The Strong Coupling in MSHT fits

Lucian Harland-Lang, University College London

Workshop on Extracting the Strong Coupling at the EIC and other Future Colliders, May 7 2025

In collaboration with MSHT colleagues - Robert Thorne and Tom Cridge, and others - N. Armesto, F. Giuli, P. Newman, B. Schmooker, K. Wichmann

...and thanks to T. Cridge for stolen slides!

MSHT Global PDF Fitting

- Global fit of collinear unpolarised PDFs. More than 60 different datasets - Fixed Target, HERA DIS, neutrinos, Drell-Yan, Tevatron, LHC. 6 neutrinos, 2 fixed target DY, 8 HERA, 8 Tevatron, 27 LHC.
- Almost 5000 datapoints included over wide range of (x, Q^2): $10^{-4} \lesssim x \lesssim 0.8$ and $2\ \mathrm{GeV}^2 \lesssim Q^2 \lesssim 10^6\ \mathrm{GeV}^2$.
- Robust methodology with developments on all three fronts:
  1. Theoretical - Vast majority of processes included have full NNLO QCD theory, with NLO EW where relevant. Recent extension to approximate N3LO with theoretical uncertainties for first time. TC et al, 2207.04739
  2. Experimental - Many new datasets, more precise, more channels, more differential.
  3. Methodological - Extended parameterisation, 52 PDF parameters - allow fitting to accuracy $< 1\%$. Closure tests performed to examine central value and uncertainties. Harland-Lang, TC, Thorne, 2407.07944

What can the EIC contribute to this? $\Rightarrow$ Precise, new DIS data.

MSHT20 PDF Overview

- MSHT20 - New PDF set from MSHT group for precision LHC era (2012.04684). More data, extended methodology, improved theory.
- Global fit $\Rightarrow$ 61 different datasets, $\gtrsim$ 4500 datapoints - 10 Structure Func., 6 neutrinos, 2 fixed target DY, 8 HERA, 8 Tevatron, 27 LHC.
- Many developments since, only relevant ones shown:

  1. Extraction of $\alpha_S(M_Z^2)$ at NLO and NNLO: 2106.10289. $\rightarrow$ alphas-2022 workshop + Snowmass
  2. Approximate N3LO (aN3LO) PDFs with theoretical uncertainties: 2207.04739. $\rightarrow$ (world first!)
  3. Top mass determination in MSHT at NNLO: 2306.14885. $\rightarrow$ α_S-m_t correlation
  4. Impact of Jet, Dijet and Z p_T data at up to aN3LO in MSHT: 2312.12505. $\rightarrow$ Added new data
  5. Determination of $\alpha_S(M_Z^2)$ at up to aN3LO. **2404.02964**

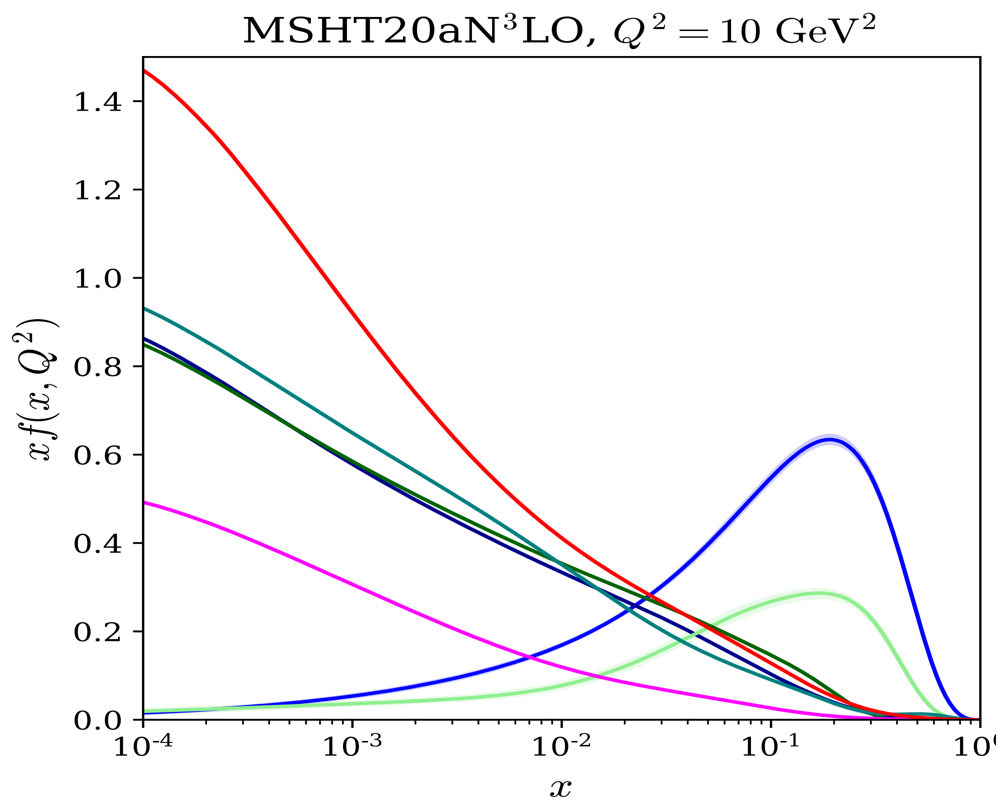
- Also QED PDFs, PDF experiment sensitivities, EIC study, PDF4LHC21, etc.

N. Armesto et al., arXiv:2309.11269, and in prep.

MSHT20 Approximate N3LO PDF Overview

- As experiments become more precise, need to improve accuracy and precison of theoretical predictions.
- In particular as PDFs become more precise we need:
  1. Move to higher orders (N3LO) in QCD.
  2. Inclusion of theoretical uncertainties.

  $\Rightarrow$ we can address both in one go! $\Rightarrow$ **MSHT20aN3LO PDFs**.
- Idea is to include known N3LO effects already into PDFs and to parameterise remaining unknown pieces via nuisance parameters.
- Variation of these remaining unknown N3LO pieces then provides a theoretical uncertainty within an approximate N3LO fit (aN3LO).
- Vary *actual unknown higher order pieces to get MHOU uncertainty*, rather than rely on scale variations as a proxy for this.



More information J. McGowan, T.C., L.A. Harland-Lang, R.S. Thorne: 2207.04739

See also - NNPDF4.0aN3LO!

N3LO - What do we know?

- **Approximate** $\neq$ **poorly known**!

$$P(x,\alpha_s) = \alpha_S P^{(0)}(x) + \alpha_S^2 P^{(1)}(x) + \alpha_S^3 P^{(2)}(x) + \alpha_S^4 P^{(3)}(x) + \ldots$$

★ **Splitting functions:** a wealth of information. Moments & various limits, with much recent further progress.

$$F_2(x,Q^2) = \sum_{\alpha\in H,q,g;\beta\in q,H} \left(C_{\beta,\alpha}^{VF,n_f+1} \otimes A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)\right)$$

★ **DIS:** massless coefficient functions known (+ massive high Q^2). Massive low Q^2 approx. known.

$$f_\alpha^{n_f+1}(x,Q^2) = [A_{\alpha i}(Q^2/m_h^2) \otimes f_i^{n_f}(Q^2)](x)$$

★ **Heavy Flavour:** again wealth of information. Moments & various limits, with much recent progress.

$$\sigma = \sigma_0 + \sigma_1 + \sigma_2 + \sigma_3 + \ldots \equiv \sigma_{N3LO} + \ldots$$

★ **Hadronic Cross Sections:** while much progress made, thus far not useable in PDF fits.

- Indeed, now (only v. recently) evolution effectively known exactly for pheno purposes. Remaining ingredients - massive DIS + hadronic cross sections will not arrive soon.
- Approximation + uncertainty required for these, but it is appropriate to move to aN3LO now.

Determination of the Strong Coupling at up to aN3LO

Extracting the strong coupling in a PDF fit

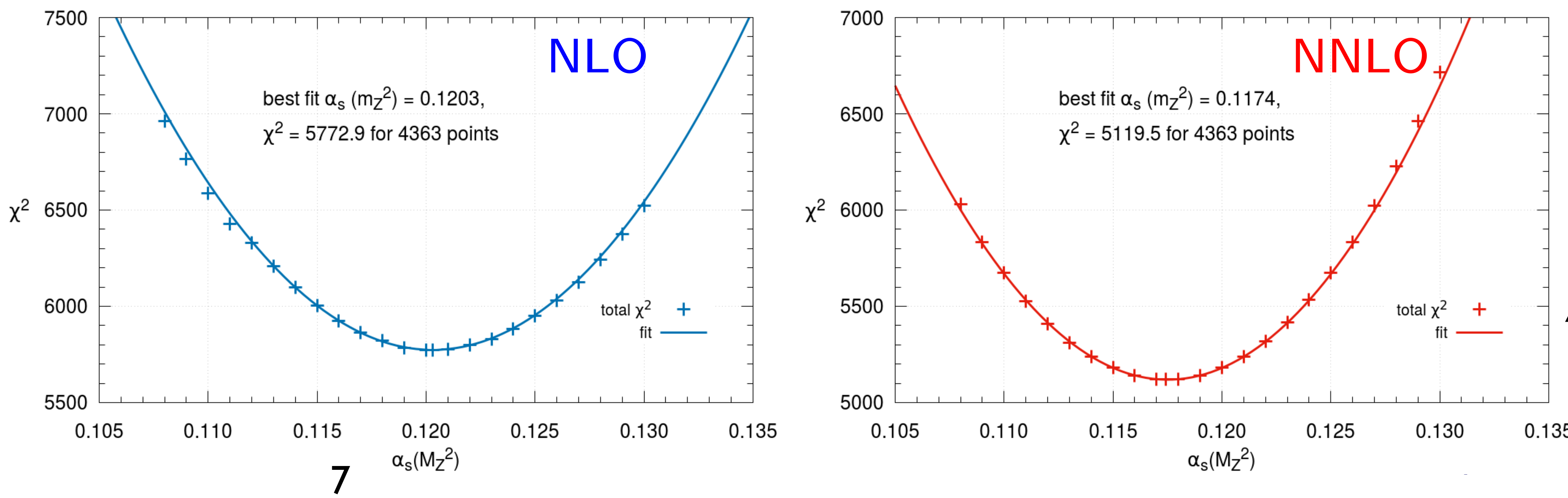
$P(x,\alpha_s) = \alpha_S P^{(0)}(x) + \alpha_S^2 P^{(1)}(x) + \alpha_S^3 P^{(2)}(x) + \alpha_S^4 P^{(3)}(x) + \ldots$

- Global PDF fit sensitive to value of strong coupling through impact on **evolution** and **cross sections**.
- While baseline sets often provided with $\alpha_S = 0.118$, can allow it be to free parameter and see what we find.
- Individual datasets have different α_S dependencies, but **global** determination provides robust fit.
- Determination of α_S and PDFs highly correlated. Only completely consistent way to include impact of a (PDF sensitive) hadronic measurements is via full refit. **S. Forte and Z. Kassabov, arXiv: 2001.04896**

$\alpha_{S,\mathrm{NNLO}}(M_Z^2) < \alpha_{S,\mathrm{NLO}}(M_Z^2)$
as NNLO corrections +ve, so
fitting same data $\Rightarrow$ lower α_S.

$$\boxed{\alpha^{prev}_{S,\mathrm{NLO}}(M_Z^2) = 0.1203 \qquad \alpha^{prev}_{S,\mathrm{NNLO}}(M_Z^2) = 0.1174}$$

- In original (up to) NNLO MSHT20 fit, the best fit values were found to be:
- What about aN3LO?



Nice Quadratic χ^2 profile ✓

Note we provide the $\Delta\chi^2$ changes with α_S $\Rightarrow$ can use this info!

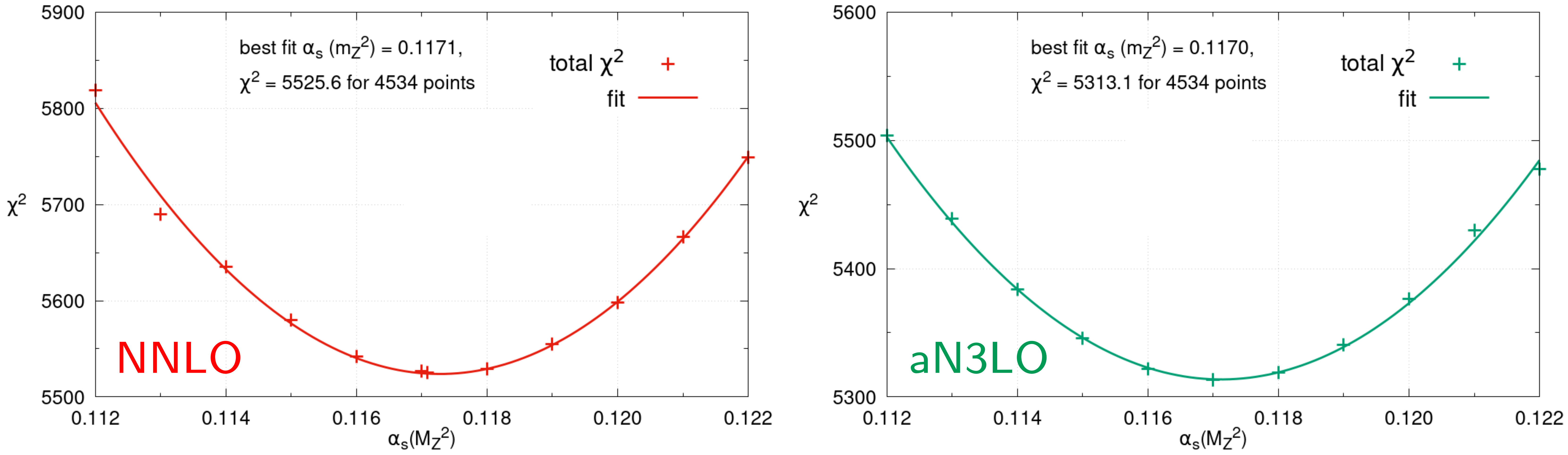
MSHT, 2404.02964

The strong coupling at aN3LO

Minor updates + ATLAS 8 TeV jets

- Can now extend this analysis to aN3LO. Baseline very similar (not identical) to MSHT20. Find:

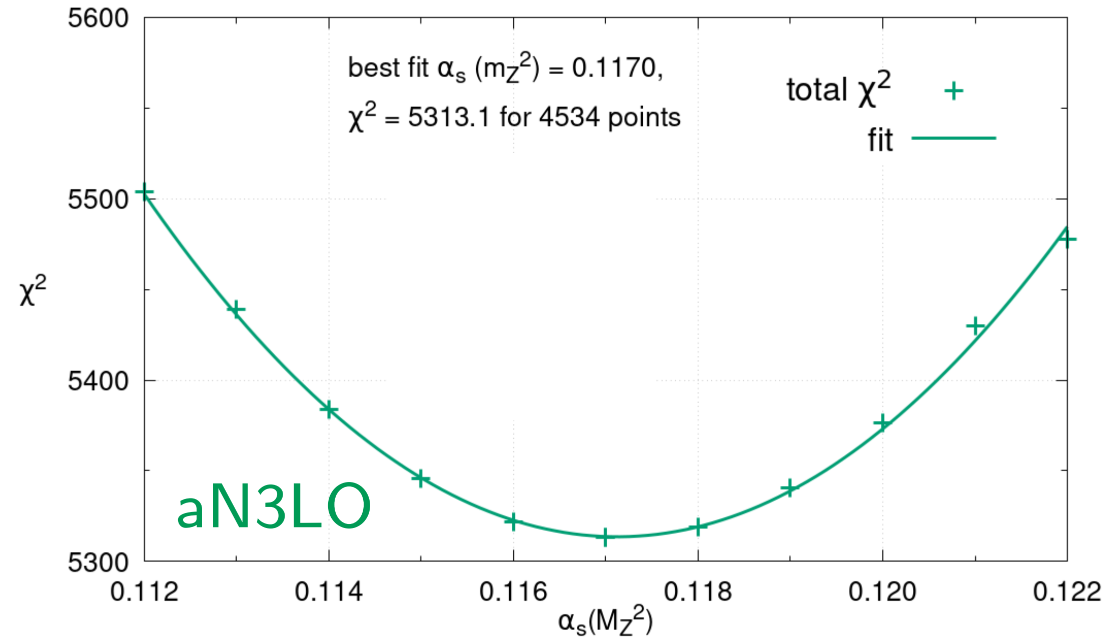
$$\alpha_{S,\mathrm{NNLO}}^{new}(M_Z^2) = 0.1171 \qquad \alpha_{S,\mathrm{aN3LO}}^{new}(M_Z^2) = 0.1170$$



Nice Quadratic χ^2 profile ✓

- NNLO: similar to previous result (0.1174).
- Very good perturbative convergence to aN3LO, and both consistent with world average. 0.1180 ± 0.0009
- Confirmed that more recent aN3LO splitting function information gives v. similar result ($\ll$ uncertainty)
- Looking in more detail…

- Find that global χ^2 profile built up of different competing pulls…



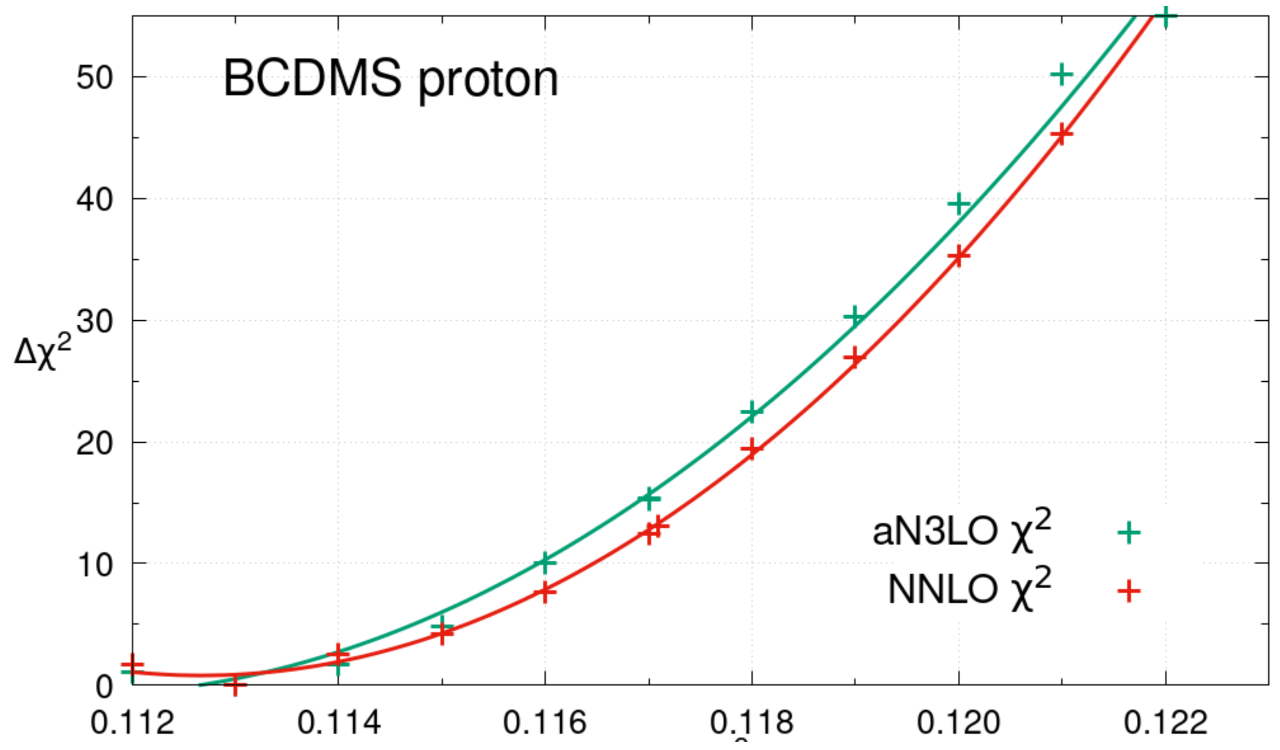
★ **Fixed target data**. DIS in particular sensitive through impact on evolution

