α_s from event shapes

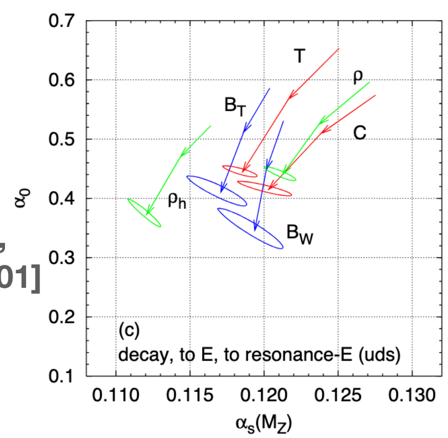
Miguel Benitez

Universidad de Salamanca

IUFFyM
Instituto Universitario de Física Fundamental y Matemáticas

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

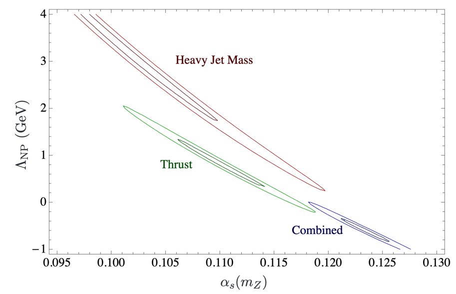


[Salam, Wicke 2001]

Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNNLL resummation with NNLO matching



Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

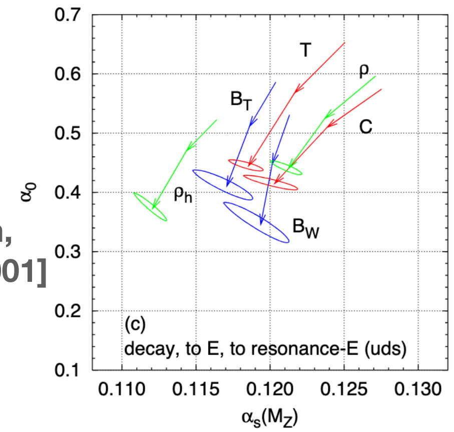
These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$

2001-2010

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

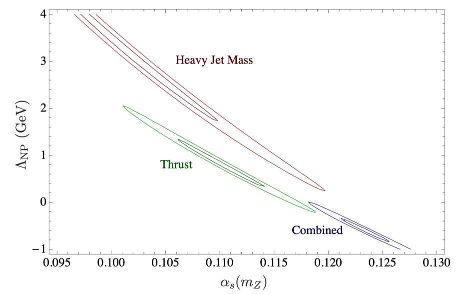
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

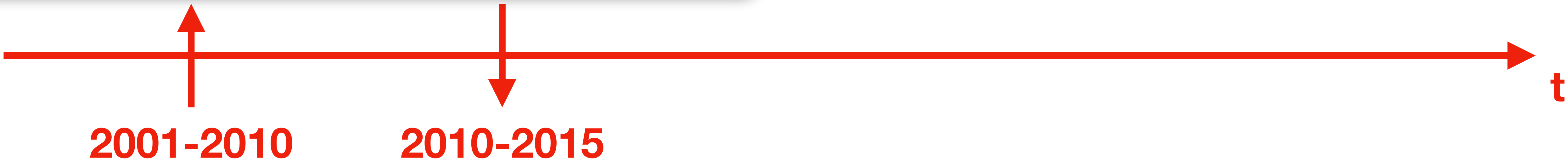
NNNLL resummation with NNLO matching



Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$



Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$
[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

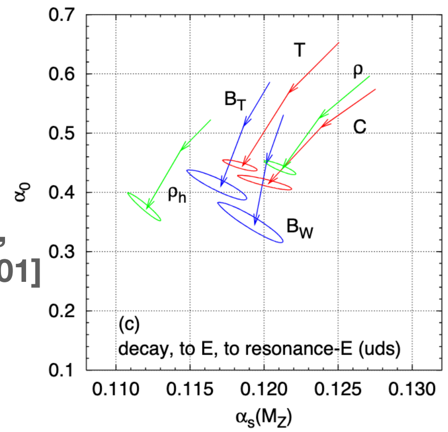
Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution
[Gehrmann, Luisoni, Monni 2012]

Precise Determination of α_s from the C-parameter Distribution
[Hoang, Kolodrubetz, Mateu, Stewart 2015]

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

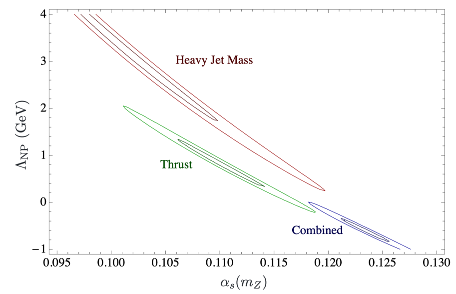
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNNLL resummation with NNLO matching

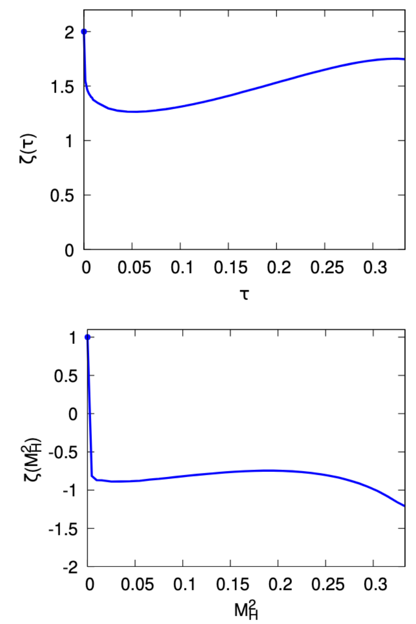


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$

[Caola et al. 2021, 2022]
[Nason, Zanderighi 2023, 2025]



Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$
[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

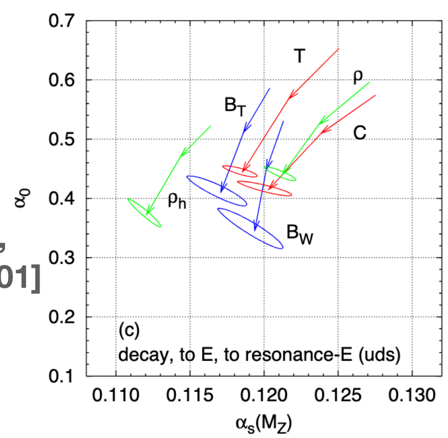
Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution
[Gehrmann, Luisoni, Monni 2012]

Precise Determination of α_s from the C-parameter Distribution
[Hoang, Kolodrubetz, Mateu, Stewart 2015]

α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

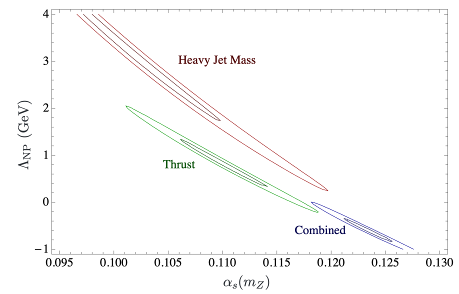
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNNLL resummation with NNLO matching

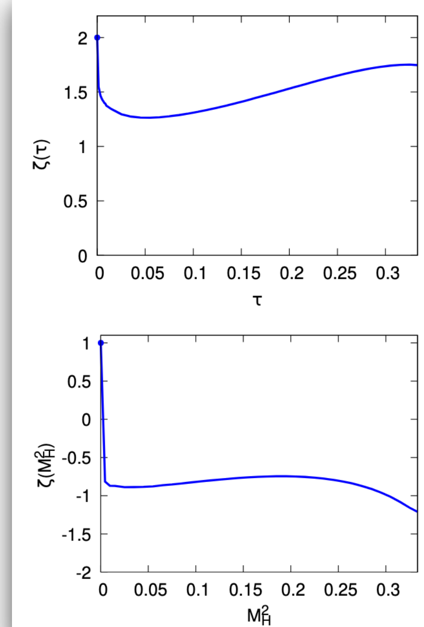


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$

[Caola et al. 2021, 2022]
[Nason, Zanderighi 2023, 2025]



Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$
[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution
[Gehrmann, Luisoni, Monni 2012]

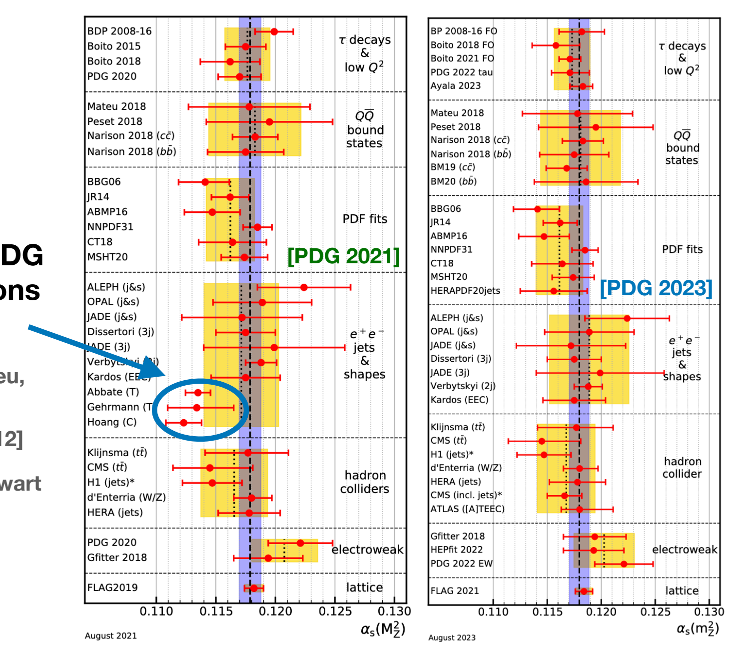
Precise Determination of α_s from the C-parameter Distribution
[Hoang, Kolodrubetz, Mateu, Stewart 2015]

The last edition of the PDG removed three extractions from event shapes

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

[Gehrmann, Luisoni, Monni 2012]

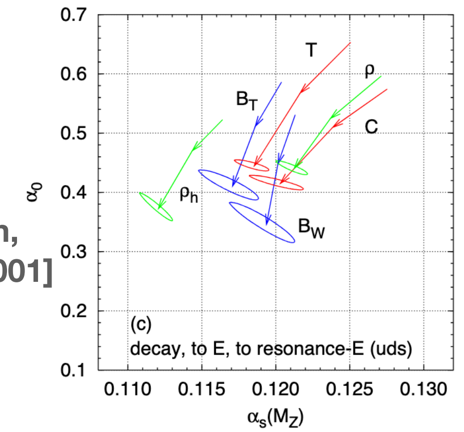
[Hoang, Kolodrubetz, Mateu, Stewart 2015]



α_s from e^+e^- event shapes

[See also prior work of Catani, Trentadue, Turnock, Webber 1993]

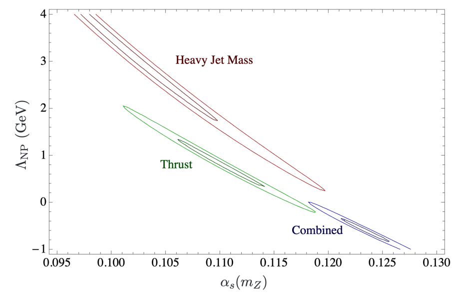
[Salam, Wicke 2001]



Secondly fits for the heavy-jet mass (a very non-inclusive variable) lead to values for α_s which are about 10% smaller than for inclusive variables like the thrust or the mean jet mass. This needs to be understood. It could be due to a difference in the behaviour of the perturbation series at higher orders.

