New developments in collinear and TMD probability density functions

Extracting the Strong Coupling at the EIC and other Future Colliders
CFNS, Stony Brook University
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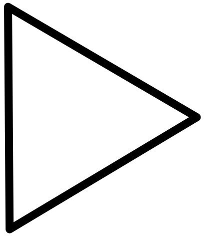
Agenda:

Introduction
& Motivation

Parton Branching
Method

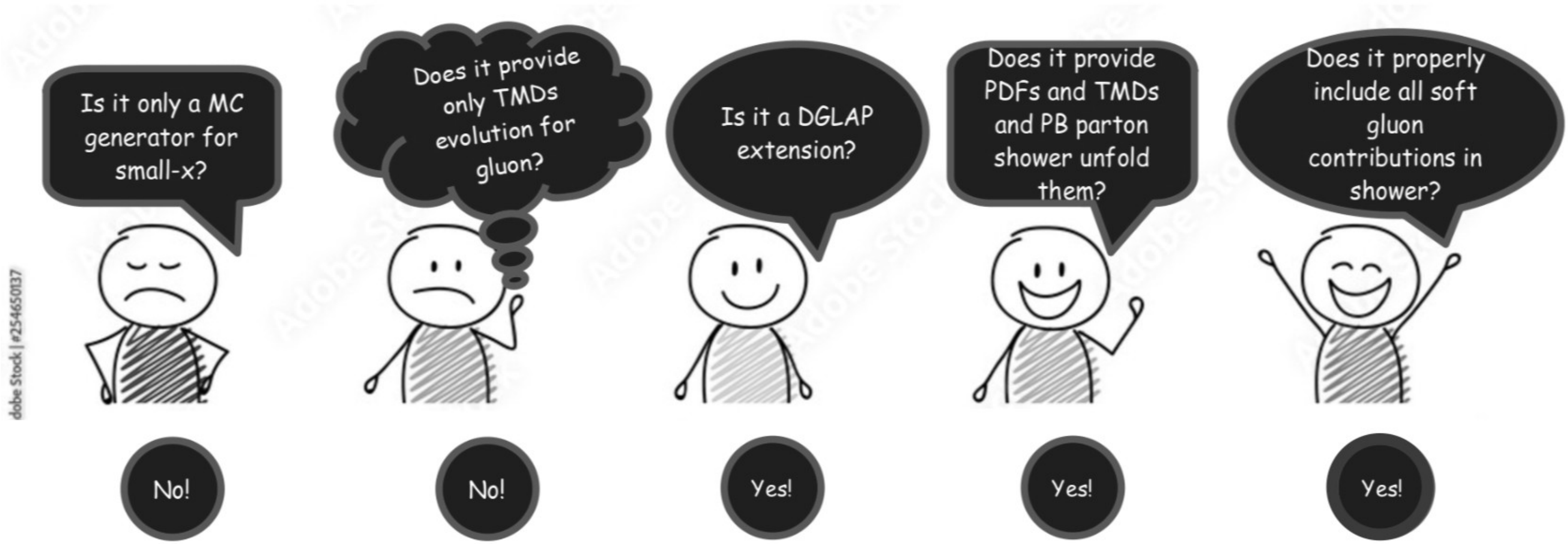


TMDs
developments

Conclusions

The Parton Branching Method in a nutshell

[JHEP 09 060 (2022)]
[Phys. Rev. D 100 (2019) no.7, 074027
[Eur.Phys.J.C 82 (2022) 8, 755]
[Eur.Phys.J.C 82 (2022) 1, 36]
[Phys. Lett. B 822 136700 (2021)]



[Taken from S.Taheri Monfared at DIS2024]

The Parton Branching Method

We obtain (angular ordered evolution of) collinear PDFs and parton densities in terms of the transverse momentum: k of the propagating parton: Transverse Momentum Dependent (TMD) parton distributions A(x, k, μ)):

Starting energy scale: μ0

$$\tilde{A}_a(x, k_{\perp,0}^2, \mu_0^2) = x f_a(x, \mu_0^2) \cdot \frac{1}{q_s^2} \exp\left(-\frac{k_{\perp,0}^2}{q_s^2}\right)$$

μ

$$\begin{aligned} \mathcal{A}_a(x, \mathbf{k}, \mu^2) &= \Delta_a(\mu^2)\, \mathcal{A}_a(x, \mathbf{k}, \mu_0^2) + \sum_b \int \frac{d^2\mathbf{q}'}{\pi \mathbf{q}'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(\mathbf{q}'^2)}\, \Theta(\mu^2 - \mathbf{q}'^2)\, \Theta(\mathbf{q}'^2 - \mu_0^2) \\ &\times \int_x^{z_M} \frac{dz}{z} P_{ab}^{(R)}(\alpha_s, z)\, \mathcal{A}_b\left(\frac{x}{z}, \mathbf{k} + (1-z)\mathbf{q}', \mathbf{q}'^2\right) \quad , \end{aligned}$$

These TMDs are linked to the **collinear parton densities:** also called "integrated TMDs" (iTMDs) by:

$$f_a(x, \mu^2) = \int \mathcal{A}_a(x, \mathbf{k}, \mu^2) \frac{d^2\mathbf{k}}{\pi}$$

Key element in both PDFs and TMDs is the soft gluon resolution scale z_M because it separates **resolvable** and **non-resolvable** emissions

For collinear PDFs : when $z_M \rightarrow 1$ we recover DGLAP results

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PDFs and TMDs : at a glance

How to perform the evolution?