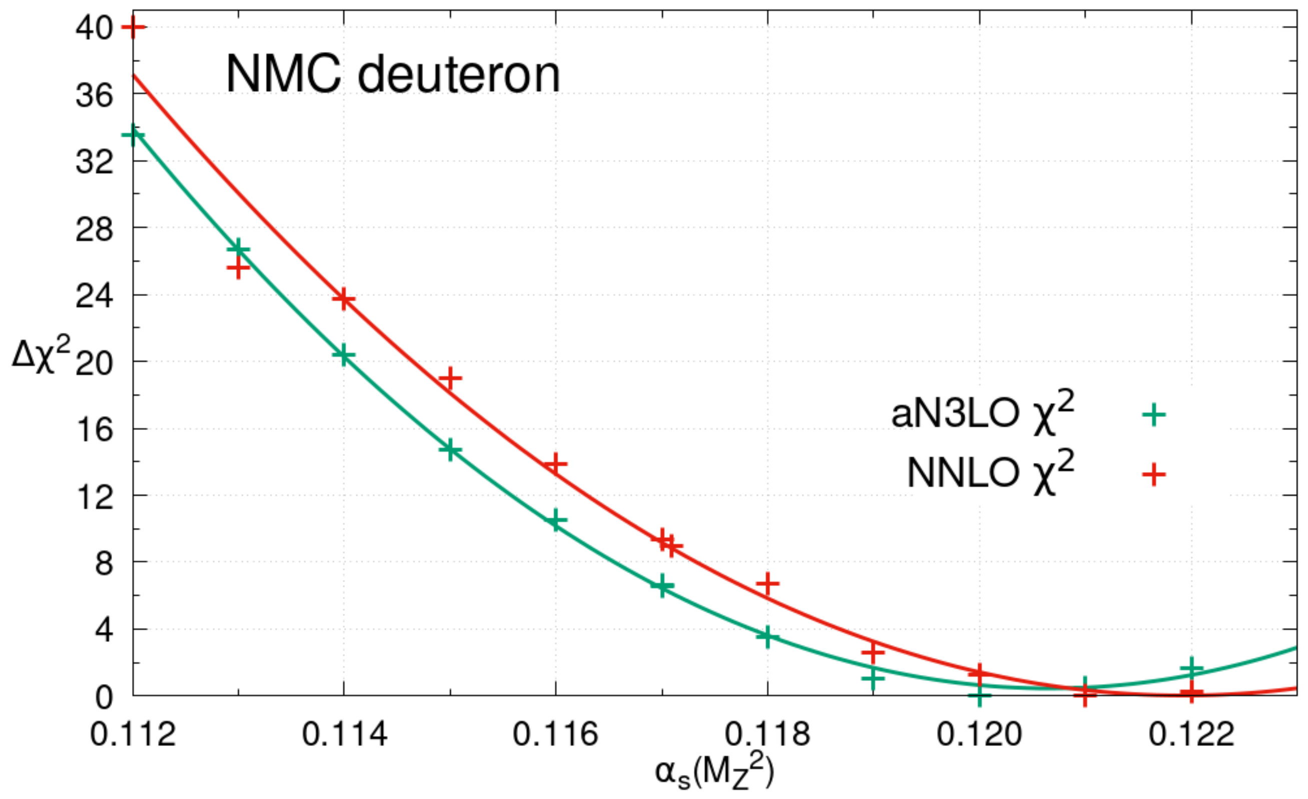
★ **LHC DY**. Due to high precision provide reasonable constraints

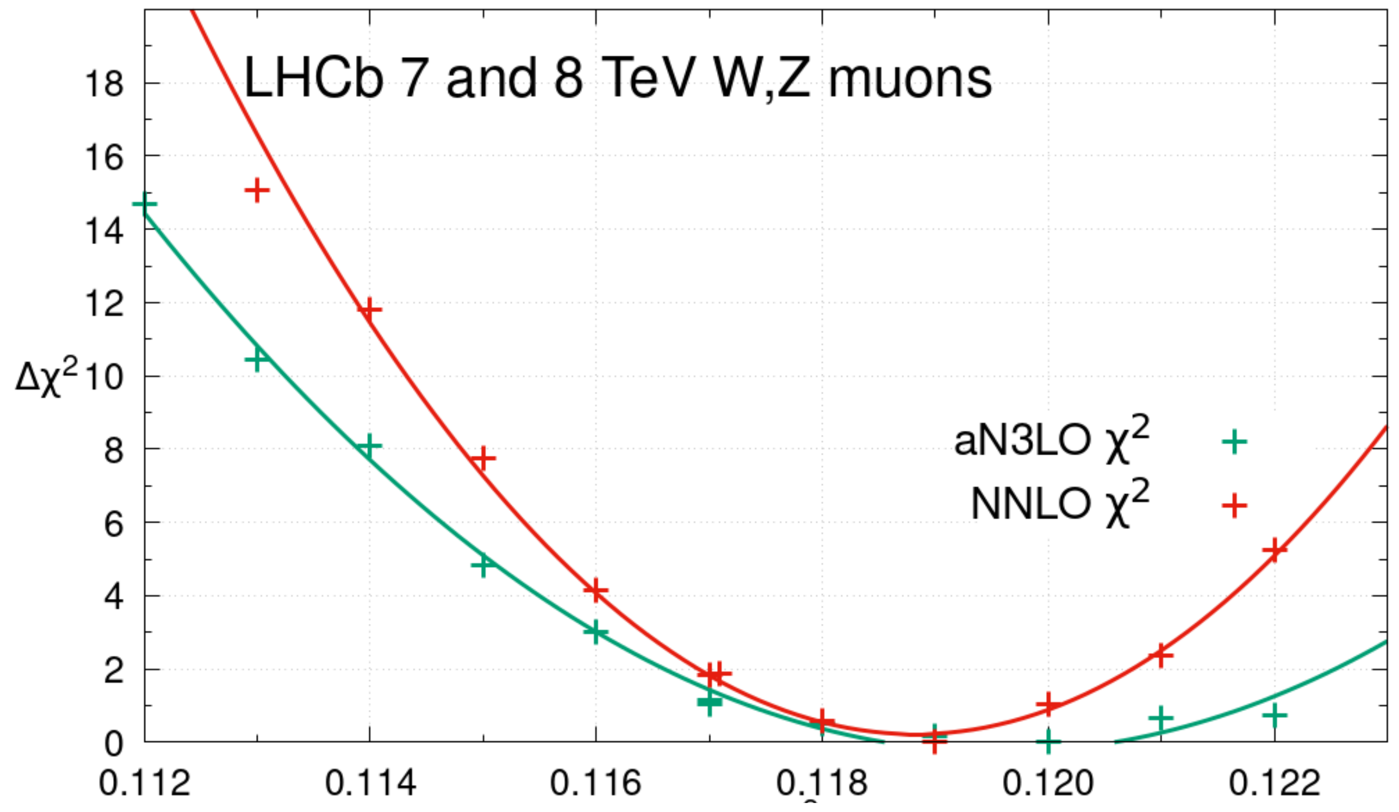
★ **LHC Jets**. Clear direct constraints. Some difference between datasets…

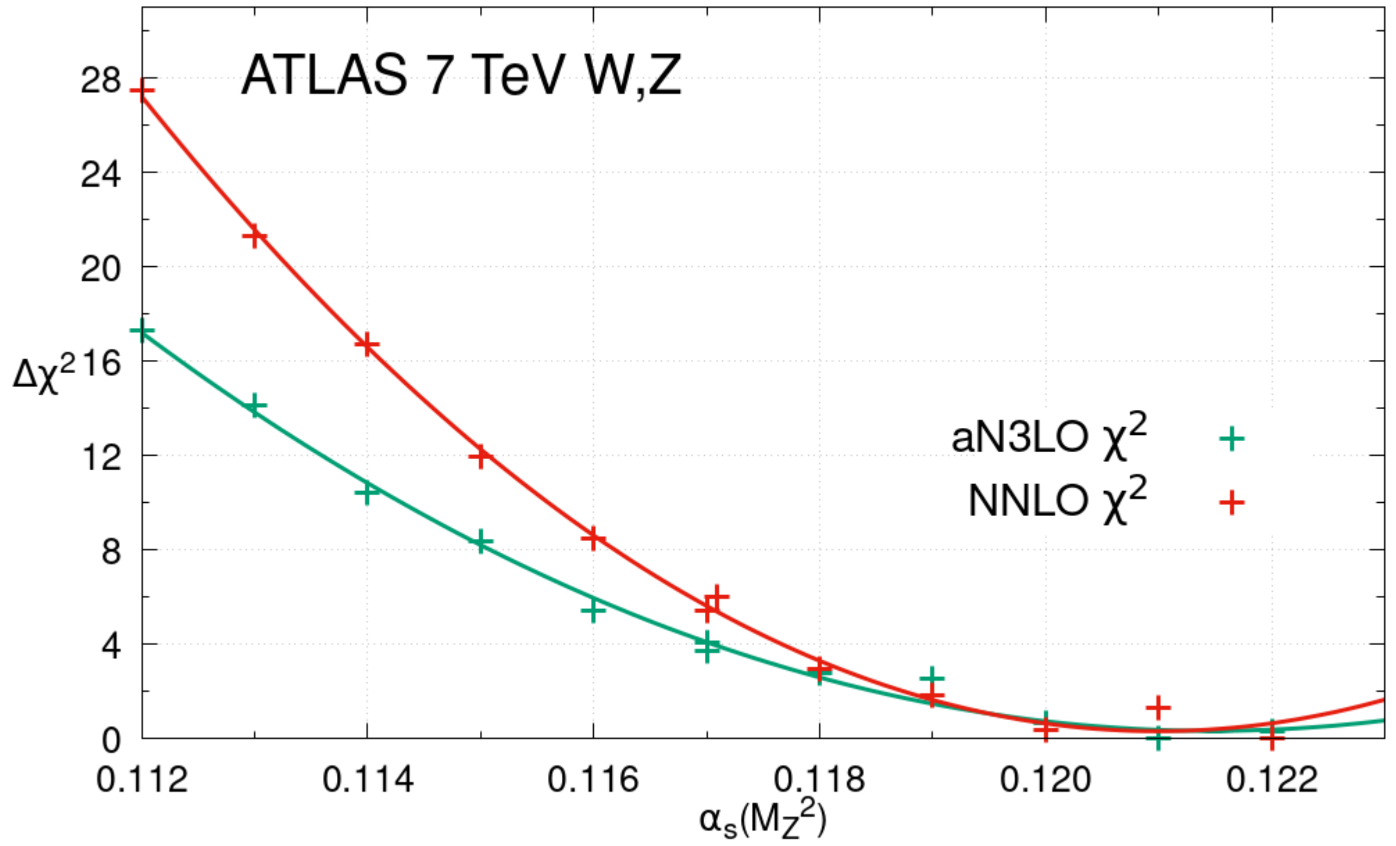
NNLO
aN3LO

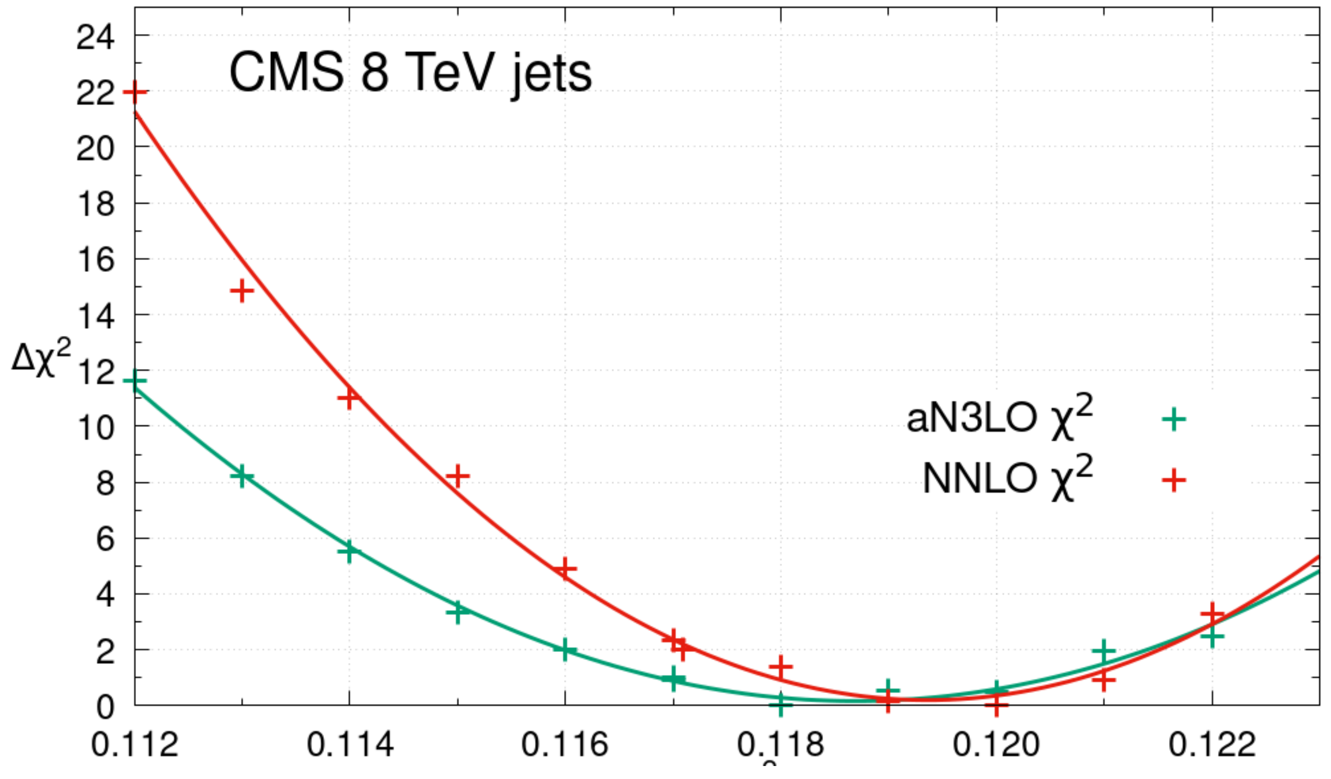
BCDMSp prefers lower α_S to slow fall of structure function with Q^2.

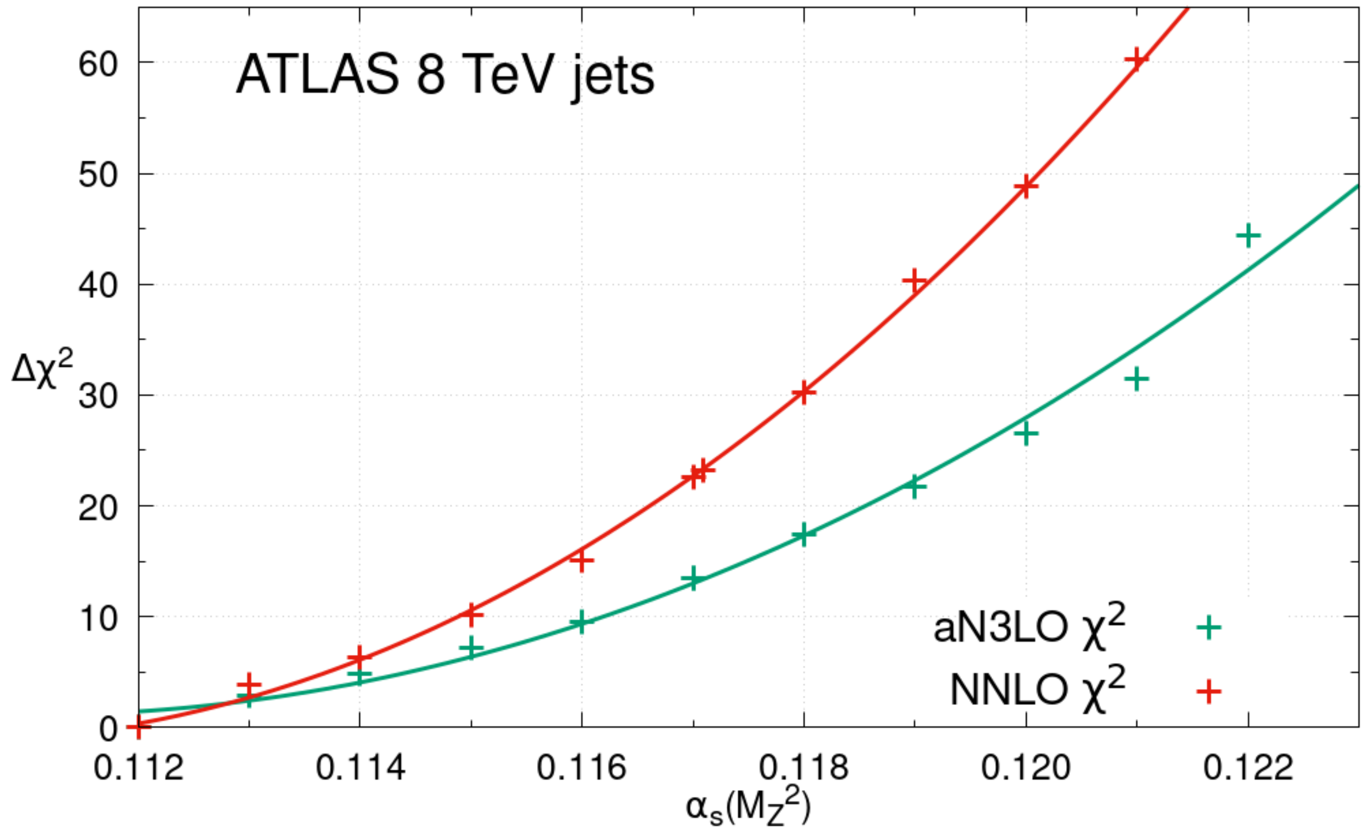












- Trends all rather similar between NNLO and aN3LO (i.e. convergence).

Jets vs. Dijets?

T.C., L.A. Harland-Lang,
R.S. Thorne 2312.12505.

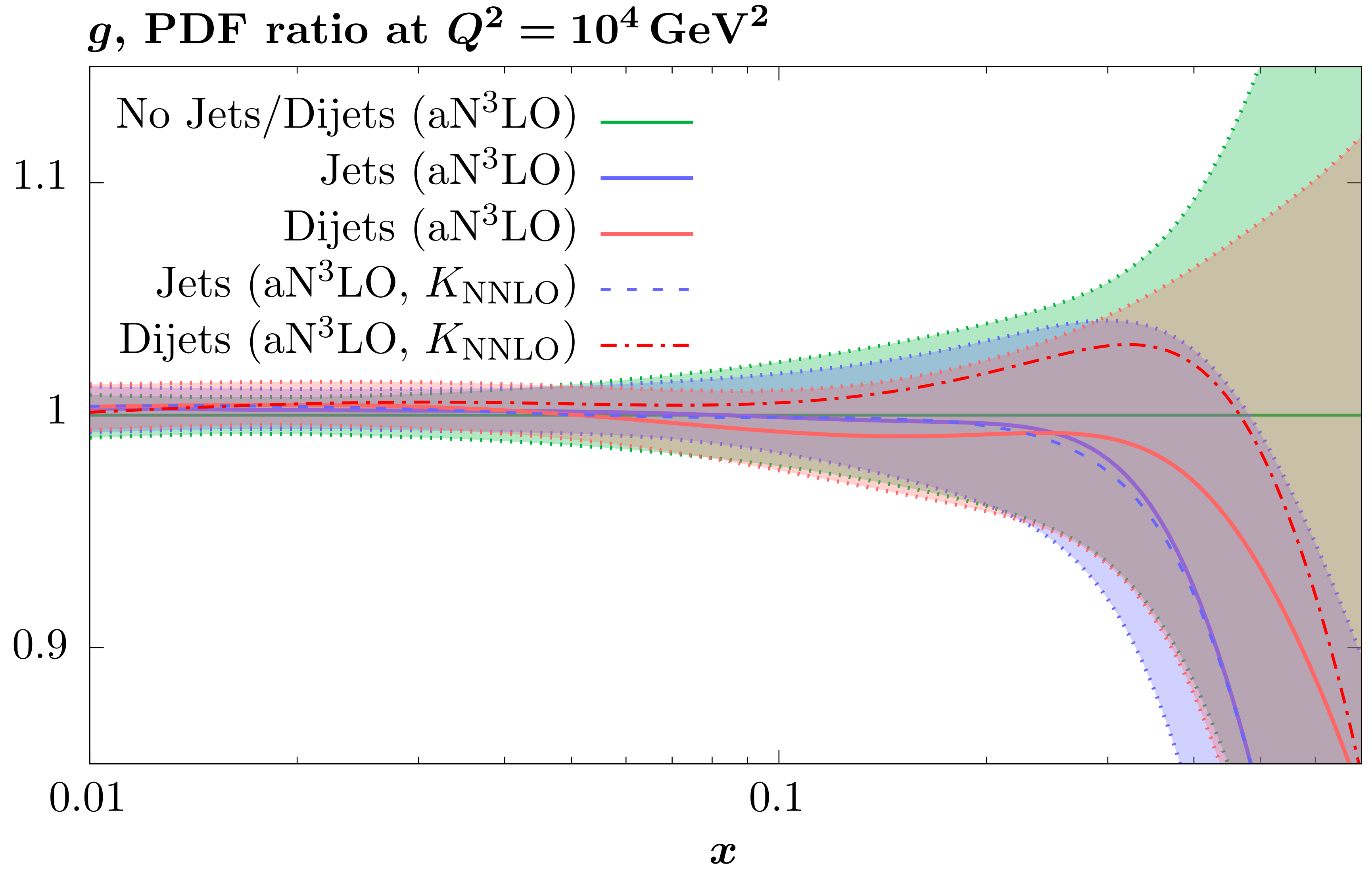
- Studied in detail in recent paper. Worth briefly mentioning here. Bottom line:

★ Potentially general reasons to prefer dijet data: non-unitary nature of inclusive jets, and potential for 3D distributions in dijets (more constraining!).

