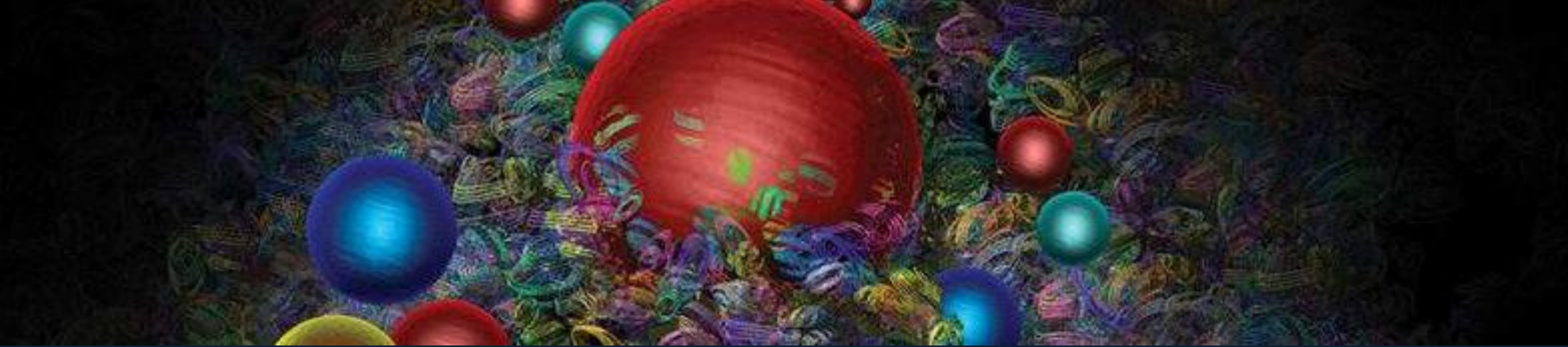


Study the nucleon tomography through Generalized parton distributions at the Electron-Ion Collider

Yuxun Guo

U.C. Berkeley/ Lawrence Berkeley Laboratory

Disclaimer: Not a review

Probing the frontiers of nuclear physics with AI at the EIC (II)

CFNS, Stony Brook University, NY

March 19 – 21st., 2025

Center for Frontiers in Nuclear Science

QUARK-GLUON TOMOGRAPHY COLLABORATION

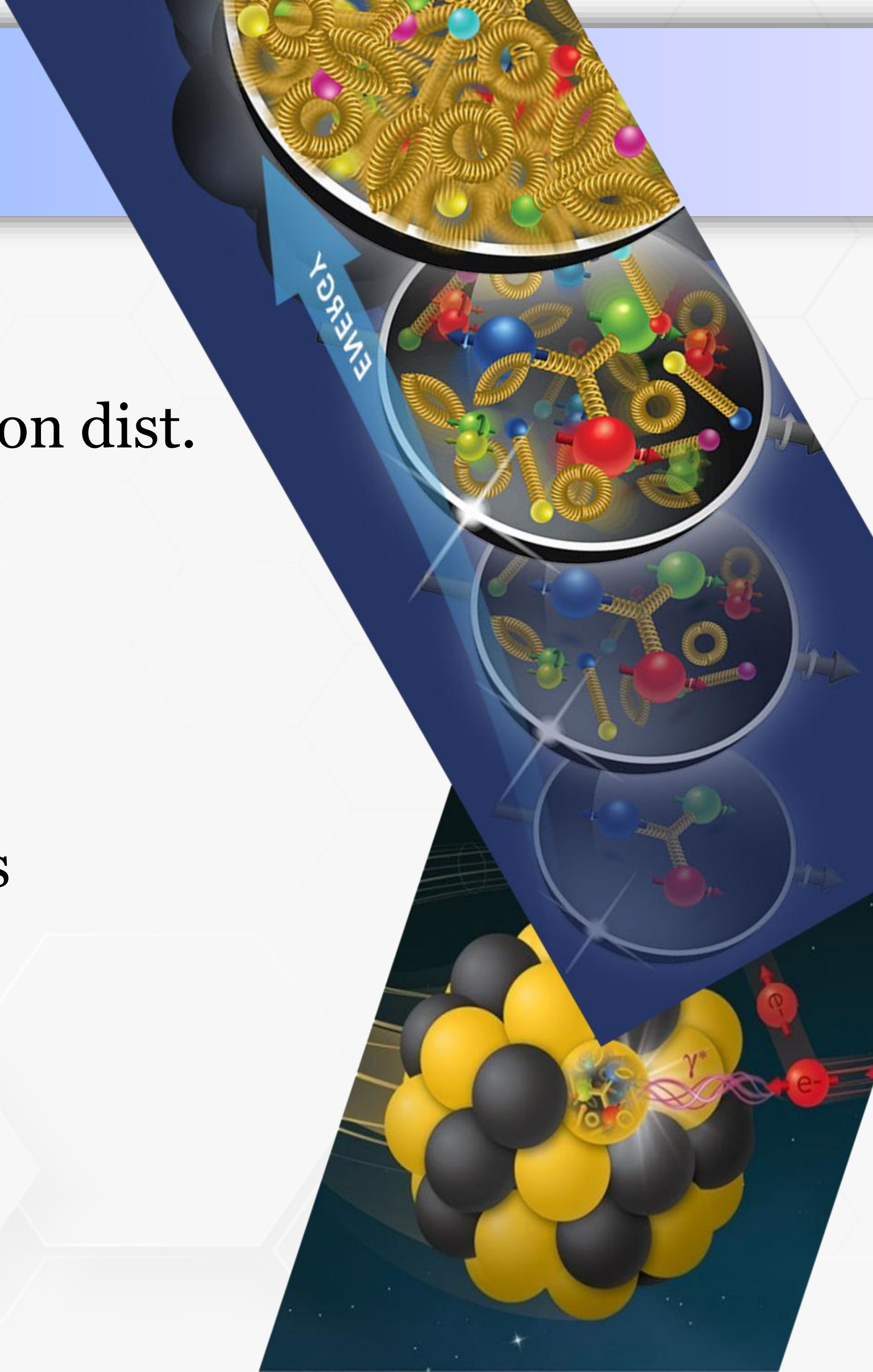
Outline

» Intro to nucleon tomography and Generalized parton dist.

» What limits their phenomenological extraction?

» Some developments regarding the GPD extractions

» Summary

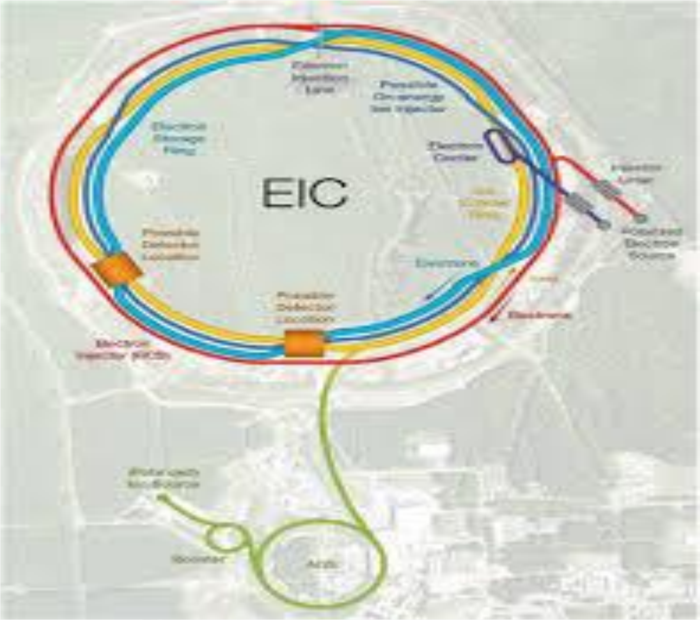


Nucleon tomography and GPDs

Internal structure of the nucleon

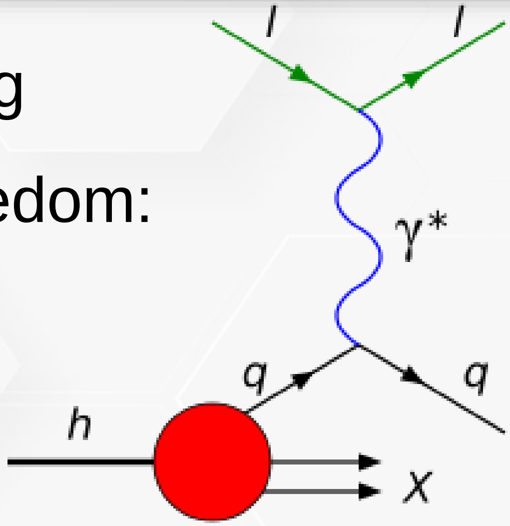
Ever since we realized that nucleons are not fundamental particles, understanding their internal structures had become one of the most important topics.