Parameterise collinear PDF at μ_0^2

Produce PB kernels individually for collinear and TMD densities for quarks and gluons

(uPDFevolv2 package)

Extract initial parametrization by fitting to experimental data (xFitter package)

(collinear coefficient functions at NLO)

Store the PDFs & TMDs in grid format for later use

(x. ex. in CASCADE3)

$$\tilde{A}_a(x, k_{\perp,0}^2, \mu_0^2) = x f_a(x, \mu_0^2) \cdot \frac{1}{q_s^2} \exp\left(-\frac{k_{\perp,0}^2}{q_s^2}\right)$$

Collinear distribution

obtained from fit to inclusive DIS data

q_s

constrained from low pT DY data

<u>TMDs developments</u>

- Soft gluon treatment
- Consistency between PDFs and TMDs
- Photon and Z boson PDFs & TMDs

TMDs developments

- Soft gluon treatment
  arXiv:2309.11802 [M. Mendizabal, F. Guzman, H. Jung, S. Taheri Monfared]
  arXiv:2404.04088 [I. Bubanja, H. Jung, A. Lelek, N. Raicevic, S. Taheri Monfared]

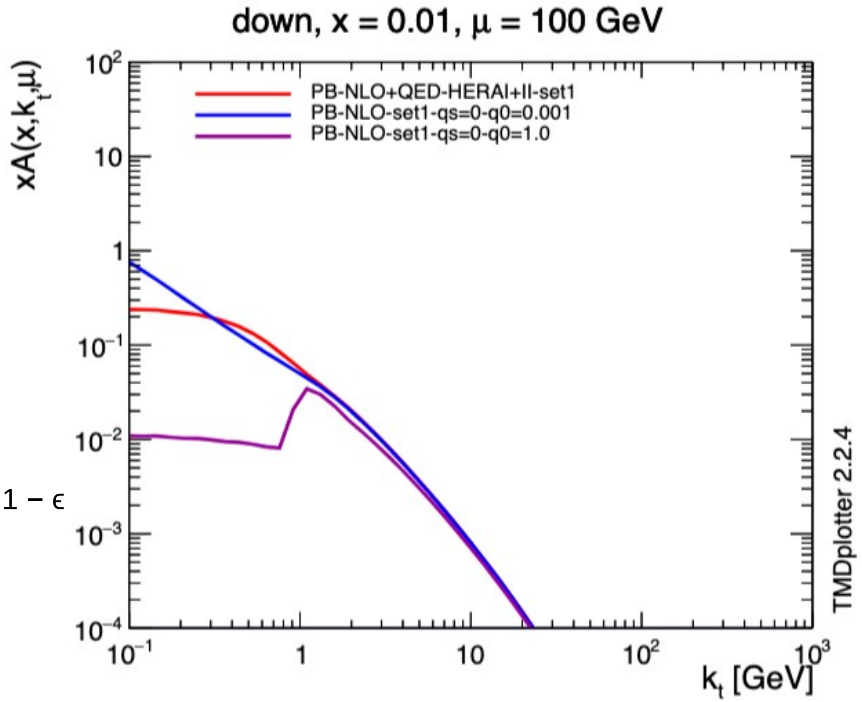
Evolution with PB method with and without z_M cut-off

$$\Delta_a^S(\mu^2,\mu_0^2,\epsilon) = \exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{dq^2}{q^2}\left[\int_0^{z_{\mathrm{dyn}}(q)} dz\frac{k_a(\alpha_s)}{1-z} - d_a(\alpha_s)\right]\right) \times \exp\left(-\int_{\mu_0^2}^{\mu^2}\frac{dq^2}{q^2}\int_{z_{\mathrm{dyn}}(q)}^{z_M} dz\frac{k_a(\alpha_s)}{1-z}\right) = \Delta_a^{(\mathrm{P})}\left(\mu^2,\mu_0^2,q_0^2\right)\cdot\Delta_a^{(\mathrm{NP})}\left(\mu^2,\mu_0^2,\epsilon,q_0^2\right)$$

, with ϵ defined via $z_M = 1 - \epsilon$

Red curve: PB-NLO-2018 Set1 (including intrinsic-k_T, $z_M \to 1$, $q_0 = 0.5$ GeV)
Blue curve: prediction without including intrinsic-k_T distributions ($q_s = 0$)
Purple curve: prediction applying $z_M = z_{dyn} = 1 - q_0/q$ with $q_0 = 1.0$ GeV without including intrinsic-k_T distributions.

Difference between curves illustrates the importance of soft contributions (to have proper cancellation of virtual and real emissions)



Transverse momentum distributions of down quarks at μ = 100 GeV from the PB-approach

Parton shower developments

- Parton shower and parton densities consistency
  [arXiv:2504.10243[H. Jung, L. Lönnblad, M. Mendizabal, S. Taheri Monfared]

Issue: Using NLO collinear parton densities but LO splitting functions within the parton shower leads to significant inconsistencies!

