Joint Factorization of QCD and QED Radiation in Lepton-Hadron Scattering

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CFNS Workshop: New Opportunities for Beyond-the-Standard Model Searches at the EIC
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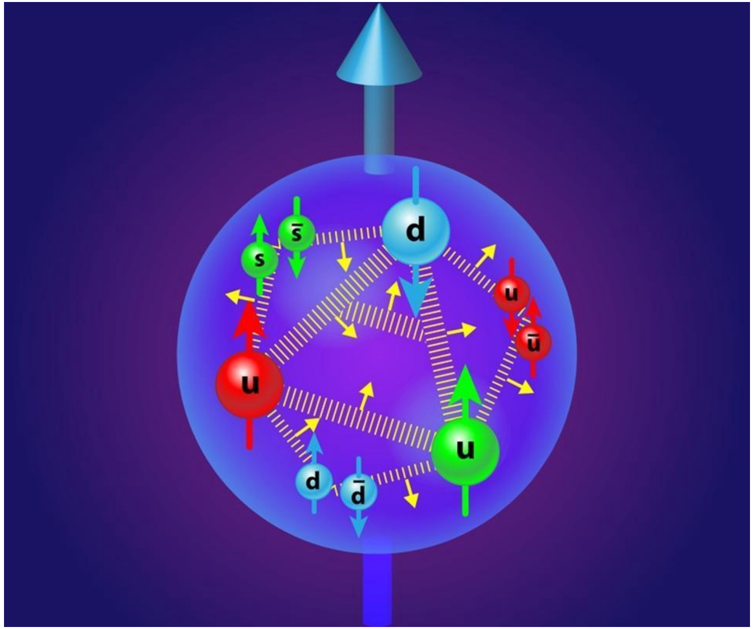
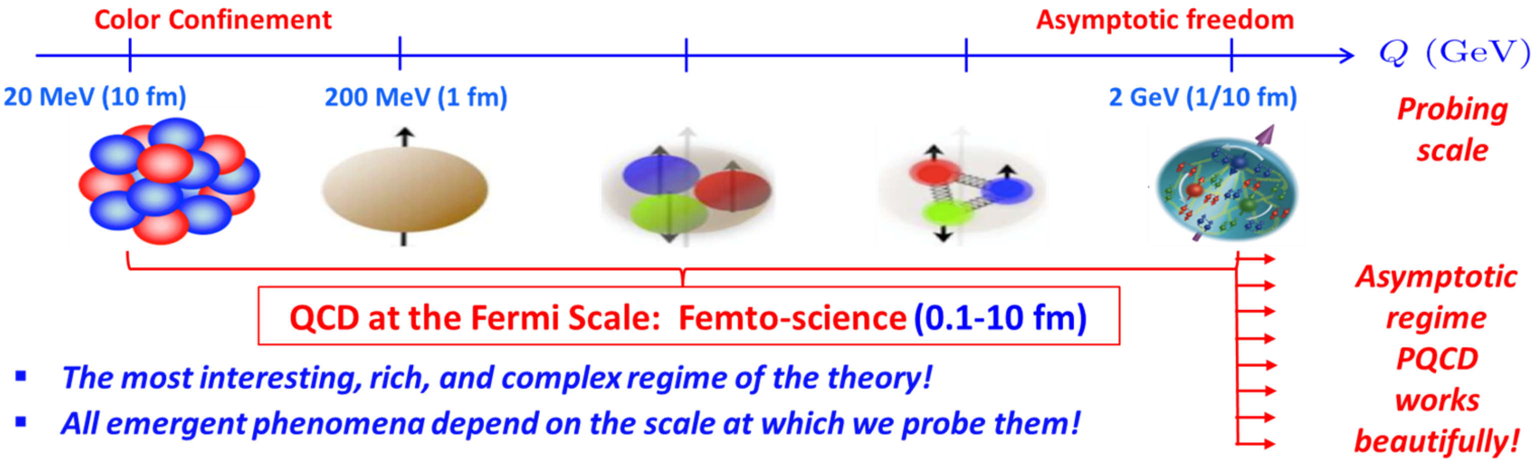
Outline

- **Motivation**
- Deep Inelastic Scattering
  - Calculations
- Semi-Inclusive Deep Inelastic Scattering
  - Standard Approach
  - Puzzles from Standard Approach
  - Cross Section Comparison



Hadronic Structure

- Structure is an emergent property of QCD

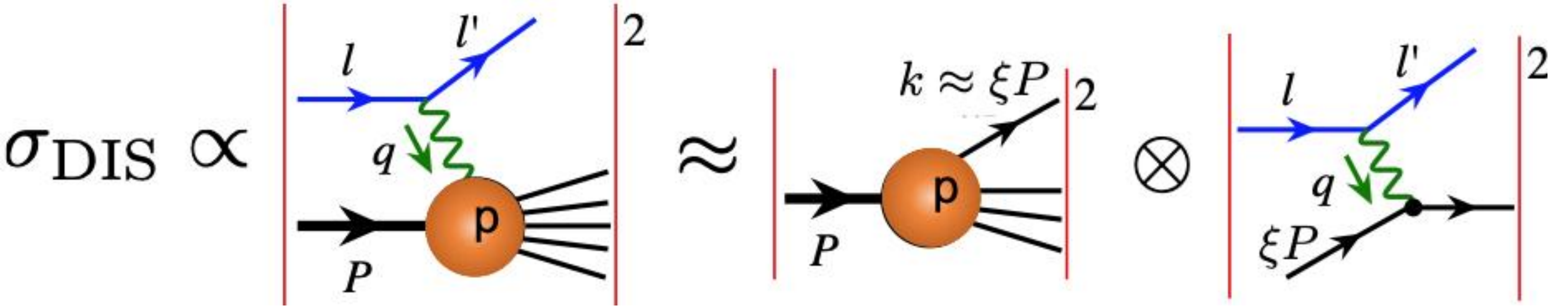


- Identified hadronic observables are non-perturbative QCD

Theoretical Tools

- Perturbative QCD Factorization
  - Approximation at Feynman diagram level
- Effective field theory (EFT)
  - Approximation at the Lagrangian level
- Lattice QCD
  - Approximation for finite lattice spacing
- Other approaches
  - Light-cone perturbation theory, constituent quark models, etc

Perturbative QCD Factorization



$$E' \frac{d\sigma_{ep \to e'X}}{d^3 l'} \approx \sum_i \int d\xi \, f_{i/p}(\xi) \, E' \frac{d\hat{\sigma}_{ei \to e'X}}{d^3 l'} = \sum_i e_i^2 \left\{ \frac{2\alpha^2}{Q^2 s} \left[\frac{1 + (1-y)^2}{y^2} \right] \right\} f_{i/p}(x)$$

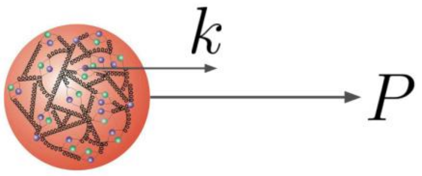
- Separates process into non-perturbative QCD effects (PDFs) and perturbative hard scattering process

Parton Distribution Functions

- Parton Distribution Functions (PDFs): describe probability of finding parton with given momentum fraction inside hadron

$$f_i(\xi) = \int \frac{\mathrm{d}w^-}{4\pi} e^{-i\xi p^+ w^-} \langle N|\bar{\psi}_i(0, w^-, \mathbf{0}_\mathrm{T})\gamma^+\psi_i(0)|N\rangle$$

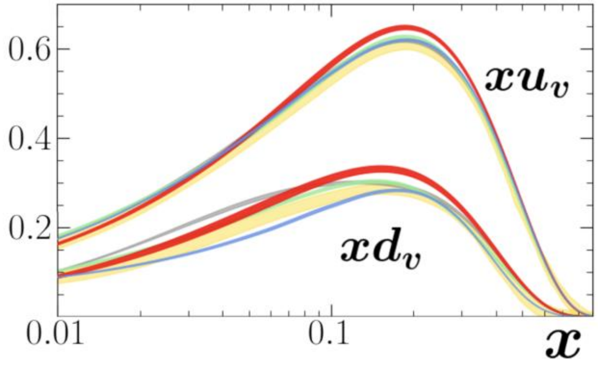
$$\xi = \frac{k^+}{P^+}$$



- In the free field approximation:

$$\psi_i(x) = \sum_{k,\alpha} b_{k,\alpha}(x^+) u_{k,\alpha} e^{-ik^+x^- + ik_\mathrm{T}\cdot x_\mathrm{T}} + d^\dagger_{k,\alpha}(x^+) u_{k,-\alpha} e^{ik^+x^- - ik_\mathrm{T}\cdot x_\mathrm{T}}$$

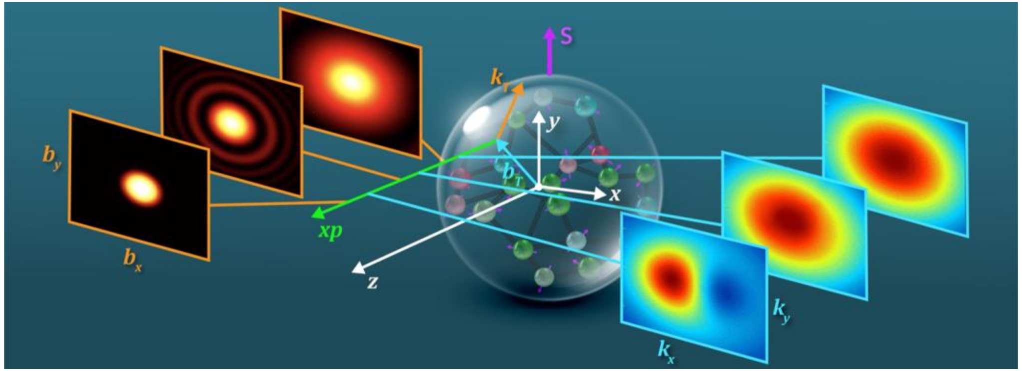
$$f_i(\xi) \sim \sum_\alpha \int \mathrm{d}^2k_\mathrm{T} \langle N| \underbrace{b^\dagger_{k,\alpha} b_{k,\alpha}(\xi p^+, k_\mathrm{T}, \alpha)}_{\text{number operator}} |N\rangle$$



PDFs from JAM Collaboration

3D Structure of the Proton

- Transverse Momentum Dependent PDFs: Intrinsic transverse motion of parton considered $f(\xi, k_{\mathrm{T}})$
- Generalized Parton Distribution Functions: Position space distribution compared to center
- Need two-scale observables



Predictive Power of QCD Factorization

- Non-perturbative hadron structure is universal with calculable matching coefficients:
  - lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{EXP}_{l+P\to l+X} = C_{l+k\to l+X} \otimes \mathrm{PDF}_P + O(Q_s^2/Q^2)$$

$$\sigma^{EXP}_{l+P\to l+H+X} = C_{l+k\to l+k+X} \otimes \mathrm{PDF}_P \otimes \mathrm{FF}_H + O(Q_s^2/Q^2)$$

  - hadron-hadron reactions (LHC)

$$\sigma^{EXP}_{P+P\to l+\bar{l}+X} = C_{k+k\to l+\bar{l}+X} \otimes \mathrm{PDF}_P \otimes \mathrm{PDF}_P + O(Q_s^2/Q^2)$$

  - lepton-lepton reactions (Belle)

$$\sigma^{EXP}_{l+\bar{l}\to H+X} = C_{l+\bar{l}\to k+X} \otimes \mathrm{FF}_H + O(Q_s^2/Q^2)$$

- Combine theory, experiment, and phenomenology to get hadron structure
  - Factorization provides good observables
  - Measurements provide reliable data
  - Global analysis extracts the universal structure information

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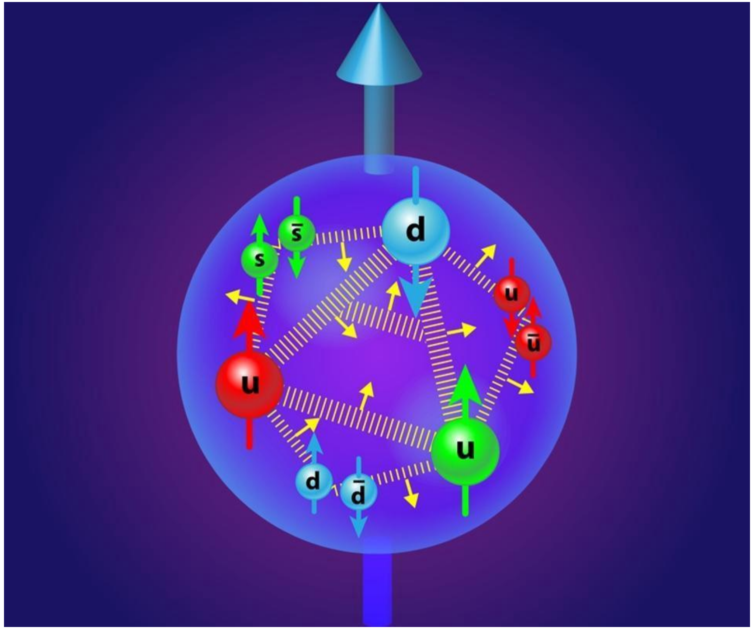
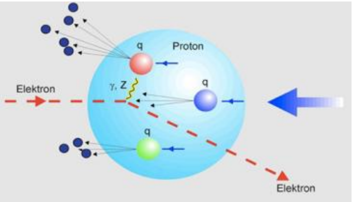
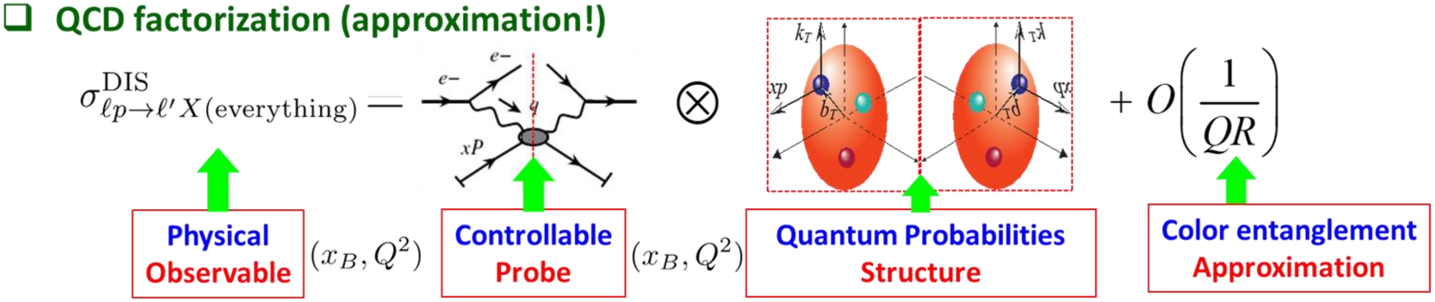


