Extracting the gluon density at the Electron-Ion Collider from F_L measurements

Javier Jiménez-López

Physics and Mathematics department, University of Alcalá

Extracting the Strong Coupling at the EIC and other Future Colliders

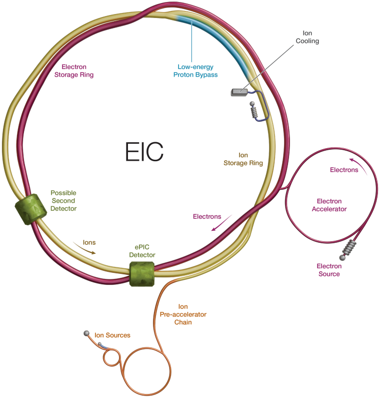
$5 - 7$ May 2025

Outline of the Talk

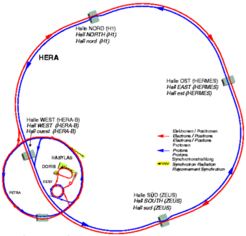
- **The Electron-Ion Collider**
- **The Altarelli-Martinelli relation**
- **Extracting F_L at EIC**
- **The strong coupling constant α_s**
- **Results for the gluon density $xg(x, Q^2)$ at EIC**
- **Early measurements of F_L at EIC**
- **Conclusions**

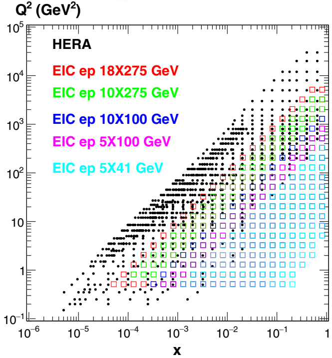
The Electron-Ion Collider

The Electron-Ion Collider



ePIC





The Altarelli-Martinelli relation

The Altarelli-Martinelli relation

The gluon PDF plays a crucial role in unveiling the inner structure of the proton.

Even though it is common to obtain it via PDF sets, such as HERAPDF or NNPDF, there is a deep connection between the longitudinal structure function F_L and $xg(x, Q^2)$.
This was formalized by Altarelli and Martinelli. Unfortunately, it is not exactly solvable but it is approximated by:

$$xg(x, Q^2) \approx 1.77 \frac{3\pi}{2\alpha_s(Q^2)} F_L(x, Q^2) \tag{1}$$

Then, we need:

- F_L extracted at EIC.
- The strong coupling constant at different energies.

Extracting F_L at EIC

Outcome of the DESY 2024 Summer Student Program

PHYSICAL REVIEW D **111,** 056014 (2025)

Prospects for measurements of the longitudinal proton structure function F_L at the Electron Ion Collider

Javier Jiménez-López*

Universidad de Alcalá, Departamento de Física y Matemáticas, Facultad de Ciencias, 28805 Alcalá de Henares, Madrid, Spain

Paul R. Newman

School of Physics and Astronomy, University of Birmingham, Birmingham B15 2TT, United Kingdom

Katarzyna Wichmann

Deutsches Elektronen-Synchrotron DESY, Germany

Phys. Rev. D 111, 056014

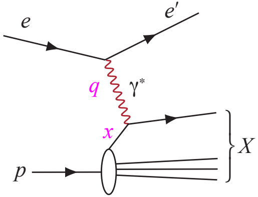
The longitudinal structure function F_L

The reduced cross-section of NC DIS for $Q^2 \ll M_Z^2$ is:

$$\sigma_r(x, Q^2, y) = F_2(x, Q^2) - \frac{y^2}{Y_+} F_L(x, Q^2) \quad (2)$$

where:

- x: Bjorken scaling variable,
- y: inelasticity,
- Q^2: virtuality of the process,
- $Y_+ = 1 + (1 - y)^2$.



As x and Q^2 are known $\Rightarrow$ F_L and F_2 can be obtained with a linear fit.

This is the well-known Rosenbluth-type separation technique, used by multiple research groups.

