Extracting α_s using spin Structure function at ECCE

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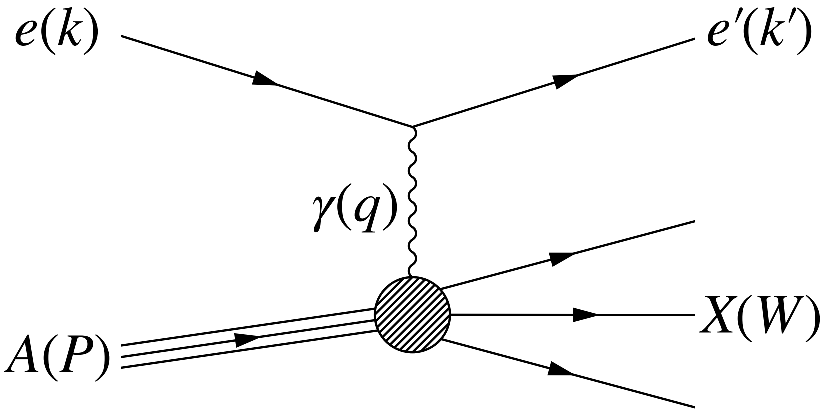
CFNS, Stony Brook University, May 6th, 2025

Part of this work done with: T. Kutz (MIT), J. R. Pybus (MIT), D. W. Upton (UVA, ODU), C. Cotton (UVa), A. Deshpande (CFNS, Stony Brook U.), W.B. Li (CFNS, Stony Brook U.), M. Nycz (UVa), X. Zheng (UVa), and the former ECCE Consortium (now part of the ePIC Collaboration).

Inclusive Deep inelastic scattering



Resolution
4-momentum transfer square
$Q^2 = -q^{\mu} q_{\mu} = |\vec{q}|^2 - \omega^2$

Number involved nucleons
$x_b = Q^2/2m_p\omega$

$$\sigma = \sigma_{\mathrm{Mott}}[\alpha F_1(x,Q^2) + \beta F_2(x,Q^2) + \gamma g_1(x,Q^2) + \varpi g_2(x,Q^2)]$$

pointlike scattering×(spin independent + spin dependent)

Structure functions:

- ❑ $F_{1,2}$: Unpolarized Structure functions
- ❑ $g_{1,2}$: Polarized Structure functions

Approaches to extract α_s from spin structure functions

- Q^2-evolution of $g_1(x, Q^2)$: Complex task
    - Involves DGLAP global fit,
    - non-perturbative inputs: quark and gluon distributions,
    - possibly higher-twists for low Q^2/ large-x data.

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- Q^2-evolution of moment $\int g_1(x, Q^2)dx$: Simpler
  - No x-dependence,
  - Non-perturbative inputs: more-or-less well measured axial charges a_0, a_3 and a_8 +) possibly higher-twists for low-Q^2 data).
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- Q^2-evolution of iso-vector moment $\int g_1^{p-n}(x, Q^2)dx$: Simplest
  - Axial charge $a_3 = g_A$ precisely measured ($g_A = 1.2762 \pm 0.0005$)
  - DGLAP-evolution known to higher order than single nucleon case
  - No gluon contribution.
  - Issue: But low-x issue and demands measurement on polarized p and n.

Bjorken Sum Rule

$$\Gamma_1^{p-n} \equiv \int g_1^{p-n} dx = \frac{1}{6} g_A [1 - \frac{\alpha_s}{\pi} - 3.58(\frac{\alpha_s}{\pi})^2 - 20.21(\frac{\alpha_s}{\pi})^3 - 175.7(\frac{\alpha_s}{\pi})^4 - \sim 893(\frac{\alpha_s}{\pi})^5]$$