[See also Dissertori et al 2007]

NNNLL resummation with NNLO matching

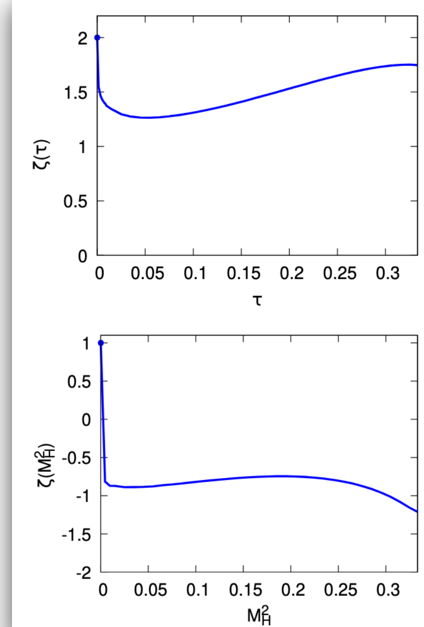


Event Shape	$\alpha_s(m_Z)$	Λ_{NP} (GeV)	χ^2/d.o.f.
Thrust	0.1101	0.821	66.9/47
Heavy Jet Mass	0.1017	3.17	60.4/43
Combined	0.1236	-0.621	453/92

[Chien, Schwartz 2010]

These studies used, in parts, only 25% of data bins: $0.08 < \rho < 0.18$

[Caola et al. 2021, 2022]
[Nason, Zanderighi 2023, 2025]



On Determining $\alpha_s(m_Z)$ from Dijets in e^+e^- Thrust

Miguel A. Benitez,[a] André H. Hoang,[b] Vicent Mateu,[a] Iain W. Stewart[b,c] and Gherardo Vita[d]

A Precise Determination of α_s from the Heavy Jet Mass Distribution

Miguel A. Benitez,[1] Arindam Bhattacharya,[2] André H. Hoang,[3] Vicent Mateu,[1] Matthew D. Schwartz,[2] Iain W. Stewart,[3,4] and Xiaoyuan Zhang[2]

Thrust at N3LL with Power Corrections and a Precision Global Fit for $\alpha_s(m_Z)$
[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

Power corrections in the dispersive model for a determination of the strong coupling constant from the thrust distribution
[Gehrmann, Luisoni, Monni 2012]

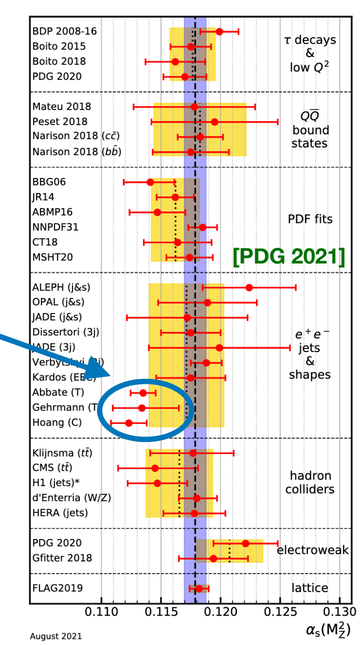
Precise Determination of α_s from the C-parameter Distribution
[Hoang, Kolodrubetz, Mateu, Stewart 2015]

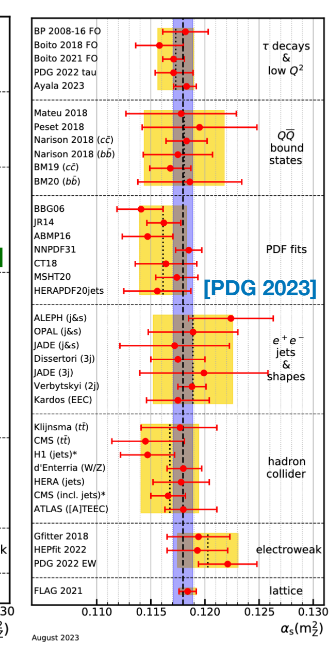
The last edition of the PDG removed three extractions from event shapes

[Abbate, Fickinger, Hoang, Mateu, Stewart 2010]

[Gehrmann, Luisoni, Monni 2012]

[Hoang, Kolodrubetz, Mateu, Stewart 2015]





Outline

How should you think of this talk?

- Summary of analyses that determined/discussed α_s from „classical" e^+e^- event shape observables

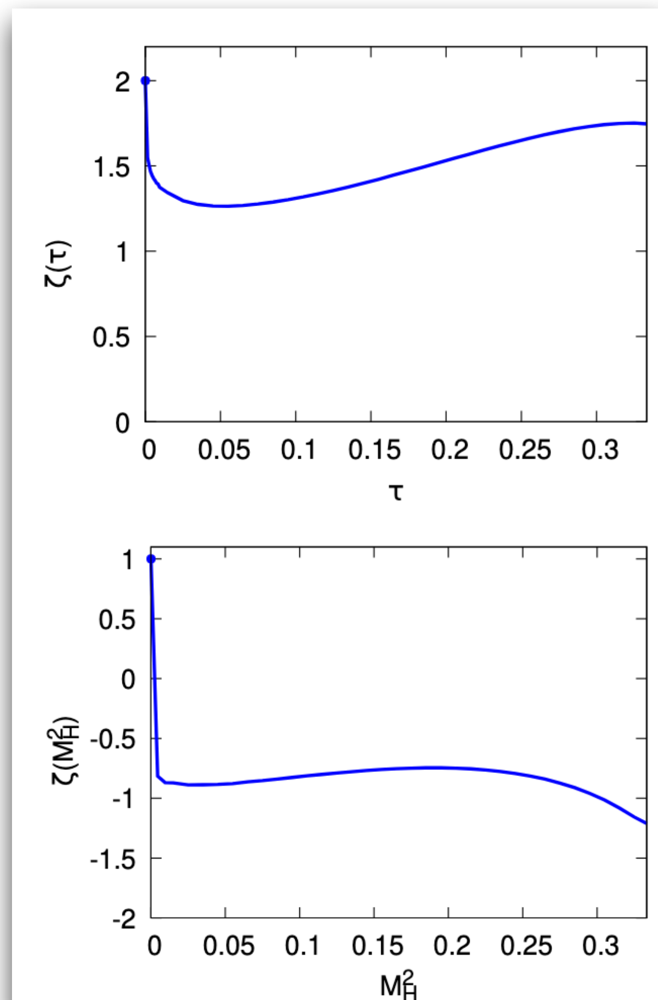
What this talk will not provide for you

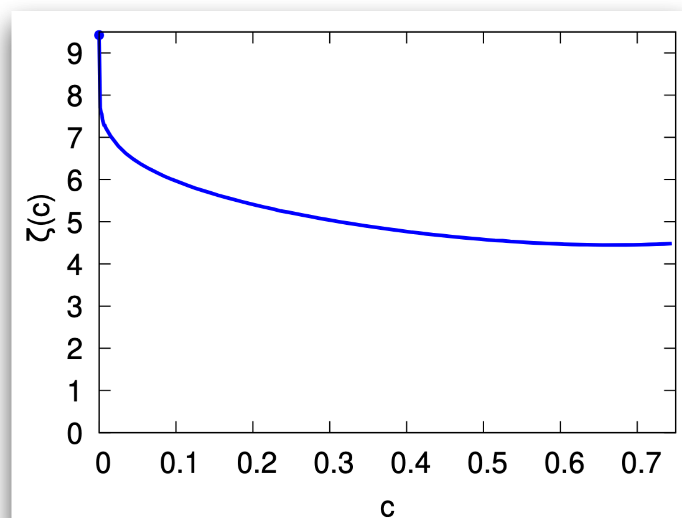
- Results for e^+e^- involving Monte Carlo methods for NP corrections
- ENCs
- Technical details on
  - Renormalon analysis
    [Caola et al. 2021, 2022]
  - Dijet resummation using Soft-Collinear Effective Theory (SCET)
  - Sudakov Shoulder resummation

Linear power corrections in the 3jet region

[Caola et al. 2021, 2022]

- Investigated the structure of linear renormalons in the three jet region by computing cross section for the process $\gamma^* \to q\bar{q}\gamma$
- added contributions arising from each one of the final state color dipoles to go from $\gamma^* \to q\bar{q}\gamma$ to $\gamma^* \to q\bar{q}g$





+ others

α_s extractions based on Fixed-Order

[Nason, Zanderighi 2023, 2025]

- Simultaneous fit to C-parameter, thrust and y_3 for α_s and NP parameter
- Analyzed different effects in addition to NP contribution

	$\alpha_s(M_Z)$							
	CTy3		C		T		y_3	
Variation	$\zeta(v)$	$\zeta(0)$	$\zeta(v)$	$\zeta(0)$	$\zeta(v)$	$\zeta(0)$	$\zeta(v)$	$\zeta(0)$
default	0.1181	0.1161	0.1169	0.1139	0.1168	0.1158	0.1155	0.1154
$\mu_R = \mu_0/2$	0.1167	0.1155	0.1141	0.1105	0.1159	0.1128	0.1122	0.1131
$\mu_R = 2\mu_0$	0.1167	0.1150	0.1212	0.1184	0.1208	0.1191	0.1157	0.1161
std scheme	0.1173	0.1153	0.1164	0.1118	0.1152	0.1148	0.1150	0.1149
p scheme	0.1160	0.1141	0.1164	0.1118	0.1152	0.1148	0.1137	0.1135
D scheme	0.1199	0.1173	0.1190	0.1153	0.1205	0.1170	0.1168	0.1166
$C_{\rm ll} = 1.5$	0.1165	0.1143	0.1151	0.1116	0.1154	0.1133	0.1142	0.1142
$C_{\rm ll} = 3$	0.1177	0.1159	0.1221	0.1116	0.1180	0.1172	0.1156	0.1154
non-pert scheme (b)	0.1193	0.1163	0.1191	0.1176	0.1185	0.1184	0.1154	0.1154
non-pert scheme (c)	0.1189	0.1167	0.1195	0.1172	0.1192	0.1191	0.1154	0.1154
minus non-pert error	0.1187	0.1161	0.1173	0.1139	0.1165	0.1158	0.1157	0.1154
plus non-pert error	0.1189	0.1161	0.1172	0.1139	0.1172	0.1158	0.1153	0.1154