Pseudo-data simulation

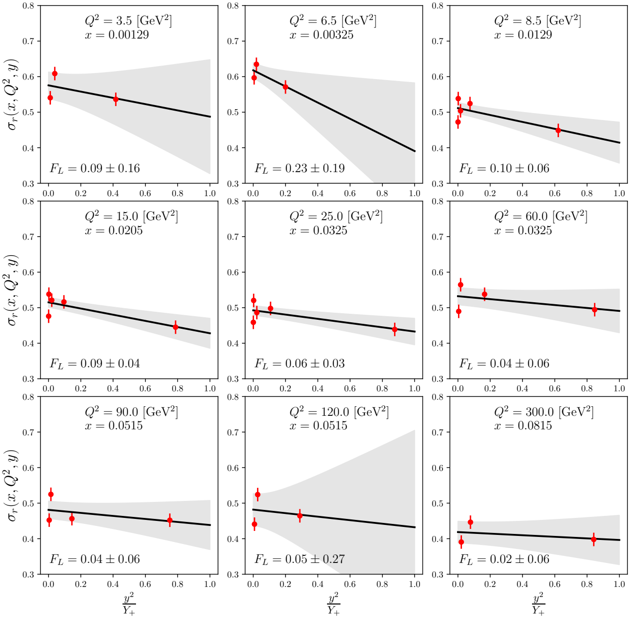
The extraction was done with the 5 main beam configurations:

e-beam energy (GeV)	p-beam energy (GeV)	$\sqrt{s}$ (GeV)	Integrated lumi (fb^{-1})
18	275	141	15.4
10	275	105	100
10	100	63	79.0
5	100	45	61.0
5	41	29	4.4

- Reduced cross-sections generated with HERAPDF2.0 NNLO.
- Smearing procedure for two different uncertainty scenarios:
  - Conservative scenario: $1.9\,\%$ of correlated systematics and $3.4\,\%$ of uncorrelated systematics $\implies$ total uncertainty of $3.9\,\%$,
  - Optimistic scenario: total uncertainty of $1\,\%$.

Note that the uncertainty due to the normalisation between different beam energies is not considered.

Example fits



MC replica method

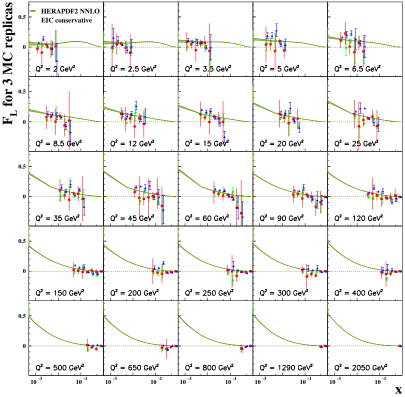
In order to sample the distribution of possible outcomes for F_L values and uncertainties, 1000 replicas of the data set are analysed.

The method was adapted from: Diffractive longitudinal structure function at the Electron Ion Collider, Phys. Rev. D 105, 074006

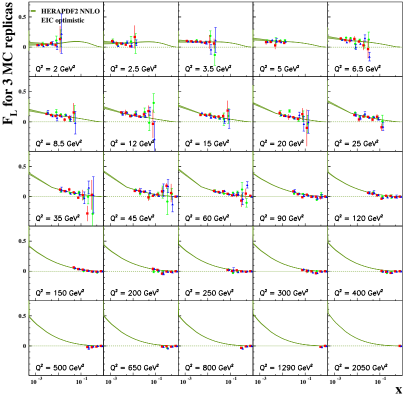
For each pseudo generated data point:

- Performed Gaussian smearing 1000 times,
- Bins whose absolute uncertainty where larger than 0.3 are not considered.
    - This criterion removes around $30\,\%$ of the points for the conservative scenario and $20\,\%$ for the optimistic scenario.
    - These points might be recovered when real data is available.