Nucleon's First spin structure function

Nucleon Axial charge $[\Gamma_1^{p-n}(Q^2 \to \infty)]$

pQCD radiative correction

Bjorken Sum Rule

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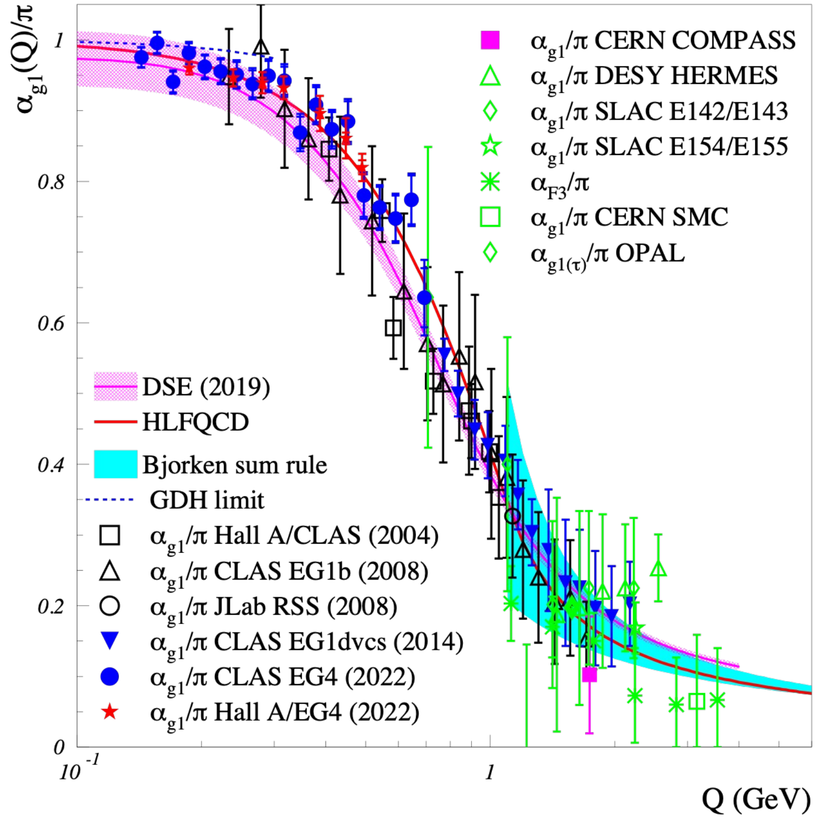
Two possibilities to extract $\alpha_s(M_z)$

- ❑ Do an absolute measurement of Γ_1^{p-n} and solve BJSR for $\alpha_s(Q^2)$
  - One α_s per Γ_1^{p-n} experimental data point
  - Poor systematic uncertainty
  - Typical $\frac{\Delta\alpha_s}{\alpha_s} \sim 10\%$



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Two possibilities to extract $\alpha_s(M_z)$

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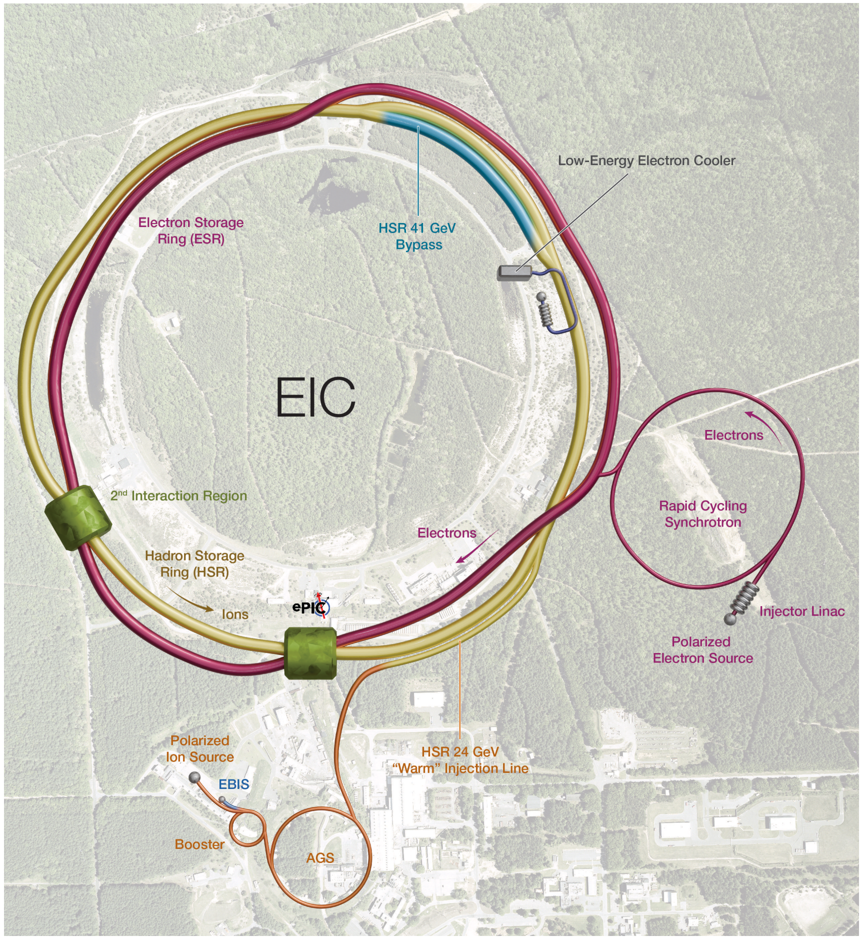
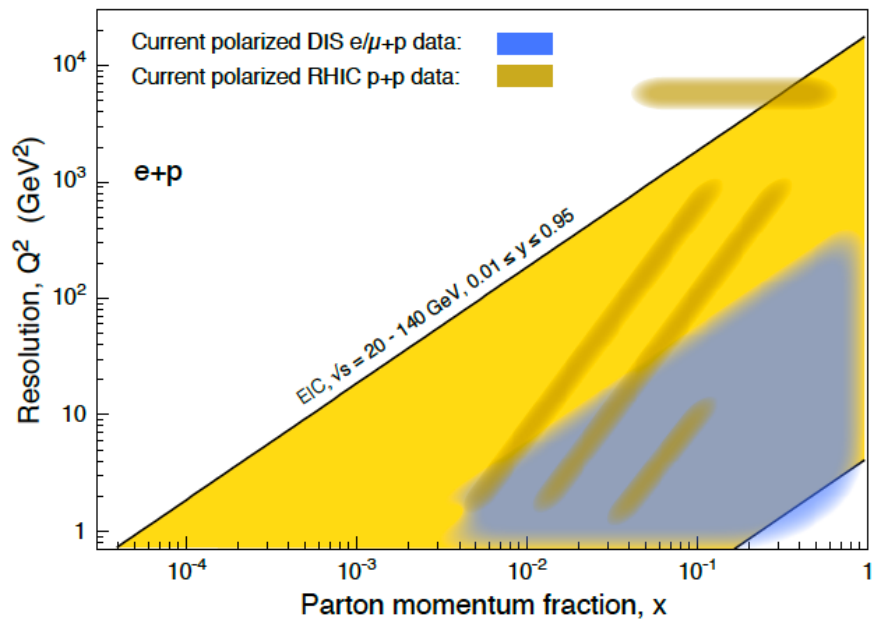
- Measurement of Q^2 dependence of $\Gamma_1^{p-n}(Q^2)$

