Gauge theory dynamics with tensor networks

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Work done in collaboration with

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arXiv:2501.18301v3
arXiv:25xx.xxxxx

The Electron-Ion Collider

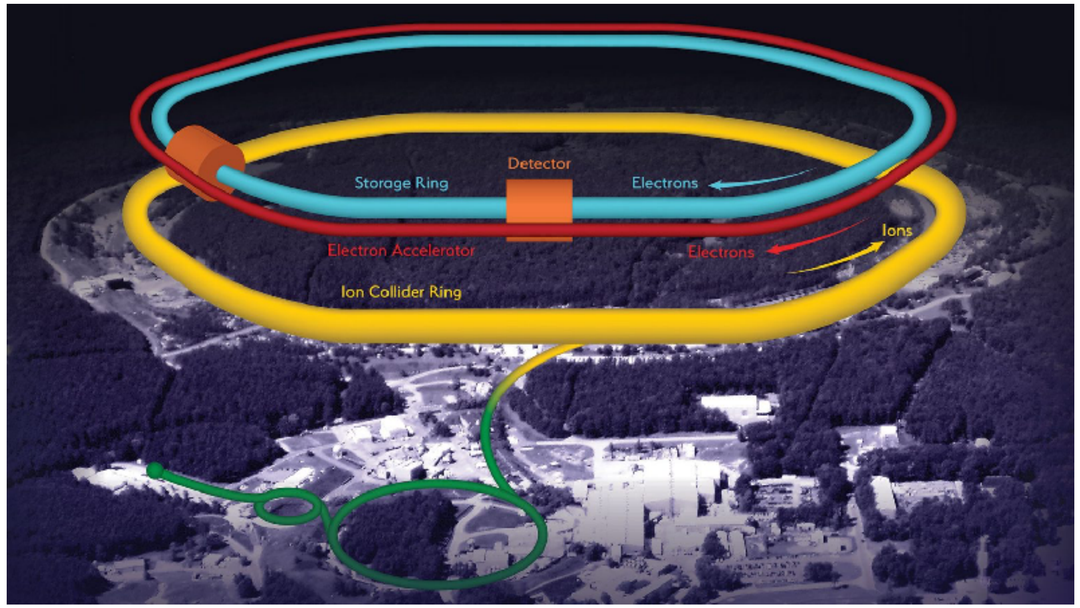


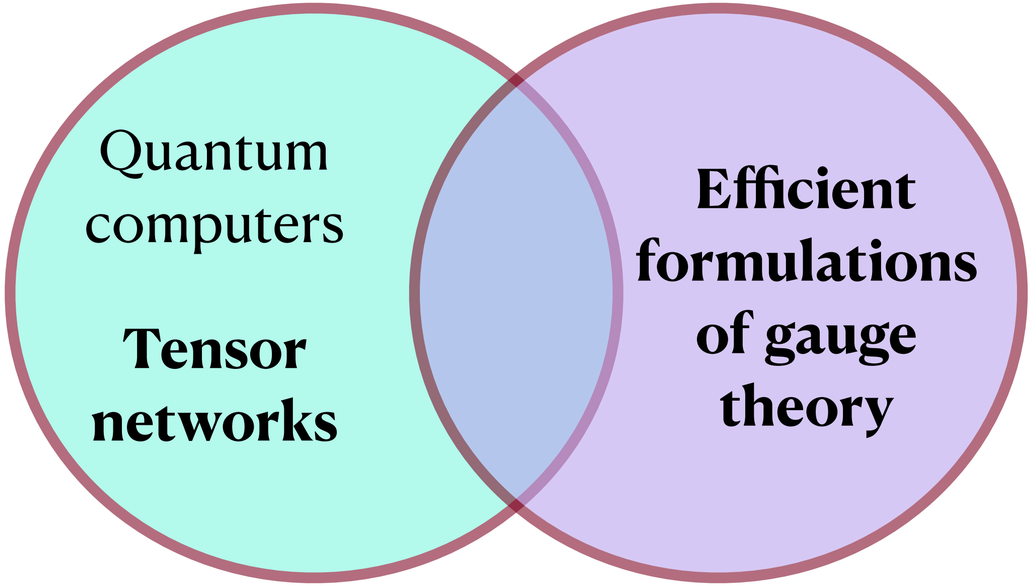
Image courtesy of Brookhaven National Lab

- The goals of the EIC are to study: Deep inelastic scattering, PDFs, Multidimensional imaging (GPDs, TMDs etc.), hadronization, fragmentation etc.
- A theoretical/ computational program will go hand-in-hand with experiments.
- These quantities involve Minkowski-time matrix elements.
- While Euclidean/ EFT methods exist to compute them, Hamiltonian simulations can offer a powerful complementary approach.
- Such Hamiltonian methods are especially useful to study real-time dynamics from first principles.

Hamiltonian simulations are challenging due to the exponentially large Hilbert space.

Gauge theory simulation is especially tricky due to complicated gauge constraints and degrees of freedom.

Powerful techniques emerge at the intersection of efficient quantum many-body ansatzes and efficient formulations of gauge theory.

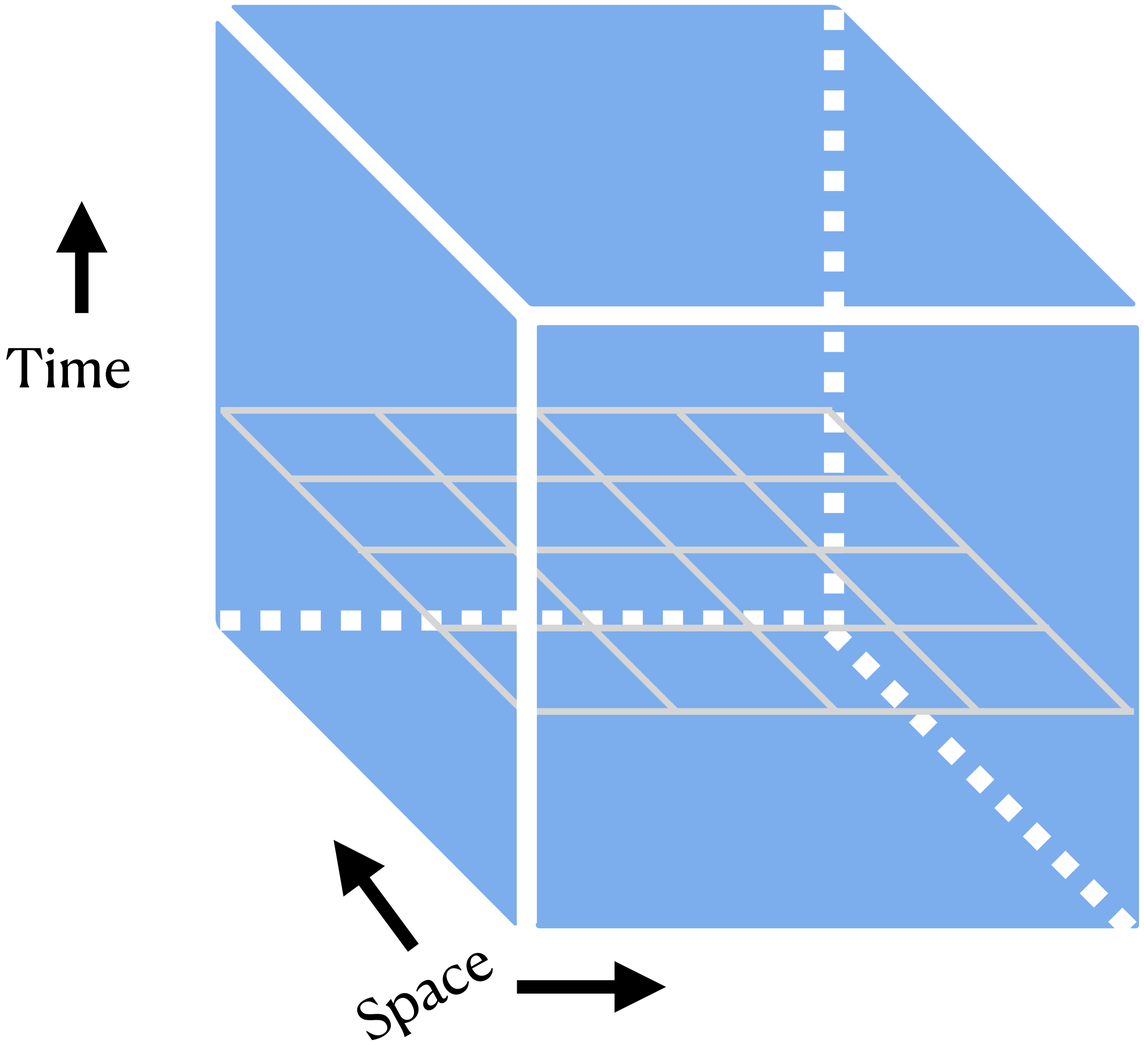


While classical tensor network computations may not get us all the way to full QCD, they are useful testbeds and can eventually inform larger-scale quantum computations!

Overview

- **Hamiltonian formulations of gauge theory:**
  From Kogut-Susskind to the loop-string-hadron (LSH) formulation
- **Matrix product states for the LSH formulation**
- **Computation of static and dynamical properties:**
  - Ground state computations
  - Simulation of dynamical string breaking

The Kogut-Susskind formulation for $\mathrm{SU}(n)$ LGT



- Quantize gauge theory on equal time slices.
- Pick the temporal gauge $A_0 = 0$.
- Discretize space.
- Time stays continuous.

Kogut, John, and Leonard Susskind. "Hamiltonian formulation of Wilson's lattice gauge theories." Physical Review D 11.2 (1975): 395.

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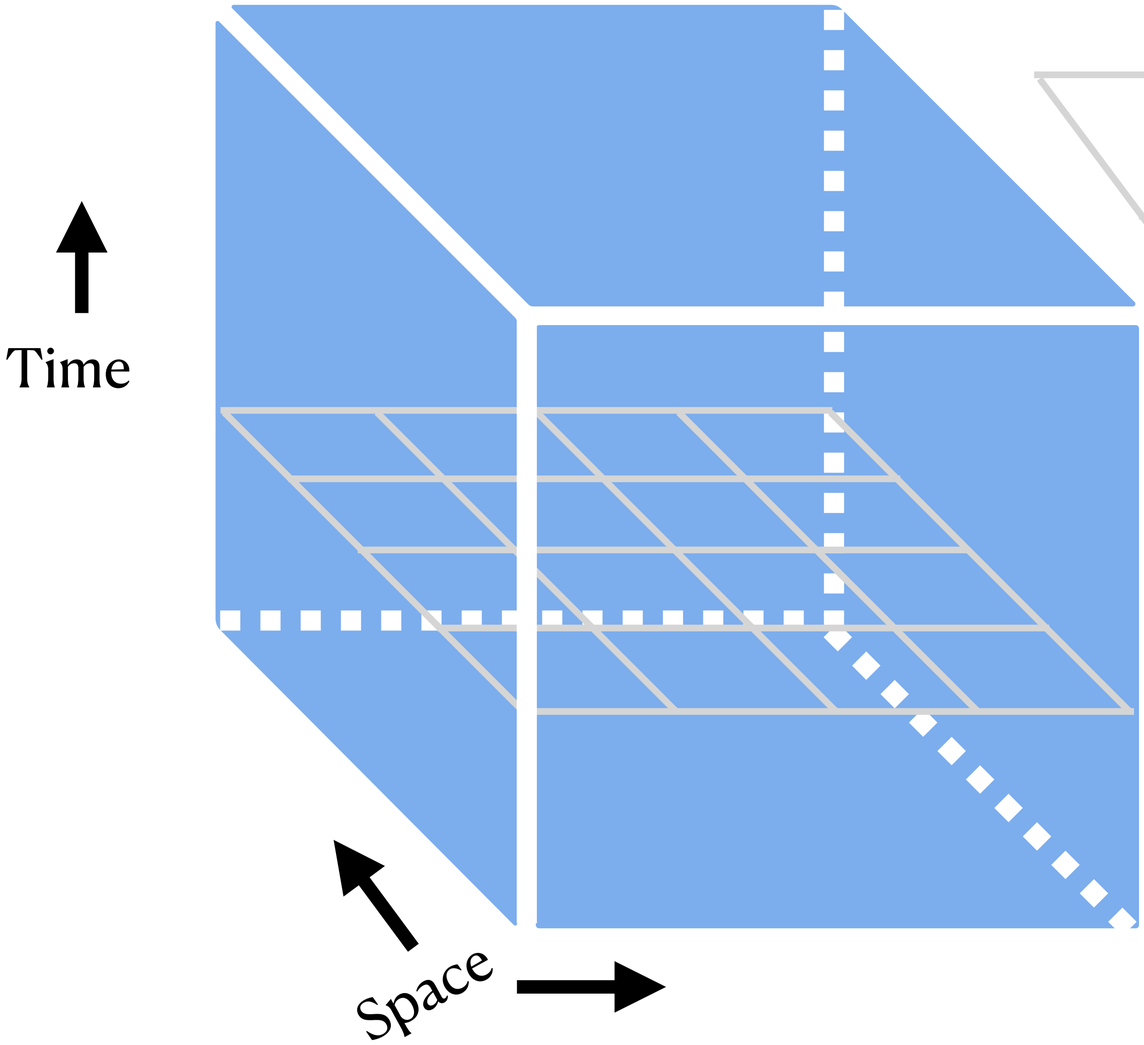

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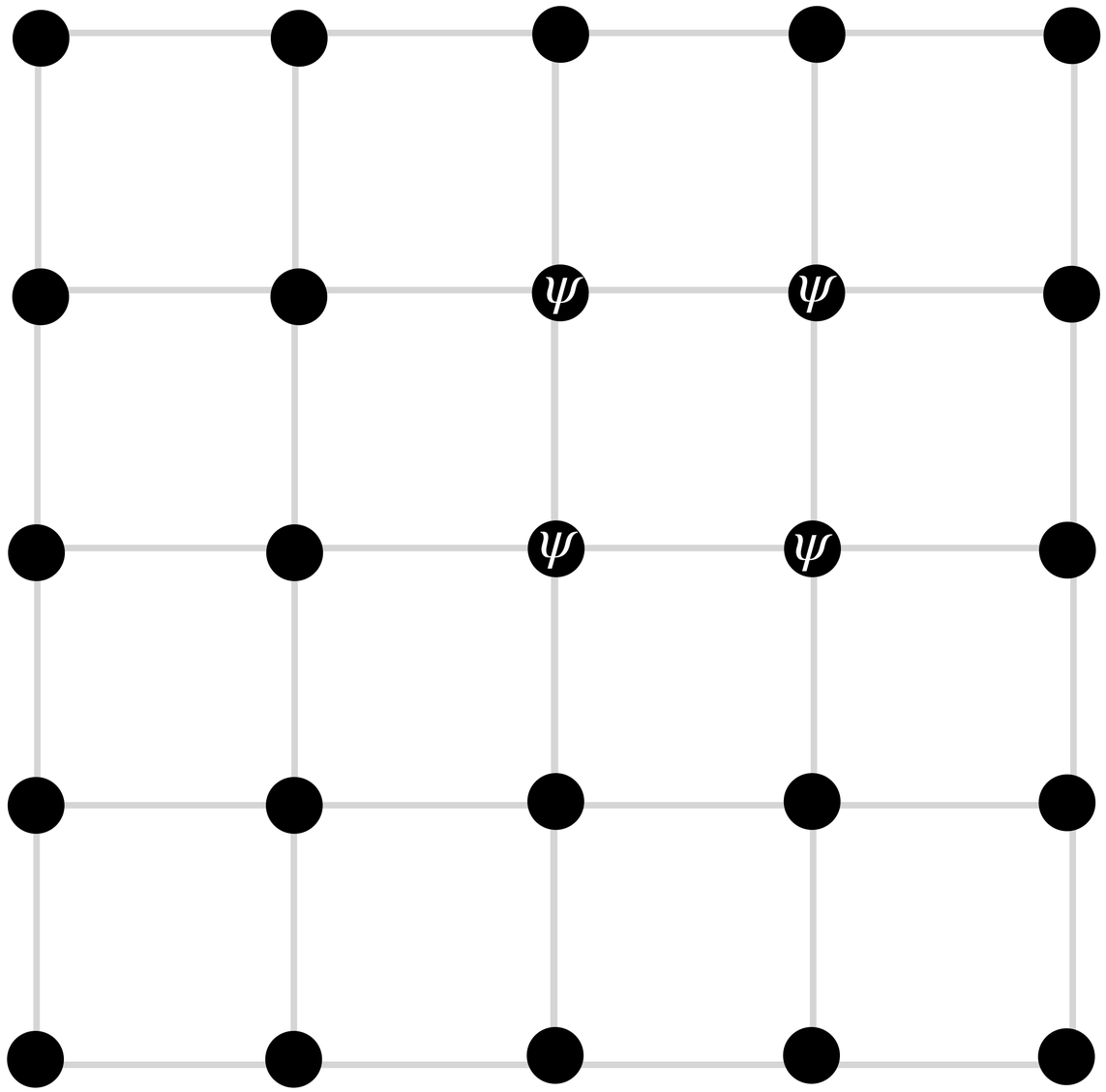

At sites,

- Staggered matter ψ^α, $\alpha \in \{1,\ldots,n\}$
- ψ^α is a fundamental (anti-fundamental) SU(n) fermion on even (odd) sites
- Hilbert space described by fermion occupation $|f^\alpha\rangle$, $f^\alpha \in \{0,1\}$

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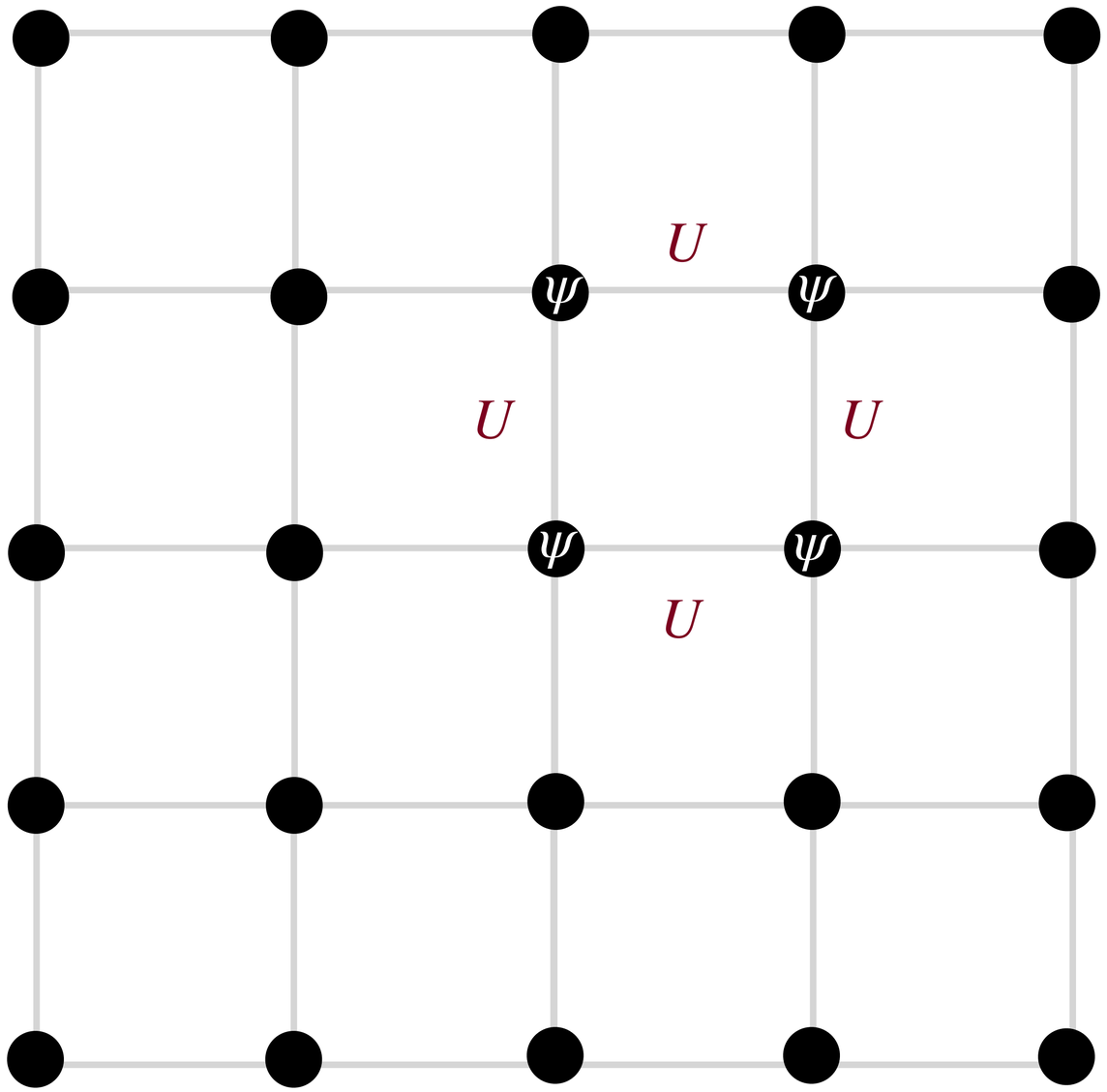

At links,

- Gauge holonomy $U^{\alpha\beta},\ \alpha,\beta \in \{1,\ldots,n\}$

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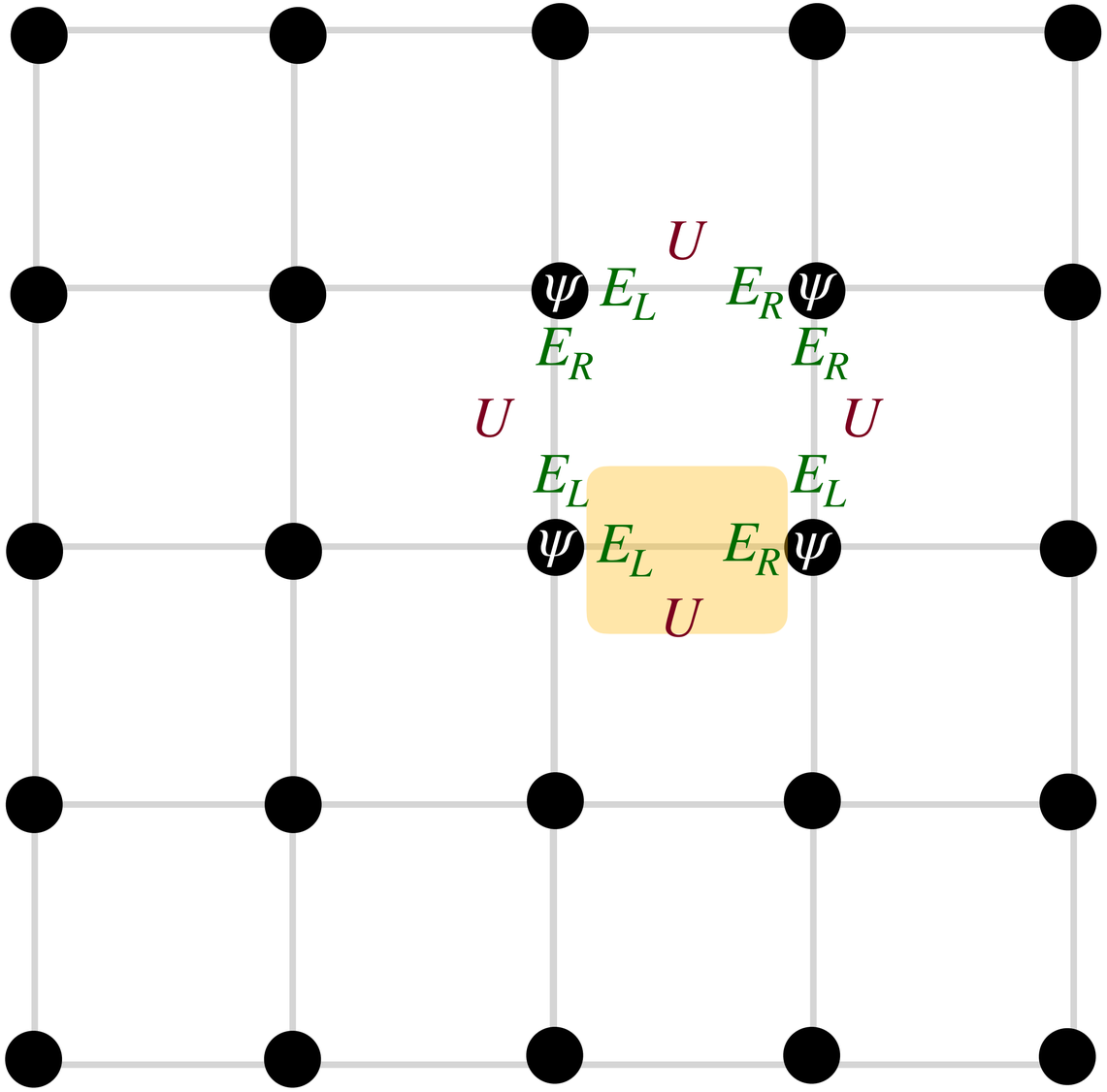

