Structure Functions for the Electron-Ion Collider

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New Opportunities for BSM Searches at the EIC
Stony Brook University
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Outline

1. Structure Functions at the Precision Frontier
2. Photon content of nucleon and nuclei
3. Charged-current structure functions

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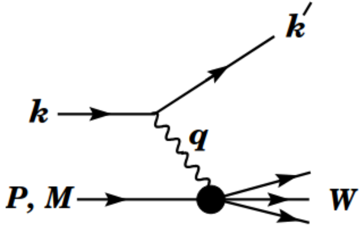
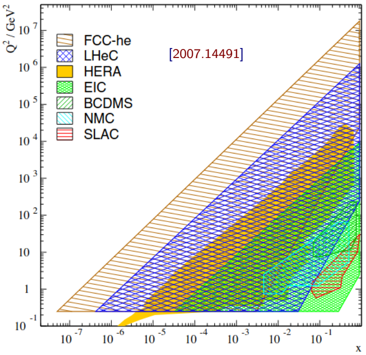
1 Structure Functions at the Precision Frontier
2 Photon content of nucleon and nuclei
3 Charged-current structure functions

Deep Inelastic Scattering: Kinematics





In the nucleon's rest frame

- $\nu = \frac{q\cdot P}{M} = E - E'$: lepton's energy loss
- $Q^2 = -q^2 = 2(EE' - \vec{k}\cdot\vec{k}') - m_\ell^2 - m_{\ell'}^2 \approx 2EE'(1-\cos\theta)$: photon virtuality (momentum transfer)
- $x = \frac{Q^2}{2M\nu}$: Bjorken x, parton momentum fraction at the LO
- $y = \frac{q\cdot P}{k\cdot P} = \frac{\nu}{E} = 1 - \frac{E'}{E}$: inelasticity, lepton's energy loss fraction
- $W^2 = (P+q)^2 = M^2 + 2M\nu - Q^2$: squared mass of the X system
- $s = (k+P)^2 = \frac{Q^2}{xy} + M^2 + m_\ell^2$: center-of-mass energy square

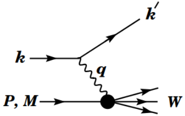
DIS conditions:

- Deep: $Q^2 \gg M^2$
- Inelastic: $W^2 \gg M^2$ with a lower bound $(M+m_\pi)^2$

DIS cross section and Structure Functions

Schematically

$$d\sigma \sim \sum_X \left| \quad \right|^2 \sim L^{\mu\nu} W_{\mu\nu}$$



- $L^{\mu\nu}$ Leptonic tensor, calculable with the electroweak interaction
- $W_{\mu\nu}$ Hadronic tensor, constrained by Lorentz invariance

Cross section

$$\frac{d^2\sigma^i}{dx\,dy} = \underbrace{\frac{4\pi\alpha^2}{xyQ^2}\eta^i}_{\text{Leptonic}} \underbrace{\left\{ \left(1 - y - \frac{x^2y^2M^2}{Q^2}\right) F_2^i + y^2 x F_1^i \mp \left(y - \frac{y^2}{2}\right) x F_3^i \right\}}_{\text{Hadronic}}$$

- Neutral Current: $\eta^{\text{NC}} = 1$ for unpolarized $e^{\pm}$,

$$F_2^{\text{NC}} = F_2^{\gamma} - (g_V^e \pm \lambda g_A^e)\,\eta_{\gamma Z}\, F_2^{\gamma Z} + \left({g_V^e}^2 + {g_A^e}^2 \pm 2\lambda g_V^e g_A^e\right)\eta_Z\, F_2^Z$$

$$xF_3^{\text{NC}} = -(g_A^e \pm \lambda g_V^e)\,\eta_{\gamma Z}\, xF_3^{\gamma Z} + \left[2 g_V^e g_A^e \pm \lambda\left({g_V^e}^2 + {g_A^e}^2\right)\right]\eta_Z\, xF_3^Z$$

- Charged Current: $\eta^{\text{CC}} = (1 \pm \lambda)^2 \eta_W$

$$F_1^{\text{CC}} = F_1^W, \quad F_2^{\text{CC}} = F_2^W, \quad xF_3^{\text{CC}} = xF_3^W$$

Quark-Parton model

Bjorken Scaling: In the Bjorken limit $Q^2, \nu \to \infty$, $F_i(x, Q^2) \to F_i(x)$

- Neutral Current

$$\left[F_2^{\gamma}, F_2^{\gamma Z}, F_2^{Z}\right] = x \sum_q \left[e_q^2,\, 2e_q g_V^q,\, (g_V^q)^2 + (g_A^q)^2\right](q + \bar{q}),$$

$$\left[F_3^{\gamma}, F_3^{\gamma Z}, F_3^{Z}\right] = \sum_q \left[0,\, 2e_q g_A^q,\, 2g_V^q g_A^q\right](q - \bar{q}),$$

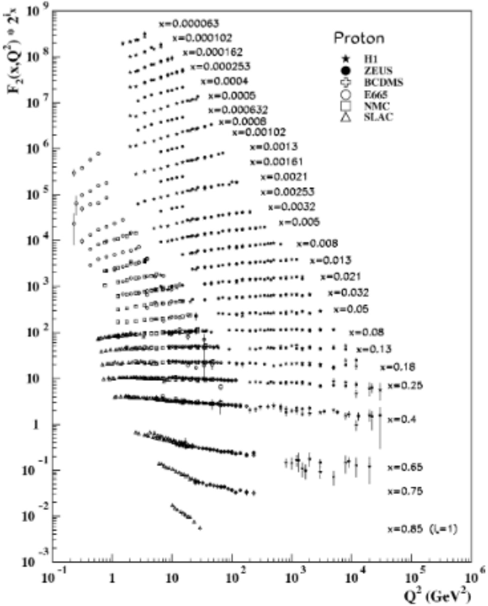
- Charged Current

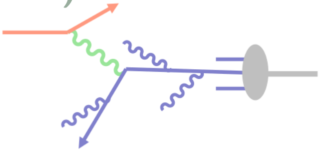
$$F_2^{W^-} = 2x\left(u + \bar{d} + \bar{s} + c \ldots\right),$$

$$F_3^{W^-} = 2\left(u - \bar{d} - \bar{s} + c \ldots\right),$$

- Callan-Gross relation: $F_2^i = 2xF_1^i$.

Scaling violation and QCD (Improved Parton Model)





- Factorization of the Structure Functions

$$F = \sum_i C_i \otimes f_i + \mathscr{O}(M^2/Q^2),$$

$$C \otimes f = \int_x^1 \frac{dy}{y} C(y) f\left(\frac{x}{y}\right).$$

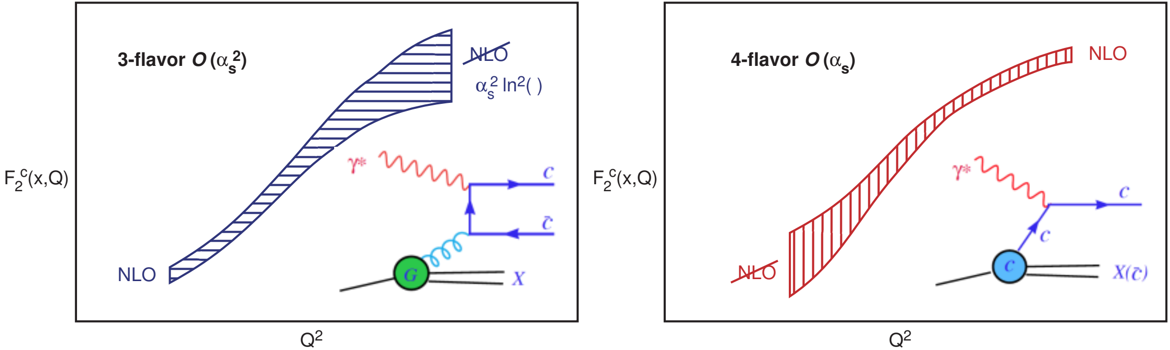
- DGLAP evolution:

$$\frac{\partial f_i}{\partial \ln \mu^2} \sim \frac{\alpha_s(\mu^2)}{2\pi} \sum_b (P_{ij} \otimes f_j),$$

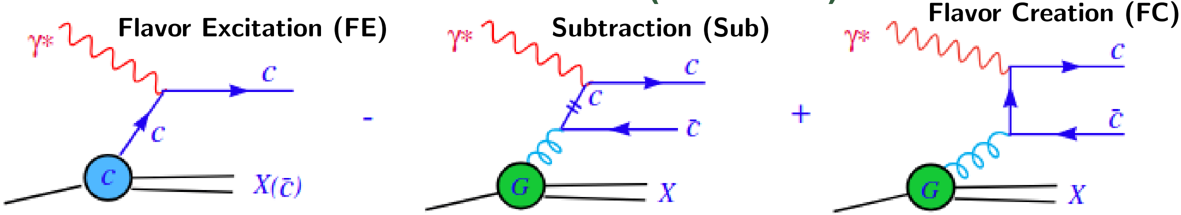
- DIS scale: $\mu^2 \sim Q^2$
- Also: $F_L = F_2 - 2xF_1 \neq 0$.