$\zeta(\nu)$ = dipole model

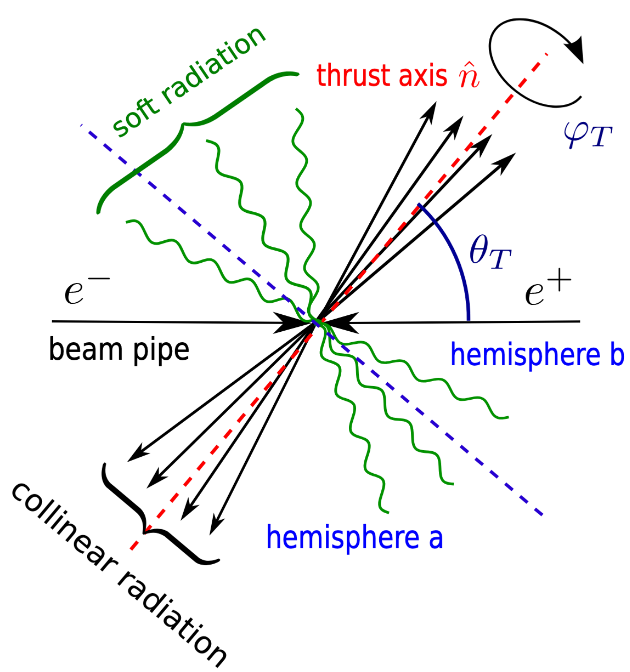
$\zeta(0)$ = flat NP correction

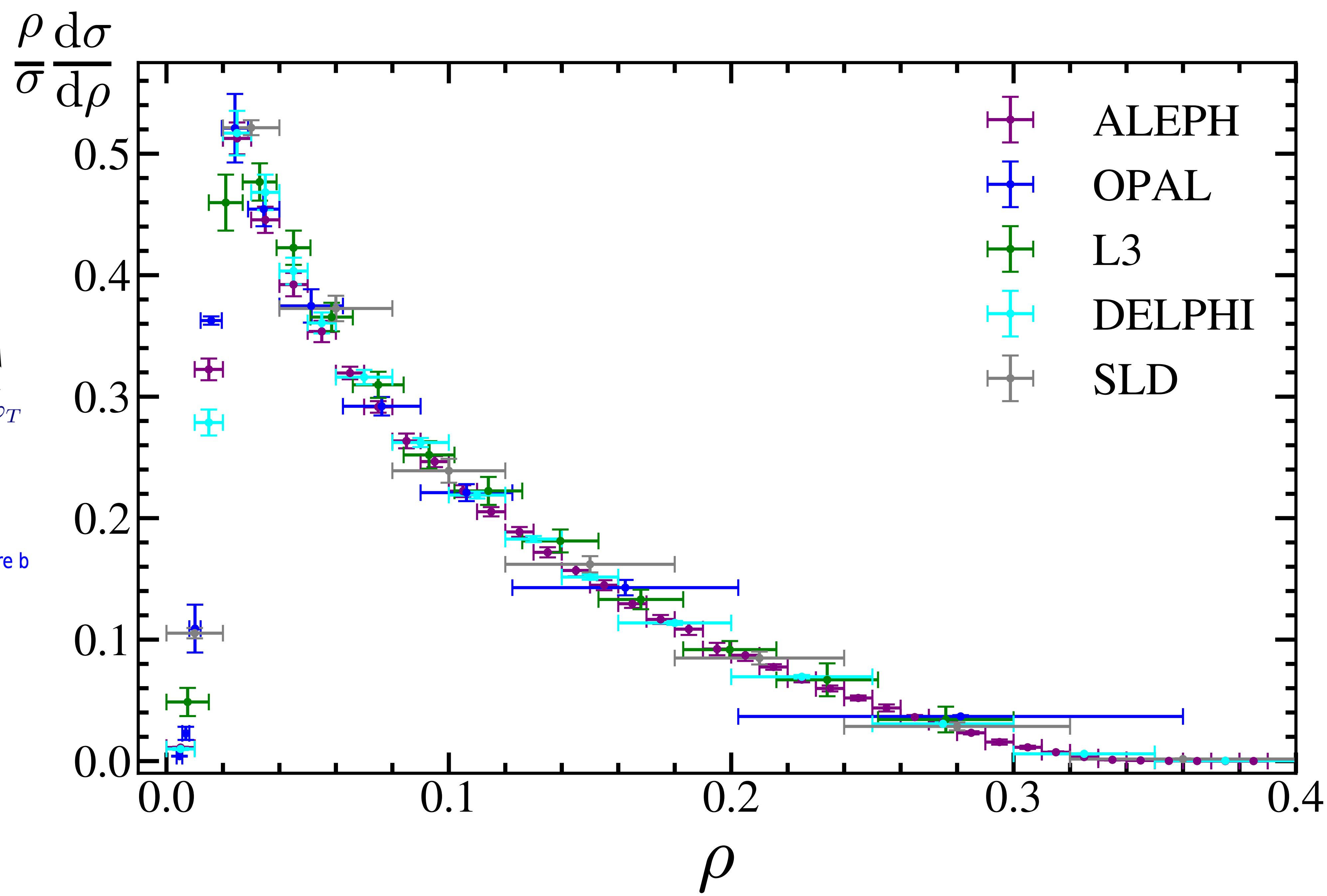
Conclusion of analyses: Uncertainties accompanying earlier α_s extractions using analytic methods to determine NP correction were underestimated

Event Shapes using SCET resummation

[MB, Hoang, Mateu, Stewart, Vita 2024]
[MB, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, Zhang 2025]

- Scaling of observable important for theory prediction





Event Shapes using SCET resummation

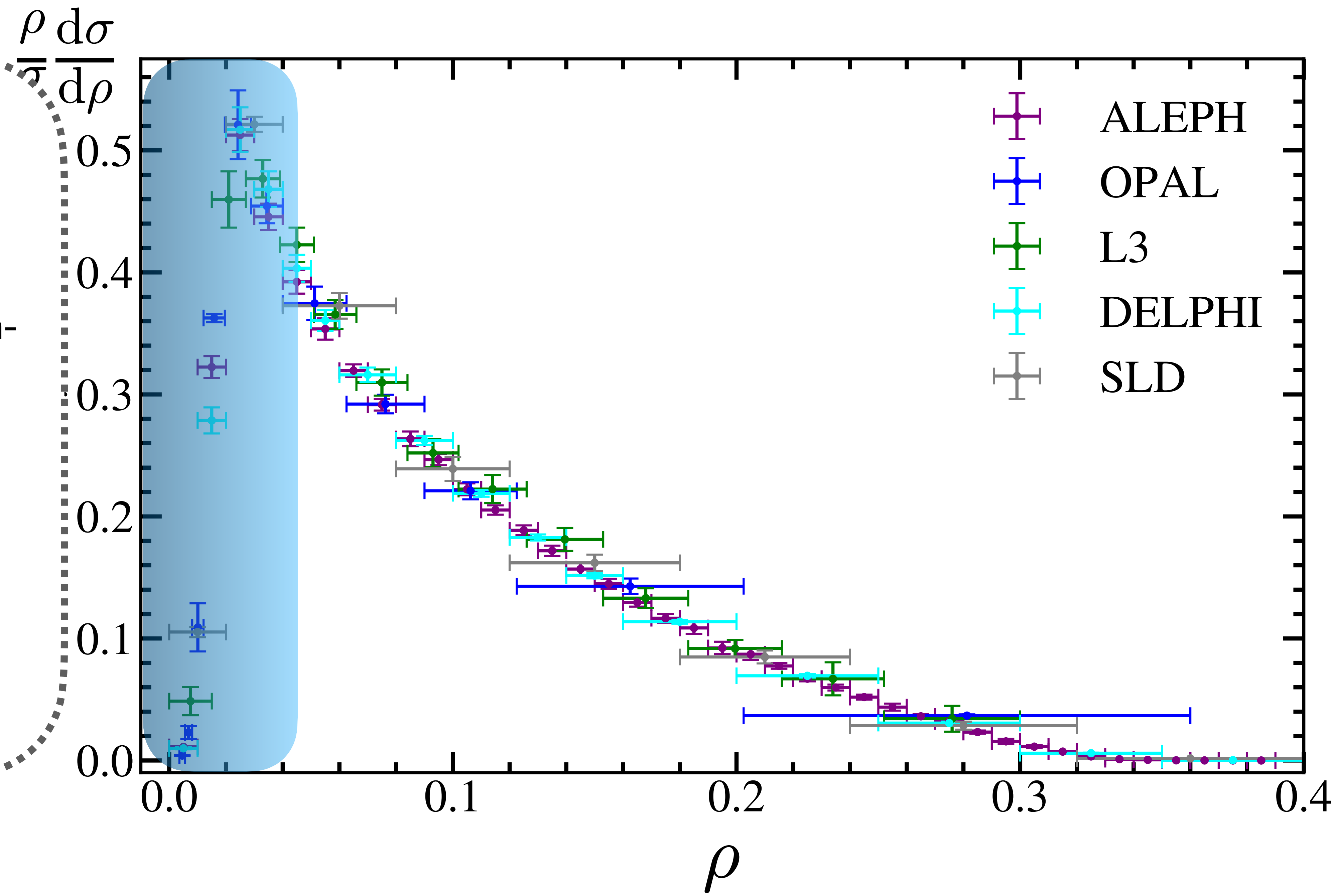
[MB, Hoang, Mateu, Stewart, Vita 2024]
[MB, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, Zhang 2025]

- Scaling of observable important for theory prediction

Peak region

$$s_i \sim Q\Lambda_{\text{QCD}}$$

Heavily affected by non-perturbative dynamics



Event Shapes using SCET resummation

[MB, Hoang, Mateu, Stewart, Vita 2024]
[MB, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, Zhang 2025]

