^{ijk}\left(\psi_a^\dagger\sigma^j\mathrm{i}\sigma^2\psi_b^*\right)\left(\chi_c^t\mathrm{i}\sigma^2\sigma^k\chi_d\right)\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} \\
O^{\alpha\beta;(2)}_{\bar{\mathbf{3}}\otimes\mathbf{3}} &= [\psi_a^t(\mathrm{i}\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(\mathrm{i}\sigma^2)\chi_d^*]\,\Gamma^{\alpha\beta;mn}\,\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}, \\
\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}} &:= \frac{1}{2\sqrt{3}}(\delta^{ac}\delta^{bd}-\delta^{ad}\delta^{bc}), \quad \mathcal{C}^{ab;cd}_{\mathbf{6}\otimes\bar{\mathbf{6}}} := \frac{1}{2\sqrt{6}}(\delta^{ac}\delta^{bd}+\delta^{ad}\delta^{bc}) \\
\Gamma^{kl;mn} &:= \frac{1}{2}(\delta^{km}\delta^{ln}+\delta^{kn}\delta^{lm}-\frac{2}{3}\delta^{kl}\delta^{mn})
\end{aligned}
$$

NRQCD Operators

- The operators manifest the correct C/P-parity under the charge conjugation/parity transformations

$$\psi \to \mathrm{i}\left(\chi^\dagger \sigma^2\right)^t, \quad \chi \to -\mathrm{i}\left(\psi^\dagger \sigma^2\right)^t$$

$$\psi(t,\mathbf{r}) \to \psi(t,-\mathbf{r}), \quad \chi(t,\mathbf{r}) \to -\chi(t,-\mathbf{r})$$

- We use the basis in which the diquark and anti-diquark in the color-triplet and color-sexet, respectively. The operators can also be constructed from quark-antiquark pairs in the color-singlet and color-octet.
- These NRQCD operators can also be inferred by performing the Foldy-Wouthuysen-Tani transformation from the QCD interpolating currents in QCD sum rules.
  H.-X. Chen, et al.,2020

Perturbative Matching

Since the SDCs are insensitive to the long-distance physics, one can use the perturbative matching procedure to determine the SDCs.

- Replace the physical tetraquark state T^J_{4c} with a free 4-quark state
- Calculate both sides of factorization formula in perturbative QCD and perturbative NRQCD
- Solving the factorization formula to determine the SDCs.

E.g.,

$$\left|\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\right\rangle = \frac{1}{2}\sum_{s_*,\lambda_*}\left\langle\frac{1}{2}\lambda_1\frac{1}{2}\lambda_2\middle|1s_1\right\rangle\left\langle\frac{1}{2}\lambda_3\frac{1}{2}\lambda_4\middle|1s_2\right\rangle\langle 1s_1 1s_2|Jm_j\rangle$$

$$\mathcal{C}^{ab;cd}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\left|c^{\lambda_1}_a(q_1)c^{\lambda_2}_b(P-q_1)\bar{c}^{\lambda_3}_c(q_2)\bar{c}^{\lambda_4}_d(Q-P-q_2)\right\rangle$$

$$\Rightarrow\left\langle\mathcal{T}^{J,m_j}_{\bar{\mathbf{3}}\otimes\mathbf{3}}(Q)\middle|\varepsilon(m_j)\cdot O^{(J)\dagger}_{\bar{\mathbf{3}}\otimes\mathbf{3}}\middle|0\right\rangle = 4$$

Factorization Formula

- For tetraquark production at EIC, p_{T} is relatively small and fixed-order calculation should be reliable.
- inclusive production cross section of a hadron T_{4c} at the EIC can be written as

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\,\mathrm{d}p_{\mathrm{T}}} = \sum_i \int_{x_\gamma^{\min}}^1 \mathrm{d}x_\gamma \frac{2x_i p_{\mathrm{T}}}{z(1-z)} f_{\gamma/e}(x_\gamma)\, f_{i/p}(x_i)\, \frac{\mathrm{d}\hat{\sigma}(\gamma+i\to T_{4c}+X,\mu)}{\mathrm{d}\hat{t}},$$

