Brookhaven National Laboratory

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Gravitational form factors

Yoshitaka Hatta

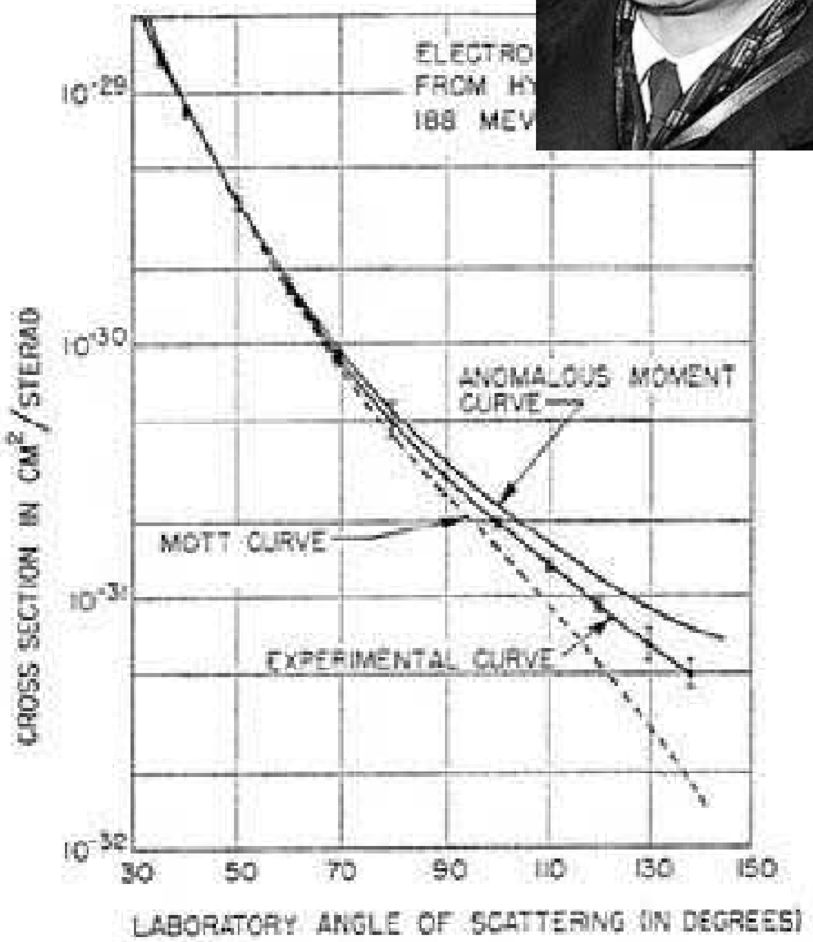
BNL/RIKEN BNL

New opportunities for BSM model searches at the EIC, July 21-24, 2025

Proton electromagnetic form factors

EM form factors from elastic scattering

$$\langle p'|J^{\mu}(0)|p\rangle = \bar{u}(p')\left[\gamma^{\mu}F_1 + \frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m}F_2\right]u(p)$$



Electric form factor

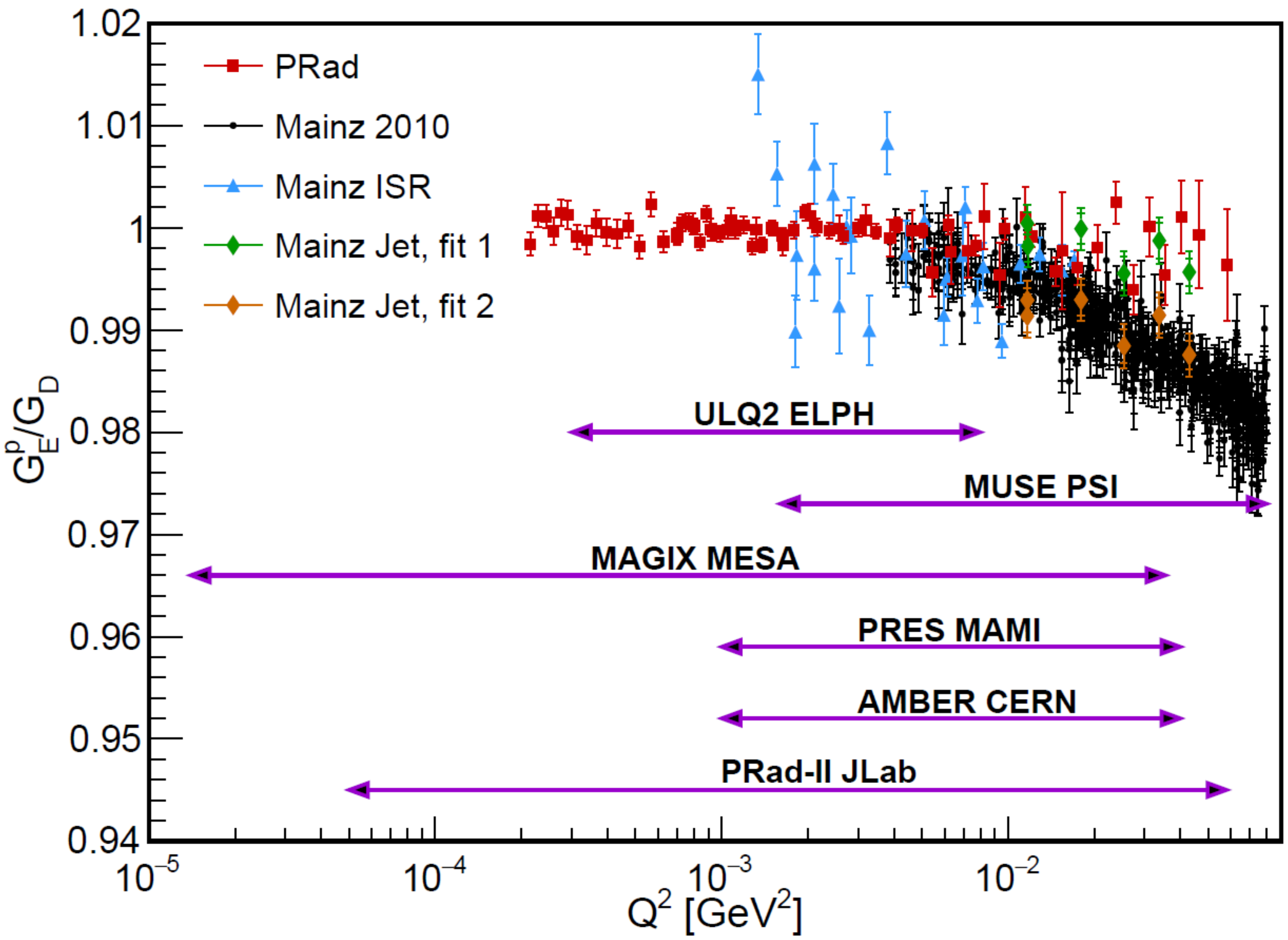
$$G_E(t) = F_1(t) + \frac{t}{4M^2}F_2(t)$$

Proton charge radius

$$\langle r^2\rangle = 6\frac{dG_E(t)}{dt}\bigg|_{t=0}$$

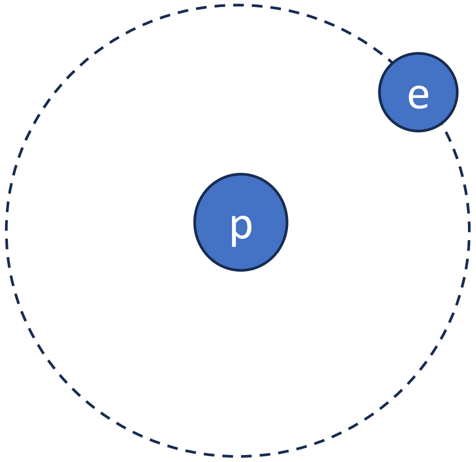
Elastic scattering 70 years later

Xiong, Peng, 2302.13818



Charge radius from hydrogen atom spectrum

Coulomb potential modified in a hydrogen atom



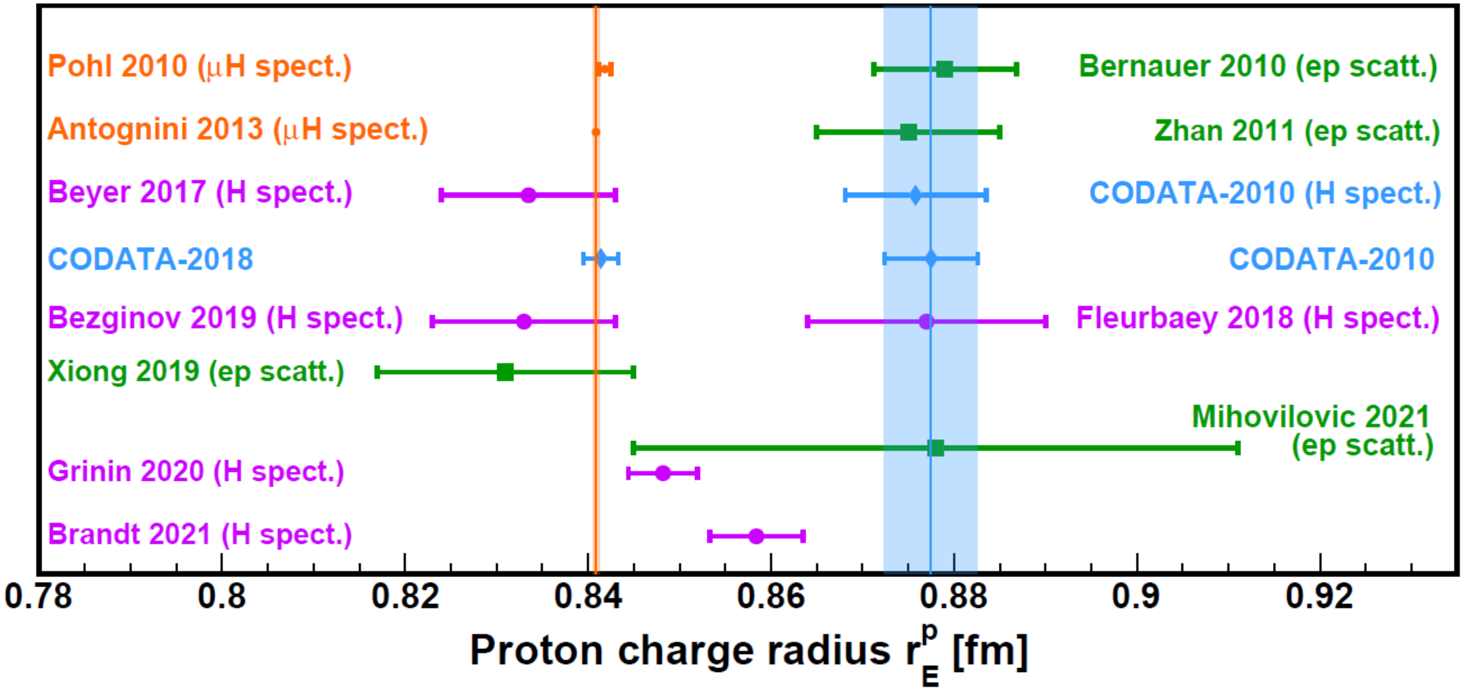
$$V(r) = \frac{-e^2}{4\pi r} \Rightarrow -e^2 \int \frac{d^3k}{(2\pi)^3} \frac{G_E(k^2) e^{-i\vec{k}\cdot\vec{r}}}{k^2}$$

$$\Delta E_{l=0} = \frac{2\alpha^4 m^3}{3n^3} \langle r^2 \rangle$$ ← Proton charge radius!

Part of the Lamb shift.

Enhanced in the muonic hydrogen! $m_\mu \approx 200 m_e$

Proton radius puzzle?