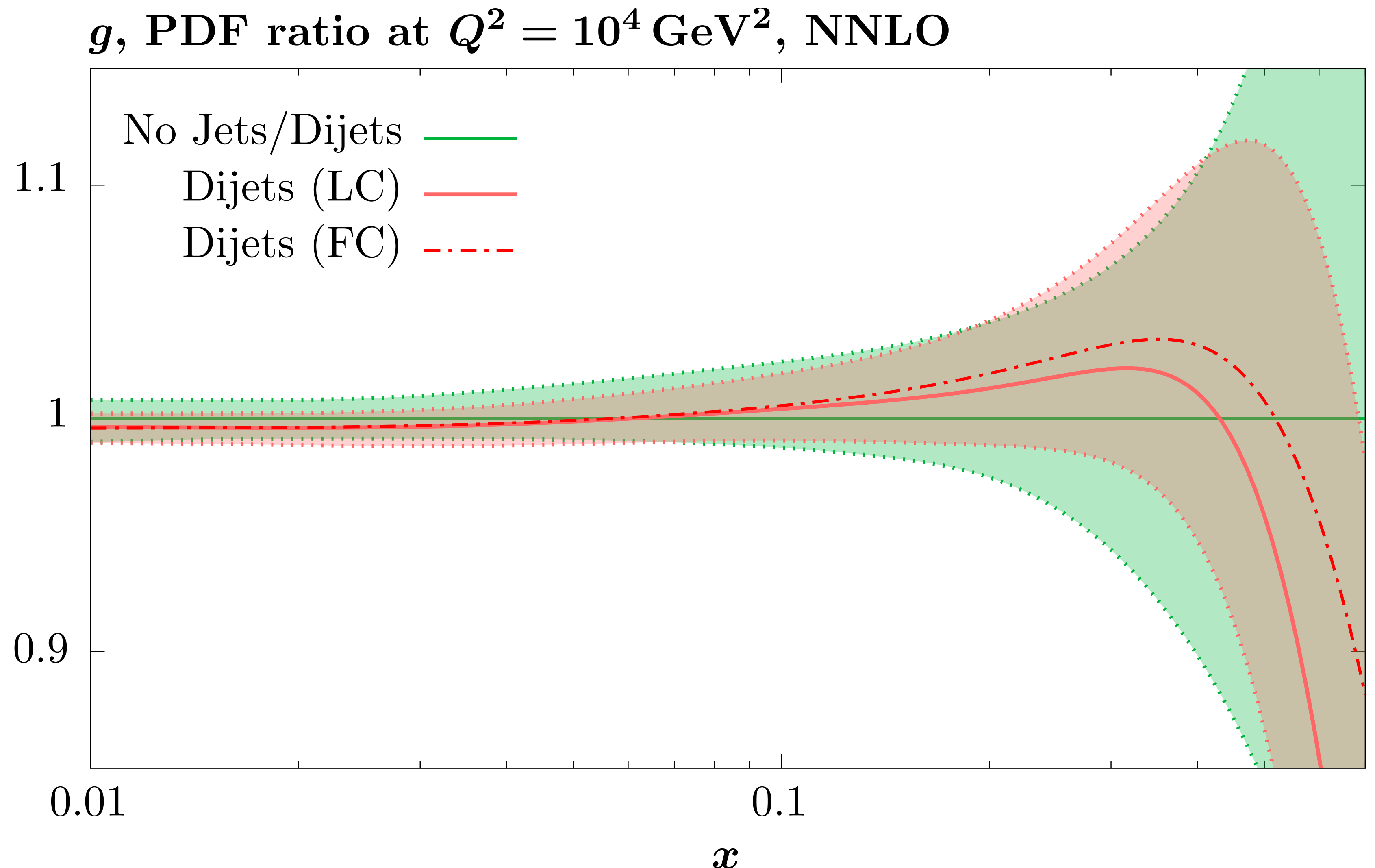
★ Supported by our study: fit quality better in dijet case at both NNLO and aN3LO

★ Some difference in pull on gluon at NNLO, better consistency at aN3LO.

Inclusive Jets	N_{pts}	χ^2/N_{pts} NNLO	χ^2/N_{pts} aN3LO	Dijets	N_{pts}	χ^2/N_{pts} NNLO	χ^2/N_{pts} aN3LO
Total	472	1.39	1.43	Total	266	1.12	1.04
Total (+ATLAS 8 TeV jets)	643	1.67	1.61	Total	266	1.12	1.04



★ Impact of full colour mild…

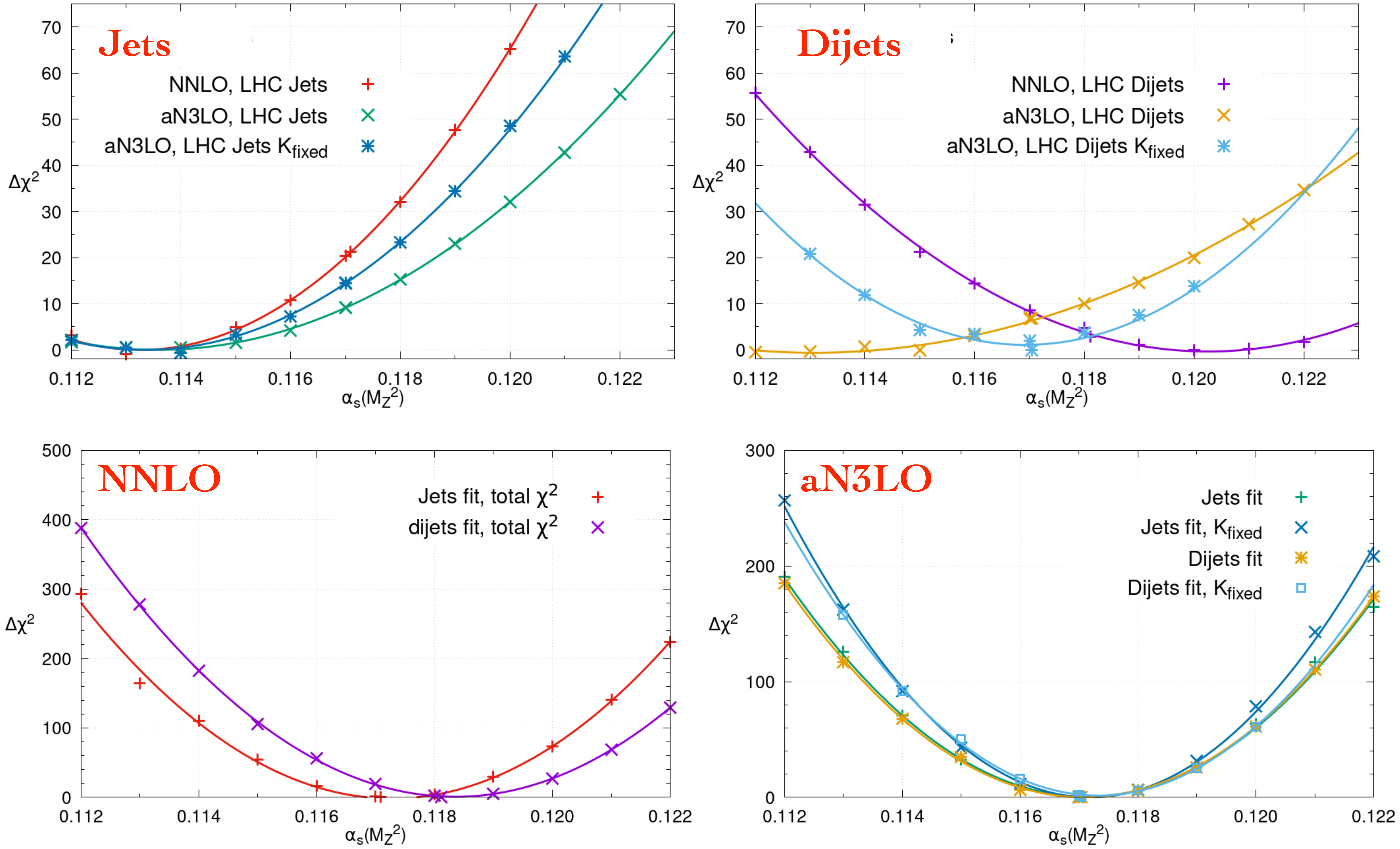


- What about α_S ?

  ★ NNLO: pull very different between jets and dijets.

  ★ aN3LO: this stabilises!

★ Translates into difference in **global** profiles:



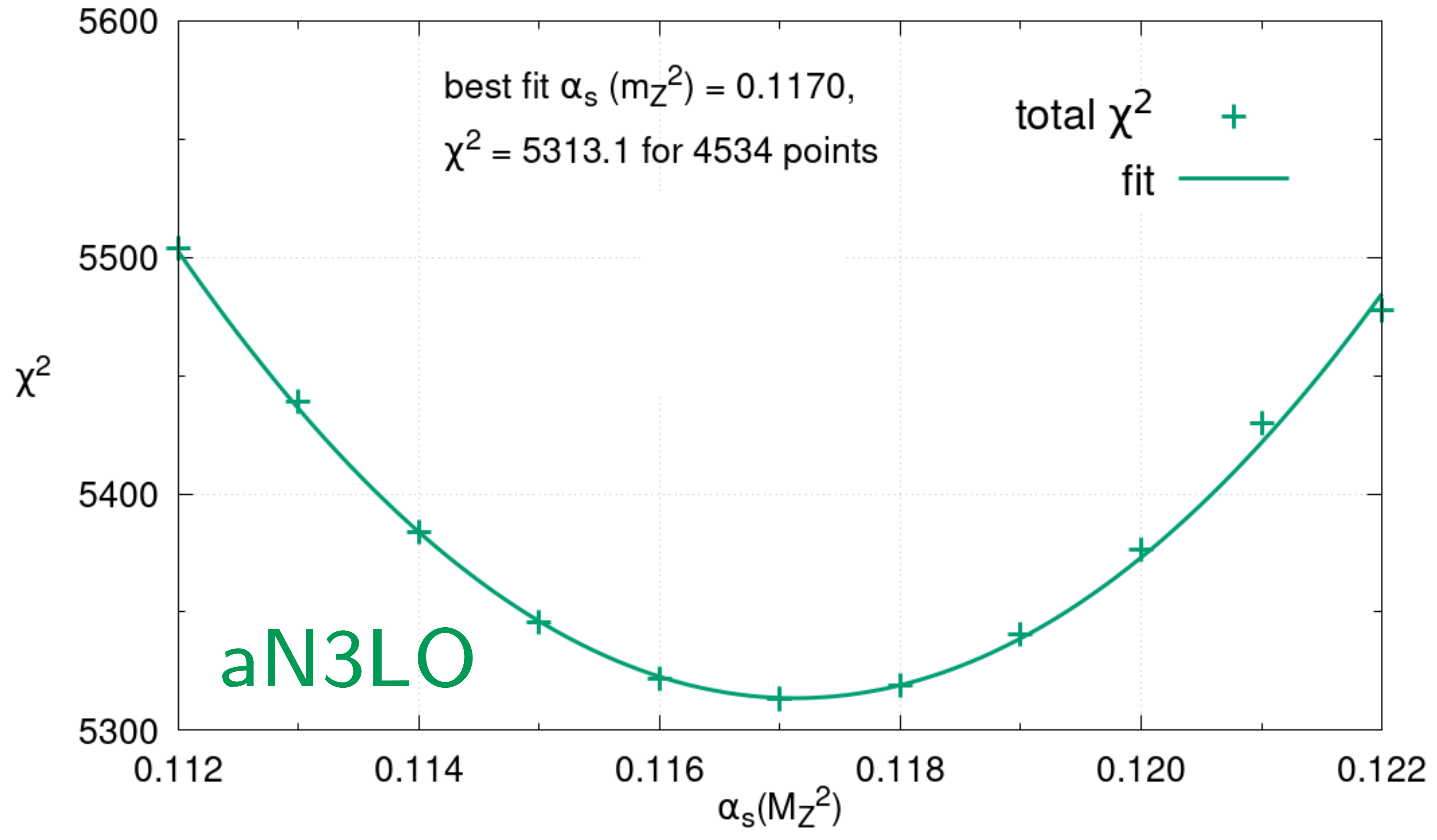
★ Much better consistency at aN3LO, though at NNLO consistent within (dynamic tolerance) uncertainties:

$$\alpha_S(M_Z^2)(\text{Dijet, NNLO}) = 0.1181 \pm 0.0012 \qquad \alpha_S(M_Z^2)(\text{Jet, NNLO}) = 0.1171 \pm 0.0015$$

★ But in tension with $T^2 = 1$: ~ 0.0004 $\quad \sim 0.0005$

$$\alpha_S(M_Z^2)(\text{Dijet, aN}^3\text{LO}) = 0.1170 \pm 0.0013$$

Uncertainty Evaluation



- In textbook case, would simply take $\Delta\chi^2 = 1$ from minimum, to give (roughly):

$$\alpha_S = 0.1170 \pm 0.0005$$

- However from discussion before, expect to be too aggressive. Enlarged tolerance needed.

- In MSHT apply `dynamic tolerance' criterion. Briefly:
  - ★ Evaluate individual χ^2 profile for each dataset
  - ★ Deviation with α_S increasing/decreasing monitored and limited such that this does not exceed 'hypothesis testing' criterion $\Delta\chi^2 \lesssim \sqrt{2N}$ i.e. remains good according to this measure.
  - ★ In toy model can show given two datasets in tension that* PDF uncertainty $\propto$ difference (unlike $T = 1$).
  - ★ Will result in one dataset setting most stringent upper/lower limits, but find many others with similar limits, i.e. uncertainty not driven by a single (potentially problematic) dataset.
  - ★ Broadly corresponds to $T \sim 3 - 4$.

MSHT, 2407.07944

***For experts: in reality, limit is rescaled by best fit value:** $\Delta\chi^2 \lesssim \chi^2_{n,0}\left(\frac{\xi_{68}}{\xi_{50}} - 1\right)$

T. Cridge and M. Lim, arXiv:2306.14885

Though reasons to suggest we could…

F_2^c provides upwards bound of: $\Delta\alpha_S(M_Z^2) = +0.0020$.

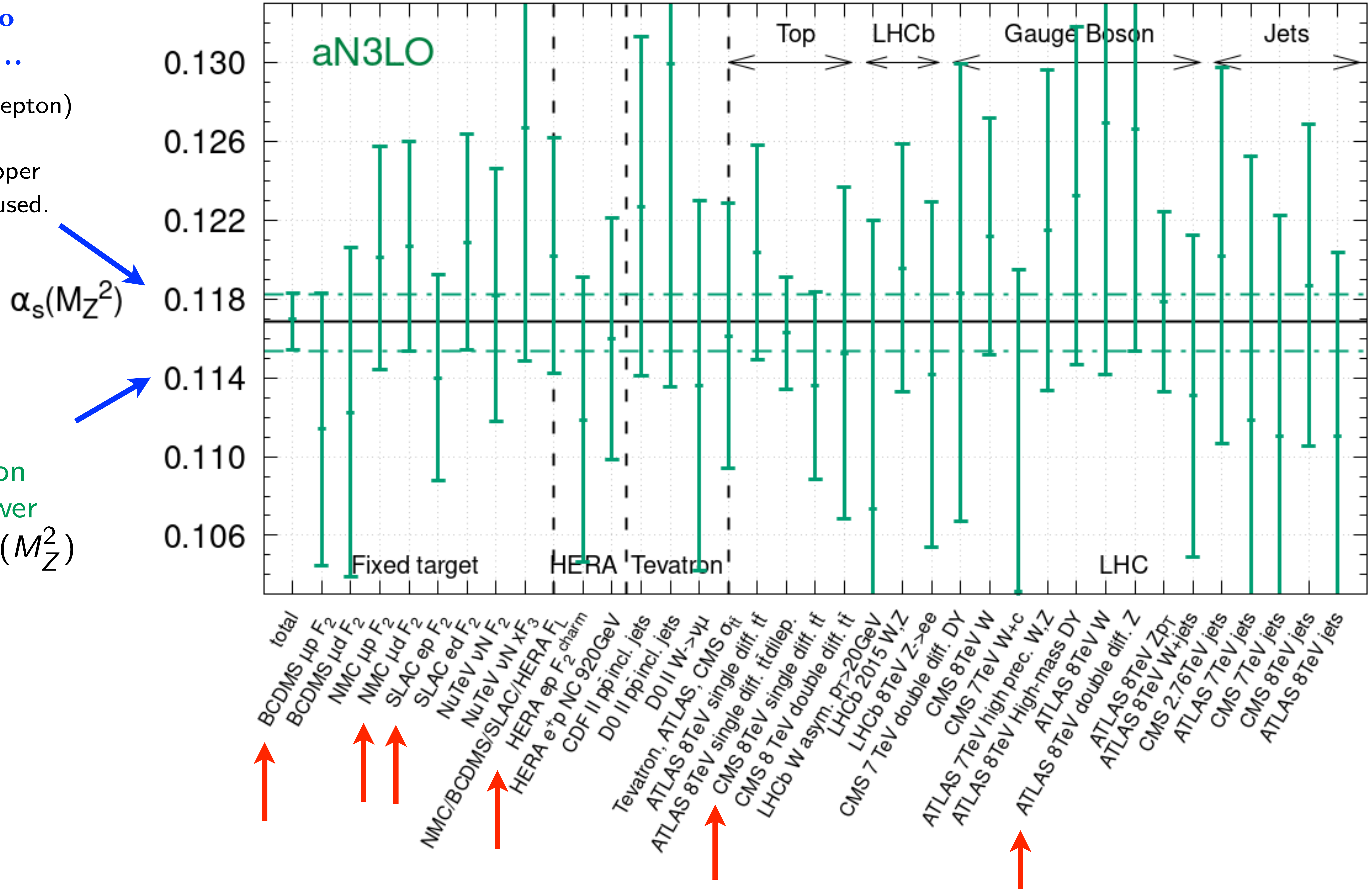
CMS and ATLAS (dilepton) $t\bar{t}$ single diff. would give slightly higher upper α_S bounds, but not used.

(Consistent with other fixed target DIS (p), and ~ known N3LO)

BCDMSp data strongest constraint upwards: $\Delta\alpha_S(M_Z^2) = +0.0013$.

NMC deuteron, ATLAS 8 TeV Z both give lower bounds of $\Delta\alpha_S(M_Z^2) = -0.0017$.

SLAC deuteron data gives lower bound: $\Delta\alpha_S(M_Z^2) = -0.0016$.



- Putting together and suitably symmetrising, we quote:

$$\alpha_{S,\mathrm{aN3LO}}(M_Z^2) = 0.1170 \pm 0.0016$$

Consistent with (NNLO) World Average of 0.1180 ± 0.0009.