However, finding a probe is quite hard.



Nucleon tomography and 3D structures

A prominent feature of the strong interaction is the asymptotic freedom:



$$k^+ = xP^+$$

Parton Distribution Function (PDF) $f_{q/g}(x)$

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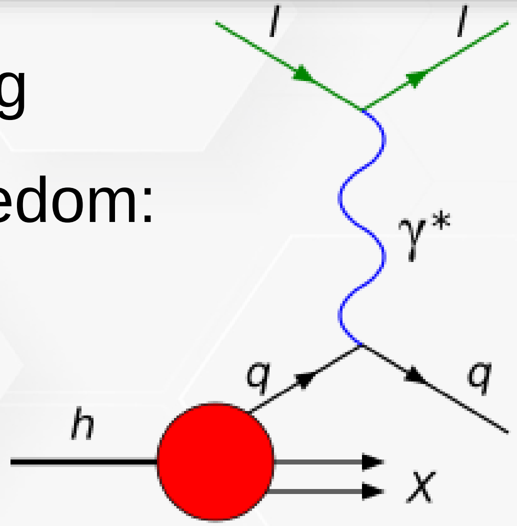
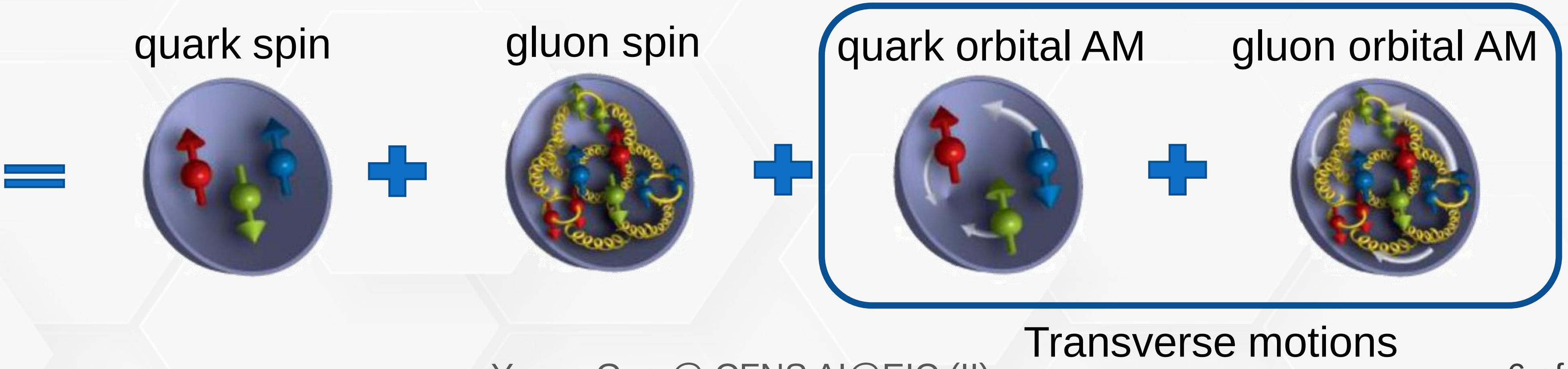


$$k^+ = xP^+$$

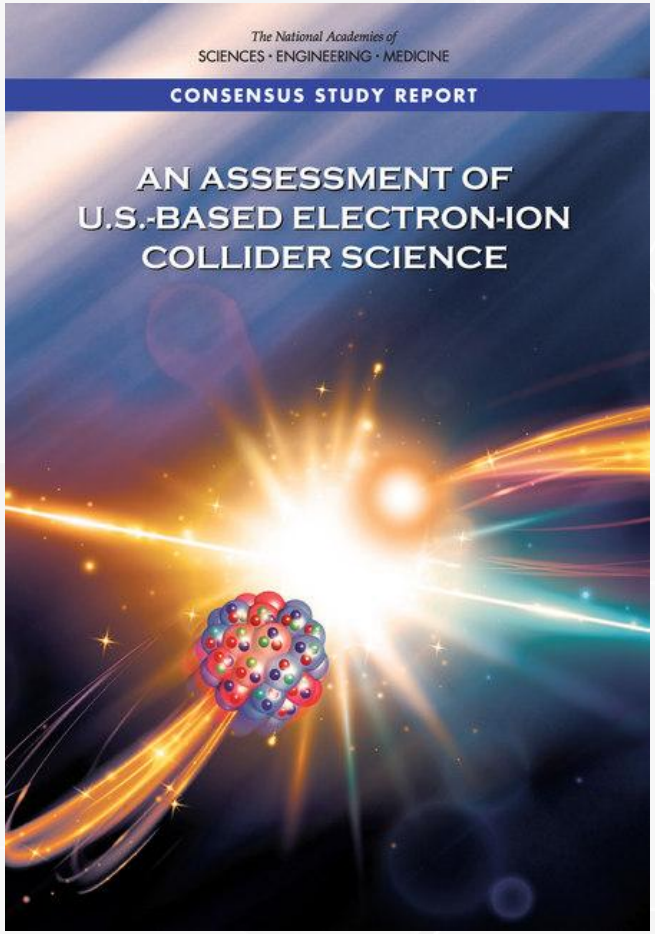
Parton Distribution Function (PDF) $f_{q/g}(x)$

There are many good reasons we want the transverse dynamics as well

$$\frac{\hbar}{2} =$$



Nucleon structures with EIC



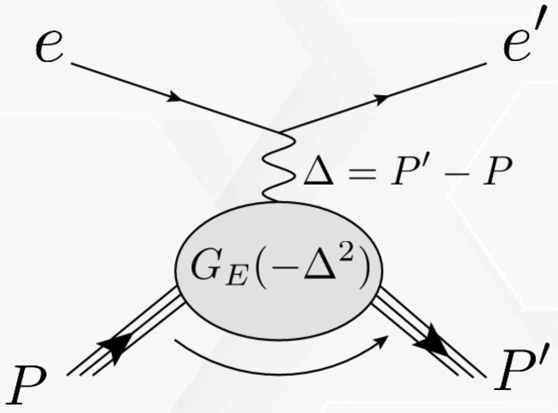
Finding 1:

An EIC can uniquely address three profound questions about nucleons — neutrons and protons — and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

3D distributions of quarks and gluons

The spatial distributions of quarks and gluons can be accessed with proton recoil.



elastic scattering

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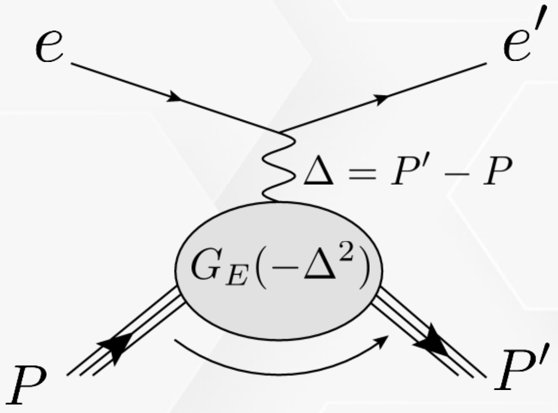