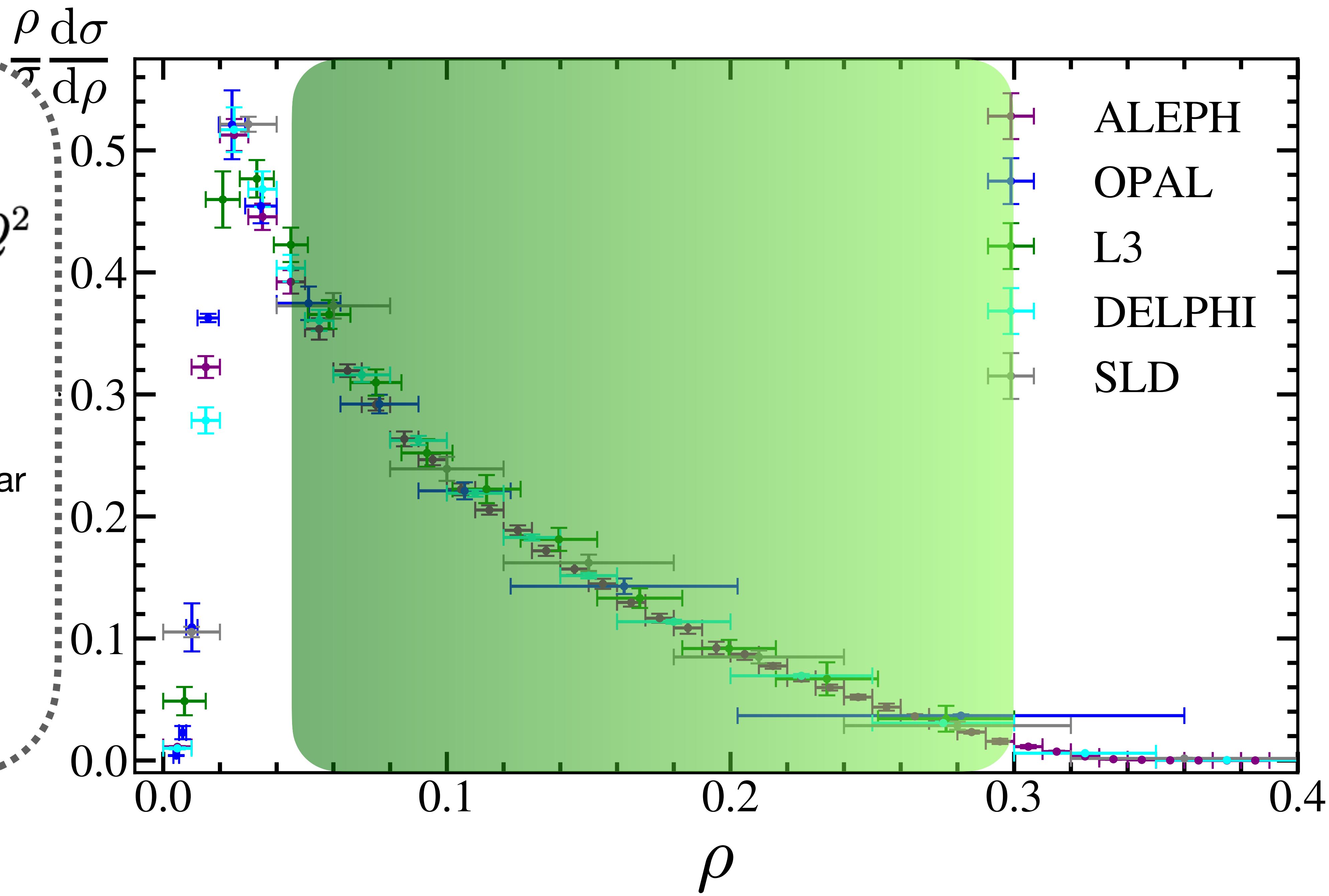
- Scaling of observable important for theory prediction

Tail region

$$Q\Lambda_{\rm QCD} \ll s_i \ll Q^2$$

Soft and collinear radiation dominates

$\Lambda_{\rm QCD} \ll s_i$ provides clear separation between perturbative and non-perturbative modes



Event Shapes using SCET resummation

[MB, Hoang, Mateu, Stewart, Vita 2024]
[MB, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, Zhang 2025]

- Scaling of observable important for theory prediction

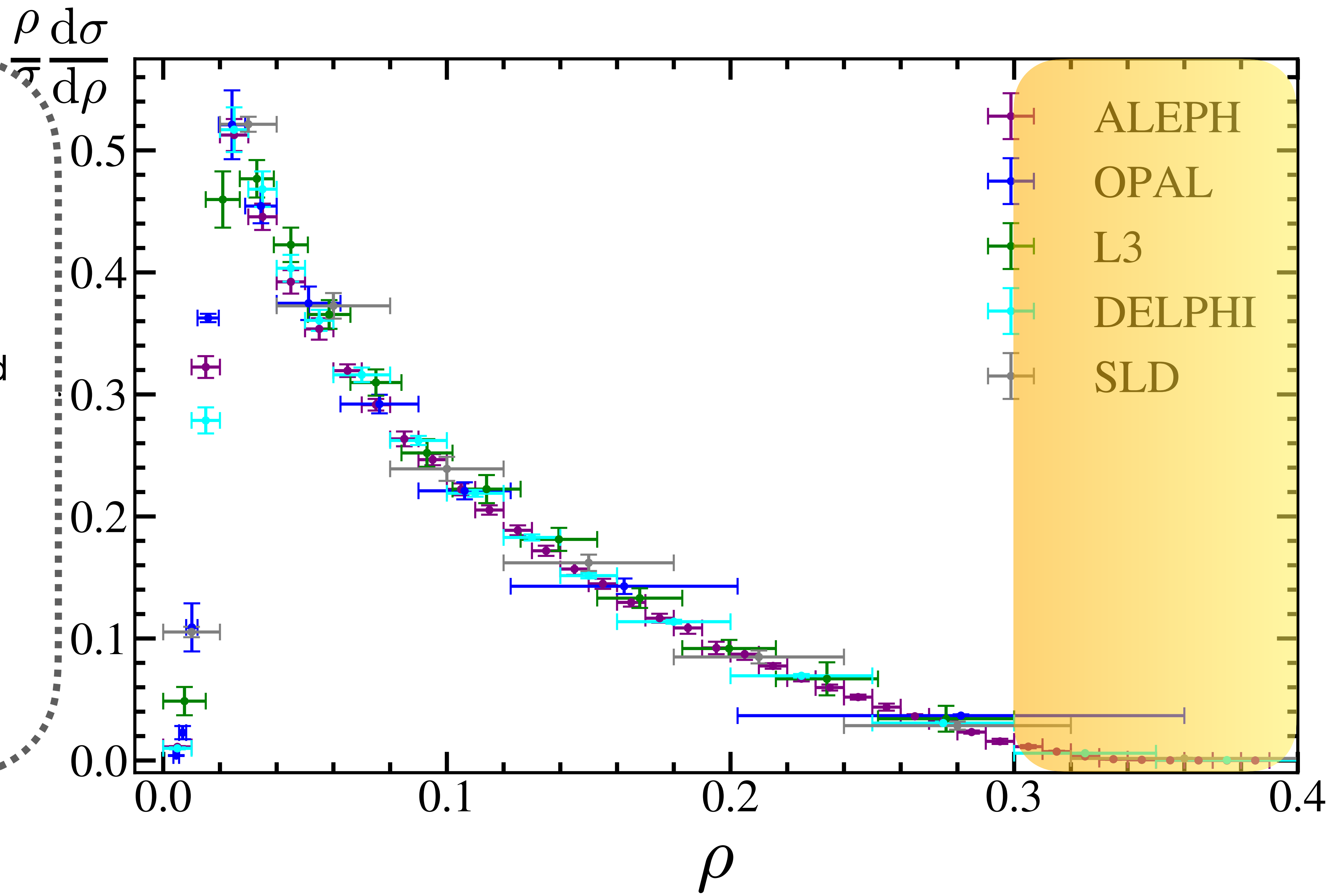
Far-Tail region

$$s_i \sim Q^2$$

No clear distinction between hard scale and observable

Distribution well approximated by FO

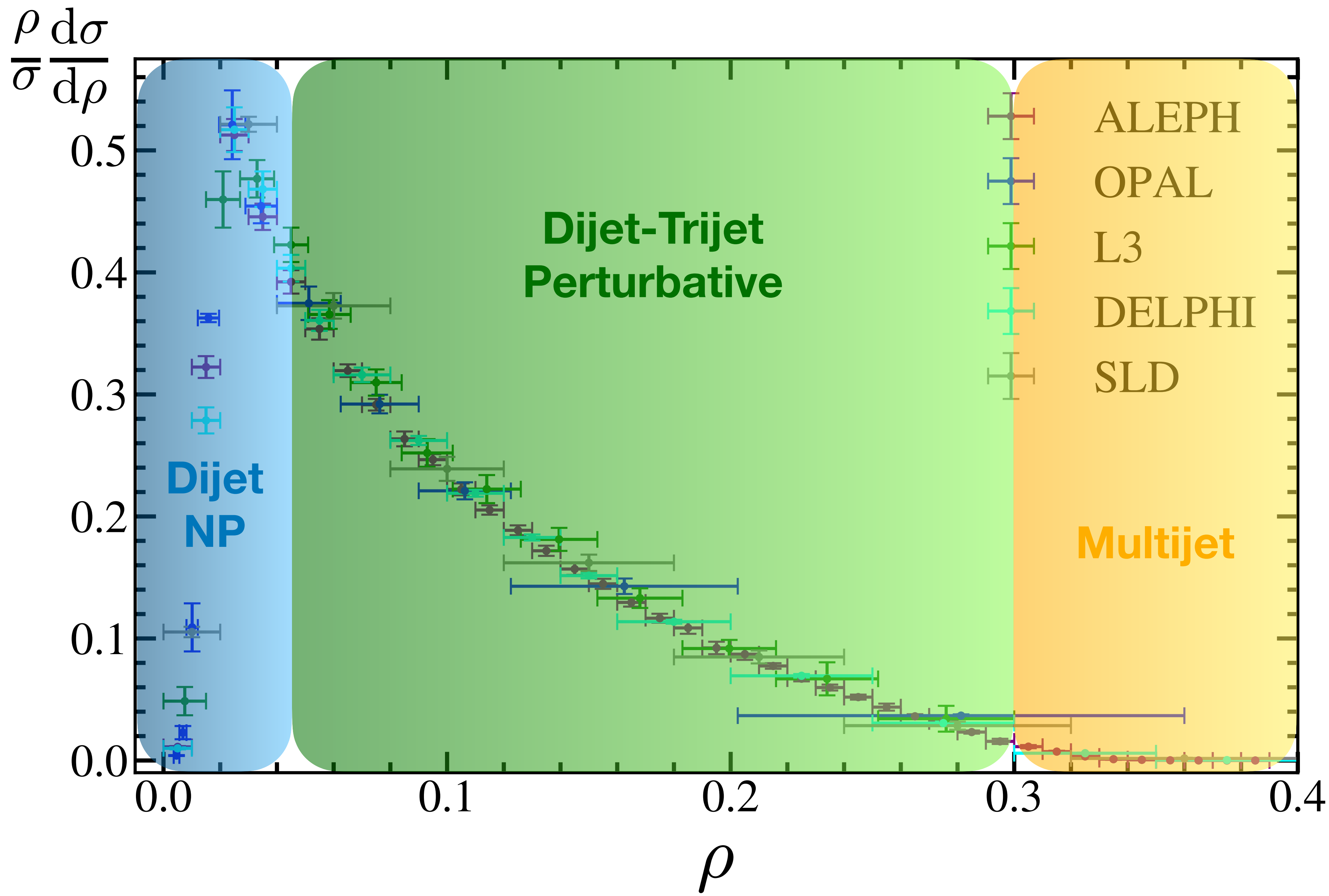
Small non-perturbative corrections



Event Shapes using SCET resummation

[MB, Hoang, Mateu, Stewart, Vita 2024]
[MB, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, Zhang 2025]