Heavy Flavor Structure Functions

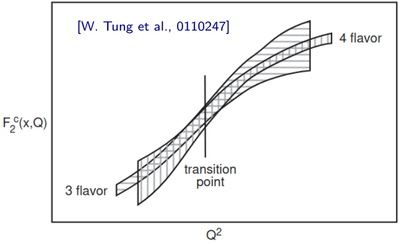
- Inclusive structure functions: $F = \sum_{i=h,l} F_i$
- Heavy-flavor SFs (e.g., $F_{h=c,b}$) can be directly measured: HERA, EMC
- Fixed-flavor-number (FFN) scheme vs Zero-mass (ZM) scheme
- Heavy-quark mass and resummation (heavy-flavor PDF)



General-mass variable-flavor-number (GM-VFN) scheme



$$F_h(x,Q) = \Bigg[\underbrace{f_h(\chi,\mu)}_{\text{FE}} - \underbrace{\alpha_s(\mu)\ln\left(\frac{\mu}{m_h}\right)\int_\chi^1 \frac{dz}{z} P_{hg}\left(\frac{\chi}{z}\right) g(z,\mu)}_{\text{Subtraction}}\Bigg]\omega^0 + \underbrace{\alpha_s(\mu)\int_\chi^1 g(z,\mu)\,\omega^1\left(\frac{\chi}{z},\frac{m_h}{Q}\right)}_{\text{FC}}$$



- Composite scheme: FFN (3f) + ZM (4f)
- The double-counting between FC and FE is subtracted out through an asymptotic term
  - when $Q \gtrsim 2m_Q$, FE≃Sub, FC remain→ 3-flavor (FFN)
  - when $Q \gg 2m_Q$, FC≃Sub, FE remain→ 4-flavor (ZM)

The ACOT scheme series

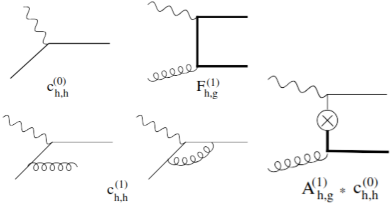
- Aivazis-Collins-Olness-Tung [PRD1994] introduce an asymptotic subtraction term
$$\text{Sub} = \underbrace{\alpha_s(\mu)\ln\left(\frac{\mu}{m_h}\right)\int_\chi^1 \frac{dz}{z} P_{hg}\left(\frac{\chi}{z}\right) g(z,\mu)}_{\tilde{f}_h(x,\mu)}\,\omega^0,$$
sharing the same Wilson coefficient as FE ω^0.
- Simplified-ACOT scheme [J. Collins PRD1998, M. Kramer et al., PRD2000] treats heavy-quark as massless in Flavor Excitation. Warning: instability in the cancellation between SB and FE around the switching point.
- The S-ACOT-χ scheme [W. Tung et al., 0110247] introduces rescaling variable $\chi = x(1+4m_Q^2/Q^2)$ to capture the mass threshold effect.
It stabilizes the perturbative convergence near the switching point by enforcing energy-momentum conservation in all scattering contributions.
- It is extended to the NNLO [M. Guzzi, 1108.5112], and adopted in the CTEQ-TEA (CT) PDF global fitting.
- The S-ACOT-m_T scheme [I. Helenius et al., 1804.03557]
- The S-ACOT-MPS [K. Xie et al., 2108.03741] scheme extends the S-ACOT-χ method to hadron-hadron collisions
- We also introduces a subtraction PDF $\tilde{f}_h(x,\mu)$ to simplify the calculation, up to NLO [K. Xie et al., 2410.03876]. The NNLO is under development.

Flavor decomposition

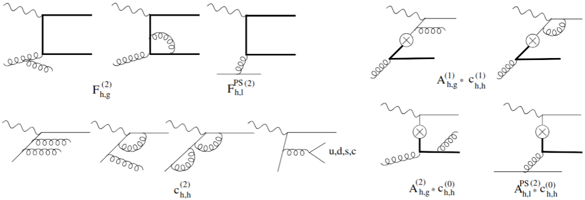
- Light singlet and non-singlet
$$\Sigma = \sum_i (q_i + \bar{q}_i),\ \ q_{\rm ns}^V = \sum_i (q_i - \bar{q}_i),\ \ q_{ij}^{\pm} = (q_i \pm \bar{q}_i) - (q_j \pm \bar{q}_j).$$
with the DGLAP equation ($L = \log \mu^2$)
$$\frac{\mathrm{d}}{\mathrm{d}L} q_{ij}^{\pm} = P_{\rm ns}^{\pm} \otimes q_{ij}^{\pm},\ \ \frac{\mathrm{d}}{\mathrm{d}L} q_{\rm ns}^V = P_{\rm ns}^V \otimes q_{\rm ns}^V$$
$$\frac{\mathrm{d}}{\mathrm{d}L} \begin{pmatrix} \Sigma \\ g \end{pmatrix} = \begin{pmatrix} P_{\Sigma\Sigma} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ g \end{pmatrix}.$$
- Flavor $SU(N_f)$ decomposition of heavy-flavor structure functions
$$F_h = \sum_i C_{h,i} \otimes f_i = C_{h,h,\mathsf{ns}} \otimes f_h^{\mathsf{ns}} + C_{h,l} \otimes \Sigma + C_{h,g} \otimes f_g$$
- Heavy-flavor non-singlet: $f_h^{\mathsf{ns}} = f_h + f_{\bar{h}}$, where h is the $(N_f + 1)$-th flavor

Subtraction up to NNLO [M. Guzzi, 1108.5112]

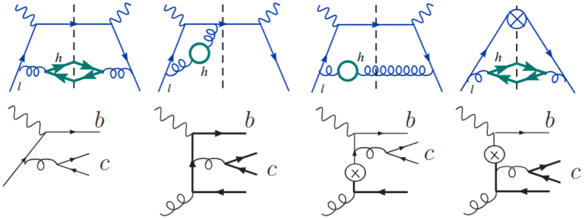
- NLO

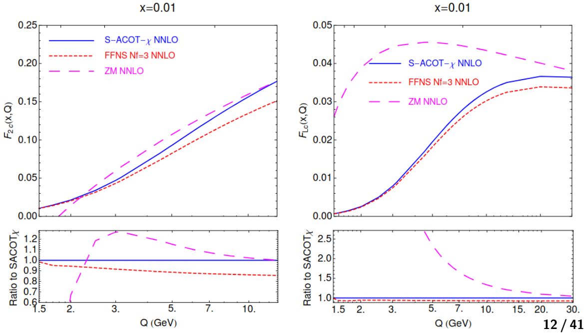


- NNLO

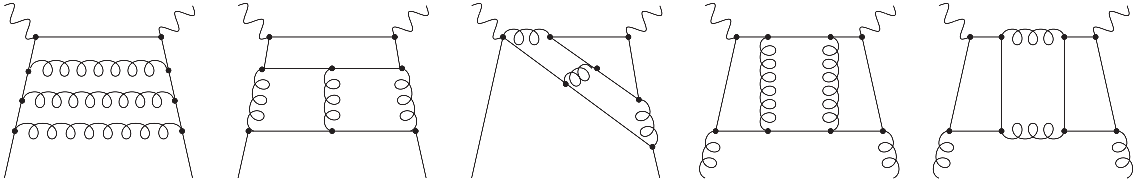


- Single and double masses





The flavor classes up to N3LO [0504242]



(a) FC_2 (b) FC_{02} (c) FC_{11} (d) FC_2^g (e) FC_{11}^g

Flavor class Flavor factor	FC_2 fl_2	FC_{11} fl_{11}	FC_{02} fl_{02}	FC_2^g fl_2^g	FC_{11}^g fl_{11}^g
Flavor structure	$\hat{Q}^2$	$\hat{Q}\,\mathrm{Tr}\,\hat{Q}$	$\bar{I}\,\mathrm{Tr}\,\hat{Q}^2$	$\mathrm{Tr}\,\hat{Q}^2$	$(\mathrm{Tr}\,\hat{Q})^2$
Non-Singlet	1	$3\langle e\rangle$	0	–	–
Singlet Σ	1	$\langle e\rangle^2/\langle e^2\rangle$	1	1	$\langle e\rangle^2/\langle e^2\rangle$