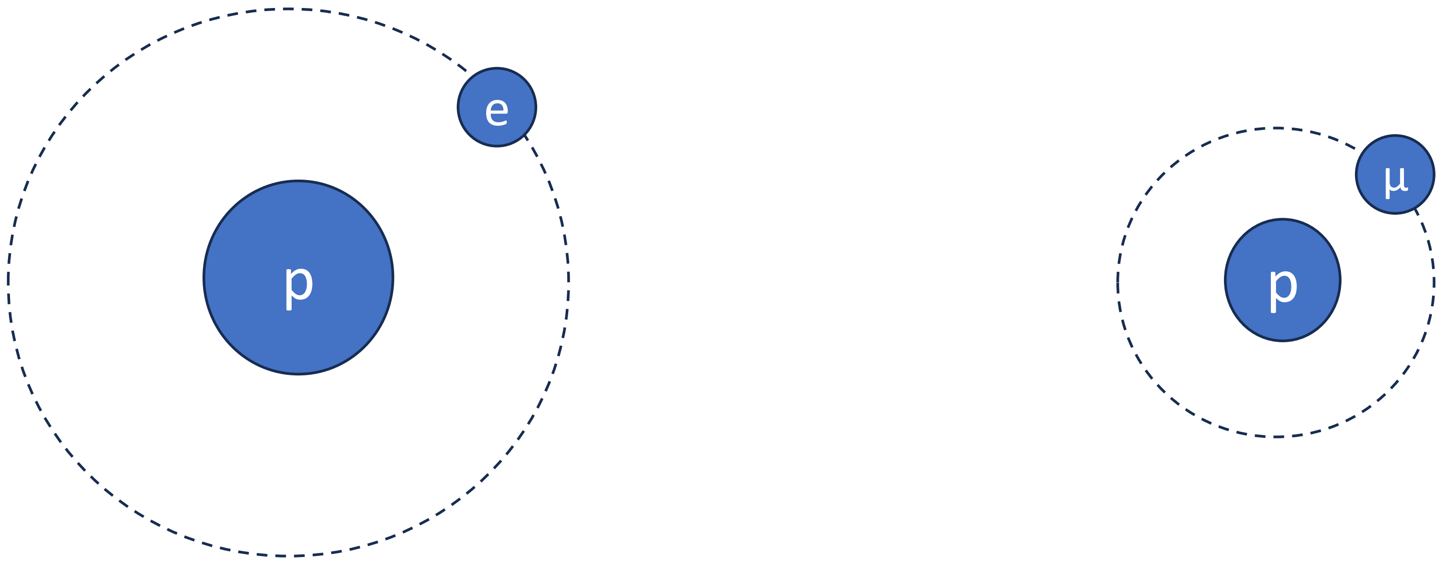
Both CODATA and PDG now recommend the smaller value ~0.84fm.

Several future experiment planned, aim for less than 1% precision

PRad (2019) $r_p = 0.831 \pm 0.007_{\text{stat}} \pm 0.012_{\text{syst}}$

Lepton universality

Can the charge radius be different in ordinary hydrogen and muonic hydrogen?



Target precision $\Delta r < 0.005\,\mathrm{fm}$

Efforts to reduce theory uncertainties (higher order QED, two-photon exchange, …)

New BSM interactions necessarily affect muon $g-2$

Radius zoo

2312.12984

Charge radius

$$\langle r^2\rangle_c = \frac{\int d\boldsymbol{x} x^2 \rho_c(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_c(\boldsymbol{x})} = \frac{6}{G_E(0)} \frac{dG_E(t)}{dt}\bigg|_{t=0}$$

Magnetic radius

$$\langle r^2\rangle_M = \frac{6}{G_M(0)} \frac{dG_M(t)}{dt}\bigg|_{t=0}$$

Baryon number radius

$$\langle r^2\rangle_B = \frac{\int d\boldsymbol{x} x^2 \rho_B(\boldsymbol{x})}{\int d\boldsymbol{x} \rho_B(\boldsymbol{x})}$$

Mass radius

$$\langle r^2\rangle_m = \frac{\int d\boldsymbol{x}\, x^2 T^{00}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{00}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{3D(0)}{2M^2}$$

Scalar radius

$$\langle r^2\rangle_s = \frac{\int d\boldsymbol{x}\, x^2 T^{\mu}_{\mu}(\boldsymbol{x})}{\int d\boldsymbol{x} T^{\mu}_{\mu}(\boldsymbol{x})} = 6\frac{dA(t)}{dt}\bigg|_{t=0} - \frac{9D(0)}{2M^2}$$

Tensor radius

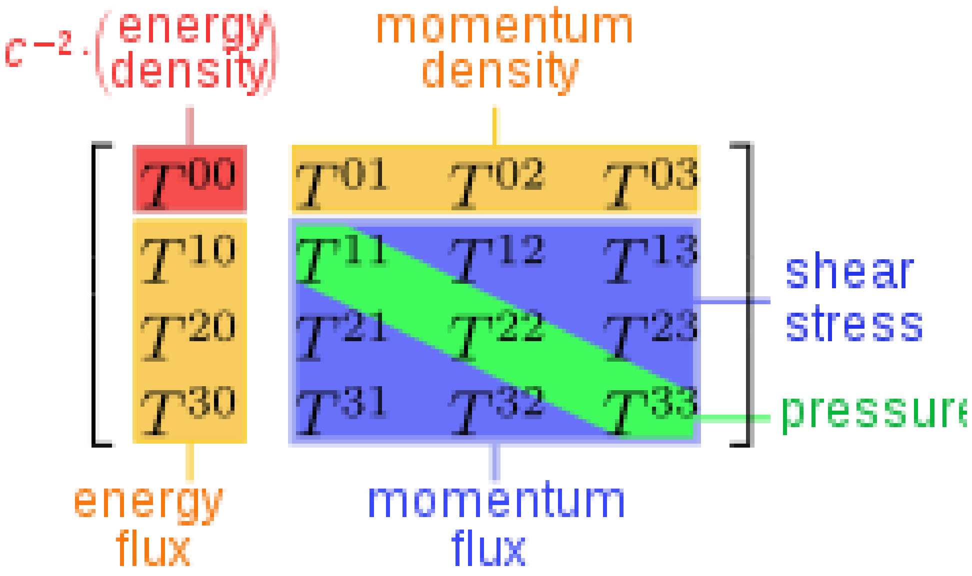
$$\langle r^2\rangle_t \equiv \frac{\int d\boldsymbol{x}\, x^2 \left(T^{00}(\boldsymbol{x}) + \frac{1}{2}T_{ii}(\boldsymbol{x})\right)}{\int d\boldsymbol{x} \left(T^{00} + \frac{1}{2}T_{ii}\right)} = 6\frac{dA(t)}{dt}\bigg|_{t=0}$$

Mechanical radius

$$\langle r^2\rangle_{mech} = \frac{\int d\boldsymbol{x} x^2 \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})}{\int d\boldsymbol{x} \frac{x_i x_j}{x^2} T_{ij}(\boldsymbol{x})} = \frac{6D(0)}{\int_{-\infty}^{0} dt D(t)}$$

…

Gravitational form factors



QCD energy momentum tensor

$$T^{\mu\nu} = \sum_f \bar{\psi}_f \gamma^{(\mu} i D^{\nu)} \psi_f - F^{\mu\rho} F^{\nu}{}_{\rho} + \frac{g^{\mu\nu}}{4} F^{\alpha\beta} F_{\alpha\beta}$$

Associated form factors

$$\bar{P} = \frac{P + P'}{2}, \quad \Delta = P' - P$$

$$\langle P' | T^{\mu\nu} | P \rangle = \bar{u}(P') \left[A(t) \gamma^{(\mu} \bar{P}^{\nu)} + B(t) \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M} \right] u(P)$$

$$A(0) = 1, \quad B(0) = 0 \qquad D(0) = ??$$

D-term—the last global unknown

$$\langle P'|T^{ij}|P\rangle \sim (\Delta^i\Delta^k - \delta^{ik}\Delta^2)D(t)$$

$D(t=0)$ is a conserved charge of the nucleon, similar to the magnetic moment

Fourier transform $\vec{\Delta} \to \vec{r}$ can be interpreted as `pressure' inside a nucleon Polyakov (2003)

$$T^{ij}(r) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij} p(r)$$

$$p(r) = \frac{1}{6M}\int \frac{d\mathbf{\Delta}}{(2\pi)^3} e^{i\mathbf{\Delta}\cdot\mathbf{r}} t D(t) \qquad D = M\int d^3r\, r^2 p(r)$$

Conjecture: All stable hadrons must have $D < 0$

Pion GFFs

Spin-0 hadron → 2 GFFs $\langle p'|T^{\mu\nu}|p\rangle = 2A(t)P^\mu P^\nu + \frac{D(t)}{2}(\Delta^\mu\Delta^\nu - \Delta^2 g^{\mu\nu})$

Soft pion theorem:

In the chiral limit of QCD, $D(0) = -1$

Moreover, $D_i(0) = -A_i(0)$ for each parton species $i = u, d, s, g, \cdots$

Nucleon D-term in the Sakai-Sugimoto model

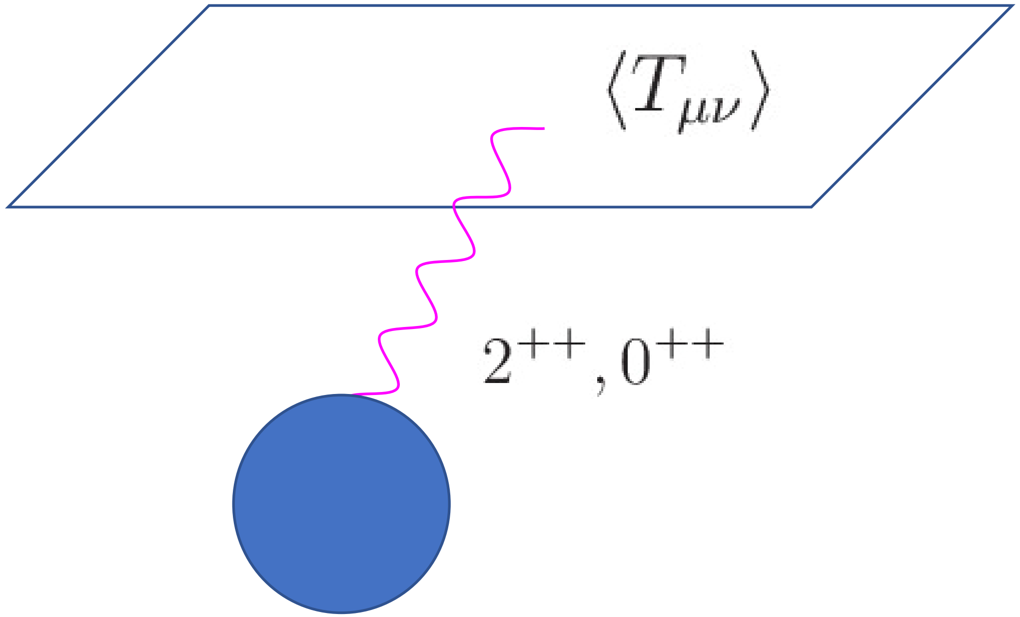
Fujita, YH, Sugimoto, Ueda (2022)

Baryons = instantons on D8 branes in type-IIA superstring

QFT energy momentum tensor from holographic renormalization

Graviton in 7D AdS = QCD glueballs

Glueball dominance in large-Nc QCD



$$D(|\vec{k}|) \sim \sum_{n=1}^{\infty} \frac{c_n^{\mathrm{T}}(|\vec{k}|)}{\vec{k}^2 + (m_n^{\mathrm{T}})^2} + \sum_{n=1}^{\infty} \frac{c_n^{\mathrm{S}}(|\vec{k}|)}{\vec{k}^2 + (m_n^{\mathrm{S}})^2}$$

see also, Mamo, Zahed (2021)