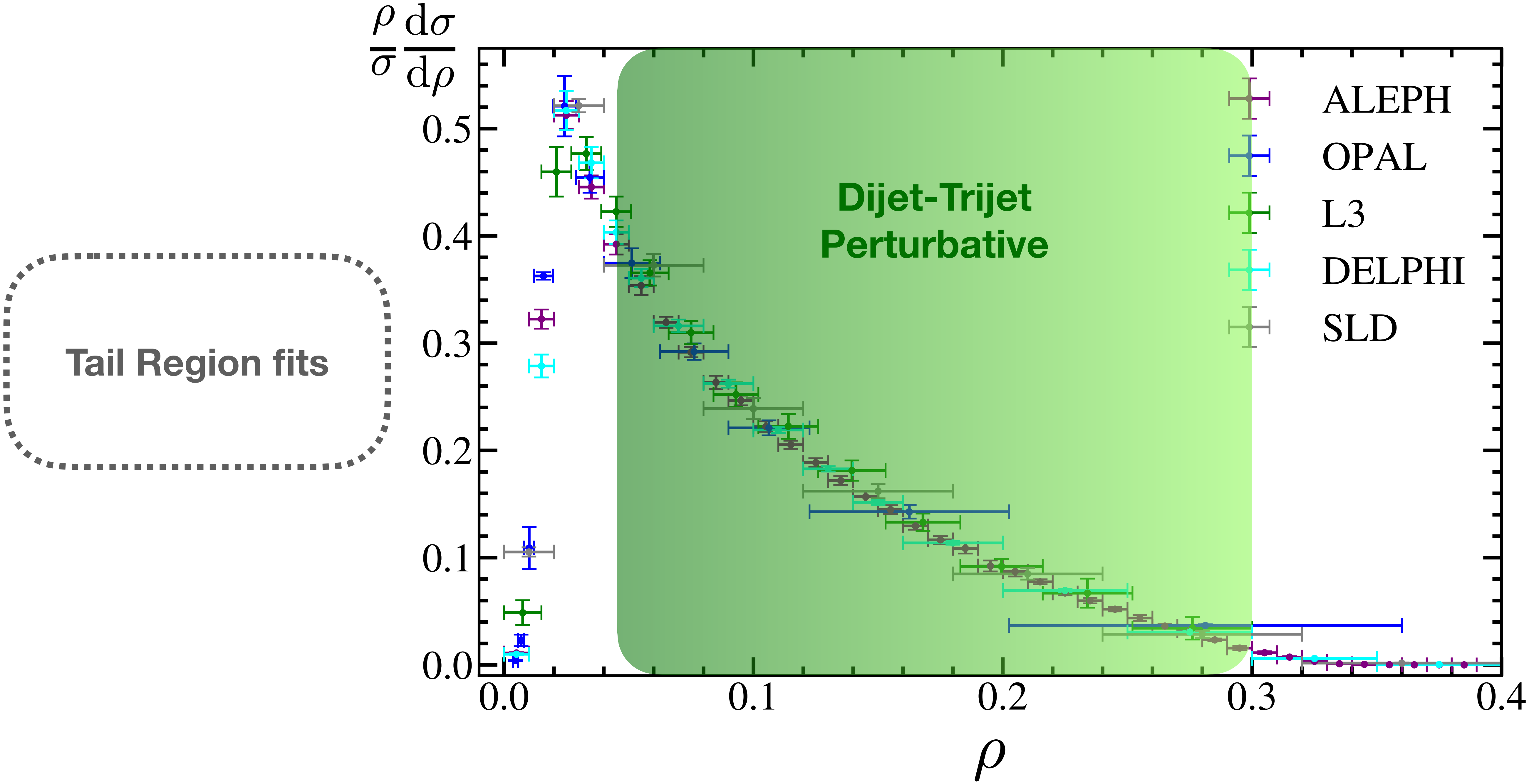
- Scaling of observable important for theory prediction



Event Shapes using SCET resummation

[MB, Hoang, Mateu, Stewart, Vita 2024]
[MB, Bhattacharya, Hoang, Mateu, Schwartz, Stewart, Zhang 2025]

- Scaling of observable important for theory prediction



The HJM distribution — dijet + shoulder resummation

Same methodology applies for thrust*

- HJM differential cross section factorizes in the dijet limit into global hard factor times two-dimensional convolution of two one-dimensional jet functions and two-dimensional soft and shape functions

$$\mathrm{d}\sigma_{\mathrm{dij}} = H_{\mathrm{dij}} \times J_1 \times J_2 \otimes S_{1,2} \otimes F^{\Xi}_{1,2}(\Omega_1^{\rho})$$

- Around symmetric trijet limit $\rho \to 1/3$, distribution factorizes as

$$\mathrm{d}\sigma_{\mathrm{sh}}^{\mathrm{pert}} = H_{\mathrm{sh}} \times J_1 \times J_2 \times J_3 \otimes S_{1,2,3}$$

- Matching between the dijet, fixed-order and shoulder regions done by writing full cross section as

$$\mathrm{d}\sigma = \left[\mathrm{d}\sigma_{\mathrm{dij}} - \mathrm{d}\sigma_{\mathrm{dij}}^{\mathrm{sing}}\right] + \mathrm{d}\sigma_{\mathrm{FO}} + \left[\mathrm{d}\sigma_{\mathrm{sh}} - \mathrm{d}\sigma_{\mathrm{sh}}^{\mathrm{sing}}\right]$$

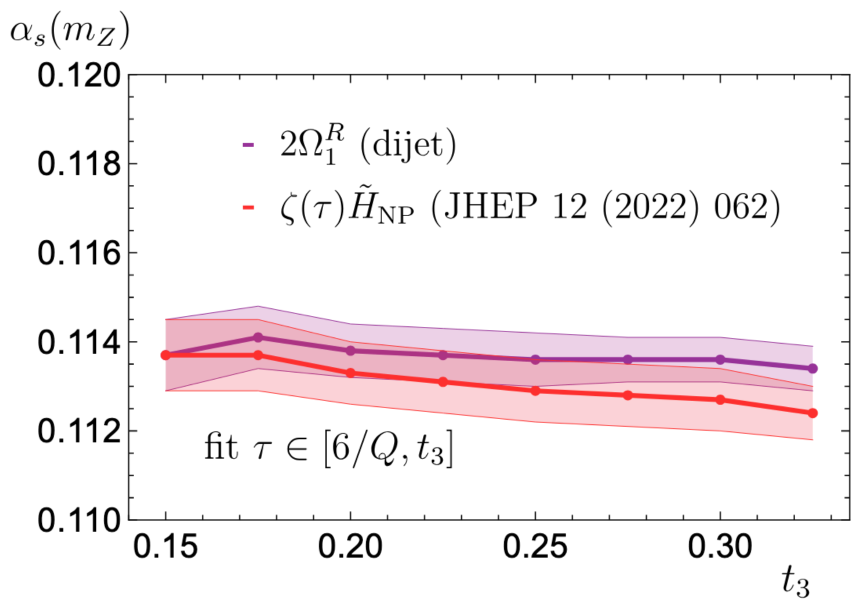
- Model power corrections around the symmetric trijet limit with non-perturbative shift parameter Θ_1

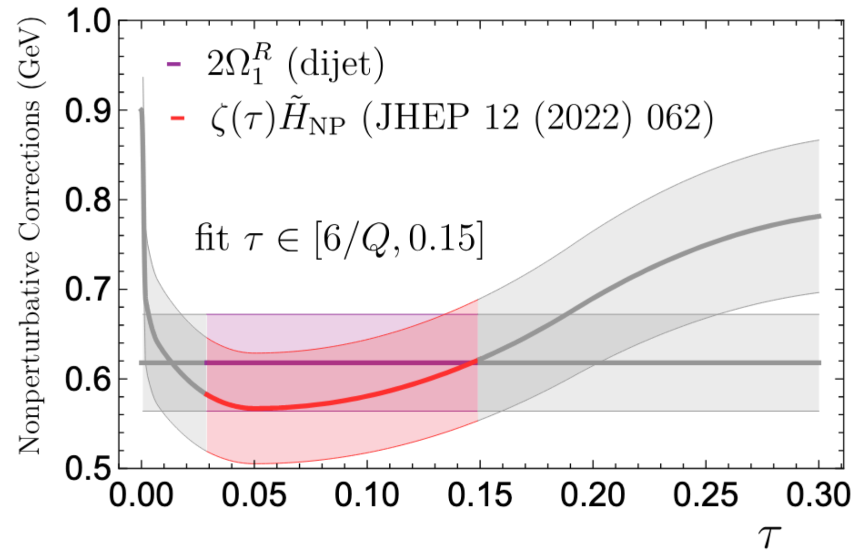
$$\frac{\mathrm{d}\sigma_{\mathrm{sh}}}{\mathrm{d}\rho}(\rho) = \frac{\mathrm{d}\sigma_{\mathrm{sh}}^{\mathrm{pert}}}{\mathrm{d}\rho}\left(\rho - \frac{\Theta_1}{Q}\right)$$

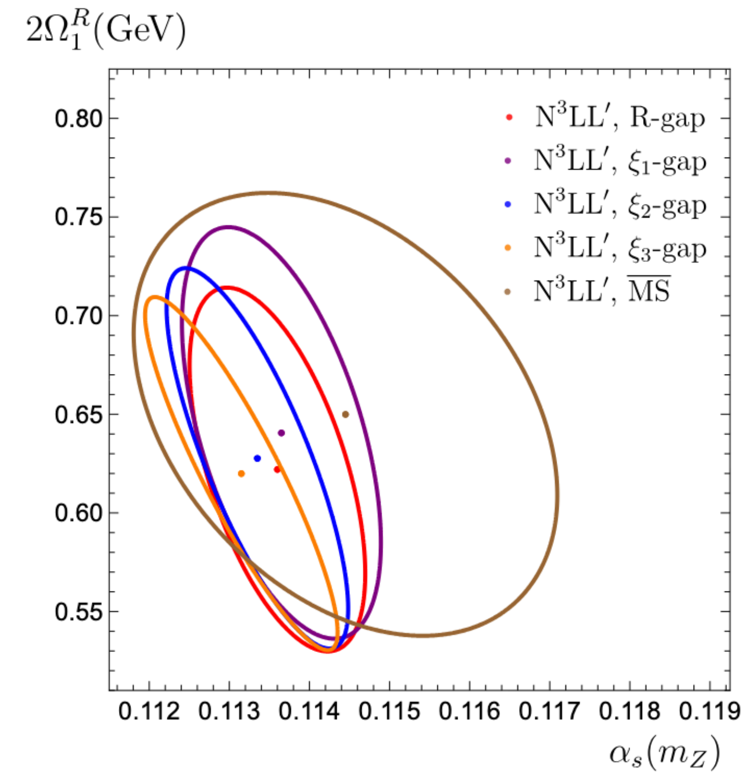
***1d soft and shape functions, no shoulder**