At links,

- Gauge holonomy $U^{\alpha\beta}$, $\alpha, \beta \in \{1, \ldots, n\}$
- Left and right electric fields E_L^a, E_R^a, $a \in \{1, \ldots, n^2 - 1\}$

$$[E_L^a, E_L^b] = if^{abc} E_L^c$$
$$[E_R^a, E_R^b] = if^{abc} E_R^c$$
$$[E_R^a, E_L^b] = 0$$

$$[E_L^a, U] = -T^a U$$
$$[E_R^a, U] = U T^a$$

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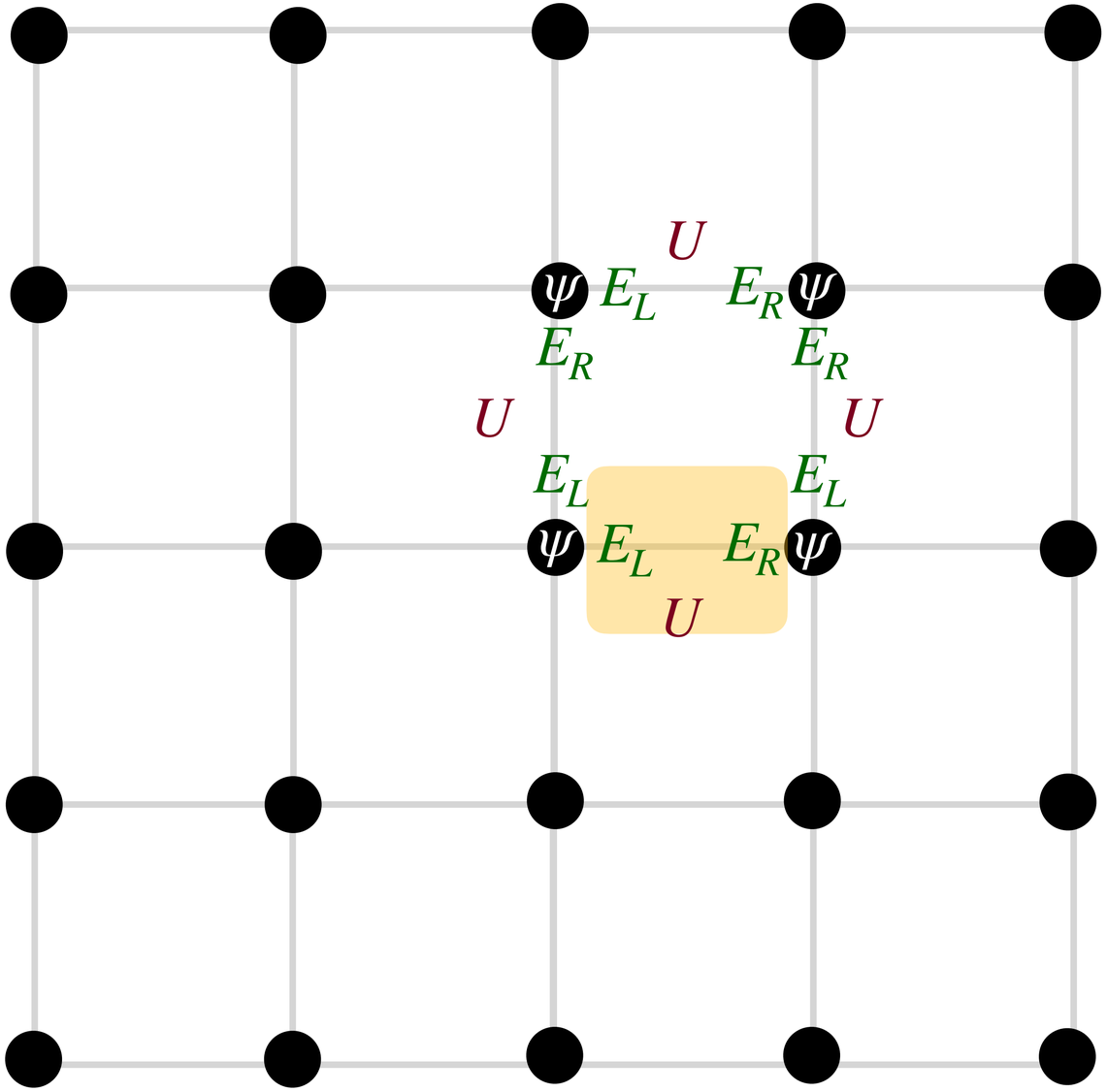

At links,

- Hilbert space described by group states
- Labelled by the irrep j and identifiers m_L and m_R at the two ends of the link:
  $|jm_L\rangle\,|jm_R\rangle = |jm_Lm_R\rangle$
- $E_L^2\,|jm_Lm_R\rangle_r = E_R^2\,|jm_Lm_R\rangle = f(j)\,|jm_Lm_R\rangle$
- U^{ab} mixes different irreps in a group-dependent fashion.

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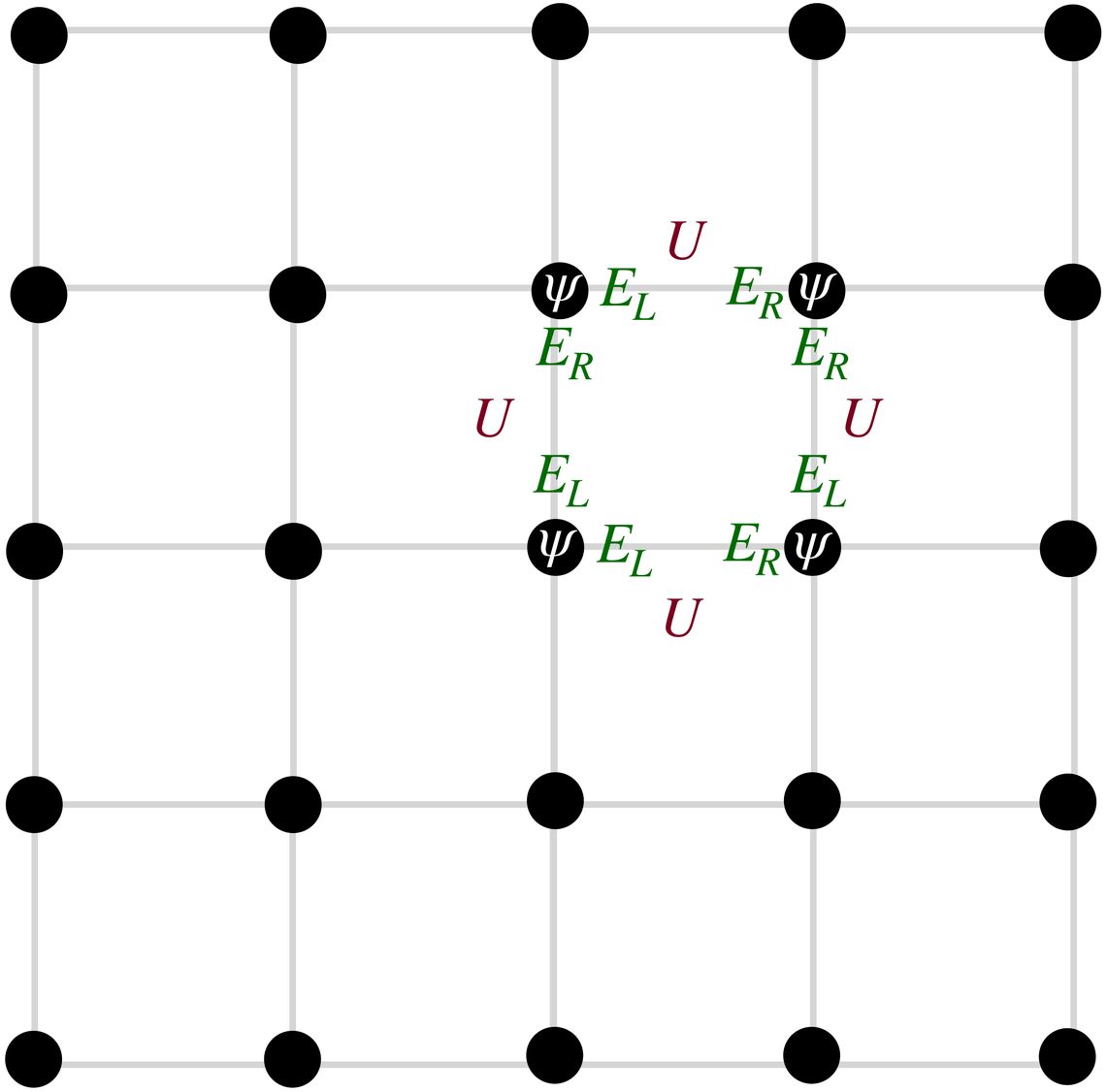

$$H = \frac{g^2 a}{2} \sum_{\text{links}} E^2 - \frac{1}{g^2 a^3} \sum_{\text{plaquettes}} \left(\mathrm{Tr}(\square) + \mathrm{h.c.} \right)$$

$$+ m \sum_{\text{sites}} (-1)^{S_1} \times \psi^\dagger \psi$$

$$+ \frac{1}{2a} \sum_{\text{links}} \left((-1)^{S_2}\ \psi^\dagger U \psi + \mathrm{h.c.} \right)$$

- S_1 and S_2 are site-dependent staggering phases.

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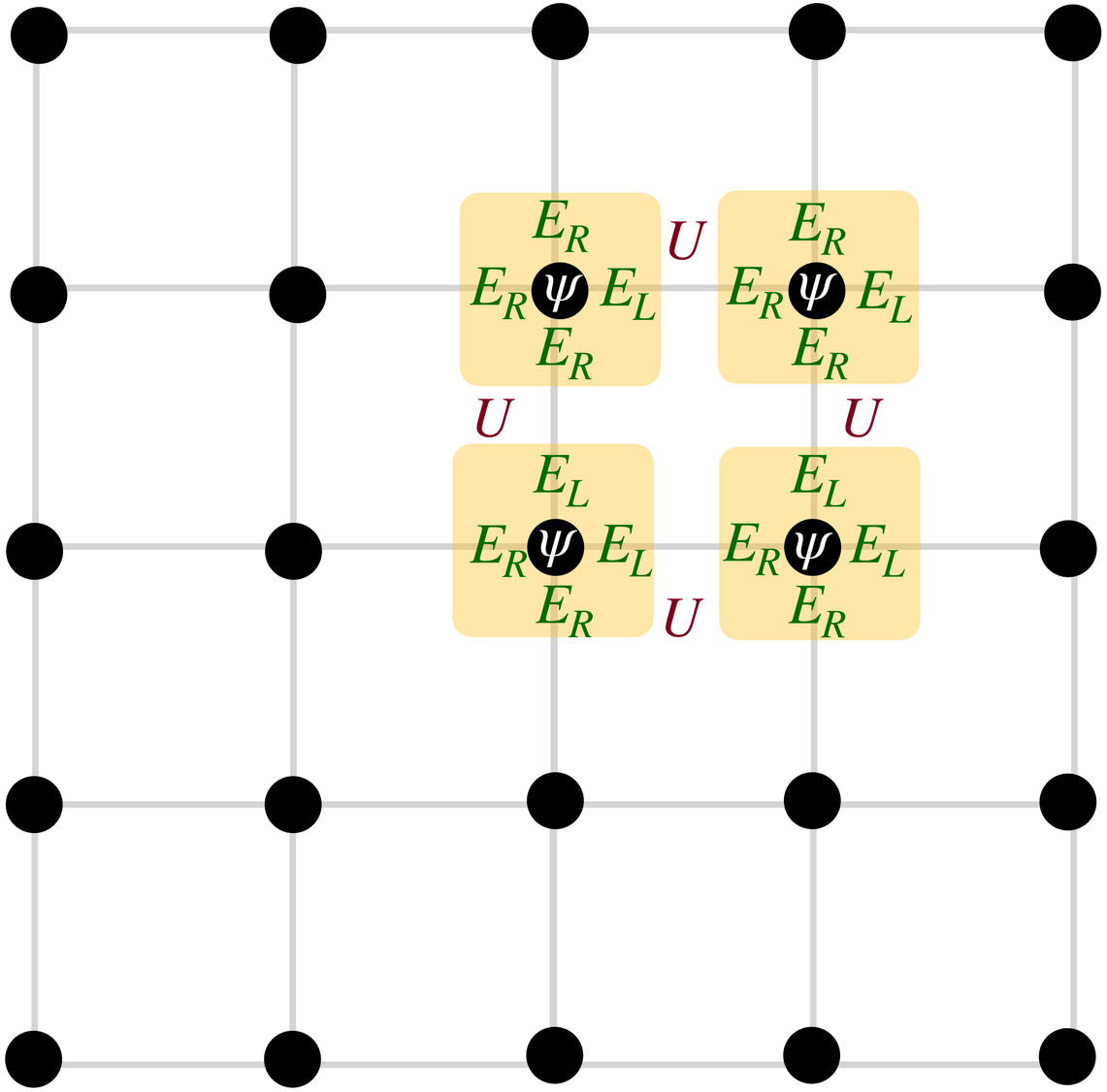

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- Physical states $|\phi\rangle$ obey Gauss law:

$$G^a |\phi\rangle = 0,$$

$$G^a = \sum_{\text{directions } i} \left(E^a_{L,i} + E^a_{R,i} \right) + \psi^\dagger T^a \psi,$$

$$[G^a, G^b] = i f^{abc} G^c,$$

at each site and for all $a \in \{1, \ldots, n^2 - 1\}$.

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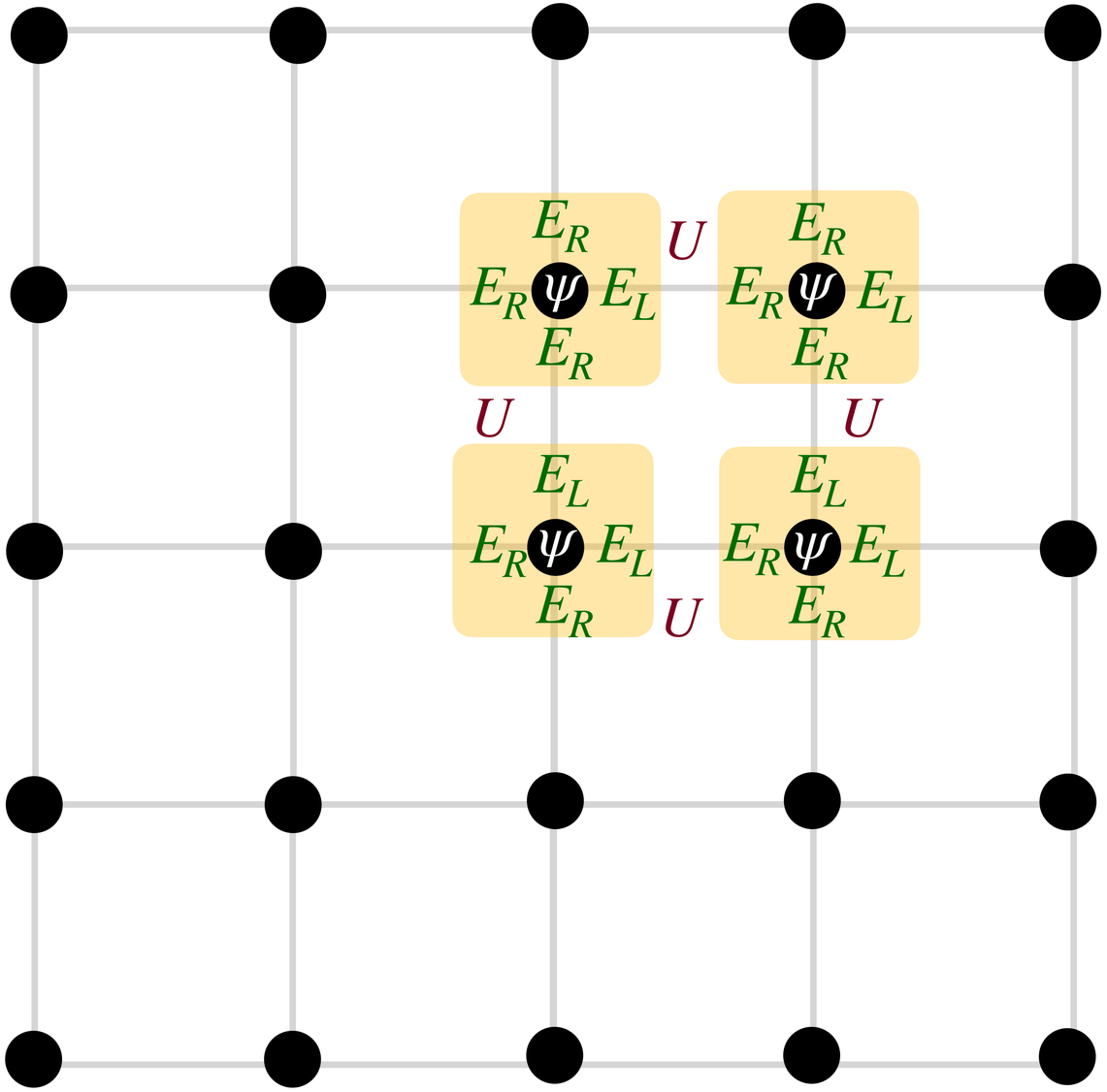

- No gauge fixing beyond temporal gauge - non-commuting Gauss laws at each site.
- Full tensor product Hilbert space much larger than physical Hilbert space.
- Several basis and truncation choices for link Hilbert space.

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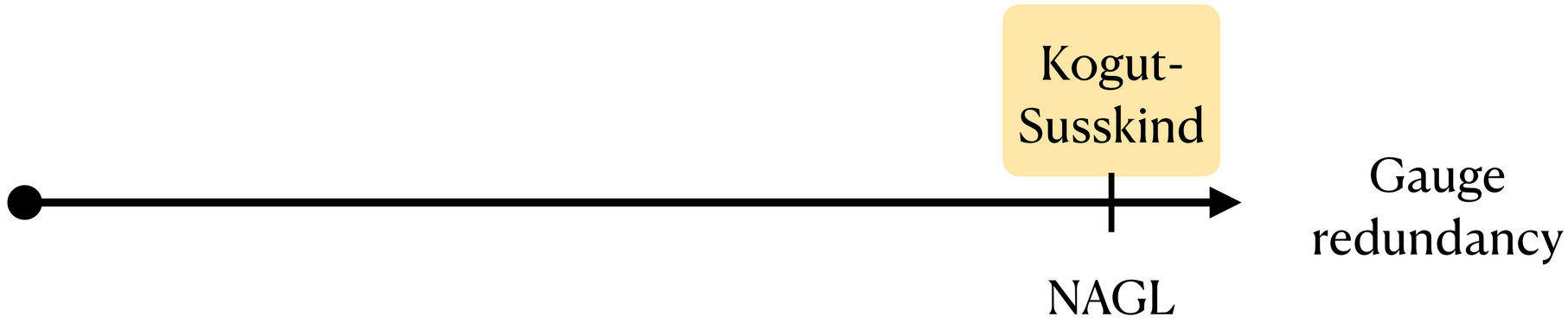
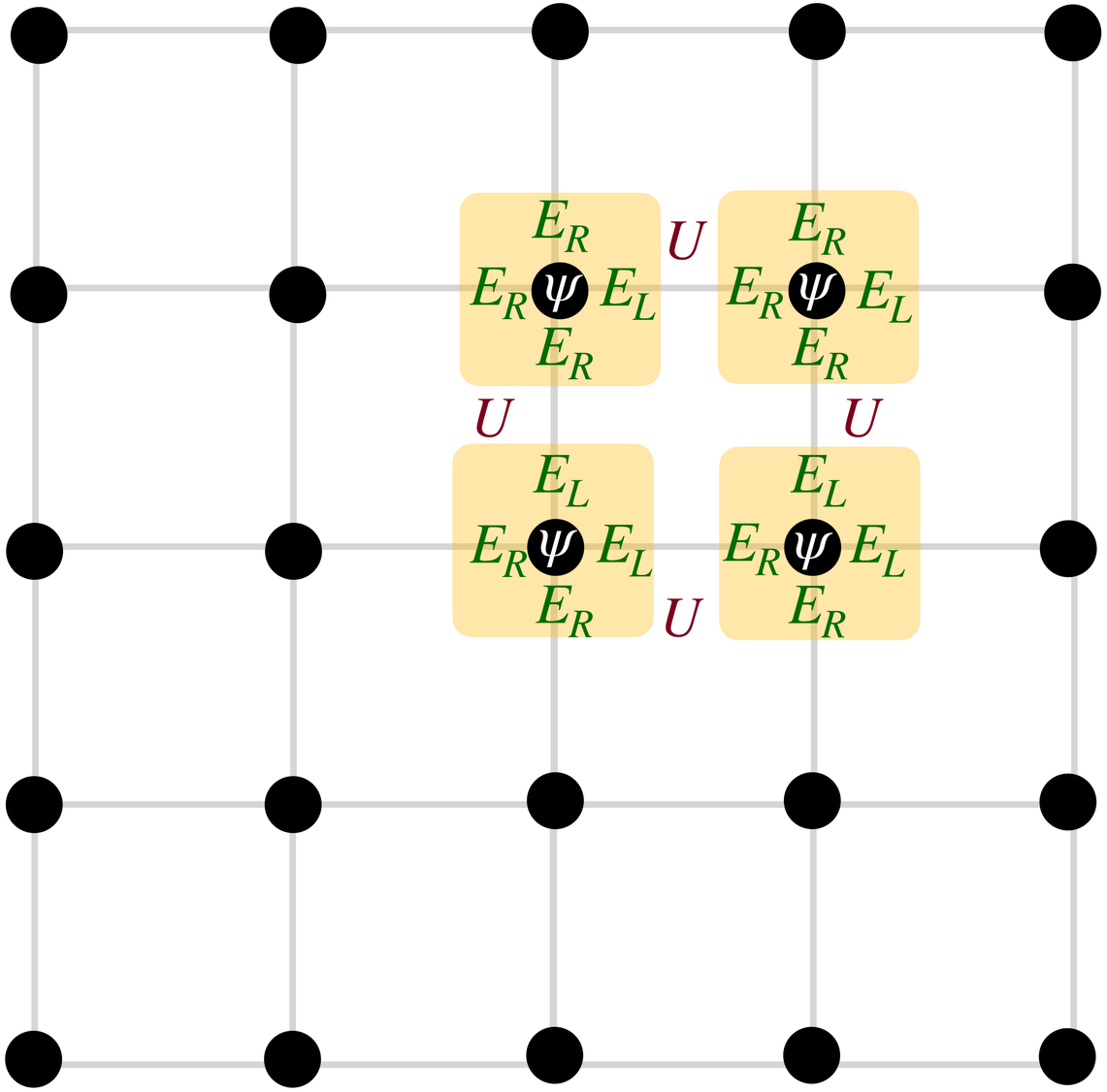