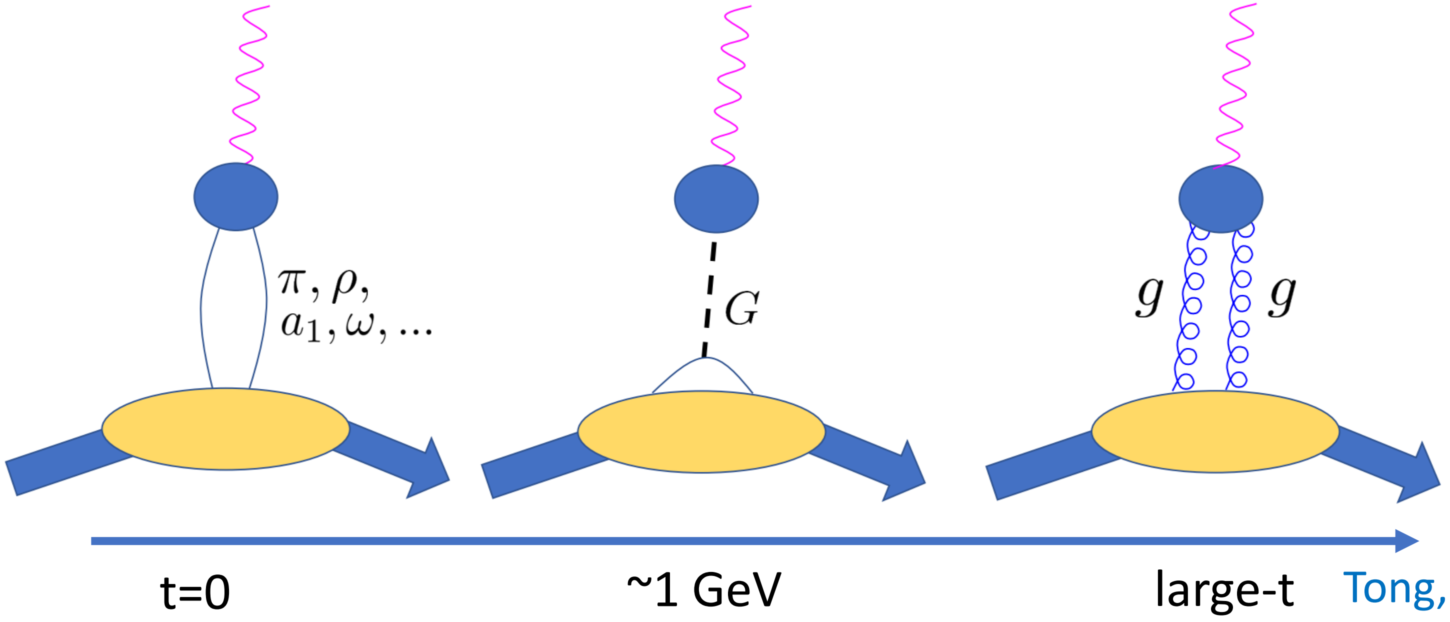
At $t = |\vec{k}|^2 = 0$, the infinite sum can be performed in a closed form

Numerical result (revised in Sugimoto, Tsukamoto, 2503.19492)

$$D(0) = -3.42 + 1.36 = -2.06$$

Negative (attractive) contribution from isovector mesons $\pi, \rho, a_1, \cdots$

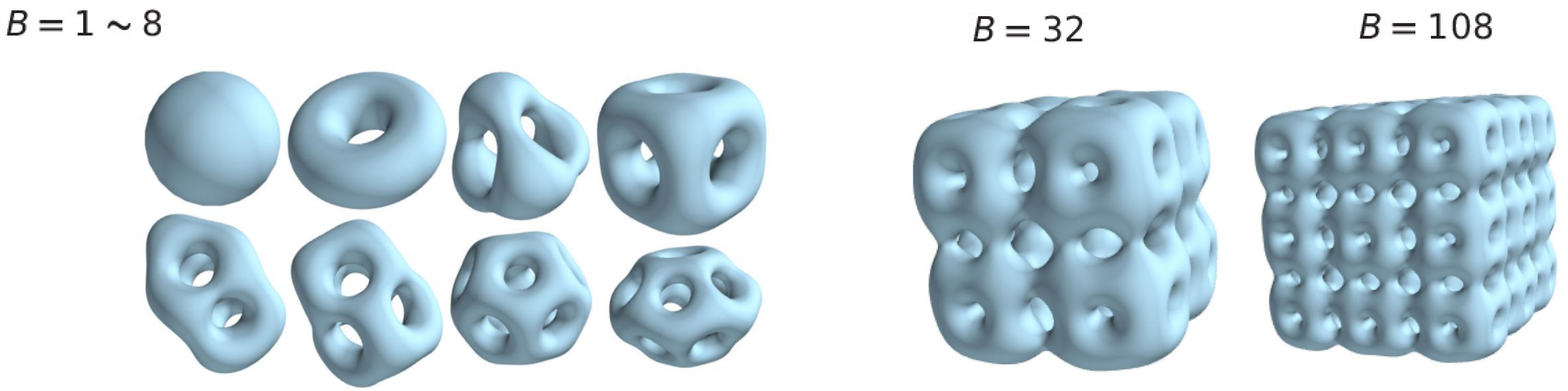
Positive (replusive) contribution from isoscalar mesons ω



Tong, Ma, Yuan (2021)

D-term of atomic nuclei in the Skyrme model

Martin-Caro, Huidobro, YH 2304.05994;
2312.12984



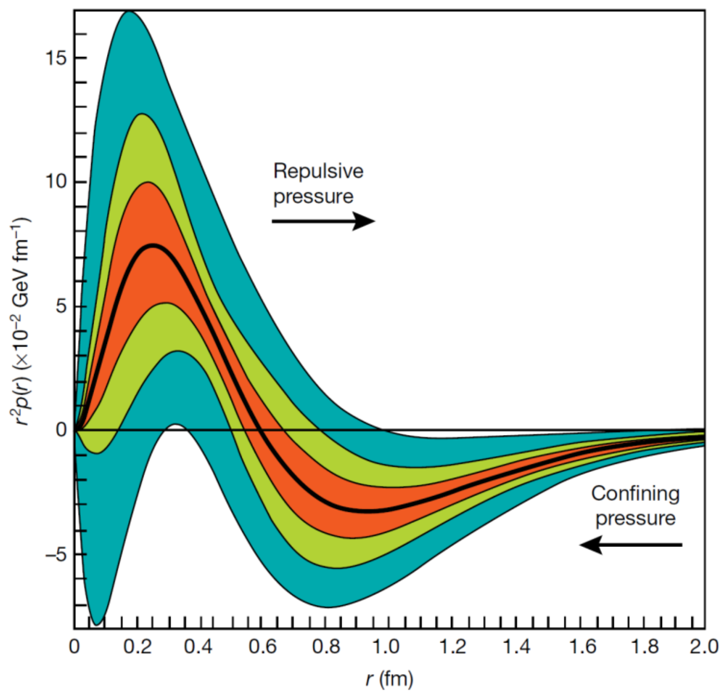
B	1	2	3	4	5	6	7	$8a$	$8b$	32	108
$D(0)$	-3.701	-13.126	-26.757	-43.304	-62.72	-85.95	-106.596	-128.368	-140.816	-1.874×10^3	-2.152×10^4

The value D(0) grows quickly with increasing B

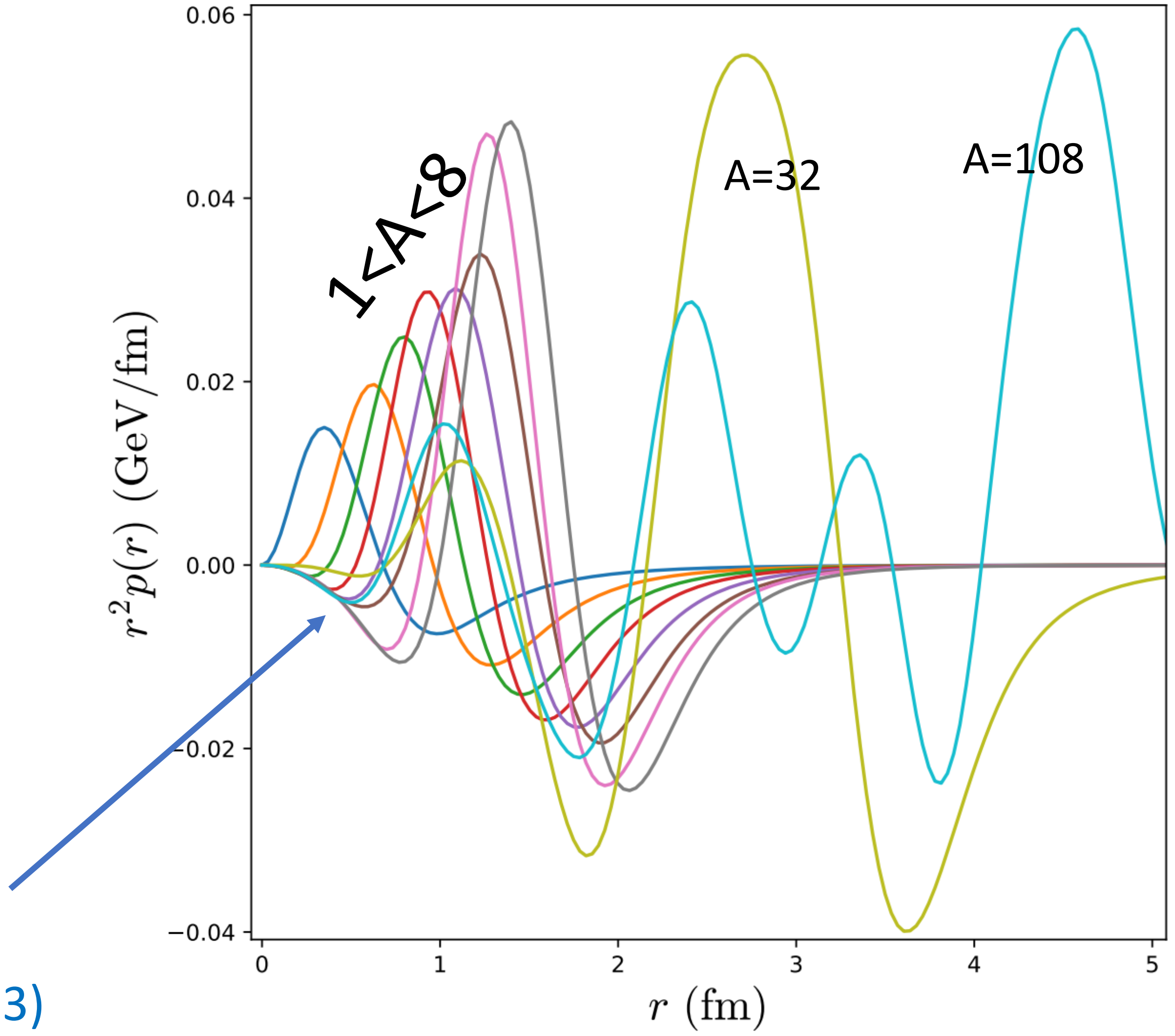
cf. Polyakov (2003); Liuti, Taneja (2005); Guzey, Siddikov (2005)

`Pressure' inside nucleon and nuclei

Burkert, Elouadrhiri, Girod (2018)



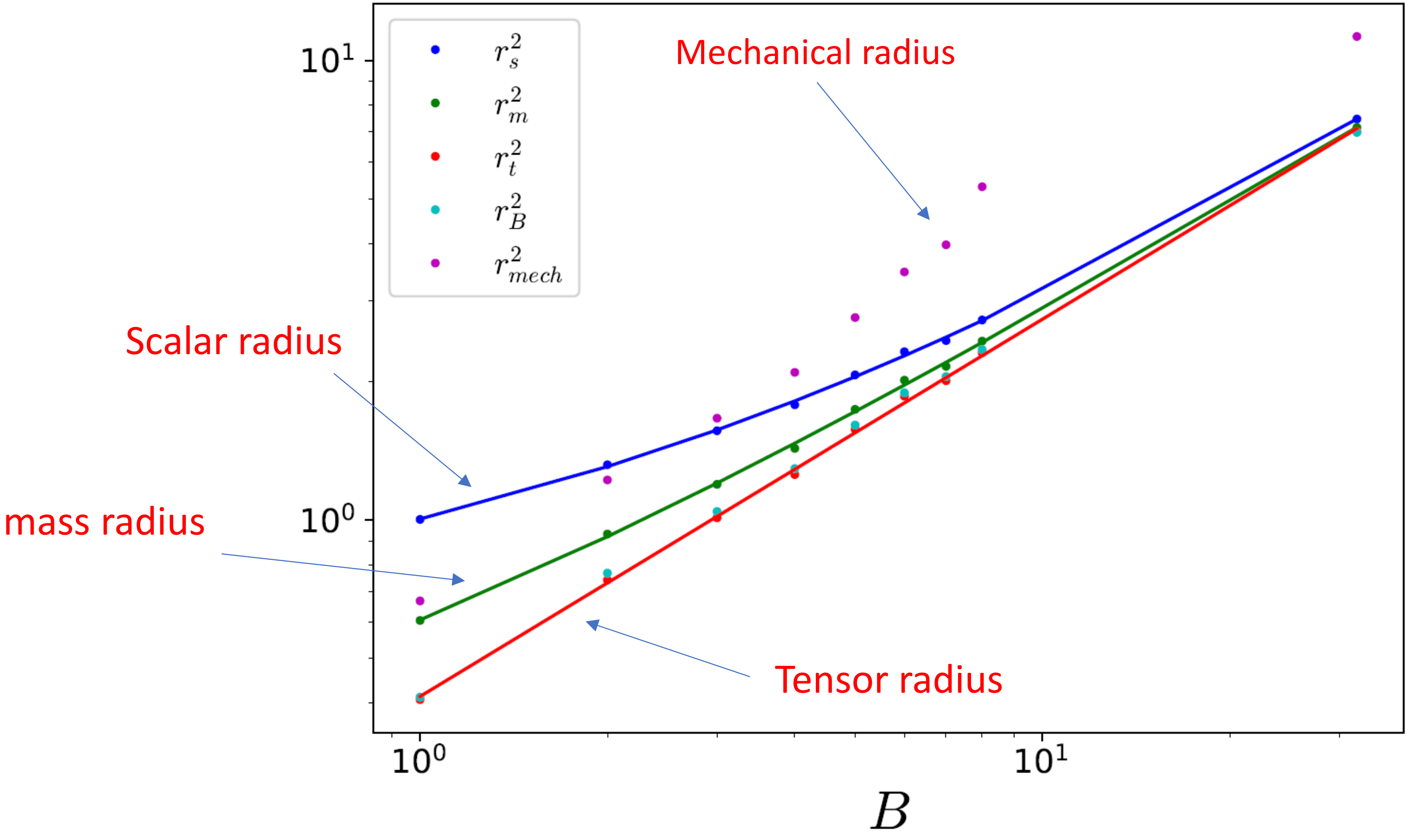
Martin-Caro, Huidobro, YH, 2312.12984



Negative pressure near the core for nuclei A>1
see also, Freese, Cosyn (2022), He, Zahed (2023)