Example replicas



Conservative scenario



Optimistic scenario

Averaging over MC replicas

In order to get a final measurement of F_L and it's uncertainties, we apply the following averaging procedure:

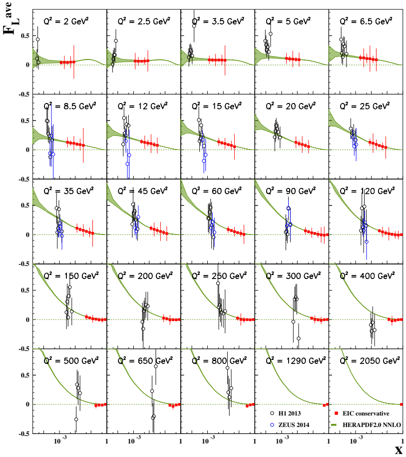
$$\overline{v} = \mathcal{S}_1 \,/\, N \qquad (\Delta v)^2 = \frac{\mathcal{S}_2 - \mathcal{S}_1^2 \,/\, N}{N-1} \tag{3}$$

where:

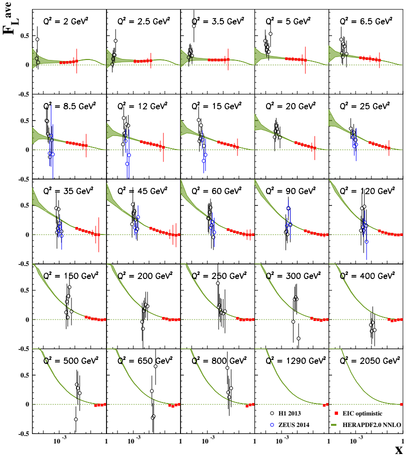
$$\mathcal{S}_n = \sum_{i=1}^{N} v_i^n \tag{4}$$

and v_i stands for extracted value of F_L in the i-th MC replica.

F_L averaged over 1000 MC replicas

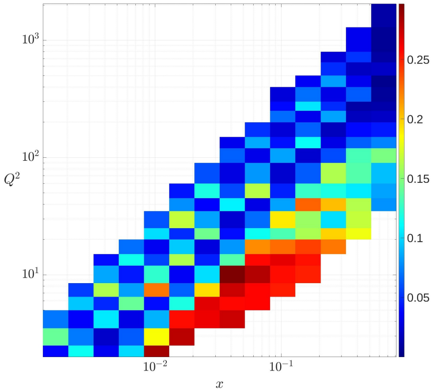


Conservative scenario

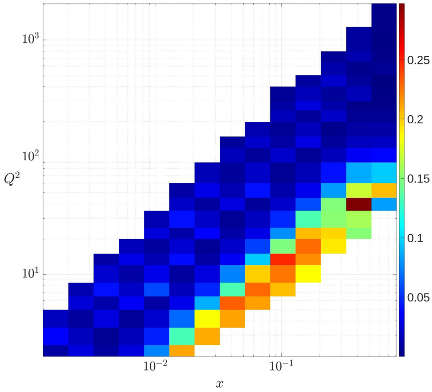


Optimistic scenario

F_L uncertainties with the MC replica method



Conservative scenario

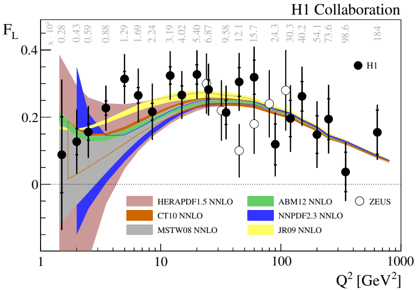


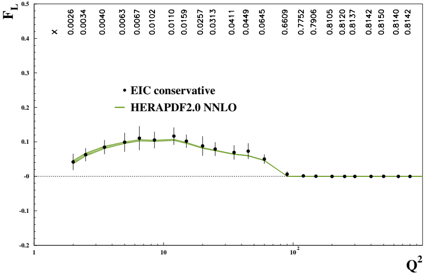
Optimistic scenario

Great precision on F_L measurements, further improved by the optimistic scenario

$F_L(Q^2)$: comparison with HERA

HERA and EIC data cannot be directly compared (x is very different for both).
However, we can compare uncertainties (plots are in the same scale):





Unprecedented precision of F_L measurements at EIC

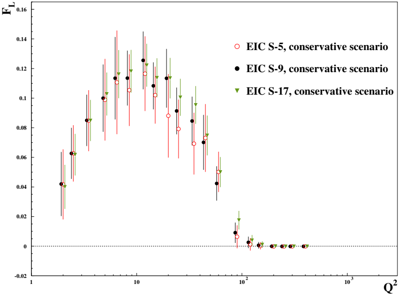
Possible beam energy configurations at EIC