  - Need Γ_1^{p-n} at several Q^2 points, Only one (or few) value of α_s
  - Good accuracy
  - 1990's CERN/SLAC data yielded: $\alpha_s(M_z) = 0.12 \pm 0.009$
    Nucl.Phys. B496 337 (1997)

Possible future extractions of α_s from $\Gamma_1^{p-n}(Q^2)$



Nucl. Phys. A 1026, 122447



Extracting α_s using spin structure function at ECCE

See Win Lin's talk for ePIC simulation

Event generator
[DJANGOH]

↓

EIC simulation
[ECCE]

↓

Pseudo-data
[pDIS, ep, e3He]

↓

Spin structure function
[g_1^p, g_1^n]

↓

Brorken Sum rule
[$\Gamma_1^{p-n}(Q^2)$]

↓

Extract α_s

Extracting α_s using spin structure function at ECCE

- Simulated data
  - DIS events generated using DJANGOH for both *e-p* and *e-*^{3}*He*
  - For e-p: 5x41 GeV, 10x100 GeV, 18x275 GeV
  - For e-3He: 5x41 GeV, 10x100 GeV, 18x166 GeV
  - Integrated luminosity of 10 fb^{-1}

Note: Neutron structure function is extracted from e-3He using double tagging measurement that significantly minimize the nuclear correction.

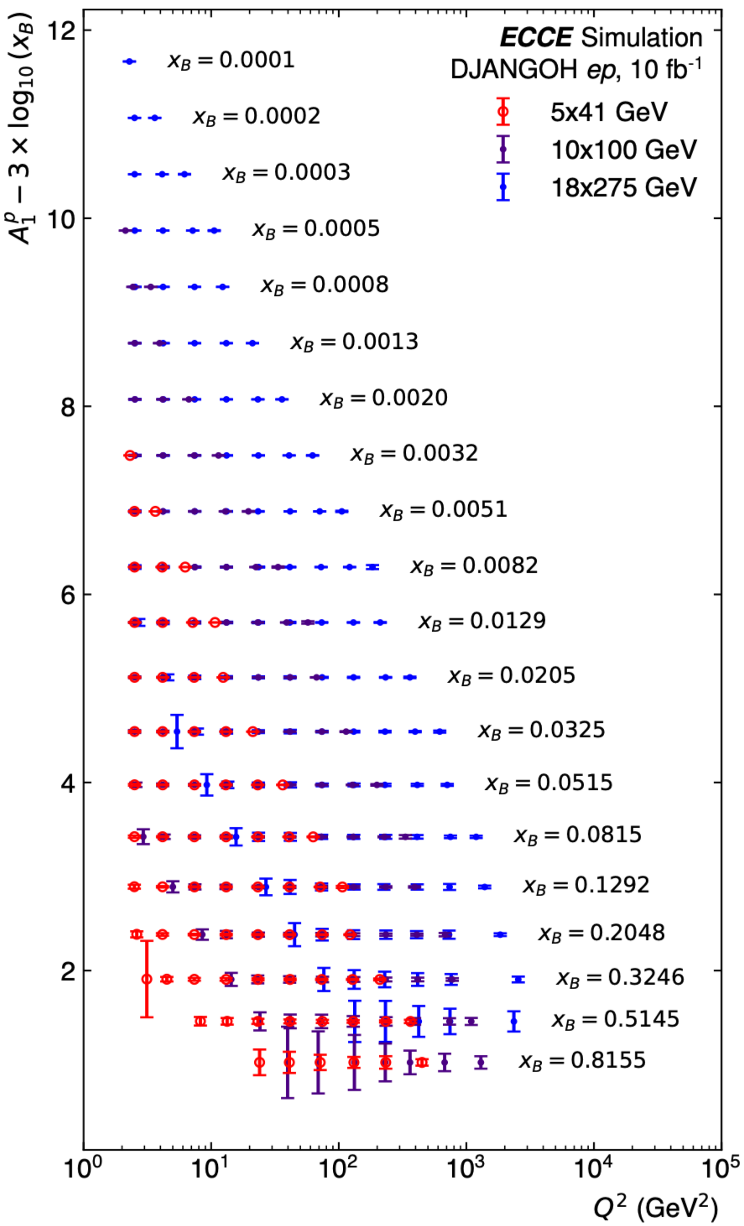
- Monte Carlo simulation using ECCE configuration
  - Including detector effects: acceptance, resolution, efficiencies
  - Provide psesudo – data for analysis
- Data analysis
  - Asymmetries: $A_{||,\perp}$, A_1
  - Polarized structure function: g_1^p, g_1^n
  - Bjorken Sum Rule: $[\Gamma_1^{p-n}(Q^2)]$
  - Ready for fitting