Nuclear radii

Martin-Caro, Huidobro, YH, 2312.12984



$$\langle r^2 \rangle_s = \langle r^2 \rangle_m - \frac{3D(0)}{M^2}$$

$$\frac{D(0)}{M^2} \propto B^{\beta-2}$$

GFFs for quarks and gluons

Separately defined for quarks and gluons (Ji 1996)

hidden form factor

$$\langle P'|T^{\mu\nu}_{q,g}|P\rangle = \bar{u}(P')\Big[A_{q,g}\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\frac{\bar{P}^{(\mu}i\sigma^{\nu)\alpha}\Delta_\alpha}{2M} + D_{q,g}\frac{\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2}{4M} + \bar{C}_{q,g}M\eta^{\mu\nu}\Big]u(P)$$

Relation to generalized parton distribution (GPD)

$$\int_{-1}^{1} dx\, x H_q(x,\eta,t) = A_2^q(t) + \eta^2 D^q(t)$$

$\bar{C}_q + \bar{C}_g = 0$ because the total EMT is conserved.

$$\langle P|(T_{q,g})^\mu_\mu|P\rangle = 2M^2(A_{q,g} + 4\bar{C}_{q,g})$$

Connection to the trace anomaly and gluon condensate $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ → Origin of hadron masses

Relation between $\bar{C}_{q,g}(0)$ and $\langle P|F^{\mu\nu}F_{\mu\nu}|P\rangle$ in MSbar

1 loop
2 loop } YH, Rajan, Tanaka (2018)
3 loop Tanaka (2019)
4 loop Ahmed, Chen, Czakon (2022)

$$\left\langle \mathrm{Tr}\left([\Theta_g]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}}\ \left(-0.437676\,\alpha_s - 0.261512\,\alpha_s^2 - 0.183827\,\alpha_s^3 - 0.256096\,\alpha_s^4\right)$$
$$+\ \langle [O_m]_R\rangle_{\mathrm{P}}\ \left(0.495149\,\alpha_s + 0.776587\,\alpha_s^2 + 0.865492\,\alpha_s^3 + 0.974674\,\alpha_s^4\right)\ ,$$

$$\left\langle \mathrm{Tr}\left([\Theta_q]_R^{\overline{\mathrm{MS}}}\right)\right\rangle_{\mathrm{P}} = \langle [O_F]_R\rangle_{\mathrm{P}}\ \left(0.079578\,\alpha_s + 0.058870\,\alpha_s^2 + 0.021604\,\alpha_s^3 + 0.013675\,\alpha_s^4\right)$$
$$+\ \langle [O_m]_R\rangle_{\mathrm{P}}\ \left(1\ +\ 0.141471\,\alpha_s - 0.008235\,\alpha_s^2 - 0.064351\,\alpha_s^3 - 0.065869\,\alpha_s^4\right)$$

Experimental study of GFFs?

- Introduced theoretically in the 60s.
- Received far less attention than EM form factors, not because they are less interesting/important.
- The obvious reason: We cannot measure them directly!

One-graviton exchange cross section $\frac{d\sigma}{dt} \sim G_N^2 \frac{s^2}{t^2}$

$$G_N \sim 1/M_P^2 \qquad M_P \sim 10^{19} \text{ GeV}$$

- But theorists don't give up…

Direct measurement of GFFs?

- Graviton exchange suppressed by the Planck energy $M_P \sim 10^{19}\ \mathrm{GeV}$

- But in some BSM scenarios, the effective Planck energy could be in the TeV region.
  e.g. extra dimension models. These models typically predict massive gravitons.

- Long history of tests of Newton's inverse-square law

Adelberger, Heckel, Nelson hep-ph/0307284

$$V(r) = -G\frac{m_1\ m_2}{r}\left[1 + \alpha\, e^{-r/\lambda}\right]$$

Linearized massive gravity

Quadratic action

$$S \approx \frac{1}{2\kappa^2}\int dx\left[-\frac{1}{4}h^{\mu\nu}\partial^2 h_{\mu\nu} - \frac{1}{2}h^{\mu\nu}\partial_\mu\partial_\nu h + \frac{1}{4}h\partial^2 h + \frac{h^{\mu\nu}}{2}\partial_\lambda\partial_\mu h^\lambda_\nu\right]$$

Add a mass term

$$S_m = \frac{1}{2}\int dx(m_1^2 h_{\mu\nu}h^{\mu\nu} + m_2^2 h^2) \qquad m_1^2 = -m_2^2$$

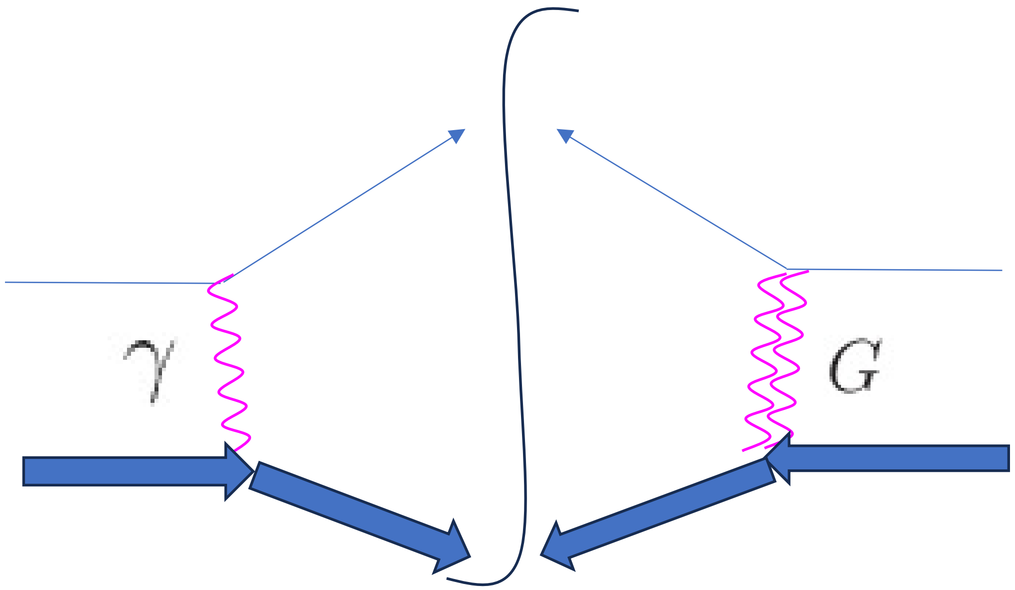
(Fierz-Pauli theory)

→ Massive spin-2 field (transverse, traceless)

Coupling to SM particles $\delta\mathcal{L} = \kappa h_{\mu\nu}T^{\mu\nu}$ assume $\kappa \sim 1\,\mathrm{TeV}^{-1}$

TeV-scale elastic ep, eA scattering

YH, 2311.14470



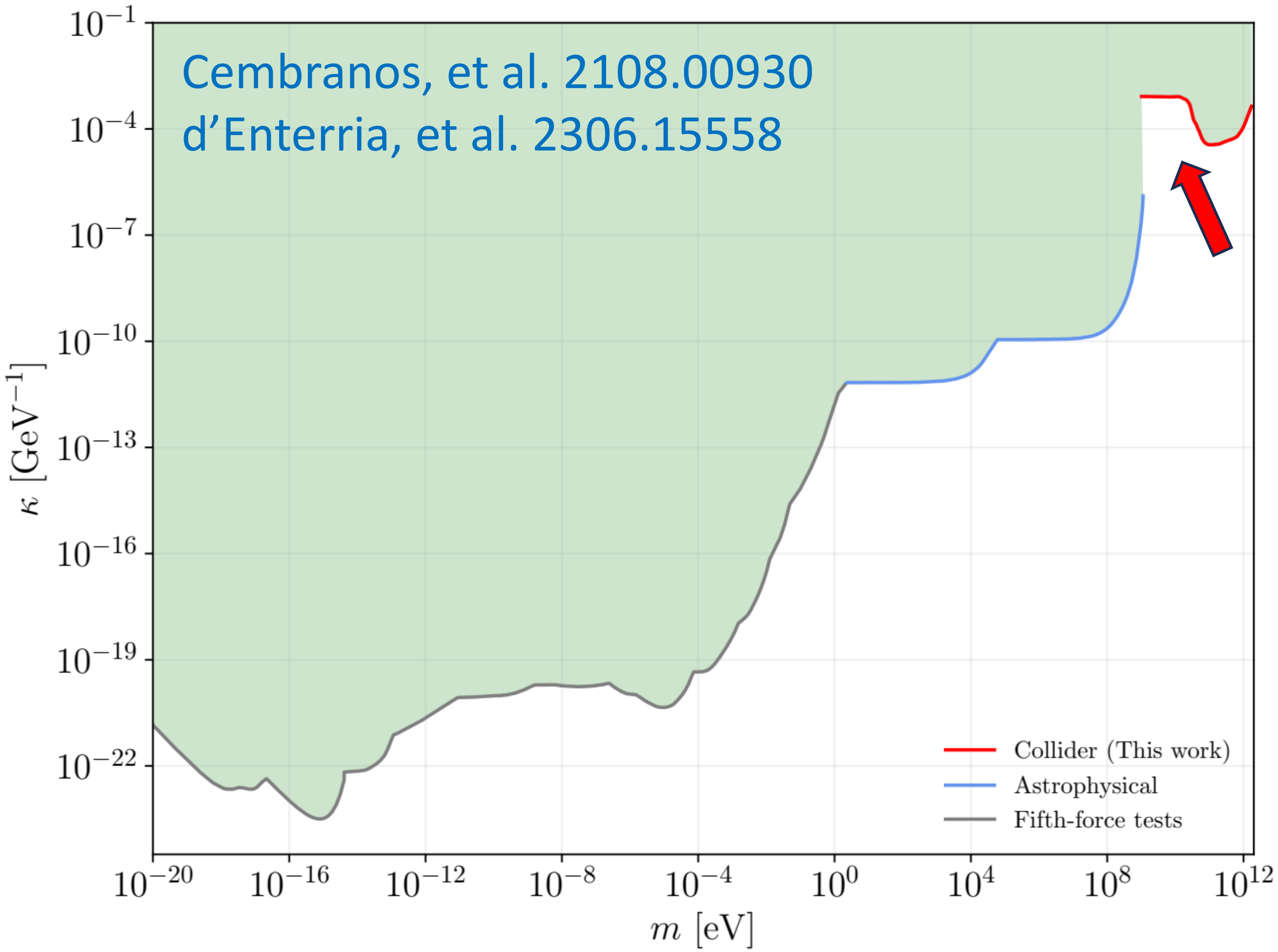
Look for the graviton-photon interference in elastic scattering

Rosenbluth

$$\frac{d\sigma}{dt} = \frac{4\pi\alpha_{em}^2}{t^2}\left\{\left(1+\frac{t-2M^2}{s}+\frac{M^4}{s^2}\right)\left(F_1^2(t)-\frac{tF_2^2(t)}{4M^2}\right)+\frac{t^2}{2s^2}(F_1(t)+F_2(t))^2\right\}$$
$$+\frac{\alpha_{em}\kappa^2 s}{t(t-m^2)}\left\{\left(1+\frac{3(t-2M^2)}{2s}\right)\left(A(t)F_1(t)-\frac{tB(t)F_2(t)}{4M^2}\right)+\mathcal{O}(s^{-2})\right\}+\mathcal{O}(\kappa^4)$$