Consequence:
We explore how to treat parton showers through a toy model

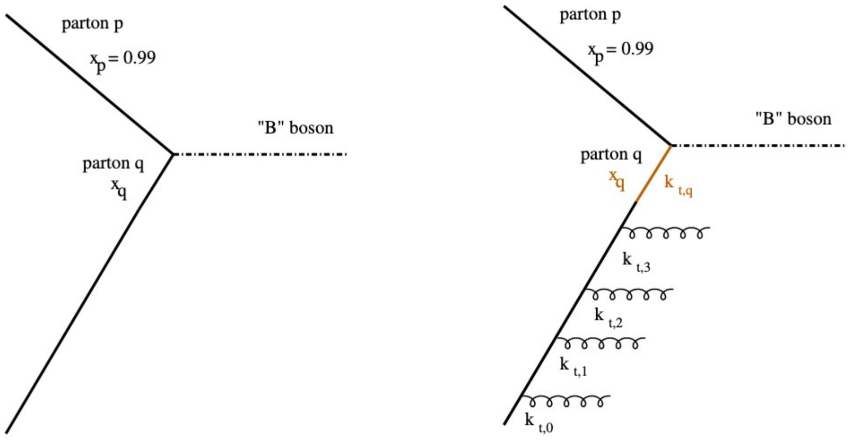
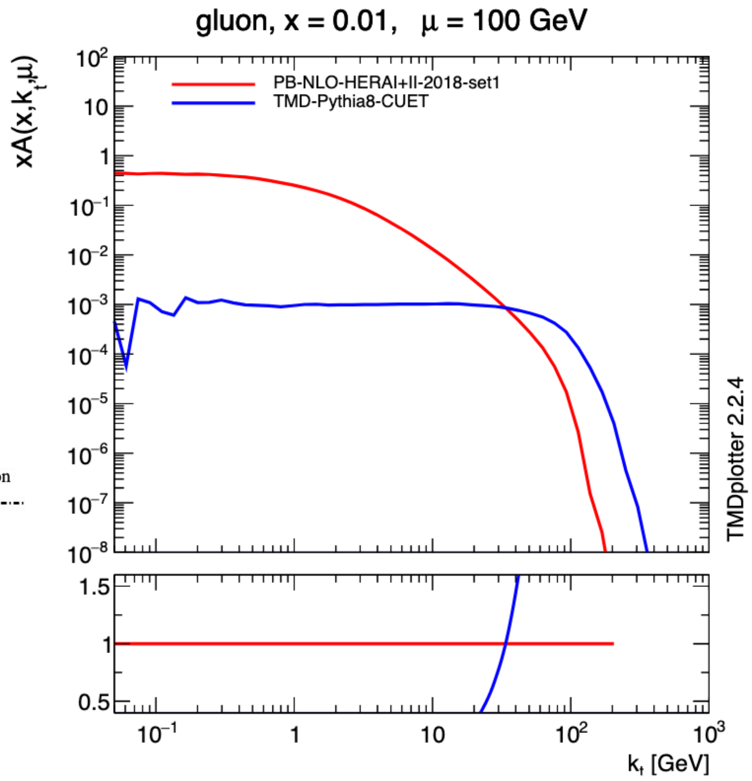


Illustration of the toy process: p + q → B: (left) bare process; (right) including initial-state parton shower



TMD for gluons at scale μ = 100 GeV (PB-NLO-2018 Set1) from PB-method and PYTHIA8 (CUET tune)

Parton shower developments

LO scenairo

- LO parton densities + LO splitting functions

NLO scenairo

- NLO parton densities + NLO splitting functions
- Proper treatment of negative contributions from splitting functions at large z

- Angular ordering
- Kinematic limits
- Choice of α_s scale
- Same kinematic frame for k_T

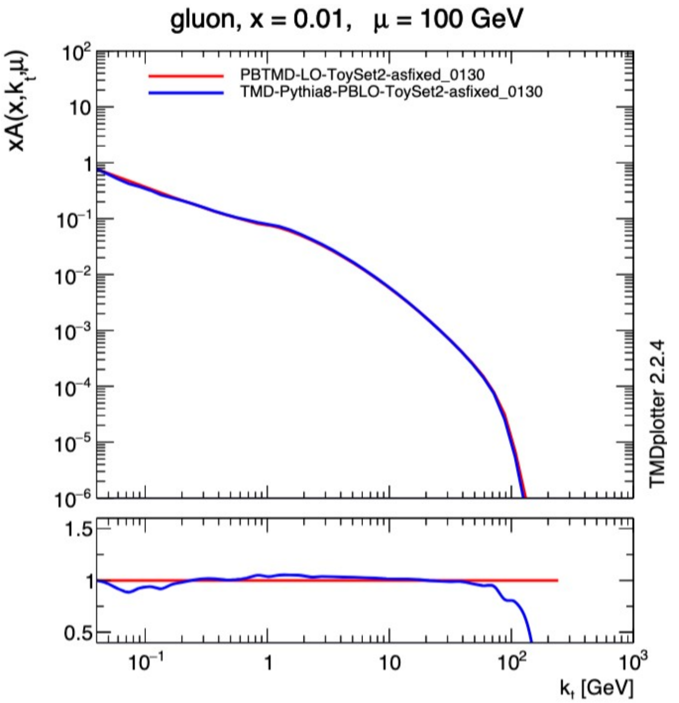
Consistency between PB-approach and PS2TMD approach

Result: With the correct approach **we can construct TMD parton densities from any parton shower event**:

1st PDF2ISR: We construct the initial-state radiation (ISR)
2nd PS2TMD: We reconstruct TMD parton distributions from that parton shower

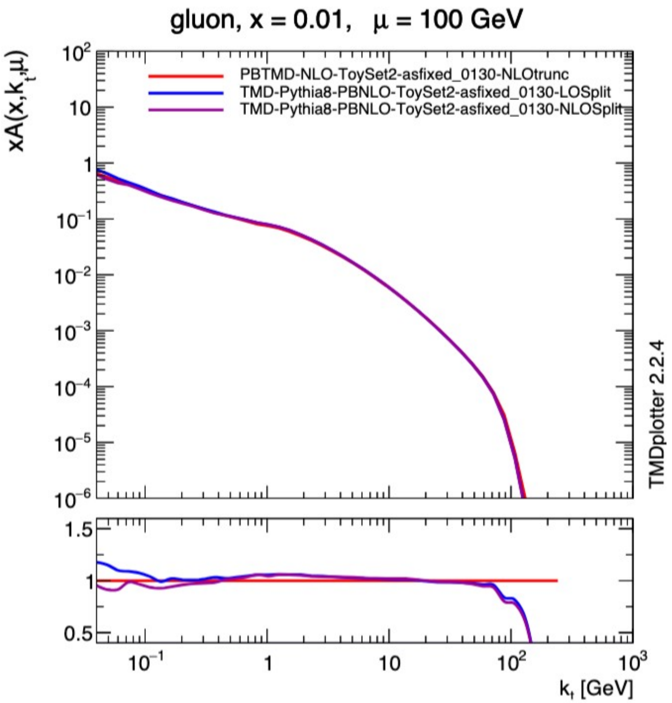
Parton shower developments

LO TMD parton distributions



TMD for gluons at μ = 100 GeV from PB-Toy Set2 and PYTHIA8 PDF2ISR at **LO**

NLO TMD parton distributions



TMD for gluons at μ = 100 GeV from PB-NLO-2018 Set2 and PYTHIA8 PDF2ISR at **NLO**

Red curve: PB-NLO-2018 predictions at NLO (with NLO $\alpha_s(m_Z) = 0.118$).