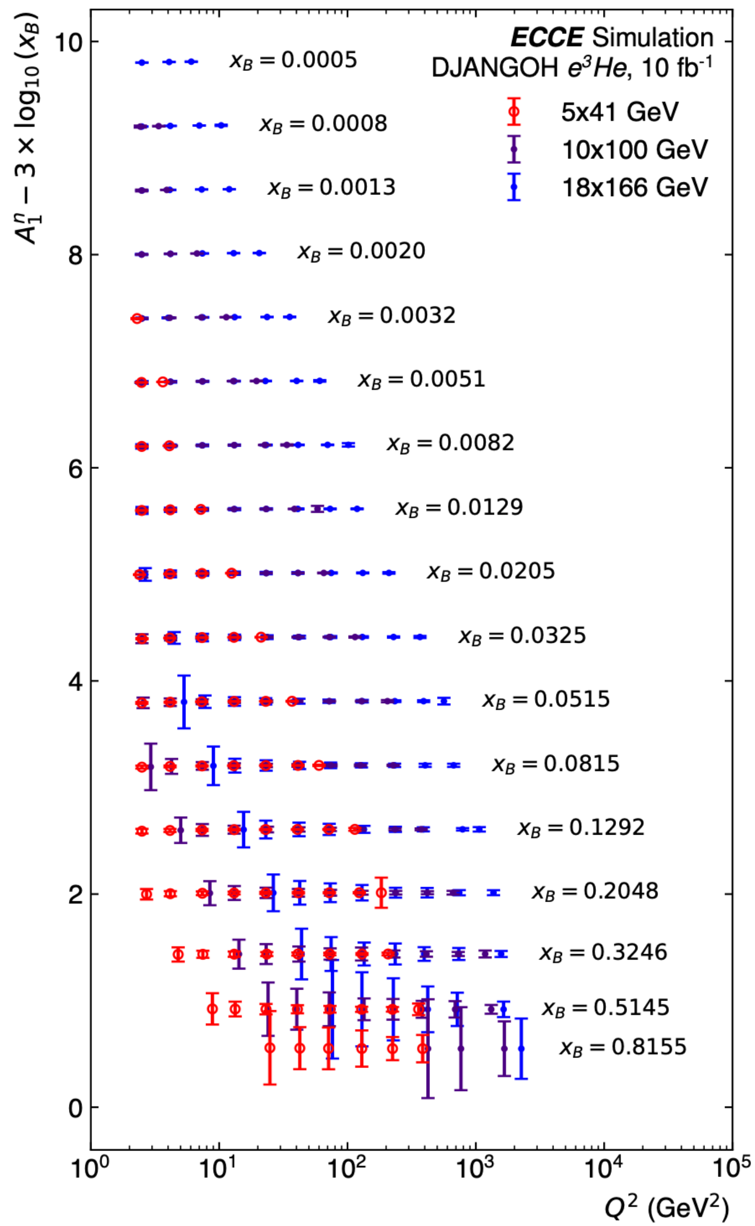
Uncertainties

- Statistics

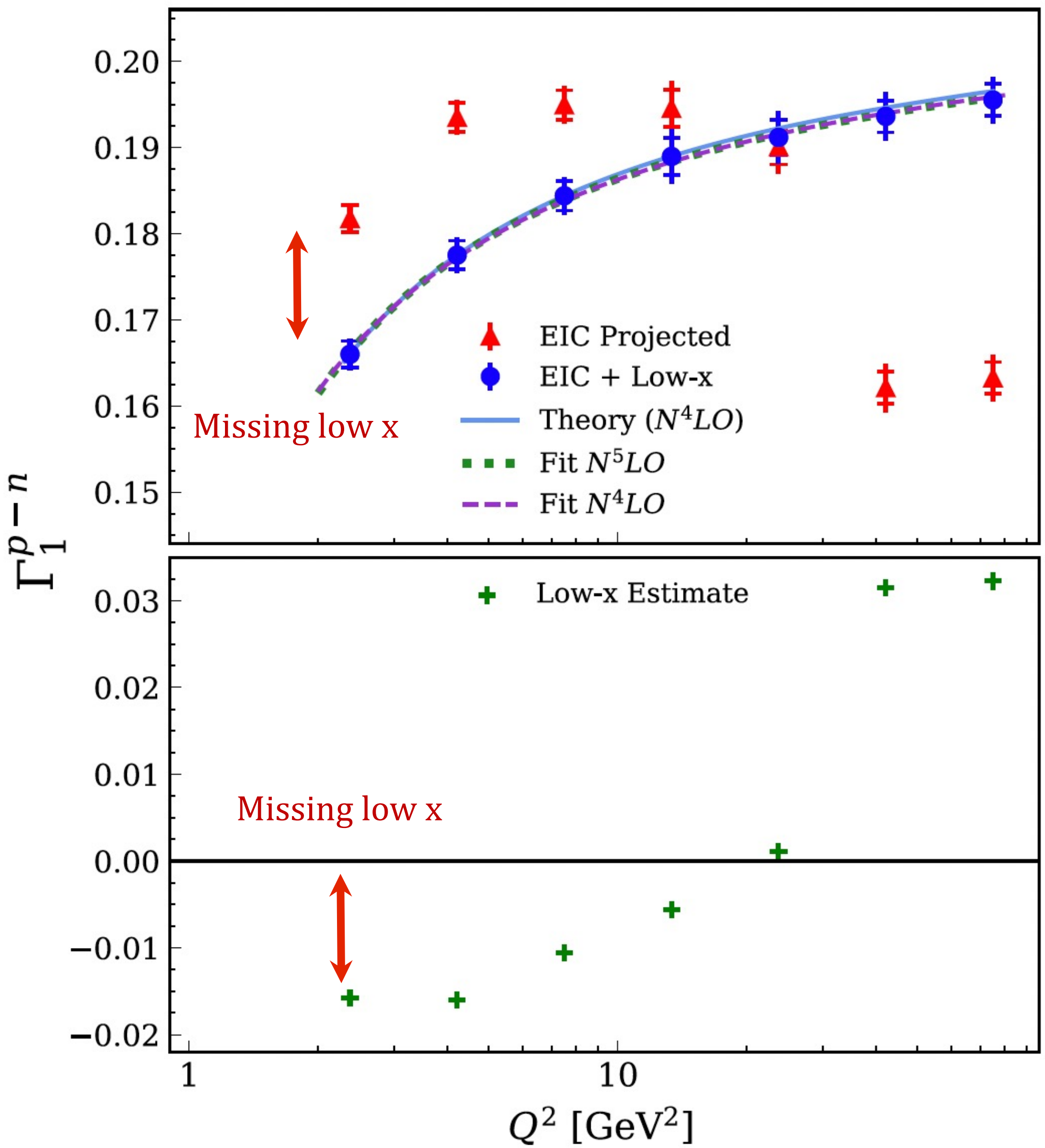
- Systematics
  - Detector effects,
  - Beam polarimetries,
  - Radiative corrections,
  - Missing high- and low-x part,
  - PDF parameterizations;
  - Negligible: neutron information

EIC: generated pseudo-data





Measured fraction of the Bjorken sum $\Gamma_1^{p-n}(Q^2)$



Extraction of $\alpha_s(M_Z)$

Fit and procedure:

- Main fit function: Bjorken sum approximant at N^{4}LO with α_s at 4-loop (i.e β_3), for main result.

$$\int\Gamma_1^{p-n} = \frac{1}{6}g_A[1 - \frac{\alpha_s}{\pi} - 3.58(\frac{\alpha_s}{\pi})^2 - 20.21(\frac{\alpha_s}{\pi})^3 - 175.7(\frac{\alpha_s}{\pi})^4]$$