α_s from Thrust

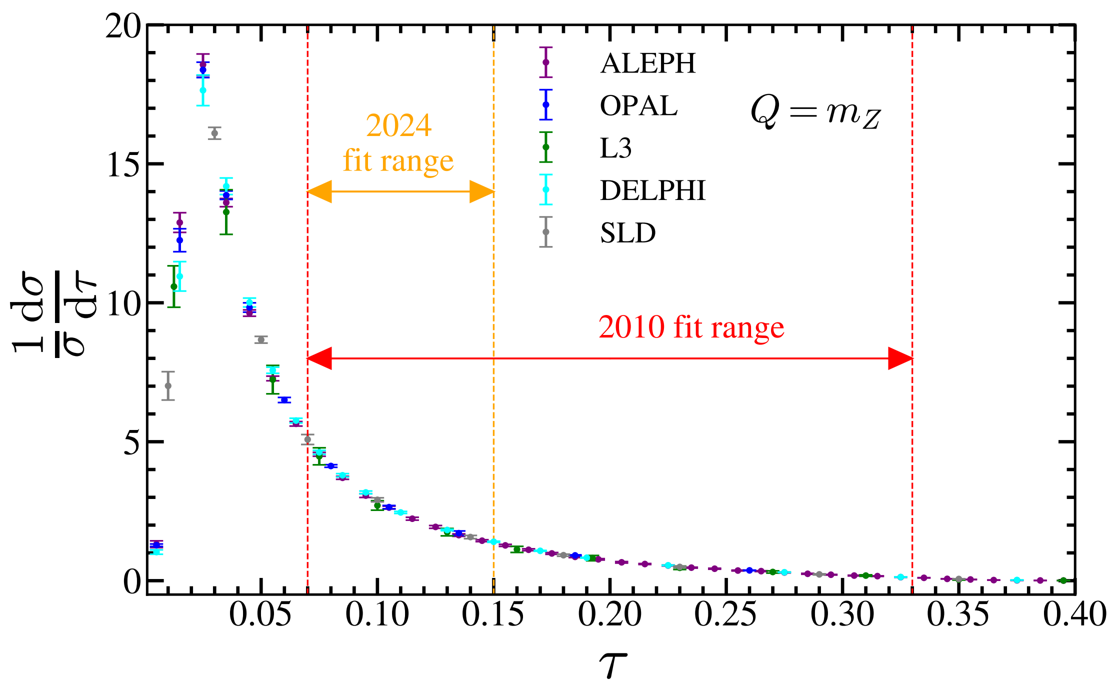
- Impact of power corrections and renormalon-subtraction scheme

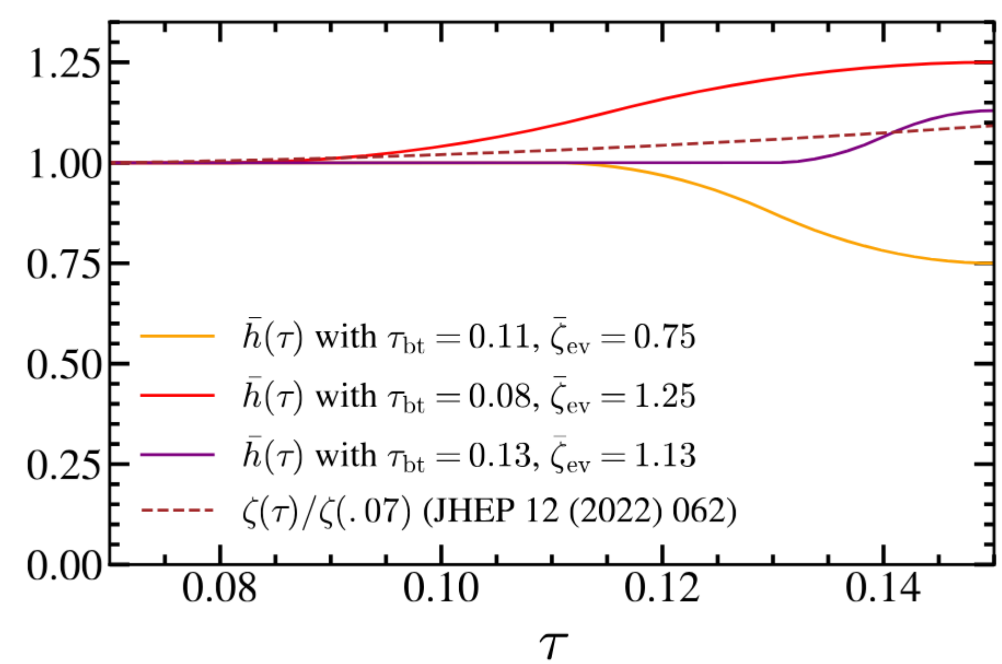






- Deviation from dijet treatment of power corrections — decrease of fit range





α_s from Thrust

- Summary of all effects analyzed

	$\delta\alpha_s(m_Z)$	$\delta\Omega_1^R$	Included in [50]
Experiment	0.0003	0.010	✓
Ω_1/α_s	0.0007	0.026	✓
Total Experiment + $\boldsymbol{\Omega_1/\alpha_s}$	**0.0008**	**0.028**	✓
Ω_2 hadronization	0.0002	0.013	✓
3jet hadronization	0.0002	0.010	
Subleading power dijet	0.0002	0.004	
Total subleading hadronization	**0.0003**	**0.017**	
Perturbative	**0.0008**	**0.037**	✓
Total	**0.0012**	**0.049**	

Considering all effects addressed in recent literature,
total uncertainty experiences only slight increase

- Final result

$$\alpha_s(m_Z) = 0.1136 \pm 0.0012_{\text{tot}}$$
$$\Omega_1^R = 0.311 \pm 0.049_{\text{tot}}\ \text{GeV}$$
$$\chi^2/\text{dof} = 0.86$$

Modified fit procedure for HJM

- Use χ^2 function including theoretical and experimental uncertainties

Experiment

35 GeV < Q < 207 GeV
(700 experimental datapoints)

Minimal Overlap Model treats correlations of systematic uncertainties on experimental measurements

$$\sigma_{ij}^{\rm exp} = \delta_{ij}(\Delta_i^{\rm stat})^2 + \delta_{D_i D_j}\min(\Delta_i^{\rm sys}, \Delta_j^{\rm sys})^2$$

Theory

Theory uncertainties assessed though renormalization scale variation
→ not Gaussian + highly correlated

Employ flat random scan: M = 5000 sets of k ≤ 17 parameters generated, each produces theory prediction for data-point x_i

Determine $\bar{x}_i = (x_i^{\rm max} + x_i^{\rm min})/2$ and $\Delta_i^{\rm theo} = (x_i^{\rm max} - x_i^{\rm min})/2$

Correlation coefficient r_{ij} among bins $r_{ij}^{\rm theo} = \dfrac{\langle (x_i - \bar{x}_i)(x_j - \bar{x}_j)\rangle}{\sqrt{\langle (x_i - \bar{x}_i)^2\rangle}\sqrt{\langle (x_j - \bar{x}_j)^2\rangle}}$

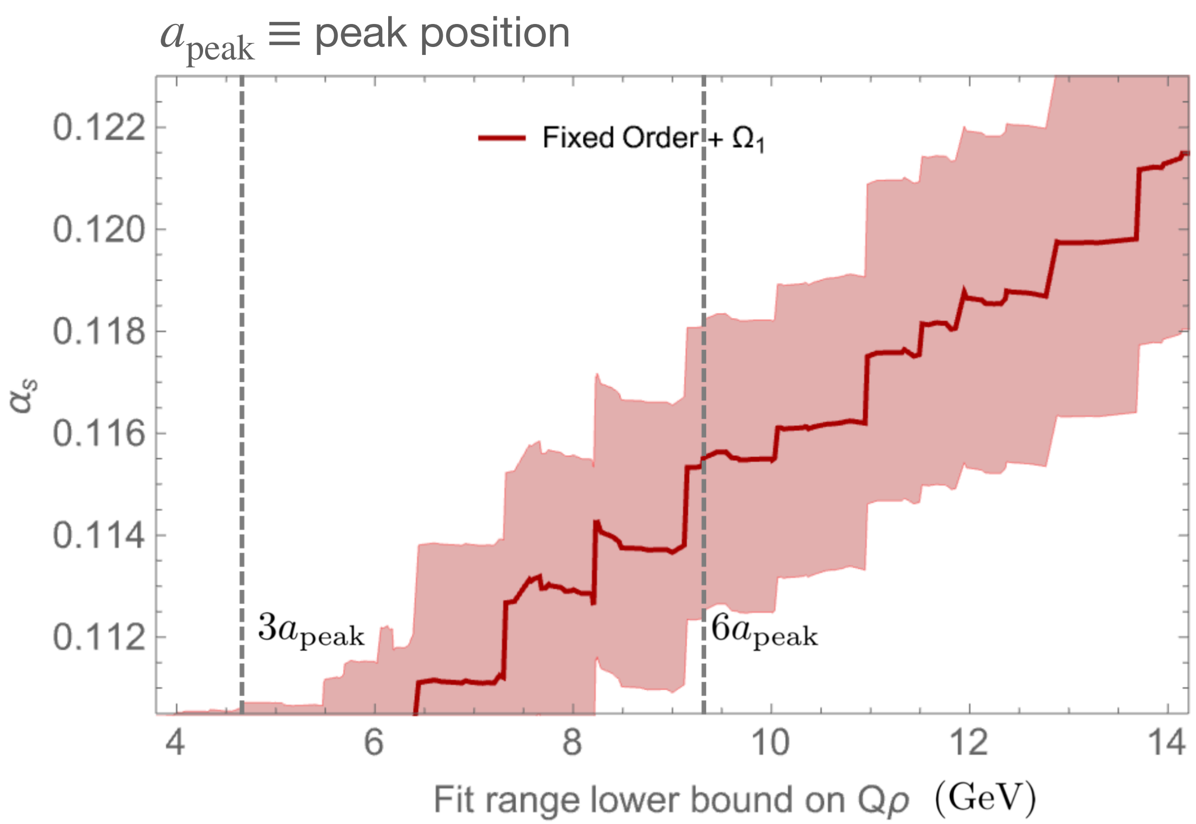
Theory covariance matrix results from scaling correlation coefficient by $1-\sigma$ uncertainties

$$\sigma_{ij}^{\rm theo} = \Delta_i^{\rm theo}\,\Delta_j^{\rm theo}\,r_{ij}^{\rm theo}$$

- Total covariance matrix = sum of theoretical and experimental: $\sigma_{ij}^{\rm tot} = \sigma_{ij}^{\rm theo} + \sigma_{ij}^{\rm exp}$
- χ^2 reads: $$\chi^2 = \sum_{i,j=1}^{N_{\rm bins}} (\bar{x}_i - x_i^{\rm exp})\,(\bar{x}_j - x_j^{\rm exp})\,(\sigma_{\rm tot}^{-1})_{ij}$$