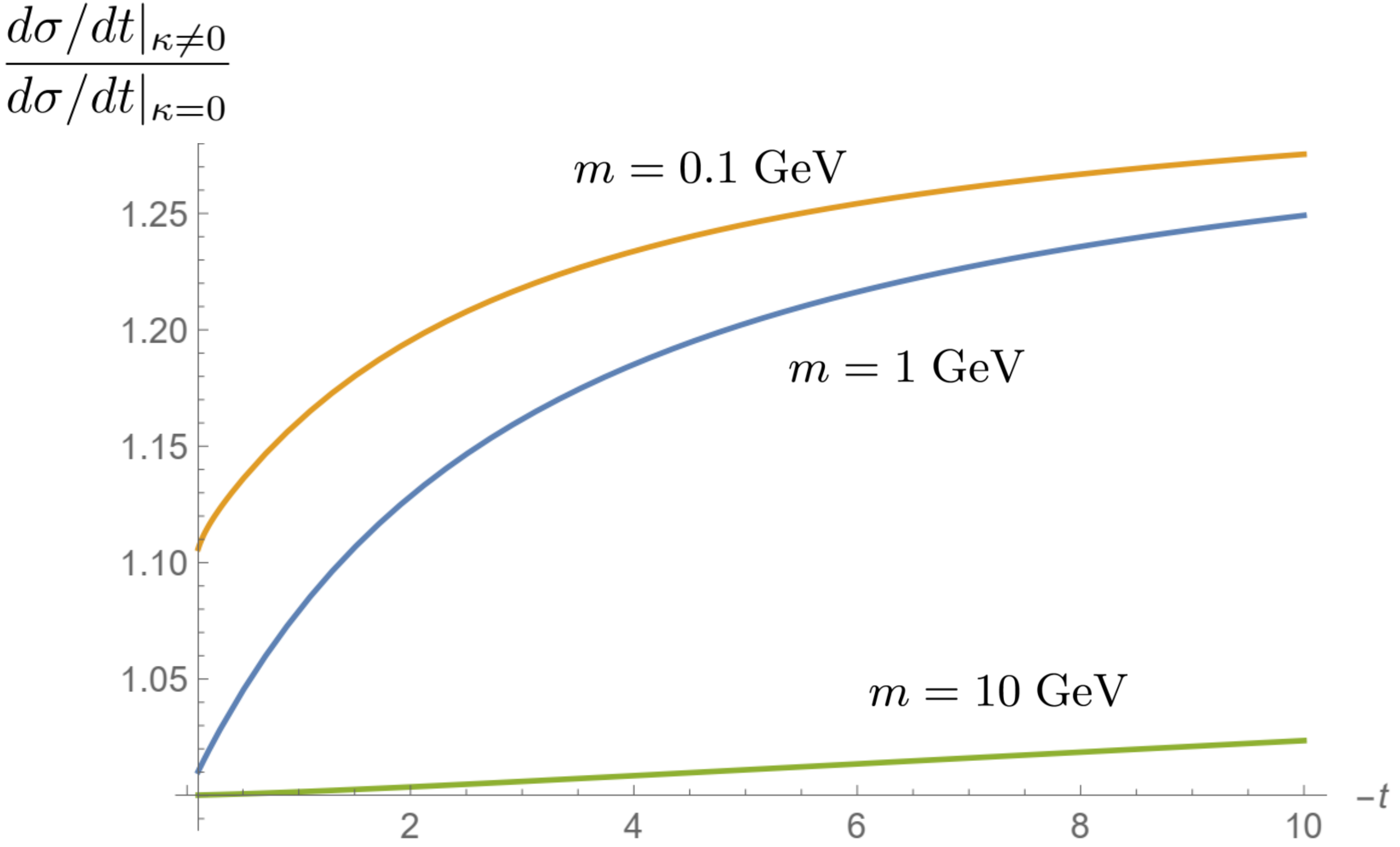
Notice that the D-form factor drops out.

Evading the LHC constraints



Low mass region
→ Kepler motion, Casimir force, etc.

High mass region
→ LHC graviton production



Input:

$$\kappa = 10^{-4}\ \mathrm{GeV}^{-1}$$

$$\sqrt{s} = 1\ \mathrm{TeV}$$

$$G_E(t) = \frac{G_M(t)}{\mu_p} = \frac{1}{\left(1 - \frac{t}{0.71\mathrm{GeV}^2}\right)^2}$$

$$A(t) = \frac{1}{\left(1 - \frac{t}{M_A^2}\right)^2}$$

Upward deviation from the QED prediction (EM and gravitational forces are both attractive)
Easily extended to atomic nuclei

Where to look for?

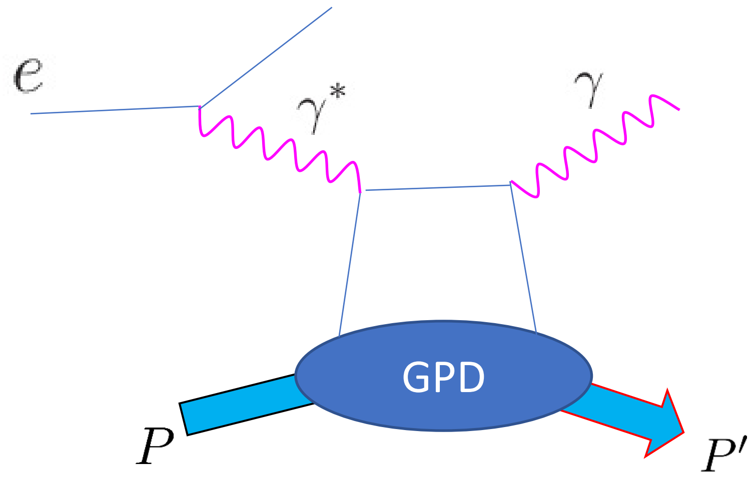
MuIC : a future TeV-scale Muon-ion collider at BNL Acosta, Li 2107.02073

Indirect measurement of GFF

Spin-2 particle not readily available.
But if there are two spin-1 particles (two photons, two gluons),
can they mimic a spin-2 exchange?

1+1=2

For example, Deeply Virtual Compton Scattering (DVCS)

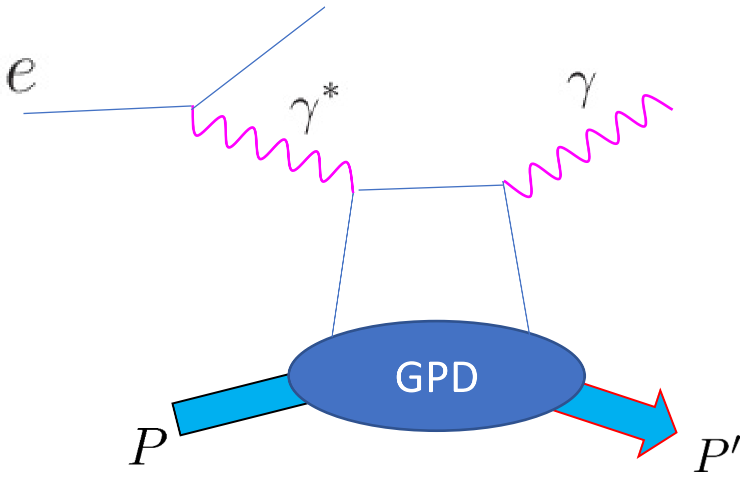


$$\mathcal{H}_q = \int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

Skewness $\xi = \dfrac{P^+ - P'^+}{P + +P'^+}$

Quark D-term from DVCS

$$D = D_u + D_d + D_s + D_g + \cdots$$



$D_{u,d}$ related to the subtraction constant in the dispersion relation for the Compton form factor Teryaev (2005)

$$\mathrm{Re}\mathcal{H}_q(\xi,t) = \frac{1}{\pi}\int_{-1}^{1} dx \mathrm{P}\frac{\mathrm{Im}\mathcal{H}_q(x,t)}{\xi - x} + 2\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

$$\int_{-1}^{1} dz z D_q(z,t) = D_q(t)$$

1 graviton $\approx$ 2 photons

After all, 1 graviton $\neq$ 2 photons

$$\int_{-1}^{1} dz \frac{D_q(z,t)}{1-z}$$

what is measurable

$$\int_{-1}^{1} dz z D_q(z,t)$$

what we want

2-photon state couples to operators with arbitrary spin.
How can one isolate the spin-2 component?

$$\frac{1}{1-z} = 1 + z + z^2 + z^3 + \cdots$$

spin-2 (EMT) (z) spin-4 (z^3)

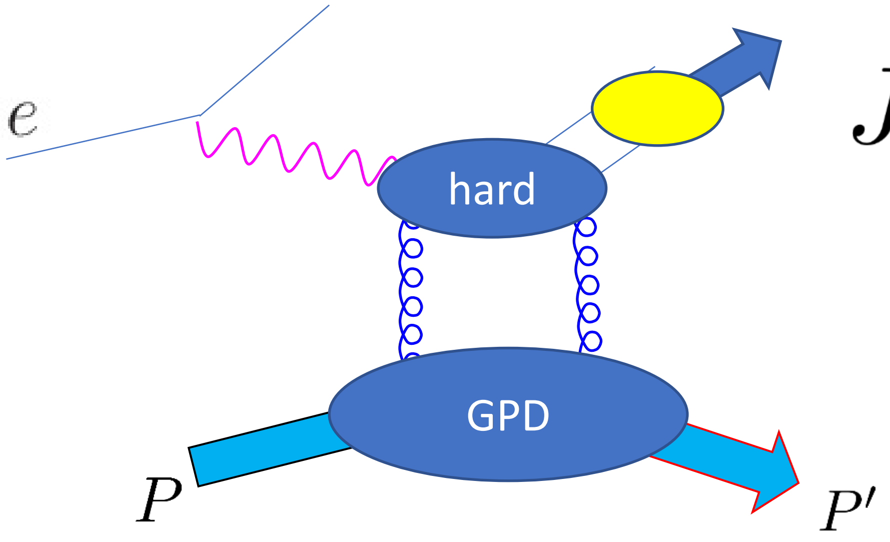
1+1= anything

$$d_1^{uds}(t=0, 2\ \mathrm{GeV}^2) = -1.7 \pm 21$$
$$d_3^{uds}(t=0, 2\ \mathrm{GeV}^2) = 0.7 \pm 15$$
$$d_1^{g}(t=0, 2\ \mathrm{GeV}^2) = -2 \pm 30$$
$$d_3^{g}(t=0, 2\ \mathrm{GeV}^2) = 0.1 \pm 2.3$$

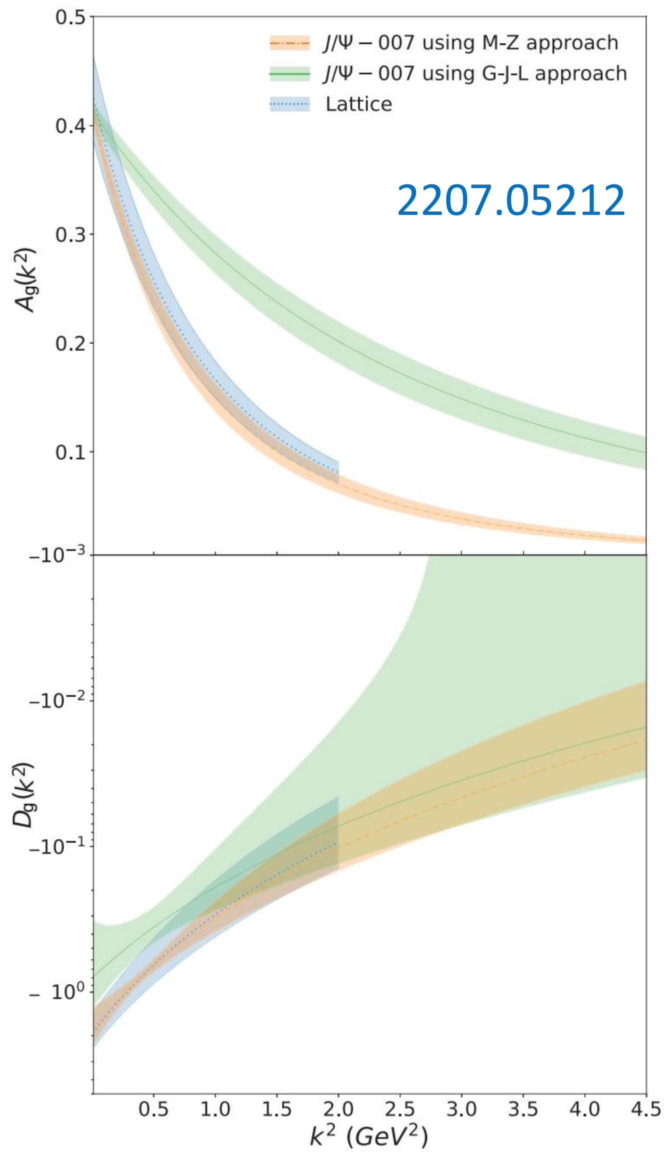
(NLO n=3 radiativ

Dutrieux, Meisgny, Mezrag, Moutarde (2024)