$$\alpha_s^{\overline{\mathrm{MS}}}(Q) = \frac{4\pi}{\beta_0\ln(Q^2/\Lambda_s^2)}[1 - \frac{\beta_1}{\beta_0^2}\frac{\ln(\ln(Q^2/\Lambda_s^2))}{\ln(Q^2/\Lambda_s^2)} +$$
$$\frac{\beta_1^2}{\beta_0^4\ln^2(Q^2/\Lambda_s^2)}(\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1 + \frac{\beta_2\beta_0}{\beta_1^2}) +$$
$$\frac{\beta_1^3}{\beta_0^6 ln^3(Q^2/\Lambda_s^2)}(-\ln^3(\ln(Q^2/\Lambda_s^2)) + \frac{5}{2}\ln^2(\ln(Q^2/\Lambda_s^2)) + 2\ln(\ln(Q^2/\Lambda_s^2)) -$$
$$\frac{1}{2} - 3\frac{\beta_2\beta_0}{\beta_1^2}\ln(\ln(Q^2/\Lambda_s^2)) + \frac{\beta_3\beta_0^2}{2\beta_1^3}) + \frac{\beta_1^4}{\beta_0^8 ln^4(Q^2/\Lambda_s^2)}(\ln^4(\ln(Q^2/\Lambda_s^2)) -$$
$$\frac{13}{3}\ln^3(\ln(Q^2/\Lambda_s^2)) - \frac{3}{2}\ln^2(\ln(Q^2/\Lambda_s^2)) + 4\ln(\ln(Q^2/\Lambda_s^2)) + \frac{7}{6} +$$
$$\frac{7}{6} + \frac{3\beta_2\beta_0}{\beta_1^2}(2\ln^2(\ln(Q^2/\Lambda_s^2)) - \ln(\ln(Q^2/\Lambda_s^2)) - 1) -$$
$$\frac{\beta_3\beta_0^2}{\beta_1^3}\left(2\ln(\ln(Q^2/\Lambda_s^2)) + \frac{1}{6}\right)]$$

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a)