NAGL = Non-Abelian Gauss law

The Kogut-Susskind formulation for SU(n) LGT



- No gauge fixing beyond temporal gauge - non-commuting Gauss laws at each site.
  Fixing gauge beyond temporal gauge.
- Full tensor product Hilbert space much larger than physical Hilbert space.
  Working with smaller subspaces
- Several basis and truncation choices for link Hilbert space.
  Determining appropriate bases and truncations

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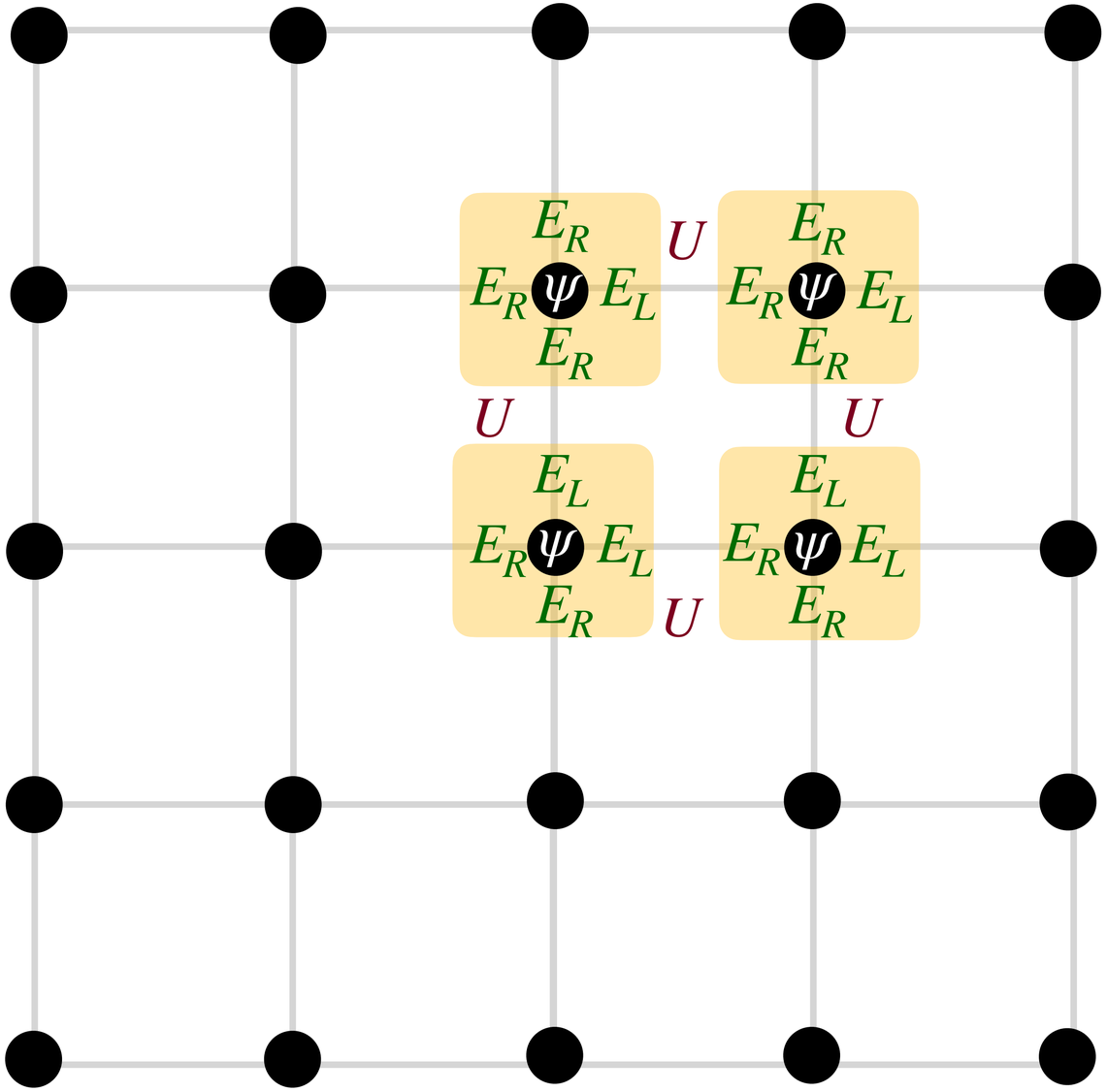

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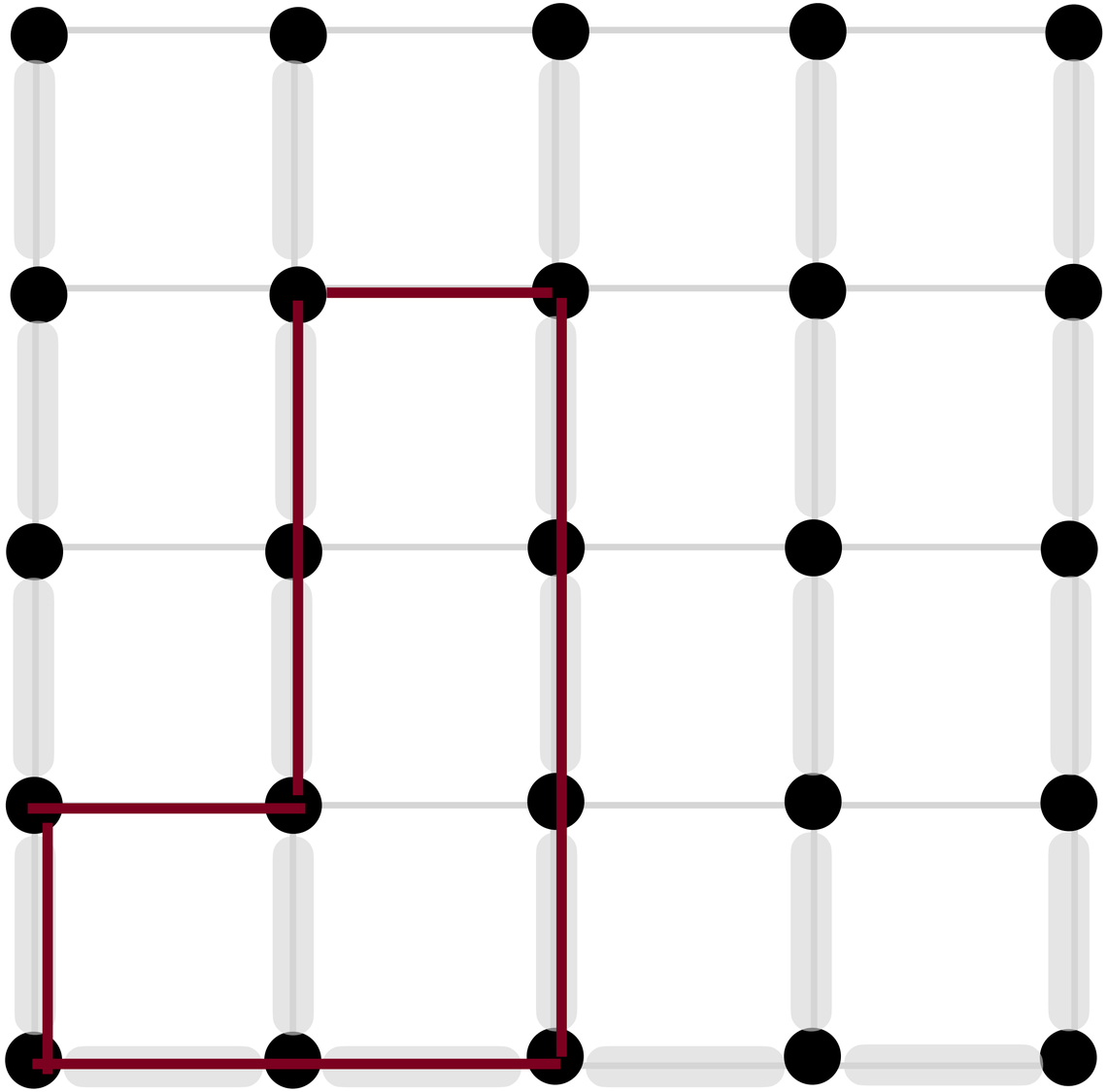
Gauge fixing beyond temporal gauge: Maximal trees



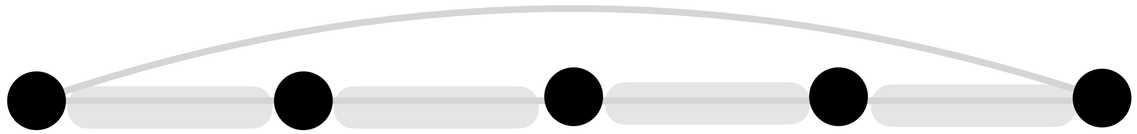
- Link variables U set to identity along maximal tree.
- Non-local loops (shown in red) serve as link degrees of freedom
- Non-local Hamiltonian

Pros: Fewer degrees of freedom + constraints

Cons: Non-locality + More complicated residual Gauss law + Trickier to handle matter

Ligterink, N. E., Niels R. Walet, and R. F. Bishop. "Toward a many-body treatment of Hamiltonian lattice SU (N) gauge theory." Annals of Physics 284.2 (2000): 215-262.
D'Andrea, I., Bauer, C. W., Grabowska, D. M., & Freytsis, M. (2024). New basis for Hamiltonian SU (2) simulations. Physical Review D, 109(7), 074501.
Grabowska, Dorota M., Christopher F. Kane, and Christian W. Bauer. "A Fully Gauge-Fixed SU (2) Hamiltonian for Quantum Simulations." arXiv preprint arXiv:2409.10610 (2024).

Gauge fixing beyond temporal gauge: The $(1 + 1)$D case

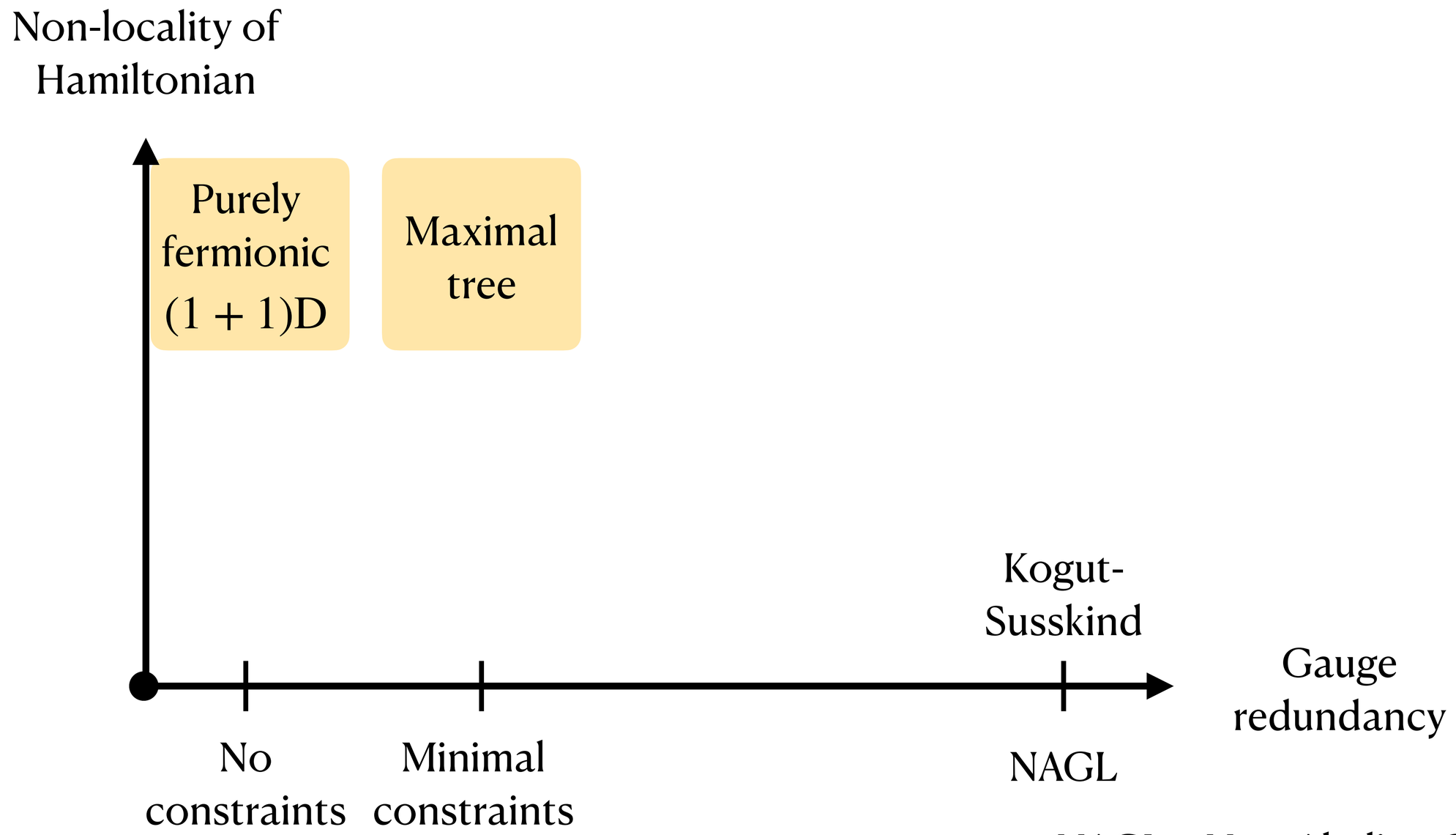


- Gauge degree of freedom can be eliminated completely (except the boundary mode)
- Straightforward solution of constraints allows a theory with only matter

Pros: Purely matter theory with a single boundary or no constraint

Cons: Non-locality + Only works for $(1 + 1)$D

Davoudi, Zohreh, Indrakshi Raychowdhury, and Andrew Shaw. "Search for efficient formulations for Hamiltonian simulation of non-Abelian lattice gauge theories." Physical Review D 104.7 (2021): 074505.



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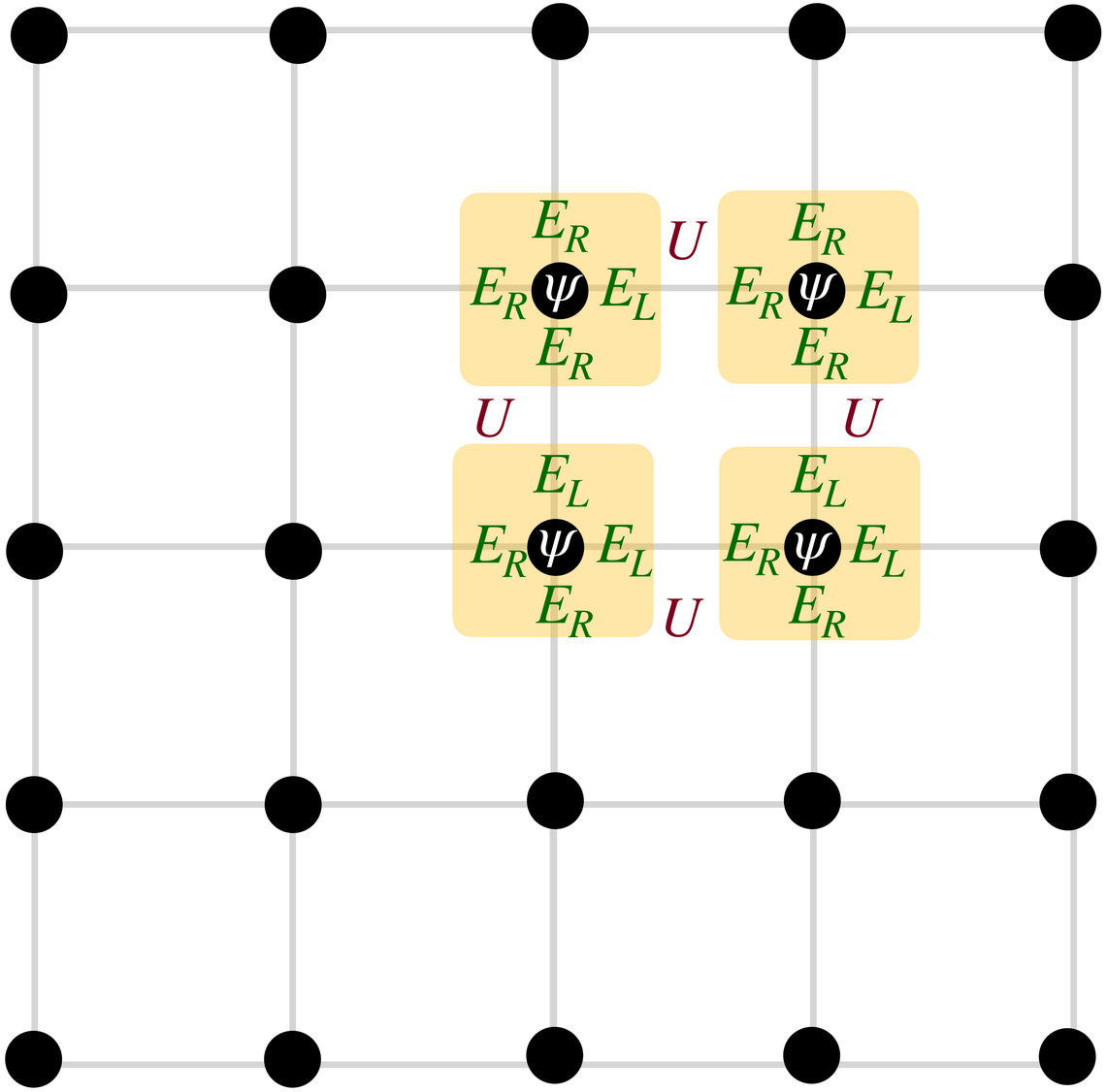

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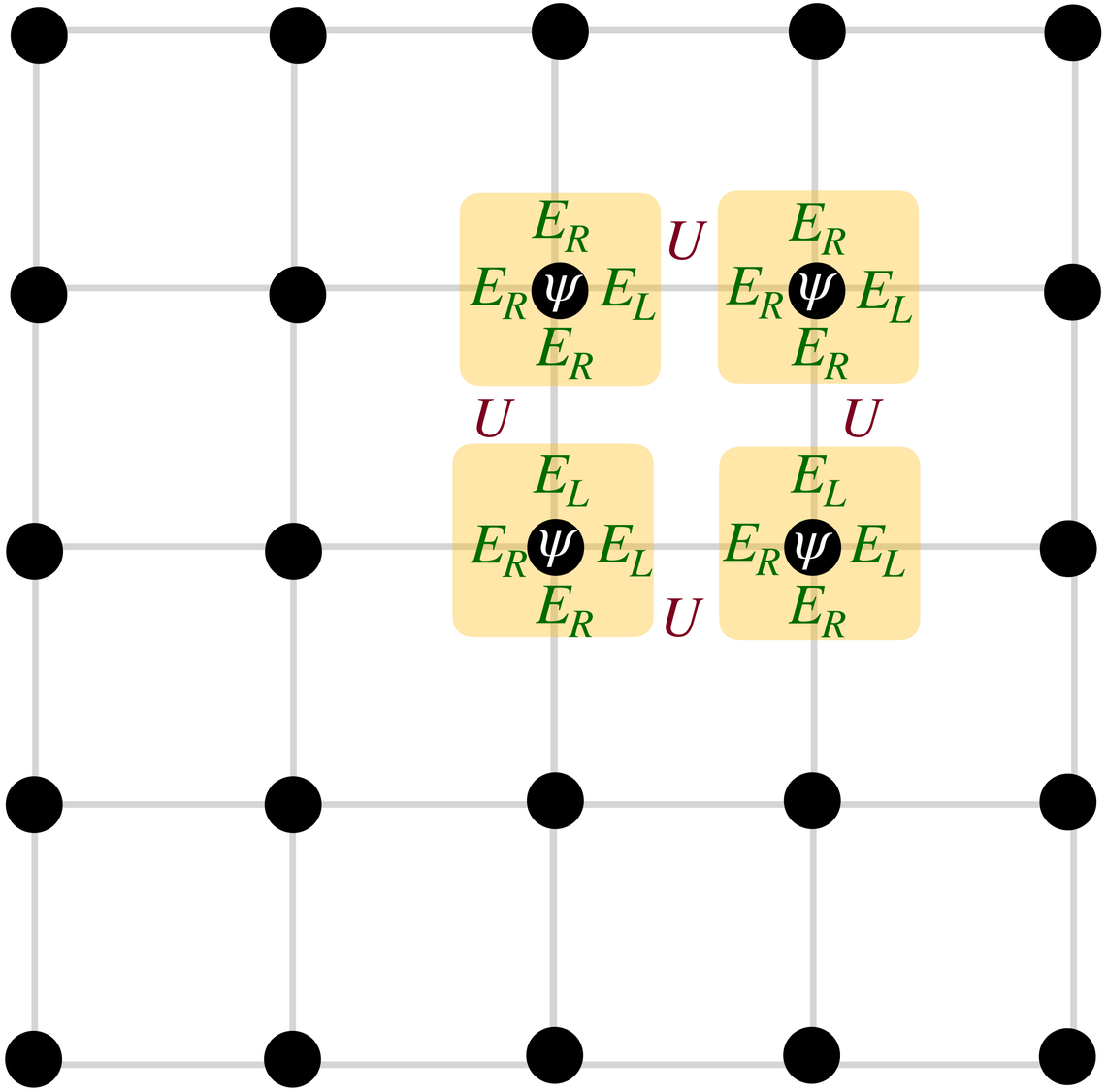

- No gauge fixing beyond temporal gauge - non-commuting Gauss laws at each site.
  Fixing gauge beyond temporal gauge.
- Full tensor product Hilbert space much larger than physical Hilbert space.
  Can we work with smaller subspaces while avoiding non-locality?
- Several basis and truncation choices for link Hilbert space.
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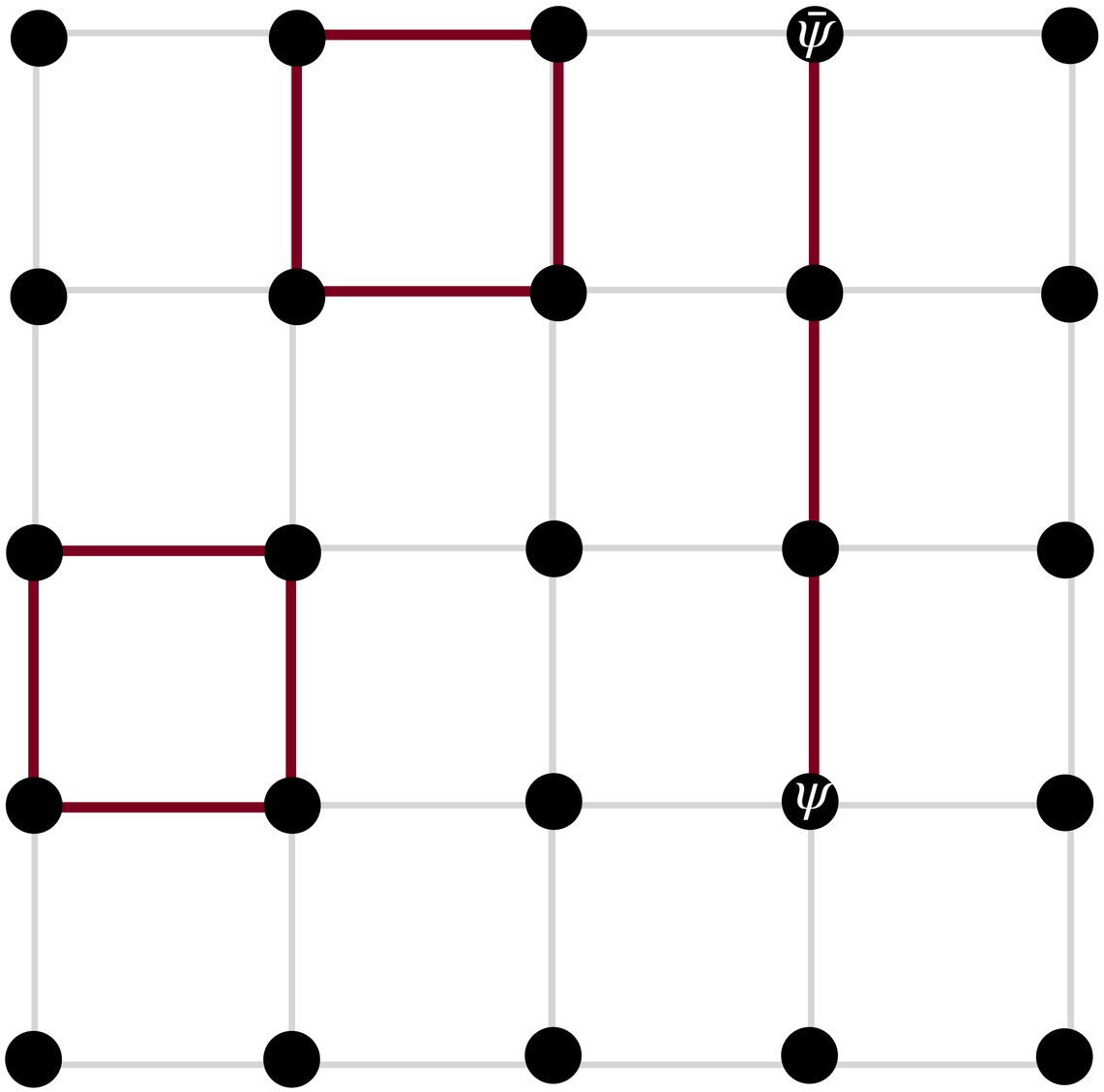
Working with smaller subspaces: Basis of physical subspace

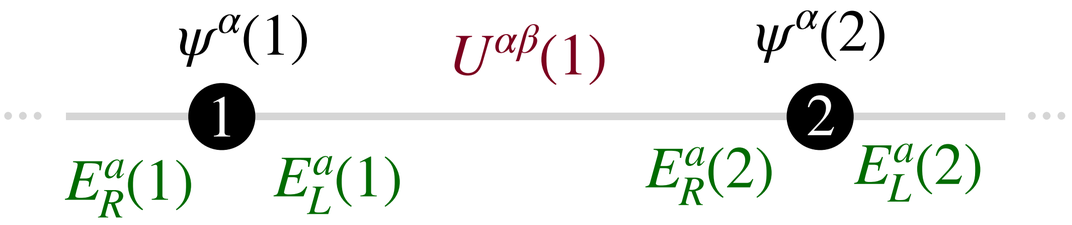


- Ideally: Work with basis of gauge-invariant states which only spans physical Hilbert space
- However: set of gauge-invariant states is over-complete.
- Determining a linearly independent and complete basis for physical subspace is challenging
- Resulting basis would be non-local

Mandelstam, S. "Charge-monopole duality and the phases of non-Abelian gauge theories." Physical Review D 19.8 (1979): 2391.