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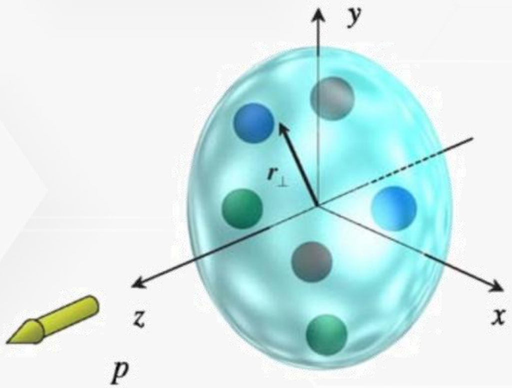

charge distribution

$$\rho_{\rm NR}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} G_E(-\boldsymbol{\Delta}^2)$$

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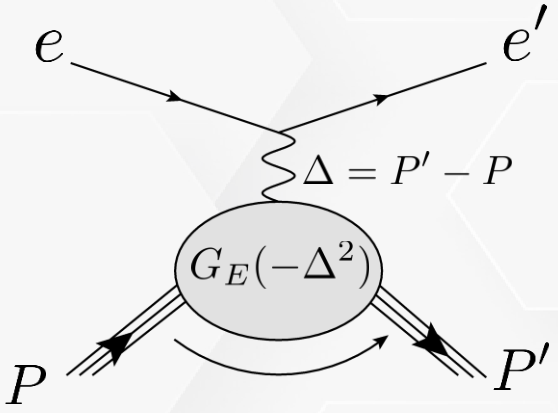

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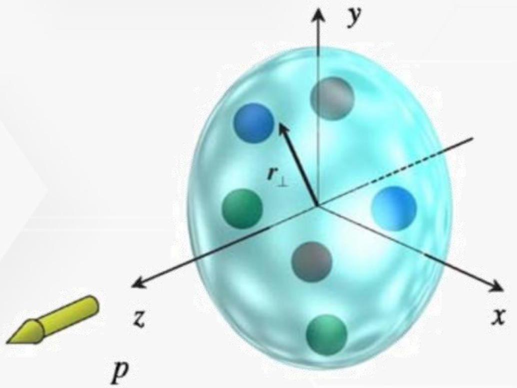

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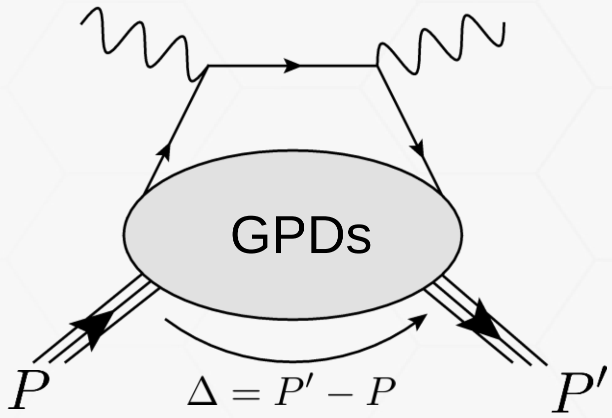

generalized parton distribution

D. Muller et. al. Fortsch.Phys. 42 101 (1994)

X. Ji Phys. Rev. Lett. 78, 610 (1997)

$$\rho_{\rm NR}(\boldsymbol{r}) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}} G_E(-\boldsymbol{\Delta}^2)$$

GPDs correspond to the quark/gluon distributions localized in the impact parameter space.

3D mass & spin structures with GPDs

GPDs are 3D distributions unifying parton distributions and form factors

$$F(x, \Delta^\mu) = F\,(x, \xi, t)$$

x : parton momentum fraction

ξ : skewness parameter – longitudinal momentum transfer $\xi \equiv -n \cdot \Delta/2$

t : total momentum transfer squared $t \equiv \Delta^2$

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GPDs reduce to form factors when integrated over *x* X. Ji, J. Phys. *G* 24 1181-1205 (1998)

Charge FFs

$$\int \mathrm{d}x H(x,\xi,t) = F_1(t)$$

$$\int \mathrm{d}x E(x,\xi,t) = F_2(t)$$

Gravitational FFs

$$\int \mathrm{d}x\, xH(x,\xi,t) = A(t) + (2\xi)^2 C(t)$$

$$\int \mathrm{d}x\, xE(x,\xi,t) = B(t) - (2\xi)^2 C(t)$$

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GPDs also provide an intuitive 3D image of nucleon:

M. Burkardt, Int. J. Mod. Phys. A 18 173-208 (2003)

$$\rho_q^{\mathrm{Unp}}(x, \boldsymbol{b}) = \int \frac{\mathrm{d}^2\boldsymbol{\Delta}}{(2\pi)^2} e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{b}} H_q(x, -\boldsymbol{\Delta}^2) = \mathscr{H}_q(x, \boldsymbol{b})$$

which contains information of nucleon spin structure, e. g. transverse spin

$$J_q^T(x) = \int \mathrm{d}^2\boldsymbol{b}(b^y \times xP^+)\rho_q^T(x, \boldsymbol{b})$$

Y. Guo et. al. Nucl. Phys. B 969 115440 (2021)

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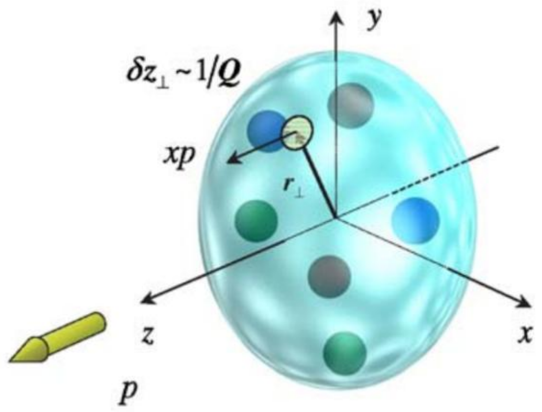

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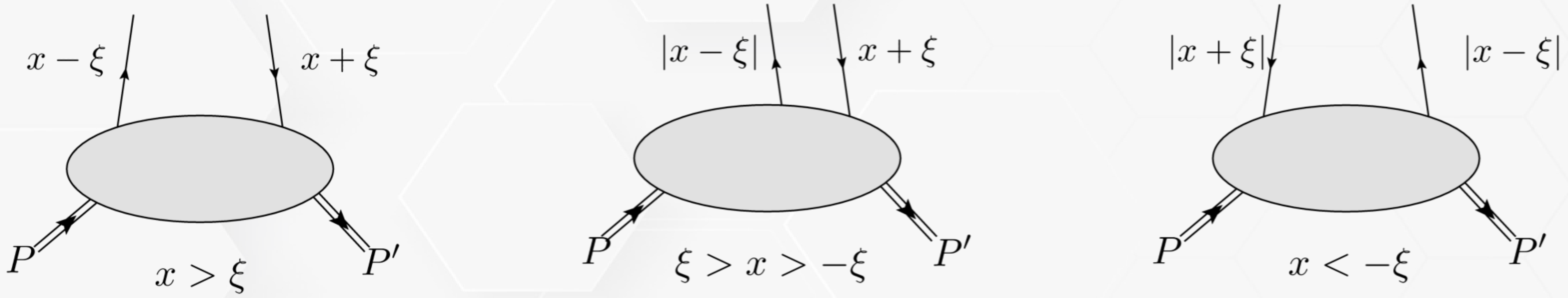
Constraints on GPD

Physical constraints on GPDs

Generally, GPDs are defined on: $x \in [-1,1], \xi \in [0,1], t \in (-\infty, 0]$