Blue curve: PYTHIA8 PDF2ISR with NLO splitting functions

Purple curve: shows the simulation when α_s is calculated from PYTHIA8

TMD distributions from the PB-approach in a forward evolution are identical to those from the backward evolution parton shower with PYTHIA8 PDF2ISR

Photon and Z boson PDFs & TMDs developments

- Photon and Z boson PDFs & TMDs
  [paper in progress: H. Jung, K. Moral Figueroa, S. Taheri]

Improve data analysis → new questions! → x.ex.:

What if the Z boson and photons were considered to be in the evolution?

Future accelerators

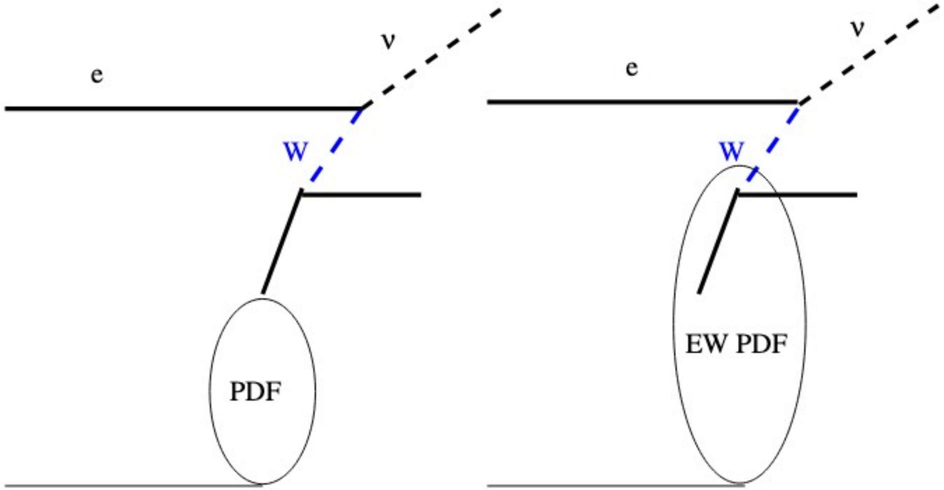
Technical development

Transverse momentum dependent parton distribution functions for Z boson & photons

Theory development

Better measurement predictions

we can **validate** this!



Photon & Z boson "PDF"

DIS diagrams and its role in QCD PDFs (left) and EWK PDFs (right)

Photon and Z boson PDFs & TMDs developments

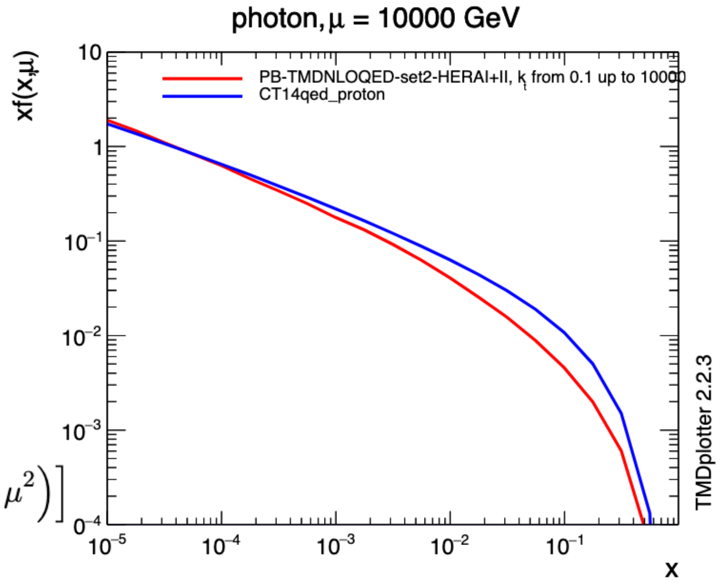
PB solution to DGLAP: General PDF

$$x f_a(x,\mu^2) = \Delta_a(\mu^2) x f_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{dq'^2}{q'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \int_x^{z_M} dz\, P_{ab}^{(R)}(z,\alpha_s) \frac{x}{z} f_b\left(\frac{x}{z}, q'^2\right)$$

Photon PDF

- The QED evolution is performed assuming the photon is generated dynamically **only from photon radiation off the quarks → intrinsic photon contribution is neglected ≡ no elastic component**
- We constrain the QCD partons by a fit to HERA data
- We perform the evolution

$$\mu^2 \frac{\partial x f_\gamma(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{\rm em}}{2\pi} \int_x^1 \frac{dz}{z} \left(1+(1-z)^2\right) \sum_{u,d} e_i^2 \left[\frac{x}{z} F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z} \bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$