Lepton-Hadron Deep Inelastic Scattering



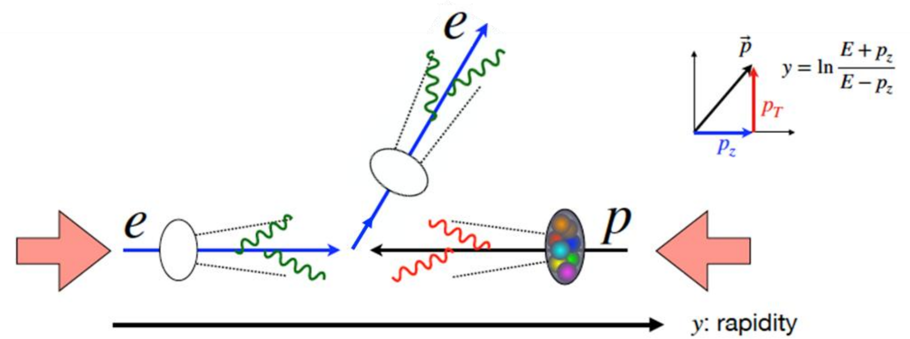
- SLAC experiments led to discovery of quarks and development of QCD
- Best clean probe for extracting hadron structure (JLab and EIC)

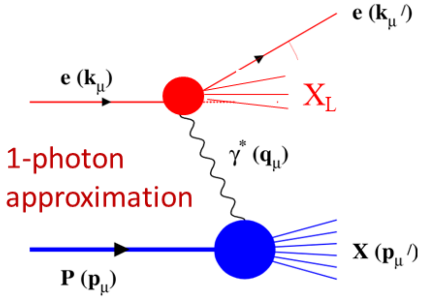


- Localized probe ($\frac{1}{Q^2} \ll 1$ fm) and two variables (Q^2 and x_B)

Induced QCD and QED Radiation

- 1 photon approximation and radiative corrections used to handle contributions

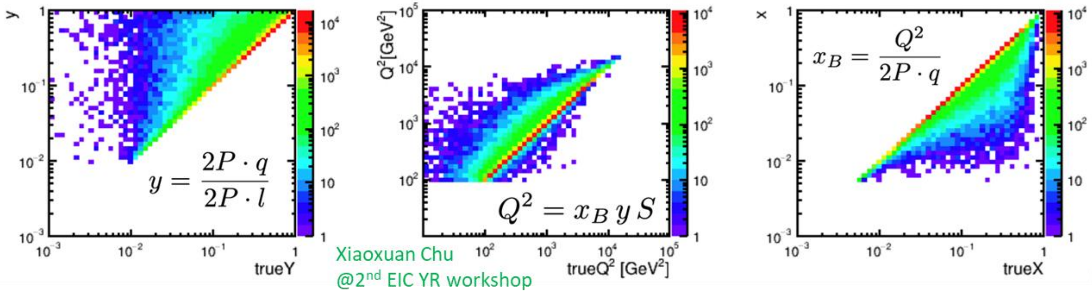




$$(x_B, Q^2) \rightarrow (\hat{x}_B, \widehat{Q}^2)$$

- Monte Carlo programs for the radiative corrections with “cutoff” masses to keep exchanged photon virtual

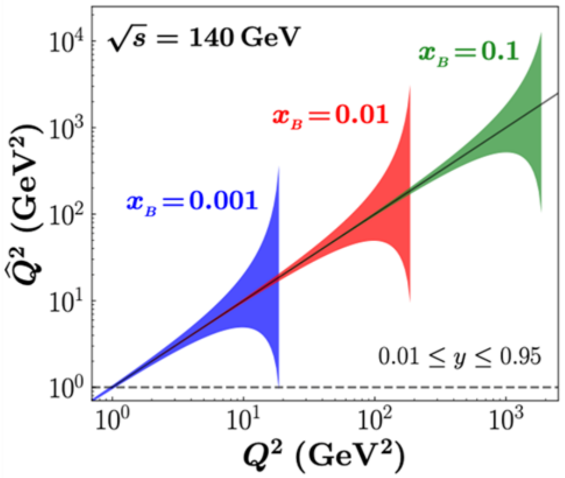
Broadened Kinematics from QED Radiation



$$x_B \to \hat{x}_B \in [x_B, 1]$$

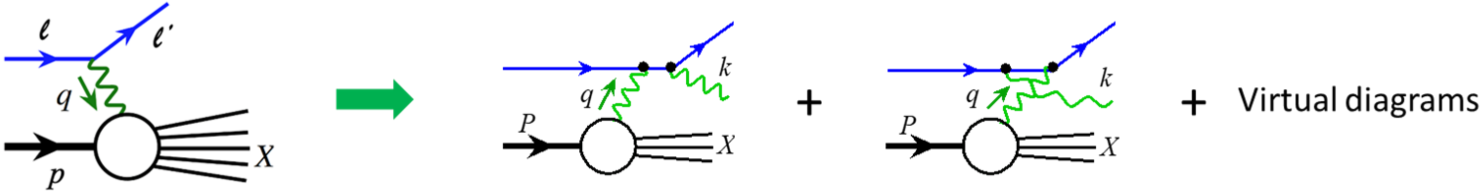
$$\widehat{Q}^2_{\text{min}} = Q^2\,\frac{(1-y)}{(1-x_B\,y)}$$

$$\widehat{Q}^2_{\text{max}} = Q^2\,\frac{1}{(1-y+x_B\,y)}$$

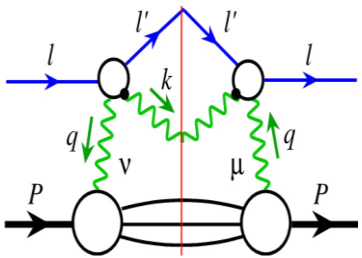


Parameter Dependence of Traditional RC Approach

- Leading Order RC:



- Cut Diagram Notation:



$$E' \frac{d\sigma^{\mathrm{RC}}_{eh\to eX}}{d^3\ell'} \propto \int d^4q \left[W^{\mu\nu}(P,q) \frac{1}{q^2+i\epsilon} L^{(1)}_{\mu\nu}(\ell,\ell',q) \frac{1}{q^2-i\epsilon} \right]$$

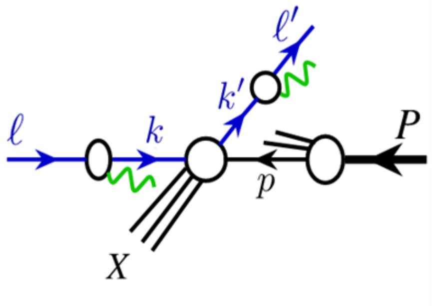
$$\to \infty$$

- Pinch of the pole if phase space unrestricted

Joint QCD and QED Factorization for DIS

Liu, Melnitchouk, Qiu, Sato, Phys.Rev.D 104 (2021) 094033
JHEP 11 (2021) 157
Cammarota, Qiu, Watanabe, Zhang [2408.08377]

- No “1-photon” approximation necessary



$$E_{k'}\frac{d\sigma_{kP\to k'X}}{d^3k'} = \frac{1}{2s}\sum_{i,j,a}\int_{\zeta_{\min}}^1 \frac{d\zeta}{\zeta^2}\int_{\xi_{\min}}^1 \frac{d\xi}{\xi} D_{e/j}(\zeta,\mu^2)\, f_{i/e}(\xi,\mu^2) \times \int_{x_{\min}}^1 \frac{dx}{x} f_{a/N}(x,\mu^2)\, \widehat{H}_{ia\to jX}(\xi k, xP, k'/\zeta,\mu^2) + \cdots$$

$$x_{\min} = \frac{\xi\, x_B\, y}{\xi\zeta + y - 1}, \quad \xi_{\min} = \frac{1-y}{\zeta - x_B y}, \quad \zeta_{\min} = 1-(1-x_B)y,$$

- Hard Part counts in powers of both $\alpha^m\alpha_s^n$

$$\widehat{H}^{(2,0)}_{eq\to eX} = (2\hat{s})\left[E_{k'}\frac{d\sigma^{(\mathrm{LO})}_{eq\to eq}}{d^3k'}\right] = e_q^2\,(4\alpha_{em}^2)\left[\frac{\hat{s}^2+\hat{u}^2}{\hat{t}^2}\right]\delta\left(\hat{s}+\hat{t}+\hat{u}\right)$$

$$= e_q^2\,(4\alpha_{em}^2)\,\frac{x^2\zeta\left[(\zeta\xi s)^2+u^2\right]}{(\xi t)^2\,(\zeta\xi s+u)}\,\delta(x-x_{\min})$$

$$s = (\ell+P)^2, \quad u = (\ell'-P)^2 = (y-1)s, \quad t = (\ell-\ell')^2 = -Q^2$$

Recovering the Born Expression

- Resumming collinear radiation recovers 1-photon approximation

$$\frac{d^2\sigma_{\ell P\to\ell' X}}{dx_B dy} \approx \int_{\zeta_{\min}}^{1} \frac{d\zeta}{\zeta^2} \int_{\xi_{\min}}^{1} d\xi\, D_{e/e}(\zeta,\mu^2)\, f_{e/e}(\xi,\mu^2) \left[\frac{Q^2}{x_B}\frac{\hat{x}_B}{\widehat{Q}^2}\right] \times \frac{4\pi\alpha^2}{\hat{x}_B\,\hat{y}\,\widehat{Q}^2}\left[\hat{x}_B\hat{y}^2 F_1(\hat{x}_B,\widehat{Q}^2) + \left(1-\hat{y}-\frac{1}{4}\hat{y}^2\hat{\gamma}^2\right)F_2(\hat{x}_B,\widehat{Q}^2)\right]$$

$$\frac{d^2\sigma_{kP\to k' X}}{d\hat{x}_B d\hat{y}}$$

- When LDF and LFF are delta functions, recover the Born cross section

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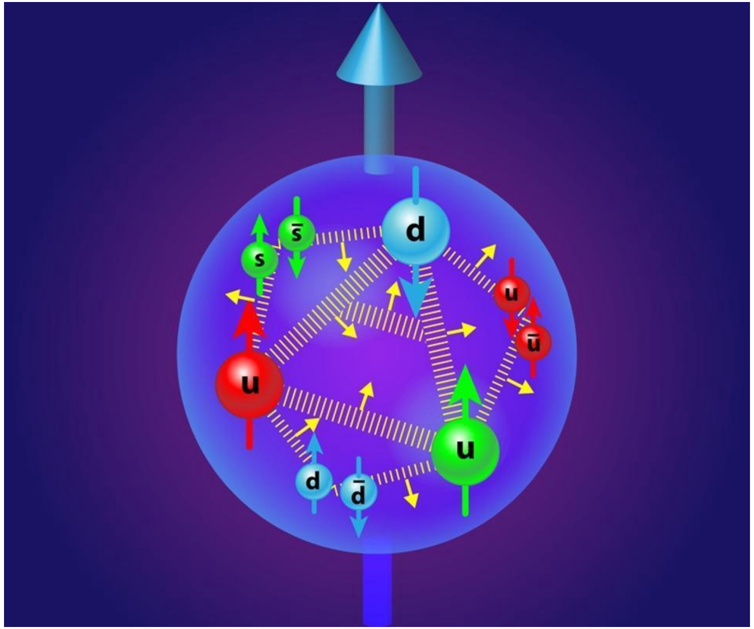
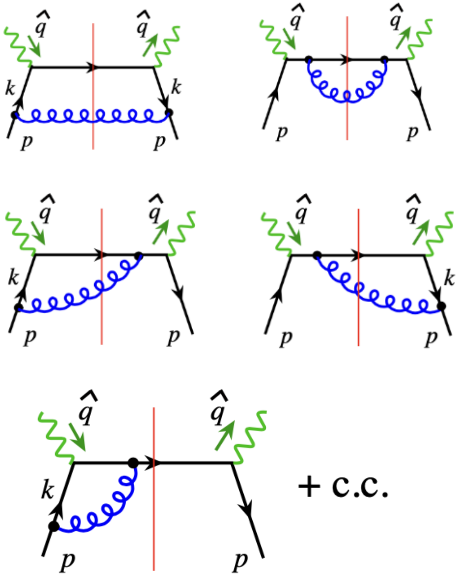


Calculation of NLO QCD Contributions

Cammarota, Qiu, Watanbe, Zhang [2408.08377] and [2505.23487]

$$
\begin{aligned}
\widehat{H}^{(2,1)}_{eq\to eX}(\hat{s},\hat{t},\hat{u}) = {} & e_q^2(4\alpha_{em}^2)\left(\frac{\alpha_s}{2\pi}\right)\frac{1}{\hat{Q}^2} \\
& \times\left\{\frac{1+(1-\hat{y})^2}{\hat{y}^2}\left[P^{(0,1)}_{q/q}(\hat{x}_B)\ln\left[\frac{\hat{Q}^2}{\mu^2}\right]\right.\right. \\
& + C_F\left((1+\hat{x}_B^2)\left[\frac{\ln(1-\hat{x}_B)}{1-\hat{x}_B}\right]_+ - \frac{3}{2}\left[\frac{1}{1-\hat{x}_B}\right]_+\right. \\
& \left.\left. - \frac{1+\hat{x}_B^2}{1-\hat{x}_B}\ln(\hat{x}_B) + 3 - \left[\frac{9}{2}+\frac{\pi^2}{3}\right]\delta(1-\hat{x}_B)\right)\right] \\
& \left. + \frac{1-\hat{y}}{\hat{y}^2}\Big[C_F\,(4\hat{x}_B)\Big]\right\}
\end{aligned}
$$