		E_p [GeV]					
		41	100	120	165	180	275
E_e [GeV]	5	**29**	**45**	49	**57**	60	74
	10	40	**63**	69	**81**	**(85)**	**105**
	18	54	**85**	93	**109**	114	**141**

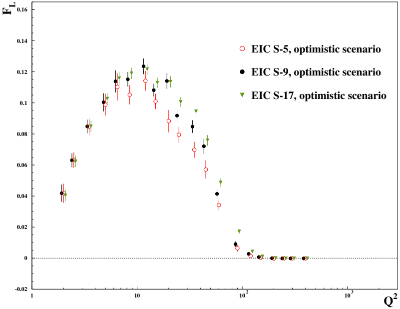
- S-5 is the baseline and is illustrated in green,
- S-9 is obtained by adding the red values,
- S-17 is obtained by adding the rest except for 10×180 $[\text{GeV}^2]$.

Possible improvements

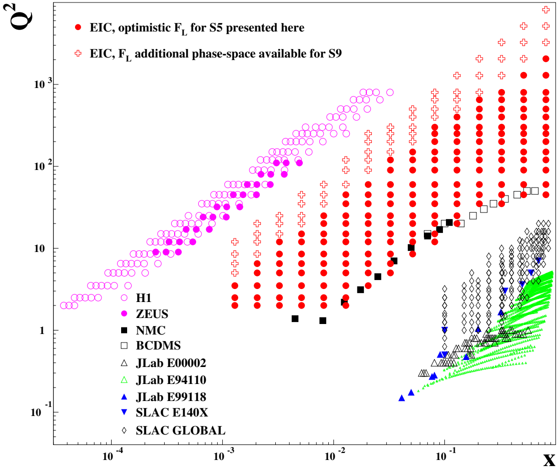
- ➤ F_L measurements will improve significantly with more beam configurations available.
- ➤ The greatest improvement comes from reducing the uncertainties rather than adding more beam energies.



Conservative scenario



Optimistic scenario



EIC has great potential to measure the F_L structure function in unexplored kinematic phase space

The strong coupling constant α_s

The strong coupling constant α_s

The strong coupling constant can be calculated as a function of the energy scale μ, in the $\overline{\text{MS}}$ renormalization scheme, solving the following differential equation:

$$\mu^2 \frac{d}{d\mu^2} \alpha_s^{(n_f)}(\mu) = -\sum_{i \geq 0} \beta_i^{(n_f)} \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi} \right)^{i+2} \tag{5}$$

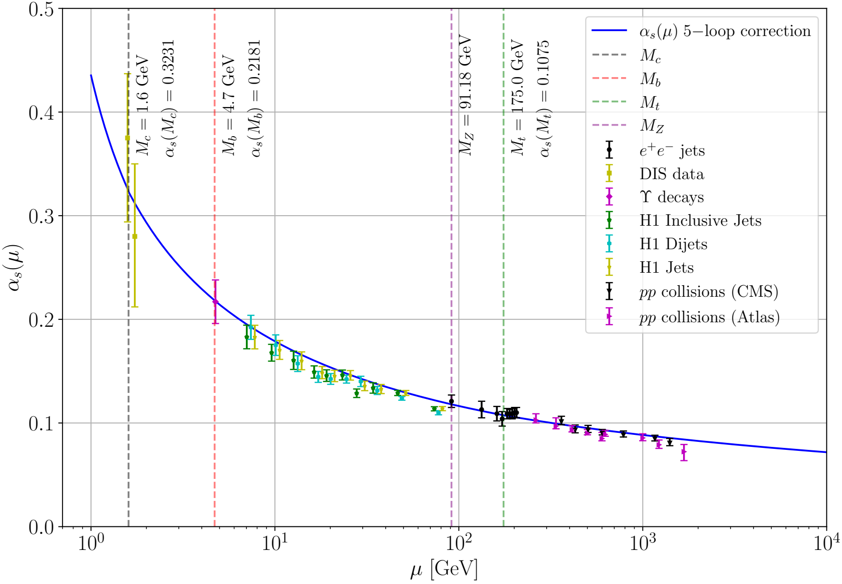
where n_f is the number of active flavours and the coefficients $\beta_i^{(n_f)}$ have been taken from: Phys. Rev. Lett. 118, 082002.