Z boson PDF

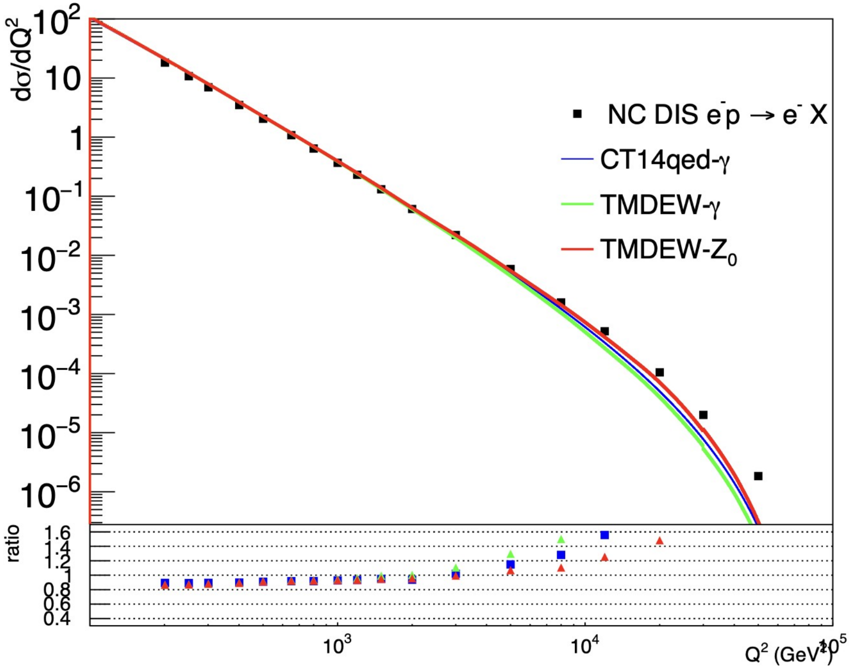
- We can obtain the Z boson PDF by incrementing the energy scale Q^2 and carrying on producing them dynamically
- Similarly to how quark splitting functions are extracted from DIS, we can obtain splitting functions for the photon and Z boson within the standard model: **W^- approximation** (S. Dawson, "The Effective W Approximation", Nucl. Phys. B 249 (1985) 42)
- Effective couplings are extracted from DIS cross-sections and the Z boson mass is treated as an **additional factor** related to the energy range Q^2

$$\mu^2 \frac{\partial x f_Z(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{\rm em}}{8\pi \sin^2\theta_W \cos^2\theta_W} \int_x^1 \frac{dz}{z} \left(1+(1-z)^2\right) \sum_{u.d} \left(V_i^2 + A_i^2\right) \left[\frac{x}{z} F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z} \bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

Photon and Z boson PDFs & TMDs developments

The photon and Z densities can be directly validated using measured DIS neutral current cross-sections

- ✓ Z^0 parton density best fits the DIS data from Neutral Current (NC) with e-
- ✓ At low energies the photon contribution is dominant, while at higher energies it is the Z^0 one



DIS neutral current cross-section from HERA compared to the photon and Z boson calculation

Photon: $$\frac{d\sigma_{NC}}{dQ^2} = \frac{4\pi^2\alpha_{em}}{Q^2}\,\frac{dx\,f_\gamma(x,Q^2)}{dQ^2}$$

Z boson: $$\frac{d\sigma_{NC}}{dQ^2} = \frac{4\pi^2\alpha_{em}\,Q^2\,(V_e^2+A_e^2)}{(Q^2+M_Z^2)^2}\,\frac{dx\,f_Z(x,Q^2)}{dQ^2}$$

PDFs (remainder)

$$\mu^2\frac{\partial x f_\gamma(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_{em}}{2\pi}\int_x^1\frac{dz}{z}\left(1+(1-z)^2\right)\sum_{u,d}e_i^2\left[\frac{x}{z}F_i\left(\frac{x}{z},\mu^2\right)+\frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

$$\mu^2\frac{\partial x f_Z(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_{em}}{8\pi\sin^2\theta_W\cos^2\theta_W}\int_x^1\frac{dz}{z}\left(1+(1-z)^2\right)\times\sum_{u,d}\left(V_i^2+A_i^2\right)\left[\frac{x}{z}F_i\left(\frac{x}{z},\mu^2\right)+\frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

Conclusions

Soft gluon treatment

- We have confirmed the need to properly treat soft gluon radiation in order to preserve consistency

Parton shower and parton densities consistency

- We have developed a method (PDF2ISR) which obtains an initial-state parton shower fully consistent with LO and NLO collinear parton densities and it is easily extended at NNLO
- This has been successfully validated through the comparison of TMDs from the PDF2ISR method applied on Pythia8 against the PB-TMDs

Photon and Z boson PDFs & TMDs

- We obtained for the first time the photon and Z boson PDFs and TMDs.
- This is the first time Z boson PDFs are both available and obtained from a full evolution calculation
- We successfully validated the photon and Z boson densities with neutral current DIS cross-sections from HERA, currently extending it to W boson.

Outlook

- We have a full and consistent description of the standard model. This allows us to obtain collinear and TMD parton densities.
- This offers a wide range of applications like VBF at the LHC, future muon colliders etc

Backup

In pQCD the proton is described in terms of

parton density functions f(x).