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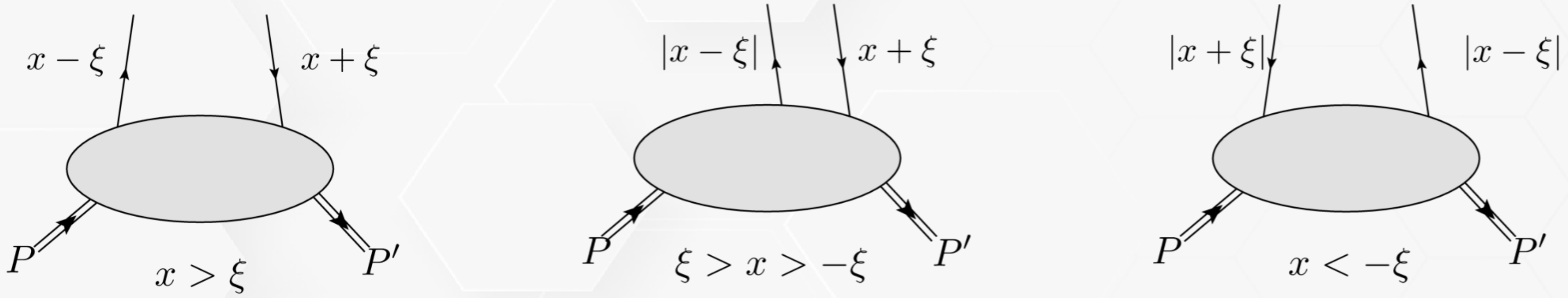
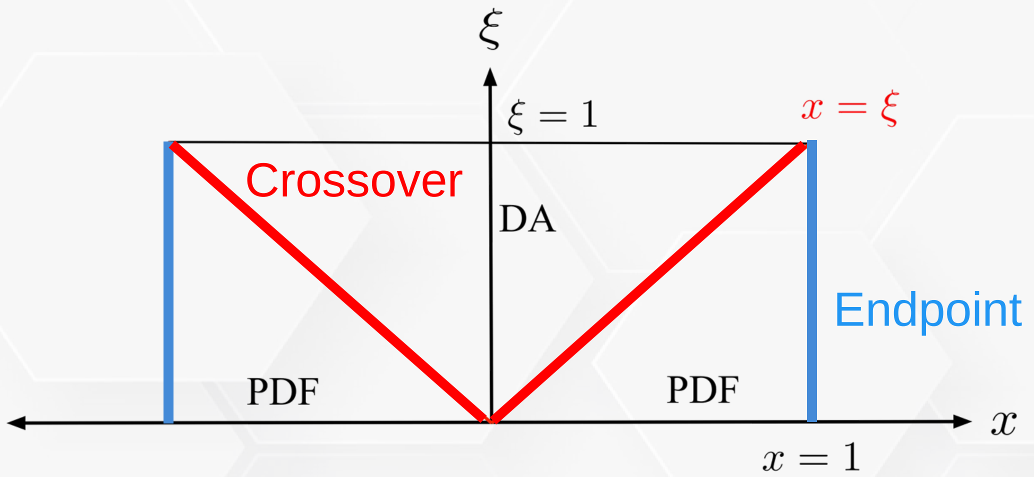


"Phase diagram" of GPDs with crossover lines



Physical constraints on GPDs

Moreover, GPDs are subject to integral constraints:

$$\int_0^1 \mathrm{d}x x^{2n} F_g(x,\xi,t) = \sum_{i=0}^{n} (2\xi)^{2i} A_g^{(2n+2,2i)}(t) + (2\xi)^{2n+2} C_g^{(2n+2)}(t)$$

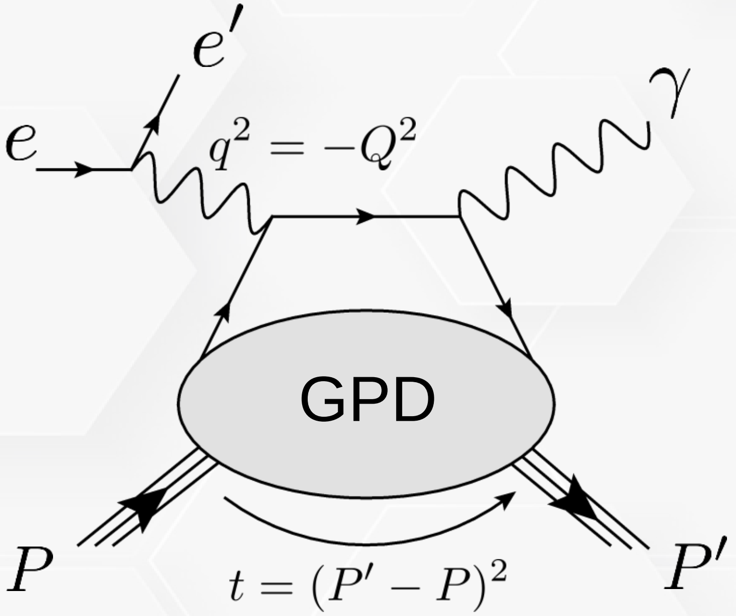
Polynomiality: The 2nth-moment of GPD must be polynomials of xi or order 2n+2

GPDs are functions of **three variables** that are **not completely smooth**

and subject to **boundary** and **integral** constraints.

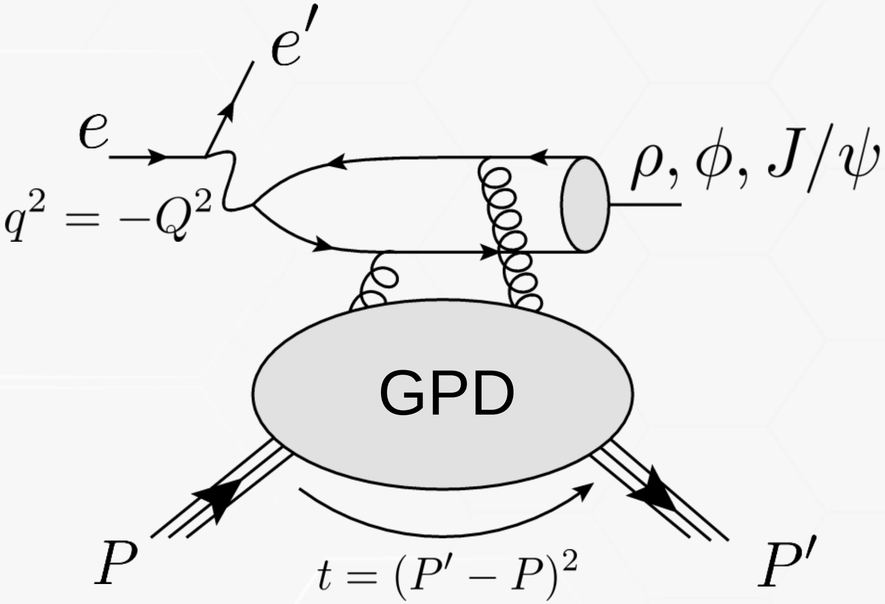
Deeply virtual processes

Diffractive processes provide us access to the GPDs and 3D structures.



Deeply virtual Compton scattering

X. Ji, Phys. Rev. D 55, 7114 (1997)



Deeply virtual meson production

A.V. Radyushkin Phys. Lett. B 385 333-342 (1996)
J. C. Collins et. al. Phys. Rev. D 56 2982-3006 (1997)

The two-fold inverse problem of unfolding GPDs

The challenge of extracting GPDs lies in the unfolding processes.

- Axial/polarized GPD contributions even in unpolarized cross-sections.