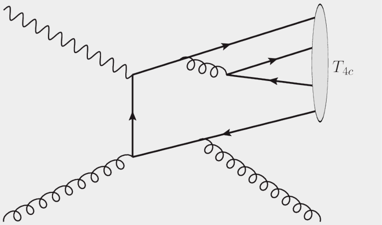
- $z := P_{T_{4c}}\cdot P_p / P_\gamma \cdot P_p$: elasticity parameter;
- $x_\gamma^{\min} = \frac{M_T^2 - m_H^2 z}{s\,z(1-z)}$

NRQCD Factorization

- LO partonic channel: $\gamma + g \to T_{4c} + g$
- C-parity conservation $\rightsquigarrow$ vector tetraquark state 1^{+-}

$$\frac{\mathrm{d}\hat{\sigma}(\gamma g \to T_{4c}^{(1)} + X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}} F_{3,3}^{(1)}(\hat{s},\hat{t}) \left\langle O_{3,3}^{(1)} \right\rangle$$

- more than 300 tree-level Feynman diagrams



SDC

$$
\begin{aligned}
&F^{(1)}_{3,3}(\hat{s},\hat{t}) = \pi^3 e_c^2 \alpha_s^4 r_s^2 \big[72 r_t^8 \left(5445 - 5298 r_t + 1462 r_t^2 - 184 r_t^3 + 49 r_t^4\right) - 432 r_t^7 \left(-5445 + 9879 r_t - 5524 r_t^2 + 1030 r_t^3 - 112 r_t^4 + 42 r_t^5\right) r_s + 2 r_t^6 (3332340 - 8427078 r_t \\
&+8454303 r_t^2 - 4101132 r_t^3 + 1115650 r_t^4 - 253810 r_t^5 + 43627 r_t^6) r_s^2 + 2 r_t^5 (5880600 - 17892198 r_t + 25180533 r_t^2 - 22035111 r_t^3 + 12807704 r_t^4 - 4945126 r_t^5 \\
&+1195291 r_t^6 - 138533 r_t^7) r_s^3 + r_t^4 \left(14113440 - 49523400 r_t + 83600442 r_t^2 - 101112318 r_t^3 + 94409051 r_t^4 - 60657225 r_t^5 + 24055510 r_t^6 - 5305354 r_t^7 + 505879 r_t^8\right) r_s^4 \\
&+ r_t^3 \left(11761200 - 49523400 r_t + 95733756 r_t^2 - 135804348 r_t^3 + 164472260 r_t^4 - 151209848 r_t^5 + 91395217 r_t^6 - 33278237 r_t^7 + 6611864 r_t^8 - 555538 r_t^9\right) r_s^5 + r_t^2 (6664680 \\
&-35784396 r_t + 83600442 r_t^2 - 135804348 r_t^3 + 186897370 r_t^4 - 206629419 r_t^5 + 164091573 r_t^6 - 86266517 r_t^7 + 27956171 r_t^8 - 5016861 r_t^9 + 381715 r_t^{10}) r_s^6 + r_t (2352240 \\
&-16854156 r_t + 50361066 r_t^2 - 101112318 r_t^3 + 164472260 r_t^4 - 206629419 r_t^5 + 187216756 r_t^6 - 119518674 r_t^7 + 52323094 r_t^8 - 14762980 r_t^9 + 2381419 r_t^{10} - 165406 r_t^{11}) r_s^7 \\
&+ (392040 - 4267728 r_t + 16908606 r_t^2 - 44070222 r_t^3 + 94409051 r_t^4 - 151209848 r_t^5 + 164091573 r_t^6 - 119518674 r_t^7 + 59925804 r_t^8 - 20969265 r_t^9 + 4946107 r_t^{10} - 698919 r_t^{11} \\
&+43850 r_t^{12}) r_s^8 + (-381456 + 2386368 r_t - 8202264 r_t^2 + 25615408 r_t^3 - 60657225 r_t^4 + 91395217 r_t^5 - 86266517 r_t^6 + 52323094 r_t^7 - 20969265 r_t^8 + 5682942 r_t^9 - 1042547 r_t^{10} \\
&+119941 r_t^{11} - 6480 r_t^{12}) r_s^9 + (105264 - 444960 r_t + 2231300 r_t^2 - 9890252 r_t^3 + 24055510 r_t^4 - 33278237 r_t^5 + 27956171 r_t^6 - 14762980 r_t^7 + 4946107 r_t^8 - 1042547 r_t^9 \\
&+ 135646 r_t^{10} - 10512 r_t^{11} + 408 r_t^{12}) r_s^{10} + (-13248 + 48384 r_t - 507620 r_t^2 + 2390582 r_t^3 - 5305354 r_t^4 + 6611864 r_t^5 - 5016861 r_t^6 + 2381419 r_t^7 - 698919 r_t^8 + 119941 r_t^9 \\
&-10512 r_t^{10} + 324 r_t^{11}) r_s^{11} + \left(2 - 3 r_t + r_t^2\right)^2 \left(882 - 1890 r_t + 13277 r_t^2 - 21970 r_t^3 + 14354 r_t^4 - 4032 r_t^5 + 408 r_t^6\right) r_s^{12}\big] \\
&\times \left\{1327104 \left(3 - r_s\right)^2 \left(2 - r_s\right)^2 \left(1 - r_s\right)^2 \left(r_s\left(2 - r_t\right) - 2 r_t\right)^2 \left(3 - r_t\right)^2 \left(2 - r_t\right)^2 \left(1 - r_t\right)^2 \left(r_s + r_t\right)^2 \left(r_s\left(3 - 2 r_t\right) - 3 r_t\right)^2\right\}^{-1}, \quad r_s := 16 m_c^2/\hat{s},\ r_t := 16 m_c^2/\hat{t}
\end{aligned}
$$