+

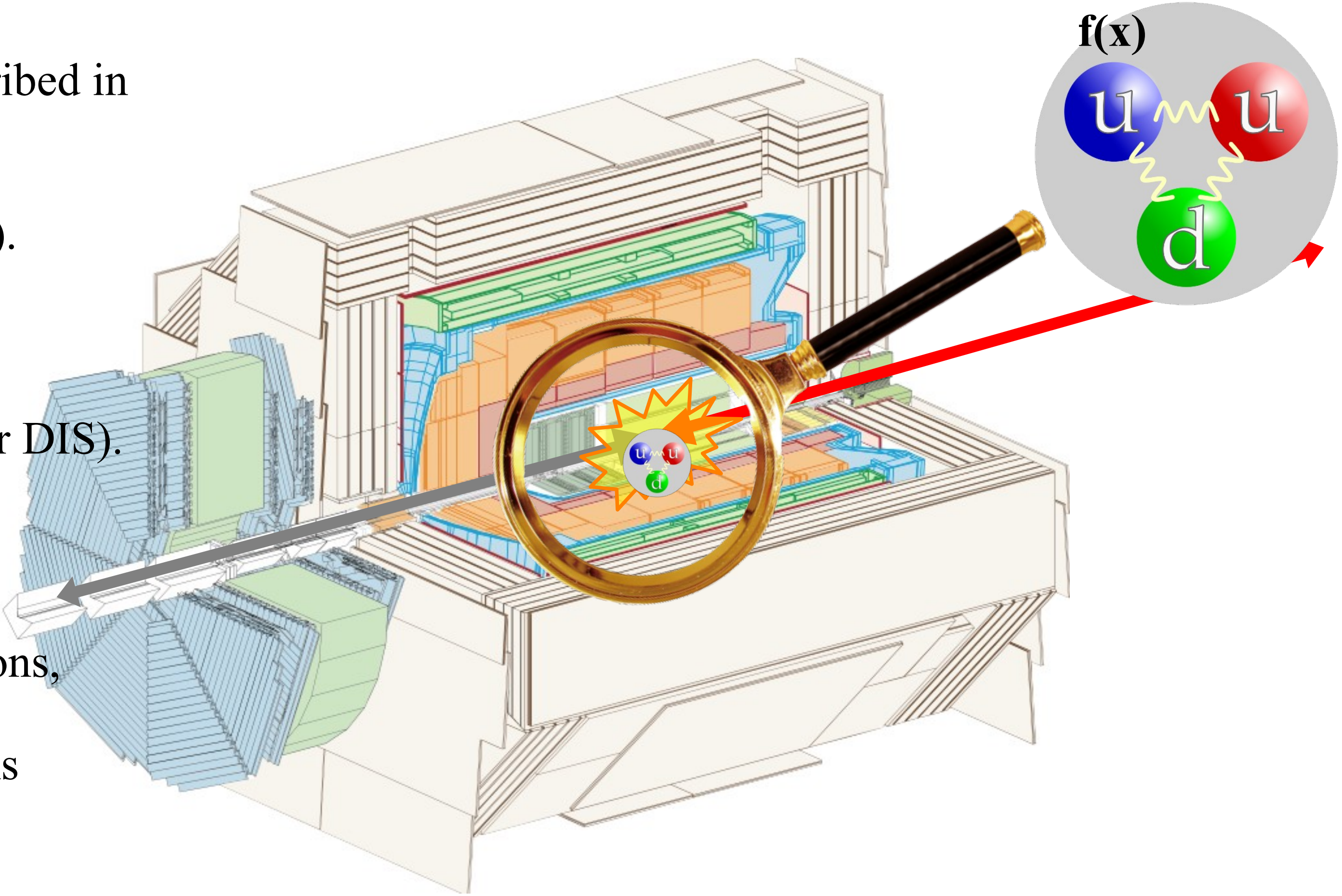
Dependence on:

factorisation scale, μ^2 (Q^2 for DIS).

↕

Parton momentum distributions, xf (x):

Parton **D**istribution **F**unctions (**PDFs**)



The H1 detector. Image credit: DESY

HERA dataset

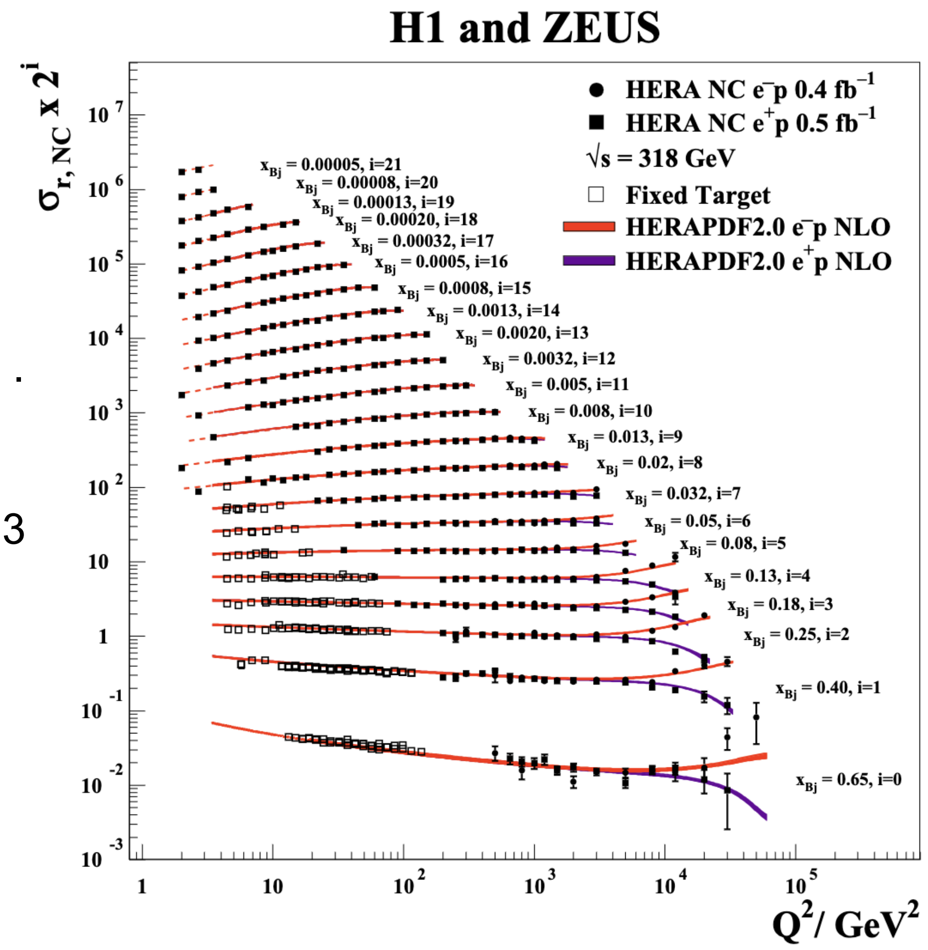
Deep inelastic scattering (DIS) of electrons on protons at centre-of-mass energies of up to $\sqrt{s} \simeq 320$ GeV at HERA covering:

- Neutral Current (NC) for $0.045 \leq Q^2 \leq 50000$ GeV^2 and $6 \cdot 10^{-7} \leq x_{Bj} \leq 0.65$ at inelasticity y between 0.005 and 0.95
- Charged Current (CC) for $200 \leq Q^2 \leq 50000$ GeV^2 and $1.3 \cdot 10^{-2} \leq x_{Bj} \leq 0.40$ at y between 0.037 and 0.76

H1 and ZEUS employed different experimental techniques, (detectors, kinematic reconstruction…)

↕

Reduced systematic uncertainty.