Vector current $\bar{\psi}\left(-\frac{\lambda n}{2}\right)\gamma^+\psi\left(\frac{\lambda n}{2}\right)$ Axial-vector current $\bar{\psi}\left(-\frac{\lambda n}{2}\right)\gamma^+\gamma^5\psi\left(\frac{\lambda n}{2}\right)$

- The off-forward kinematics - more GPD species

$$\left\langle\bar{\psi}\left(-\frac{\lambda n}{2}\right)\gamma^+\psi\left(\frac{\lambda n}{2}\right)\right\rangle \sim \bar{u}(P',S')\left[\not{n}H(x,\xi,t)+\frac{i\sigma^{\mu\nu}n_\mu\Delta_\nu}{2M}E(x,\xi,t)\right]u(P,S)$$

- The amplitudes have imaginary parts (twice of real variables)

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$$F_{UU}=4\left[(1-\xi^{2})\left(h^{\mathrm{U}}\mathcal{H}^{*}\mathcal{H}+\tilde{h}^{\mathrm{U}}\widetilde{\mathcal{H}}^{*}\widetilde{\mathcal{H}}\right)-\frac{t}{4M^{2}}\left(h^{\mathrm{U}}\mathcal{E}^{*}\mathcal{E}+\xi^{2}\tilde{h}^{\mathrm{U}}\widetilde{\mathcal{E}}^{*}\widetilde{\mathcal{E}}\right)\right.$$
$$\left.-\xi^{2}\left(h^{\mathrm{U}}\mathcal{E}^{*}\mathcal{E}+h^{\mathrm{U}}(\mathcal{E}^{*}\mathcal{H}+\mathcal{H}^{*}\mathcal{E})+\tilde{h}^{\mathrm{U}}(\widetilde{\mathcal{E}}^{*}\widetilde{\mathcal{H}}+\widetilde{\mathcal{H}}^{*}\widetilde{\mathcal{E}})\right)\right],$$

Deconvolution of Compton form factors

DVCS probes the GPDs via the Compton form factors (similarly for DVMP)

$$\mathcal{H}_{CFF}(\xi,t) = -\sum_q Q_q^2 \int_{-1}^{1} \mathrm{d}x \left(\frac{1}{x-\xi+i0} + \frac{1}{x+\xi-i0}\right) H_q(x,\xi,t) \ ,$$

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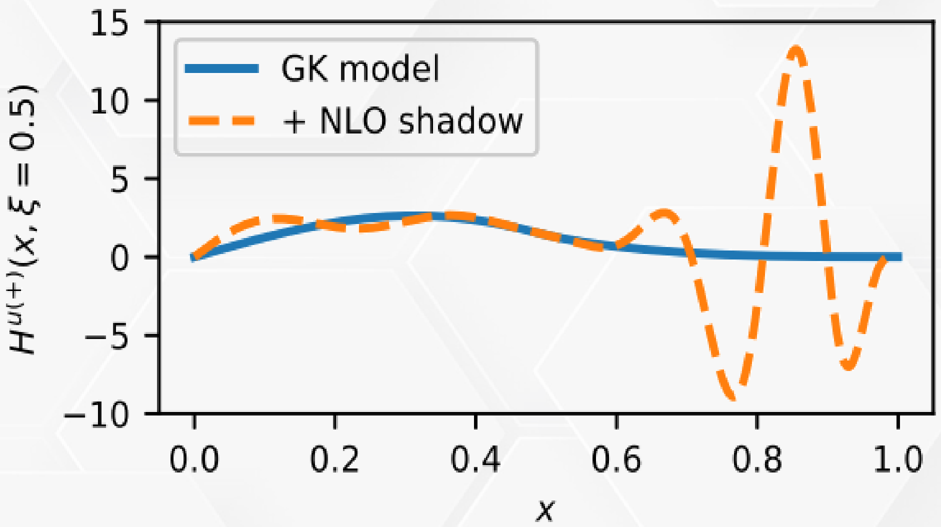
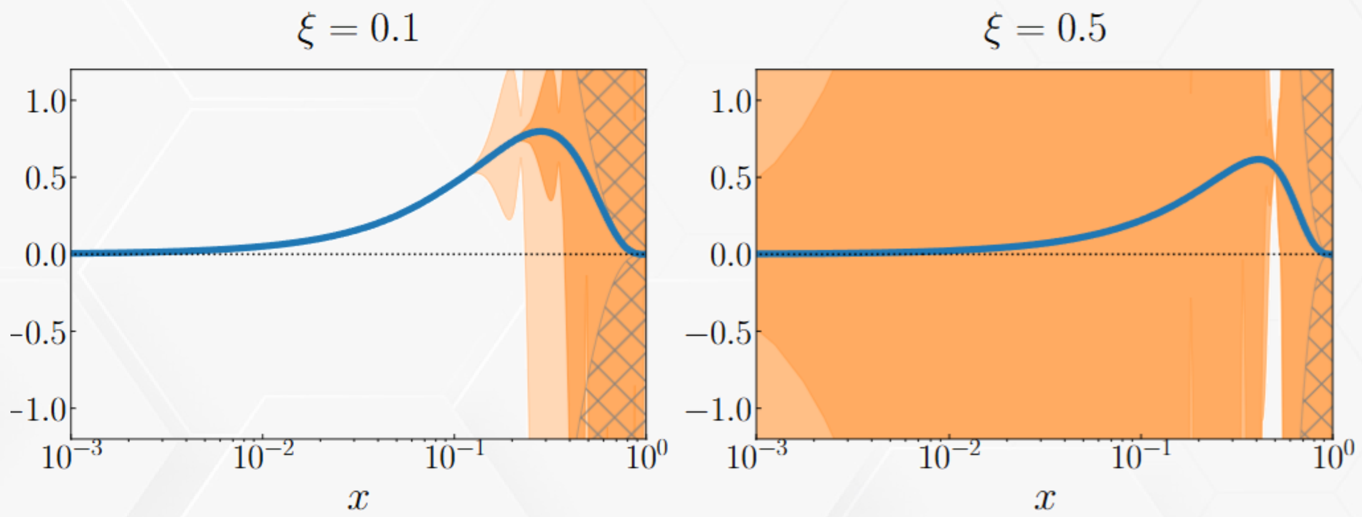
2nd inverse problem: deconvolution does not give a unique solution