The initial condition to solve the ODE was:

$$\alpha_s(\mu = M_Z) = 0.1180 \pm 0.0009 \tag{6}$$

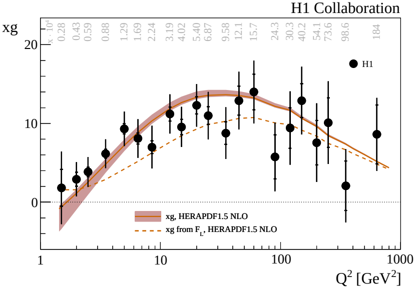
One might also find this paper interesting: RunDec: a Mathematica package for running and decoupling of the strong coupling and quark masses

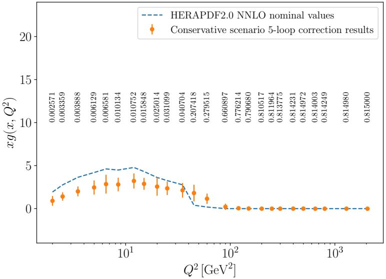
5-loop order $\alpha_s(\mu)$



Results for the gluon density $xg(x, Q^2)$ at EIC

$xg(x, Q^2)$ at EIC



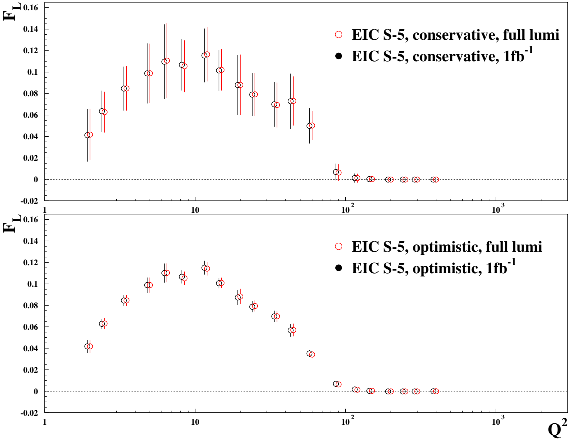


$$\delta\, xg(x, Q^2) = \frac{3}{2}\frac{1.77\pi}{\alpha_s(Q^2)} \sqrt{\alpha_s^2(Q^2)\,(\delta F_L)^2 + F_L^2\,(\delta\alpha_s(Q^2))^2} \qquad (7)$$

The results for $xg(x, Q^2)$ are still preliminary and a final version will be given in the future

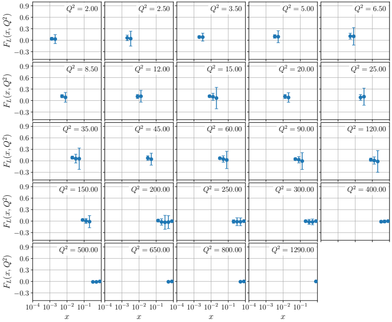
Early measurements of F_L at EIC

F_L with only 1 fb^{-1} (Full HERA statistics)

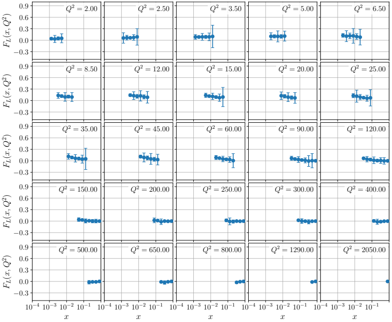


We don't need high luminosity to get $F_L \implies$ Possible early measurement of F_L

F_L averaged bins for low luminosity (conservative scenario)

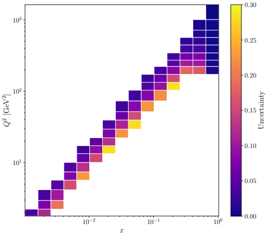


Data obtained with the 5×100, 10×100 and 10×275 [GeV^2] beam energy configurations