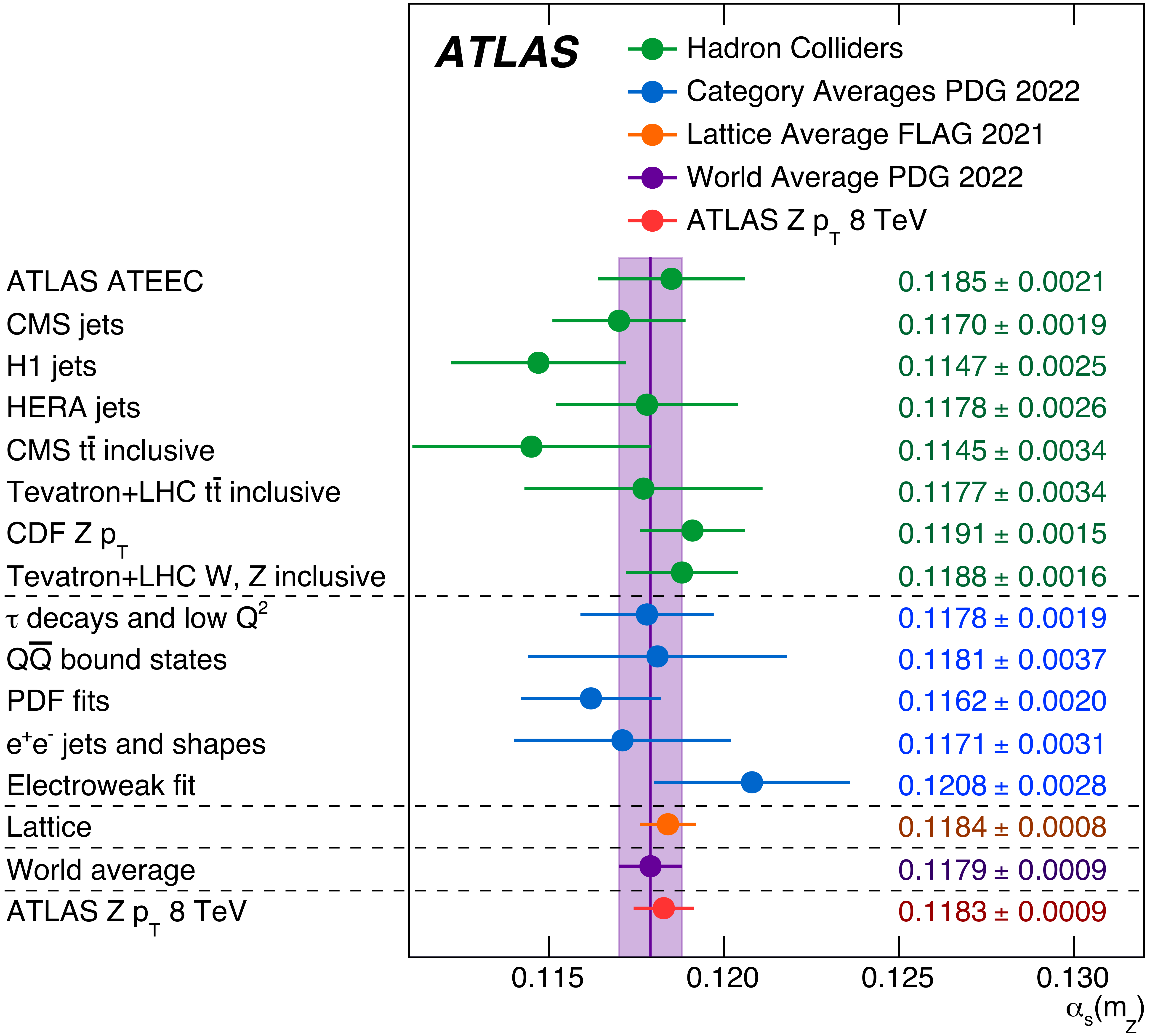
With thanks to T. Cridge

Comparison to other results

$$\alpha_{S,\text{aN3LO}}(M_Z^2) = 0.1170 \pm 0.0016$$

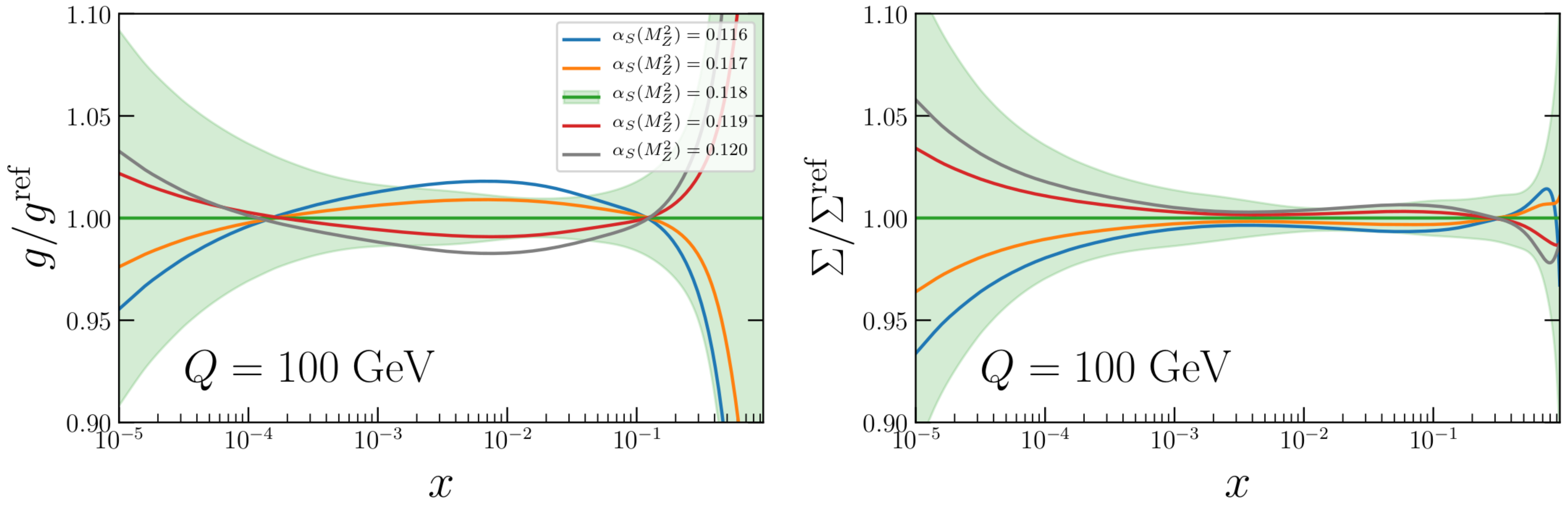
- Consistent with world average and recent ATLAS measurement.
- Uncertainty larger but similar order. ~ larger than NNLO (in part MHOUs).
- Again, if we took $\Delta\chi^2 = 1$ would be factor of ~ 2 smaller, but v. good reasons to believe that is too aggressive.

$\alpha_S = 0.1170 \pm 0.0005$

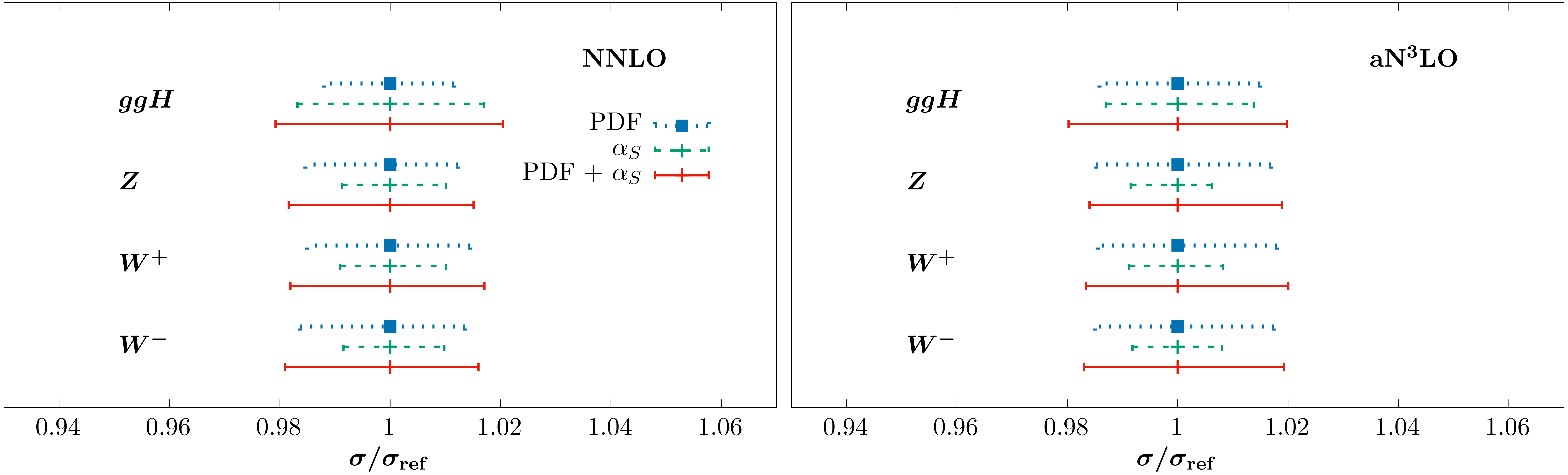


PDFs

- ★ Clear correlation between PDFs and α_S, as expected.
- ★ Change generally within PDF uncertainties for $\Delta\alpha_S = \pm 0.001$ though close to edge for gluon.
- ★ **Gluon** anticorrelated with α_S for $x \lesssim 0.1$ to maintain $\mathrm{d}F_2/\mathrm{d}Q^2 \sim \alpha_S g$. Correlation at high $x \gtrsim 0.1$ from sum rule.
- ★ Less impact on **quarks** - reduced/increased at high/low x from splitting.



Cross Sections



★ Impact on cross sections includes α_S variation in matrix elements + PDFs - non-trivial interplay to get final result. Important to treat these together!

★ For LHC **Higgs** the anticorrelation between gluon and α_S compensates larger direct uncertainty.

★ For **DY** direct α_S uncertainty small, and largest effect from change in PDF.

★ Combined PDF + α_S broadly leads to at most moderate increase over PDF uncertainty alone.

The impact of EIC (pseudo)-data in the MSHT fit

Pseudodata Study

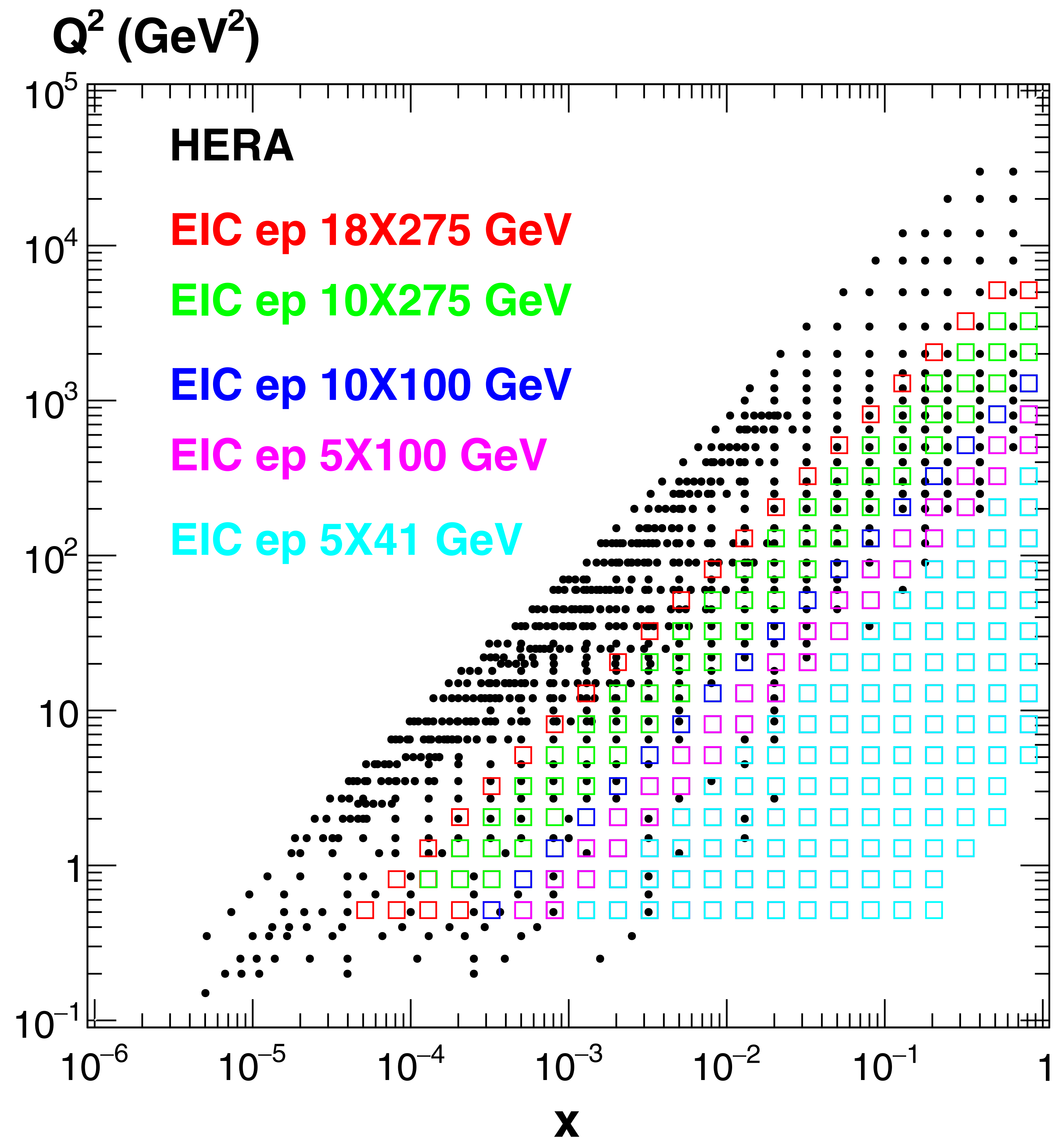
N. Armesto et al., arXiv:2309.11269

- Previous collaborative study: generate pseudodata (PD) with updated beam energies, configurations, lumis and uncertainty projections.
  - Two (low/high) acceptance cases considered.
  - Kinematic coverage (including MSHT cuts)

$$Q^2 > 2\ \mathrm{GeV}^2,\ 0.01 < y < 0.95,$$
$$W^2 > 15\ \mathrm{GeV}^2.$$

TABLE I. Beam energies, center-of-mass energies and annual integrated luminosities of the different configurations considered for the EIC.

e-beam energy (GeV)	p-beam energy (GeV)	$\sqrt{s}$ (GeV)	Integrated lumi (fb^{-1})
18	275	141	15.4
10	275	105	100
10	100	63	79.0
5	100	45	61.0
5	41	29	4.4

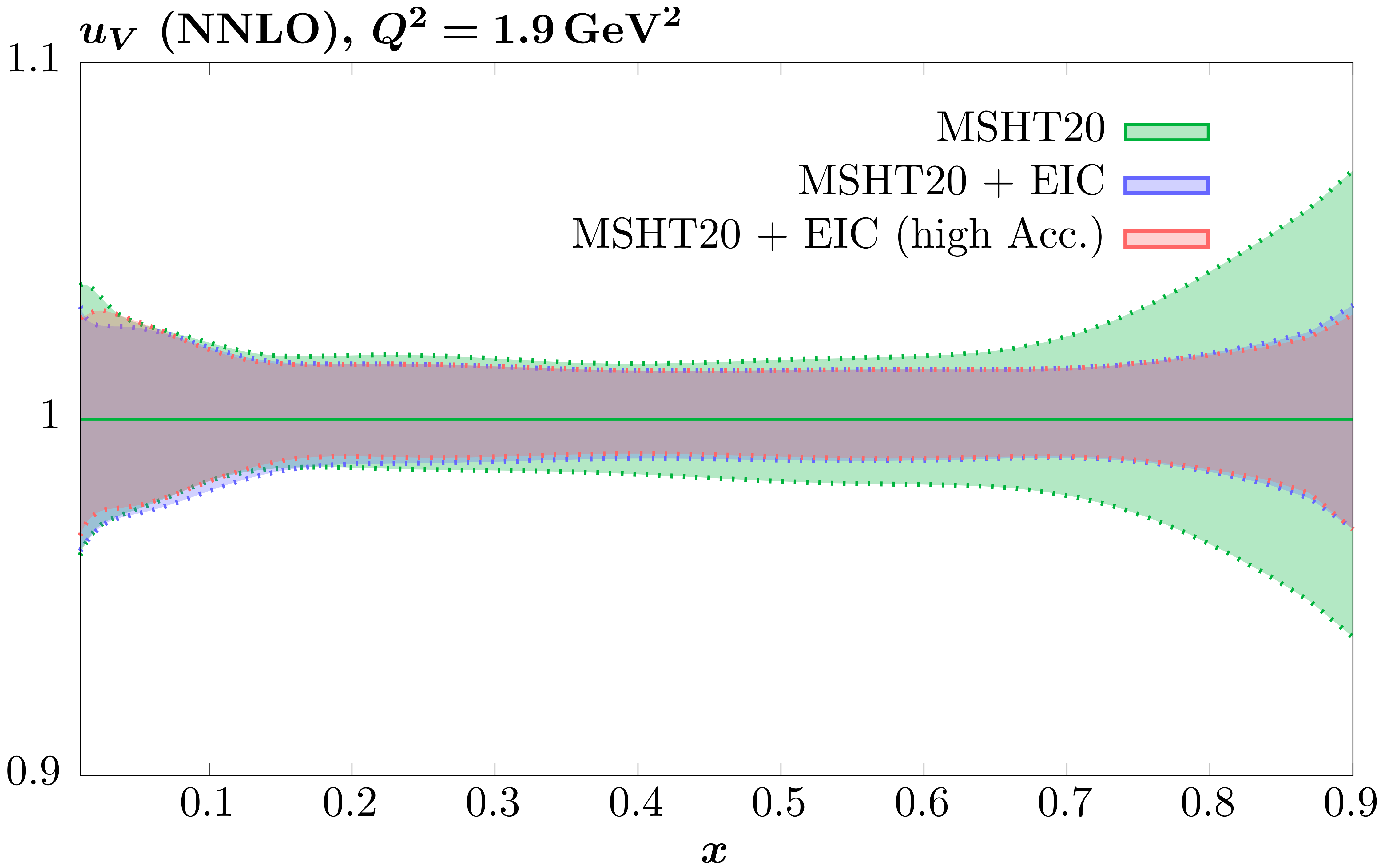


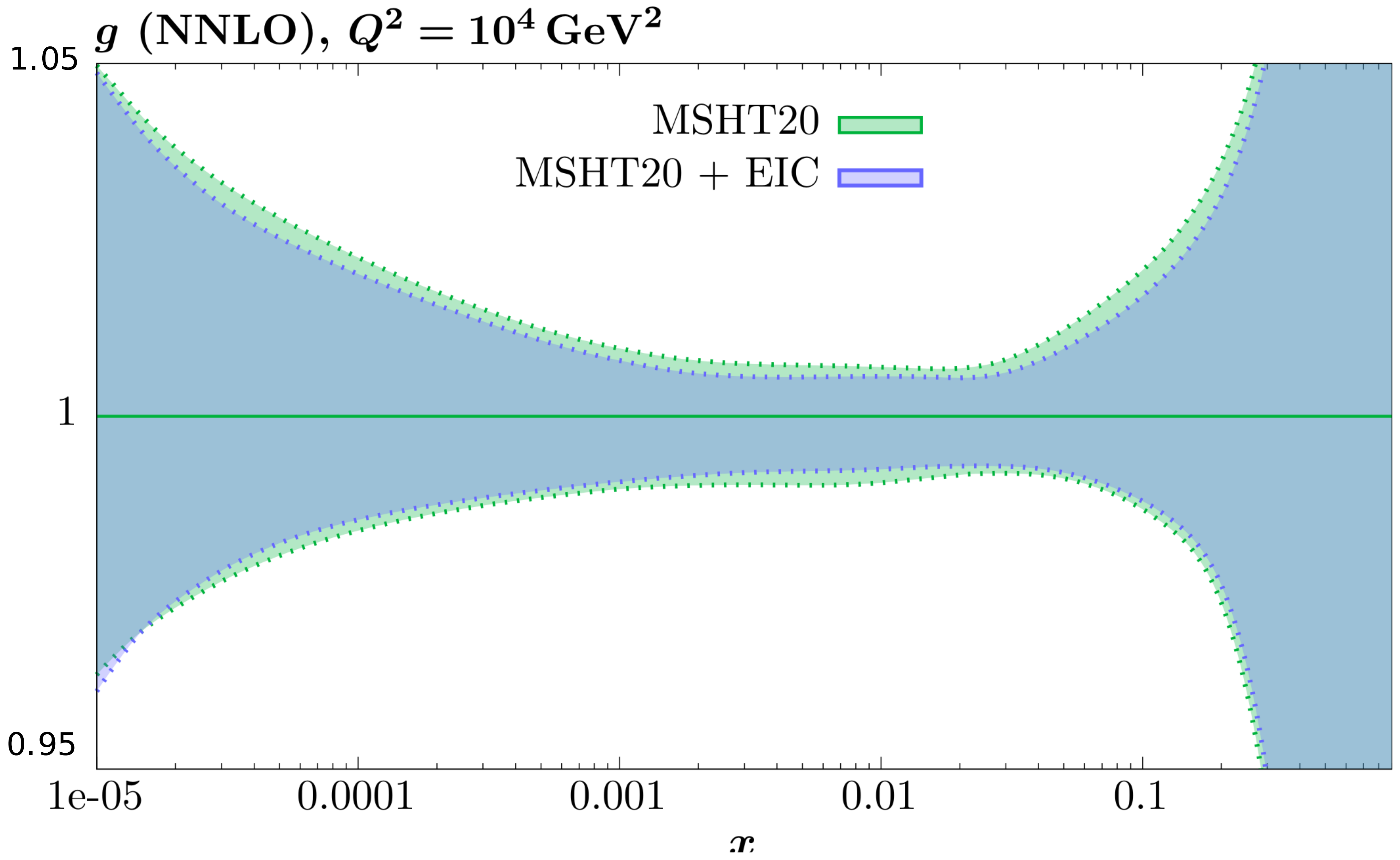
- Including sensible projections for main uncertainty sources.

→ 1.5-2.5% point-to-point uncorrelated
→ 2.5% normalisation (uncorrelated between different $\sqrt{s}$)

- NB: Semi-inclusive DIS not included!