V. Bertone et. al. SciPost Phys.Proc. 8 (2022) 107

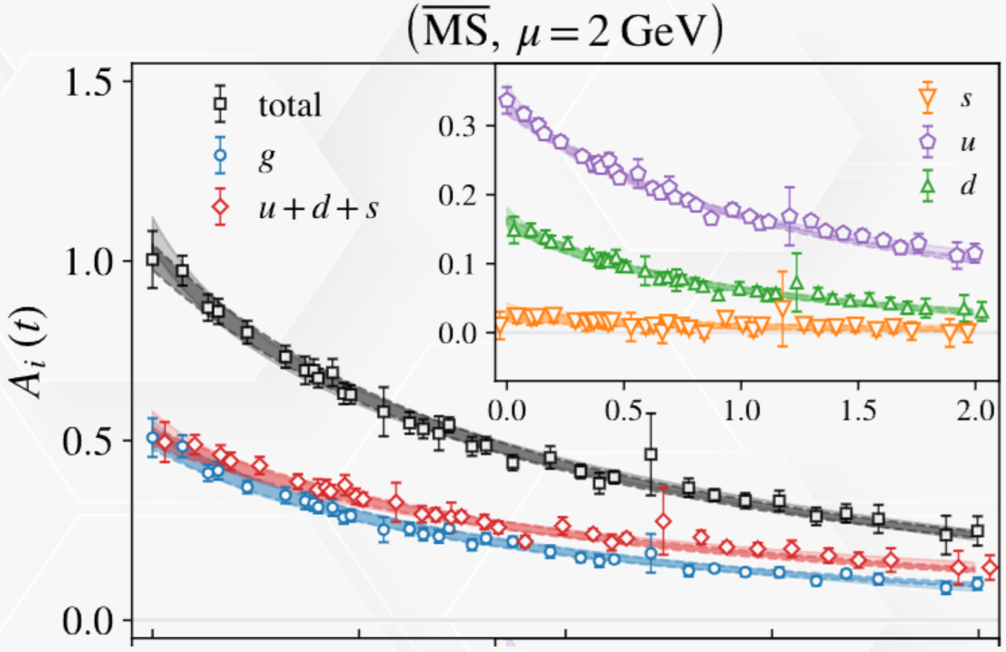


E. Moffat et. al Phys. Rev. D 108 3, 036027 (2023)

Lattice QCD input

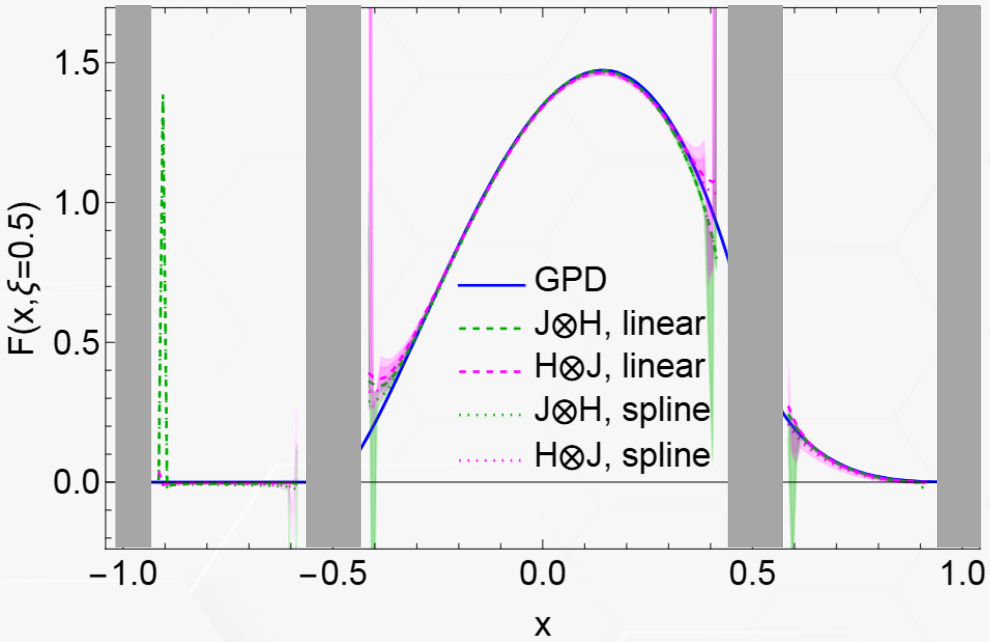
Lattice QCD provides complimentary inputs: both moments and x-space

Lattice simulation of Gravitational form factors



D. Hackett et. al Phys. Rev. Lett. 132 25, 251904 (2024)

Resummation of LaMET calculation of GPDs



J. Holligan et. al arXiv: 2501.19225

$$[-1 + x_0, -\xi - x_0] \cup [-\xi + x_0, \xi - x_0] \cup [\xi + x_0, 1 - x_0]$$

Phenomenological extraction

Global analysis of GPDs

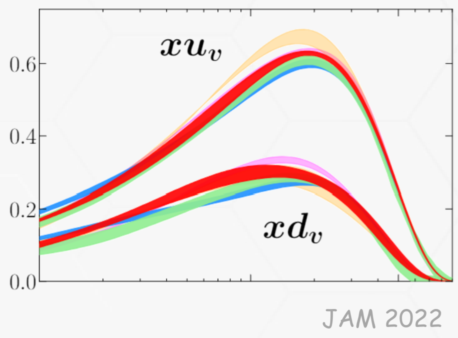
Experimental data and constraints

- ❑ Polarized and unpolarized PDFs from global analysis
  - ▪ Alternatively, one can fit to (polarized) DIS directly
- ❑ Neutron/ Proton charge form factors from global analysis
- ❑ Deeply virtual Compton scattering data at JLab/HERA & future EIC
- ❑ Deeply virtual meson productions data at HERA & future EIC

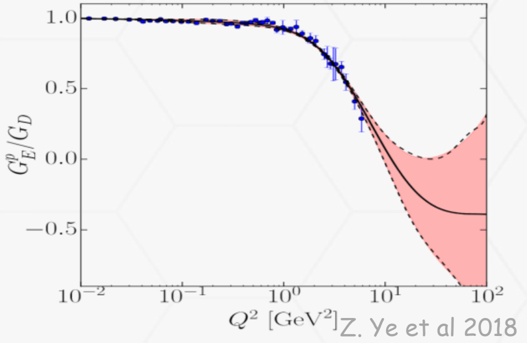
Lattice QCD simulations

- ❑ Lattice simulations of nucleon generalized form factors
- ❑ Lattice simulations of unpolarized and helicity GPDs at zero and non-zero ξ (skewness)

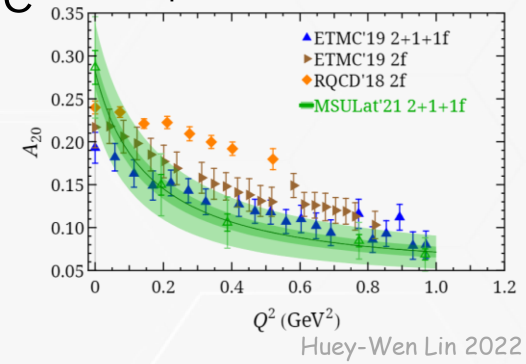
Example of global PDF fit



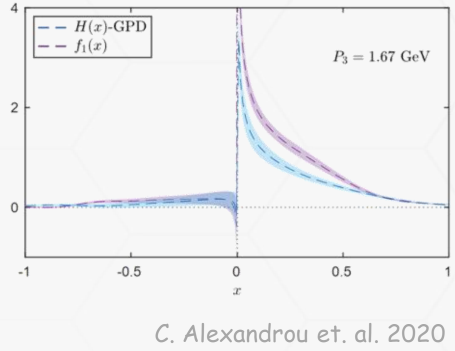
Example of charge form factor fit



Example of lattice form factors



Example of lattice GPD



- JAM (2022) PDF global analysis results
- Globally extracted electromagnetic form factors (*Z. Ye et al* 2018)
- Lattice GPDs (*Alexandrou et al* 2020) and form factors (*Alexandrou et al* 2022)
- DVCS measurements from JLab (*CLAS* 2019 & 2021, *Hall A* 2018 & 2022) and HERA (*H1* 2010)

Comprehensive global analysis programs are needed!

Moment-space parameterization of GPD

We could project GPDs onto a set of basis functions:

$$F(x,\xi,t)=\sum_{j=0}^{\infty}(-1)^j p_j(x,\xi)\mathcal{F}_j(\xi,t)$$

D. Mueller and A. Schafer
Nucl. Phys. B 739 1-59 (2006)

$p_j(x,\xi)$: Orthogonal basis $\mathcal{F}_j(\xi,t)$: Moments of GPDs to be parameterized

GPDs are functions of **three variables** that are **not completely smooth** and subject to **boundary** and **integral** constraints.

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GPDs are functions of **three variables** that are **not completely smooth** and subject to **boundary** and **integral** constraints.

The QCD evolution of GPD indicates Gegenbauer polynomials as the basis.

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}F\left(x,\xi,t,Q^2\right)=\frac{\alpha_s(Q)}{2\pi}\int_{-1}^{1}\frac{\mathrm{d}x'}{|\xi|}\left[V\left(\frac{x}{\xi},\frac{x'}{\xi}\right)\right]_+F\left(x',\xi,t,Q^2\right)$$

Moment resummation and GUMP

Unfortunately, such moment expansion is generally divergent.

$$F(x,\xi,t) = \frac{1}{2i}\int_{c-i\infty}^{c+i\infty} dj\, \frac{p_j(x,\xi)}{\sin(\pi[j+1])}\mathcal{F}_j(\xi,t)\ ,$$

We need to parameterize all the GPD moments for the resummation.