Quarkonium photo-production near threshold



J/ψ Υ



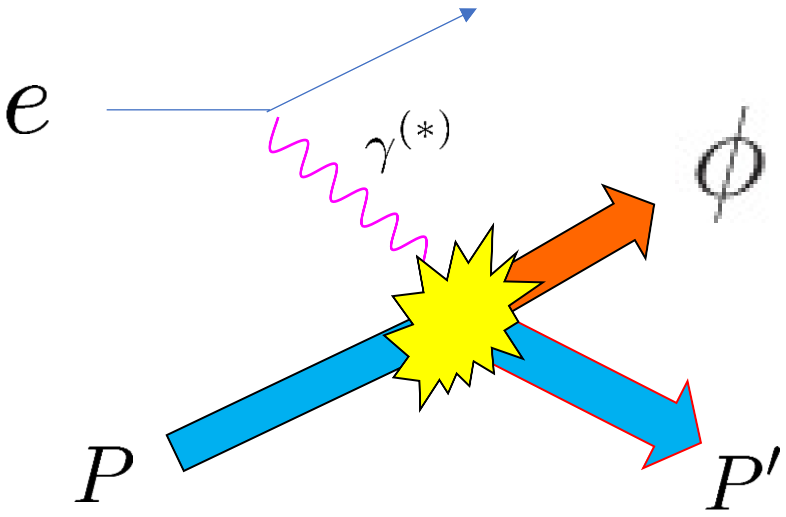
Ongoing experiments at JLab, future measurement at EIC?

Originally proposed by Kharzeev, Satz, Syamtomov, Zinovev (1997) to probe the gluon condensate.

One can also study gluon GFFs in this process YH, Yang (2018)

1 graviton $\approx$ 2 gluons

ϕ -meson electro-production near threshold



YH, Strikman (2021)
YH, Klest, Passek-K, Schoenleber (2025)

Complementary to J/ψ.
Need more than one observable for global analysis.

As sensitive to gluons as in J/ψ production (maybe even better).
Unique channel for strangeness GFFs.
Standard GPD factorization. No uncertainty from NRQCD.
Alternative scenarios for J/ψ photoproduction? Less ambiguity for ϕ electroproduction

Factorization only for the longitudinally polarized photon
L/T separation crucial → SoLID and EIC?

Again, 1 graviton $\neq$ 2 gluons

what is measurable

$$\int_{-1}^{1} \frac{dx}{x} \left(\frac{1}{\xi - x - i\epsilon} - \frac{1}{\xi + x - i\epsilon} \right) H_g(x, \xi, t)$$

what we want

$$\int_{-1}^{1} dx H_g(x, \xi, t) = A_g(t) + \xi^2 D_g(t)$$

Essentially the same problem as in the extraction of quark D-term from DVCS

HOWEVER, two important differences

Leading contribution from gluon GPD
There is a tunable skewness parameter ξ which becomes large near the threshold.

Threshold approximation

YH, Strikman 2102.12631 (Mellin moment)
Guo, Ji, Liu 2103.11506 (Mellin moment)
Guo, Ji, Yuan 2308.13006 (conformal moment)

what is measurable — what we want

$$\int_{-1}^{1} dx \frac{1}{\xi - x - i\epsilon} \begin{cases} \frac{1}{2} H^{q(+)}(x,\xi,t,\mu^2) \\ \frac{1}{x} H^{g}(x,\xi,t,\mu^2), \end{cases} \approx \frac{2}{\xi^2}\frac{5}{4}\left(A^a(t,\mu^2) + \xi^2 D^a(t,\mu^2)\right)$$

Keep only the first term in the conformal partial wave expansion

Very good approximation when $\xi = \mathcal{O}(1)$ and for gluon and strangeness GPDs
(but not for light-quark GPDs)

Recently extended to NLO

Guo, Yuan, Zhao, 2501.10532
YH, Klest, Passek-K, Schoenleber, 2501.12343
YH, Schoenleber 2502.12061

Example: NLO ϕ -electroproduction

YH, Klest, Passek-K, Schoenleber (2025)

Compare the full NLO amplitude (Muller et al. (2013)) with the truncated version, also at NLO

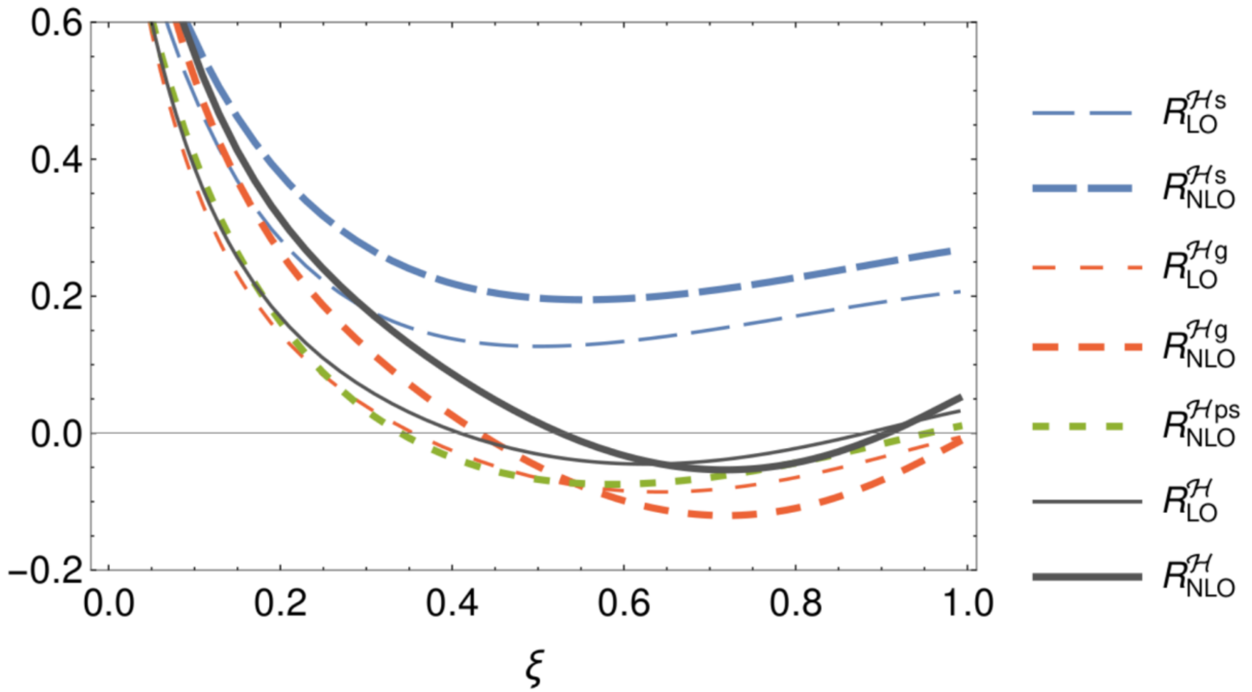
$$\mathcal{H}(\xi,t,Q^2) \approx \frac{2\kappa}{\xi^2}\frac{15}{2}\Bigg[\left\{\alpha_s(\mu)+\frac{\alpha_s^2(\mu)}{2\pi}\left(25.7309-2n_f+\left(-\frac{131}{18}+\frac{n_f}{3}\right)\ln\frac{Q^2}{\mu^2}\right)\right\}(A_s(t,\mu)+\xi^2 D_s(t,\mu))$$
$$+\frac{\alpha_s^2}{2\pi}\left(-2.3889+\frac{2}{3}\ln\frac{Q^2}{\mu^2}\right)\sum_q(A_q+\xi^2 D_q)+\frac{3}{8}\left\{\alpha_s+\frac{\alpha_s^2}{2\pi}\left(13.8682-\frac{83}{18}\ln\frac{Q^2}{\mu^2}\right)\right\}(A_g+\xi^2 D_g)\Bigg]$$

Goloskokov-Kroll (GK) model for nucleon GPD

Truncation error

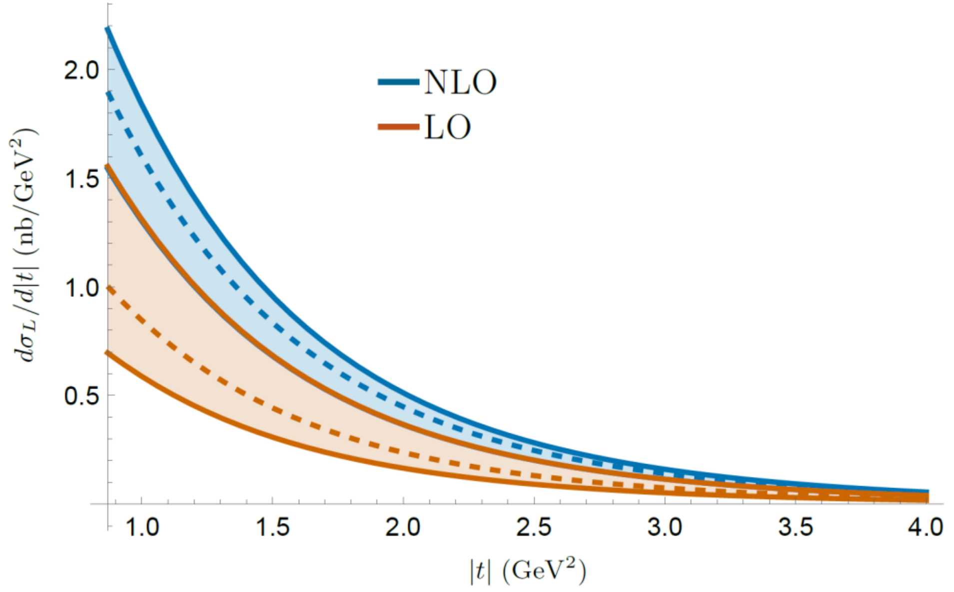
$$R = 1 - \frac{|\mathcal{H}_{\text{full}}|}{\mathcal{H}_{\text{trunc}}}$$

less than 10% for $\xi \gtrsim 0.4$



ϕ -electroproduction at NLO

YH, Klest, Passek-K, Schoenleber (2025)

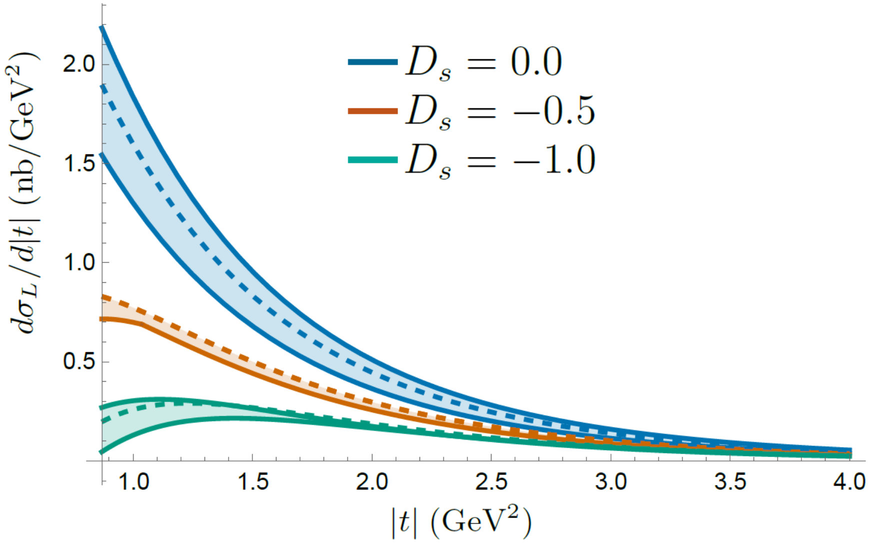


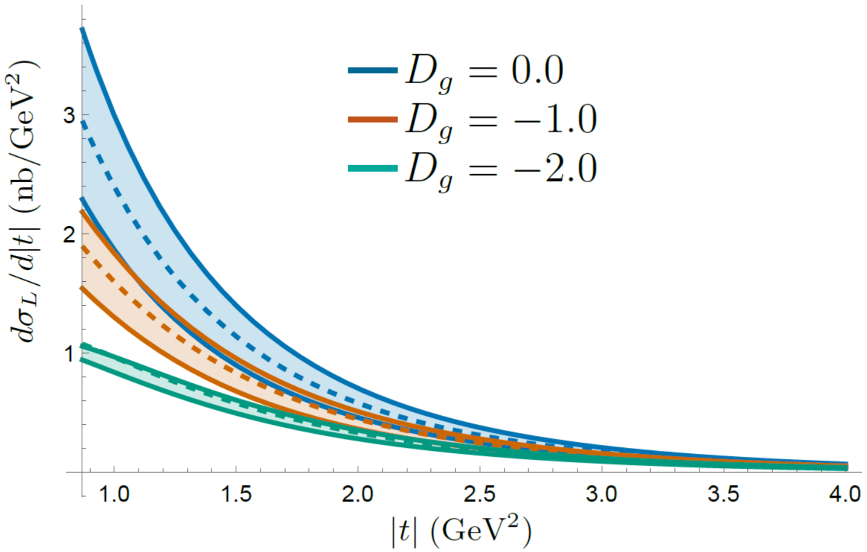
Dominated by gluons.