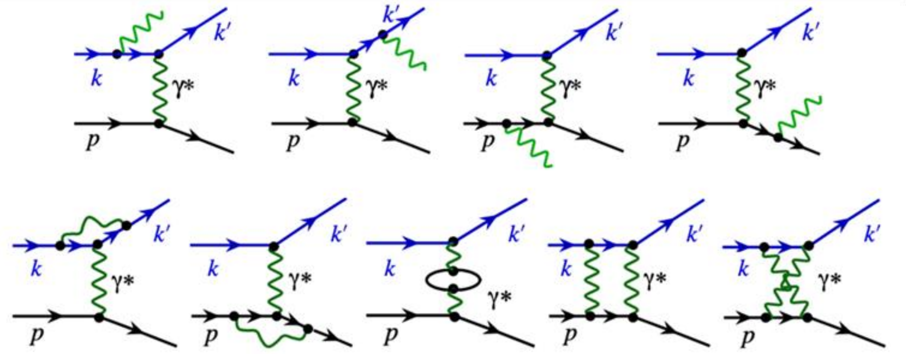
- IR and CO safe, with only free parameter factorization scale
- Uses NLO QCD structure functions

Calculation of NLO QED Contributions

Cammarota, Qiu, Watanbe, Zhang [2408.08377] and [2505.23487]

$$\widehat{H}^{(3,0)}_{eq\to eX} = \sigma^{(3,0)}_{eq\to eX} - D^{(1)}_{e/e} \otimes_\zeta \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{e/e} \otimes_\xi \widehat{H}^{(2,0)}_{eq\to eX}$$

$$- f^{(1)}_{q/q} \otimes_x \widehat{H}^{(2,0)}_{eq\to eX} - f^{(1)}_{\gamma/q} \otimes_x \widehat{H}^{(2,0)}_{e\gamma\to eX}$$



$$f^{(1)}_{i/j}(z)_{\overline{\rm MS}} = \left(-\frac{1}{\epsilon}\right)_{\rm CO} (4\pi)^\epsilon \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} P_{i/j}(z)$$

$$P_{e/e}(z) = \frac{1}{e_q^2} P_{q/q}(z) = \frac{\alpha}{2\pi}\left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right]$$

$$P_{\gamma/q}(z) = \frac{\alpha}{2\pi} e_q^2 \left[\frac{1+(1-z)^2}{z}\right]$$

NLO QED Contribution

$$\hat{H}^{(3,0)}_{eq\to eX} \propto \alpha^3\, e_q^2 \left\{ e_l^2\, \frac{2(1+\hat{v}^2)}{9\hat{v}} \left[3\ln\frac{(1-\hat{v})s}{\mu^2} - 5\right] \delta(1-\hat{w}) \right.$$

$$+ e_q \left[a_1 \delta(1-\hat{w}) + \frac{a_7}{(1-\hat{w})_+} + a_6 \right]$$

$$+ e_q^2 \left[b_1 \delta(1-\hat{w}) + b_2 \left(\frac{1}{1-\hat{w}}\right)_+ + b_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + b_4 \right]$$

$$\left. + c_1 \delta(1-\hat{w}) + c_2 \left(\frac{1}{1-\hat{w}}\right)_+ + c_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}}\right)_+ + c_4 \right\}$$

$$\hat{v} = 1 - \frac{x_B}{x}\frac{y}{\zeta}$$

$$\hat{w} = \frac{1-y}{\xi\left(\zeta - (x_B/x)\, y\right)}$$

a_1, a_6, a_7

b_1, b_2, b_3, b_4

c_1, c_2, c_3, c_4

are analytic functions of $\hat{v}$ and $\hat{w}$.

$$e_l^2 = \sum_f N_c^f\, e_f^2$$ Sum over the flavors appeared in the photon vacuum polarization

- IR and CO safe, with only free parameter factorization scale
- Important point: LDF/LFFs not pure QED and PDF/FF not pure QCD

NLO QED Contribution Details

$$\hat{H}^{(3,0)}_{eq\to eX} \propto \alpha^3\, e_q^2 \left\{ e_l^2\, \frac{2(1+\hat{v}^2)}{9\hat{v}} \left[3\ln\frac{(1-\hat{v})s}{\mu^2} - 5 \right] \delta(1-\hat{w}) \right.$$

Photon vacuum polarization

$$+ e_q \left[a_1 \delta(1-\hat{w}) + \frac{a_7}{(1-\hat{w})_+} + a_6 \right]$$

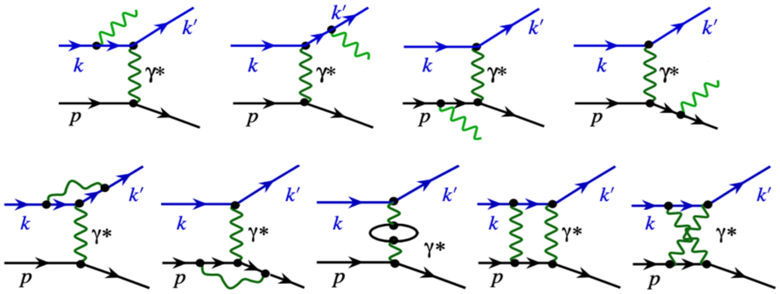
Two photon exchange

NLO “QCD” Contribution

$$+ e_q^2 \left[b_1 \delta(1-\hat{w}) + b_2 \left(\frac{1}{1-\hat{w}} \right)_+ + b_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}} \right)_+ + b_4 \right]$$

Radiative Correction Term

$$\left. + c_1 \delta(1-\hat{w}) + c_2 \left(\frac{1}{1-\hat{w}} \right)_+ + c_3 \left(\frac{\ln(1-\hat{w})}{1-\hat{w}} \right)_+ + c_4 \right\}$$



Modeling Lepton Distributions

- In QED approximation at NLO:

$$f_{e/e}^{(\mathrm{NLO})}(\xi,\mu^2) = \delta(1-\xi) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\xi^2}{1-\xi}\ln\frac{\mu^2}{(1-\xi)^2 m_e^2}\right]_+$$

$$D_{e/e}^{(\mathrm{NLO})}(\zeta,\mu^2) = \delta(1-\zeta) + \frac{\alpha_{em}}{2\pi}\left[\frac{1+\zeta^2}{1-\zeta}\ln\frac{\zeta^2\mu^2}{(1-\zeta)^2 m_e^2}\right]_+$$

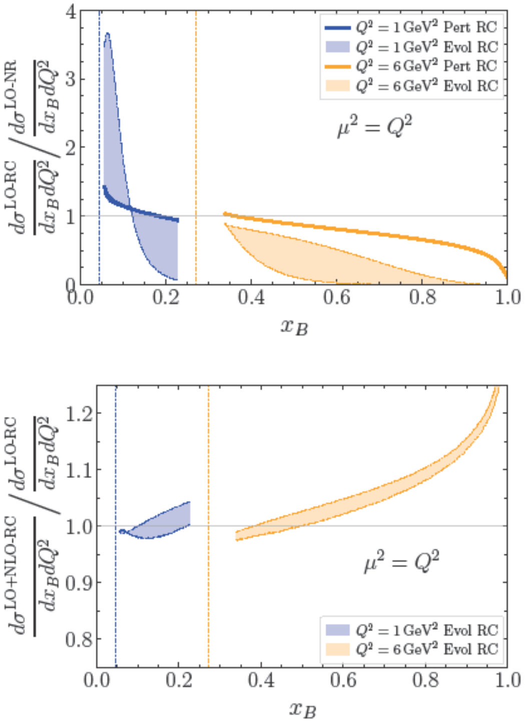
- Model distributions as

$$f_{e/e}(x) \approx D_{e/e}(x) = N_e \frac{x^\alpha(1-x)^\beta}{B(1+\alpha, 1+\beta)}$$

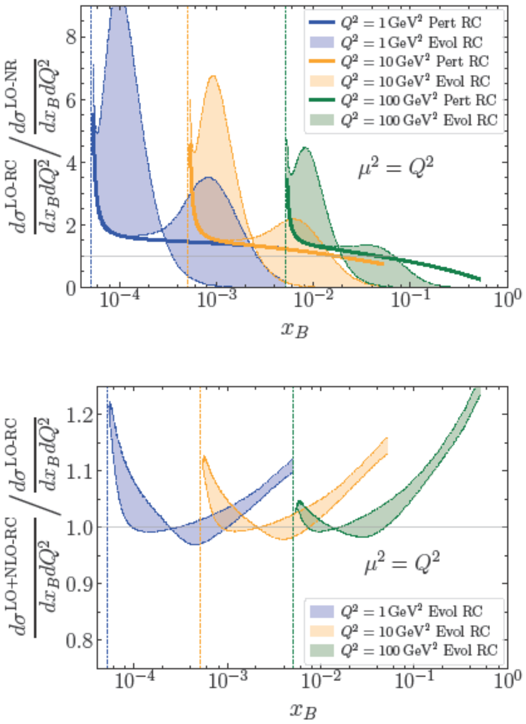
with $N_e = 1$

$$(\alpha,\beta) = (5, 1/2),\ (50, 1/8)$$

Impact of QED Factorization to Lepton-Hadron Scattering



JLab kinematics



EIC kinematics

Limits on Application

- Q^2 is not ideal for small x or beyond LO in QED

$$p_T^2 = Q^2(1-y) = Q^2\left(1 - \frac{Q^2}{x_B s}\right)$$

For EIC: $y \leq 0.95$

$$p_{T_{\min}}^2 = Q^2(1-y_{\max}) = \frac{Q^2}{20}!!!$$

- Factorization breaks down if transverse momentum too small

Other Hadron Structure Impacts

- lepton-hadron reactions (COMPASS, JLab, **EIC**)

$$\sigma^{EXP}_{l+P\to l+X} = C_{l+k\to l+X} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{LDF}_e} \otimes \boxed{\mathrm{LFF}_e}$$

$$\sigma^{\mathrm{EXP}}_{l+P\to l+H+X} = \boxed{C_{l+k\to l+k+X}} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{FF}_H} \otimes \boxed{\mathrm{LDF}_e} \otimes \boxed{\mathrm{LFF}_e}$$

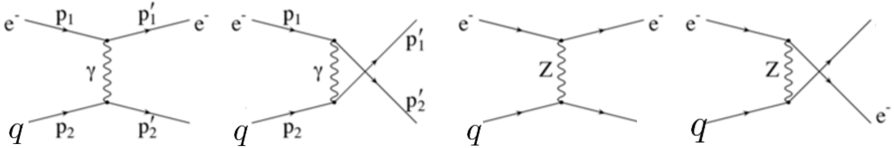
- hadron-hadron reactions (LHC)

$$\sigma^{\mathrm{EXP}}_{P+P\to l+\bar{l}+X} = \boxed{C_{k+k\to l+\bar{l}+X}} \otimes \boxed{\mathrm{PDF}_P} \otimes \boxed{\mathrm{PDF}_P} + O(Q_s^2/Q^2)$$

- lepton-lepton reactions (Belle)

$$\sigma^{\mathrm{EXP}}_{l+\bar{l}\to H+X} = \boxed{C_{l+\bar{l}\to k+X}} \otimes \boxed{\mathrm{FF}_H} \otimes \boxed{\mathrm{LDF}_e} \otimes \boxed{\mathrm{LDF}_{\bar{e}}}$$

Extensions to PVDIS



- Calculations similar to DIS
- Polarized electron beam leads to a nonzero interference axial coupling term
- Overall asymmetry proportional to desired couplings
- Electroweak symmetry requires $\mathcal{O}\left(\frac{1}{m_W}\right)$ approximation to maintain factorization

$$L^{\mu\nu}_{DIS} \propto \frac{1}{2}\mathrm{Tr}\left[\gamma^\mu(\gamma \cdot p_1)\gamma^\nu(\gamma \cdot p_1')\right]$$

$$\Delta L^{\mu\nu}_1 \propto \frac{1}{2}\mathrm{Tr}\left[\gamma^\mu(\gamma \cdot p_1)\gamma_5(c^e_v\gamma^\nu + c^e_A\gamma^\nu\gamma_5)(\gamma \cdot p_1')\right]$$

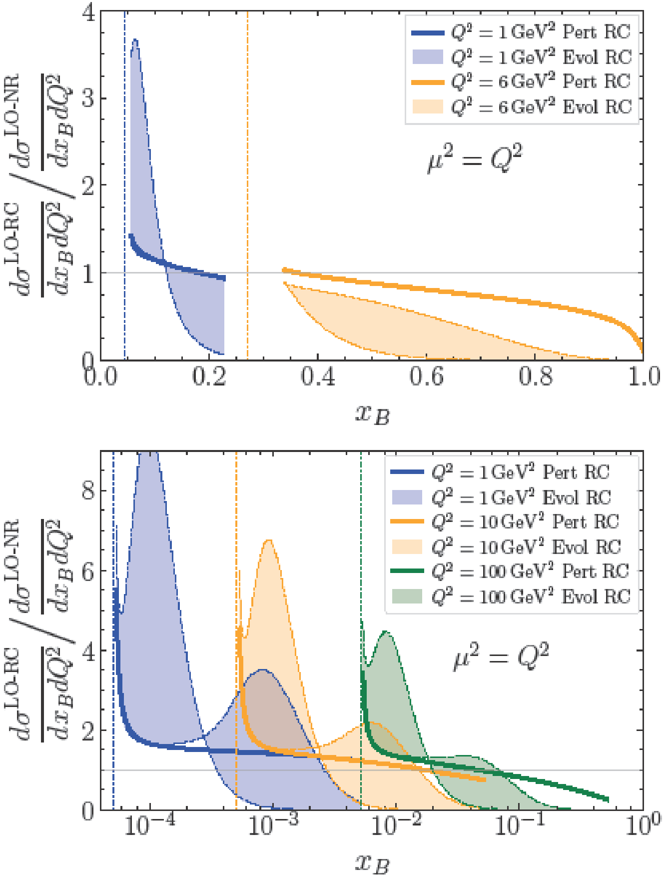
$$W_{1,\mu\nu} \propto \frac{1}{2}\mathrm{Tr}\left[\gamma_\mu(\gamma \cdot p_2)(c^q_v\gamma_\nu + c^q_A\gamma_\nu\gamma_5)(\gamma \cdot p_2')\right]$$

$$\frac{\sigma(s_e) - \sigma(-s_e)}{\sigma(s_e) + \sigma(-s_e)} \propto \sum_q c^e_A c^q_v \Delta L^{\mu\nu} W_{\mu\nu}$$