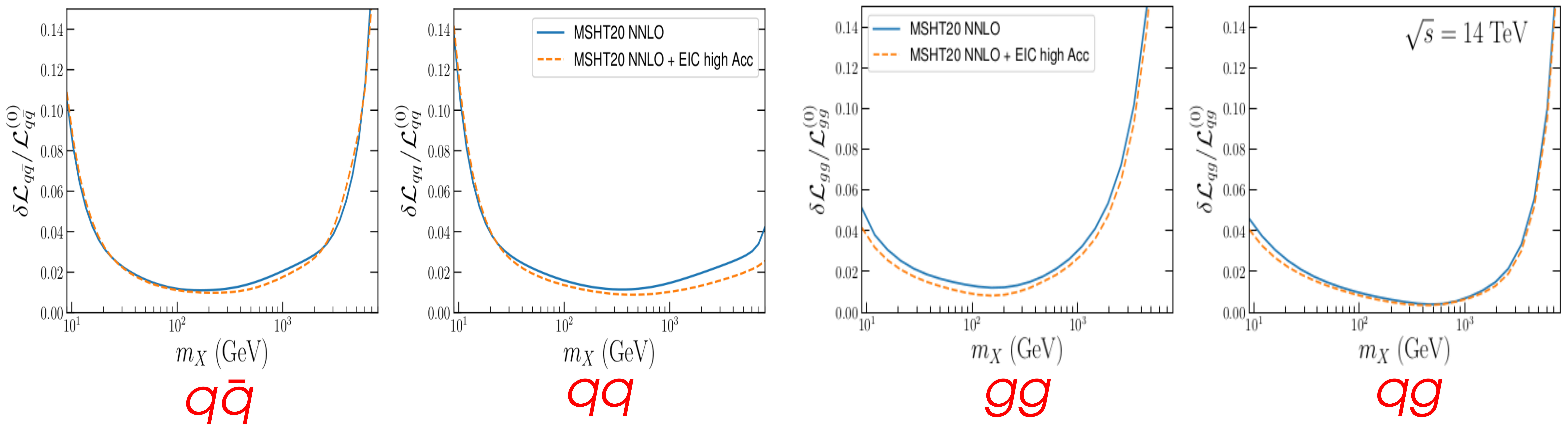
PDF impact





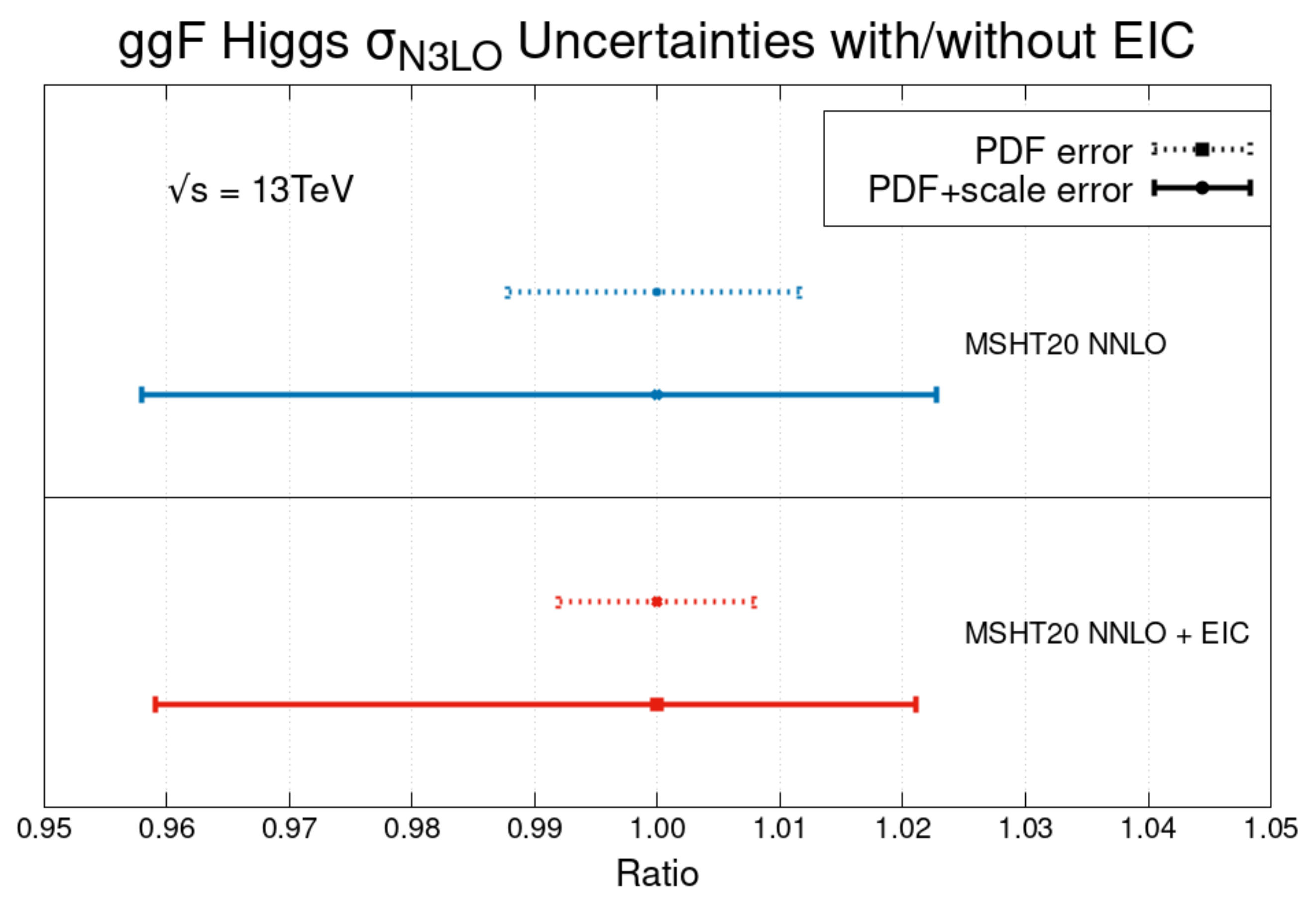
- Largest impact on u at large x as $\sigma_{e^-p}^{\mathrm{NC\ DIS}} \propto \sum_i Q_i^2 f_i(x)$.
- Though some care needed in interpretation at the highest x values: PDF extrapolation, TMC effects relevant…
- Moderate impact on gluon across x from scaling violations.
- Impact of larger y acceptance negligible as different beam energy configurations provide constraints.
- Always worth bearing in mind these tests assume consistency - in presence of tensions complementarity is key!

PDF Luminosities + Higgs



- Some knock on effect on luminosities.

- Has some effect on e.g. ggH albeit with other uncertainty sources currently dominating!



Determining the strong coupling

- $\alpha_S(M_Z^2)$ sensitivity in global PDF fit come from:
  - Direct $\alpha_S(M_Z^2)$ dependence in coefficient functions.

$$\mathbf{C}(\alpha_S) = \alpha_S^i[\mathbf{C}_0 + \alpha_S\mathbf{C}_1 + \alpha_S^2\mathbf{C}_2 + \alpha_S^3\mathbf{C}_3 + \ldots]$$

  - Indirect $\alpha_S(M_Z^2)$ dependence through PDF evolution.

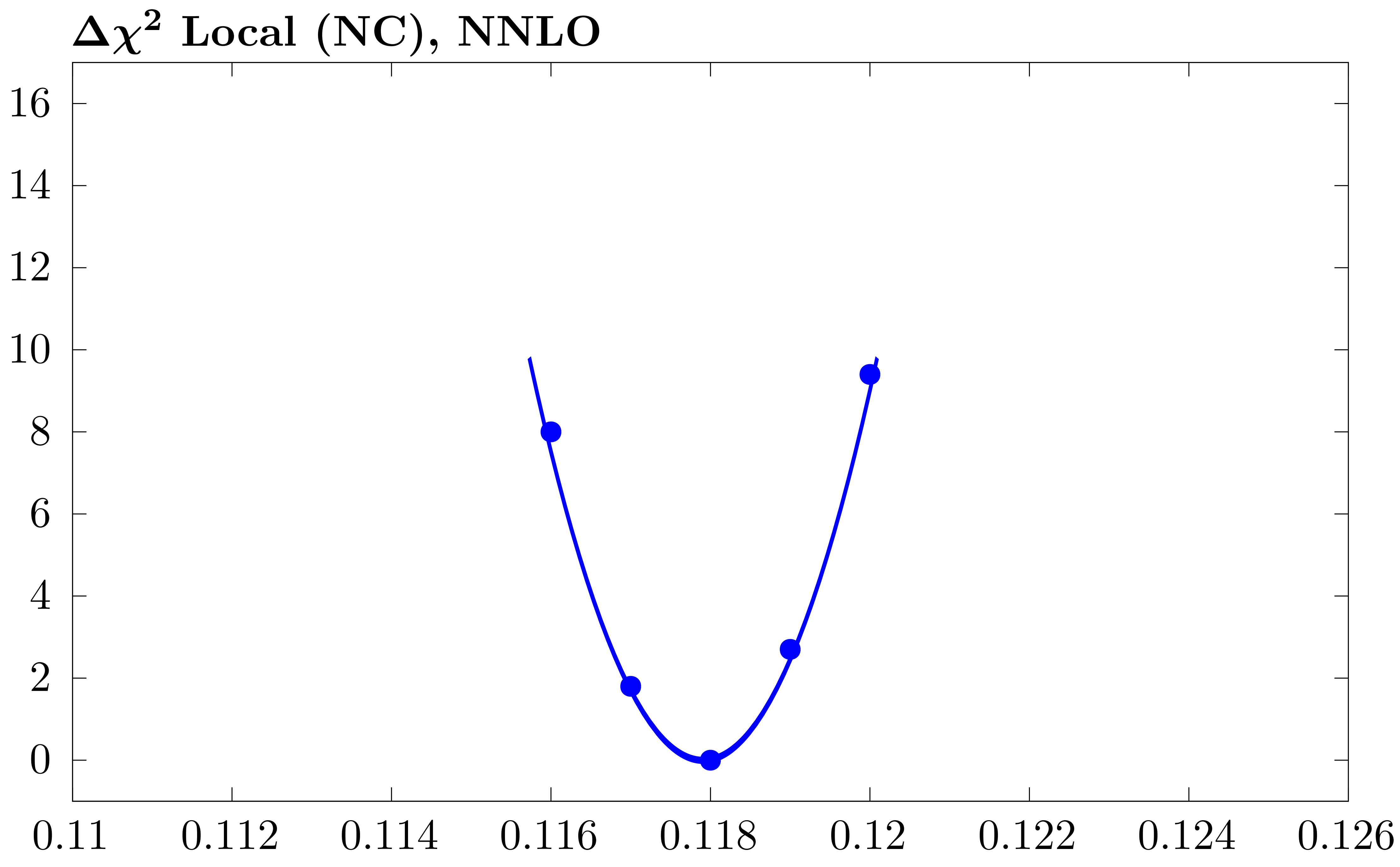
$$\frac{df}{d\log\mu_F^2} = \begin{bmatrix} P_{qq} & n_f P_{qg} \\ P_{gq} & P_{gg} \end{bmatrix} \begin{bmatrix} \Sigma \\ g \end{bmatrix}$$

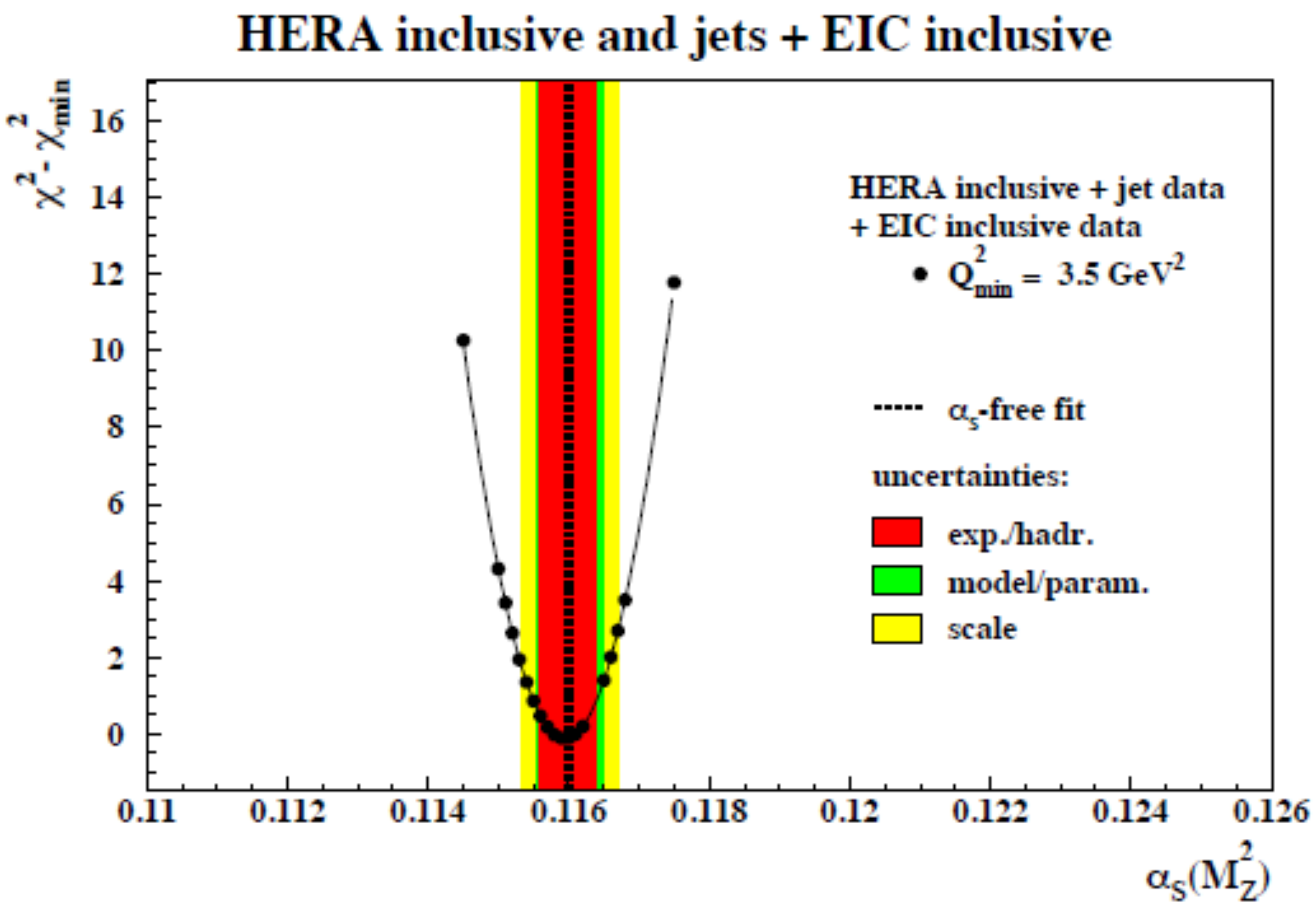
- DIS has limited sensitivity indirectly via scaling violations.
- HERA at low/intermediate x driven by gluon splitting, hard to disentangle α_S.
- EIC at higher x driven by non-singlet splitting, so α_S less correlated to g.
- Improved precision + more datapoints on structure function evolution.

★ So what do we find?

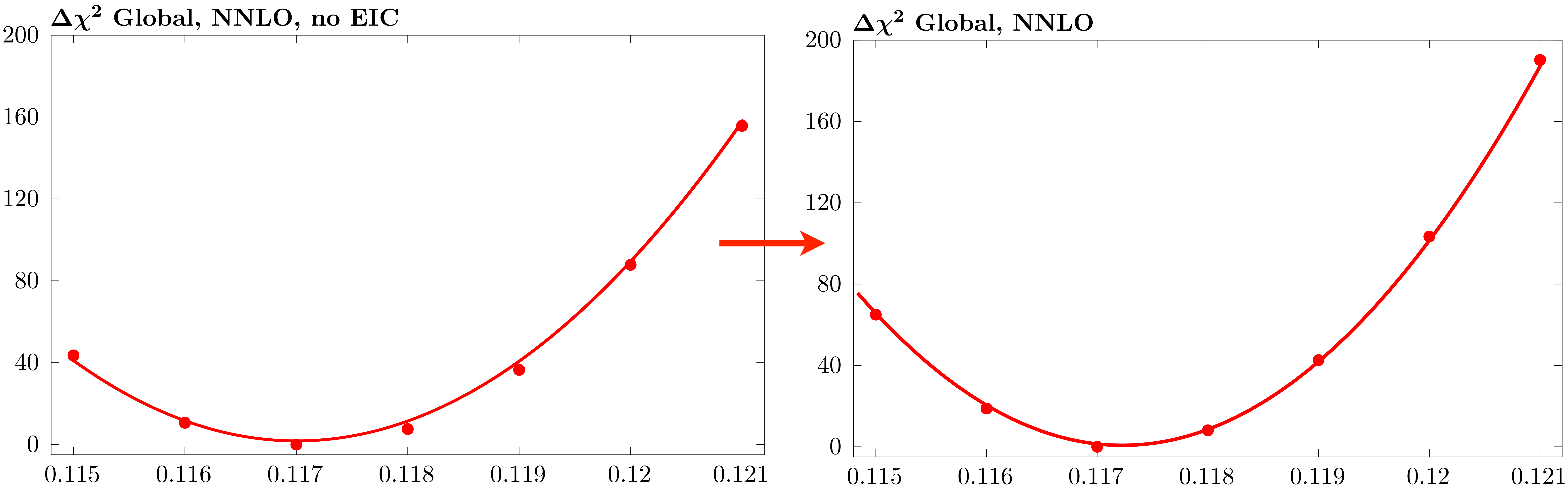
LHL et al., in prep.

NNLO Fit





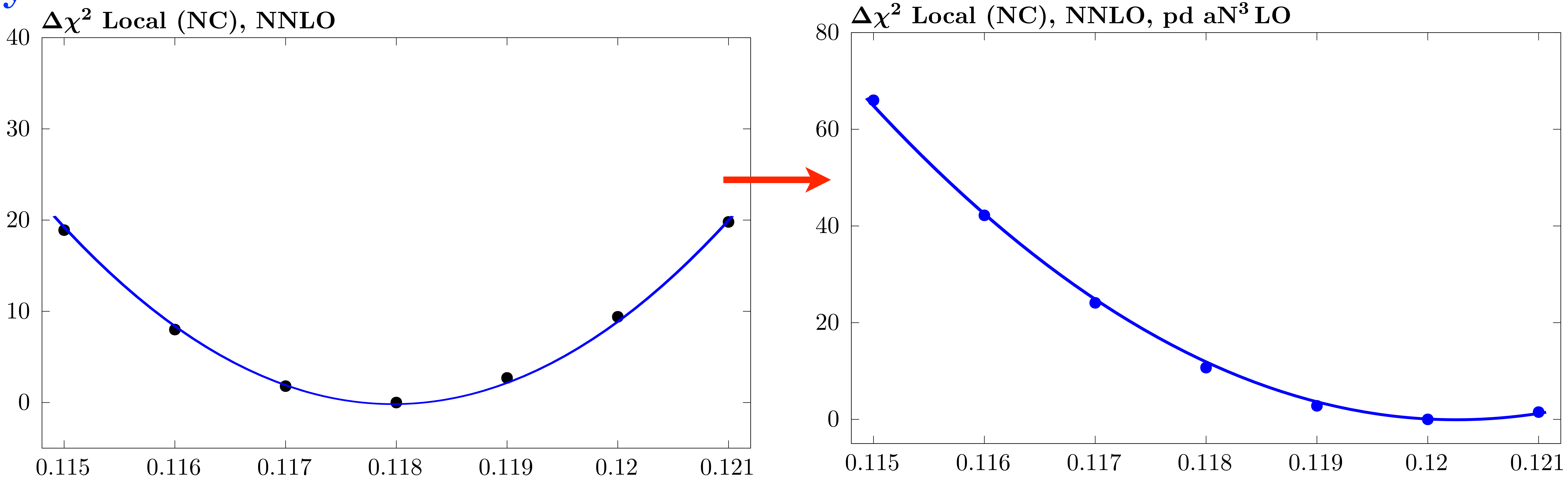
- Looking at local profile - prefers $\alpha_S(M_Z^2) = 0.118$ by construction. Shape + constraining power is in fact very similar to previous HERA + EIC study.
- But of course now it is the global impact that is relevant…



- What about **global** impact? Additional constraint clearly visible.
- Note precise impact depends on preferred previous value (~ 0.117) and that used in pseudodata generation (0.118).