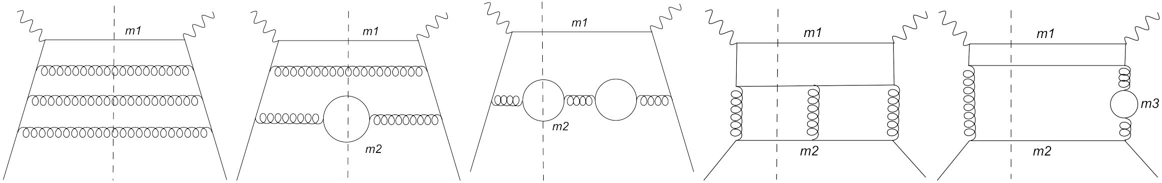
where the charge operator in the flavor basis is

$$\hat{Q} = \mathrm{diag}\left(-\tfrac{1}{3}, \tfrac{2}{3}, -\tfrac{1}{3}, \dots\right), \quad \mathrm{Tr}\,\hat{Q} = \sum_i e_i = N_f\langle e\rangle, \quad \mathrm{Tr}\,\hat{Q}^2 = \sum_i e_i^2 = N_f\langle e^2\rangle.$$

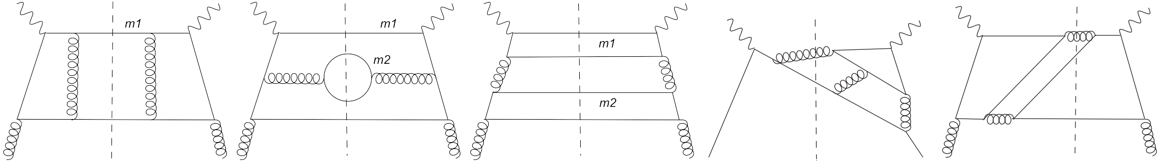
The heavy-flavor-mass dependence types [Wang, KX, Theses]

- Massless Wislon coefficients are calculated: $c_L^{(3)}$ [0411112], $c_2^{(3)}$ [0504242]

$$\sum_j^{N_f} e_j^2\, C^{\mathrm{FC}_2}_{(m_j)}\, q_j^+, \quad \sum_i^{N_{fs}} \sum_j^{N_f} e_i^2\, C^{\mathrm{FC}_{02}}_{(m_i,m_j)}\, q_j^+, \quad \sum_i^{N_{fs}} e_i^2\, C^{\mathrm{FC}_2^g}_{(m_i)}\, g, \quad \sum_j^{N_f} \sum_k^{N_{fs}^L} e_k\, e_j\, C^{\mathrm{FC}_{11}}_{(m_j,m_k)}\, q_j^+, \quad \sum_{i,k}^{N_{fs}^L} e_k\, e_i\, C^{\mathrm{FC}_{11}^g}_{(m_i,m_k)}\, g$$



(f) FC_2, T_1 (g) FC_2, T_2 (h) FC_2, T_3 (i) FC_{02}, T_1 (j) FC_{02}, T_2



(k) FC_2^g, T_1 (l) FC_2^g, T_2 (m) FC_2^g, T_3 (n) FC_{11}^g, T_1 (o) FC_{11}^g, T_2

The Intermediate-mass (IM) scheme up to N^{3}LO

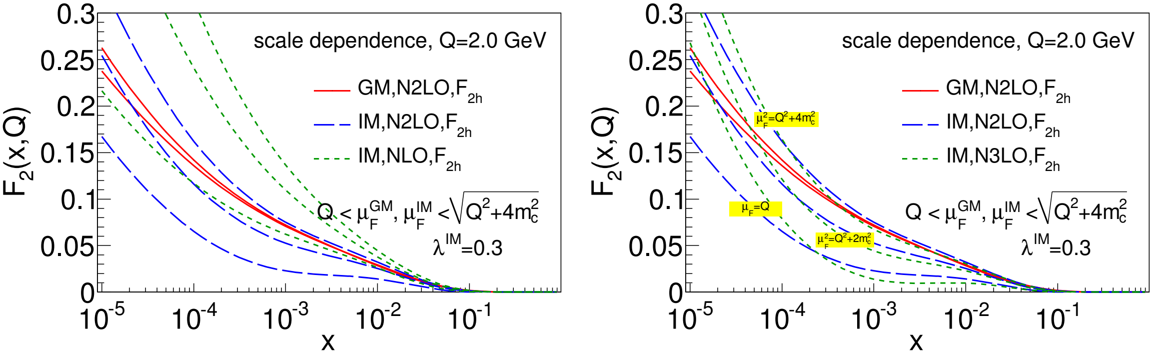
- A improved rescaling variable $\zeta(\lambda)$ to capture the mass effect in Wilson coefficients

$$x = \frac{\zeta}{1+\zeta^{\lambda}(\sum_{fs} m)^2/Q^2} \implies \zeta = \begin{cases} \chi & \lambda = 0 \\ x & \lambda = \infty \end{cases}$$

- Non-singlet $C_{\rm ns}^{(3)}$: $c^{(3)}_{\text{ns},\mathrm{FC}_2,T1} + N_f\, c^{(3)}_{\text{ns},\mathrm{FC}_2,T2} + N_f^2\, c^{(3)}_{\text{ns},\mathrm{FC}_2,T3} + f_{11}^{\text{ns}}\, N_f\, c^{(3)}_{\text{ns},\mathrm{FC}_{11}}$
- Pure-singlet $C_{\rm ps}^{(3)} = C_{\Sigma}^{(3)} - C_{\rm ns}^{(3)}$: $N_f\, c^{(3)}_{\text{ps},\mathrm{FC}_{02},T1} + N_f^2\, c^{(3)}_{\text{ps},\mathrm{FC}_{02},T2} + f_{11}^{\text{ps}}\, N_f\, c^{(3)}_{\text{ps},\mathrm{FC}_{11}}$
- Gluon $C_g^{(3)}$: $N_f\, c^{(3)}_{g,\mathrm{FC}_2^g,T1} + N_f^2\, c^{(3)}_{g,\mathrm{FC}_2^g,T2} + f_{11}^{g}\, N_f^2\, c^{(3)}_{\text{ns},\mathrm{FC}_{11}^g}$

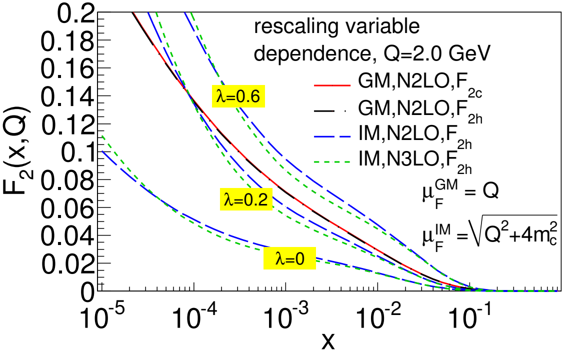
Flavor	Class	Type	$\sum_{\text{fs}} m_h$	Type	$\sum_{\text{fs}} m_h$
NS	FC_2	T_1	$2m_1$	$T_{2,3}$	$2m_1+2m_2$
	FC_{11}	–	$2m_1+2m_2$		
PS	FC_{02}	$T_{1,2}$	$2m_1+2m_2$		
	FC_{11}	–	$2m_1+2m_2$		
Gluon	FC_2^g	T_1	$2m_1$	$T_{2,3}$	$2m_1+2m_2$
	FC_{11}^g	$T_{1,2}$	$2m_1+2m_2$		

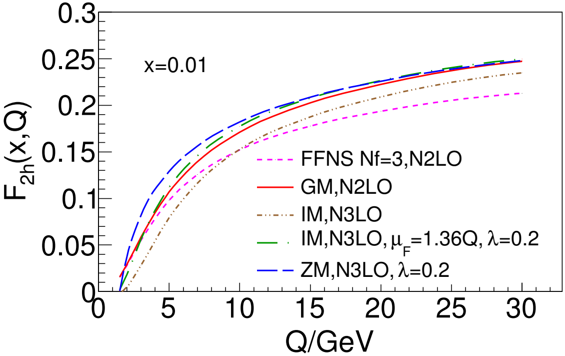
Numerical scale dependence



- The GM give a very good convergence
- The preferred scale $\mu = \sqrt{Q^2 + 2m_c^2}$

The ζ and Q dependence





- The preferred rescaling variable $\lambda = 0.2$
- The global analysis with the IM N^{3}LO in the CTEQ-TEA framework will come soon!