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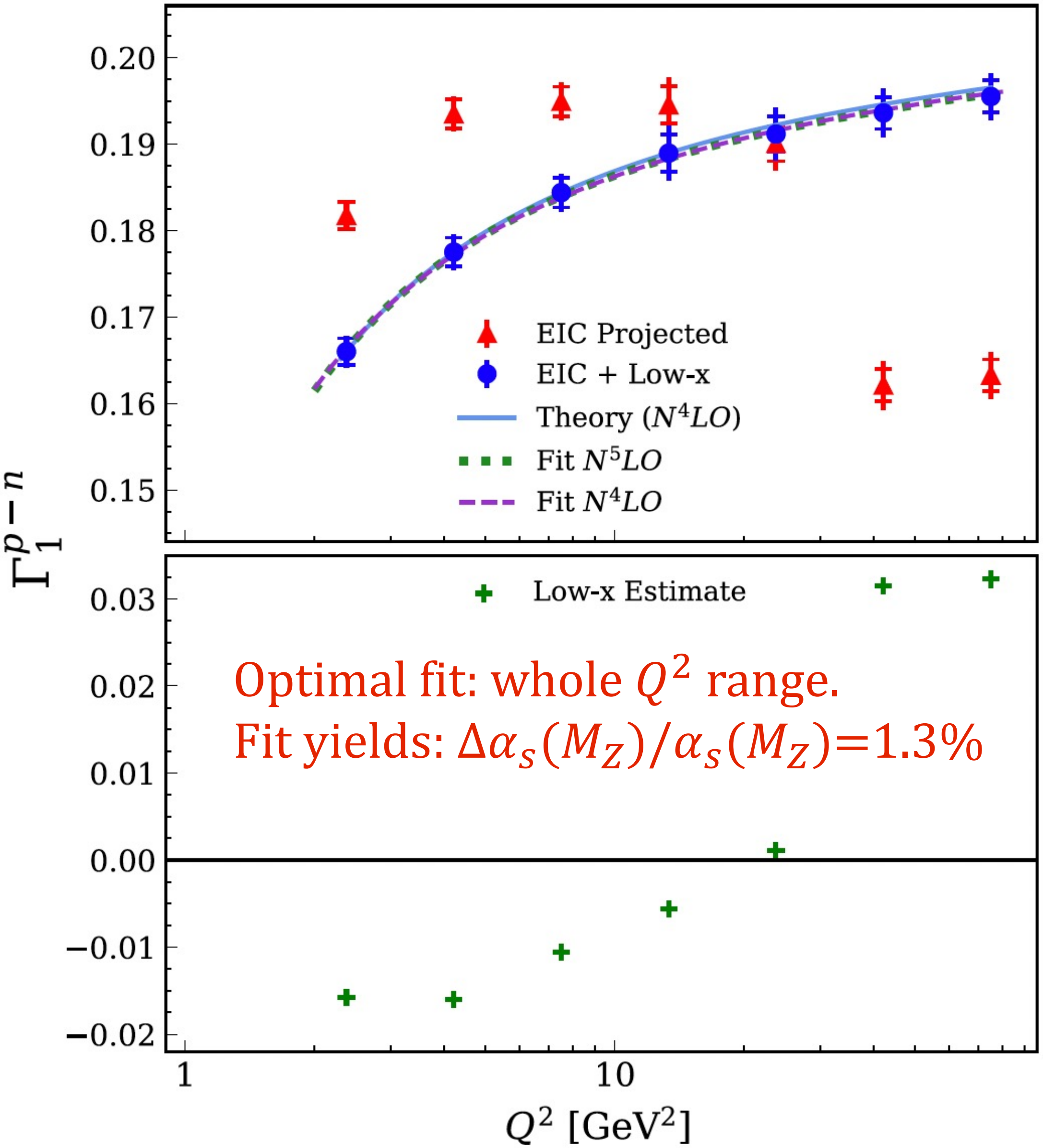
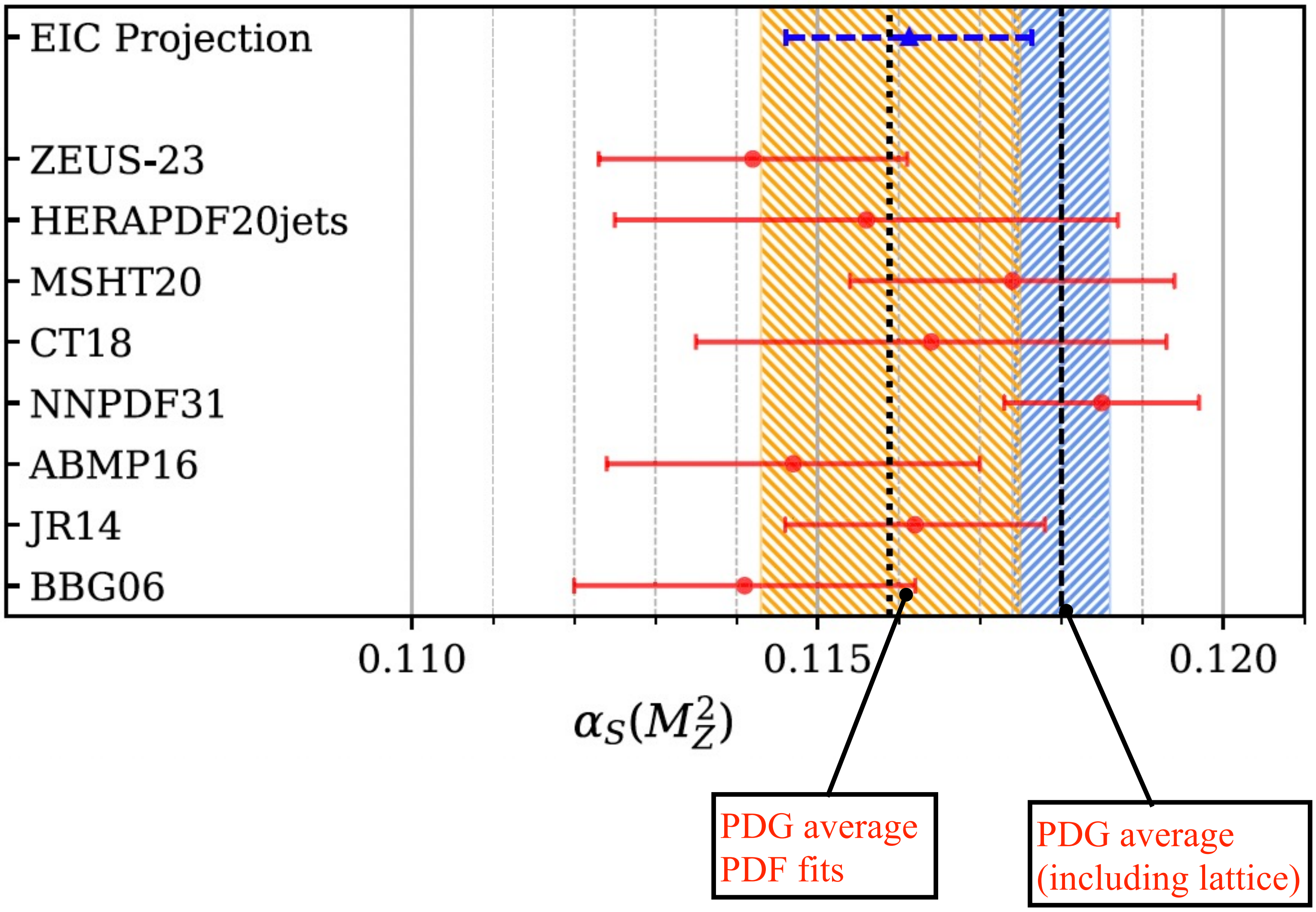
- Secondary fit at N^{5}LO and α_s at 5-loop, for pQCD truncation uncertainty.
- Systematically vary fit Q^2 range to minimize total uncertainty: Low Q^2 points have high α_s sensitivity but larger pQCD truncation error. High Q^2 points have smaller α_s sensitivity but smaller pQCD error. May not be worth including the lowest and/or highest Q^2 points. (Not worth using all points for statistics sake since stat. error is negligible.)
- 2-parameter fit:
  - Λ_s is the free parameter of interest. From it, we obtain $\alpha_s(M_z)$.
  - g_A Well-known but left as a free to account for normalization uncertainties.