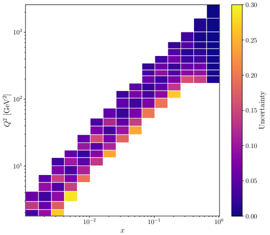


Data obtained with the 5 beam energy configurations

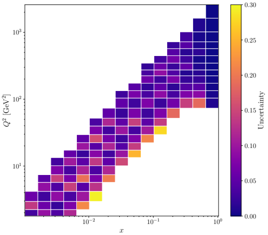
F_L uncertainties for low luminosity



5×100, 10×100 and 10×275 [GeV^2]



5×100, 10×100, 10×275 and 18×275 [GeV^2]

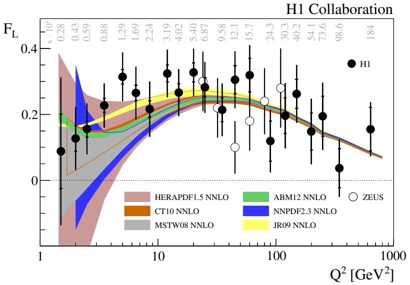


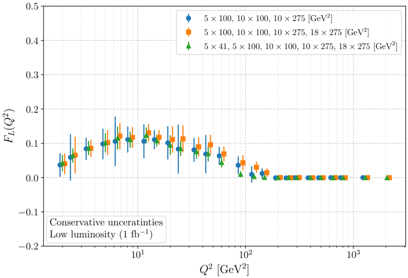
5×41, 5×100, 10×100, 10×275 and 18×275 [GeV^2]

Note: only measurements whose uncertainty is lower than 0.3 are shown.

As more configurations are added, the phase space region where the uncertainties are under our threshold grows

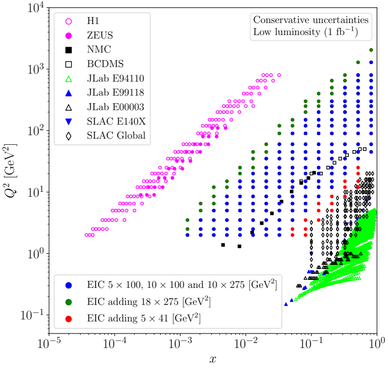
F_L averaged over x for low luminosity



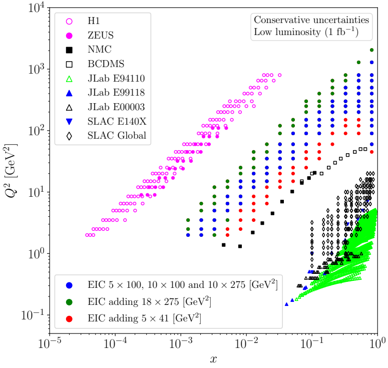


Even with a restricted amount of beam configurations, F_L can be precisely extracted at EIC

Kinematic phase space for low luminosity



Kinematic phase space without an upper bound for the uncertainty



Kinematic phase space with an upper bound for the uncertainty

Considered energies	Con FL	Opt FL	Con LL	Opt LL
5×100, 10×100 and 10×275 [GeV^2]	$65 + 79$	$90 + 54$	$66 + 78$	$90 + 54$
5×100, 10×100, 10×275 and 18×275 [GeV^2]	$99 + 78$	$128 + 49$	$101 + 76$	$127 + 50$
5×41, 5×100, 10×100, 10×275 and 18×275 [GeV^2]	$137 + 58$	$156 + 39$	$134 + 61$	$157 + 38$

In the table, green stands for measurements whose uncertainty is lower than 0.3 and red for those whose uncertainty is higher than 0.3.

The measurements with uncertainties higher than our threshold might be recovered once we have real experimental data or by extracting F_L using (y, Q^2) bins instead

Conclusions and final remarks

The upcoming Electron–Ion Collider (EIC) will revolutionize our ability to probe the longitudinal structure function and, through it, the gluon content of the proton. In particular:

- EIC will extract F_L with unprecedented precision across a vast, previously unexplored kinematic domain,
- F_L measurements will, provide an almost direct determination of the gluon density $xg(x, Q^2)$,
- F_L measurements will be possible even in the first few years of operation if we have, at least, three different configurations available.