Cancellation between LO strangeness and NLO valence

Strangeness is important if $D_s = O(1)$

Combined fit to J/psi production data desirable

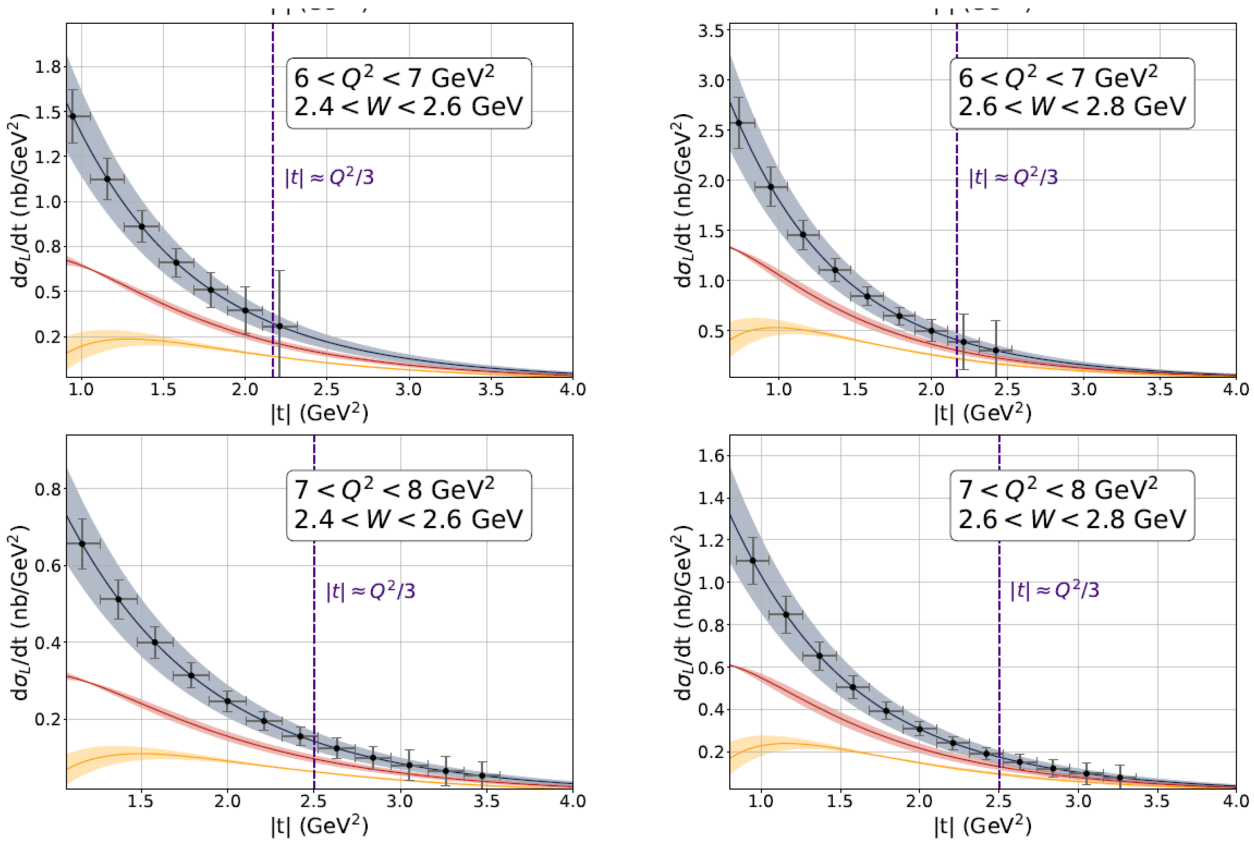


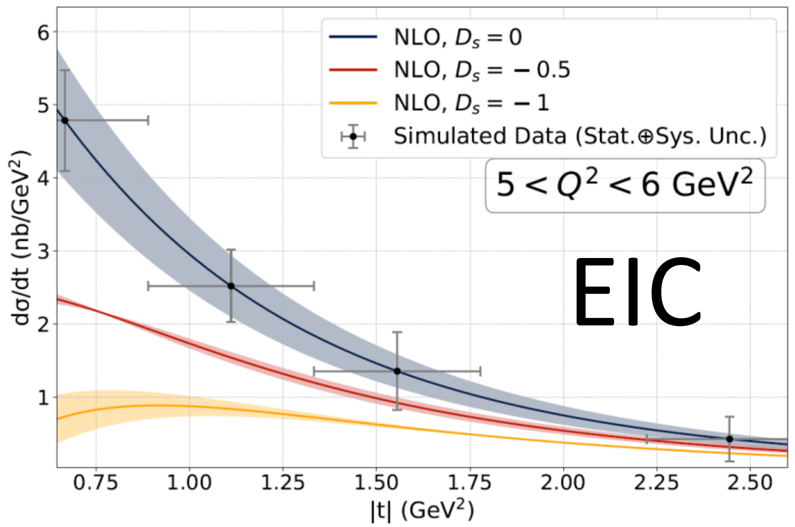


ϕ -electroproduction: Monte Carlo simulation

YH, Klest, Passek-K, Schoenleber (2025)

SoLID (Jlab)





Looks like a feasible measurement!

Pion GFFs from Sullivan process

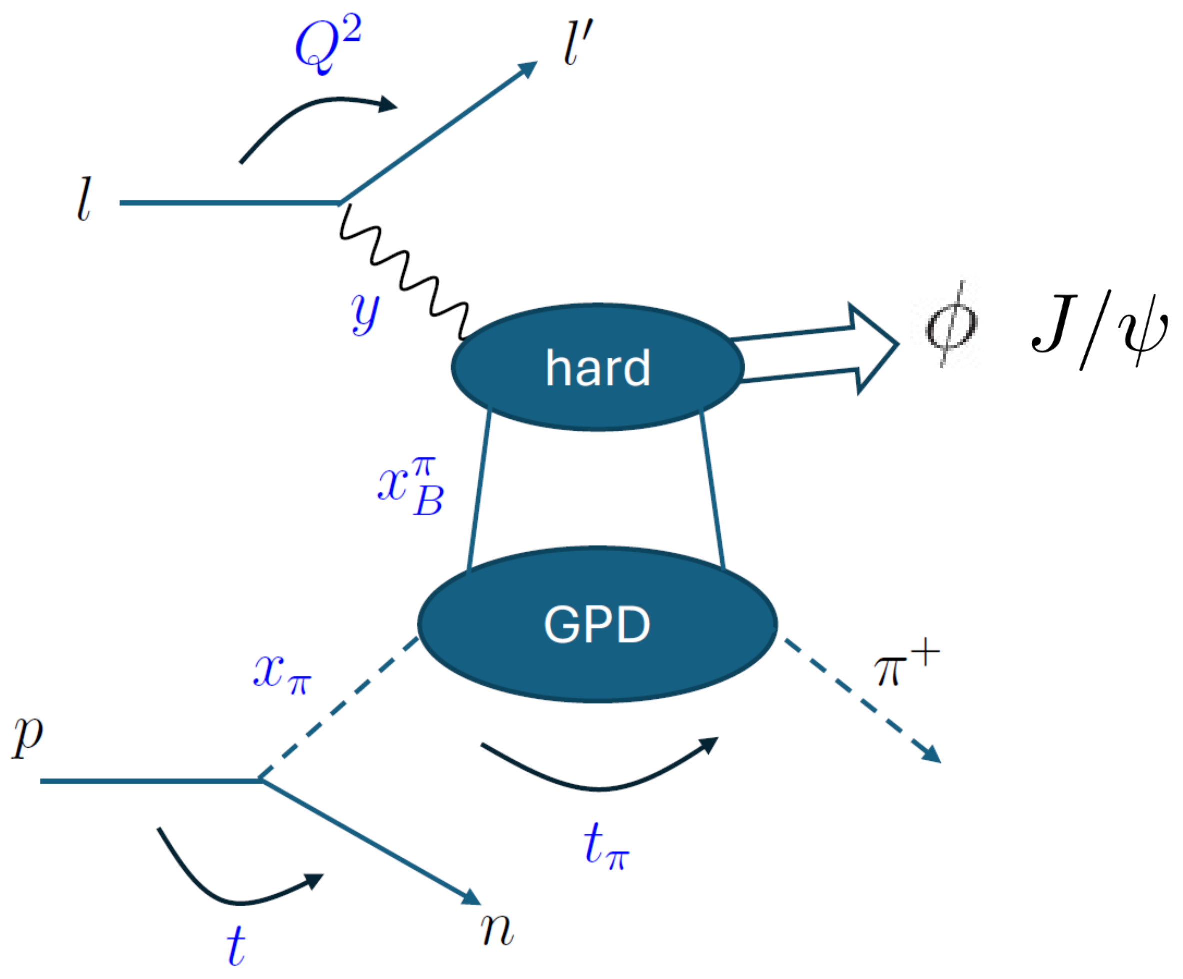
YH, Schoenleber 2502.12061 (PRL)

Originally proposed in 1972 to access the pion EM form factors

Pion GPDs from DVCS

Amrath, Diehl, Lansberg (2008)
Chavez, et al. (2022)

Pion GFFs from
J/ψ photoproduction
ϕ electroproduction
near threshold



Sullivan process near threshold

Measure the cross section $\frac{d\sigma}{dx_B dx_\pi}$

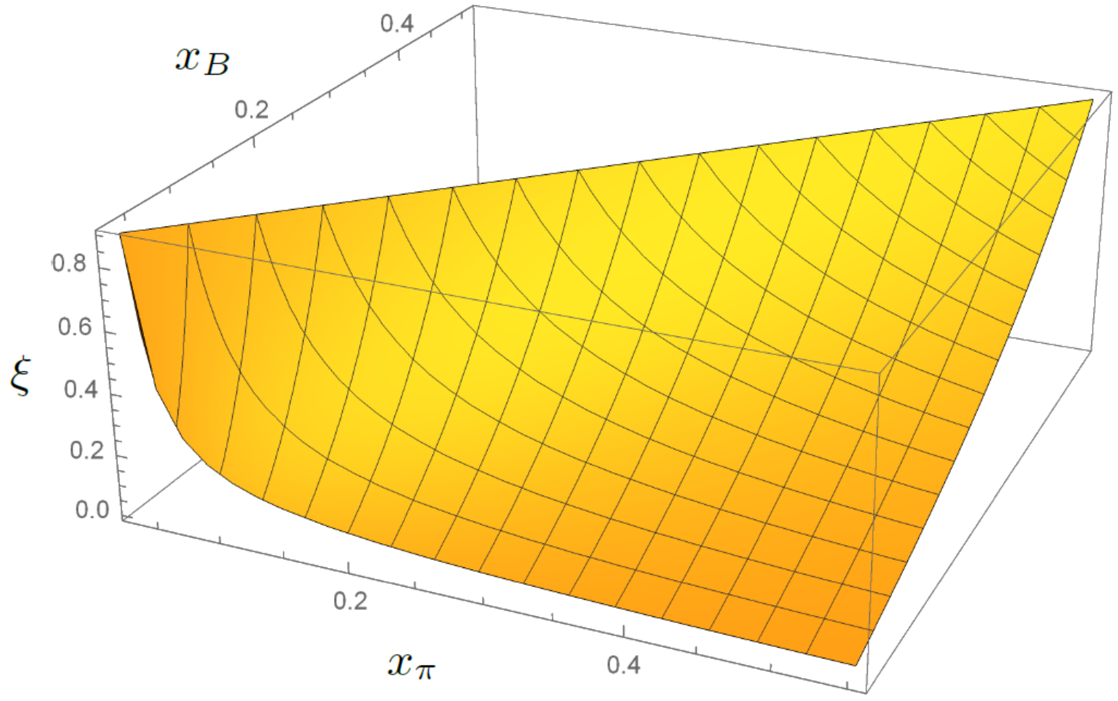
$$x_\pi = \frac{p_\pi \cdot l}{p \cdot l} \qquad x_B = \frac{Q^2}{2p \cdot q}$$