Fit results — Fixed Order

- Results for α_s using fit range $a/Q \leq \rho \leq 0.3$ for different a



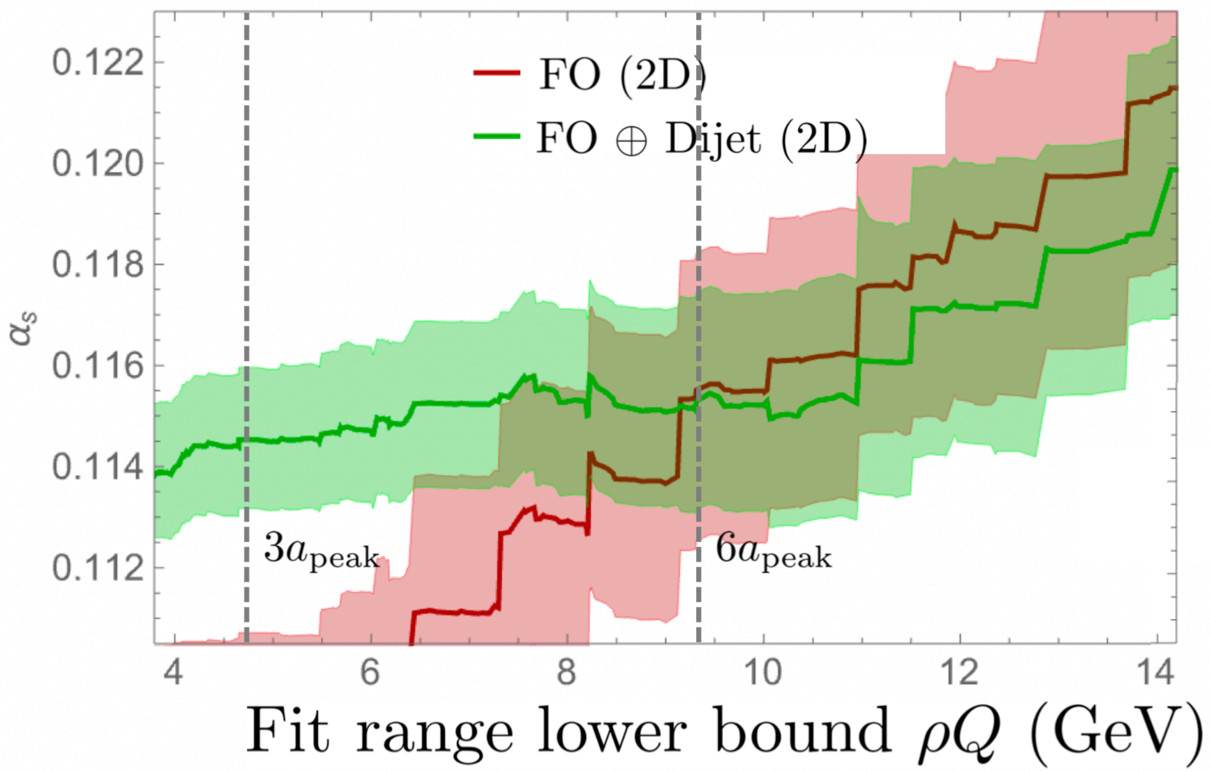
- Results for α_s very sensitive to fit range
- Large fit range uncertainty even with restriction $a \in [5a_{\rm peak}, 8a_{\rm peak}]$
- Impossible to extract sensible value of α_s without arbitrary choice of fit range

Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–

Fit results — Dijet resummation

- Results for α_s using fit range $a/Q \leq \rho \leq 0.3$ for different a

$a_{\text{peak}} \equiv$ peak position



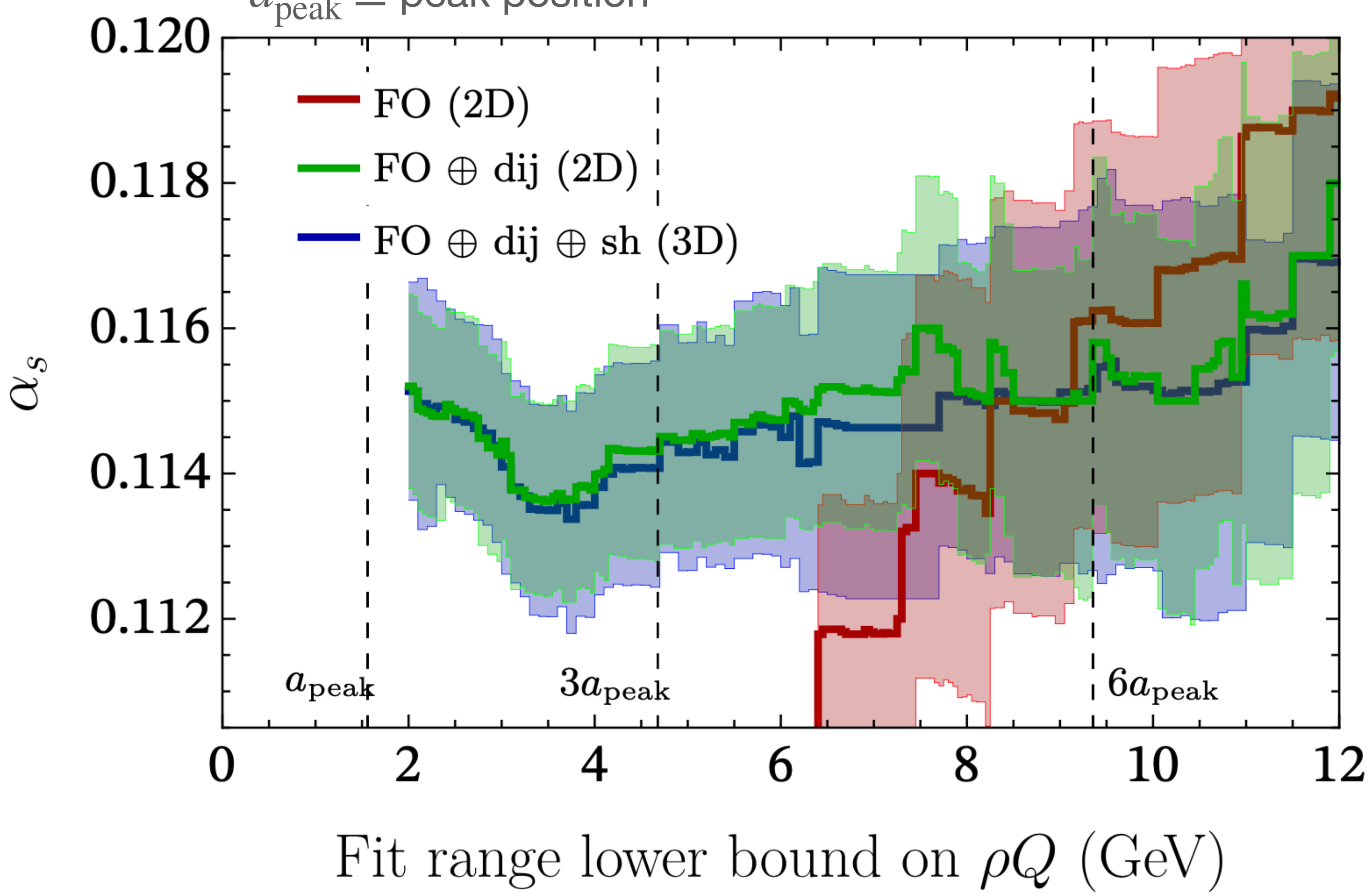
- Fit value remarkably insensitive to fit range
- Small fit range uncertainty for $a \in [3a_{\text{peak}}, 6a_{\text{peak}}]$
- Data prefers positive power correction (rightward shift of distribution)

Model	$\alpha_s(m_Z)$	th+exp	Ω_1^ρ	Θ_1	fit range	χ^2/dof	Ω_1^ρ [GeV]	Θ_1 [GeV]
Fixed Order 2D	0.1166 ± 0.0034	± 0.0014	± 0.0027	–	± 0.0015	1.108	0.06 ± 0.13	–
FO + dijet 2D	0.1148 ± 0.0018	± 0.0010	± 0.0014	–	± 0.0004	1.055	0.53 ± 0.09	–