Working with smaller subspaces: Prepotentials

For (1 + 1)D SU(2) [Generalizations exist]:



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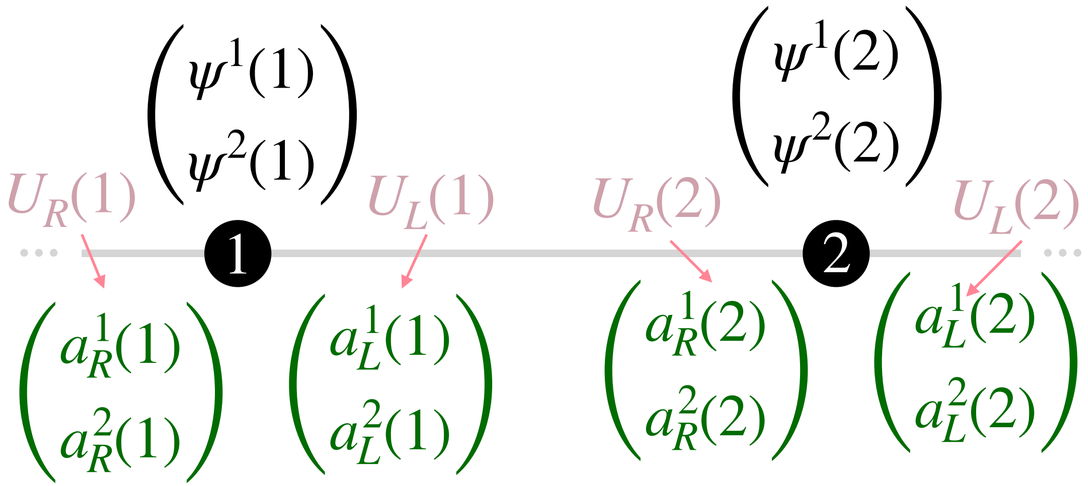

$$E^a_{L/R}(r) = a^\dagger_{L/R}(r) T^a a_{L/R}(r)$$

$$U(r) = U_L(r) U_R(r+1)$$

$$N_{L/R}(r) = a^\dagger_{L/R}(r) a_{L/R}(r)$$

$$N_\psi(r) = \psi^\dagger(r)\psi(r)$$

- All degrees of freedom are now associated with just the sites.
- The caveat of splitting the link is that the conservation of the Casimir i.e., $j = N_L(r) = N_R(r+1)$ must be separately imposed. This is referred to as the "Abelian Gauss law" (AGL).
- But now we can solve the non-Abelian Gauss law (NAGL) at each site.

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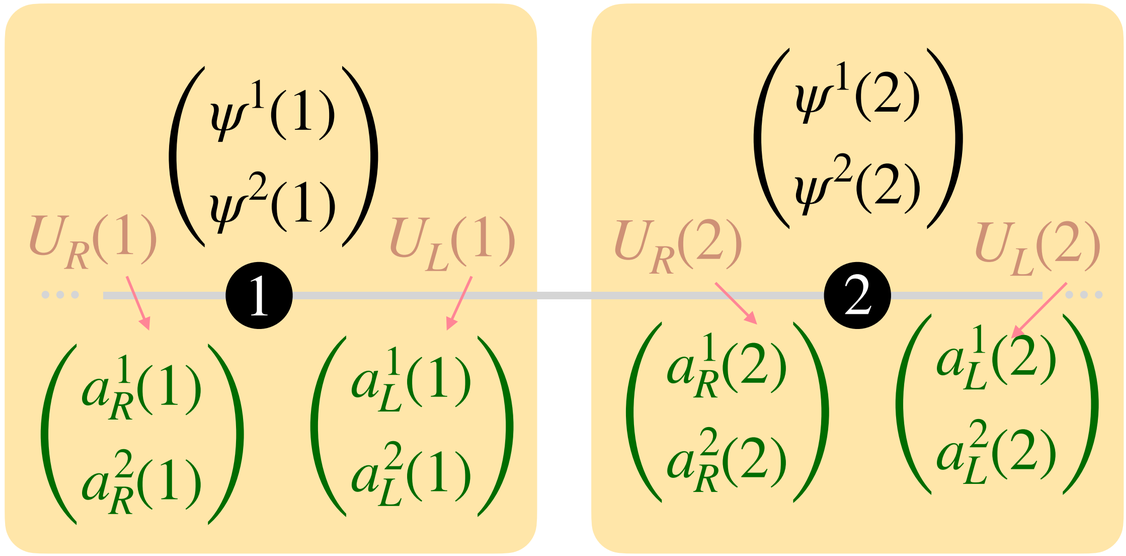
Working with smaller subspaces: Loop-string-hadron DOFs

Loop operators: Gauge-invariant combinations of a_L, a_R

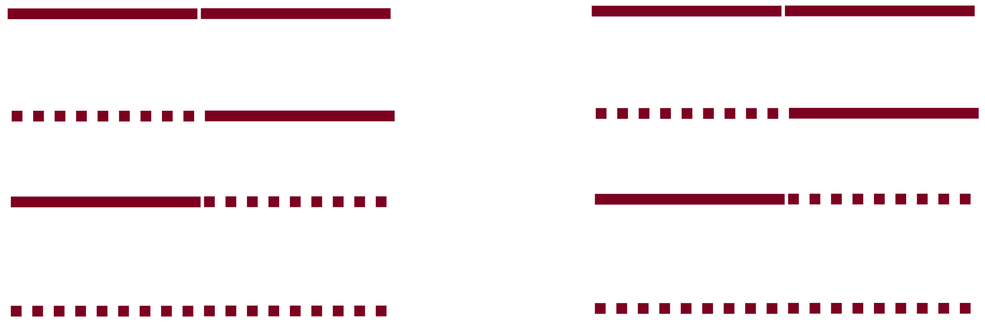


$$\mathscr{L}^{++} = a_R^\dagger \, \epsilon \, (a_L^\dagger)^T$$

$$\mathscr{L}^{--} = a_R^T \, \epsilon \, a_L$$

$$\mathscr{L}^{+-} = a_R^\dagger \, a_L$$

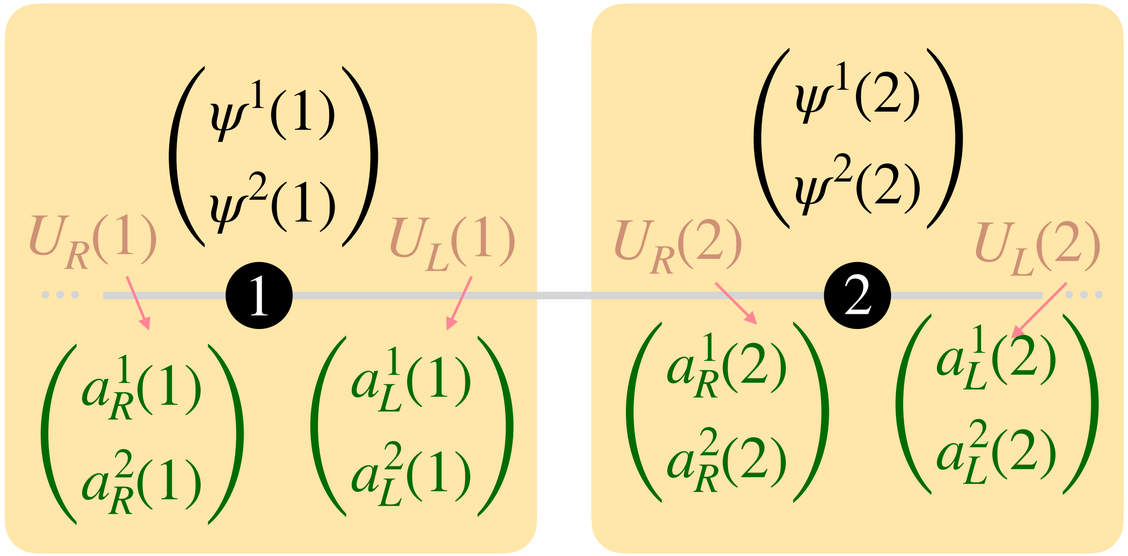
$$\mathscr{L}^{-+} = a_R^T \, (a_L^\dagger)^T$$



Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.

Working with smaller subspaces: Loop-string-hadron DOFs

String operators:
Gauge-invariant combinations of a_L, a_R, ψ, $\psi^\dagger$



$$\mathcal{S}^{++}_{\text{in}} = a_R^\dagger \, \epsilon \, (\psi^\dagger)^T$$
$$\mathcal{S}^{+-}_{\text{in}} = a_R^\dagger \, \psi$$
$$\mathcal{S}^{--}_{\text{in}} = a_R^T \, \epsilon \, \psi$$
$$\mathcal{S}^{-+}_{\text{in}} = a_R^T \, (\psi^\dagger)^T$$

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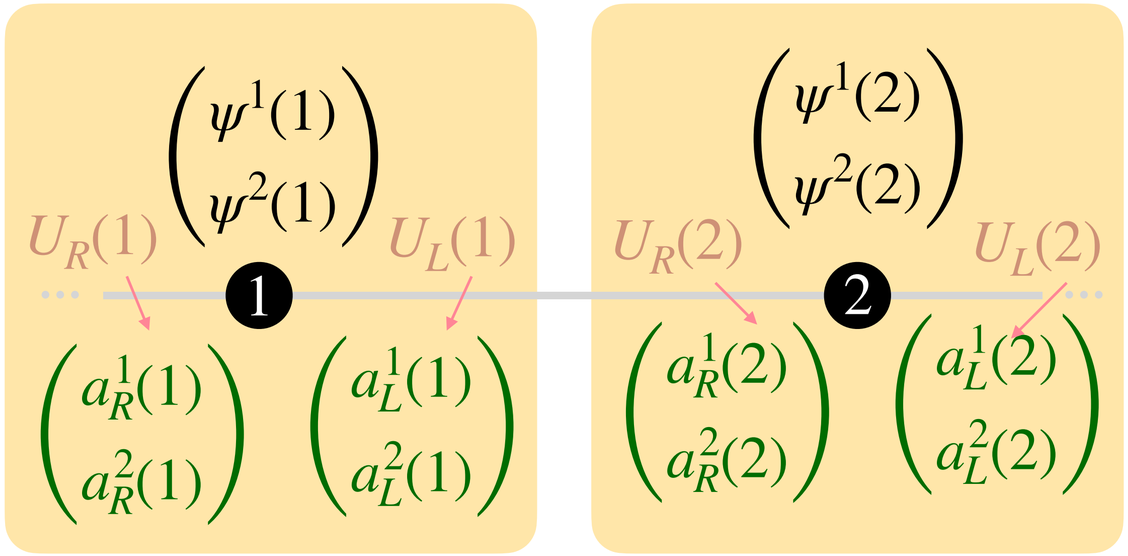

$$\mathcal{S}^{++}_{\text{out}} = \psi^\dagger \; \epsilon \; (a_L^\dagger)^T$$

$$\mathcal{S}^{-+}_{\text{out}} = \psi^T \; (a_L^\dagger)^T$$

$$\mathcal{S}^{--}_{\text{out}} = \psi^T \; \epsilon \; a_L$$

$$\mathcal{S}^{-+}_{\text{out}} = \psi^T \; (a_L^\dagger)^T$$

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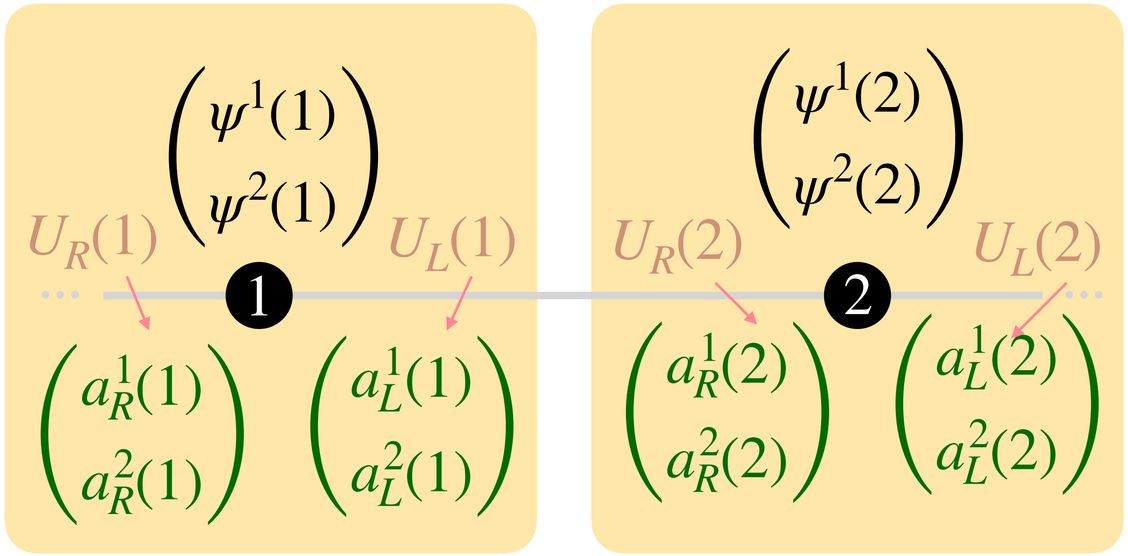


$$\mathscr{H}^{++} = -\frac{1}{2}\psi^\dagger\ \epsilon\ (\psi^\dagger)^T$$

$$\mathscr{H}^{--} = -\frac{1}{2}\psi^T\ \epsilon\ \psi$$

Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.

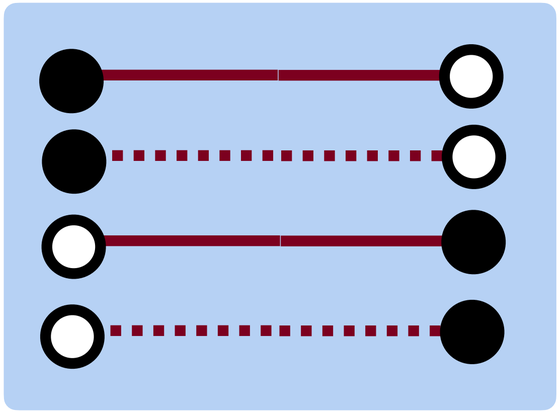
Working with smaller subspaces: Loop-string-hadron Hamiltonian

$$\frac{2}{ag^2}H = \sum_{r=1}^{N} E(r)^2 + \mu \sum_{r=1}^{N} (-1)^r \times \psi(r)^\dagger \psi(r) + x \sum_{r=1}^{N-1} \left(\psi^\dagger(r+1) U(r) \psi(r) + \text{h . c .} \right); \quad x = \frac{1}{a^2 g^2},\ \mu = 2\frac{m}{g}\sqrt{x}$$

$$\frac{2}{ag^2}H = \sum_{r=1}^{N} \frac{N_L(r)}{2}\left(\frac{N_L(r)}{2}+1\right) + \mu \sum_{r=1}^{N} (-1)^r N_\psi(r) + x \sum_{r=1}^{N-1} \left\{ \frac{1}{\sqrt{N_L(r)+1}} \left[\sum_{\sigma=+,-} S_{\text{out}}^{+\sigma}(r) S_{\text{in}}^{\sigma-}(r) \right] \frac{1}{\sqrt{N_R(r+1)+1}} + \text{hc} \right\}$$

$$N_{L/R}(r) = a_{L/R}^\dagger(r) a_{L/R}(r)$$

$$N_\psi(r) = \psi^\dagger(r)\psi(r)$$



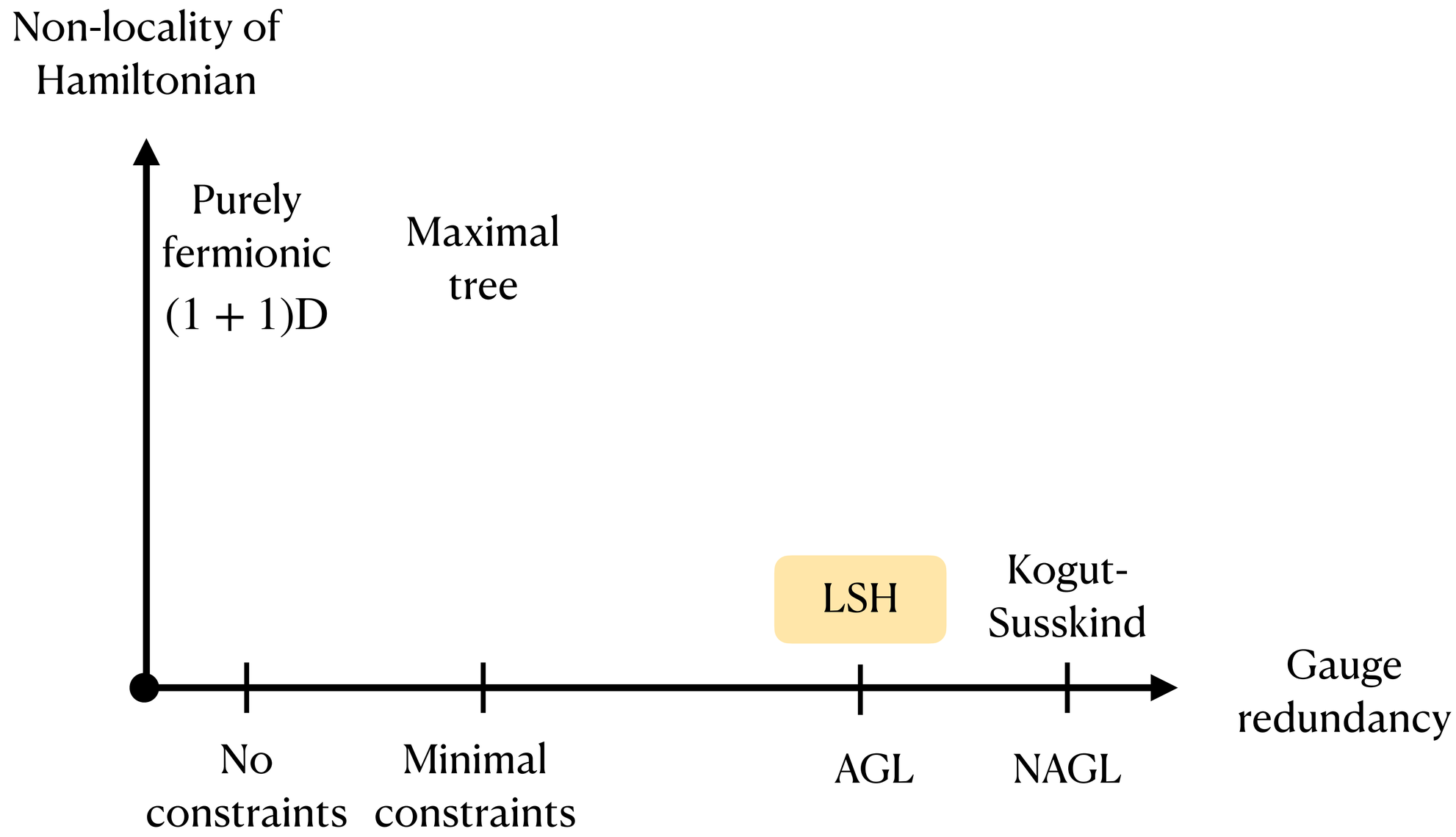
Local AGL constraint:

$$N_L(r) = N_R(r+1)$$

Global conserved charges:

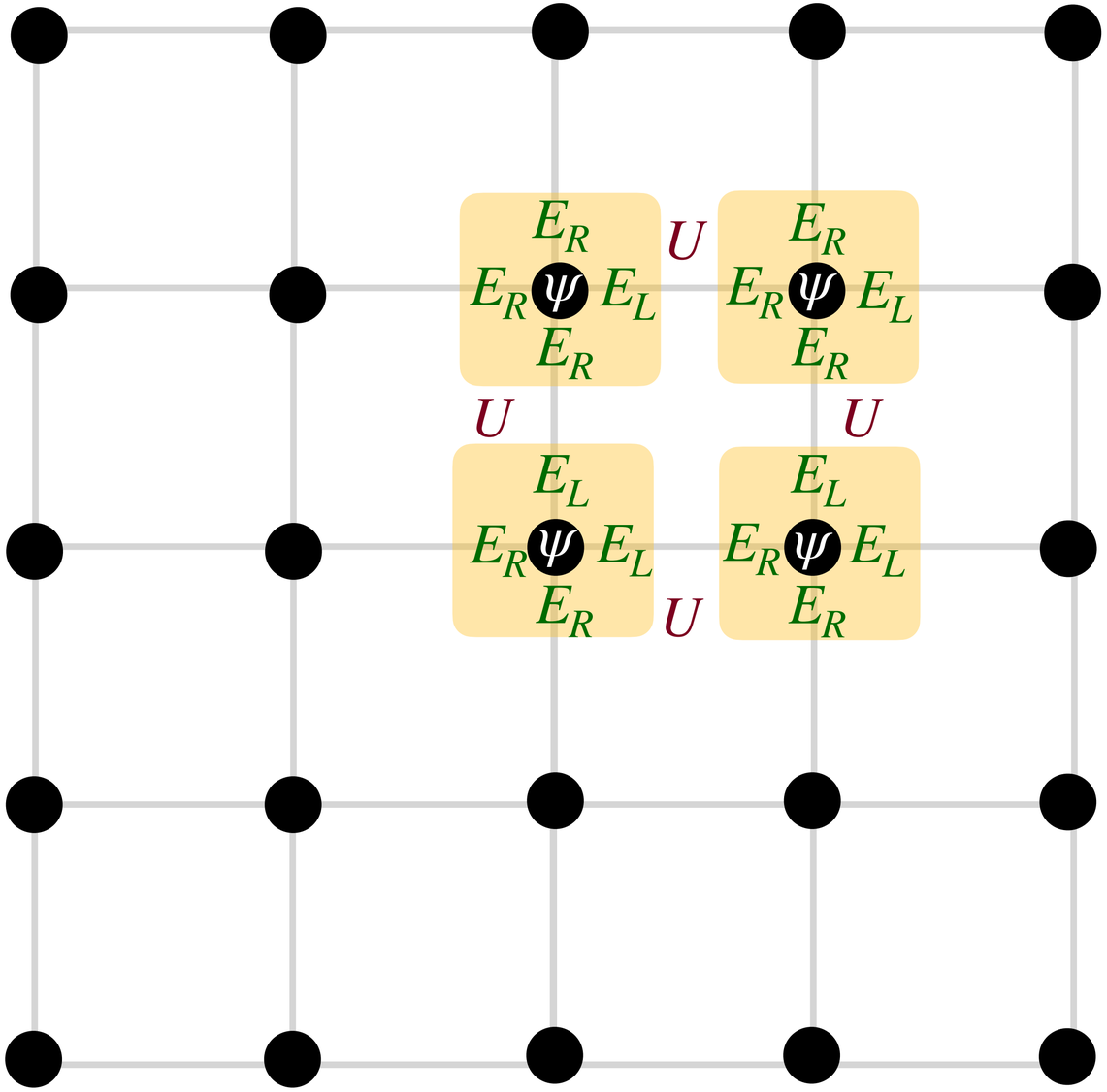
$$Q = \sum_{r=1}^{N} N_\psi(r),\ q = \sum_{r=1}^{N} \left[N_L(r) - N_R(r) \right]$$

Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.