SoLID
SOLENOIDAL LARGE INTENSITY DEVICE

DIS Conclusion

- QED radiation induced by the collision is integral to understanding process
- Without recovering all QED radiation, the photon-hadron frame is ill-defined
- Joint QED-QCD factorization scheme is a consistent and controllable approximation
- Applications to other lepton scattering processes, such as SIDIS and PVDIS



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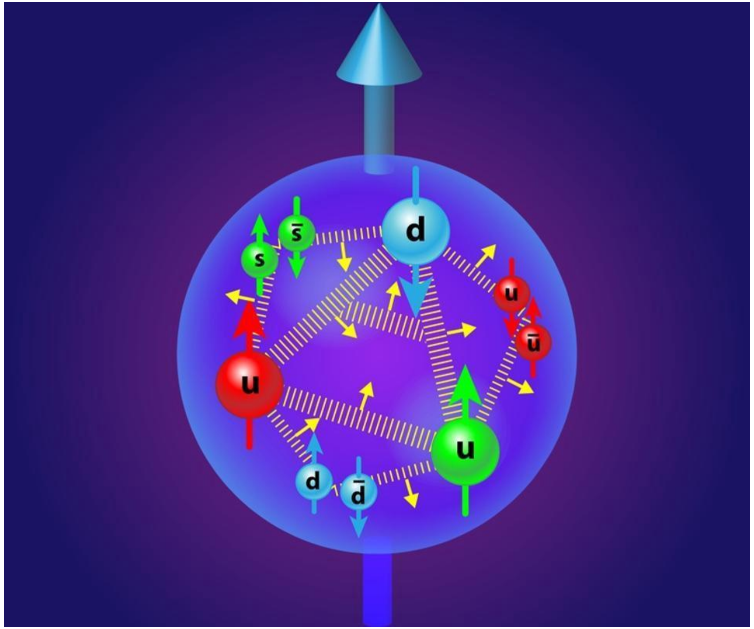
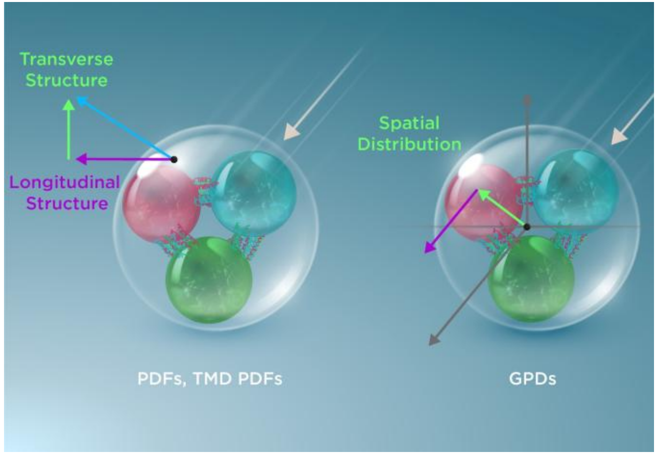
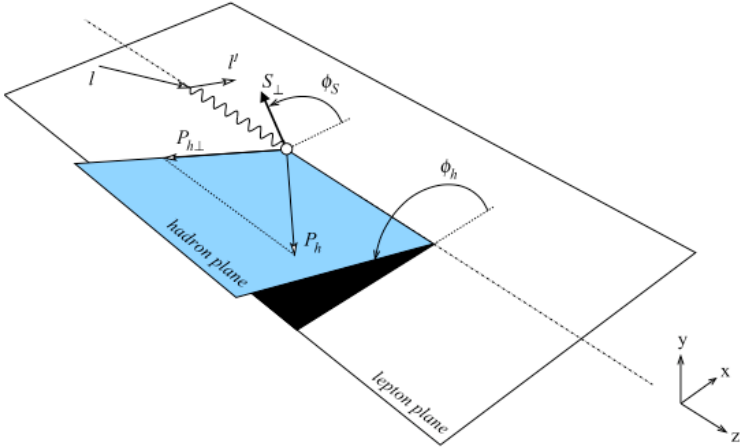


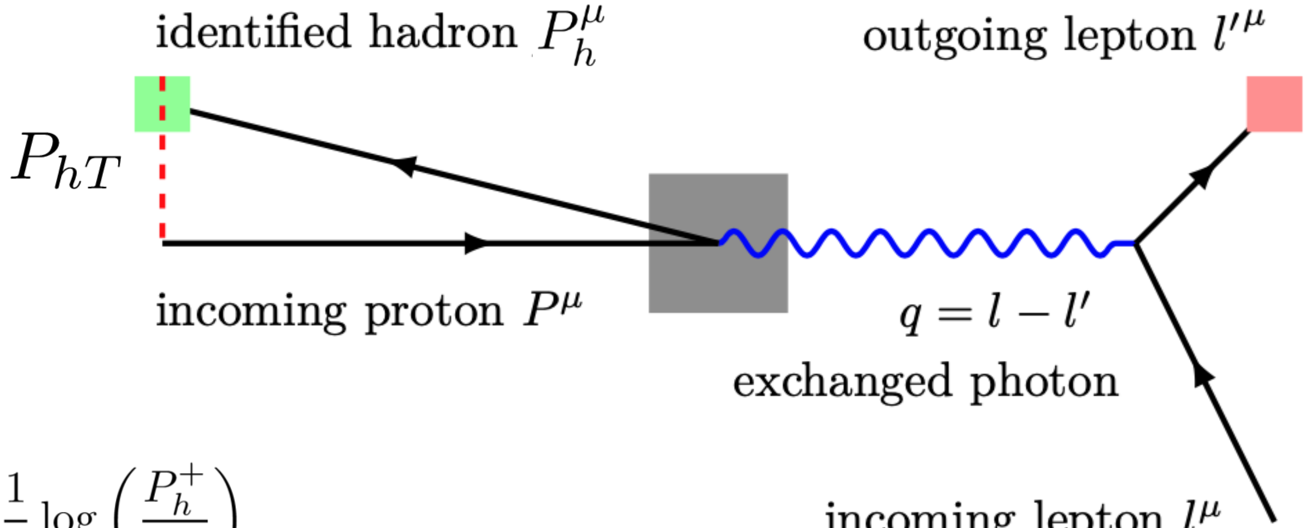
Semi-Inclusive DIS: Frontier of Hadron Structure

- Observation of two particles allows consideration of two scales of observables: $P_{h\perp}$ and Q^2
- Construction of leptonic and hadronic planes vital to separating TMDs





Breit Frame

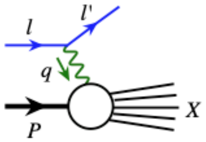


$$y_h = \frac{1}{2}\log\left(\frac{P_h^+}{P_h^-}\right)$$

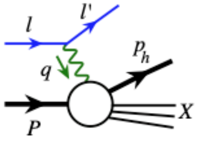
SIDIS in the Breit Frame

$$
\begin{aligned}
&\frac{d\sigma}{dx\,dy\,d\psi\,dz\,d\phi_h\,dP_{h\perp}^2} = \\
&\frac{\alpha^2}{xyQ^2}\frac{y^2}{2(1-\varepsilon)}\left(1+\frac{\gamma^2}{2x}\right)\Bigg\{F_{UU,T}+\varepsilon F_{UU,L}+\sqrt{2\varepsilon(1+\varepsilon)}\,\cos\phi_h\,F_{UU}^{\cos\phi_h} \\
&\quad+\varepsilon\cos(2\phi_h)\,F_{UU}^{\cos 2\phi_h}+\lambda_e\sqrt{2\varepsilon(1-\varepsilon)}\,\sin\phi_h\,F_{LU}^{\sin\phi_h} \\
&\quad+S_\parallel\left[\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_h\,F_{UL}^{\sin\phi_h}+\varepsilon\sin(2\phi_h)\,F_{UL}^{\sin 2\phi_h}\right] \\
&\quad+S_\parallel\lambda_e\left[\sqrt{1-\varepsilon^2}\,F_{LL}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_h\,F_{LL}^{\cos\phi_h}\right] \\
&\quad+|\boldsymbol{S}_\perp|\Big[\sin(\phi_h-\phi_S)\left(F_{UT,T}^{\sin(\phi_h-\phi_S)}+\varepsilon\,F_{UT,L}^{\sin(\phi_h-\phi_S)}\right) \\
&\qquad+\varepsilon\,\sin(\phi_h+\phi_S)\,F_{UT}^{\sin(\phi_h+\phi_S)}+\varepsilon\,\sin(3\phi_h-\phi_S)\,F_{UT}^{\sin(3\phi_h-\phi_S)} \\
&\qquad+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin\phi_S\,F_{UT}^{\sin\phi_S}+\sqrt{2\varepsilon(1+\varepsilon)}\,\sin(2\phi_h-\phi_S)\,F_{UT}^{\sin(2\phi_h-\phi_S)}\Big] \\
&\quad+|\boldsymbol{S}_\perp|\lambda_e\Big[\sqrt{1-\varepsilon^2}\,\cos(\phi_h-\phi_S)\,F_{LT}^{\cos(\phi_h-\phi_S)}+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos\phi_S\,F_{LT}^{\cos\phi_S} \\
&\qquad+\sqrt{2\varepsilon(1-\varepsilon)}\,\cos(2\phi_h-\phi_S)\,F_{LT}^{\cos(2\phi_h-\phi_S)}\Big]\Bigg\},
\end{aligned}
$$

(

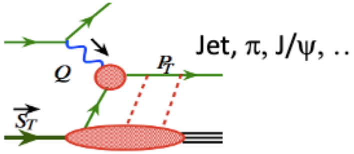


Scale: Q^2 - **PDFs**



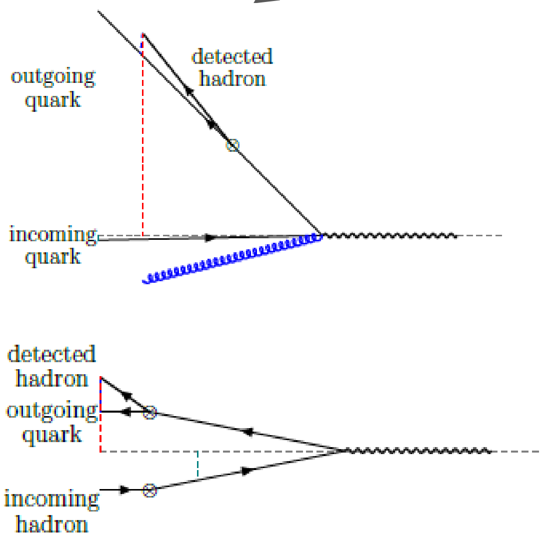
$Q^2 \gg P_{hT}^2$

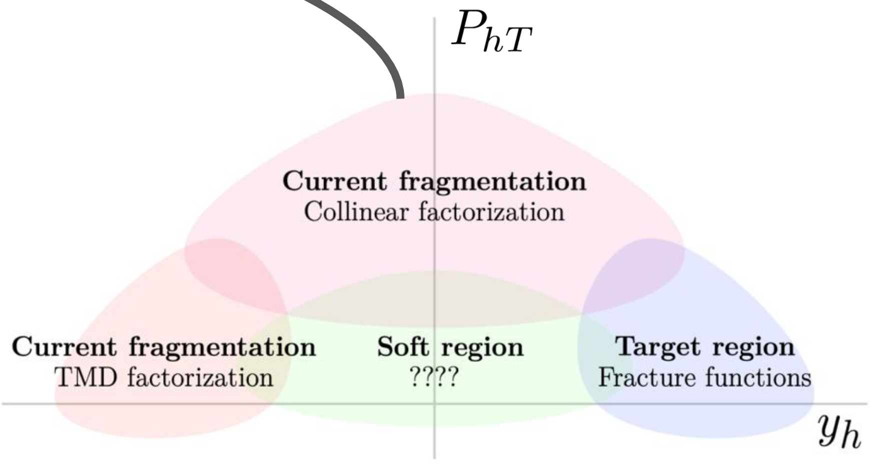
In photon-hadron frame!



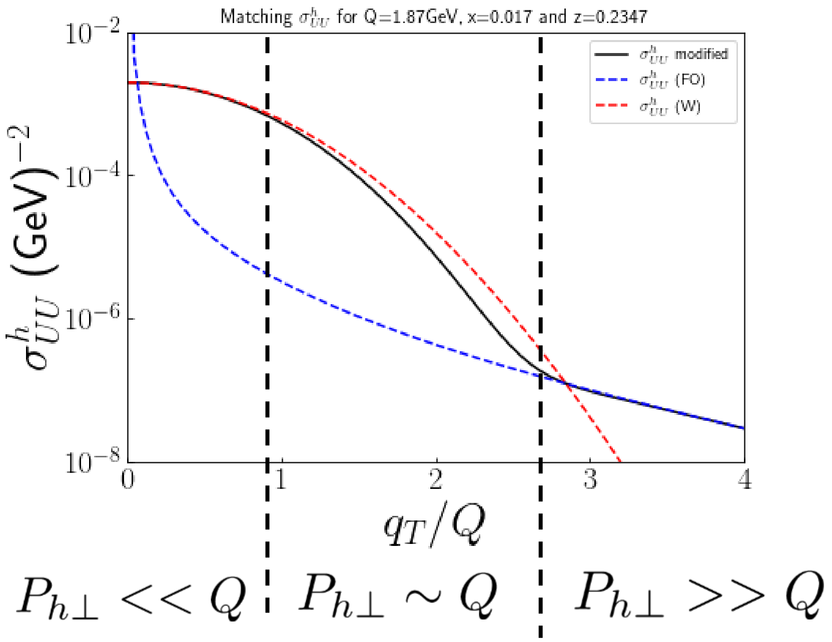
$f(x, k_T, Q)$ - **TMDs**

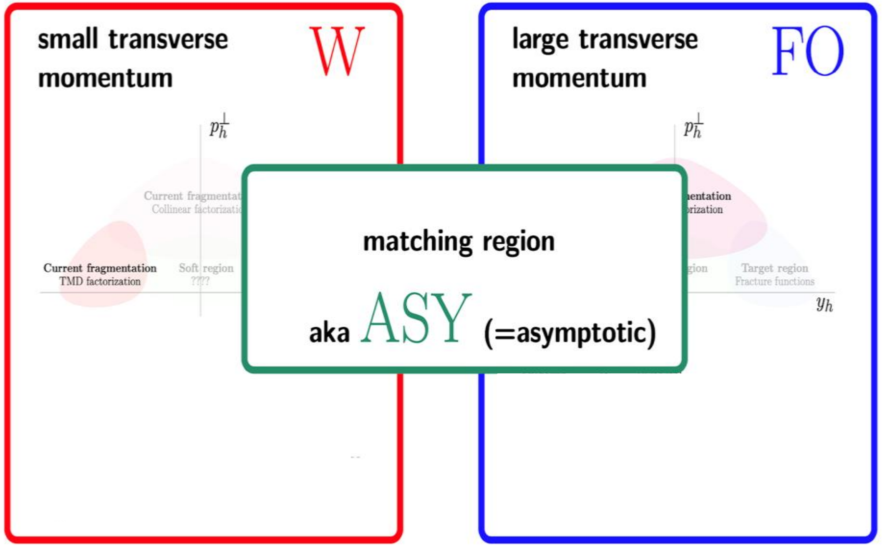
Separation of Kinematic Regions





Standard Approach





Scale Separation of Cross Section

$$\frac{d\sigma}{dx\,dQ^2\,dz\,dP_{hT}} = \mathrm{W} + \mathrm{FO} - \mathrm{ASY} + \mathcal{O}(\Lambda_{QCD}^2/Q^2)$$