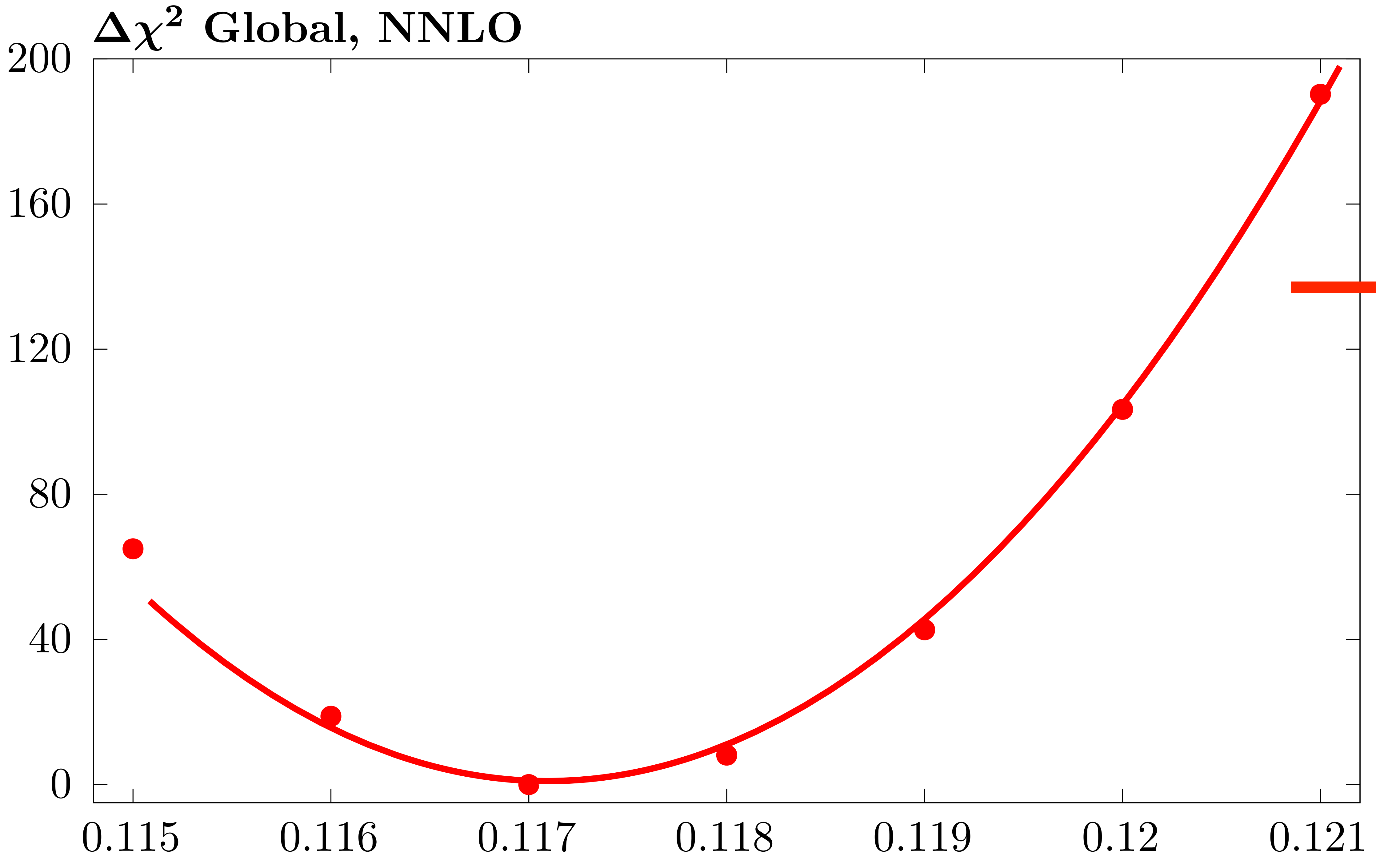
- However, care needed here! Impact of this high precision EIC pseudodata in principle significant, but need to have accuracy in mind as well.
- For example, impact of finite QCD (NNLO) order - to assess generate PD at aN3LO (with MSHTaN3LO set) but include in NNLO fit. Provides estimate of **missing higher orders**.

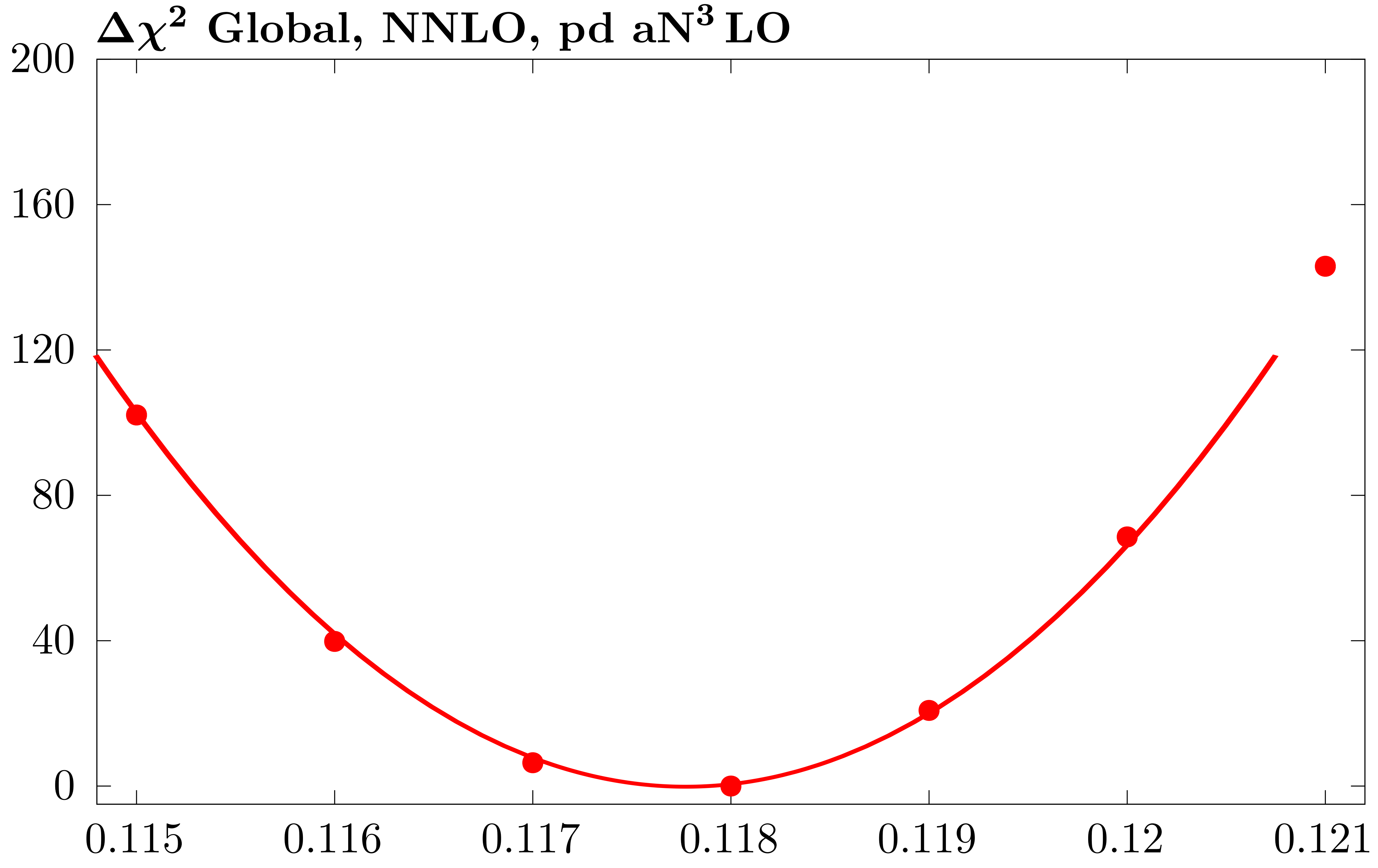
★ **Locally:**



- Even though PD generated with $\alpha_S(M_Z^2) = 0.118$ the mismatch leads to a local preference for ~ 0.120 (though quite flat). Similar effect to global preference in NLO fit.
- Note this MHO shift in global fit not the same thing as scale variation uncertainty in EIC PD alone.

★ Globally:

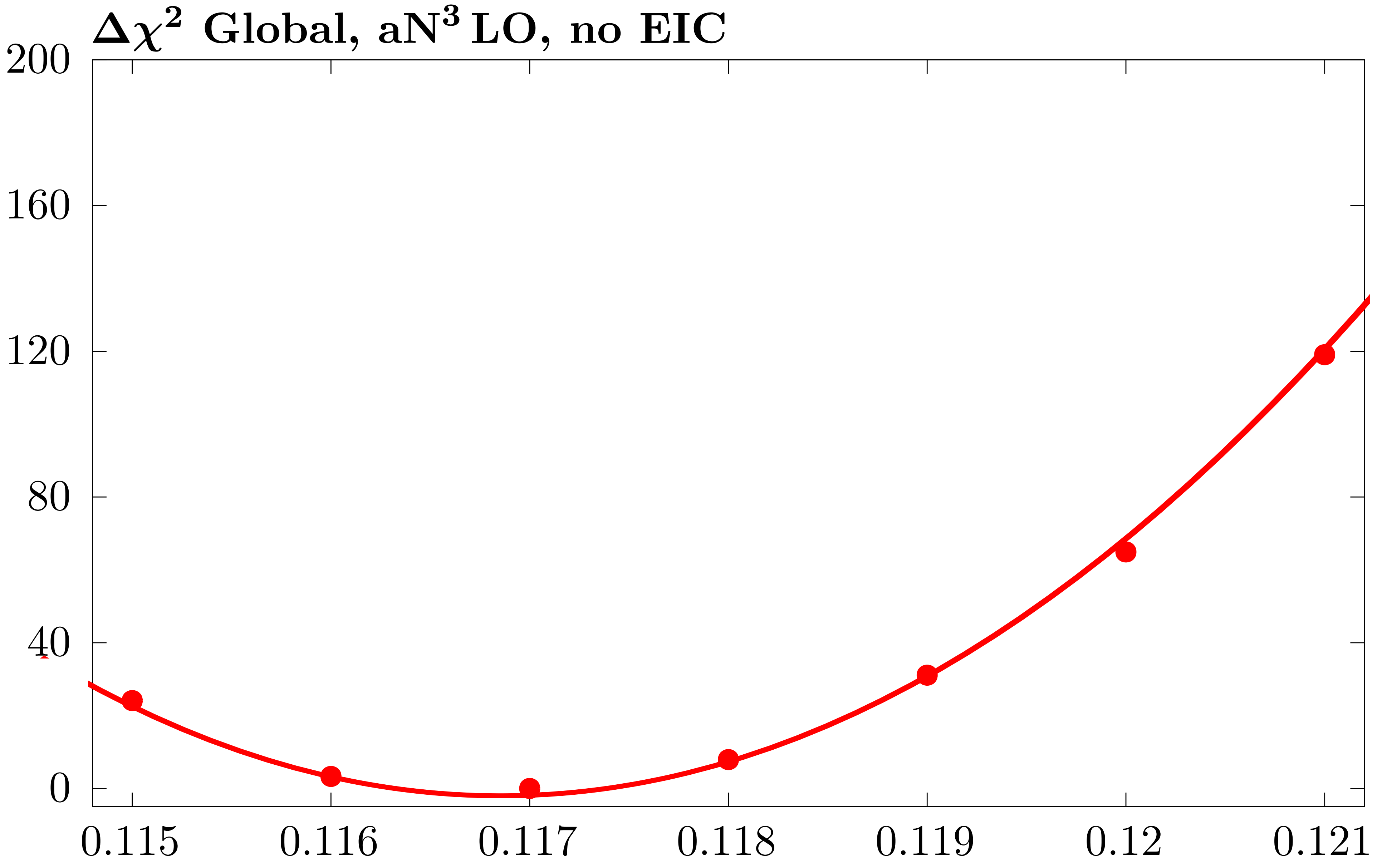




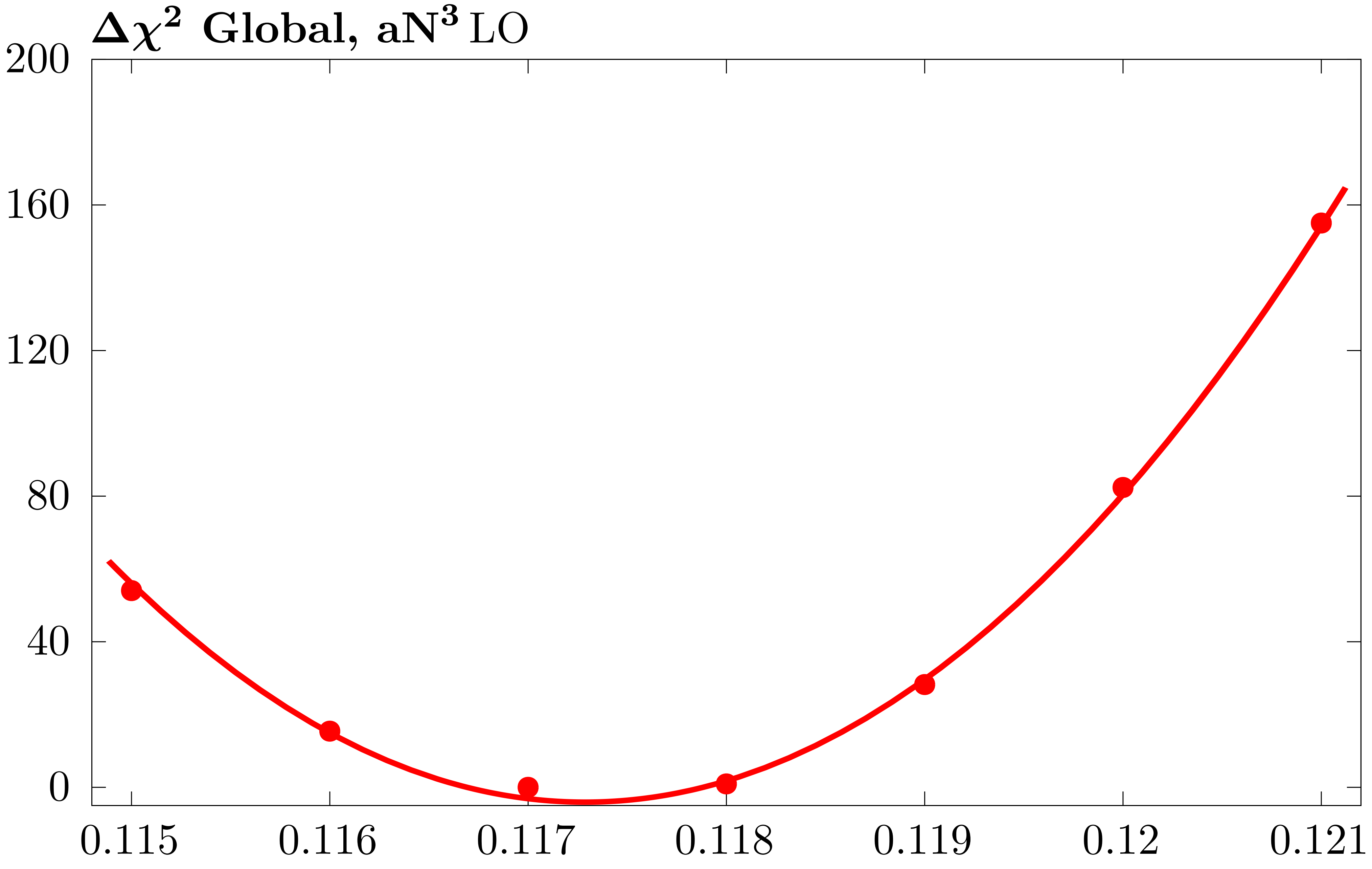
- Effect persists in global profile.

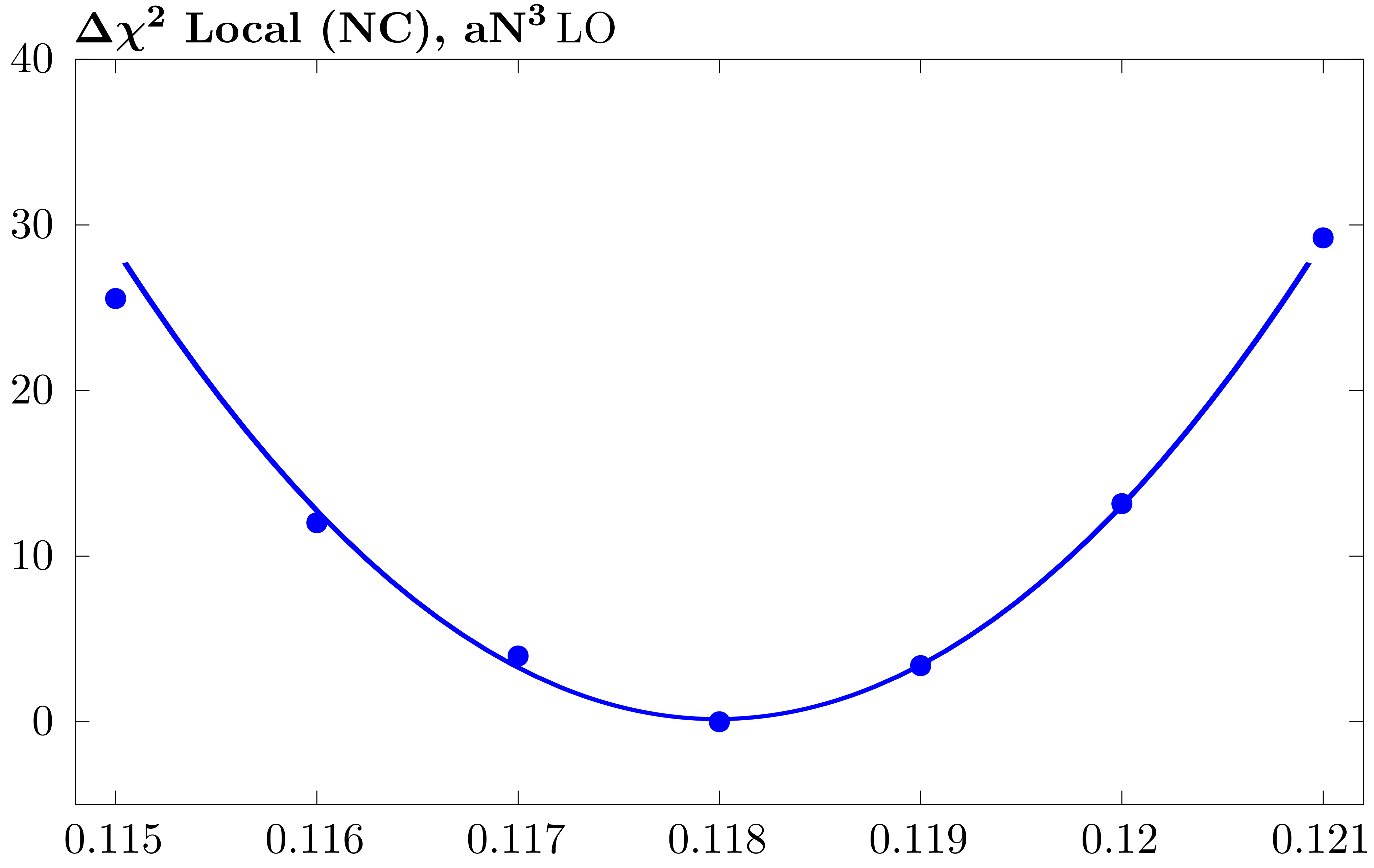
$\longrightarrow$ Any future analysis must account for this (via MHO uncertainty/error treatment). Clearly important element in uncertainty budget at **NNLO**. Points to desirability of **aN3LO**?

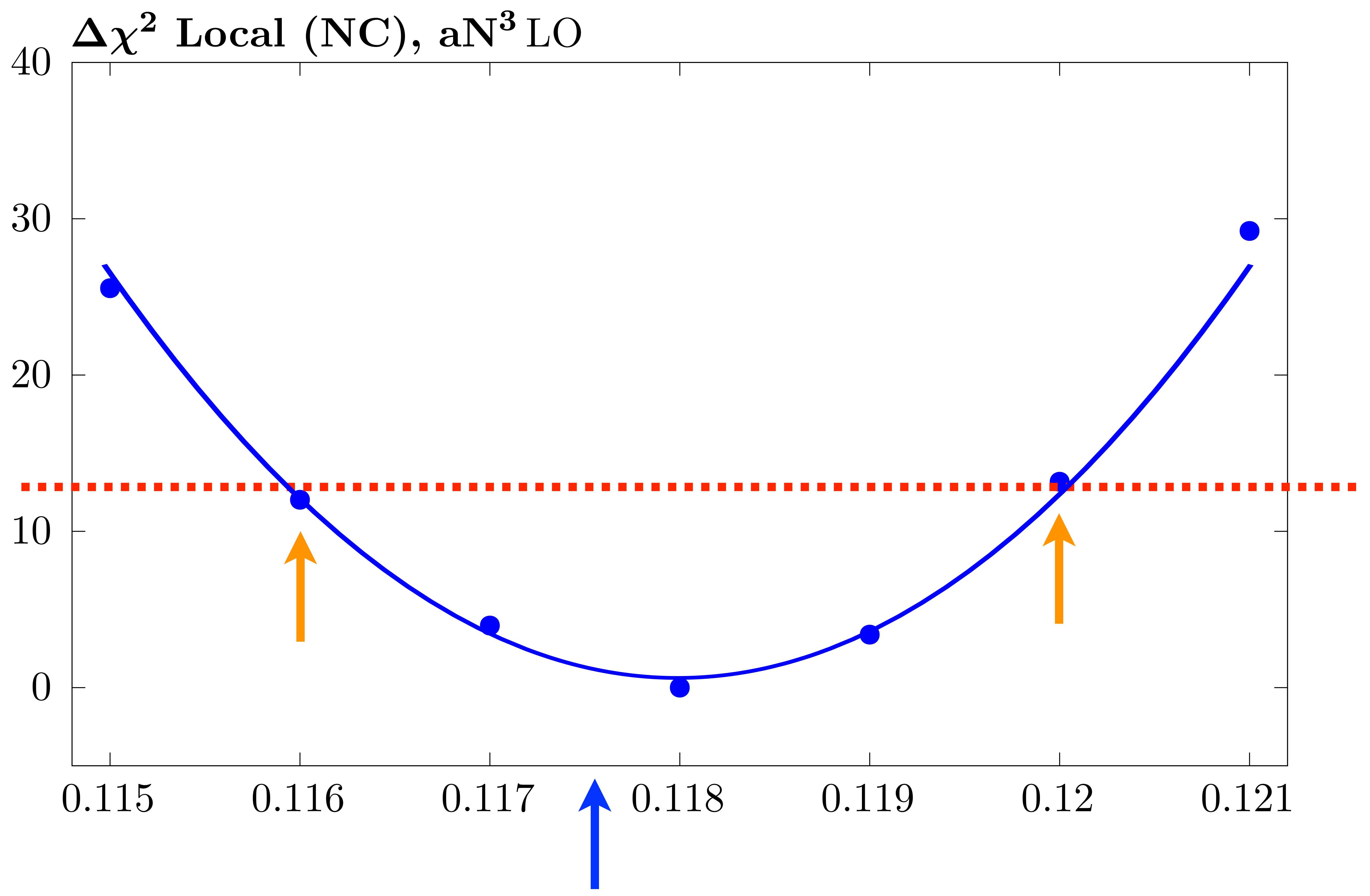
aN3LO Fit



- Repeat NNLO exercise, but now at **aN3LO**.
- Global and local impact very similar to NNLO, and as expected.