Fit results — Dijet + Shoulder resummation

- Results for α_s using fit range $a/Q \leq \rho \leq 0.3$ for different a

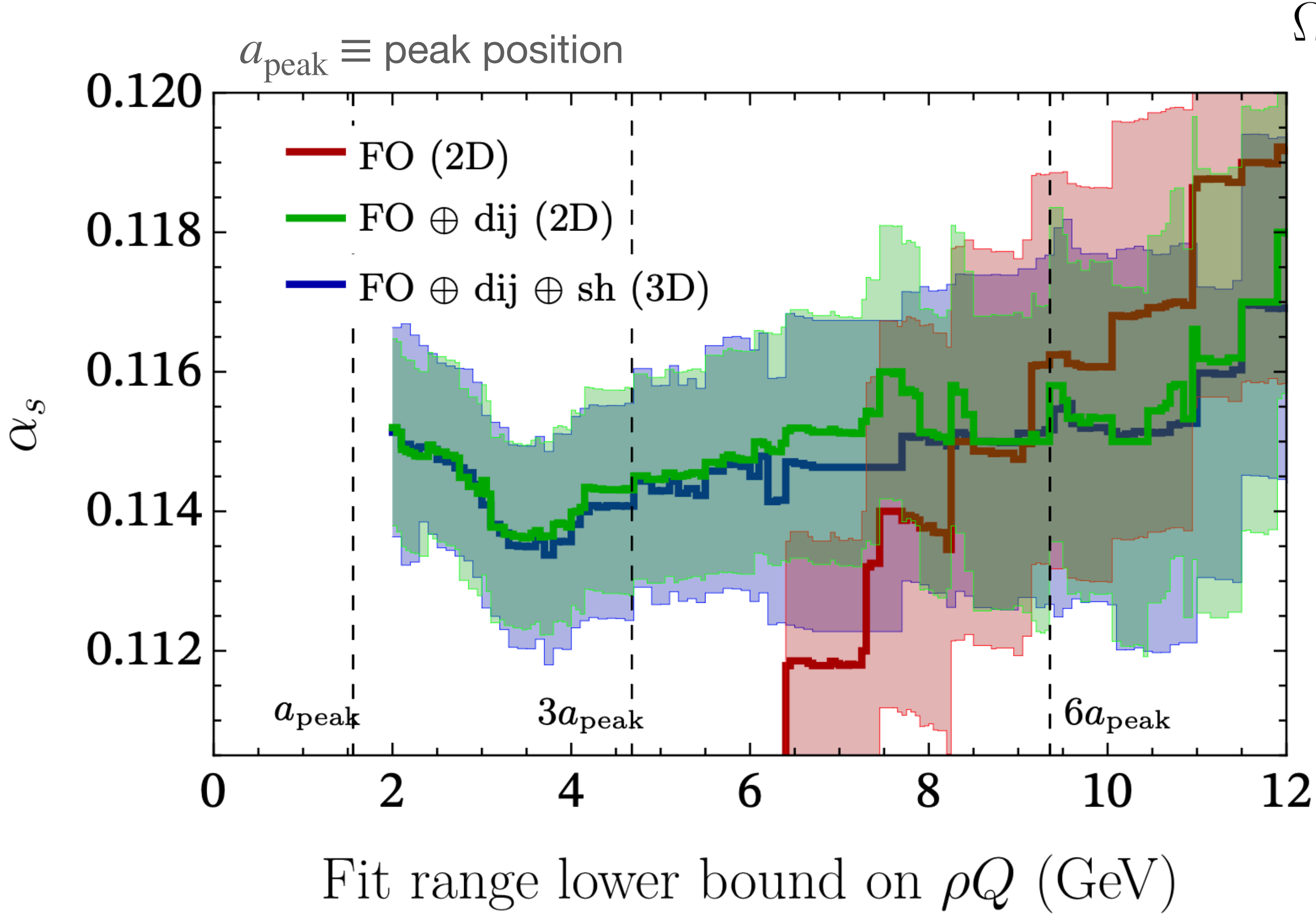
$a_{\text{peak}} \equiv$ peak position

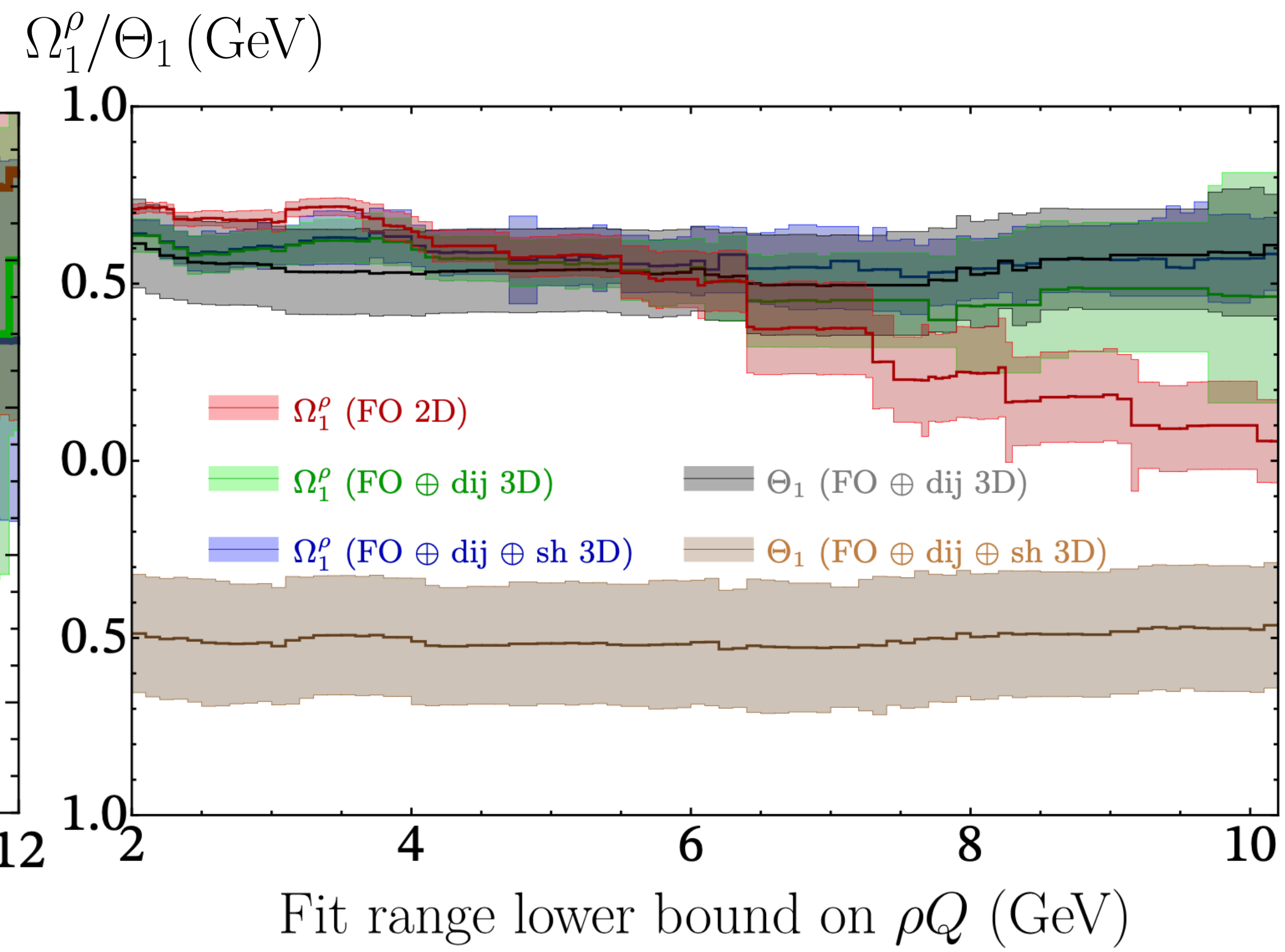


- Fit value remarkably insensitive to fit range
- Small fit range uncertainty for $a \in [3a_{\text{peak}}, 6a_{\text{peak}}]$
- **But what about the power corrections?**

Fit results — Dijet + Shoulder resummation

- Results for α_s, Ω_1^ρ and Θ_1 using fit range $a/Q \leq \rho \leq 0.3$ for different a





Data favors negative power corrections only with shoulder!

Summary of HJM analysis

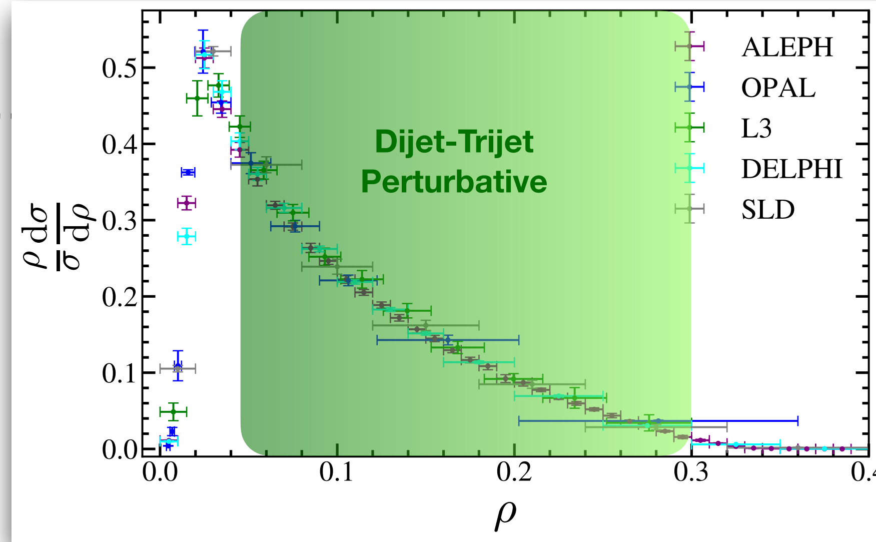
- Innovations include
  - Improved treatments of dijet/OPE and trijet/shoulder region
  - Inclusion of theory correlations during fitting
  - Careful attention to the range of data used for fitting
- Found fits are minimally sensitive to fit range when including resummation, in contrast to fixed-order perturbation theory (essentially linear dependence on lower bound)
- Found evidence for negative power correction in tail of distribution only if Sudakov shoulder resummation is included
- Extracted value is

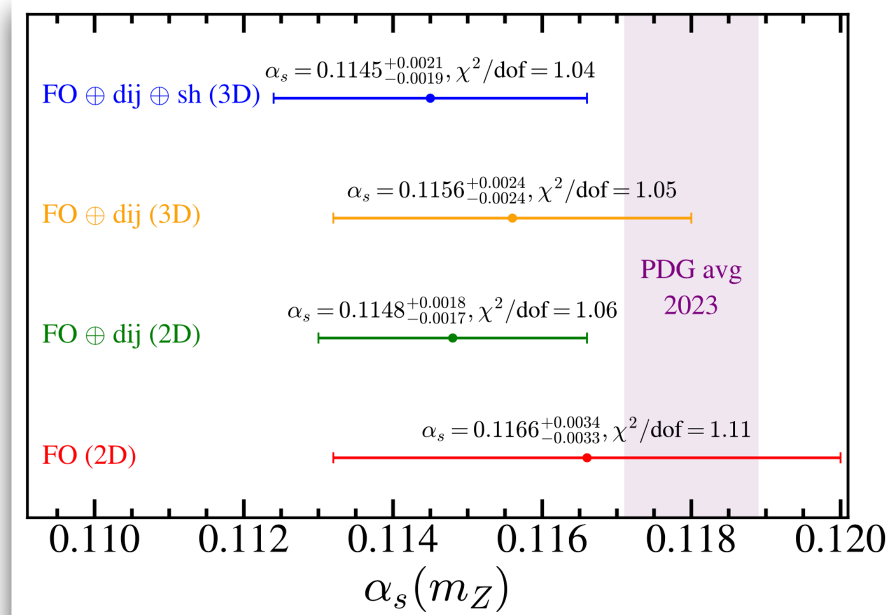
$$\alpha_s(m_Z) = 0.1145^{+0.0021}_{-0.0019}$$

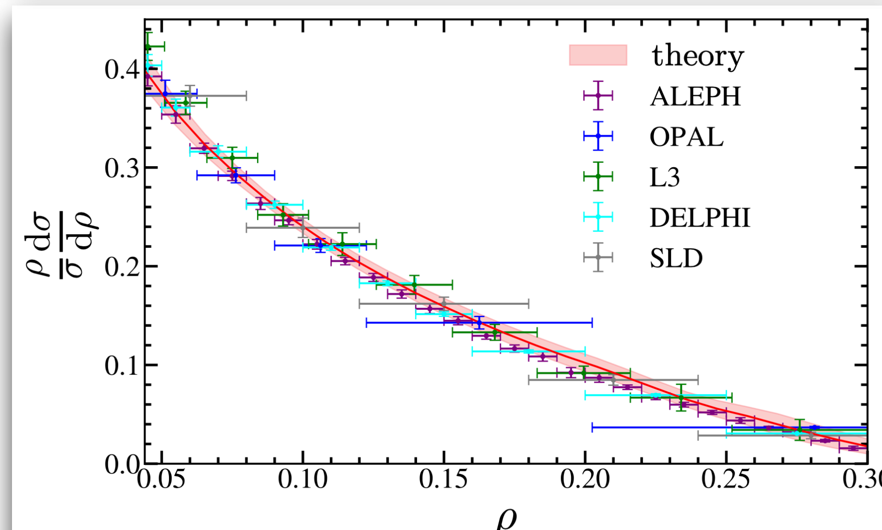
$$\Omega_1^\rho = 0.57 \pm 0.09\,\text{GeV}, \quad \Theta_1 = -0.50 \pm 0.17\,\text{GeV}$$

$$\chi^2/\text{dof} = 1.04$$

compatible with Thrust and C-parameter results







What do we learn from all of this?

- Perfectly valid to investigate impact on different sources of uncertainty, e.g. NP corrections, hadron mass effects, … However, this needs to be based on a robust theoretical description
- For this particular class of observables, a fixed-order prediction does not provide the required robust basis in the region typically used for α_s determinations
- Analytic resummation provides such a robust basis. In this case, using SCET, robust theoretical predictions are obtained
- Difference to world average may be related to statistical fluctuations — 1.6σ discrepancy can't be considered incompatible with world average
- Thorough investigation on theoretical side should be accompanied by a corresponding investigation on the experimental side

What do we learn from all of this?

- Perfectly valid to investigate impact on different sources of uncertainty, e.g. NP corrections, hadron mass effects, … However, this needs to be based on a robust theoretical de
- For this pa … rovide the required ro … s far
- Analytic re … ET, theoretical
- Difference … σ discrepanc
- Thorough ir … correspondir

Thank you for your attention!

🌧️