where

$$P_{hT} = q_T z_h$$

$$x \Leftrightarrow x_B$$

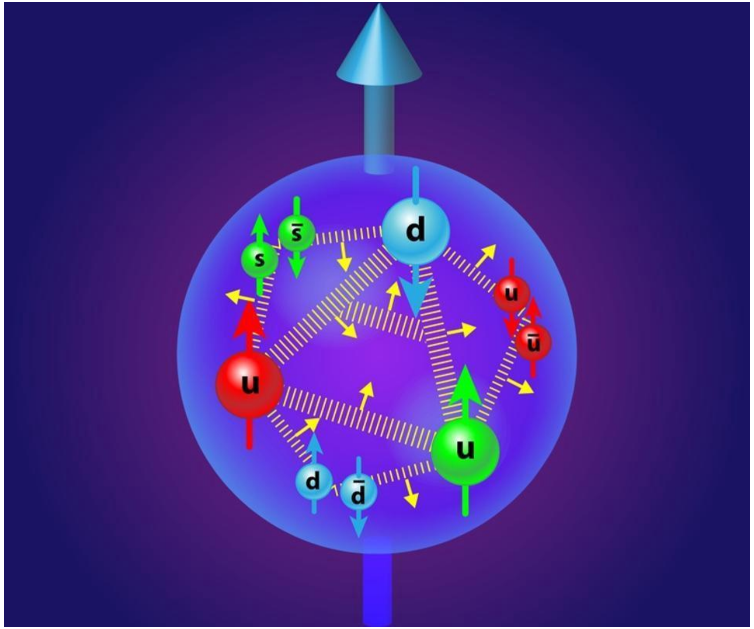
$$\sim \mathrm{W} \quad \text{for } q_{\mathrm{T}} \ll Q$$

$$\sim \mathrm{FO} \quad \text{for } q_{\mathrm{T}} \sim Q$$

- W term has resummed logarithms
- Asymptotic term perturbatively expands W term to fixed order
- Covers matching region between W and fixed order and removes non-dominant contributions

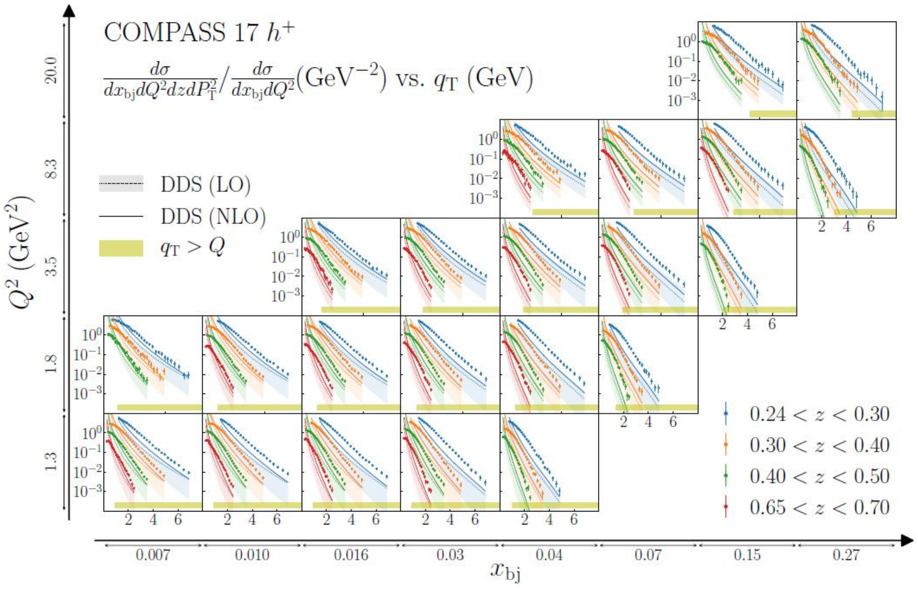
Outline

- Motivation
- Deep Inelastic Scattering
  - Calculations
- SIDIS Theory
  - Standard Approach
  - **Puzzles from Standard Approach**
  - Cross Section Comparison



Inconsistency with Data

- Large discrepancy with data at high P_{hT}^2 (example from COMPASS 2017 run, Aghasyan et al., 2018)
- P_{hT}^2 integrated theory consistent with data
- Possible Solutions
  - Higher twist corrections
  - Better constraints on PDFs and FFs
  - NLO corrections
  - Power corrections



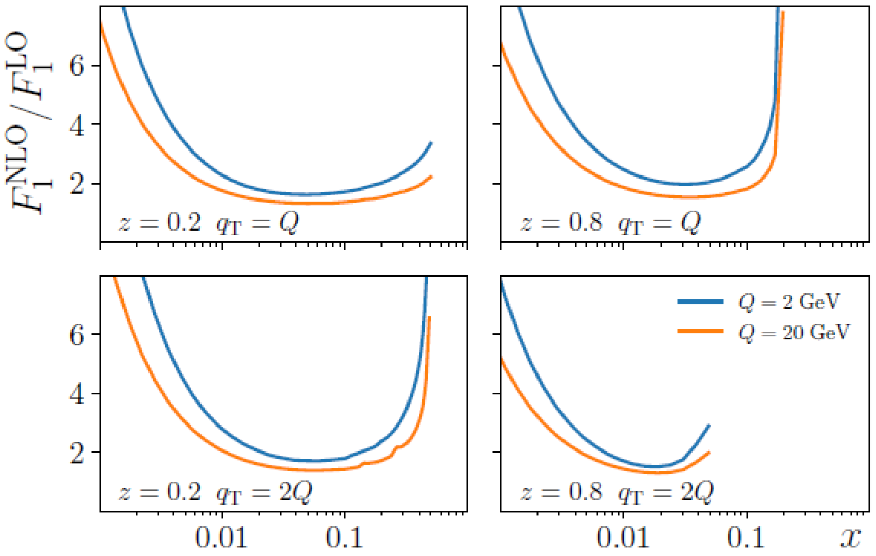
Gonzalez-Hernandez et al., 2018

Beyond Leading Order Corrections

- Order of $\alpha_s{}^2$ contributions calculated in Wang et al., 2019
- Compared *K* factor ratio for F_1
- Important at large *x*
  - Soft gluon effects near threshold
- Important at small *x*
  - Large $(P+q)^2$

$$P_1^{\mu\nu} W_{\mu\nu} = F_1$$

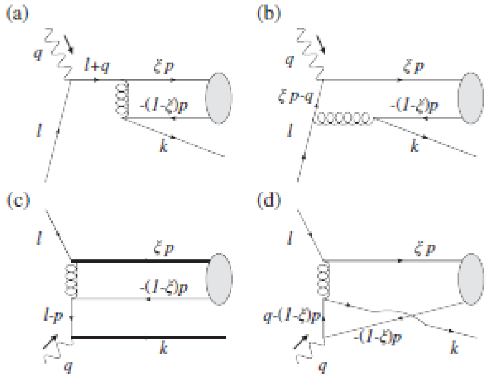
$$P_1^{\mu\nu} = \frac{\left(2(\hat{x}_B/x)^2\right) p^\mu p^\nu}{\hat{Q}^2} - \frac{1}{2} g^{\mu\nu}$$

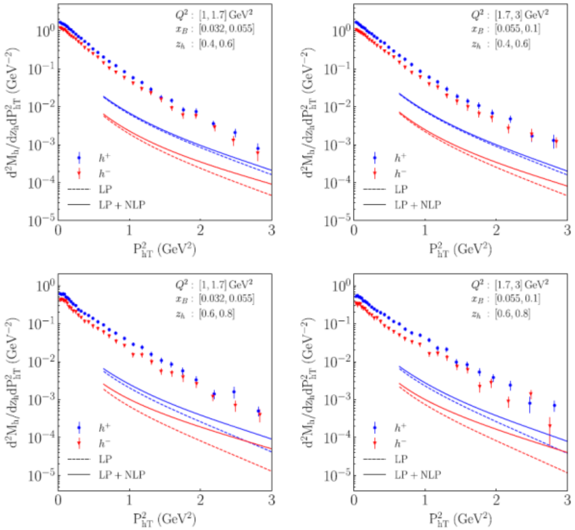


$$x \Leftrightarrow x_B$$

Power Corrections

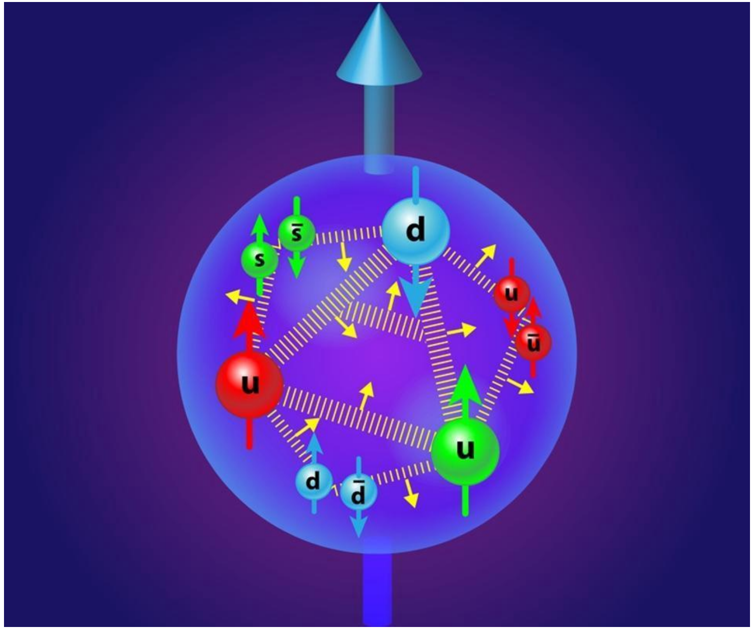
- Considered by Liu and Qiu, 2020
- Typical subleading power corrections are suppressed by large $P_{h\perp}$
- Enhancement from hadronization and edge of phase space
- Requires consideration of quark-antiquark FFs
  - Hadronization of a pair (i.e. $u\bar{d} \to \pi^{+}$)





Outline

- Motivation
- Deep Inelastic Scattering
  - Calculations
- Semi-Inclusive Deep Inelastic Scattering
  - Standard Approach
  - Puzzles from Standard Approach
  - **Cross Section Comparison**



Lorentz Transformation for QED Radiation

- From partonic momentum conservation

$$\delta^2(q_T - (p_T + k_T))$$

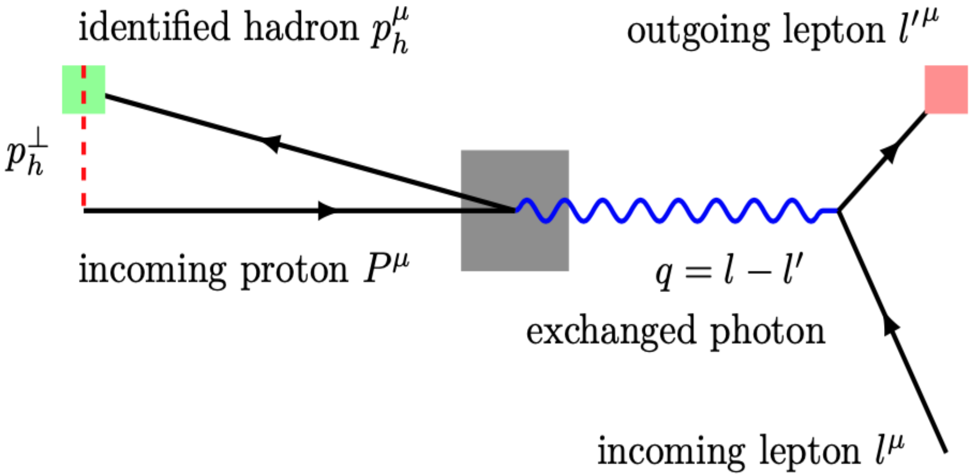
- Lepton-induced QED radiation:

$$q_T \to \hat{q}_T(\xi, \zeta)$$

$$P_{hT} \to \hat{P}_{hT}(\xi, \zeta)$$

- So there exists a Lorentz transformation:

$$\Lambda(\xi, \zeta) \cdot \hat{P}_{hT}(\xi, \zeta) = P_{hT}$$



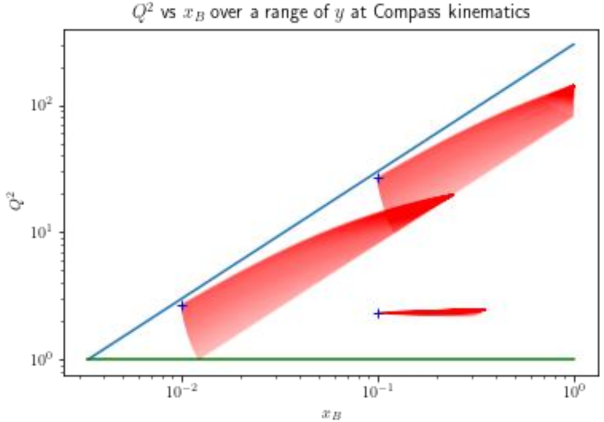
$$P_{hT} = q_T z_h$$

QED Effects on Relevant Kinematic Variables

- Explicit dependence of major kinematic variables on radiation parameters ξ,ζ

$$\hat{x}_B = \frac{\xi y x_B}{\xi\zeta - 1 + y} \qquad \hat{z}_h = \frac{\zeta y z_h}{\xi\zeta - 1 + y}$$

$$\hat{Q}^2 = \frac{\xi}{\zeta} Q^2 \qquad \hat{y} = \frac{\hat{Q}^2}{\xi S \hat{x}_B}$$