Other treatments in approximated N^{3}LO

- Alternative NNLO schemes: FONLL by NNPDF, and RT (Thorne) by MSHT
- MSHT: heavy-flavor Wilson coefficients [2207.04739]

$$C^{\mathrm{FF},(3)}_{h,\{q,g\}} = \begin{cases} C^{\mathrm{FF},(3)}_{h,\{q,g\},\mathrm{low}\text{-}Q^2}(x, Q^2 = m_h^2)\, e^{0.3(1-Q^2/m_h^2)} \\ + C^{\mathrm{FF},(3)}_{h,\{q,g\},\mathrm{low}\text{-}Q^2}(x, Q^2 \to \infty) \left(1 - e^{0.3(1-Q^2/m_h^2)}\right), & Q^2 \geq m_h^2 \\ C^{\mathrm{FF},(3)}_{h,\{q,g\},\mathrm{low}\text{-}Q^2}(x, Q^2), & Q^2 < m_h^2 \end{cases}$$

virtual heavy-flavor to light Wilson coefficients

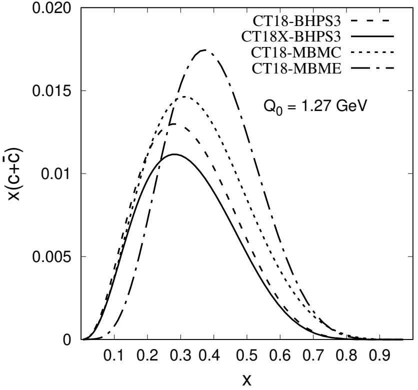
$$C^{\mathrm{FF},(3)}_{q,\{q,g\}} = \begin{cases} C^{\mathrm{FF,NS},(3)}_{q,q}(x, Q^2 \to \infty) \left(1 + e^{-0.5(Q^2/m_h^2)-3.5}\right), \\ C^{\mathrm{FF,PS},(3)}_{q,q}(x, Q^2 \to \infty) \left(1 - e^{-0.25(Q^2/m_h^2)-0.3}\right), \\ C^{\mathrm{FF},(3)}_{q,g}(x, Q^2 \to \infty) \left(1 - e^{-0.05(Q^2/m_h^2)+0.35}\right). \end{cases}$$

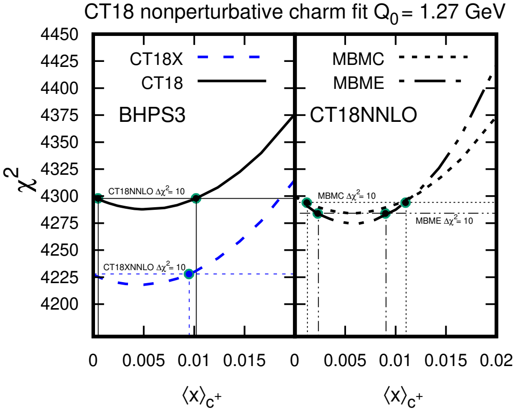
- NNPDF: threshold resummation [2402.18635]

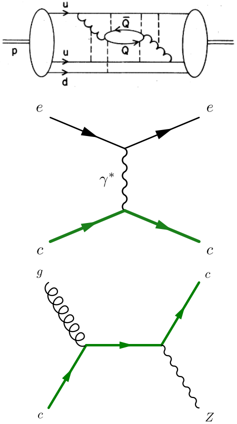
$$C^{(3)}_{i,k}(x, m_h^2/Q^2) = C^{(3),\mathrm{thr}}_{i,k}(x, m_h^2/Q^2) f_1(x) + C^{(3),\mathrm{asy}}_{i,k}(x, m_h^2/Q^2) f_2(x),$$

Intrinsic Charm (IC) puzzle

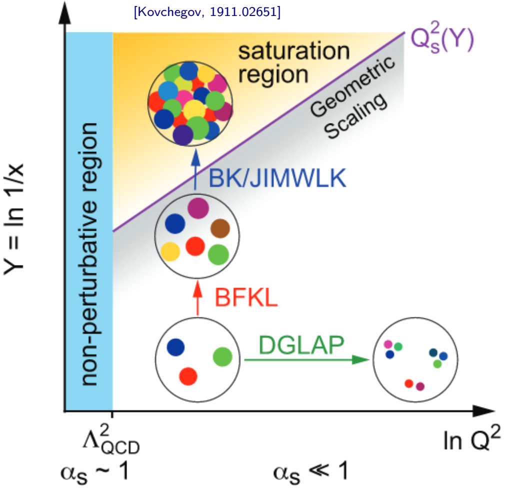
- The proton's intrinsic charm (valence-like) component: BHPS model $|uudc\bar{c}\rangle$ [Brodsky+, PLB'80, PRD'81]
- Both EMC F_2^c [CT14IC, Hou, KX+, JHEP'18] and LHCb $Z+c$ [PRL'22] data prefer an IC component
- We analyzed various uncertainties and IC models $\langle x\rangle_{\mathrm{FC}} = (4.8^{+6.3}_{-4.8})\cdot 10^{-3}$, while globally consistent with a zero IC [CT18FC, Hobbs, KX+, PLB'23], contradictory to NNPDF [Nature'22] (Unc. [Nadolsky, KX+, PRD'23])
- Precision measurement of F_2^c at EIC will help



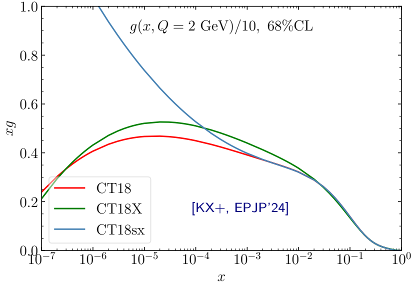


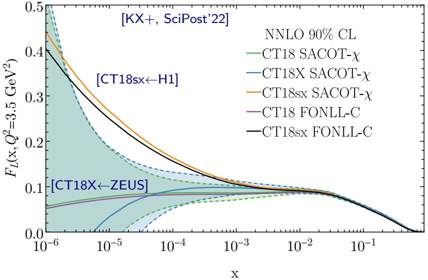


PDFs at small x



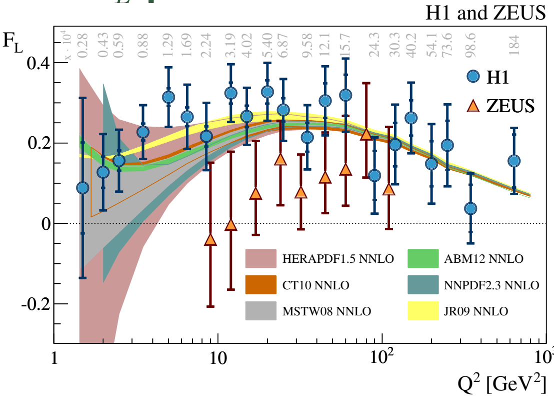
Resolve the F_L puzzle between H1 and ZEUS

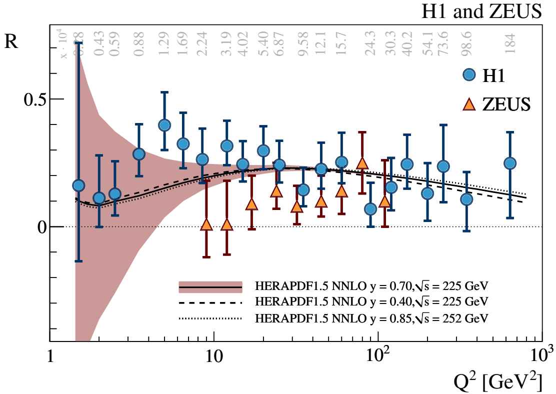




- CT18X takes a saturation model [Hou, KX+, PRD'21]
- CT18sx resum the $\ln(1/x)$ with BFKL [KX+, EPJP'24]

The F_L puzzle





[EPJC 2014]

$R = F_L/(F_2 - F_L) \approx \sigma_L/\sigma_T$ [PRD 2014]

- H1 and ZEUS do not fully agree with each other in the F_L measurement.
- H1 gives enhanced F_L, which is preferred by small-x resummation [1710.05935,1802.00064].
- ZEUS gives an opposite pull, preferred by x-scale description.
- It awaits to be resolved by the future precision measurements $\Leftarrow$ EIC