NAGL = Non-Abelian Gauss law
AGL = Abelian Gauss law

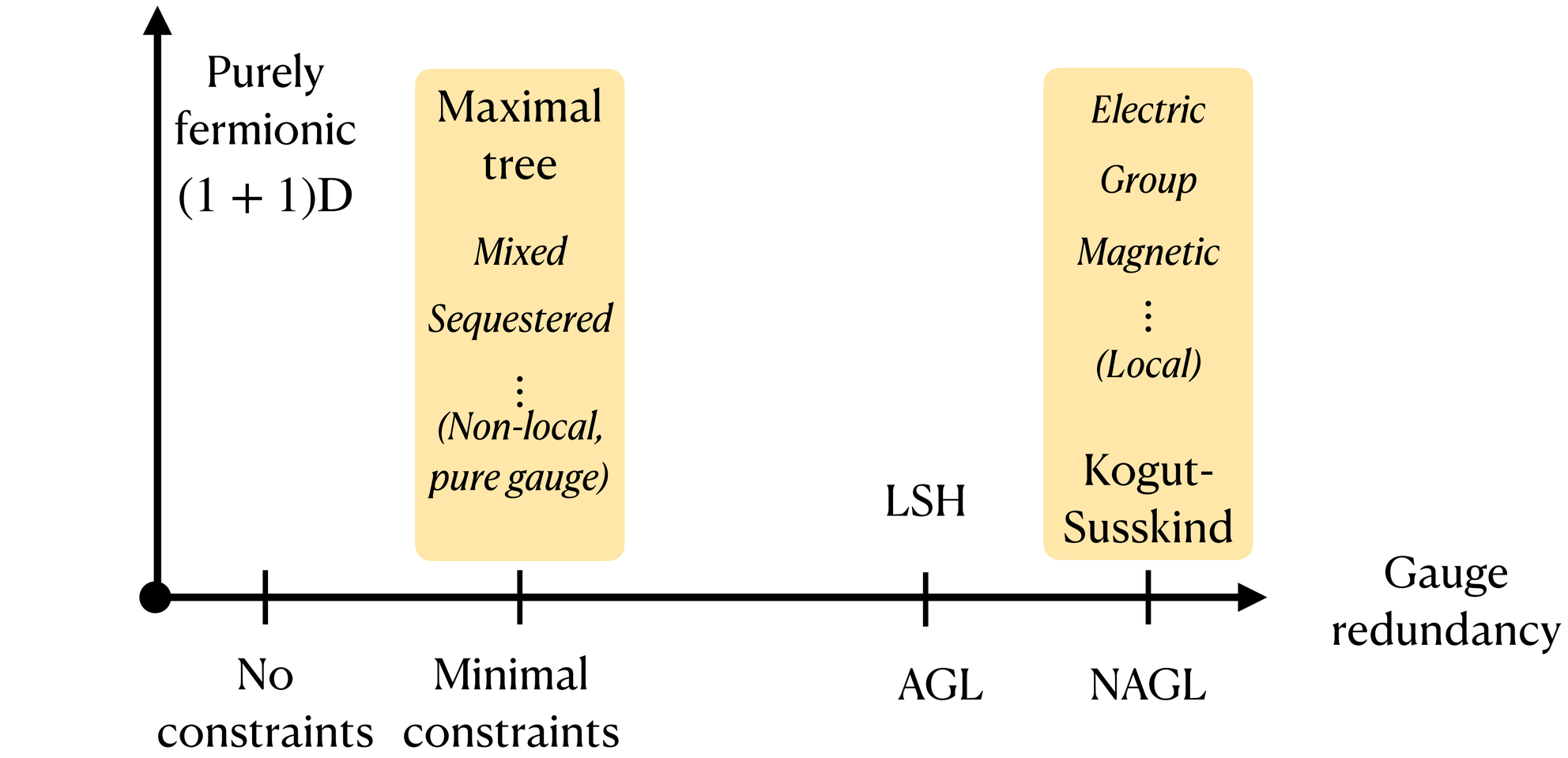
The Kogut-Susskind formulation for SU(n) LGT



- No gauge fixing beyond temporal gauge - non-commuting Gauss laws at each site.
  Fixing gauge beyond temporal gauge.
- Full tensor product Hilbert space much larger than physical Hilbert space.
  Can we work with smaller subspaces while avoiding non-locality?
- Several basis and truncation choices for link Hilbert space.
  Determining appropriate bases and truncations

Kogut, John, and Leonard Susskind. "Hamiltonian formulation of Wilson's lattice gauge theories." Physical Review D 11.2 (1975): 395.

Choice of DOFs informs choices for bases, which in turn informs truncation.



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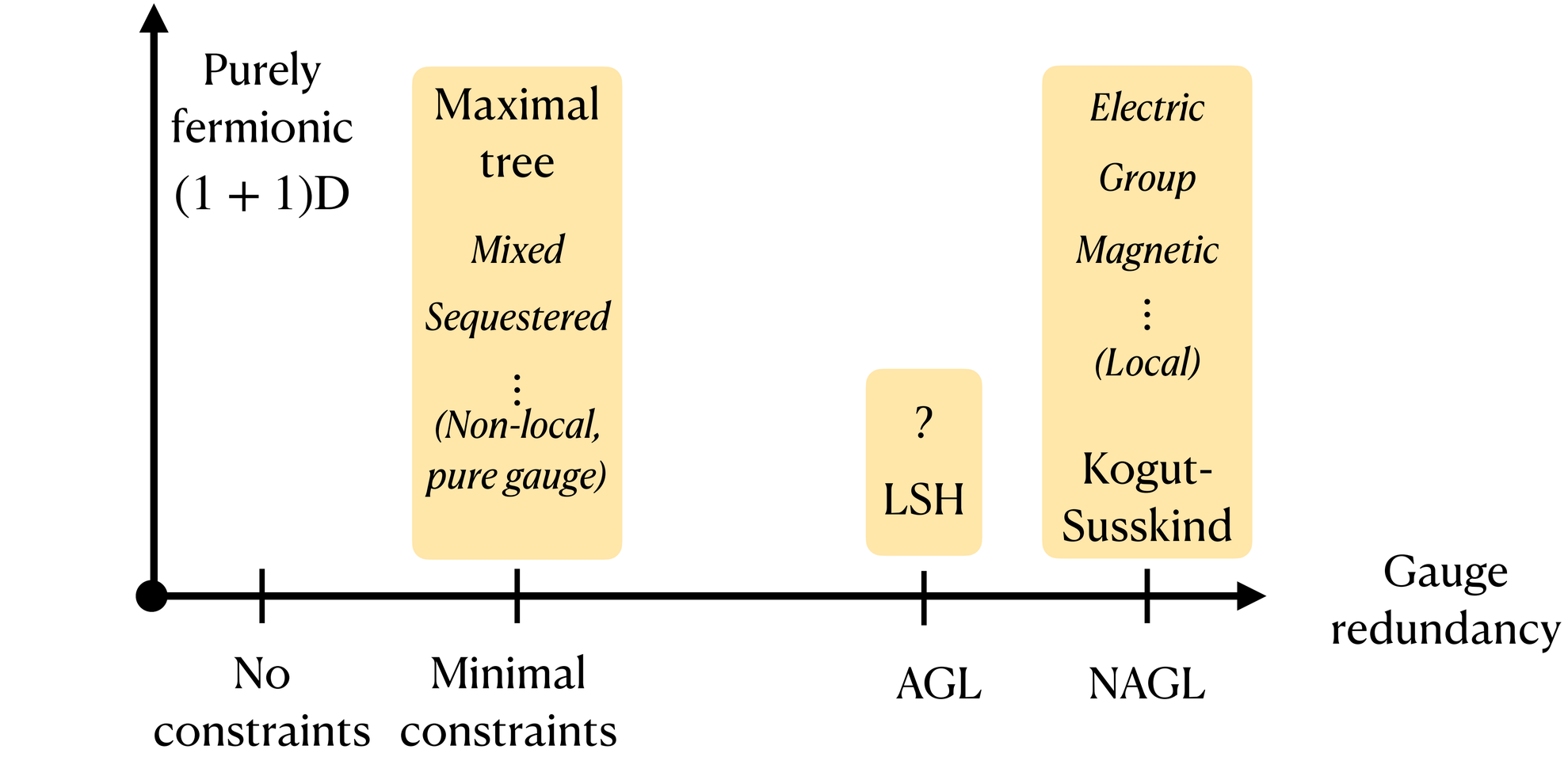

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A natural basis for $\mathrm{SU}(2)$ LSH in $(1+1)\mathrm{D}$

Local basis of NAGL obeying states

- $n_l \in \{0\} \cup \mathbb{N}$: counts number of pure flux loops
- $n_i \in \{0,1\}$: checks if incoming string
- $n_o \in \{0,1\}$: checks if outgoing string

$|n_l, n_i, n_o\rangle$

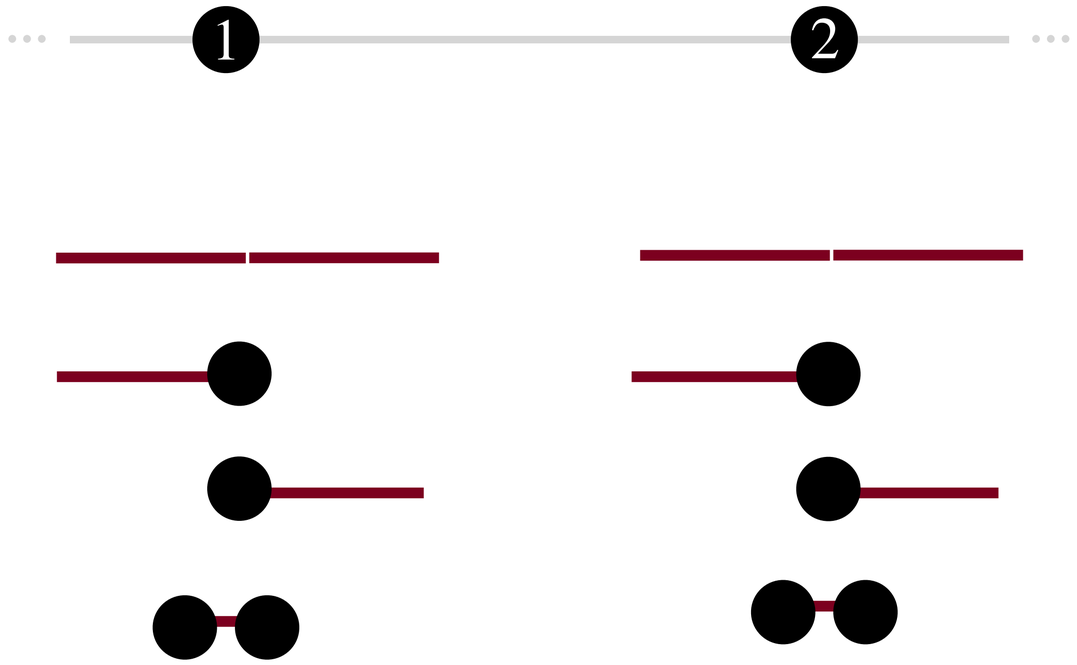
$|0\rangle \equiv |0,0,0\rangle$

$|1,0,0\rangle \propto \mathscr{L}^{++}|0\rangle$

$|0,1,0\rangle \propto \mathscr{S}^{++}_{\text{in}}|0\rangle$

$|0,0,1\rangle \propto \mathscr{S}^{++}_{\text{out}}|0\rangle$

$|0,1,1\rangle \propto \mathscr{H}^{++}|0\rangle$



Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.

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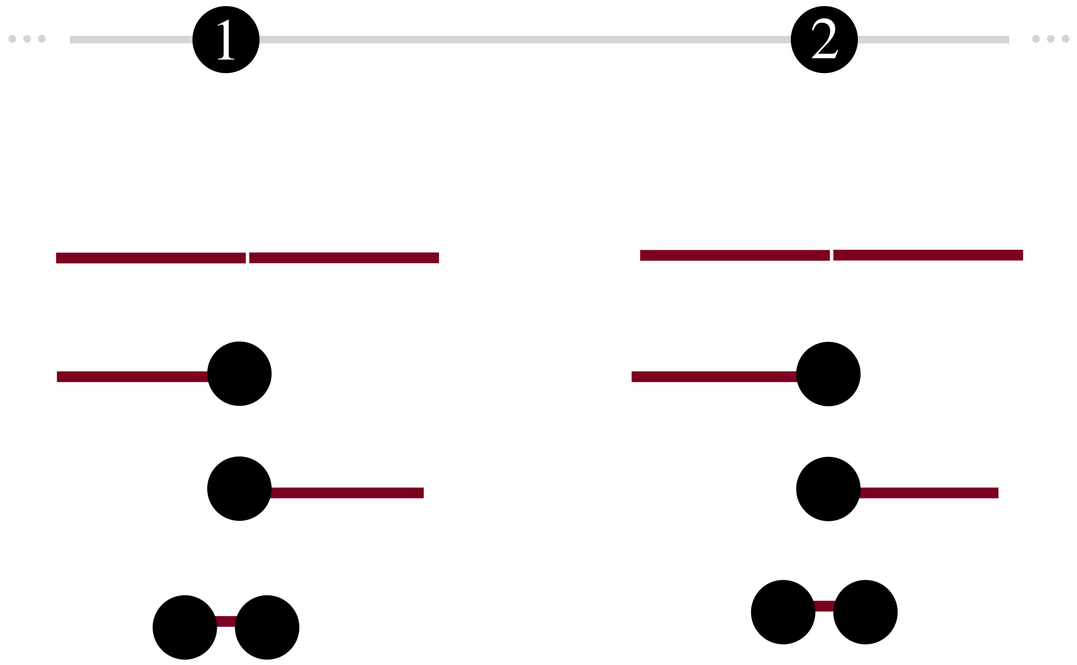

AGL:

Only global states with continuous flux lines are physical.

Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.

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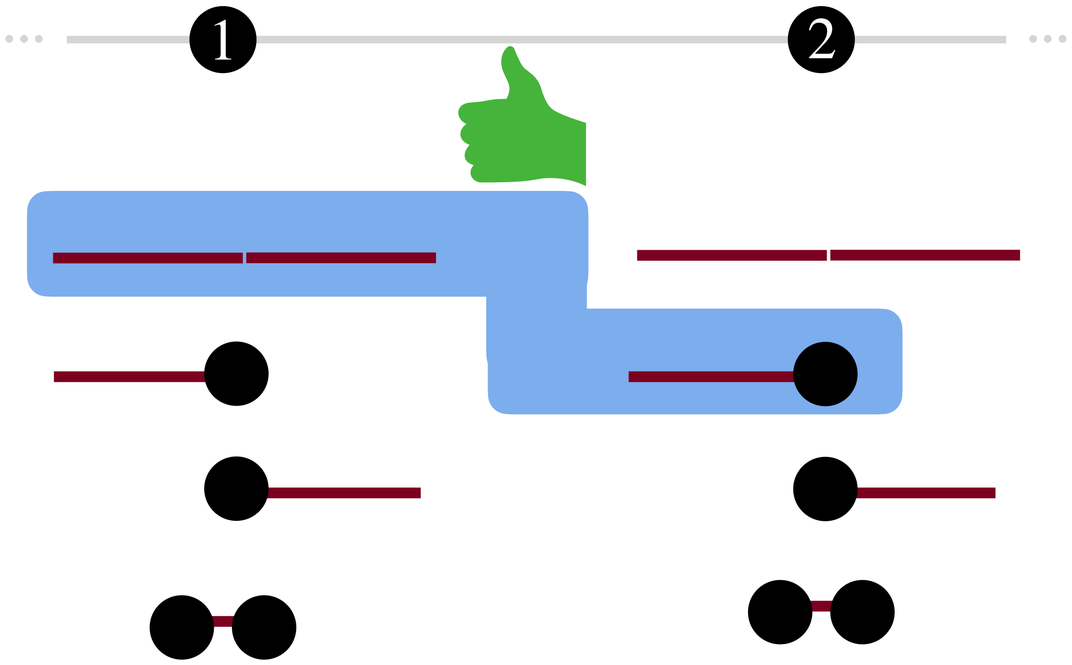

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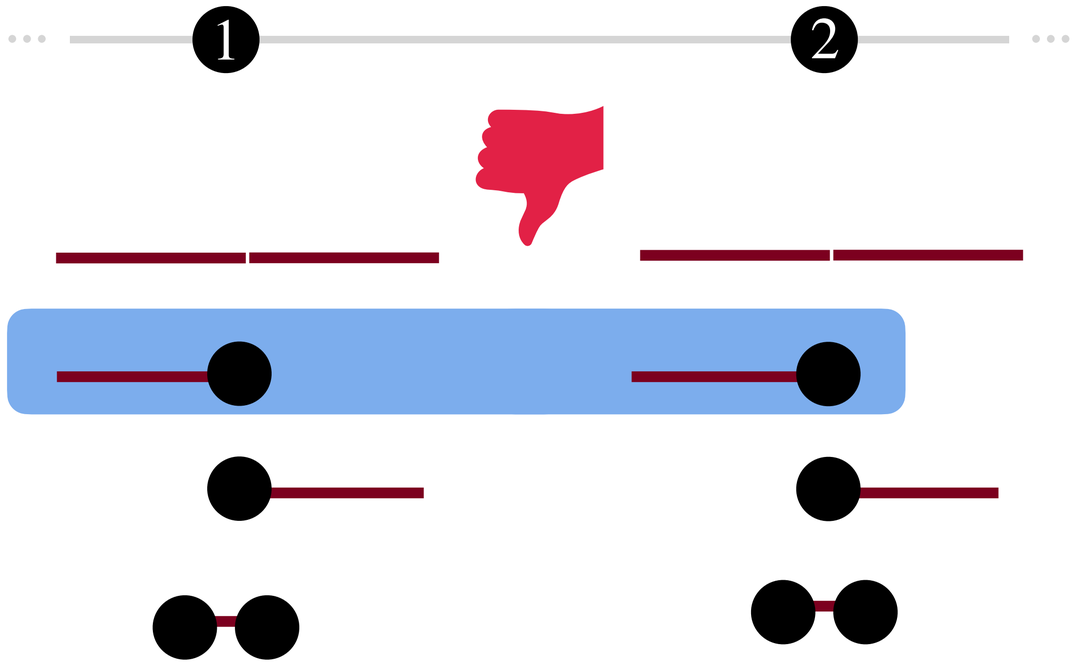

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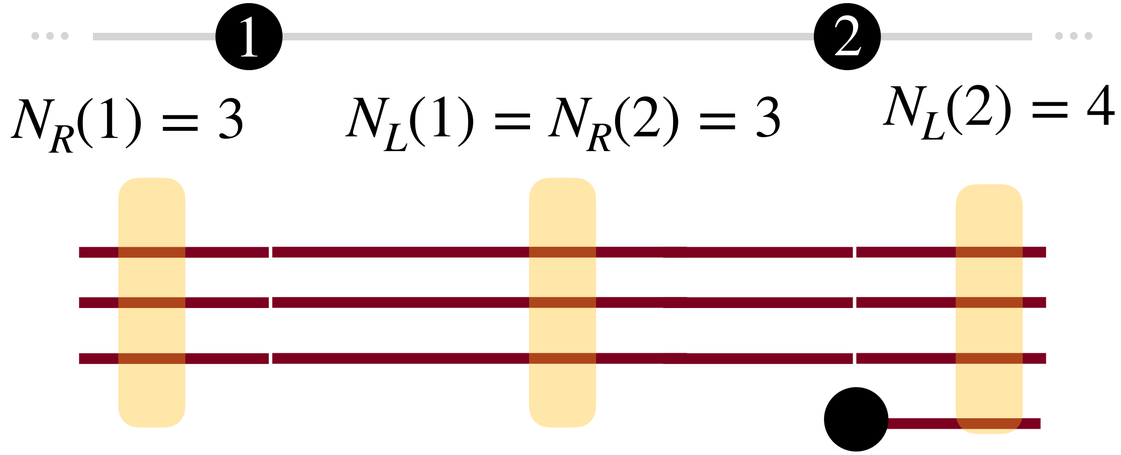


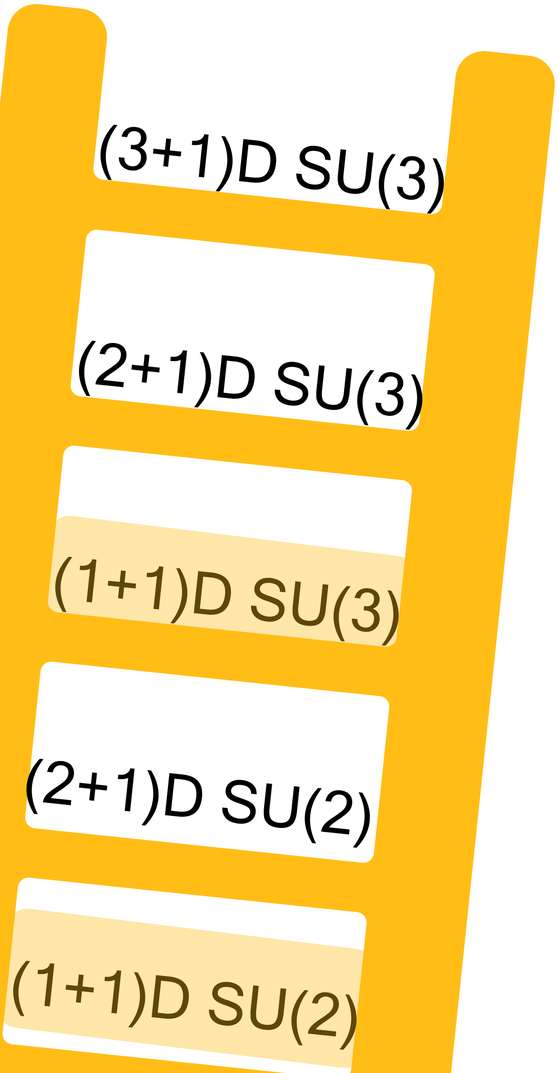
$$N_L = n_l + n_o(1 - n_i)$$
$$N_R = n_l + n_i(1 - n_o)$$

$$N_L(r) = N_R(r+1) = 2j$$

We will truncate N_L/N_R.

Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.

Loop-string-hadron formulation: Generalization to SU(3)



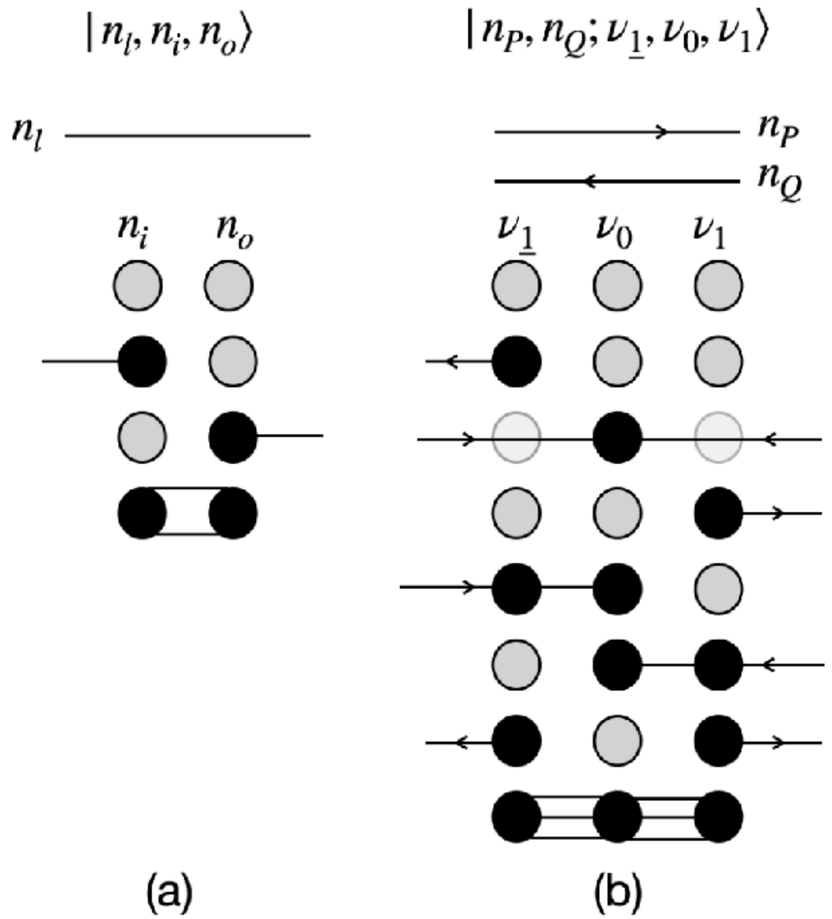


Image from:
Mathew, Emil, and Indrakshi Raychowdhury. "Protecting gauge symmetries in the the dynamics of SU (3) lattice gauge theories." arXiv preprint arXiv:2404.12158 (2024).

Raychowdhury, I. and Stryker, J.R., 2020. Loop, string, and hadron dynamics in SU (2) Hamiltonian lattice gauge theories. Physical Review D, 101(11), p.114502.

Kadam, Saurabh V., Indrakshi Raychowdhury, and Jesse R. Stryker. "Loop-string-hadron formulation of an SU (3) gauge theory with dynamical quarks." Physical Review D 107.9 (2023): 094513.

Loop-string-hadron formulation: Generalization to $(2+1)$D

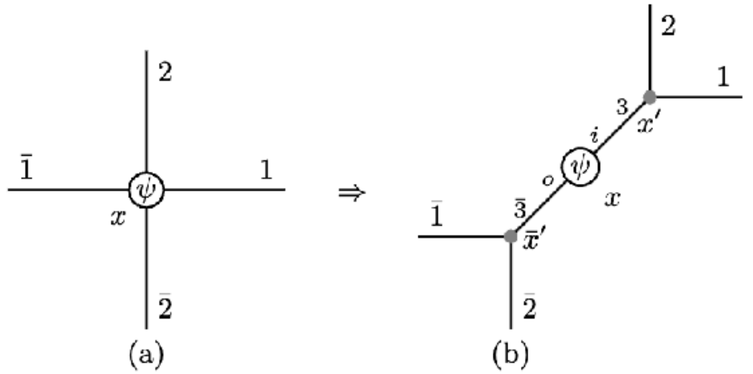
(3+1)D SU(3)

(2+1)D SU(3)

(1+1)D SU(3)

(2+1)D SU(2)

(1+1)D SU(2)



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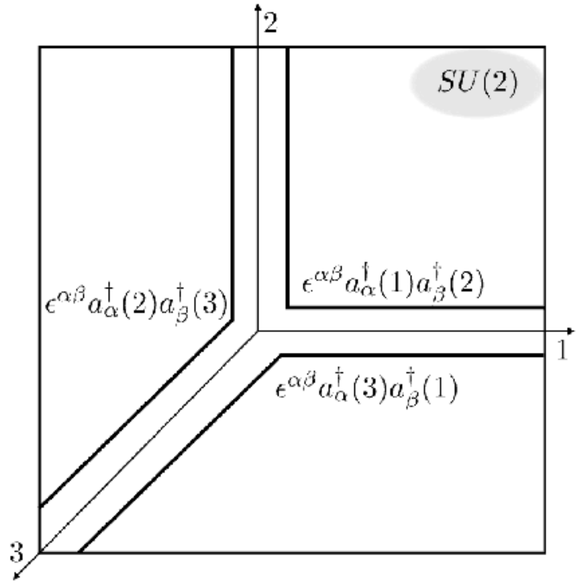
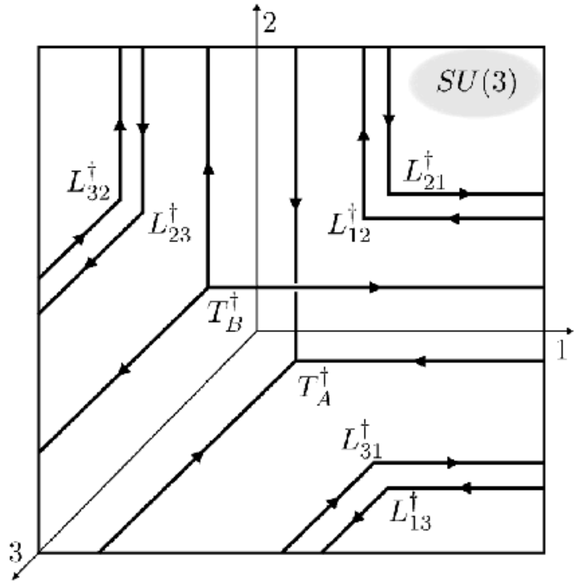




Kadam, Saurabh V., et al. "Loop-string-hadron approach to SU (3) lattice Yang-Mills theory: Gauge invariant Hilbert space of a trivalent vertex." arXiv preprint arXiv:2407.19181 (2024).

Loop-string-hadron formulation: Application to maximal tree gauge

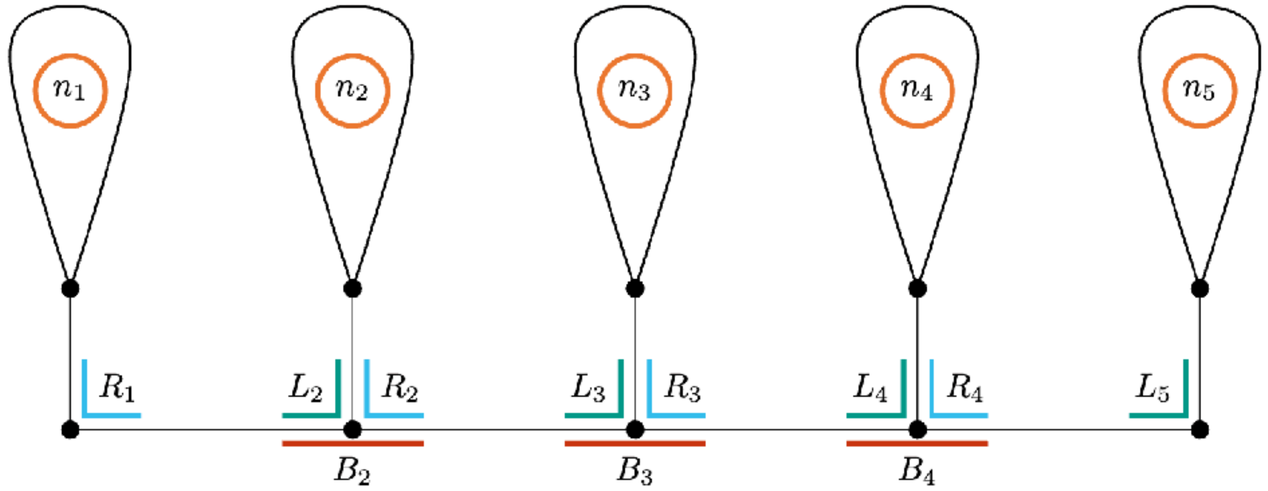
(3+1)D SU(3)

(2+1)D SU(3)

(1+1)D SU(3)

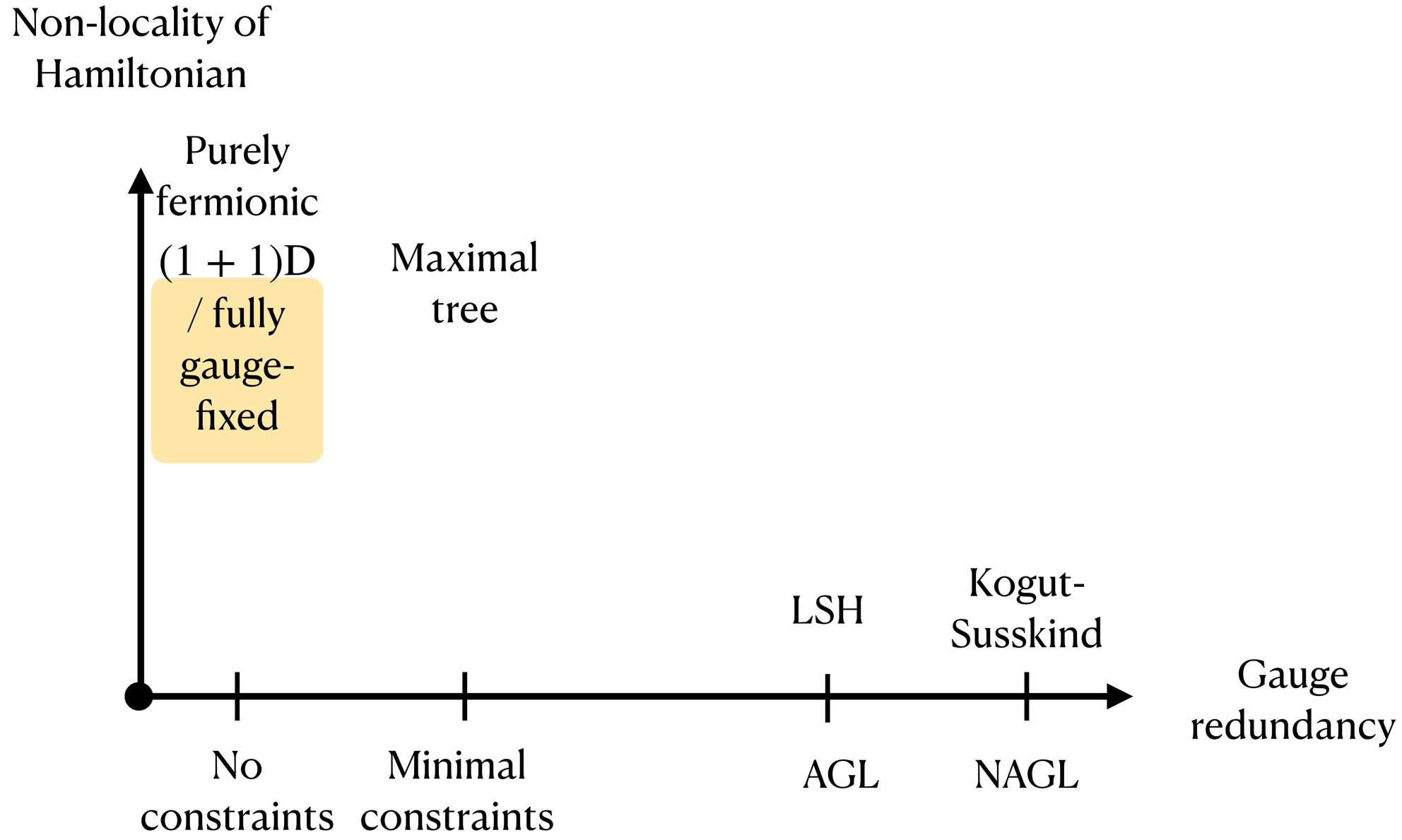
(2+1)D SU(2) (Pure gauge)

(1+1)D SU(2) (Pure gauge)



Burbano, Ivan M., and Christian W. Bauer. "Gauge Loop-String-Hadron Formulation on General Graphs and Applications to Fully Gauge Fixed Hamiltonian Lattice Gauge Theory." arXiv preprint arXiv:2409.13812 (2024).

- Fully gauge-fixed.
- Only developed for pure SU(2) gauge theory.



NAGL = Non-Abelian Gauss law
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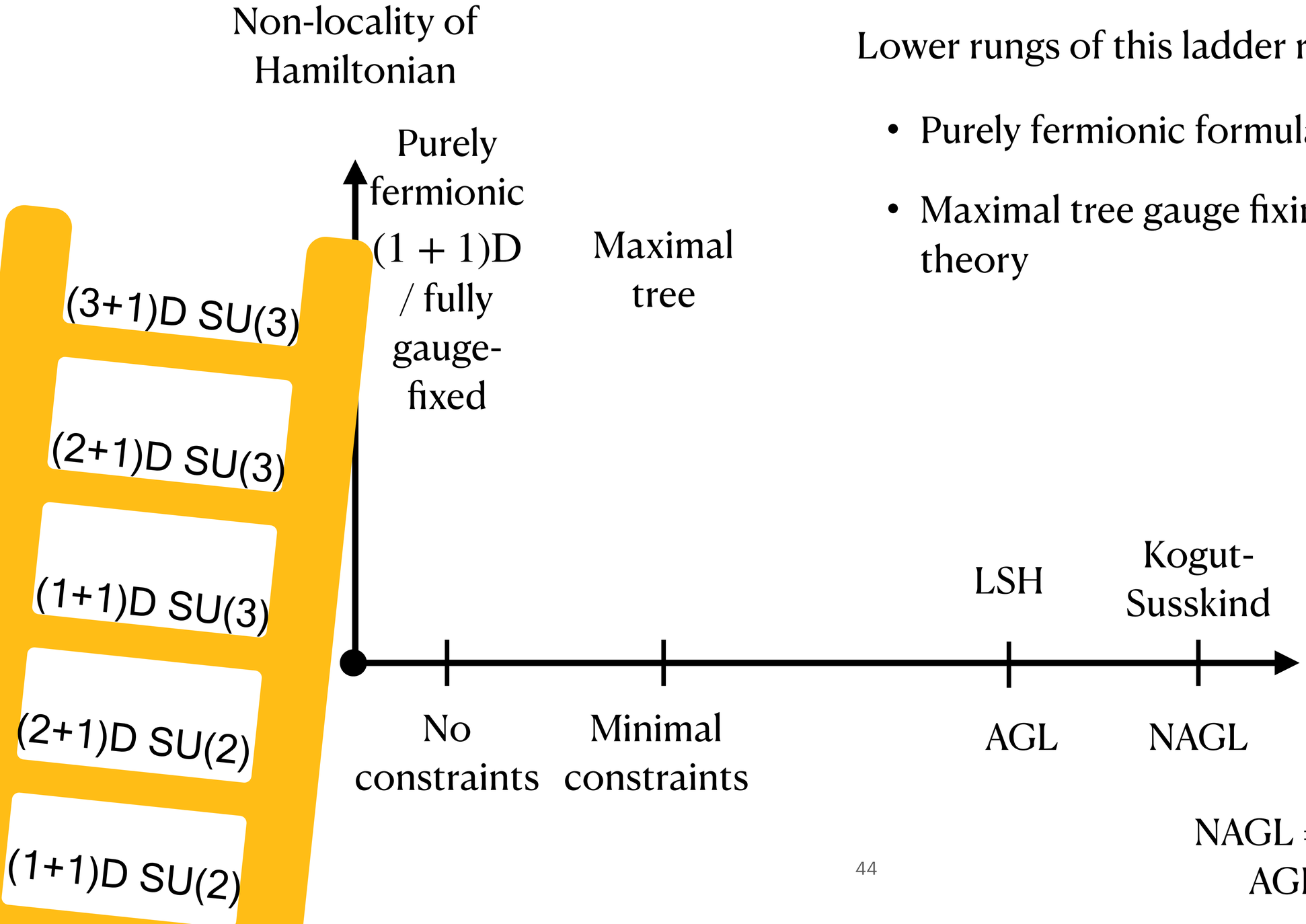
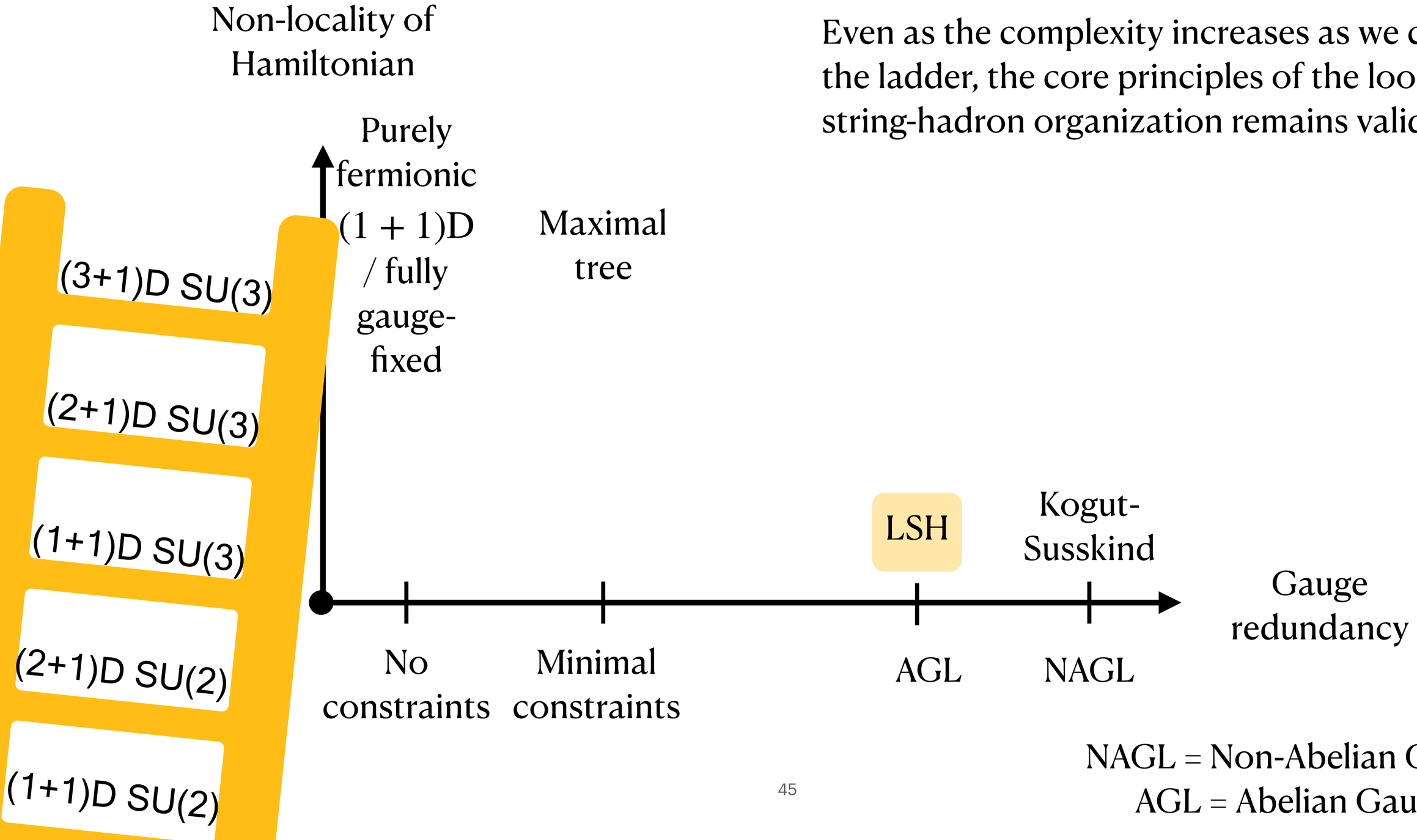


Lower rungs of this ladder reveal simplifications:

- Purely fermionic formulation in (1 + 1)D
- Maximal tree gauge fixing for pure SU(2) gauge theory

NAGL = Non-Abelian Gauss law
AGL = Abelian Gauss law

Even as the complexity increases as we climb the ladder, the core principles of the loop-string-hadron organization remains valid.