Asymptotic form of the SDC in large p_{T} :

$$
F^{(1)}_{3,3}(\hat{s},\hat{t}) = \frac{605 \pi^3 \alpha_s^4 e_c^2 m_c^8 \left(\hat{s}^2 + \hat{s}\hat{t} + \hat{t}^2\right)^2}{3458 \hat{s}^4 \hat{t}^2 (\hat{s} + \hat{t})^2} + \mathcal{O}\left(\frac{m_c^9}{p_{\mathrm{T}}^9}\right).
$$

LDMEs

- Four-body potential models are adopted to estimate the LDMEs. The results are proportional to the wave functions at the origin, where the color structure labels C_1 and C_2 indicate the color configurations $\bar{\mathbf{3}} \otimes \mathbf{3}$ or $\mathbf{6} \otimes \bar{\mathbf{6}}$.

$$\left\langle O^{(0)}_{C_1,C_2} \right\rangle \approx 16\psi_{C_1}(\mathbf{0})\psi^*_{C_2}(\mathbf{0}), \quad \left\langle O^{(1)}_{C_1,C_2} \right\rangle \approx 48\psi_{C_1}(\mathbf{0})\psi^*_{C_2}(\mathbf{0}), \quad \left\langle O^{(2)}_{C_1,C_2} \right\rangle \approx 80\psi_{C_1}(\mathbf{0})\psi^*_{C_2}(\mathbf{0}).$$

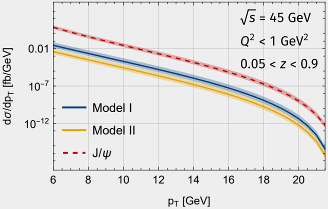
- Numerical results: $[\mathrm{GeV}^9]$

  Model I : *Lü, Chen, Dong, EPJC2020*

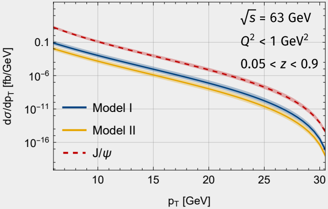
  Model II : *M.-S. Liu, F.-X. Liu, Zhong, Zhao, PRD2024*