Extraction of $\alpha_s(M_Z)$

$1.31\% = 0.83\%(\text{exp.}) \oplus 0.64\%(\text{truncation}) \oplus 0.78\%(\text{polarimetries})$



Compared to other DIS results and world average (from PDG)



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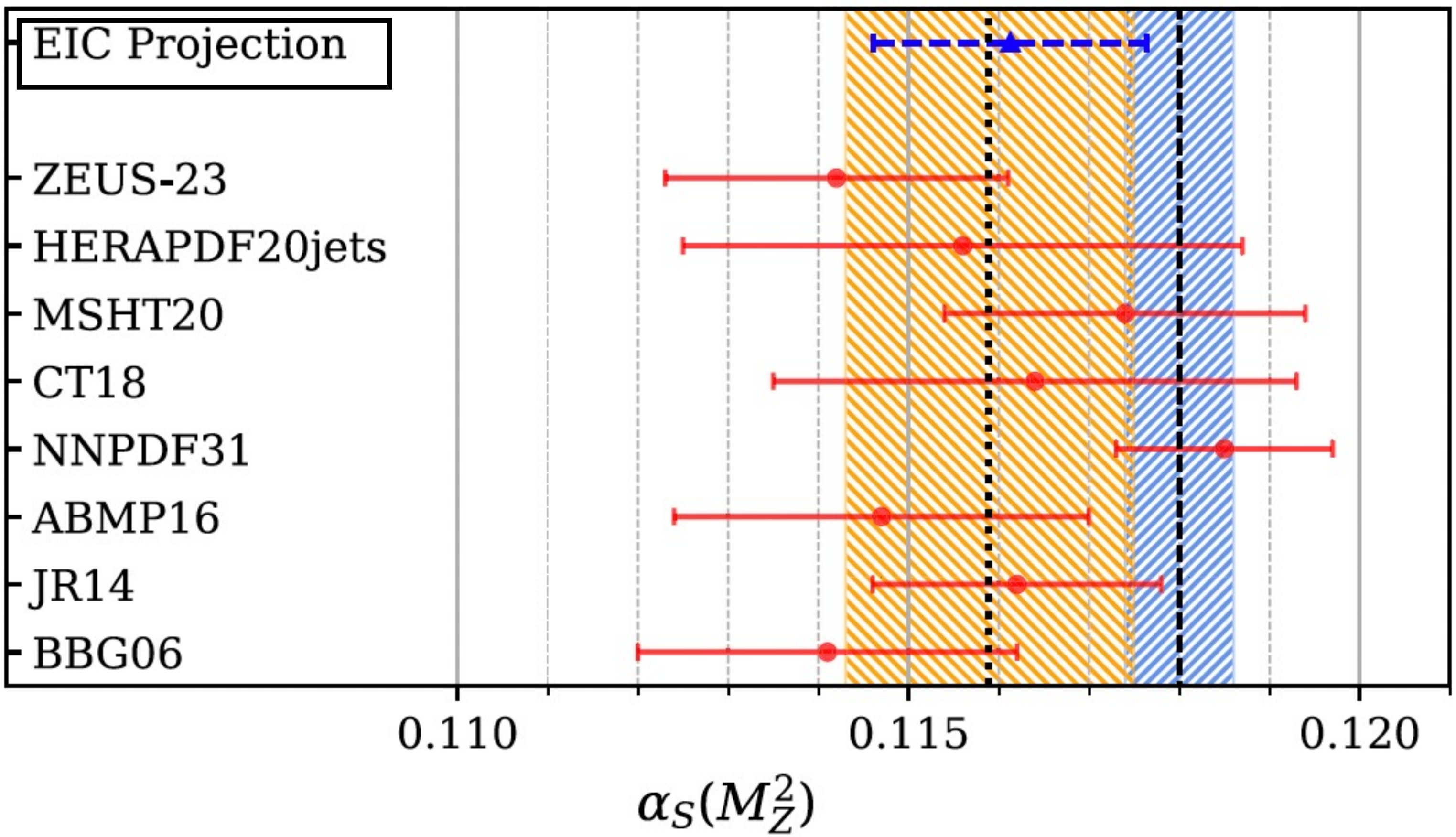

Conclusion:

- Realistic simulation shows that EIC can yield a competitive measurement.
- Just one method. Other extractions will be available, e.g.:
  - Global fits (unpolarized and polarized)
  - Inclusive neutral current reactions (EIC+HERA). S. Cerci, *et al.* EPJC, 83(11):1011, 2023: $\Delta\alpha_s(M_Z)/\alpha_s(M_Z)$=0.4%