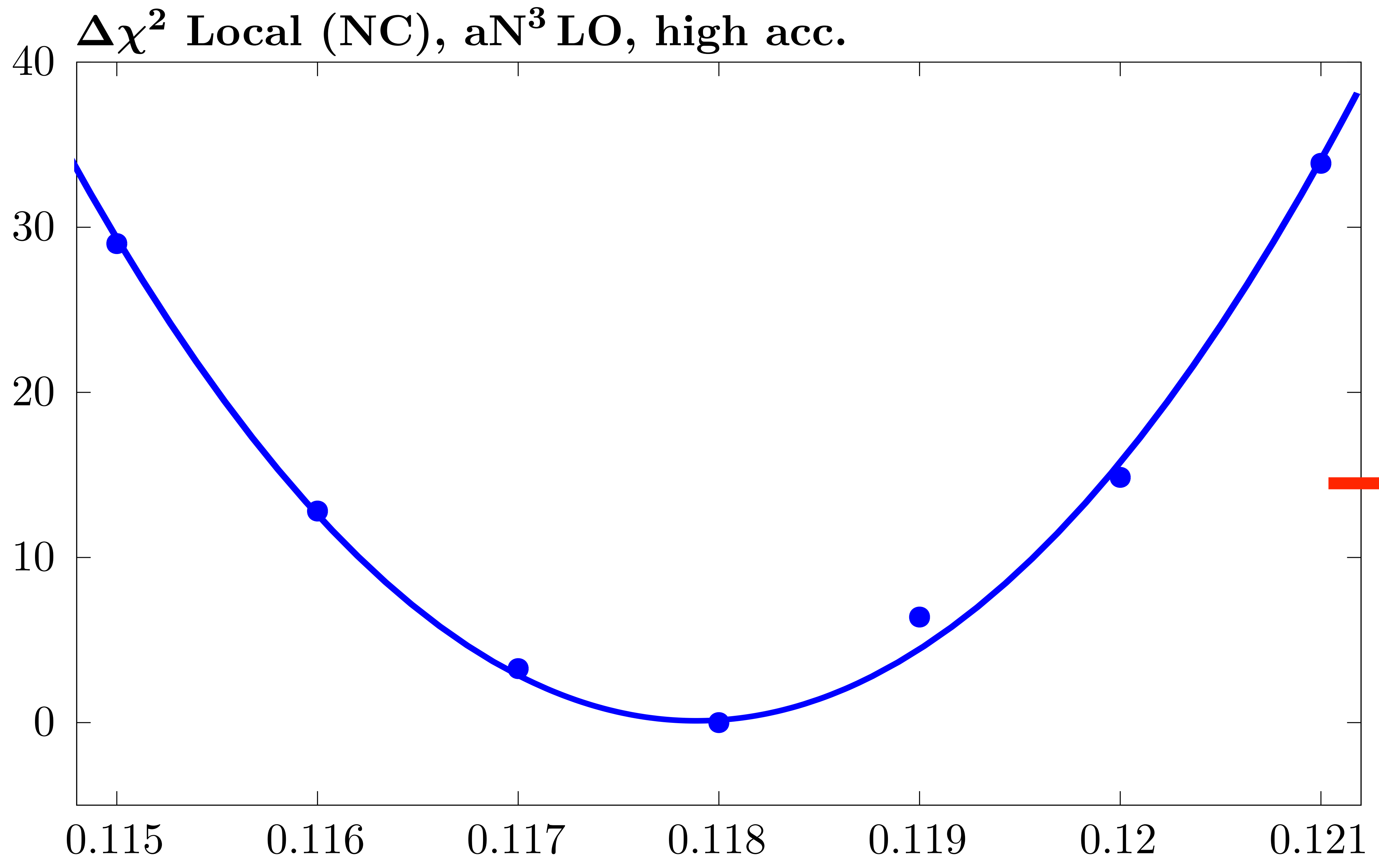


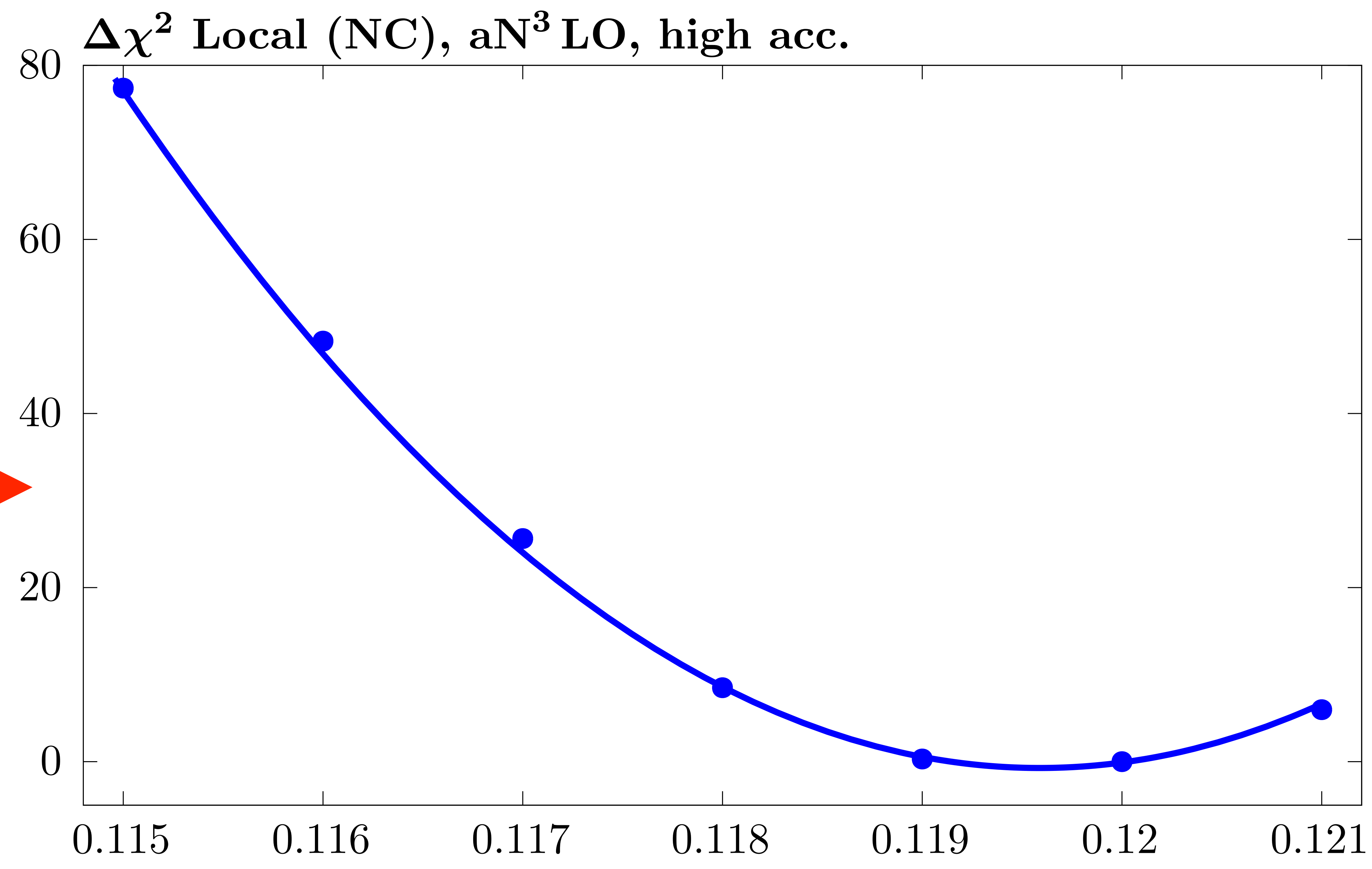
- Best fit **global** value at ~ 0.1175.
- Limit set by dynamic tolerance criterium. EIC lower limit is competitive ~ - 0.0015. Same level as other bound, from older fixed target datasets (SLAC, NMC). Persists if PD generated at 0.117.
- Note explicit missing higher order uncertainties not included here - will be less than at NNLO. Full account of these ongoing work.What other sources of uncertainty to account for?

Target Mass Corrections

- Generate PD with TMCs but fit without them.

I. Schienbein et al., 0709.1775

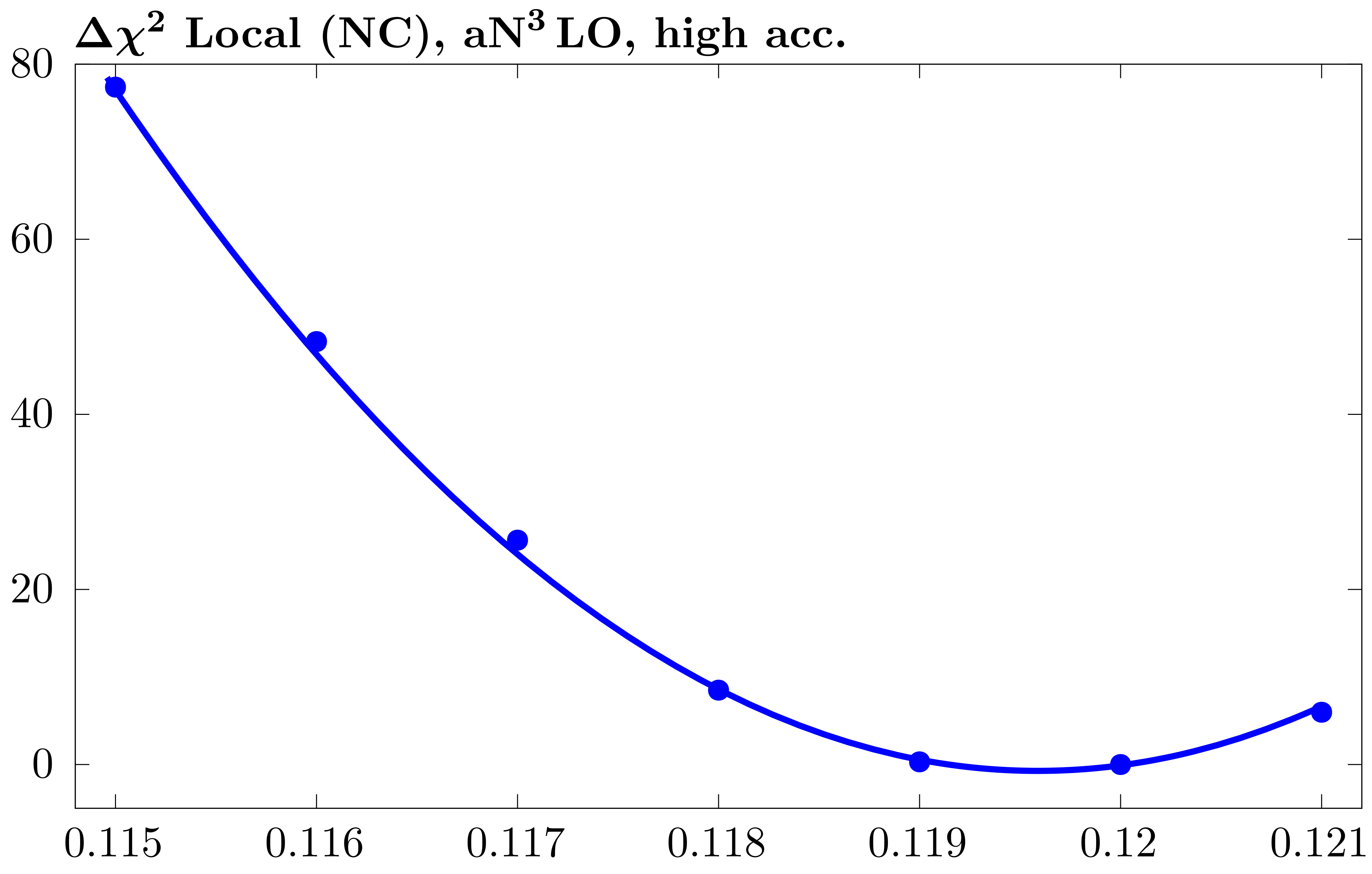


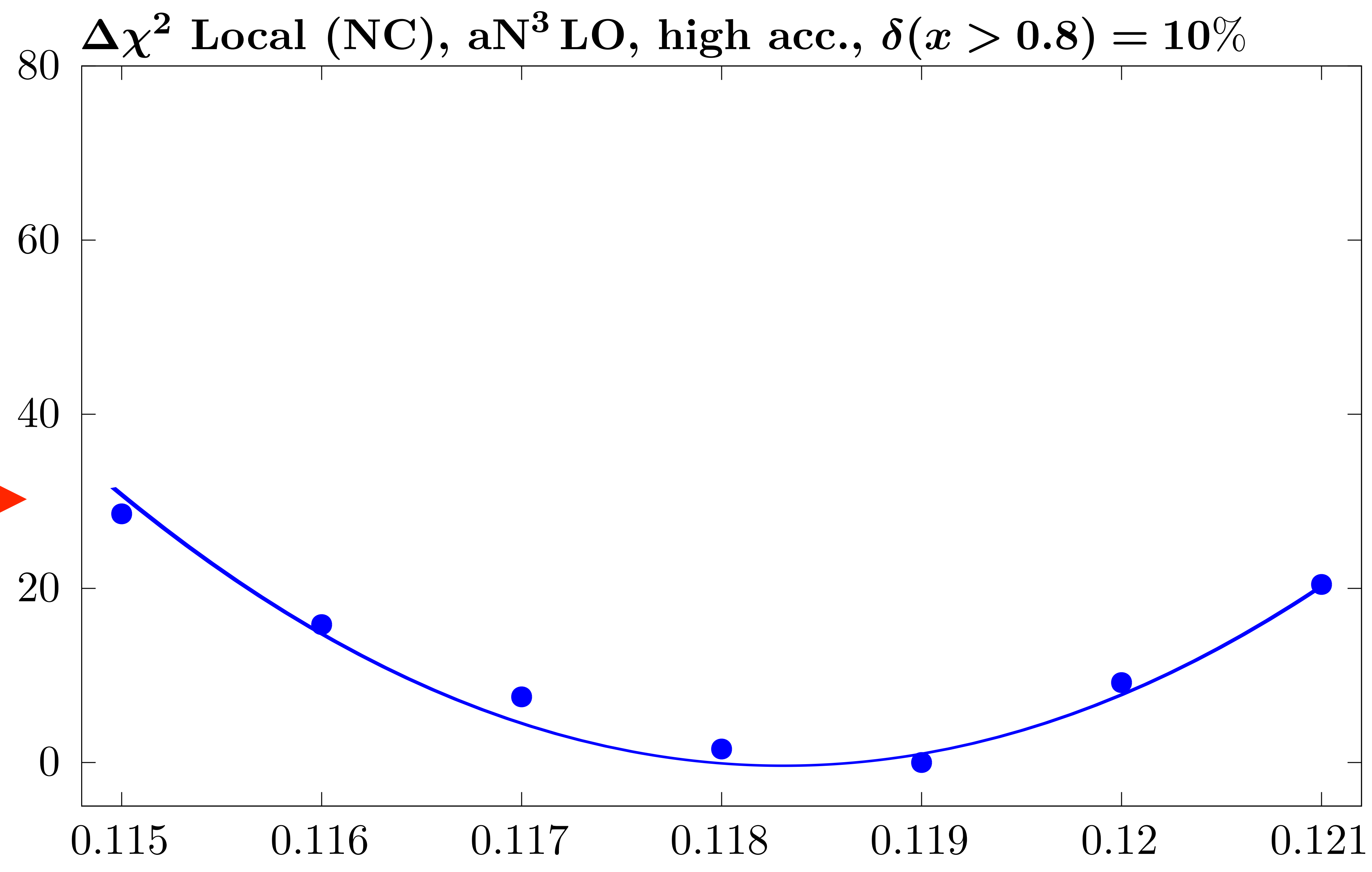


$\longrightarrow$ Full account of TMCs clearly in general necessary, though taking arguably more realistic error, or simply removing highest $x > 0.8$ points moves towards nominal result…

Target Mass Corrections

- Generate PD with TMCs but fit without them. **I. Schienbein et al., 0709.1775**



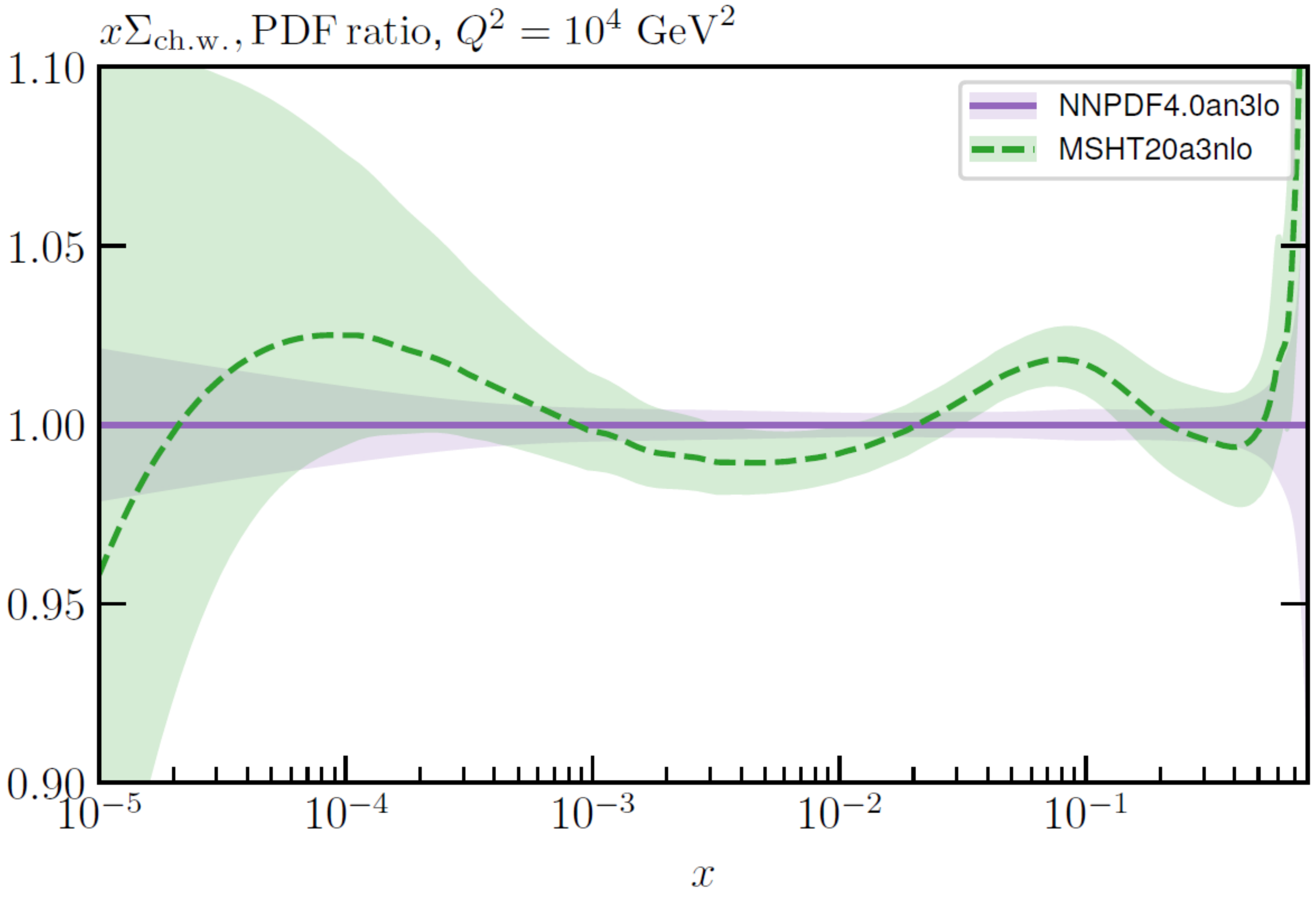


$\longrightarrow$ Full account of TMCs clearly in general necessary, though taking arguably more realistic error, or simply removing highest $x > 0.8$ points moves towards nominal result.

Other Tensions?

- Without explicit assumption about source of these, try emulating them by generating PD still at aN3LO but with the **NNPDF4.0aN3LO** set.
- Underlying MSHT20 fit (w/o EIC) ~ **MSHT20aN3LO** set, so any differences between these two will lead to tensions.