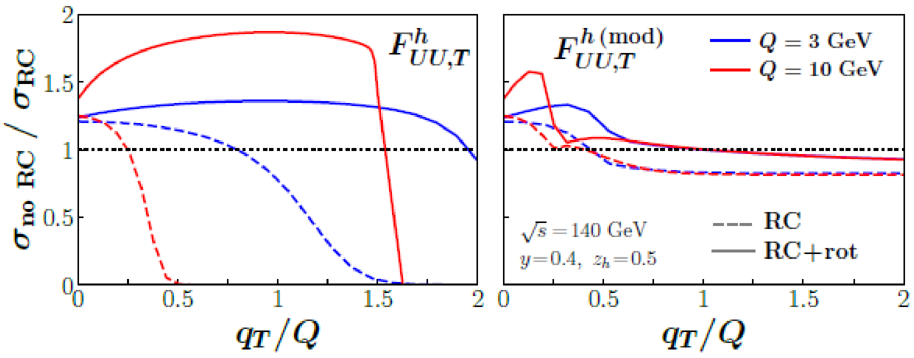


$$\hat{P}_{h\perp}^2(\xi, \zeta, \phi_h, y, z_h, Q, P_{h\perp}) = \frac{-2\cos\phi_h Q^5 z_h(\zeta\xi - 1)P_{h\perp}\sqrt{1-y}(\zeta\xi + y - 1) + Q^4 P_{h\perp}^2(\zeta\xi + y - 1)^2 - Q^6(y-1)z_h^2(\zeta\xi - 1)^2}{Q^4(\zeta\xi + y - 1)^2}$$

Previous Comparison of Unpolarized Cross Sections

- Liu et al., 2020 and 2021 established new factorization approach
- Showed effects of radiative corrections to Gaussian approximation of W term with artificial fixed order tail

$$F^h_{UU,\mathrm{Gaussian}}(x_B, y, z_h, Q^2, q_\perp) = \sum_q e_q^2 f_{q/h}(x_B, Q^2) D_{h'/q}(z_h, Q^2) \frac{e^{-(z_h q_\perp)^2/\langle (z_h q_\perp)^2 \rangle}}{\pi \langle (z_h q_\perp)^2 \rangle}$$



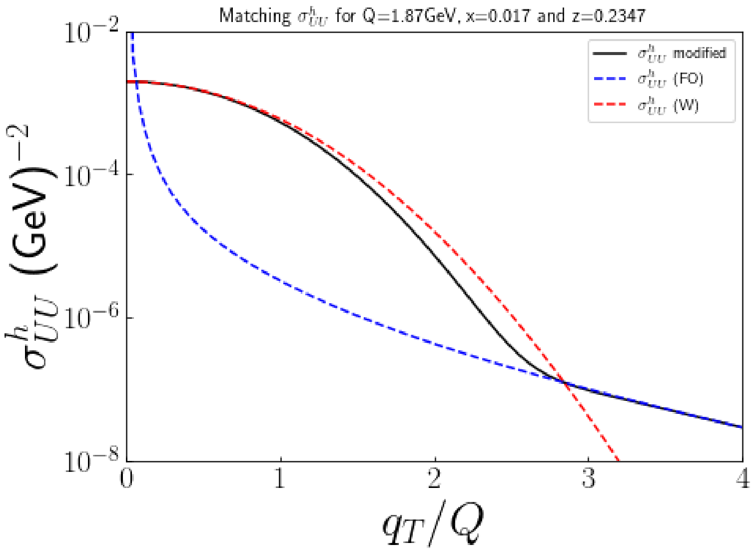
Full Spectrum Consideration Setup

- Fixed order calculation comes from Nadolsky et al., 1999 (at finite $q_T \sim Q$)
- To match W to FO, toy scheme used in this work (and in Liu et al., 2021):

$$\sigma^h_{UU,\text{Modified}} = \mathbf{W} R + (1-R)\mathbf{FO}$$

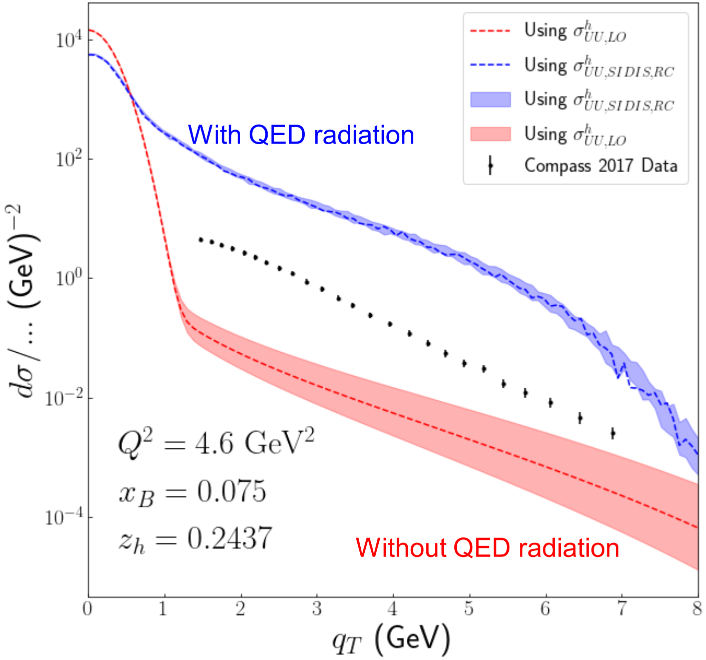
$$R = e^{-A(q_T/Q)^B}$$

A=35, B=2 from trial and error matching

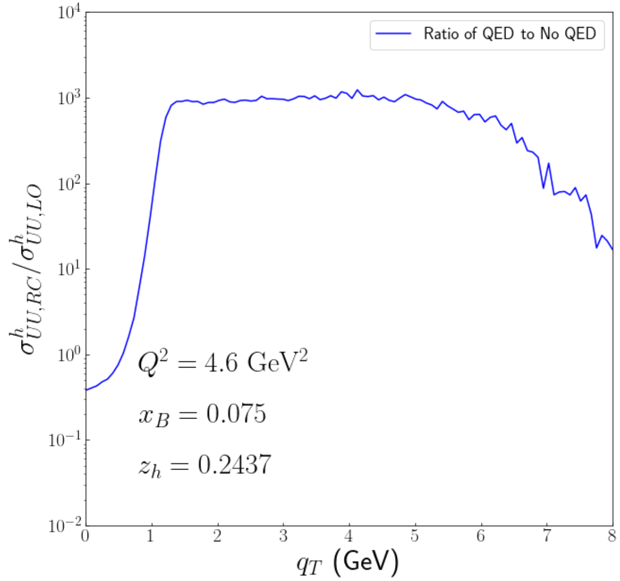


Before QED corrections

Results



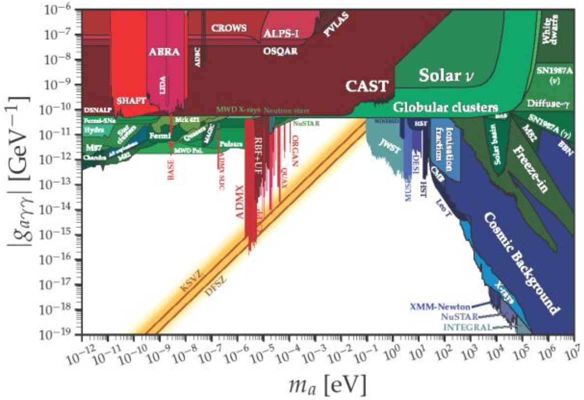
$$\frac{d\sigma}{dx_B\, dQ^2\, dz_h\, dP_{hT}^2} \Big/ \frac{d\sigma}{dx_B\, dQ^2}$$

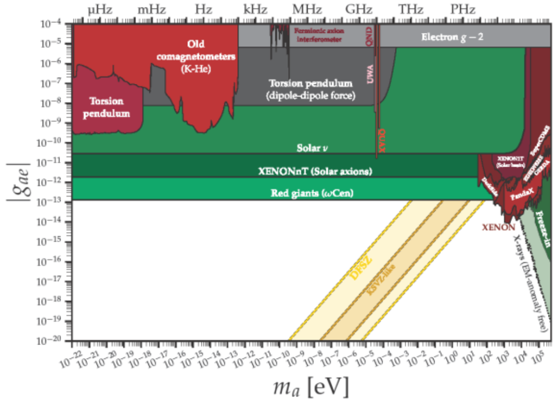


$$P_{hT} = q_T z_h$$

Extensions to Light Axion Searches

- Invisible axion emission impacts at $\mathcal{O}\left(g_{a\gamma\gamma}^{2}\right)$
- Can impact LFF, LDF at $\mathcal{O}\left(g_{aq/f}\right)$
- Avenue for constraints for axion couplings
- See Susan Gardner's talk at 9 am tomorrow: Towards new constraints on light, dark sectors through EIC studies

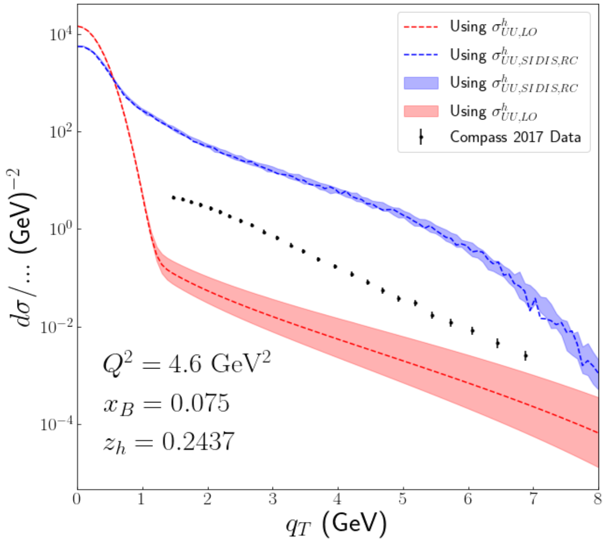




Ciaran O'Hare, https://cajohare.github.io/AxionLimits/docs/ap.html

Conclusion of SIDIS Work

- Joint QED + QCD factorization scheme for SIDIS process showed significant effects on cross section
- Full transverse momentum spectrum required to address QED effects
- Potential resolution to discrepancies between previous theory and COMPASS data



Acknowledgements

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Some images and background for the slides came from Jianwei Qiu's cake seminar slides in February 2025 at Jefferson Lab.

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Backup Slides

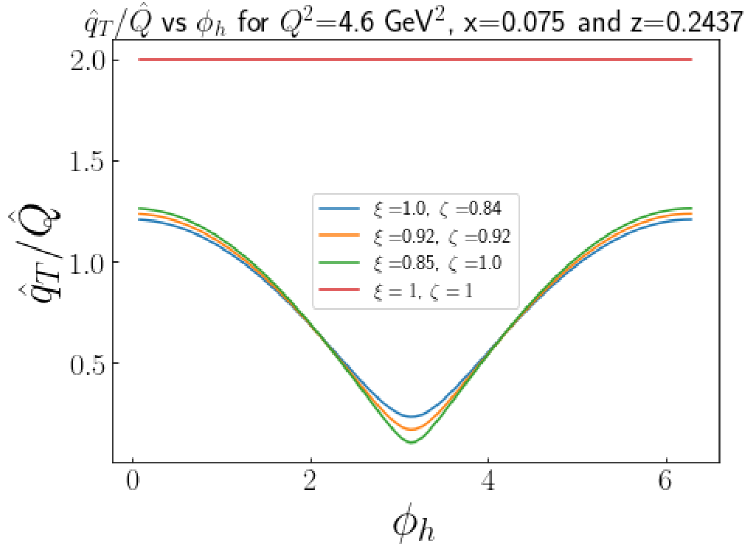
Role of Angular Effects on $\hat{P}_{hT}$

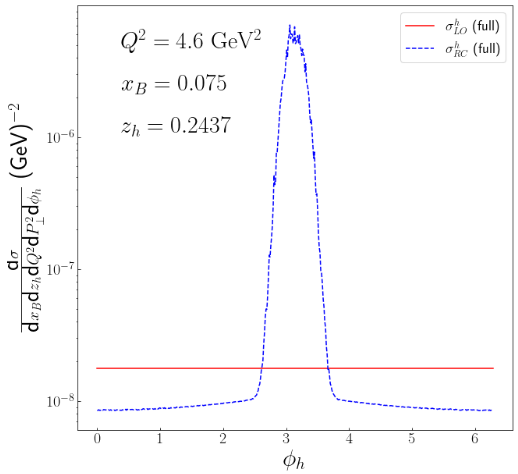
- Standard approach assumes separable angular dependence
- Lepton radiation introduces internal angular dependence
- Increases weight of back-to-back region

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_B \mathrm{d}y \mathrm{d}z_h \mathrm{d}Q^2 \mathrm{d}P_\perp^2} \propto F_{UU,T} + F_{UU,L} + \cos\phi_h F_{UU}^{\cos\phi_h} + \cos(2\phi_h) F_{UU}^{\cos 2\phi_h}$$

$$\hat{P}_{h\perp}^2(\xi, \zeta, \phi_h, y, z_h, Q, P_{h\perp}) \propto \frac{\cos\phi_h(\zeta\xi - 1)P_{h\perp}}{(\zeta\xi + y - 1)} + P_{h\perp}^2$$