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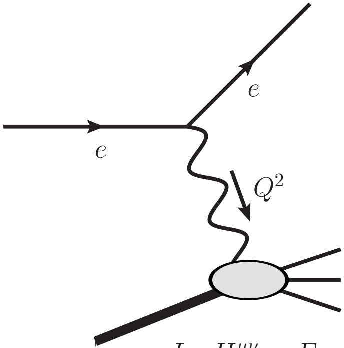
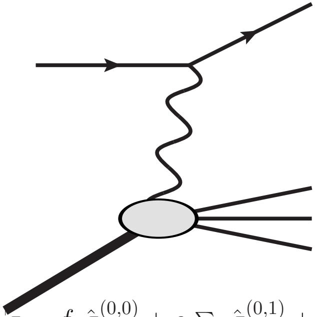
The LUXqed method [Manohar+, PRL'16, JHEP'17]



$\sigma \sim L_{\mu\nu} H^{\mu\nu} \sim F_{2,L}$

$\Leftrightarrow$

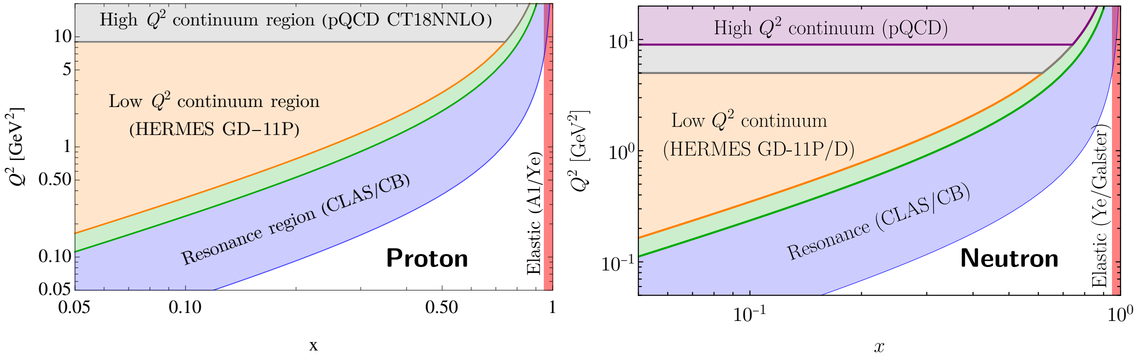


$\sigma \sim f_\gamma \hat{\sigma}^{(0,0)}_{\ell\gamma} + \alpha \sum_j \hat{\sigma}^{(0,1)}_{\ell j} + \cdots$

The master formula: Photon PDF can be determined in terms of structure functions

$$x f_\gamma(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_x^1 \frac{\mathrm{d}z}{z} \left\{ \int_{x^2 m_p^2/(1-z)}^{\mu^2/(1-z)} \frac{\mathrm{d}Q^2}{Q^2} \alpha_{\mathrm{ph}}(-Q^2) \left[\left(z p_{\gamma q}(z) + \frac{2x^2 m_p^2}{Q^2} \right) F_2\left(\frac{x}{z}, Q^2\right) - z^2 F_L\left(\frac{x}{z}, Q^2\right) \right] - \alpha(\mu^2) z^2 F_2\left(\frac{x}{z}, \mu^2\right) \right\} + \mathscr{O}(\alpha^2, \alpha\alpha_s)$$

Perturbative vs non-perturbative structure functions



- In the resonance region $W^2 = m_p^2 + Q^2(1/x - 1) < W_{\rm lo}^2 = 3\ \text{GeV}^2$, the structure functions are taken from CLAS [0301204] or Christy-Bosted [0712.3731] fits.
- In the low-Q^2 continuum region $W^2 > W_{\rm hi}^2 = 4\ \text{GeV}^2$, the HERMES GD11-P [1103.5704] fits with ALLM [PLB1991] functional form.
- In the high-Q^2 region $(Q^2 > Q^2_{\rm PDF})$, $F_{2,L}$ are determined through pQCD.
- The elastic form factors are taken from A1 [1307.6227] or Ye [1707.09063] fits of world data.

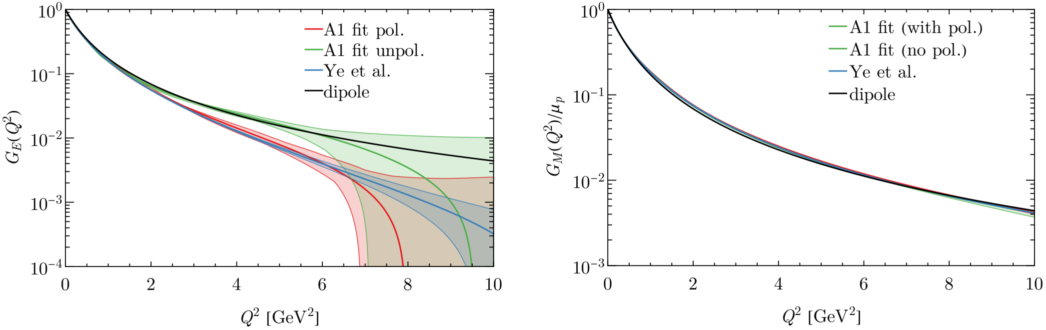
Elastic form factors

- Dipole form $G_D = 1/(1+Q^2/\Lambda^2)^2$ vs. world fits (A1 and Ye)
- The derived structure functions [9901360]

$$F_2\left(x,Q^2\right) = \frac{G_E^2 + \tau G_M^2}{1+\tau}\delta(1-x),\; F_L\left(x,Q^2\right) = \frac{\left[G_E\left(Q^2\right)\right]^2}{\tau}\delta(1-x)$$

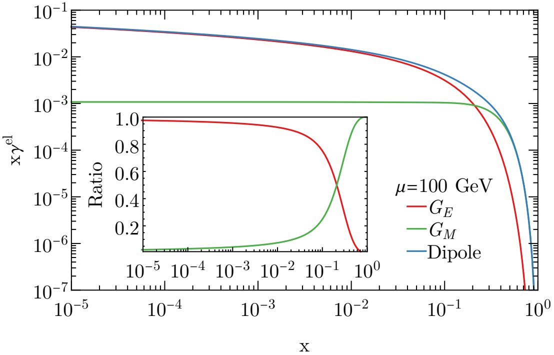
where $\tau = \frac{Q^2}{4m_\mathrm{p}^2}$ and $G_M(0) = \mu_p \approx 2.793$.

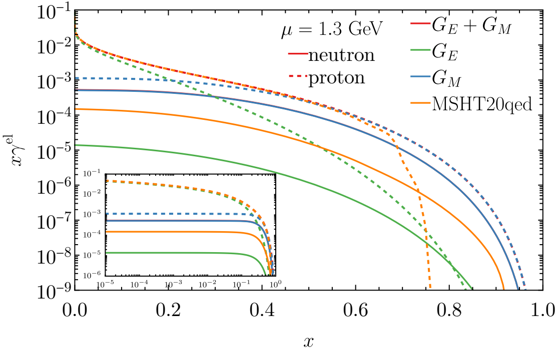
- Neutron elastic form factors can be extracted from deuterium and/or helium



Elastic photon PDF

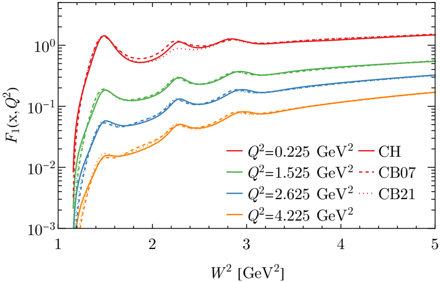
$$xf_{\gamma}^{\rm el}(x,\mu^2) = \frac{1}{2\pi\alpha(\mu^2)} \int_{\frac{x^2 m_p^2}{1-x}}^{\infty} \frac{\mathrm{d}Q^2}{Q^2}\,\alpha^2(Q^2) \left[\left(1-\frac{x^2 m_p^2}{Q^2(1-x)}\right)\frac{2(1-x)\,G_E^2(Q^2)}{1+\tau} + \left(2-2x+x^2+\frac{2x^2 m_p^2}{Q^2}\right)\frac{G_M^2(Q^2)\,\tau}{1+\tau}\right].$$