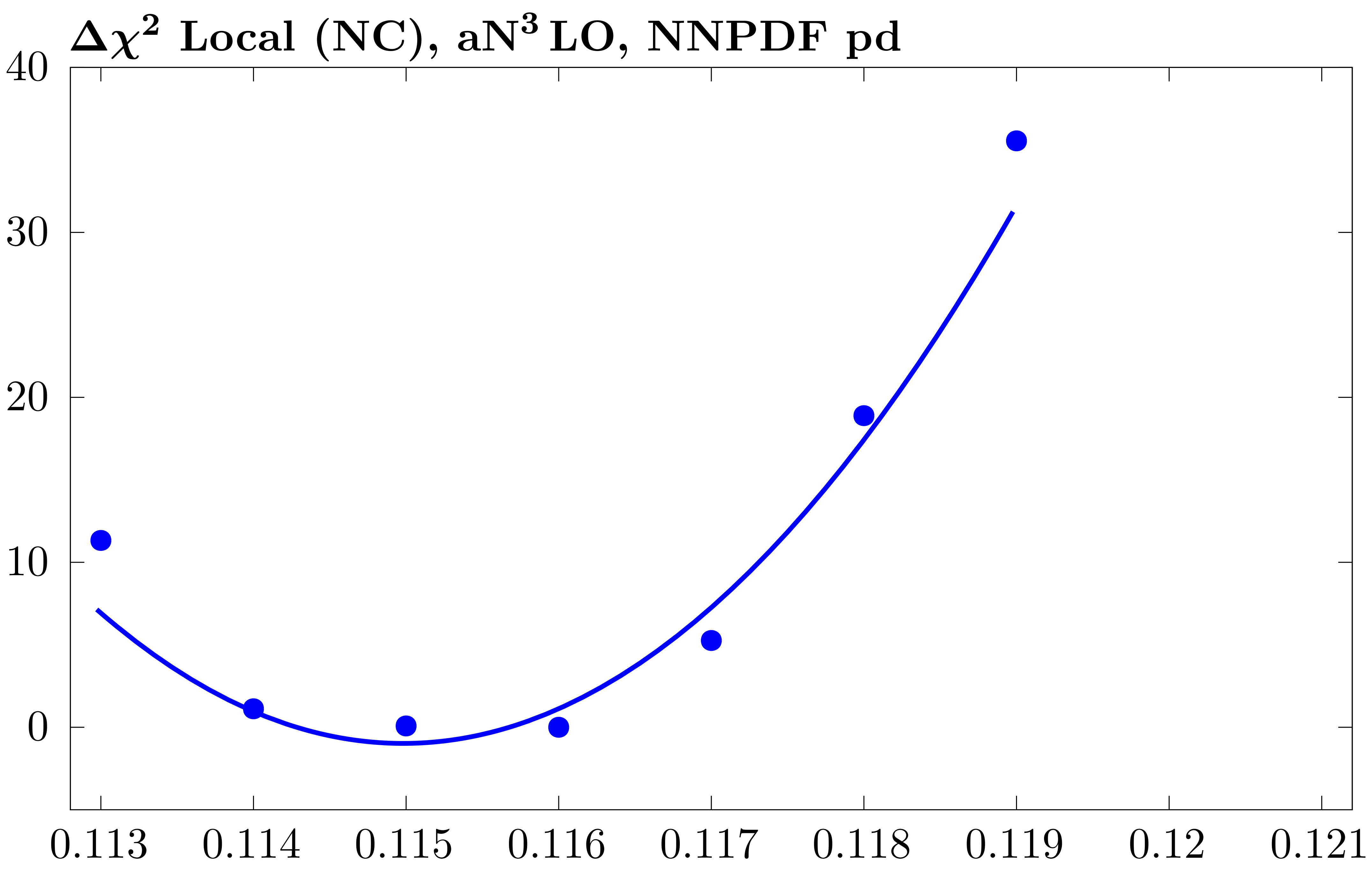
★ And these are present!

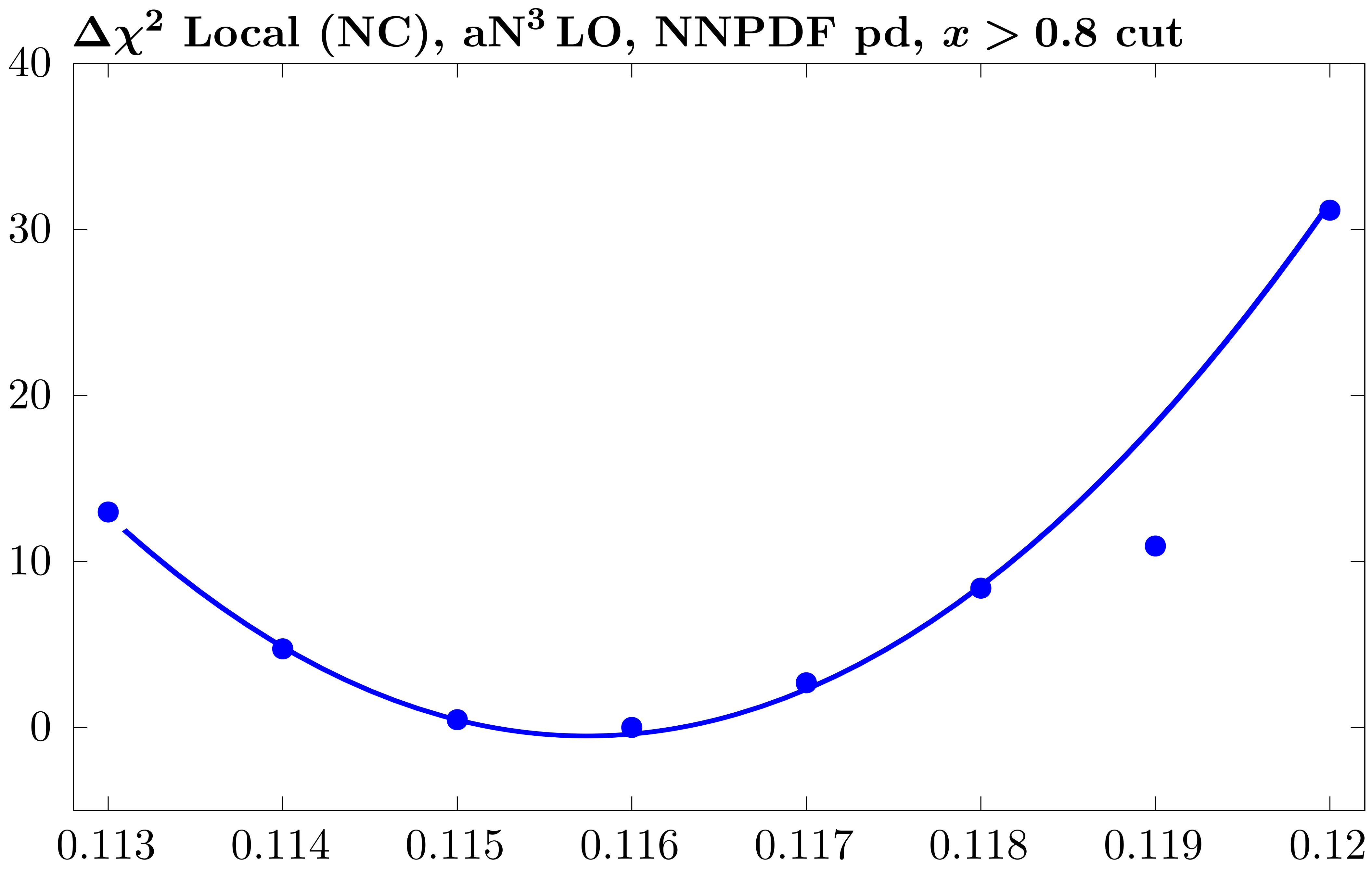


- This feeds through to **fit quality**:

	w. EIC	w EIC (NNPDF pd)
EIC	**531.6 (1.02)**	**586.8 (1.13)**
Fixed Target	**2161.1 (1.09)**	**2243.9 (1.13)**
HERA	**1573.1 (1.25)**	**1577.4 (1.25)**
Hadron Collider	**2381.7 (1.33)**	**2429.6 (1.36)**
Global (no EIC)	**6115.8 (1.21)**	**6251.0 (1.24)**
Global	**6647.4 (1.20)**	**6837.8 (1.23)**

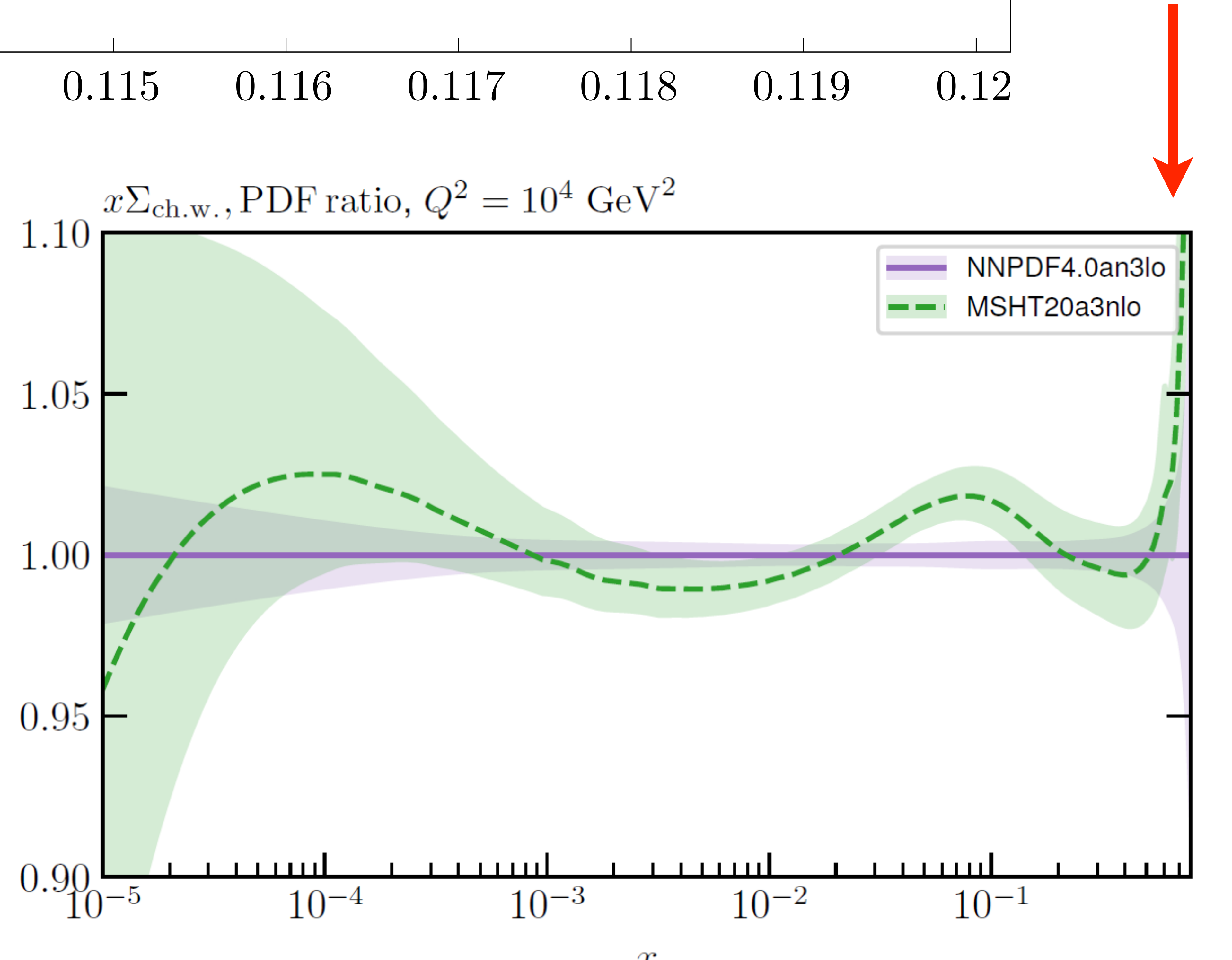
- But fit quality to EIC PD is still only $\sim 2\sigma$ from ideal, and global fit quality $\sim 1\sigma$ worse wrt baseline.
- I.e. this is not 'obviously' (post-hoc) bad at the level of the pure fit qualities.
- What about the strong coupling?

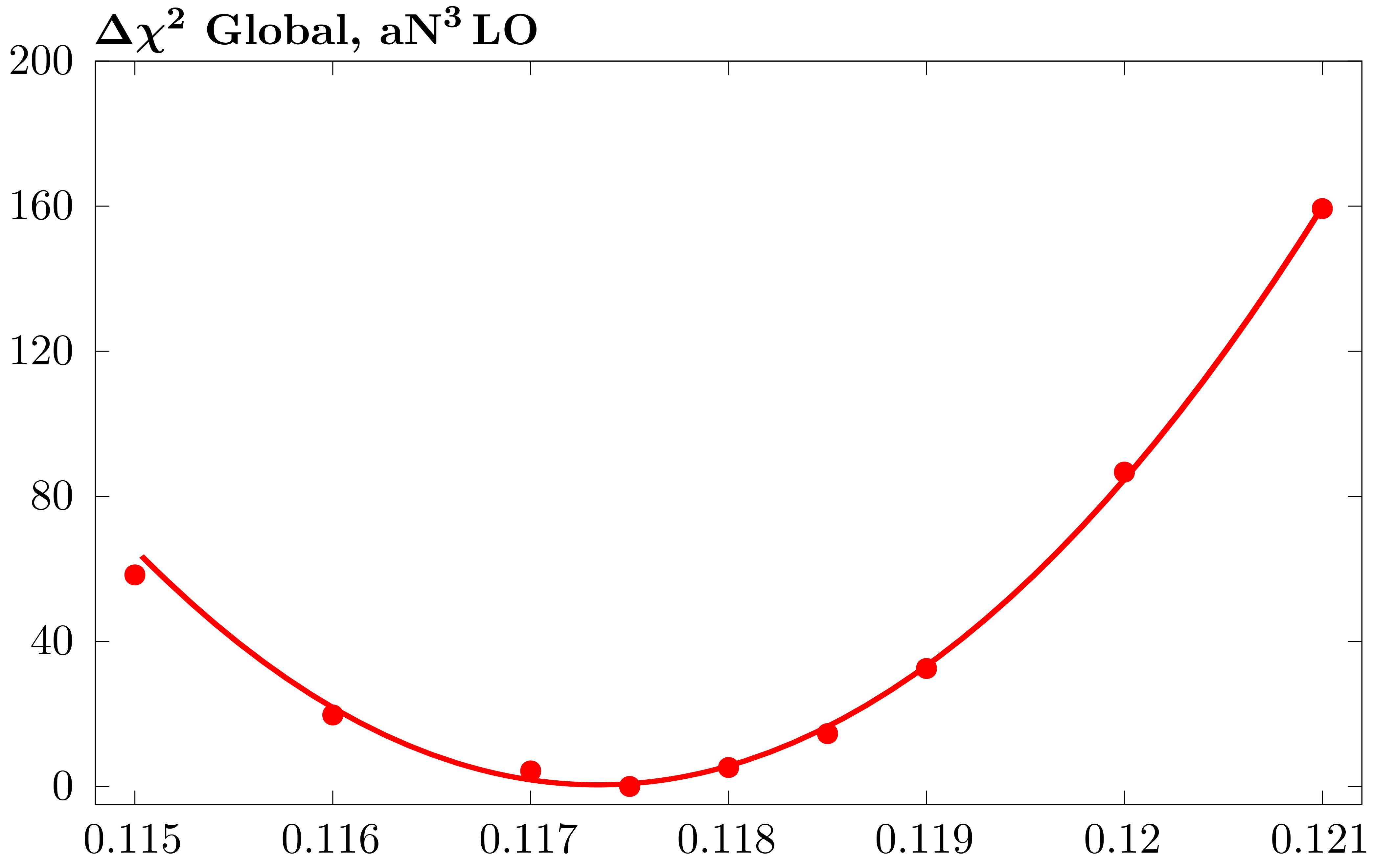


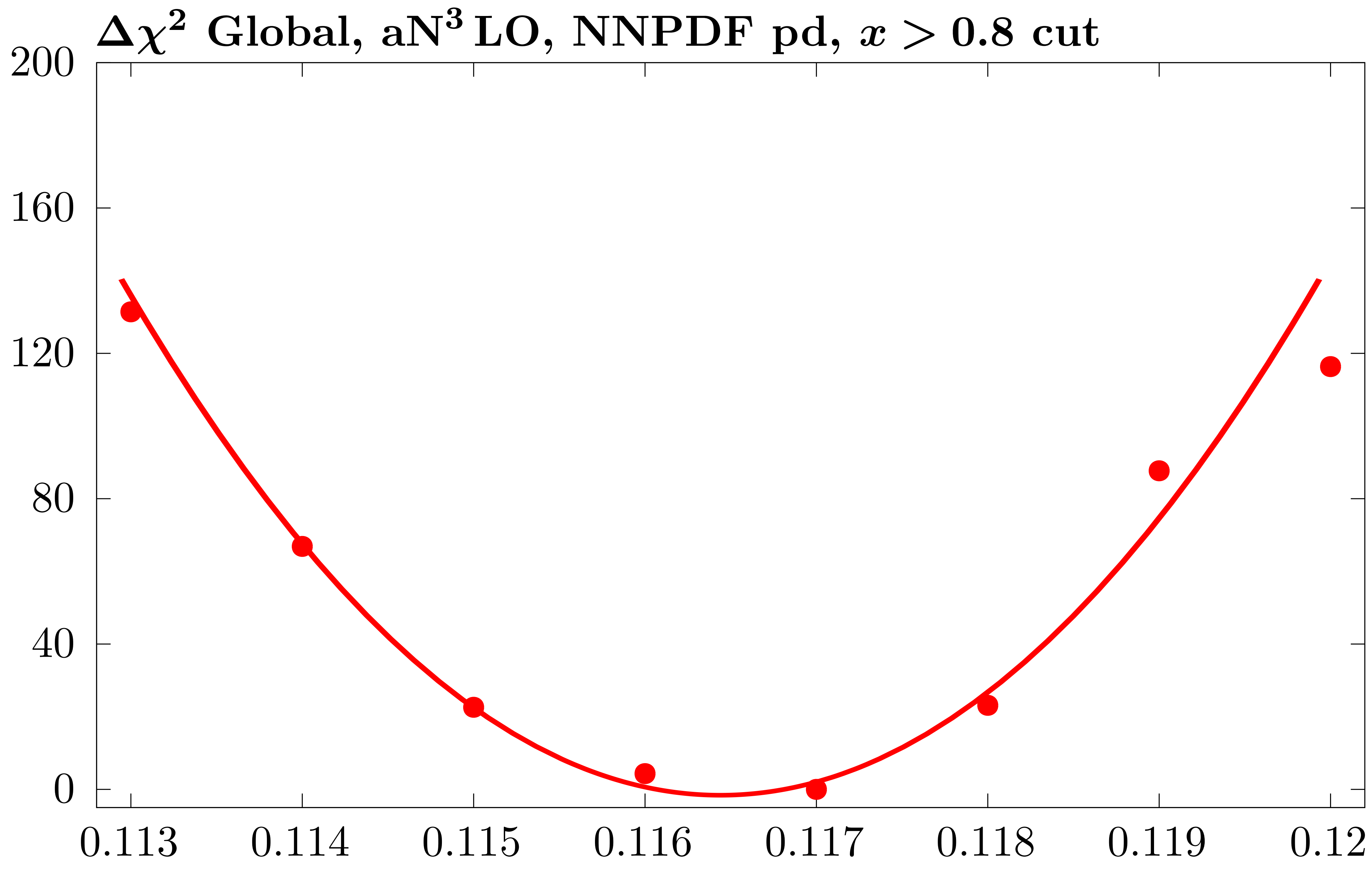


- Impact on local profiles significant! Relatively flat, but preference for lower value of ~ 0.115 then nominal 0.118.

- Cutting out highest $x > 0.8$ points reduces difference somewhat, but far from entirely.







- Global impact reasonably mild, but some reduction from ~ 0.117 value, in region of ~ 0.1175.
- This change is of the order of a $\Delta\chi^2 = 1$ uncertainty, but safely in the dynamic tolerance (~ $\pm\,0.0015$) one.
- Size of this effect (and difference in underlying PDFs in EIC-sensitive region) points to complications that inevitably arise in real analysis vs. pseudodata one.

Summary and Outlook

- ★ Fitting strong coupling simultaneously with PDFs in global MSHT fit performed for long time.
- ★ More recently, extended to aN3LO - stability achieved and consistency with NNLO.
- ★ EIC can add key complementary information about PDFs and the strong coupling.
- ★ Moderate impact on PDFs and **competitive bounds** on **strong coupling** projected.
- ★ Also investigated going beyond assumption of pseudodata/fit consistency.
- ★ NNLO fit with aN3LO pseudodata shows clear shift in locally preferred strong coupling.
- ★ Sensivity to TMCs at highest x seen.
- ★ Rough model of tensions at aN3LO also demonstrate local shifts.
- ★ Future studies should consider these effects, and will do!