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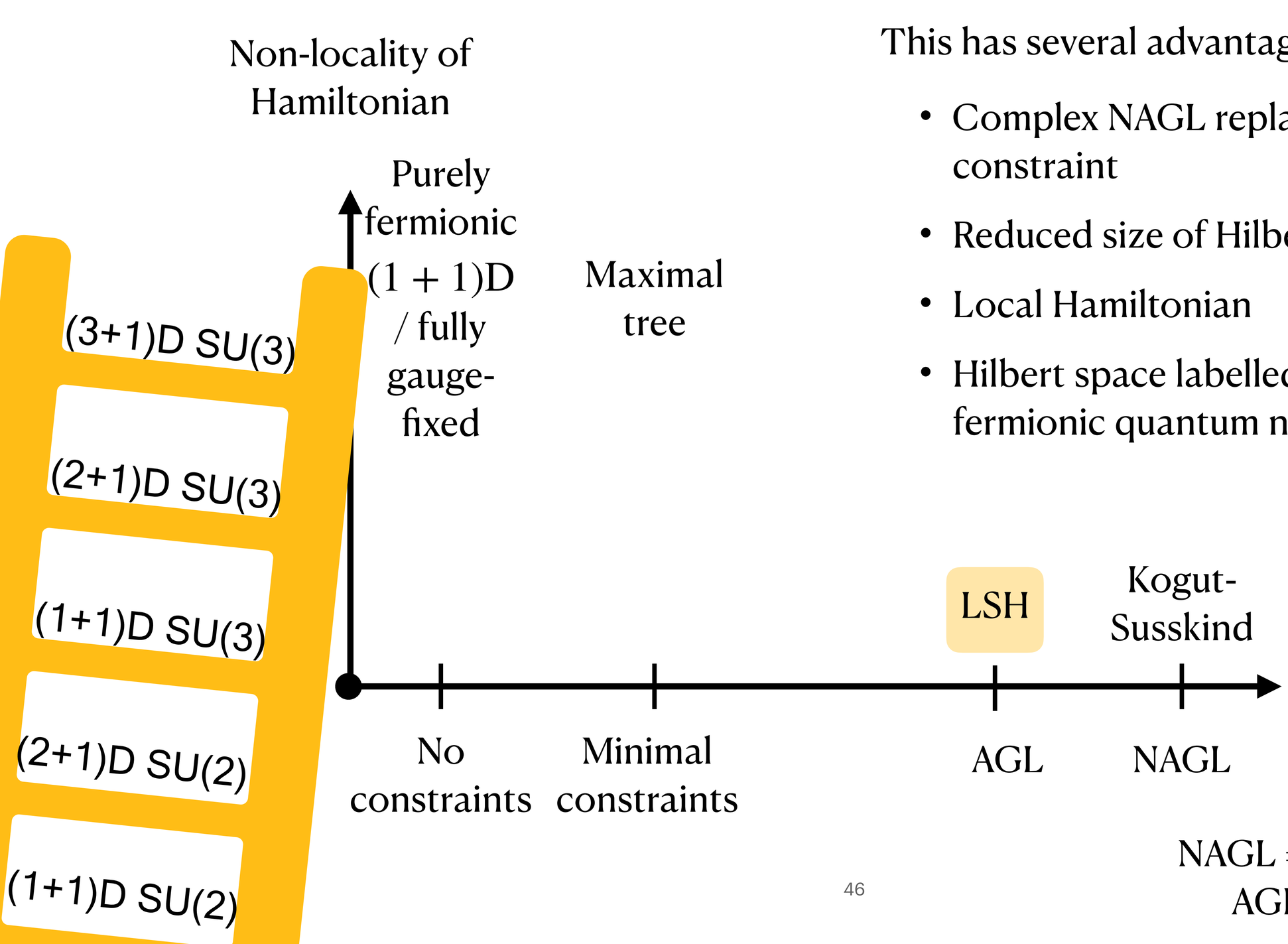
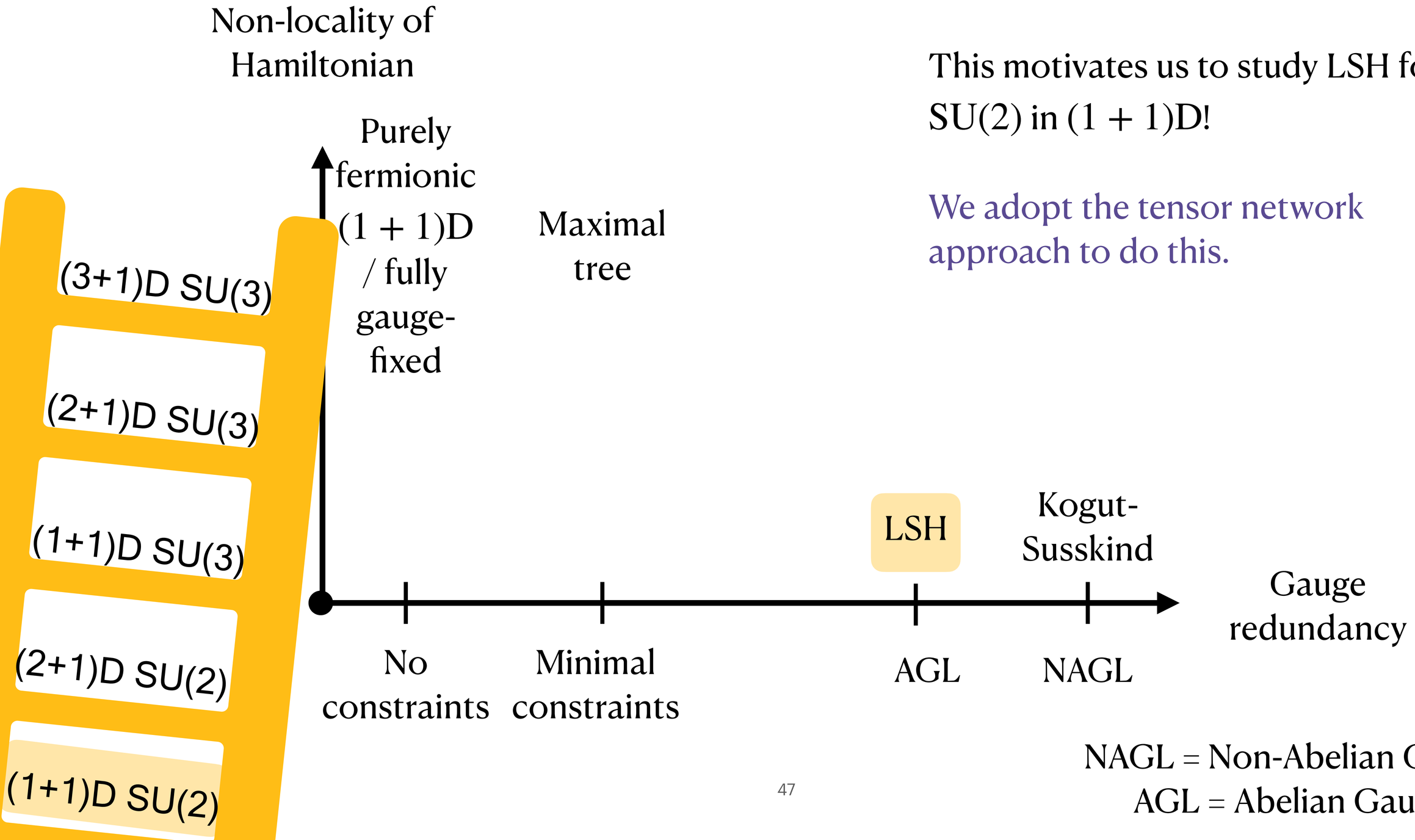


This has several advantages:

- Complex NAGL replaced by algebraic AGL constraint
- Reduced size of Hilbert space
- Local Hamiltonian
- Hilbert space labelled by simple bosonic and fermionic quantum numbers

NAGL = Non-Abelian Gauss law
AGL = Abelian Gauss law

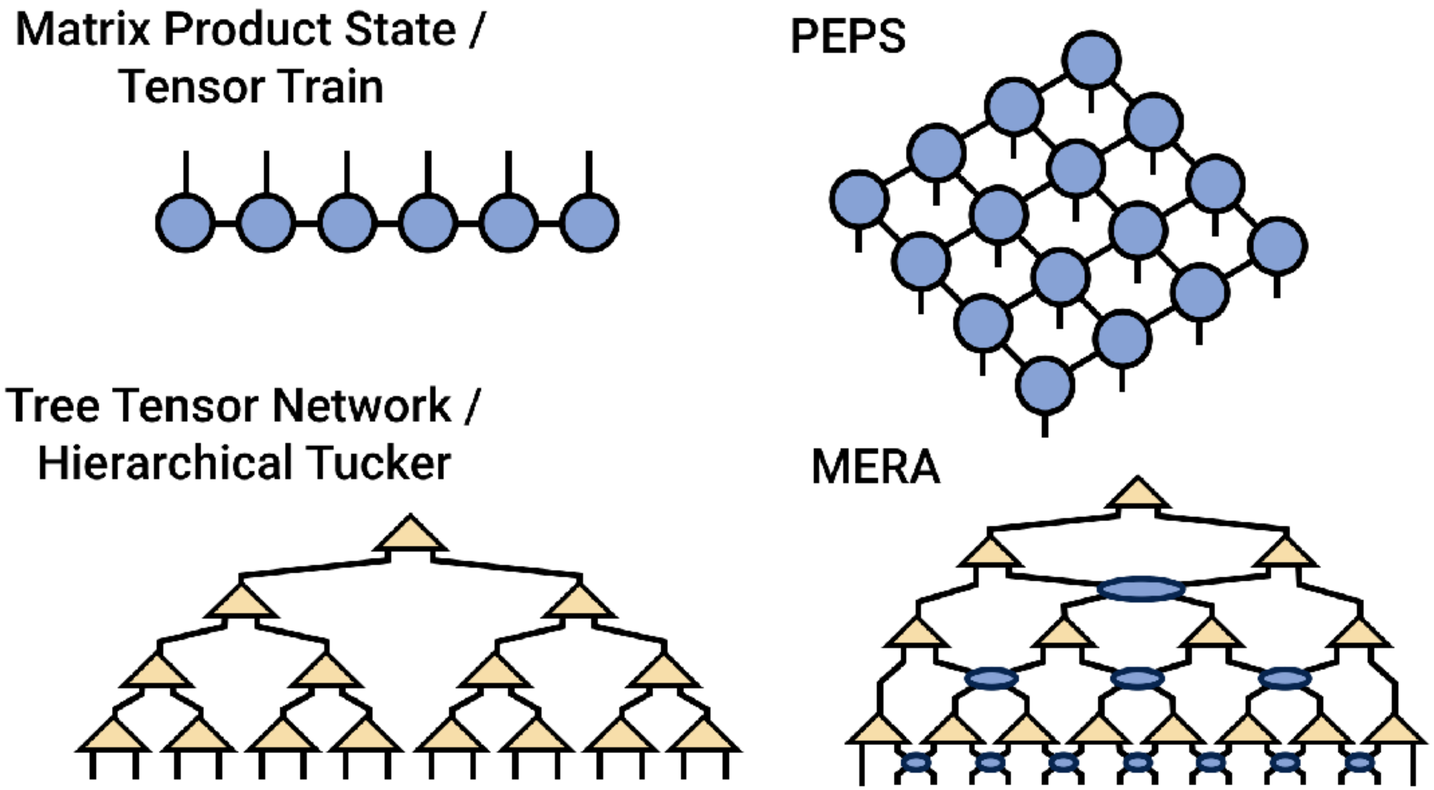


This motivates us to study LSH for SU(2) in (1 + 1)D!

We adopt the tensor network approach to do this.

NAGL = Non-Abelian Gauss law
AGL = Abelian Gauss law

Tensor networks

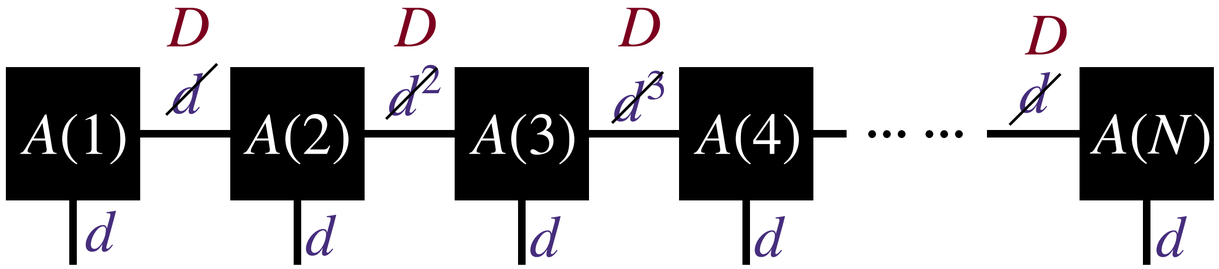


- Variational ansatzes for quantum many-body systems
- Distribute entanglement in local virtual bond degrees of freedom
- Truncation in dimension of virtual bonds enables efficient representation of low-entanglement systems
- Well-developed methods for ground state determination & time evolution

Image from tensornetwork.org

Matrix product state (MPS)

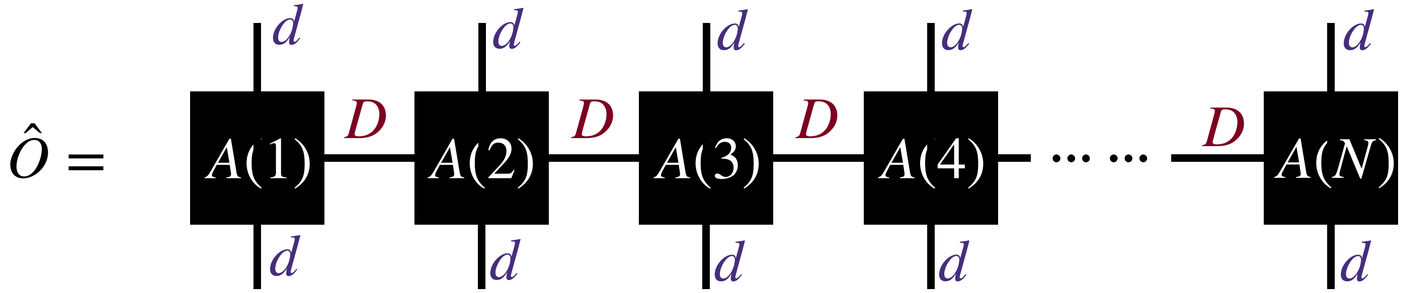
$$|\psi\rangle = \sum_{i_1,i_2,\ldots,i_N} A_{i_1}^{a_1}(1) A_{i_2}^{a_1,a_2}(2) \ldots A_{i_N}^{a_{N-1}}(N) \, |i_1, i_2, \ldots, i_N\rangle$$



- Arbitrary N-site state $|\psi\rangle$ with local Hilbert space dimension d: Exponentially growing bond dimension, d^N parameters
- Low energy states of gapped local Hamiltonians: Area law entanglement
- Can truncate bond dimension to a finite constant value without losing accuracy, $O(NdD^2)$ parameters

Orús, Román. "A practical introduction to tensor networks: Matrix product states and projected entangled pair states." Annals of physics 349 (2014): 117-158

Matrix product operators (MPOs)



- Similar decomposition possible for operators.
- For local operators, size of bond dimension determined by range of correlations, and does not scale with system size

Orús, Román. "A practical introduction to tensor networks: Matrix product states and projected entangled pair states." Annals of physics 349 (2014): 117-158

Tensor network toolkit for LSH

ITENSOR

Custom LSH site:

(n_l, n_i, n_o)

$n_i,\ n_o \in \{0,1\},$

$0 \le n_L \le 2j_{\max}$

$0 \le n_R \le 2j_{\max}$

Conservation of global QNs

$$Q = \sum_{r=1}^{N} \left(n_i(r) + n_o(r)\right)$$

$$q = \sum_{r=1}^{N} \left(n_o(r) - n_i(r)\right)$$

Site-local LSH operators on truncated Hilbert space

Parameters

System size: N

Cutoff: $j_{\max}$

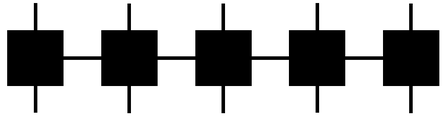
Couplings: $\frac{m}{g}, x = \frac{1}{a^2 g^2}$

MPS bond dimension: D

Q, q sector

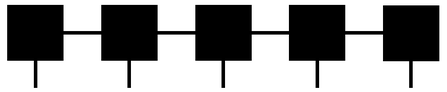
External charge sector

Hamiltonian MPO



Abelian Gauss law implemented using penalty term

Initial MPS



Density **M**atrix **R**enormalization **G**roup : Determination of ground state $|\Omega\rangle$

Time **D**ependent **V**ariational **P**rinciple: Realtime dynamics of excitations created on $|\Omega\rangle$

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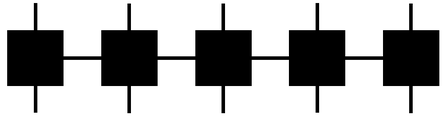
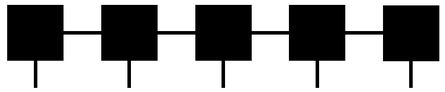


Abelian Gauss law implemented using penalty term

Initial MPS



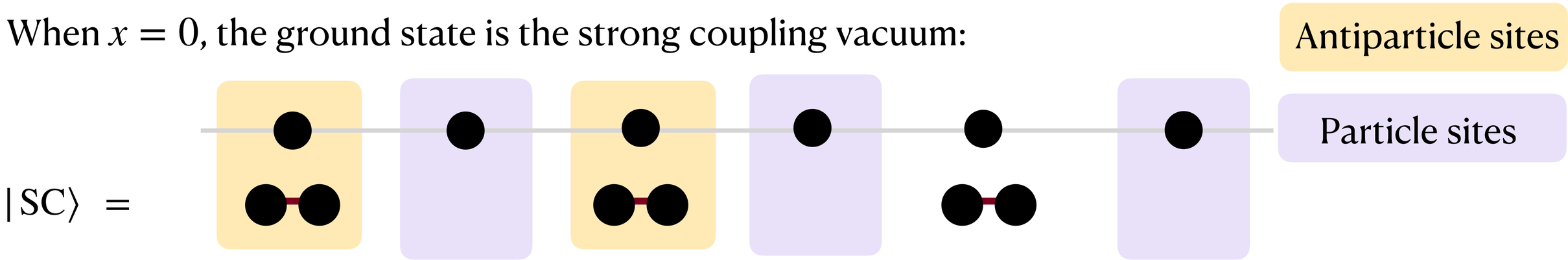
Density **M**atrix **R**enormalization **G**roup : Determination of ground state $|\Omega\rangle$

Time **D**ependent **V**ariational **P**rinciple: Realtime dynamics of excitations created on $|\Omega\rangle$

Parameter space

$$\frac{2}{ag^2}H = H_{\text{electric}} + \mu\ H_{\text{fermion mass}} + x\ H_{\text{hopping}}; \qquad x = \frac{1}{a^2g^2},\ \mu = 2\frac{m}{g}\sqrt{x}$$

When $x = 0$, the ground state is the strong coupling vacuum:



$$Q = \sum_{r=1}^{N} \left(n_i(r) + n_o(r)\right) = N \qquad q = \sum_{r=1}^{N} \left(n_o(r) - n_i(r)\right) = 0$$

We will work in this (Q, q) sector.

Parameter space

$$\frac{2}{ag^2}H = H_{\text{electric}} + \mu\ H_{\text{fermion mass}} + x\ H_{\text{hopping}} + \Lambda\ H_{\text{penalty}}; \qquad x = \frac{1}{a^2 g^2},\ \mu = 2\frac{m}{g}\sqrt{x}$$

The continuum limit is given by the following sequence of limits (at fixed m/g)

- Bond dimension $D \to \infty$
- Hilbert space truncation $2j_{\text{max}} \to \infty$
- Thermodynamic limit $N \to \infty$
- Lattice spacing $a \to 0$, i.e., $x \to \infty$

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For now, we work at a fixed $2j_{\text{max}}$. Computations for higher $2j_{\text{max}}$ are in progress.

Tensor network toolkit for LSH

ITensor

Custom LSH site:

(n_l, n_i, n_o)

$n_i,\ n_o \in \{0,1\},$

$0 \le n_L \le 2j_{\max}$

$0 \le n_R \le 2j_{\max}$

Conservation of global QNs

$$Q = \sum_{r=1}^{N} \left(n_i(r) + n_o(r)\right)$$

$$q = \sum_{r=1}^{N} \left(n_o(r) - n_i(r)\right)$$

Site-local LSH operators on truncated Hilbert space

Parameters

System size: N

Cutoff: $j_{\max}$

Couplings: $\frac{m}{g}, x = \frac{1}{a^2 g^2}$

MPS bond dimension: D

Q, q sector

External charge sector

Hamiltonian MPO

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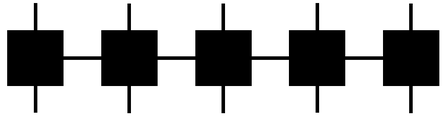

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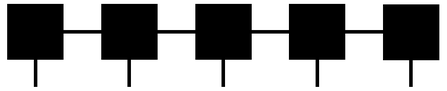

Density **M**atrix **R**enormalization **G**roup : Determination of ground state $|\Omega\rangle$

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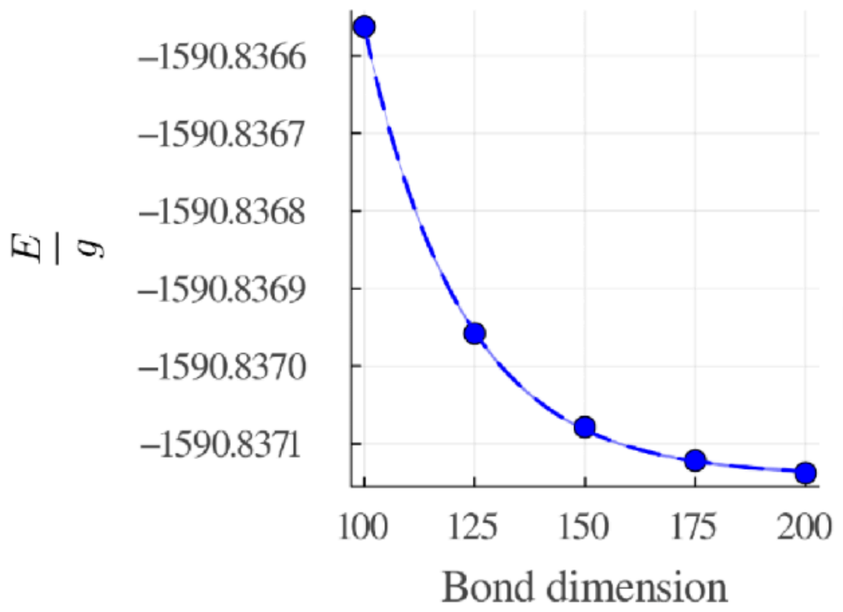
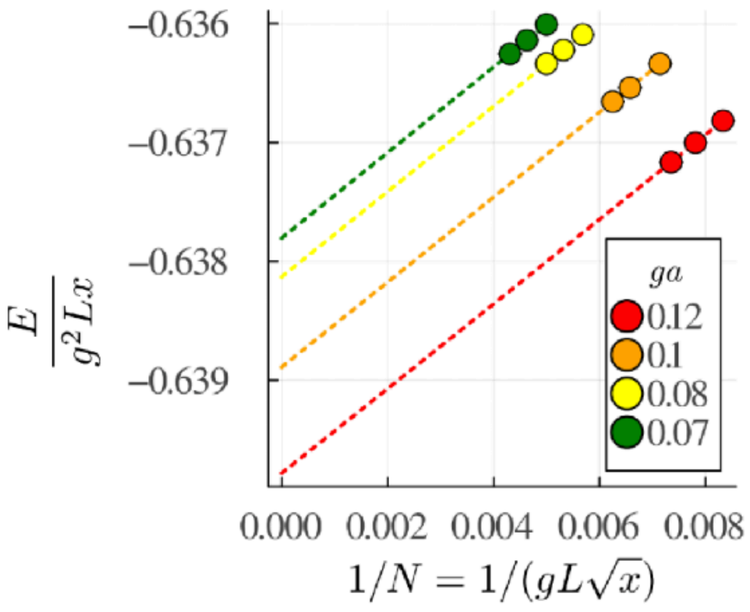
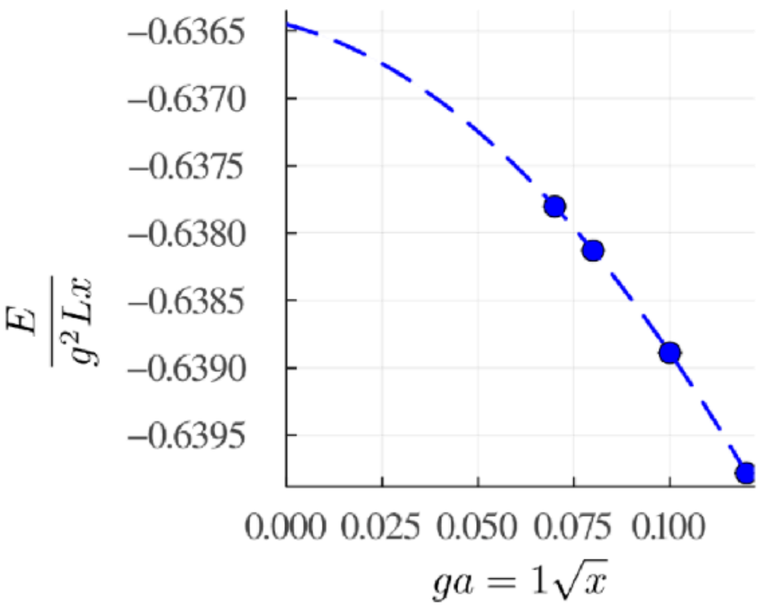
Continuum limit of ground state energy

$$m/g = 0.3,\ 2j_{\max} = 3$$

$$gL = Nga = \frac{N}{\sqrt{x}}$$



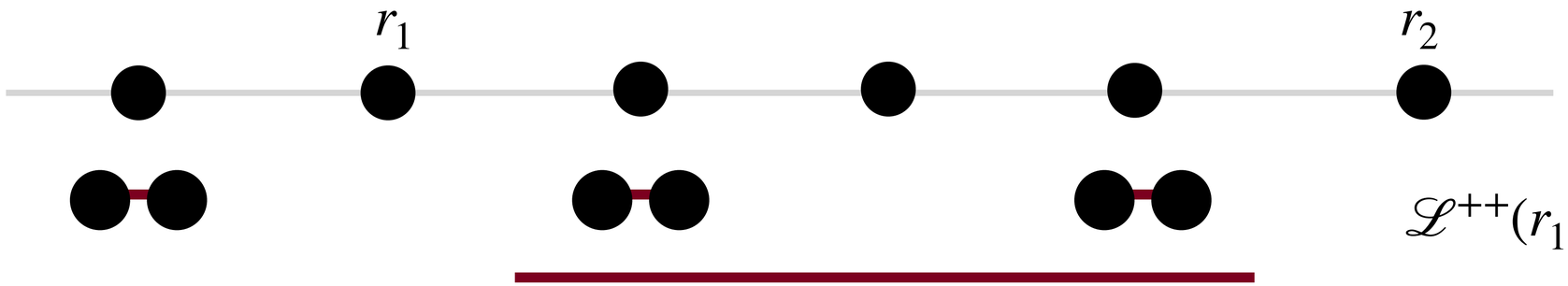




When $m/g = 0$, the continuum Hamiltonian is solvable, and we have $\lim_{\substack{L \to \infty \\ x \to \infty}} \frac{E}{g^2 L x} = -\frac{2}{\pi}$.