Importance of Angular Effects

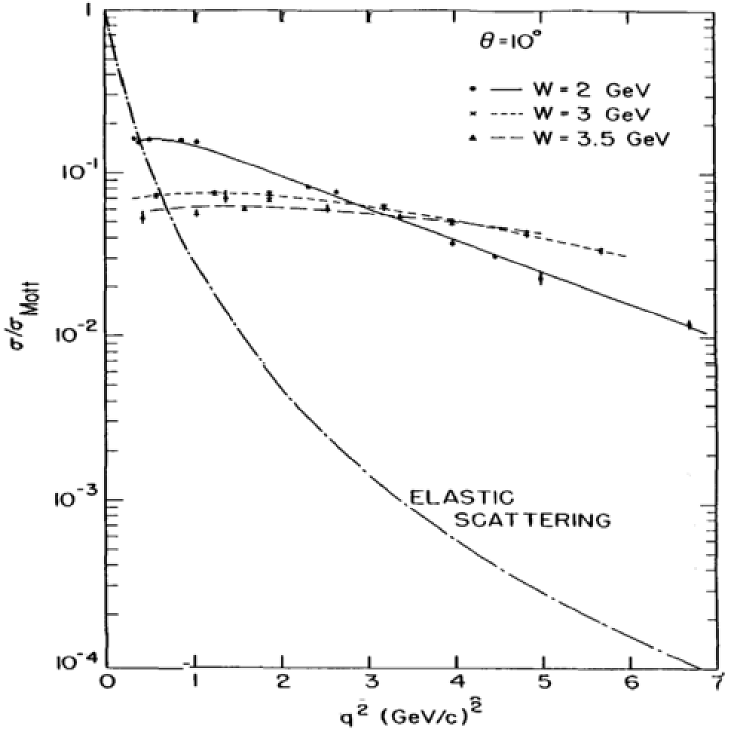




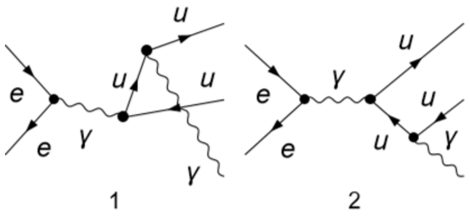
Angular modulation on internal transverse momentum has a direct impact on the P_{hT}^2 dependent cross section

Structure of the Proton

- Early SLAC experiments performed DIS measurements
- Similar to Rutherford scattering, results were not consistent with uniform distribution inside proton
- Hadrons composed of point-like partons (Feynman 1969), later determined to be quarks and gluons
- Several classes of functions defined to characterize behavior of partons



Practice Calculations



- First went through the photoproduction calculation (as in Berger et al, 1996)
- NLO real calculation (cut diagrams above)
- Hadronic tensors, with y's defined in terms of dot products of momenta

$$H_1 = -g_{\mu\nu}H^{\mu\nu} = 8(1-\epsilon)\left\{(1-\epsilon)\left[\frac{y_{13}}{y_{23}} + \frac{y_{23}}{y_{13}}\right] + \frac{2y_{12}}{y_{13}y_{23}} - 2\epsilon\right\}$$

$$H_2 = -\frac{k_\mu k_\nu}{q^2}H^{\mu\nu} = -4\left(\frac{2y_{12}}{y_{13}y_{23}} + (1-\epsilon)\left(\frac{y_{13}}{y_{23}} + \frac{y_{23}}{y_{13}}\right) - 2\epsilon\right) + \frac{4\left(y_{1\mathrm{k}}^2 + y_{2\mathrm{k}}^2\right)}{y_{13}y_{23}} - \frac{4\epsilon y_{3\mathrm{k}}^2}{y_{13}y_{23}}$$

$$E_\gamma \frac{\mathrm{d}\sigma^{(1)}}{\mathrm{d}^3\ell} = \int_0^1 e^2 N_c e_q^2 (e\mu^\epsilon)^4 \frac{1}{4}(H_1 + H_2)\mathrm{d}X_{(3)}^{PS}\mathrm{d}\hat{y}_{13}$$

$(\hat{y}_{13} = y_{13}/x_\gamma)$

Real Term Calculation

- Challenge of the NLO calculation: finding the right format to do integral (as in Van Neerven 1986)

$$I(j,l,\psi) = 2^{1-j-l}\pi\frac{\Gamma(D/2-1-j)\Gamma(D/2-1-l)}{\Gamma(D/2-1)^2\Gamma(D-2-j-1)}\,{}_2F_1(j,l;D/2-1;(1-\psi)/2)$$
$$= \int_0^\pi d\theta_1 \sin^{n-3}\theta_1 \int_0^\pi d\theta_2 \sin^{n-4}\theta_2 (1-\cos\theta_1)^{-j}(1-\cos\psi\cos\theta_1-\sin\psi\sin\theta_1\cos\theta_2)^{-l}$$

- The hypergeometric function could be simplified as

$${}_2F_1(1,1;1-\epsilon;z) = (1-z)^{-1-\epsilon}\left(1+\epsilon^2 \mathrm{Li}_2(z)\right)$$

- The power dependence of the divergent terms were expanded as

$$(1-w)^{(-\epsilon-1)} = -\frac{1}{\epsilon}\delta(1-w) + \left(\frac{1}{1-w}\right)_+ - \epsilon\left(\frac{\log(1-w)}{1-w}\right)_+$$
$$(1-w)^{(-2\epsilon-1)} = -\frac{1}{2\epsilon}\delta(1-w) + \left(\frac{1}{1-w}\right)_+ - 2\epsilon\left(\frac{\log(1-w)}{1-w}\right)_+$$

Virtual Term Calculations

- The Passarino-Veltman functions were computed as (as in Ellis 2008)

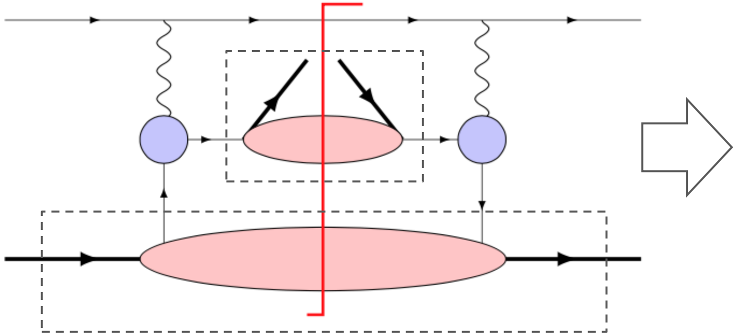
$$\mathrm{B}_0(s,0,0) = \left(\frac{\mu^2}{-s}\right)^{\epsilon}\left(\frac{1}{\epsilon}+2\right)$$

$$\mathrm{C}_0(0,0,p,0,0,0) = \left(\frac{\mu^{2\epsilon}}{\epsilon^2}\right)\left(\frac{(-p)^{-\epsilon}}{p}\right)$$

$$\mathrm{D}_0(0,0,0,0,s_1,s_2,0,0,0,0) = \left(\frac{\mu^{2\epsilon}}{s_1 s_2}\right)\left(\frac{2}{\epsilon^2}\left((-s_1)^{-\epsilon}+(-s_2)^{-\epsilon}\right)-\log^2\left(\frac{s_1}{s_2}\right)\right)$$

- After taking the real part of the series expansion (up to first order in epsilon), the epsilon dependence in the virtual term cancels with those in the real term (for the double pole) and the counter term (for the single pole)

Factorization of SIDIS



$$\mathrm{d}\sigma = \sum_{ij} H_{ij}^{\mathrm{SIDIS}} \otimes f_i \otimes d_j$$

- Up to soft gluon exchanges, provides predictive power
  - Short distance portion calculable
  - PDFs, TMDs, and FFs are universal, nonperturbative functions

Kinematic Variables Appearing in Full SIDIS Cross Section

$$x_B = \frac{Q^2}{2P \cdot q}, \quad y = \frac{P \cdot q}{P \cdot l}, \quad z = \frac{P \cdot P_h}{P \cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = -\frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = -\frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = -\frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = -\frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P \cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \quad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S \cdot q}{P \cdot q} \frac{M}{\sqrt{1+\gamma^2}}, \quad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P \cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{P \cdot q\sqrt{1+\gamma^2}}$$

Kinematic Variables with QED effects Highlighted

$$x_B = \frac{Q^2}{2P\cdot q}, \quad y = \frac{P\cdot q}{P\cdot l}, \quad z = \frac{P\cdot P_h}{P\cdot q}, \quad \gamma = \frac{2Mx_B}{Q}$$

$$\varepsilon = \frac{1 - y - \frac{1}{4}\gamma^2 y^2}{1 - y + \frac{1}{2}y^2 + \frac{1}{4}\gamma^2 y^2}$$

$$\cos(\phi_h) = \frac{l_\mu P_{h\nu} g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \cos(\phi_S) = \frac{l_\mu S_\nu g_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}},$$

$$\sin(\phi_h) = \frac{l_\mu P_{h\nu} \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 P_{h\perp}^2}}, \quad \sin(\phi_S) = \frac{l_\mu S_\nu \epsilon_\perp^{\mu\nu}}{\sqrt{l_\perp^2 S_\perp^2}}$$

$$S^\mu = S_\parallel \frac{P^\mu - q^\mu M^2/(P\cdot q)}{M\sqrt{1+\gamma^2}} + S_\perp^\mu$$

$$l_\perp^\mu = g_\perp^{\mu\nu} l_\nu, \qquad P_{h\perp}^\mu = g_\perp^{\mu\nu} P_{h\nu}$$

$$S_\parallel = \frac{S\cdot q}{P\cdot q}\frac{M}{\sqrt{1+\gamma^2}}, \qquad S_\perp^\mu = g_\perp^{\mu\nu} S_\nu$$

$$g_\perp^{\mu\nu} = g_{\mu\nu} - \frac{q^\mu P^\nu + q^\nu P^\mu}{P\cdot q(1+\gamma^2)} + \frac{\gamma^2}{1+\gamma^2}\left(\frac{q^\mu q^\nu}{Q^2} - \frac{P^\mu P^\nu}{M^2}\right)$$

$$\epsilon_\perp^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma}\frac{P_\rho q_\sigma}{P\cdot q\sqrt{1+\gamma^2}}$$

Cross Section Expression

$$E_{L'} E_{P'} \frac{\mathrm{d}\sigma}{\mathrm{d}^3\vec{L}'\,\mathrm{d}^3\vec{P}'} = \sum_q \int_{\zeta_{\min},\xi_{\min},z_{\min},0}^{1} \frac{\mathrm{d}\zeta}{\zeta^2}\frac{\mathrm{d}\xi}{\xi}\frac{\mathrm{d}z}{z^2}\frac{\mathrm{d}x}{x}\tilde{D}_{L/e}(\zeta)\tilde{f}_{e/L}(\xi)$$

$$\times \tilde{D}_{h/q}(z)\tilde{f}_{q/h}(x)\left(\frac{1}{4(2\pi)^6}\right)\frac{-8e^4\,e_q^2\,g^2\zeta}{3\xi Q^2 Sxz(\zeta\xi S+U')(S'+\zeta\xi U)}$$

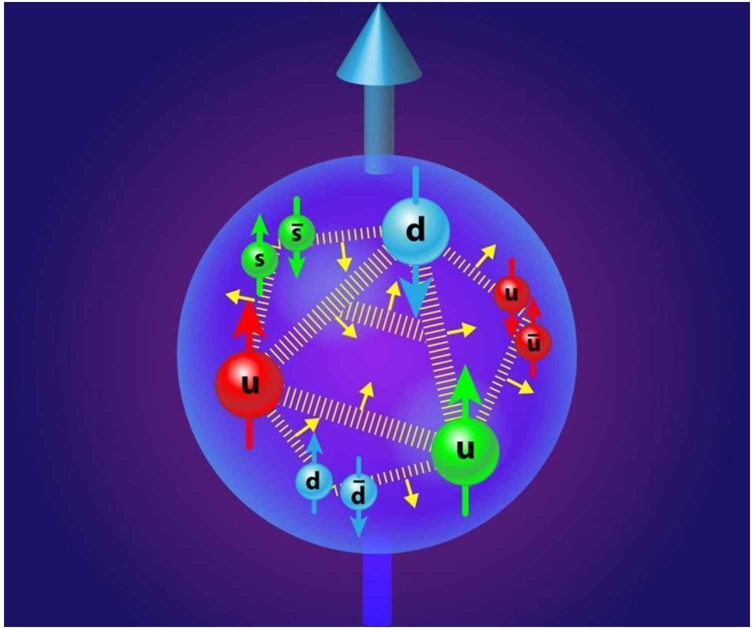
$$\left(\xi^2\left(2Q^4z^2+2\zeta Q^2Uz+\zeta^2(S^2x^2z^2+U^2)\right)+2\xi S'\left(Q^2z+\zeta(U-U')\right)\right.$$

$$\left.-2\xi U'\left(Q^2z(\xi\zeta-xz)+\zeta^2\xi U\right)+(S')^2+(U')^2\left(2\zeta^2\xi^2+x^2z^2\right)\right)$$

$$\times\ \frac{(2\pi)}{(1/\zeta)U'+\xi S+(1/z)T'}\delta\left(x-\frac{(\xi/\zeta)Q^2-(1/z\zeta)S'+(\xi/z)U}{(1/\zeta)U'+\xi S+(1/z)T'}\right)$$

Outline

- Motivation
- Deep Inelastic Scattering
  - Calculations
- Semi-Inclusive Deep Inelastic Scattering
  - Standard Approach
  - Puzzles from Standard Approach
- QED Effects
  - **Derivation of Perturbative Coefficients**
  - Cross Section Comparison