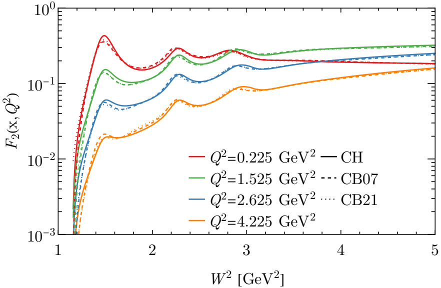


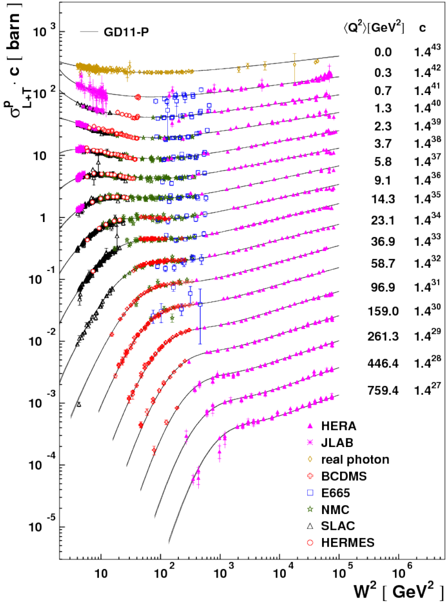


- The proton electric (magnetic) form factor dominates at low (high) x
- The neutron's elastic photon is small and mainly from magnetic form factor (zero electric charge)

Low-energy SFs: CLAS [0301204]/CB [0712.3731] and Hermes [1103.5704]

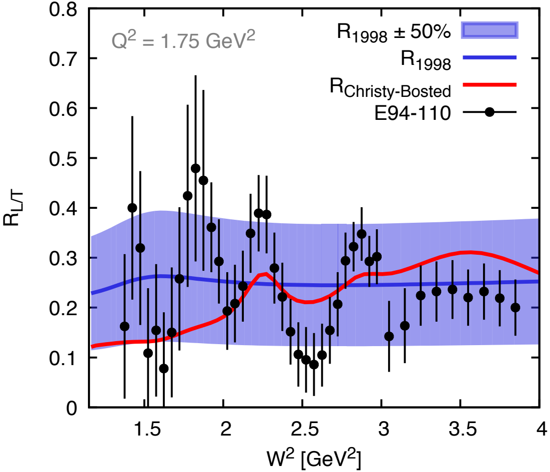


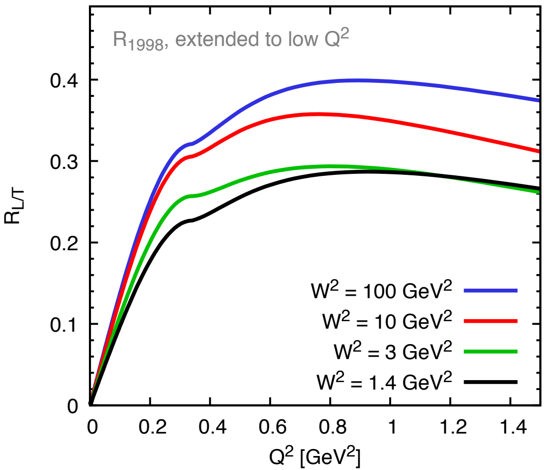




$R_{L/T} = \sigma_L/\sigma_T$ [E143, 9808028; JLab, 0410027]

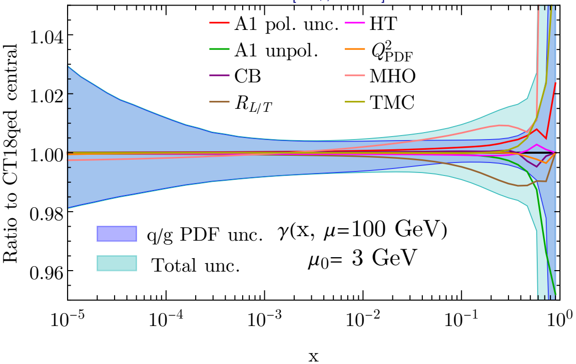
$$F_L = F_2\left(1+\frac{4m_p^2x^2}{Q^2}\right)\frac{R_{L/T}}{1+R_{L/T}},\quad F_2 = \frac{1}{4\pi^2\alpha}\frac{Q^2(1-x)}{1+4x^2m_p^2/Q^2}(\sigma_T+\sigma_L)$$



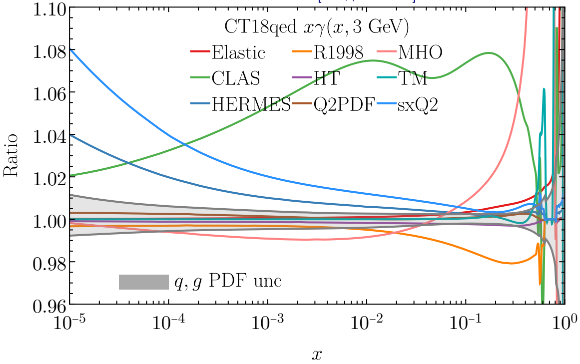


Inelastic photon: non-perturbative uncertainties

Proton [KX+, PRD'22]



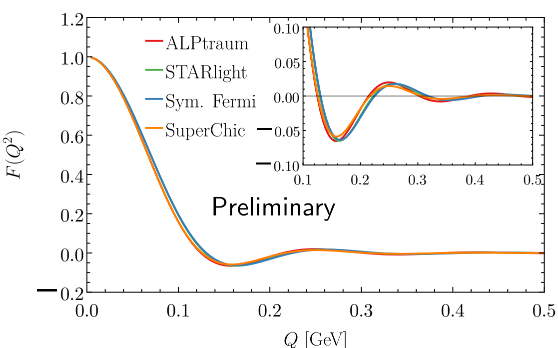
Neutron [KX+, JHEP'24]

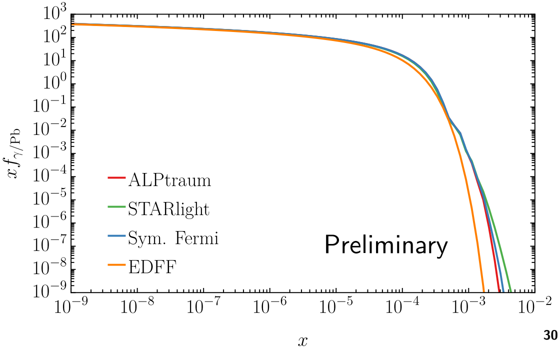


- Remaining uncertainties at large x: Higher-twist, target-mass corrections, etc [KX+, PRD'22]
- The low-Q^2 structure functions: CB/CLAS, HERMES, $R = \sigma_L/\sigma_T$
- The neutron's photon PDF suffers for a large uncertainty, due to the imprecise of SF extraction, and nuclear corrections [KX+, JHEP'24]
- Can be generated to the heavy-ion's photon.

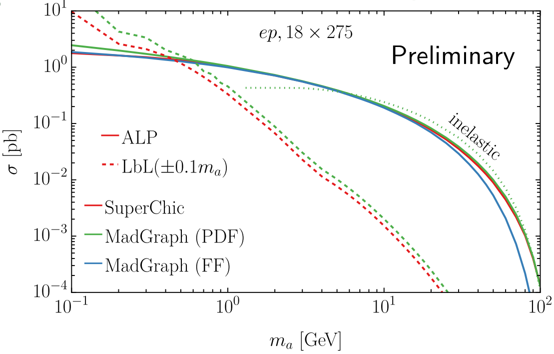
Heavy-ion (A,Z) coherent scattering

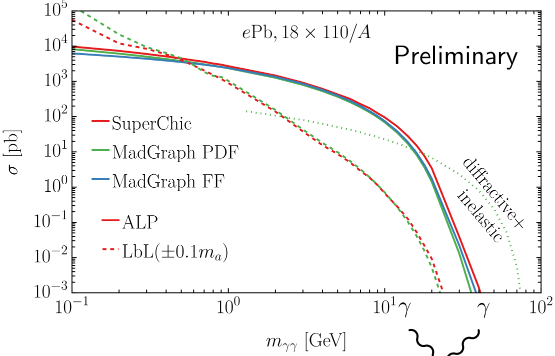
- Electric and magnetic form factors
$$G_E \sim ZF(Q^2),\ G_M \sim \mu_N F_N(Q^2) \approx 0\ ?$$
- Woods-Saxon (SuperChic)
$$F(Q^2 = |\vec{q}|^2) = \int \mathrm{d}^3 r e^{i\vec{q}\cdot\vec{r}} \rho_p(r),\ \rho_p(r) = \frac{\rho_0}{1+\exp[(r-R)/d]}$$
- Helm (ALPtraum) or STARlight or Symmetrized Fermi distribution
$$F(Q^2) = \frac{3j_1(QR_1)}{QR_1}\exp\left[-\frac{(Qs)^2}{2}\right],\ F(Q^2) = \frac{4\pi\rho_0}{ZQ^3}[\sin(QR_A) - QR_A\cos(QR_A)]\frac{1}{1+a^2Q^2}.$$