Combined HERA data for the inclusive NC e+ p and e− p reduced cross sections (fixed-target data and predictions of HERAPDF2.0 NLO)
arXiv:1506.06042v3

The Parton Branching Method

Parton Branching (PB) a method to obtain collinear PDFs and (transverse momentum dependent parton density functions) TMDs.

$$\mathcal{A}_a(x,\mathbf{k},\mu^2) = \Delta_a(\mu^2)\,\mathcal{A}_a(x,\mathbf{k},\mu_0^2) + \sum_b \int \frac{d^2\mathbf{q}'}{\pi \mathbf{q}'^2}\,\frac{\Delta_a(\mu^2)}{\Delta_a(\mathbf{q}'^2)}\,\Theta(\mu^2-\mathbf{q}'^2)\,\Theta(\mathbf{q}'^2-\mu_0^2) \times \int_x^{z_M} \frac{dz}{z}\,P_{ab}^{(R)}(\alpha_s,z)\,\mathcal{A}_b\left(\frac{x}{z},\mathbf{k}+(1-z)\mathbf{q}',\mathbf{q}'^2\right) ,$$

Splitting functions

(P_{ab}^R): probability for parton b→ a

P_{qq} & P_{gg} are divergent for z→ 1 because of high probability of soft gluons emissions

Sudakov form factors

$$\Delta_a(z_M,\mu^2,\mu_0^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\; z\, P_{ba}^{(R)}(\alpha_s,z)\right)$$

Probability of an evolution without resolvable branching between two scales

How?

Introducing a **soft-gluon resolution** scale z_M into the QCD evolution equations to separate **resolvable** and **non-resolvable** emissions:

What is z_M? $z_M \geq 1-10^{-3}$

For collinear PDFs : when $z_M \rightarrow 1$ we recover DGLAP results

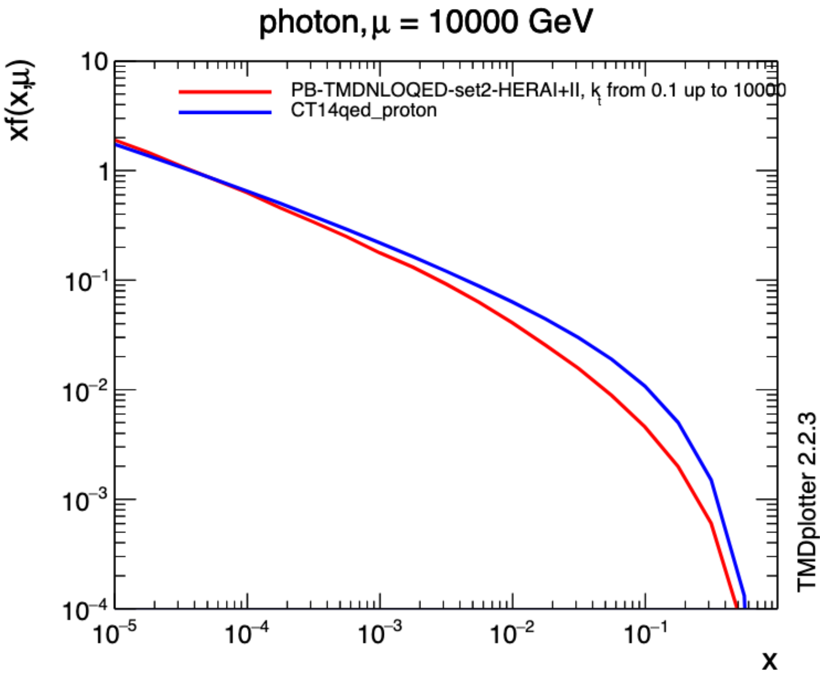
For TMD PDFs : With this z_M one is not sensitive to z_M value in angular ordering ($q_\perp^2=(1-z)^2\mu^2$)

(it's not the case for $p_\perp$ordering ($q_\perp^2=\mu^2$) and virtuality ordering ($q_\perp^2=(1-z)\mu^2$))

PDFs: Photon distribution

PB solution to DGLAP: General PDF

$$xf_a(x,\mu^2) = \Delta_a(\mu^2)xf_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{dq'^2}{q'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \int_x^{z_M} dz\, P_{ab}^{(R)}(z,\alpha_s) \frac{x}{z} f_b\left(\frac{x}{z}, q'^2\right)$$



Photon PDF

- The QED evolution is performed assuming the photon is generated dynamically **only from photon radiation off the quarks → intrinsic photon contribution is neglected ≡ no elastic component**

$$P_{qq} = e_q^2 \frac{1+z^2}{[1-z]_+} + \frac{3}{2} e_q^2 \delta(1-z) ,$$

$$P_{q\gamma} = N e_q^2 (z^2 + (1-z)^2) ,$$

$$P_{\gamma q} = e_q^2 \frac{1+(1-z)^2}{z} ,$$

$$P_{\gamma\gamma} = -\frac{N}{3} \sum_q e_q^2 \delta(1-z),$$

- We constrain the QCD partons by a fit to HERA data in the ranges $3.5 < Q^2 < 50000$ GeV^2 and $4 \cdot 10^{-5} < x < 0.65$ at NLO in QCD, for $\alpha_s(M_Z) = 0.118$, with LO QED evolution.
- We perform the evolution with Set 2 ($\alpha(p^2=\mu^2(1-z)^2)$)