Helicity Basis

- Working with helicity-based hadronic structure functions (similar to lepton case in Liu et al., 2021)
- Vectors defined as in Ji et al., 2006

$$\hat{W}^{\mu\nu} = \underbrace{\frac{1}{2}\left(\hat{X}^\mu\hat{X}^\nu + \hat{Y}^\mu\hat{Y}^\nu\right)\hat{H}_{TT}}_{\text{Transverse}} + \underbrace{\left(\hat{T}^\mu\hat{T}^\nu\hat{H}_L + \left(\hat{T}^\mu\hat{X}^\nu + \hat{X}^\mu\hat{T}^\nu\right)\hat{H}_\Delta + \dots\right)}_{\text{other, }\mathcal{O}\left(\alpha_S^2\right)}$$

$$Z^\mu = -\frac{\hat{q}^\mu}{\hat{Q}}$$

$$T^\mu = \left(\frac{1}{\hat{Q}}\right)\left(\hat{q}^\mu + 2\hat{x}_B P^\mu\right)$$

$$X^\mu = \left(\frac{1}{\vec{\hat{q}}_T}\right)\left(\frac{(P')^\mu}{z_B} - \hat{q}^\mu - \left(1 + \frac{\vec{\hat{q}}_T^2}{\hat{Q}^2}\right)\hat{x}_B P^\mu\right)$$

$$Y^\mu = \epsilon^{\mu\nu\alpha\beta} Z_\nu T_\alpha X_\beta$$

Perturbative Coefficients in Helicity Basis

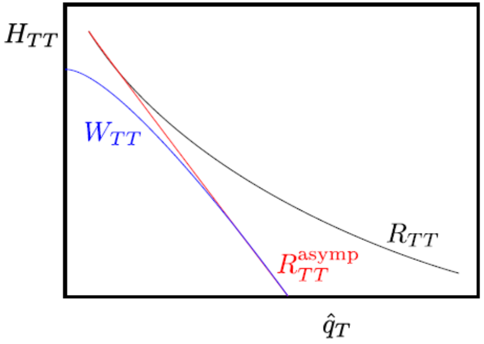
- Fixed order projection onto partonic states
- Separate into real and virtual terms
- Separate based on momentum scale

$$H_{TT} = \frac{1}{2}(X^\mu X^\nu + Y^\mu Y^\nu)W_{\mu\nu}$$

$$W_{\mu\nu} = R_{\mu\nu} + V_{\mu\nu}$$

$$H_{TT}(\vec{\hat{q}}_T) = W_{TT}(\text{small } \vec{\hat{q}}_T) + Y_{TT}(\text{large } \vec{\hat{q}}_T)$$

$$Y_{TT} = R_{TT} - R_{TT}^{\text{asymp}}$$



CSS-like Formalism

- Similar process as in CSS formalism (Collins et al., 1985)

$$\tilde{W}_{TT}(\vec{b}_T, Q) = \int \mathrm{d}^2\vec{\hat{q}}_T e^{i\vec{\hat{q}}_T \cdot \vec{b}_T} W_{TT}(\vec{\hat{q}}_T, Q) = e^{-S}[C_f \otimes f] \otimes [C_D \otimes D]$$

- Expanding in powers of α_S

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

$$S = 1 - \frac{\alpha}{\pi}\left[\frac{1}{2}A^{(1)} \ln^2 \frac{\nu_Q^2}{\mu_b^2} + B^{(1)} \ln \frac{\nu_Q^2}{\mu_b^2}\right]$$

NLO Perturbative Coefficients

$$\tilde{W}_{TT} = C_f^{(1)} C_D^{(0)} S^{(0)} + C_f^{(0)} C_D^{(1)} S^{(0)} + C_f^{(0)} C_D^{(0)} S^{(1)}$$

- Comparing the fixed order calculation with the above perturbative expansion:

$$A^{(1)} = 1$$

$$B^{(1)} = -\frac{3}{2}$$

$$C_f^{(1)}(\lambda) = \frac{1}{2\lambda}(1-2\lambda) - \frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+ \ln\frac{\mu_{\bar{M}S}}{\mu_b} - \delta(1-\lambda)$$

$$C_D^{(1)}(\eta) = \frac{1}{2\eta}(1-2\eta) - \frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+ \ln\frac{\mu_{\bar{M}S}}{\mu_b} - \delta(1-\eta)$$

Transformation Between Lab Frame and Virtual Breit Frame

- Traditionally Breit frame is photon-hadron frame
- Lepton radiation makes frame determination ambiguous
- All historical factorization formula defined in photon hadron frame
- Introduce **virtual** photon-hadron frame which is determined by a given pair of ξ,ζ under one-photon exchange approximation

$$x^\mu = (x^+, x^-, \vec{x}_\perp = (x^1, x^2))$$

$$\tilde{x}^\sigma = R^\sigma_\nu \Lambda^\nu_\mu x^\mu$$

$$y \Rightarrow \tilde{y} = y$$

$$\begin{pmatrix} x \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} \tilde{x} \\ \tilde{z} \end{pmatrix} = R(\theta(\xi, \zeta)) \begin{pmatrix} x \\ z \end{pmatrix}$$

$$\theta_{\text{No Radiation}} = \arctan\left(\frac{-E' \sin\theta_{L,L'}}{E - E' \cos\theta_{L,L'}}\right)$$

$$\theta_{\text{With Radiation}} = \arctan\left(\frac{-E' \sin\theta_{L,L'}}{(\xi\zeta)E - E' \cos\theta_{L,L'}}\right)$$

$$\phi = \tanh^{-1}\left(-\sqrt{\frac{S}{4m^2 + S}}\right)$$

Full W term

$$\tilde{W}_{TT} = \frac{4e_Q^2}{3}\left(\frac{\alpha_S}{\pi}\right)\Bigg[\delta(1-\lambda)\delta(1-\eta)\left(-\frac{1}{2}\left(\ln^2\frac{\nu_Q^2}{\mu_b^2} - 3\ln\frac{\nu_Q^2}{\mu_b^2}\right)\right)$$

$$-\ln\frac{\mu_{\bar{M}S}}{\mu_b}\left(\frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+\delta(1-\eta) + \frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+\delta(1-\lambda)\right)$$

$$+\frac{1}{2}\left(\left(\frac{1-2\lambda}{\lambda}\right)\delta(1-\eta) + \left(\frac{1-2\eta}{\eta}\right)\delta(1-\lambda)\right) - 2\delta(1-\lambda)\delta(1-\eta)$$

$$-\frac{1}{2\epsilon}\left(\frac{1}{\lambda}\left(\frac{\lambda^2+1}{1-\lambda}\right)_+\delta(1-\eta) + \frac{1}{\eta}\left(\frac{\eta^2+1}{1-\eta}\right)_+\delta(1-\lambda) + \delta(1-\lambda)\delta(1-\eta)\right)\Bigg]$$

Intuitive Model

- In order to highlight the QED radiation effects, introduce intuitive model
- Generic normalized cross section

$$\int_0^\infty \mathrm{d}q_T f(q_T) = 1$$

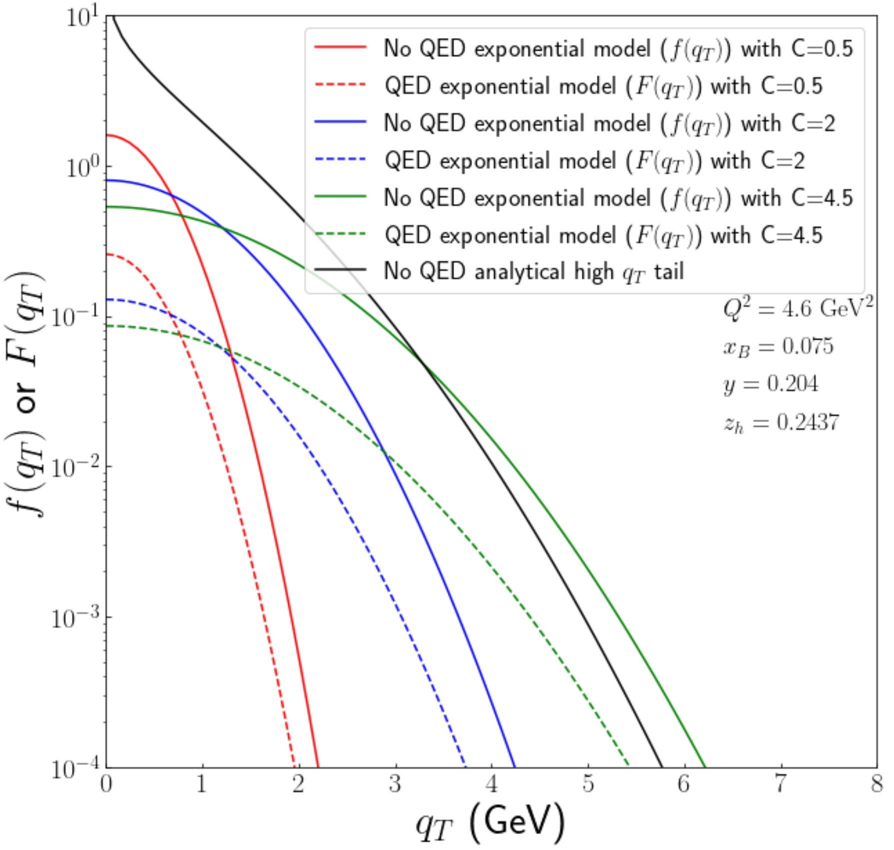
- Key ξ,ζ dependence modeled as

$$\hat{q}_T^2 = \frac{\xi}{\zeta} q_T^2$$

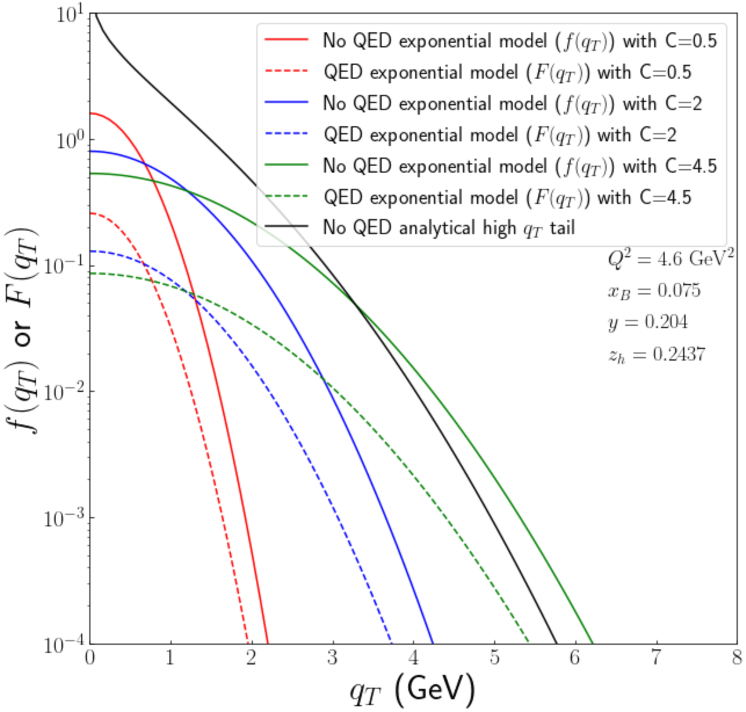
- QED cross section then

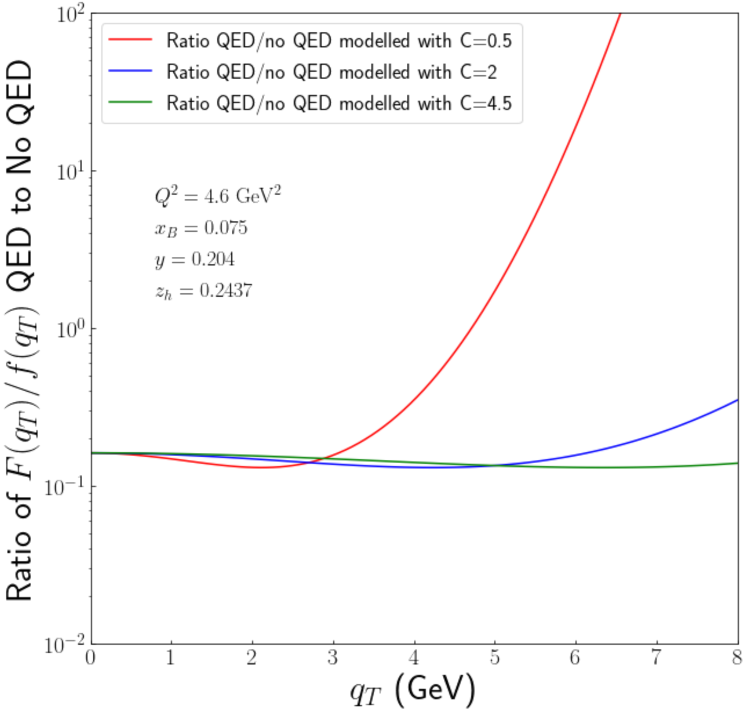
$$F(q_T) = \int \mathrm{d}\zeta \sqrt{\zeta^{-1}} \mathrm{d}\xi \sqrt{\xi} \tilde{f}(\xi) \tilde{D}(\zeta) f(\hat{q}_T)$$

- Gaussian model $f(q_T) = \frac{1}{\sqrt{\pi C}} \exp - \frac{q_T^2}{C}$



Intuitive Model Ratio



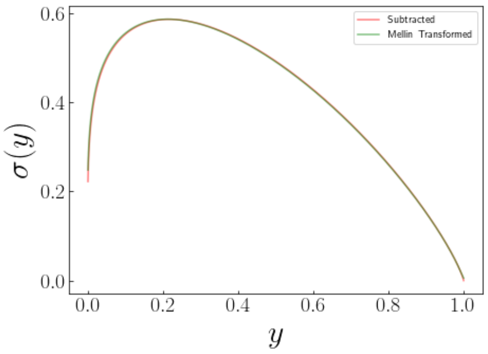


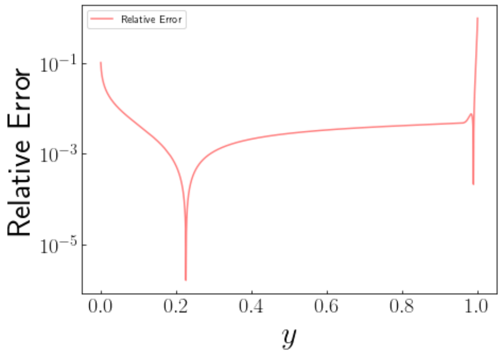
Steeper functions see a larger effect from QED radiation

Subtraction Method

- Endpoints not well behaved when computing evolution
- Separate out endpoint and use Mellin transformation to calculate those regions

$$\sigma(y) = \int_y^1 \mathrm{d}x f(x) \left(\frac{H\left(\frac{y}{x}\right)}{x} - H(y) \right) + H(y) y \frac{1}{\pi} \mathrm{Im} \left(\int_0^{c+i\infty} \mathrm{d}N y^{-N} \frac{\tilde{f}(N)}{N-1} \right)$$





Structure Functions Considered

- 4 unpolarized structure functions in SIDIS cross section
- 2 depend on hadronic angle
  - Possible to include from fixed order calculations
  - Ignored in this study as contributions are small (~1%)

Structure Function	W term	FO term
$F_{UU,T}$	Yes	Yes
$F_{UU,L}$	No	Yes
$F_{UU}^{\cos\phi_h}$	No	Possible
$F_{UU}^{\cos 2\phi_h}$	No	Possible