	0^{++}			1^{+-}	2^{++}
	$\left\langle O^{(0)}_{3,3} \right\rangle$	$\left\langle O^{(0)}_{3,6} \right\rangle$	$\left\langle O^{(0)}_{6,6} \right\rangle$	$\left\langle O^{(1)}_{3,3} \right\rangle$	$\left\langle O^{(2)}_{3,3} \right\rangle$
Model I	0.0347	0.0211	0.0128	0.0780	0.072
Model II	0.0187	−0.0161	0.0139	0.0480	0.0628

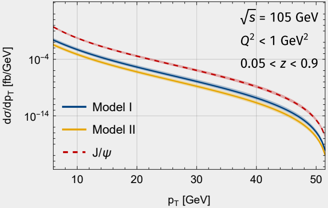
p_{T} distributions at EIC



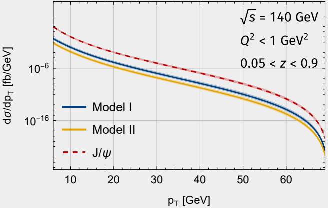
(a) $\sqrt{s} = 45$ GeV



(b) $\sqrt{s} = 63$ GeV

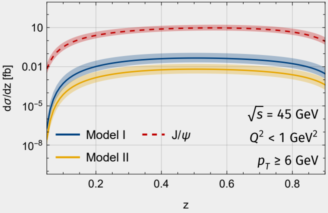


(c) $\sqrt{s} = 105$ GeV

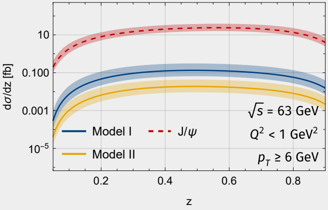


(d) $\sqrt{s} = 140$ GeV

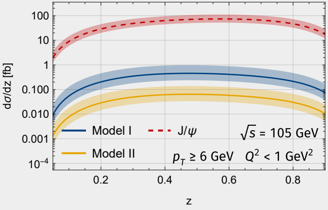
z distributions at EIC



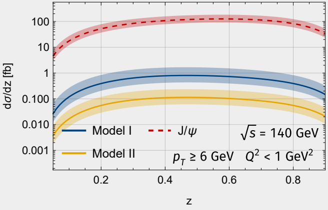
(a) $\sqrt{s} = 45$ GeV



(b) $\sqrt{s} = 63$ GeV



(c) $\sqrt{s} = 105$ GeV



(d) $\sqrt{s} = 140$ GeV

p_T-integrated cross section

- Integrated luminosity:
  $100\ \mathrm{fb}^{-1}/\mathrm{yr}$@EIC;
  $50.5\ \mathrm{fb}^{-1}/\mathrm{yr}$@EicC;
  $468\ \mathrm{pb}^{-1}$@HERA.

	$\sqrt{s}$ [GeV]	p_T range [GeV]	Model I σ [fb]	Model I N	Model II σ [fb]	Model II N
EIC	44.7	$6-20$	0.022	2.2	0.0031	0.31
	63.2	$6-20$	0.069	6.9	0.0098	0.98
	104.9	$6-20$	0.25	25.	0.035	3.5
	140.7	$6-20$	0.45	45.	0.064	6.4
HERA	319	$6-20$	1.5	0.72	0.22	0.10
EicC	20	$6-9$	0.000015	0.00076	2.1×10^{-6}	0.00011

1. QCD Factorization for Heavy Quarkonium Production

2. Production of Fully-Heavy Tetraquark

3. Summary

Summary

- Heavy quarkonium production involving more than one hard scale requires reorganization of series expansion.
- The LP contribution to quarkonium production is significant at large p_{T}, while the NLP contribution is important at lower p_{T} and is necessary for matching to fixed order calculations.
- Combined QED and QCD factorization provides a systematic framework to study the quarkonium production on ep colliders.
- NRQCD factorization could be extended to production of fully-heavy tetraquarks.

Thank you