GPDs through Universal Moment Parameterization (GUMP)

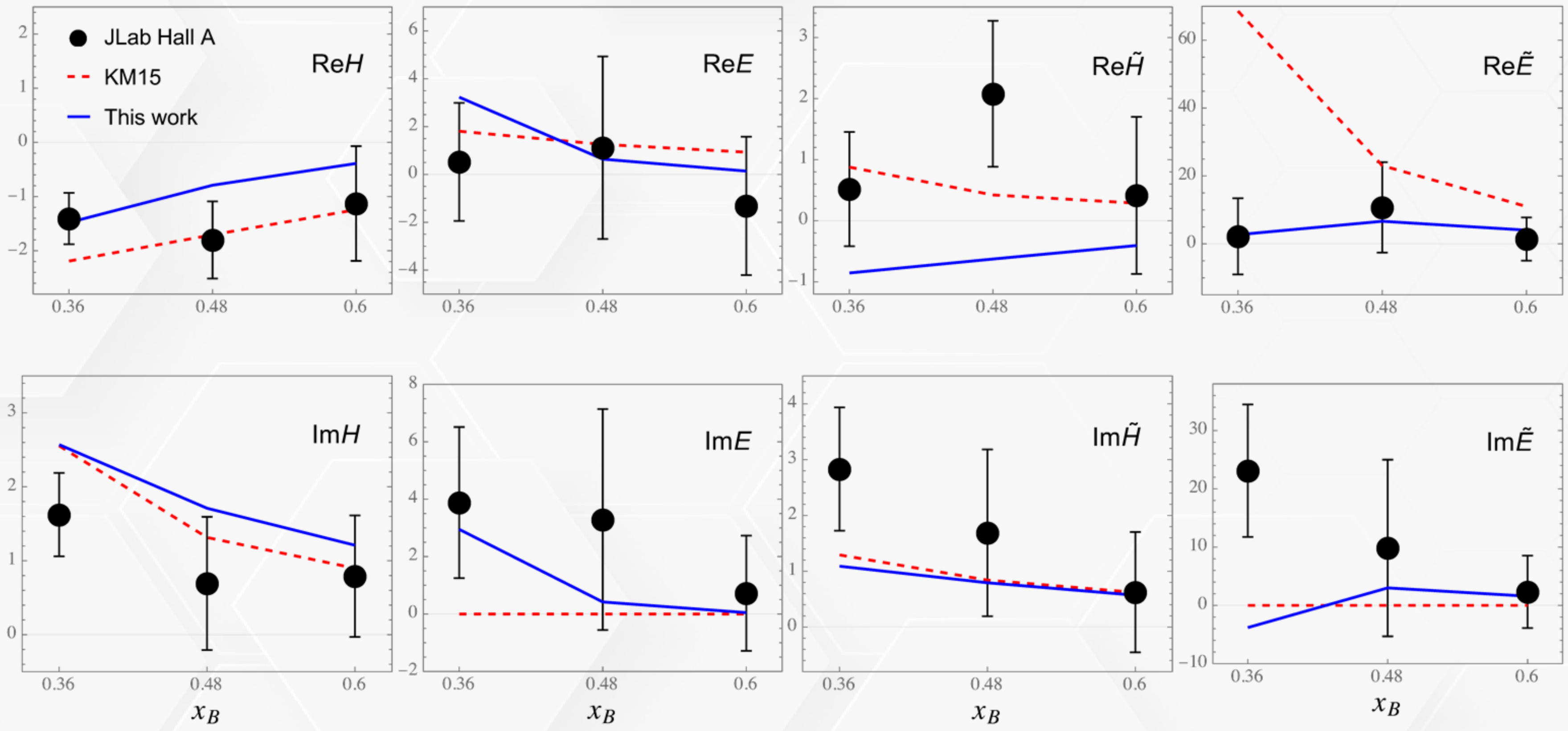
Goal: To obtain the state-of-the-art phenomenological GPDs through global analysis of both experimental data and lattice QCD simulations, utilizing a *universal moment parameterization* method.

Y. Guo et. al. arxiv: 2409.17231
Y. Guo et. al. JHEP 05 150 (2023)
Y. Guo et. al. JHEP 09 215 (2022)

Collaborators: Yuxun Guo, Xiangdong Ji, Gabriel Santiago, Fatma Aslan

Examples of phenomenological extraction

Comparison of CFFs from global and local extraction



Inverse problem of CFFs with NN

Variational autoencoder inverse mapper (VAIM) for solving CFFs

EXCLAIM Collab. arxiv: 2408.11681

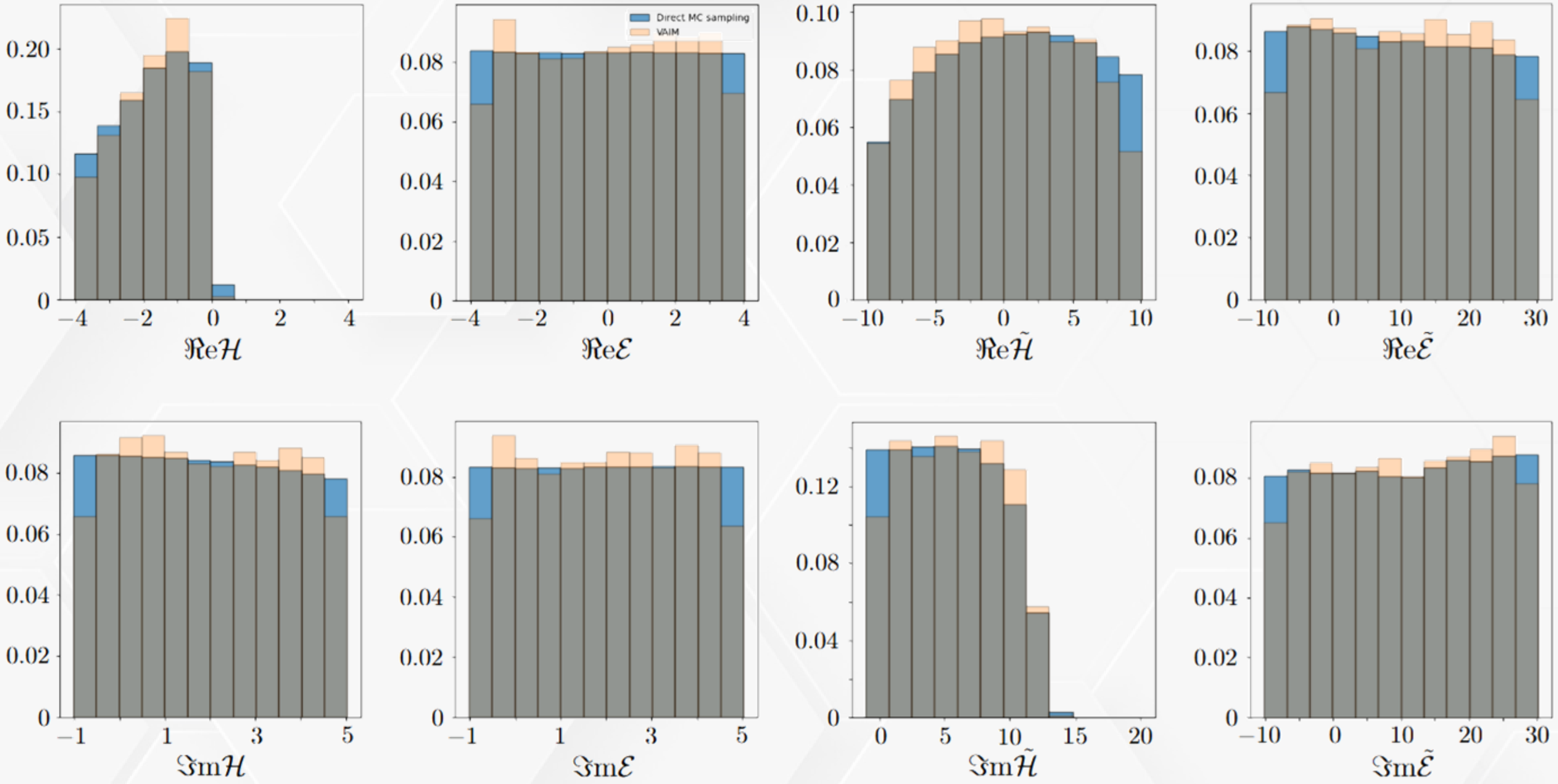
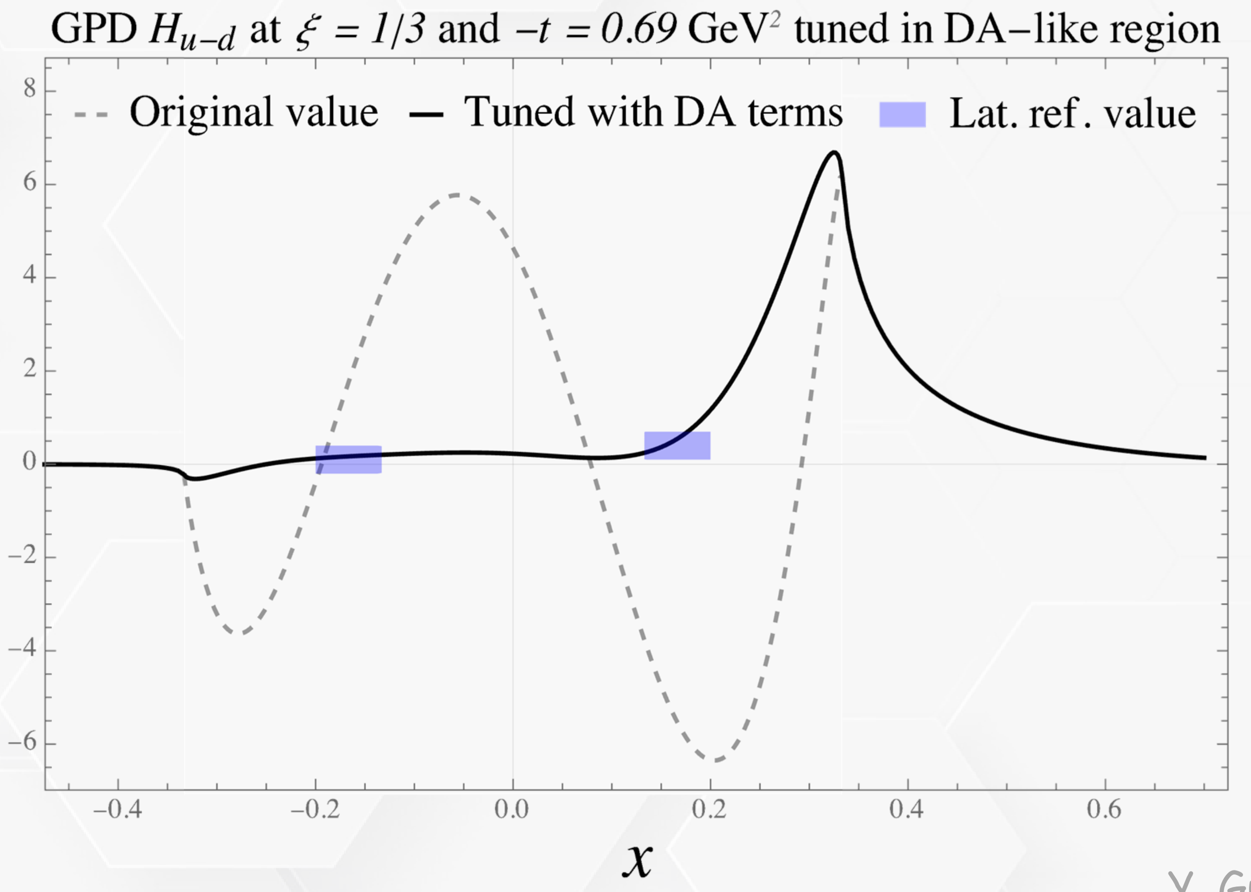