Thank you for your time !

Please, feel free to email me if you have further questions: javier.jimenezlopez240203@gmail.com

Obtaining F_L as a function of Q^2

To obtain $F_L(Q^2)$, an average was made over x of F_L for the 1000 MC samples.

The average value of F_L was obtained as:

$$\overline{F_L} = \frac{\sum_{i=1}^{N} \omega_i F_L^{(i)}}{\sum_{i=1}^{N} \omega_i} \,,$$

where ω_i are the weights defined as:

$$\omega_i = \frac{1}{\left[\Delta F_L^{(i)}\right]^2} \,.$$

Then, the uncertainty for $\overline{F_L}$ is:

$$\delta_{\text{avg}} = \sqrt{\frac{1}{\sum_{i=1}^{N} \omega_i}} \,.$$

The same procedure was applied to x using the weights already calculated for F_L.

Decoupling relation in the $\overline{\text{MS}}$ scheme

As the Appelquist-Carazzone decoupling theorem does not hold the $\overline{\text{MS}}$ scheme, the decoupling must be done "by hand" every time the number of active flavours changes during the calculation.

$$\alpha_s^{(n_f-1)}(\mu_0) = \zeta_g^2 \alpha_s^{(n_f)}(\mu_0)$$

$$\left(\zeta_g^{\overline{\text{MS}}}\right)^2 = 1 + \frac{\alpha_s^{(n_f)}(\mu)}{\pi}\left(-\frac{1}{6}L\right) + \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^2\left(\frac{11}{72} - \frac{11}{24}L + \frac{1}{36}L^2\right)$$
$$+ \left(\frac{\alpha_s^{(n_f)}(\mu)}{\pi}\right)^3\left[\frac{564731}{124416} - \frac{82043}{27648}\zeta_3 - \frac{955}{576}L + \frac{53}{576}L^2\right.$$
$$\left. - \frac{1}{216}L^3 + (n_f-1)\left(-\frac{2633}{31104} + \frac{67}{576}L - \frac{1}{36}L^2\right)\right] \text{ with } L = \log\left(\frac{\mu_0^2}{m_h^2}\right)$$

$\beta_i^{(n_f)}$ coefficients

$$\beta_0^{(n_f)} = \frac{1}{4}\left[11 - \frac{2}{3}n_f\right],$$

$$\beta_1^{(n_f)} = \frac{1}{16}\left[102 - \frac{38}{3}n_f\right],$$

$$\beta_2^{(n_f)} = \frac{1}{64}\left[\frac{2857}{2} - \frac{5033}{18}n_f + \frac{325}{54}n_f^2\right],$$

$$\beta_3^{(n_f)} = \frac{1}{256}\left[\frac{149753}{6} + 3546\zeta_3 + \left(-\frac{1078361}{162} - \frac{6508}{27}\zeta_3\right)n_f + \left(\frac{50065}{162} + \frac{6472}{81}\zeta_3\right)n_f^2 + \frac{1093}{729}n_f^3\right]$$

$\beta_i^{(n_f)}$ coefficients

$$
\begin{aligned}
\beta_4^{(n_f)} = \frac{1}{4^5}\Bigg\{ & \frac{8157455}{16} + \frac{621885}{2}\zeta_3 - \frac{88209}{2}\zeta_4 - 288090\zeta_5 \\
& + n_f\left[-\frac{336460813}{1944} - \frac{4811164}{81}\zeta_3 + \frac{33935}{6}\zeta_4 + \frac{1358995}{27}\zeta_5\right] \\
& + n_f^2\left[\frac{25960913}{1944} + \frac{698531}{81}\zeta_3 - \frac{10526}{9}\zeta_4 - \frac{381760}{81}\zeta_5\right] \\
& + n_f^3\left[-\frac{630559}{5832} - \frac{48722}{243}\zeta_3 + \frac{1618}{27}\zeta_4 + \frac{460}{9}\zeta_5\right] \\
& + n_f^4\left[\frac{1205}{2916} - \frac{152}{81}\zeta_3\right]\Bigg\}
\end{aligned}
$$