Threshold region along the diagonal line

$$x_B \approx x_\pi$$

Thanks to the light pion mass, relatively easier to achieve large skewness while keeping t small

$$t_{min} = -\frac{4\xi^2 m_\pi^2}{1-\xi^2}$$



Threshold approximation

Input: Pion GPD at $\mu^2 = 10\,\mathrm{GeV}^2$

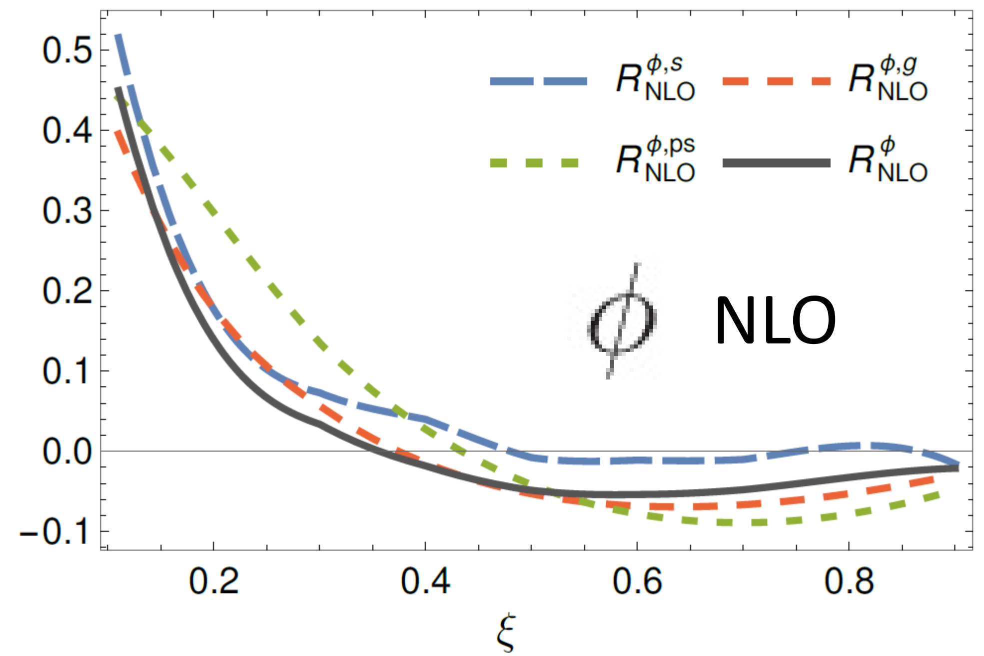
Chavez et al. 2110.06052

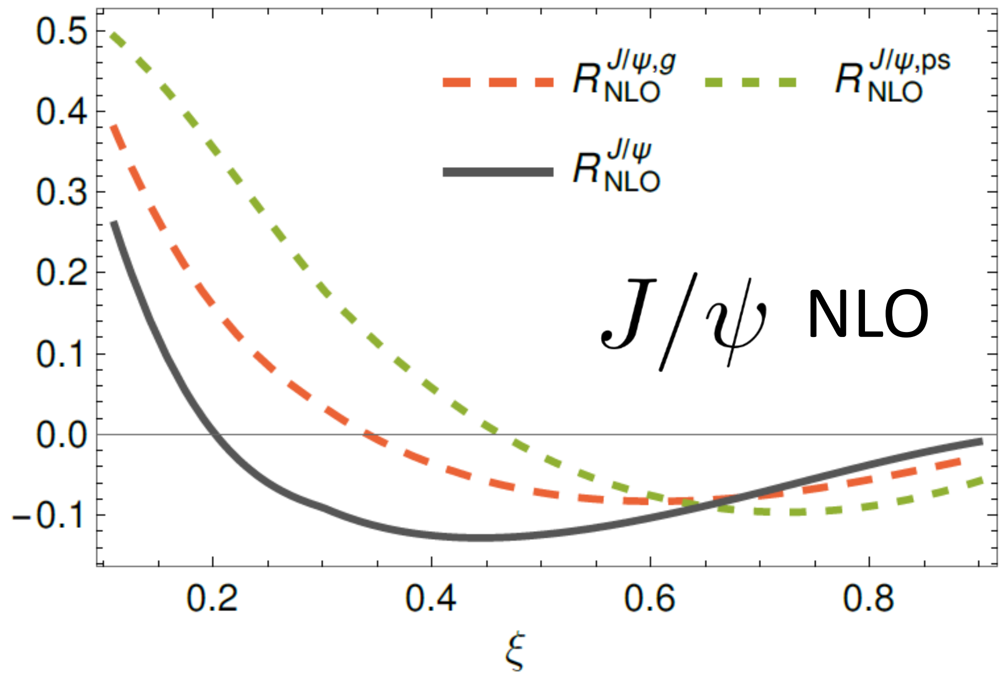
Soft pion theorem

$$D_a(0) = -A_a(0)$$

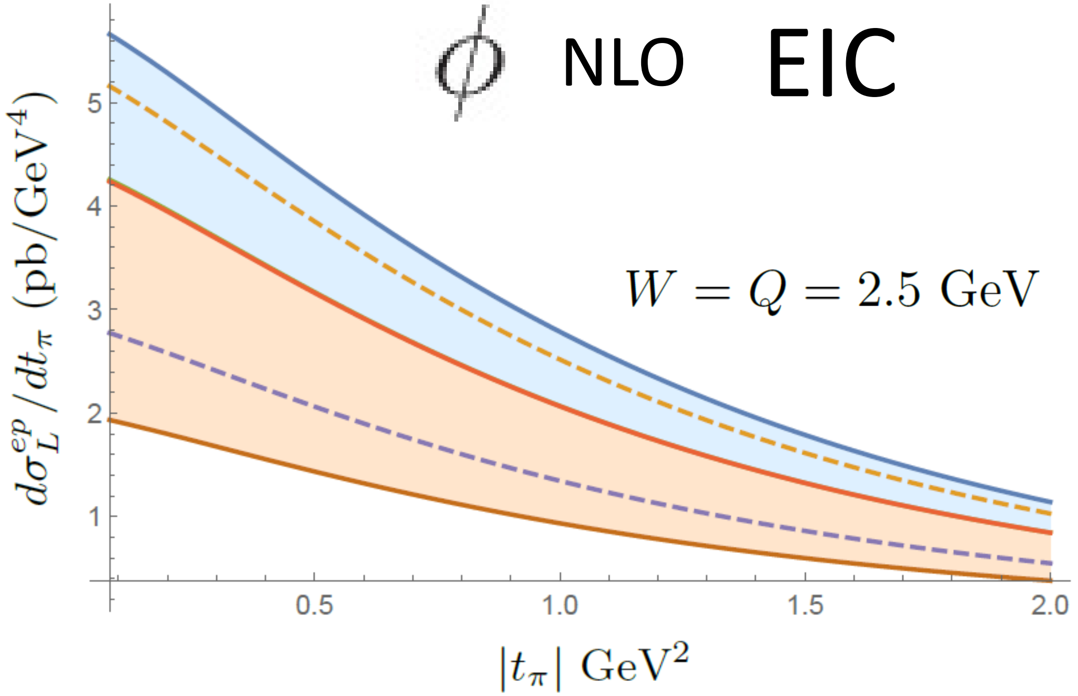
Truncation error

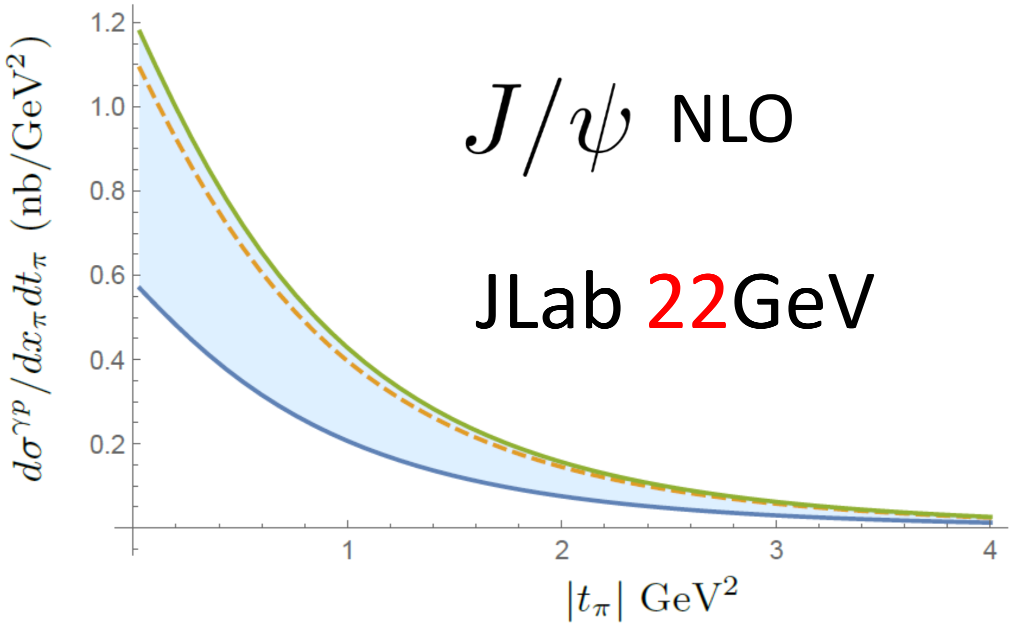
$$R = 1 - \frac{|\mathcal{H}_{\mathrm{full}}|}{\mathcal{H}_{\mathrm{trunc}}} \qquad 5 \sim 10\%$$

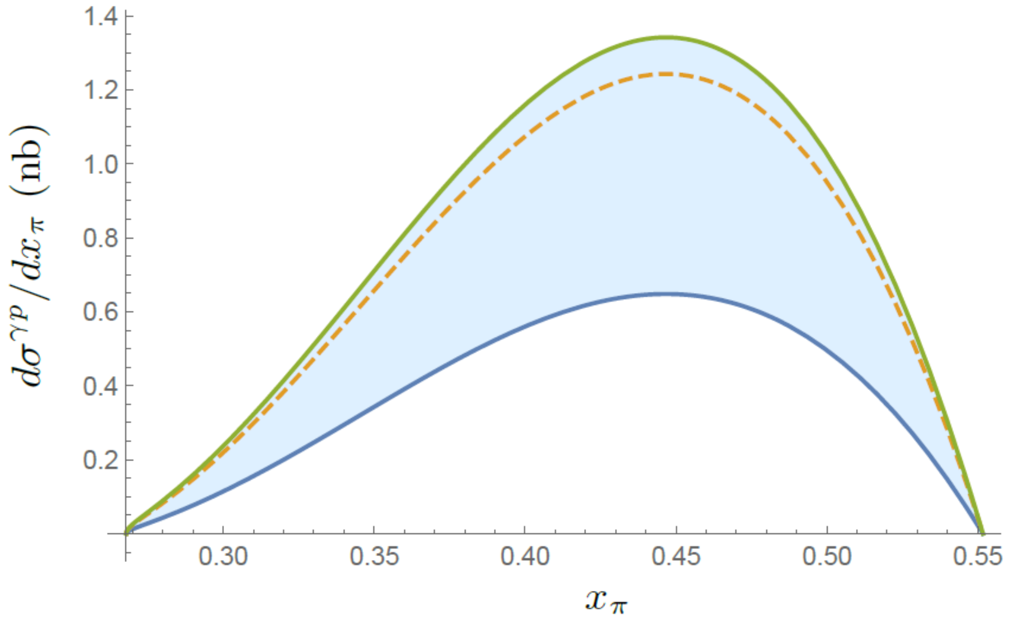




Prediction for JLab and EIC







Cross section well in the measurable range

Conclusions

- EM form factors: very active field even after 70 years, aiming for 1% precision
- GFFs: just the beginning!