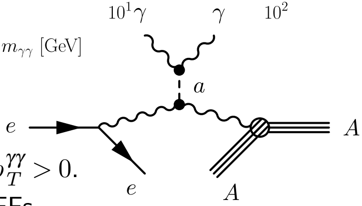




Axion-like particle or light-by-light scattering







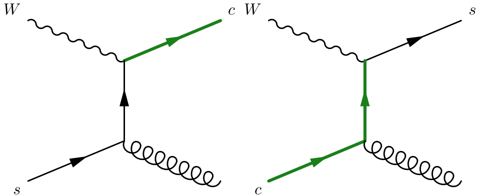
- The photon PDF corresponds to the collinear approximation, $p_T^{\gamma\gamma} \sim 0$
- It applies to the inclusive cross section
- It will fails in the off-beam-axis kinematics with recoiling boost, e.g. $p_T^{\gamma\gamma} > 0$.
- Recoiling boost requires a N-body $(N \geq 3)$ phase space, with SFs or FFs.
- A dedicated event generator is under development: elastic (diffractive and coherent), inelastic

Outline

1 Structure Functions at the Precision Frontier
2 Photon content of nucleon and nuclei
3 Charged-current structure functions

The ACOT scheme for the charged-current DIS [Gao+, 2107.00460]

- The LO Structure functions
$$F_2^{W^-} = 2x\left(u + \bar{d} + \bar{s} + c + \ldots\right),$$
$$F_3^{W^-} = 2\left(u - \bar{d} - \bar{s} + c + \ldots\right),$$
- At HO
$$F = \sum_X C_{X,j} \otimes f_j = F_l + F_Q$$

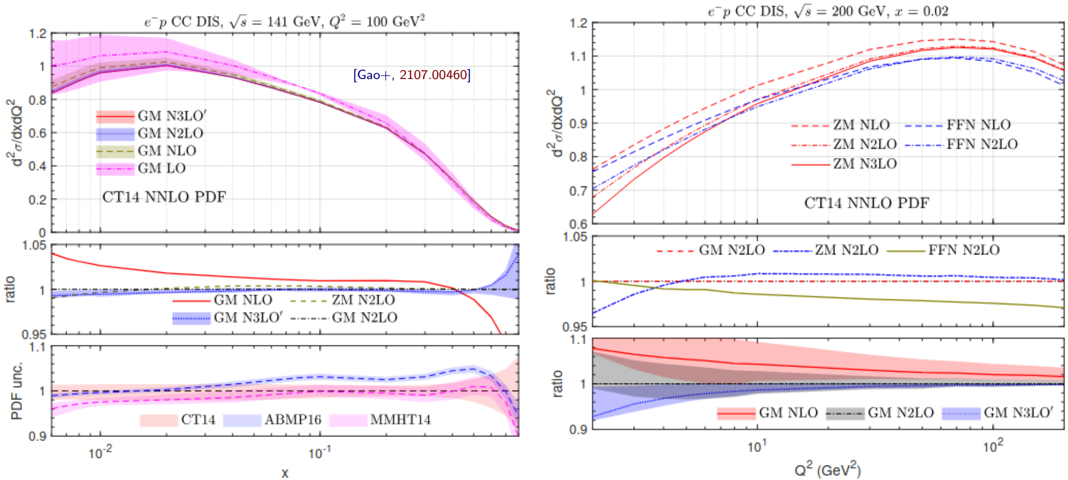


(p) Flavor Creation (q) Flavor Excitation

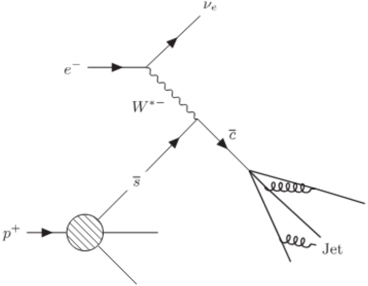
- We expand the Wilson coefficients
$$C = C^{(0)} + \alpha_s C^{(1)} + \alpha_s^2 C^{(2)} + \mathscr{O}(\alpha_s^2)$$
- The LO coefficients
$$C_{l,l}^{(0)} = \delta(1-z), \quad C_{h,h}^{(0)} = \delta(1-\chi),$$
$$C_{h,l}^{(0)} = \delta(1-\chi), \quad \chi \equiv (1 + m_c^2/Q^2)z,$$
- The NLO one
$$C_{l,l}^{(1)} = c_{l,l}^{(1)}(z), \quad C_{l,g}^{(1)} = c_{l,g}^{(1)}(z), \quad C_{l,h}^{(1)} = c_{l,l}^{(1)}(\chi),$$
$$C_{h,l}^{(1)} = H_l^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{ll}^{(1)},$$
$$C_{h,g}^{(1)} = H_g^{(1)}(z) - C_{h,l}^{(0)} \otimes A_{lg}^{(1)} - C_{h,h}^{(0)} \otimes A_{hg}^{(1)},$$
- The NNLO ones
$$C_{h,h}^{(2)} = c_{h,h}^{(2)}(\chi), \qquad C_{h,l}^{(2)} = H_l^{(2)}(z) - \Delta C_{h,l}^{(2)},$$
$$C_{h,g}^{(2)} = H_g^{(2)}(z) - \Delta C_{h,g}^{(2)}, \qquad C_{l,g}^{(2)} = c_{l,g}^{(2)}(z),$$
$$C_{l,h}^{(2)} = c_{l,h}^{(2)}(\chi), \qquad C_{l,l}^{(2)} = c_{l,l}^{(2)}(z) + \tilde{C}_{l,l}^{(\mathrm{NS},2)}(z),$$

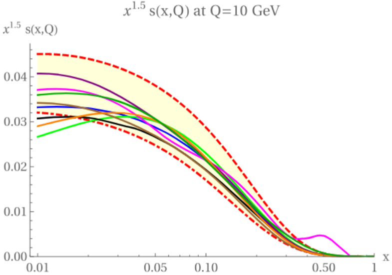
Reduced cross sections

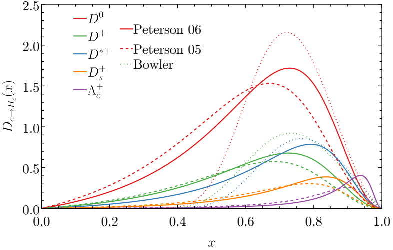
$$\frac{\mathrm{d}^2\sigma^{CC}}{\mathrm{d}x\,\mathrm{d}Q^2} = \frac{G_F^2}{4\pi x\left(1+Q^2/M_{W,Z}^2\right)^2}\left[Y_+F_2 - y^2F_L \pm Y_-xF_3\right],$$

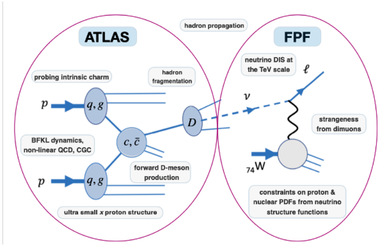


Charged-current charm production and fragmentation







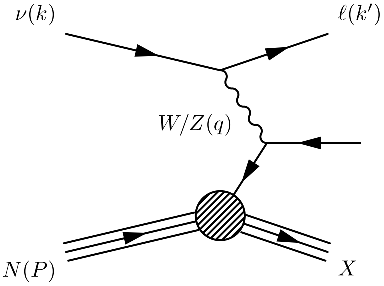


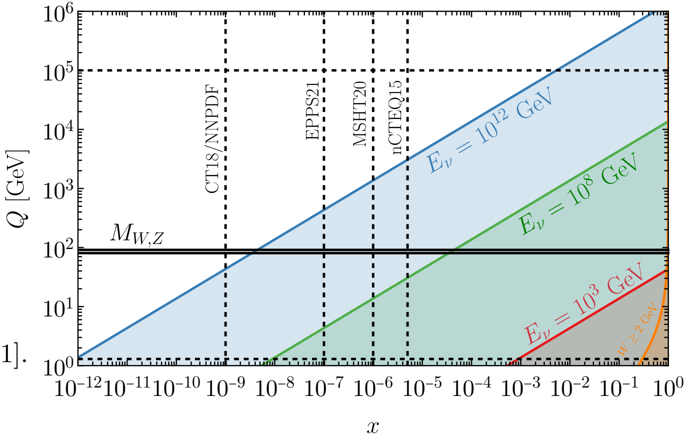
- Besides the neutral-current F_2^c (probe IC), the charge-current $F_2^{c,W}$ can probe the strangeness [Hobbs+, PRD'21]
- HERA data is very imprecise, which awaits to be improved by the EIC
- Also useful for fragmentation function, synergy with many heavy-flavor measurement, such as LHCb [KX+, SciPost'22]
- Important for the neutrino source determination at the Forward Physics Facility [KX+, Phys. Rept.'22, JPG'23]