FIG. 4. Histogram of CFFs from sampling based on VAIM versus the MCMC method. All results are shown for a specific kinematics for the unpolarized cross section at $Q^2 = 1.82$ GeV^2, $t = -0.172$ GeV^2, $x_{Bj} = 0.343$, and $E_b = 5.75$ GeV.

Complementarity of lattice input

GPDs tuned to lattice input: lattice provide direct constraints on the x-dependence.



Y. Guo et. al. JHEP 05 150 (2023)

What about x-space extraction of GPDs

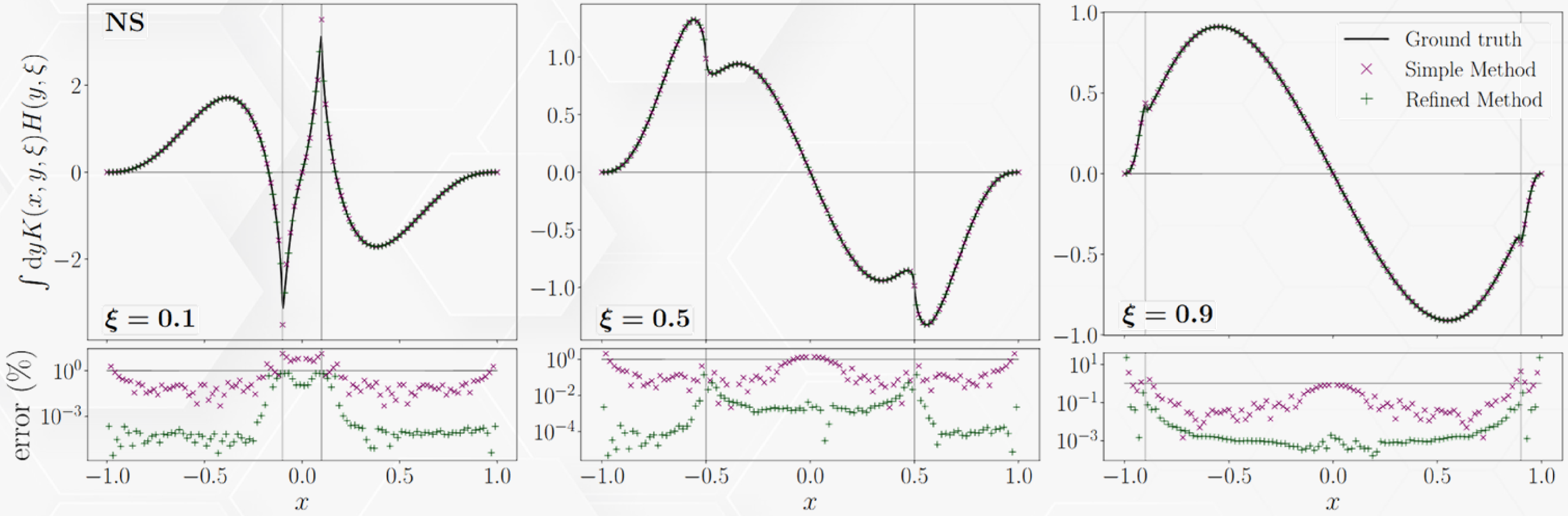


FIG. 3. Accuracy benchmark for the right-hand side of the evolution equation (1), at LO in the nonsinglet sector. This figure compares both the Simple Method (purple $\times$'s) and the Refined Method (green $+$'s) to a "ground truth," the latter determined with the GK model [29] using adaptive quadrature. For the Simple Method, we use $n_g = 5000$ Gaussian weight points and $N_x = 101$; for the Refined Method, we use $N_x = 100$. The left panel shows $\xi = 0.1$ to represent the lower limit of the intended domain of applicability for the methods; the middle panel shows $\xi = 0.5$ to represent a central ξ value; and the right panel uses $\xi = 0.9$ to represent a large ξ value.

Summary

Summary

- GPDs as the tool to explore the nucleon tomography with the future EIC.
- The inverse problems are intrinsic in the GPD extraction.
- Global analyses including lattice inputs are required to resolve the inverse problem.
- Some recent developments in the phenomenological studies of GPD.

Outlook

PHENOMENOLOGY LATTICE QCD THEORY

QUARK-GLUON TOMOGRAPHY COLLABORATION

Thank you!