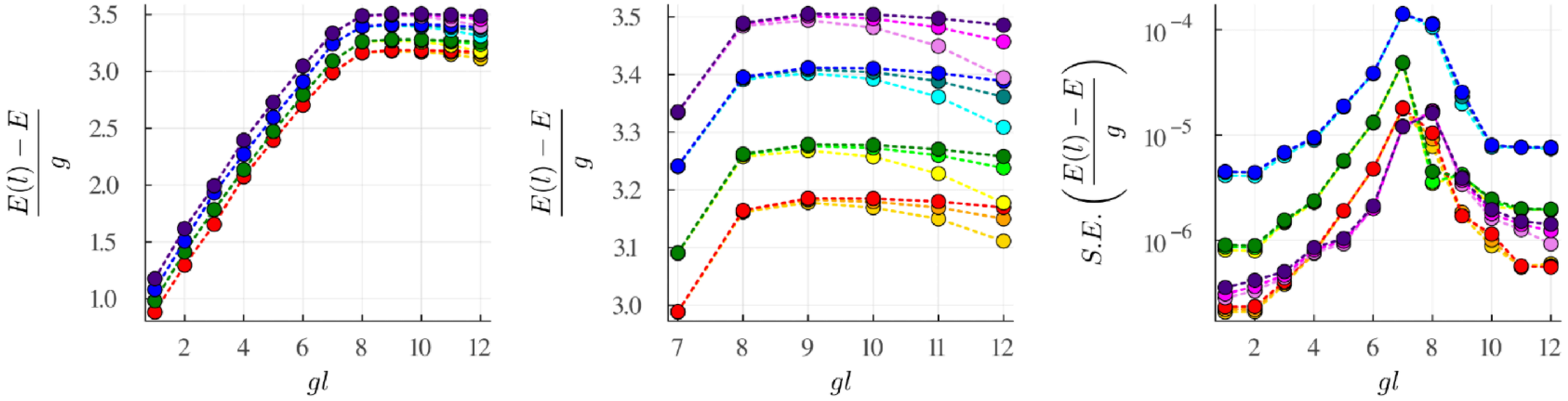
Hamer, C. J. "SU (2) Yang-Mills theory in (1+ 1) dimensions: A finite-lattice approach." *Nuclear Physics B* 195.3 (1982): 503-521.

Ground state with static charges



$$\mathscr{L}^{++}(r_1+1)\ldots\mathscr{L}^{++}(r_2-1)\,|\,\mathrm{SC}\rangle$$

$$gl = (r_2 - r_1)ga$$



(ga, gL)

(0.12,14) (0.08,14) (0.12,15) (0.08,15) (0.12,16) (0.08,16) (0.1,14) (0.07,14) (0.1,15) (0.07,15) (0.1,16) (0.07,16)

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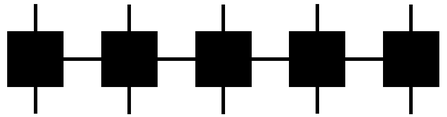

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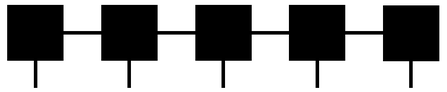

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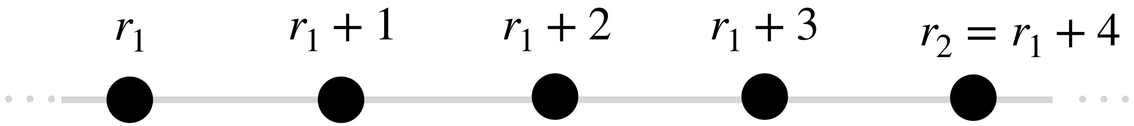
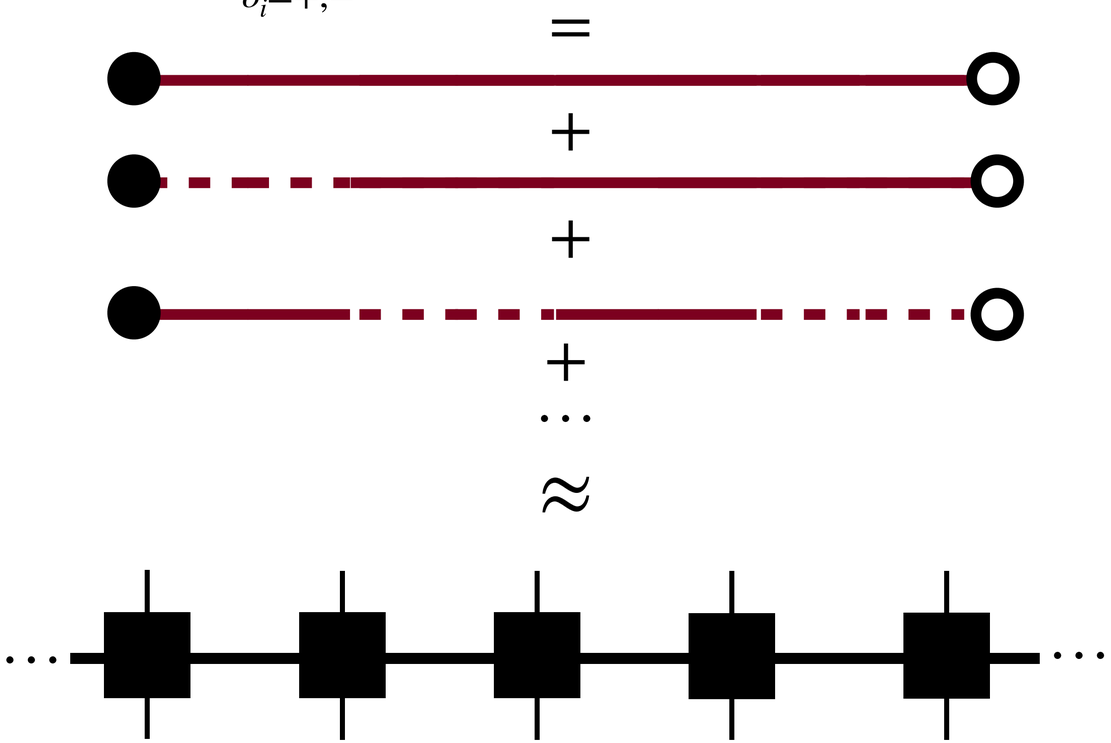
Dynamical string breaking



$$S_{r_1,r_2} = \bar{\psi}(r_1)U(r_1+1)\ldots U(r_2-1)\psi(r_2)$$
$$= \sum_{\sigma_i=+,-} S_{\text{out}}^{+,\sigma_1}(r_1)\mathscr{L}^{\sigma_1,\sigma_2}(r_1+1)\ldots\mathscr{L}^{\sigma_{\Delta r-1},\sigma_{\Delta r}}(r_2-1)S_{\text{in}}^{\sigma_{\Delta r}-}(r_2)$$



Obtain interacting ground state for some $(N, x, m/g, j_{\max})$

Act on it with the string MPO $S_{r_1,r_1+\Delta r} - S_{r_1+1,r_1+\Delta r+1}$.
r_1 is odd and Δr is even.

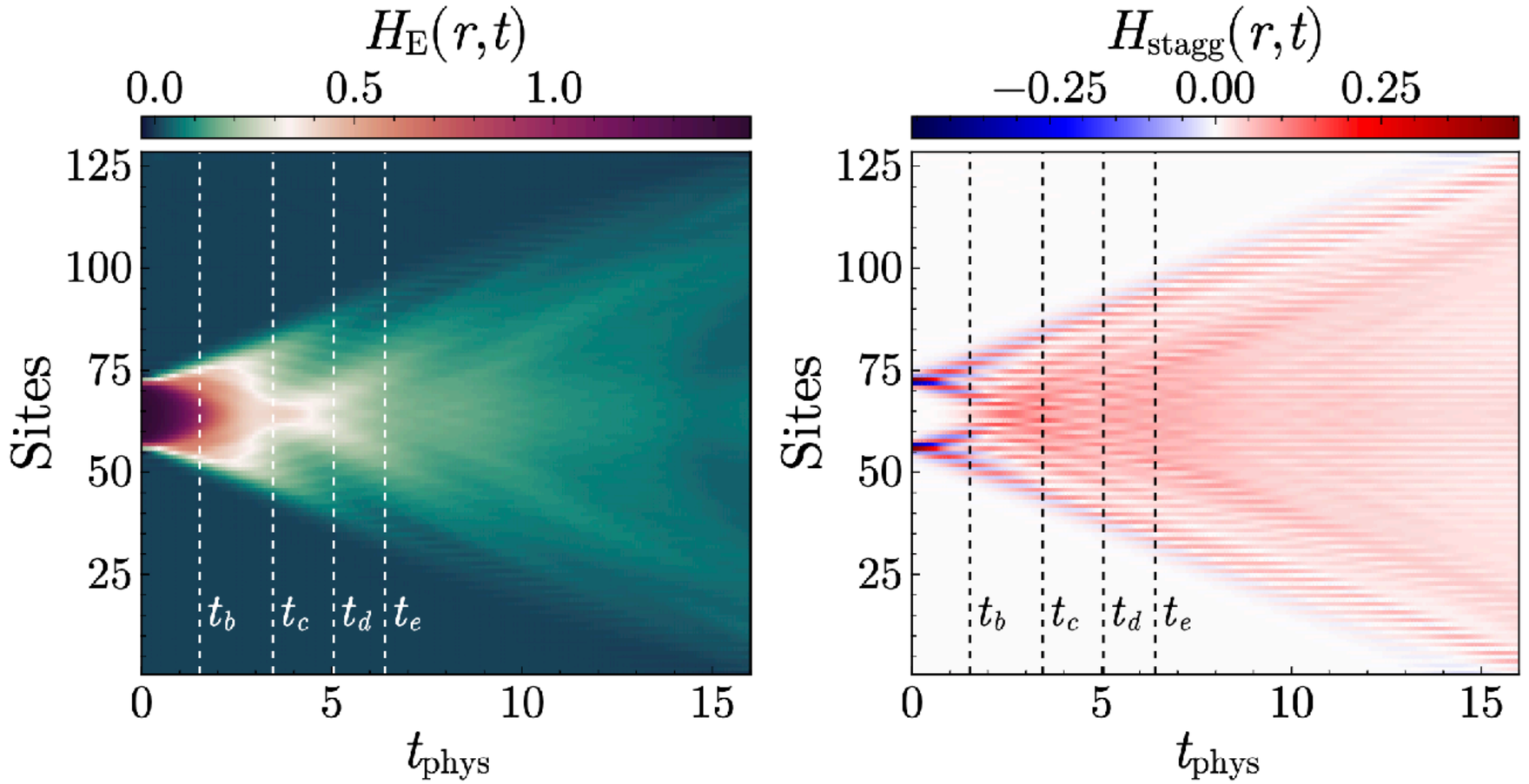
This leads to an OO-EE structure for the string, thus creating a full Dirac fermion.

Evolve it with e^{-iHt}.

Dynamical string breaking

$N = 128,\ x = 16,\ m/g = 0.2,\ j_{\max} = 5/2$

$$H_{stagg}(r) = \begin{cases} n_i(r) + n_o(r) & \text{odd sites} \\ 2 - \left(n_i(r) + n_o(r)\right) & \text{even sites} \end{cases}$$



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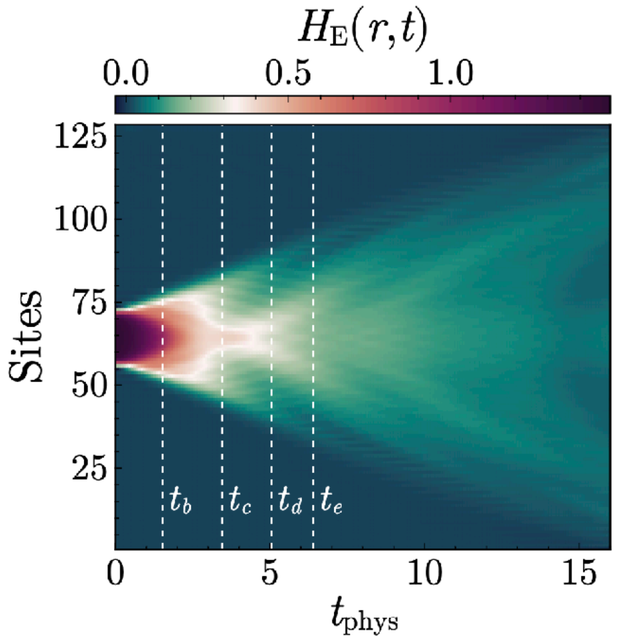

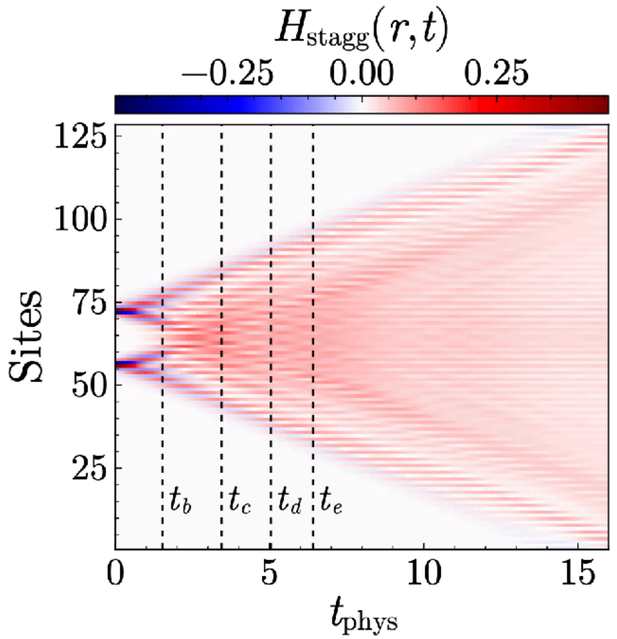

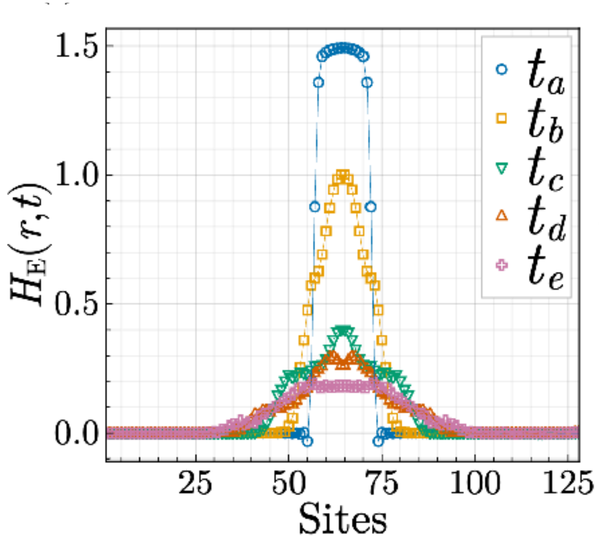

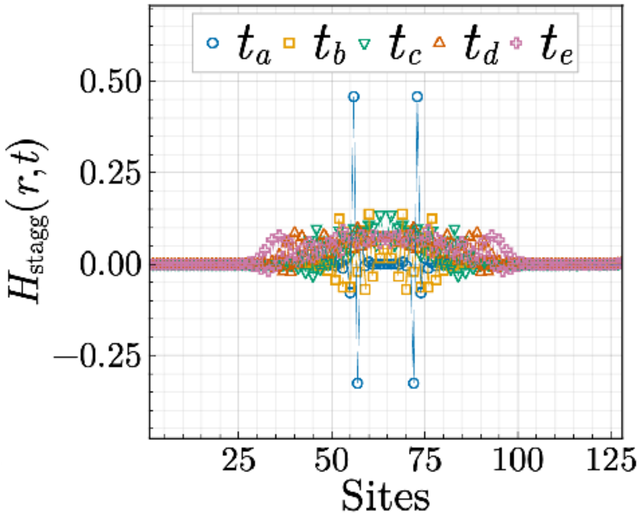

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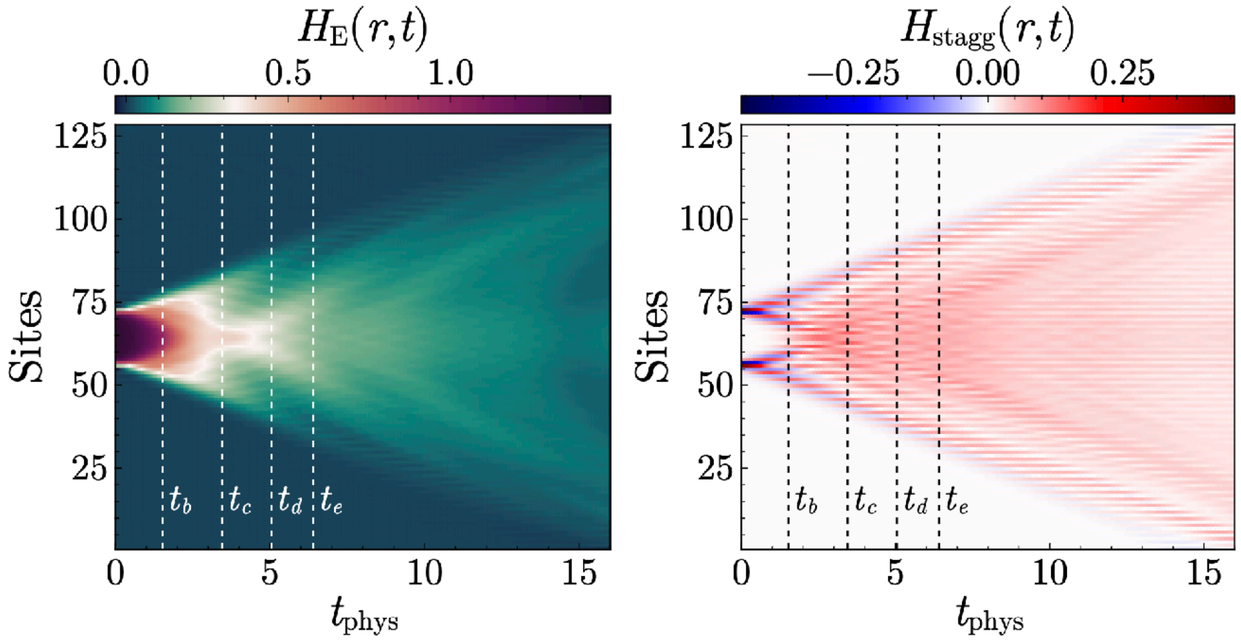
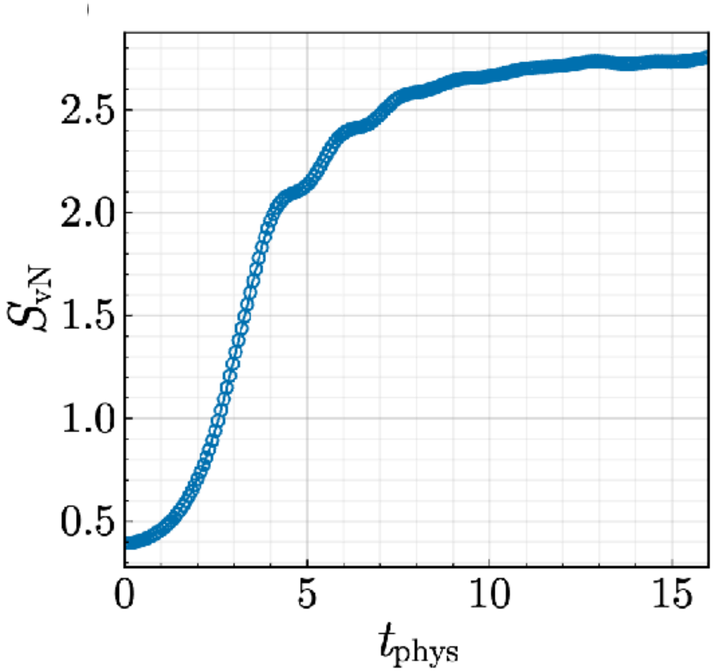




Outlook

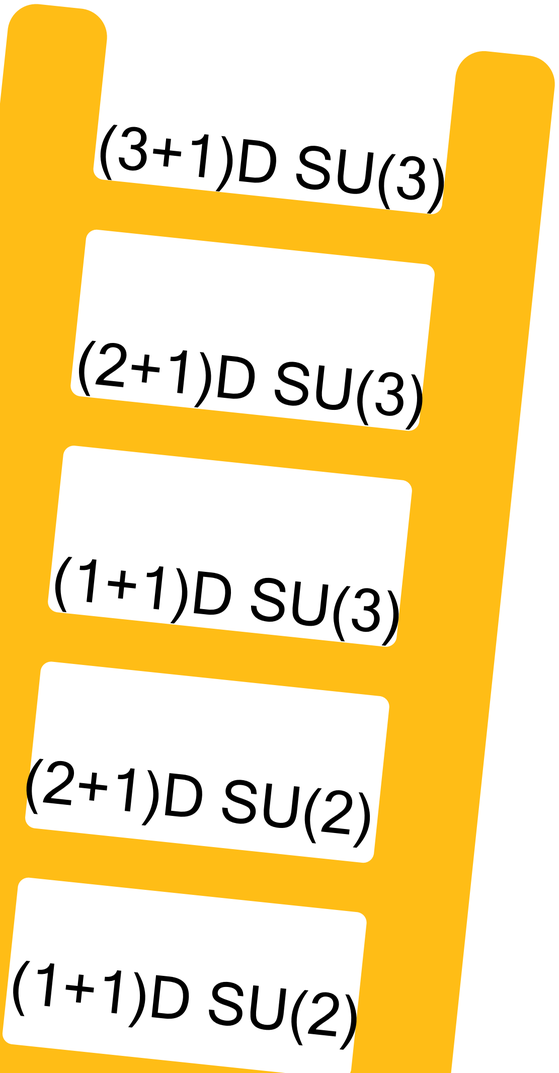
- The loop-string-hadron formulation of the SU(2) gauge theory in (1 + 1)D:
  - trades off non-commuting Gauss' laws at each site with a simple Abelian Gauss law
  - massively reduces size of the Hilbert space
  - has a local Hamiltonian
  - has a basis described by simple bosonic and fermionic quantum numbers

- The LSH formulation is readily amenable to tensor network calculations.

- We have used the LSH tensor network ansatz to compute ground state properties as well as to simulate dynamical string breaking.

- LSH can be generalized to higher dimensions and to SU(3).

Next steps



- Developing tensor network ansatzes for more complex theories, including a PEPS (Projected Entangled Pairs) ansatz for $(2 + 1)$D SU(2).
- Preparing wavepackets and simulating scattering in non-Abelian gauge theory.
- Connections with quantum computing, especially in the context of initial state preparation.
- Applications of LSH in developing neural networks for gauge theory simulations.