Neutrino scattering

Cross section can be obtained from $F^{W/Z}$

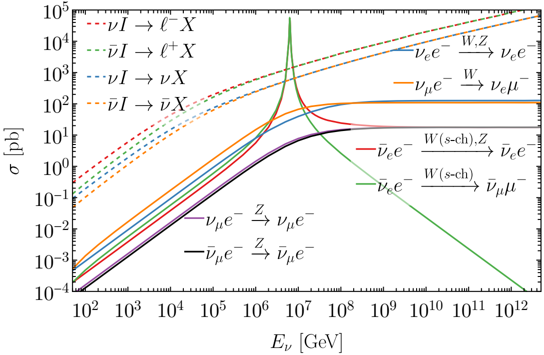
$$\sigma = \int \mathrm{d}x \mathrm{d}Q^2 \frac{G_F^2}{4\pi x \left(1 + Q^2/M_{W,Z}^2\right)^2} \left[Y_+ F_2 - y^2 F_L \pm Y_- x F_3\right]$$

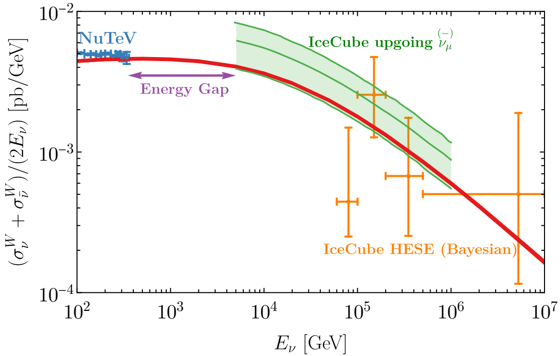


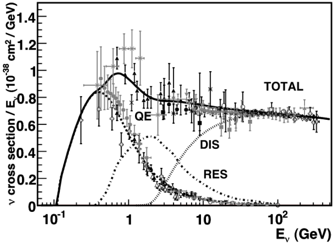


- Kinematic limits
  $Q^2 \in [Q^2_{\min}, 2ME_\nu]$, $x \in [Q^2/(2ME\nu), 1]$.
- We take extrapolation for $Q^2 < Q^2_{\min}$

Neutrino cross sections

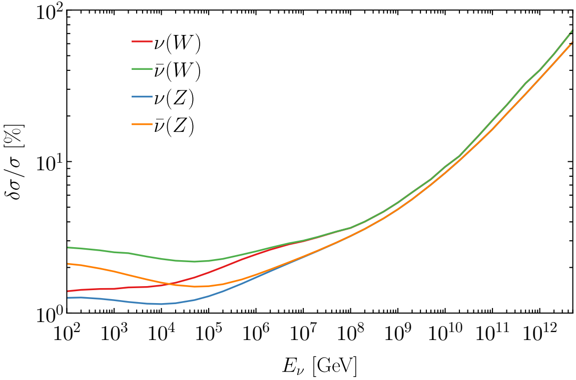


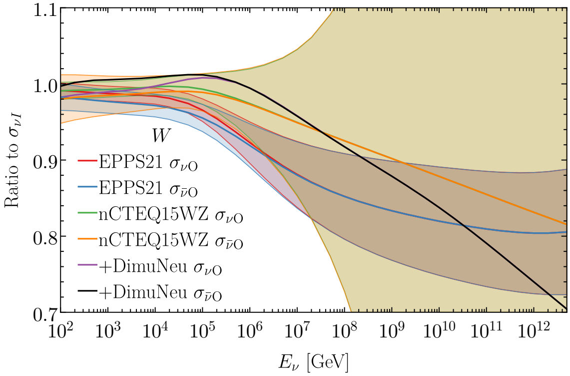




- High-energy neutrinos mostly interact with nucleon through DIS.
- High-energy neutrinos can be measured at colliders (NuTeV, FPF) and telescopes (IceCube)
- We have performed a state-of-the-art calculation (N^3LO) for the high-energy neutrino-nucleon cross sections [KX et al., 2303.13607]

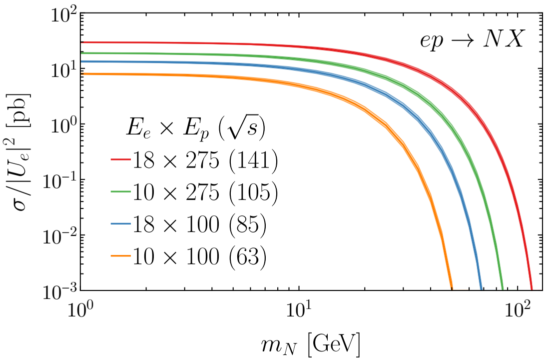
Neutrino's cross section uncertainties [KX et al., 2303.13607]

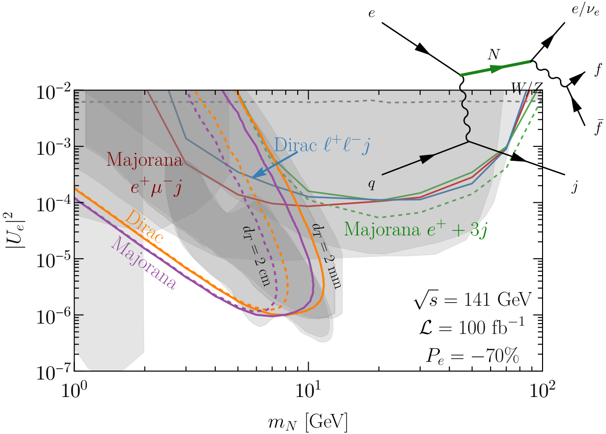




- We have investigated the flavor number up to $N_f = 6$ and heavy-quark mass effect
- The small-x is resummed up to next-to-leading logarithms
- The proton PDF uncertainty is under control
- A sizable uncertainty comes from the nuclear corrections, where the EIC can help

Heavy neutral lepton (HNL)

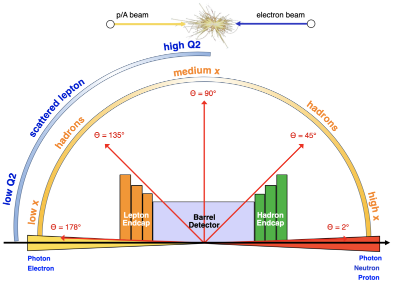


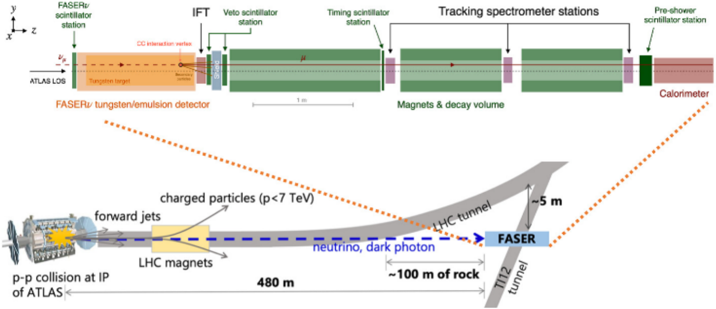


- The HNL can be produced sizeably at the EIC [KX+, 2210.09287]
- Combining the prompt decay and displaced vertex, new parameter space can be explored.
- Motivation for the 2nd detector for muon detection

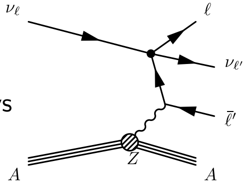
Neutrino detectors for the EIC?

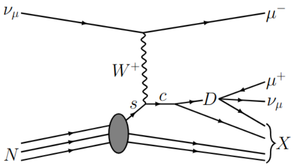
- Neutrinos from charged-current DIS $ep \to \nu_e X$
- Neutrinos from the hadron decays: FASER-like





- Neutrino trident and dimuon events
- Long-lived particles from hadron decays





Summary

- The general-mass variable-flavor-number scheme for deep inelastic scattering is developed up to the (approximated) N^3LO, with the CT global analysis incoming.
- With the LUXqed method, photon PDF of nucleon and nuclei can be precise determined through the structure functions.
- Charged-current structure functions with fully mass dependence up to NNLO, while approximated at N^3LO, with applications to neutrino scattering.

Mass schemes

Factorization schemes	Mass dependence in the FC terms	Mass dependence of the FE and subtraction terms	Introduce heavy-quark PDFs at large Q
FFN	Exact	N/A	no
ZM	None	None	yes
IM	Approximate	Approximate	yes
GM	Exact	Approximate	yes