Therefore the photon PDF is :

$$\mu^2 \frac{\partial x f_\gamma(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{em}}{2\pi} \int_x^1 \frac{dz}{z} \left(1+(1-z)^2\right) \sum_{u,d} e_i^2 \left[\frac{x}{z} F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

arXiv:2102.01494

PDFs: Heavy boson distributions

We can obtain both the Z boson and W boson PDFs by incrementing the energy scale Q^2 and carrying on producing them dynamically (strictly as radiation off the quarks according to the possible splittings).

This leads to the heavy boson mass being neglected (referred to as **scheme 0** in following slides).

Therefore, the heavy boson PDFs in this approach are by construction:

$$\mu^2 \frac{\partial x f_Z(x,\mu^2)}{\partial \mu^2} = \frac{\alpha_{\rm em}}{8\pi \sin^2\theta_W \cos^2\theta_W} \int_x^1 \frac{dz}{z}\left(1+(1-z)^2\right) \times \sum_{u,d}\left(V_i^2 + A_i^2\right)\left[\frac{x}{z}F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

Z^0 boson

Effective couplings

Similarly to how quark splitting functions are extracted from DIS, we can obtain splitting functions for the photon and heavy bosons within the standard model: valid at highest energies, when the masses of the particles can be neglected → These splittings are obtained by replacing the coupling and colour factors in the standard QCD DGLAP splitting functions: **W^- approximation** (S. Dawson, “The Effective W Approximation”, Nucl. Phys. B 249 (1985) 42)

The key difference is the **effective coupling.**

These effective couplings are extracted from the cross-section of the following processes:

$$\sigma(q\bar{q} \to \gamma^*) = \frac{1}{3}\frac{\pi}{m_{\mathrm{DY}}^2}\left[4\pi\alpha_{em} m_{\mathrm{DY}}^2\right] = \frac{1}{3}\pi\ \left[4\pi\alpha_{em}\right]$$

$$\sigma(q\bar{q} \to \mathrm{Z}) = \frac{1}{3}\frac{\pi}{m_{\mathrm{Z}}^2}\left[\frac{2G_F}{\sqrt{2}} m_{\mathrm{Z}}^4 (V_f^2 + A_f^2)\right] = \frac{1}{3}\pi\ \left[\sqrt{2} G_F m_{\mathrm{Z}}^2 (V_f^2 + A_f^2)\right]$$

The cross sections can be obtained by replacing the coupling in α_{em} in $\sigma(q\bar{q} \to \gamma^*)$ by an effective coupling α_{ef}

QCD partons	Photon	Heavy bosons
$\alpha_{eff} = \alpha_s$	$\alpha_{eff} = \alpha_s$	Z boson $\alpha_{eff} = \frac{\alpha_{em}}{4\sin^2\theta_{\mathrm{W}}\cos^2\theta_{\mathrm{W}}}(V_f^2 + A_f^2)$

Effective couplings and mass schemes

Until now we have neglected the heavy boson masses, how can we include them?

Scheme 0 In the **zero-mass**-variable-flavor scheme, the heavy boson mass is included as a mass threshold (QCD at the extremes lecture series)	**Scheme 1** The heavy boson mass is treated as an **additional factor** related to the energy range Q^2 arXiv:1803.06347	**Scheme 2** The heavy boson mass is treated **only** as a dynamical energy fraction included via a **limit** in the z integrals (**angular ordering**) arXiv:2309.11802 arXiv:1703.08562
$\alpha_{\text{eff}} = \alpha_{\text{eff}}\Theta(Q^2 - M_V^2)$	$\alpha_{\text{eff}} = \alpha_{\text{eff}}\left[\frac{Q^2}{Q^2 + M_V^2}\right]^2$	$z_M = 1 - \frac{M_V}{q}$

Limitation:
For heavy bosons: No evolution at energy scale $Q^2 < M_V^2$

Key questions: Which scheme best describes the experimental data? What’s the correct way of covering the whole energy range?

The Parton Branching Method: PDFs

Parton Branching (PB) method is a procedure to obtain collinear parton densities (PDFs) and (transverse momentum dependent) parton density functions TMDs from the DGLAP equation.

$$\mu^2 \frac{\partial x f_a(x,\mu^2)}{\partial \mu^2} = \sum_b \int_x^1 dz\, P_{ab}\left(\alpha_{\text{eff}}(\mu^2), z\right)\, \frac{x}{z} f_b\left(\frac{x}{z}, \mu^2\right)$$

P_{ab} : regularized DGLAP splitting functions for transition of parton b → a.

It can be decomposed in a sum of virtual and **real** emission branching probabilities. Therefore by applying the sum rules, we keep the real emissions:

$$P_{ab}^{(R)}(z,\alpha_s) = K_{ab}(\alpha_s)\frac{1}{1-z} + R_{ab}(z,\alpha_s)$$

And the solution to the evolution equation for the **momentum-weighted parton density $xf_a(x, \mu^2)$ at scale μ** is given by:

$$x f_a(x,\mu^2) = \Delta_a(\mu^2) x f_a(x,\mu_0^2) + \sum_b \int_{\mu_0^2}^{\mu^2} \frac{dq'^2}{q'^2} \frac{\Delta_a(\mu^2)}{\Delta_a(q'^2)} \int_x^{z_M} dz\, P_{ab}^{(R)}(z,\alpha_s) \frac{x}{z} f_b\left(\frac{x}{z}, q'^2\right)$$

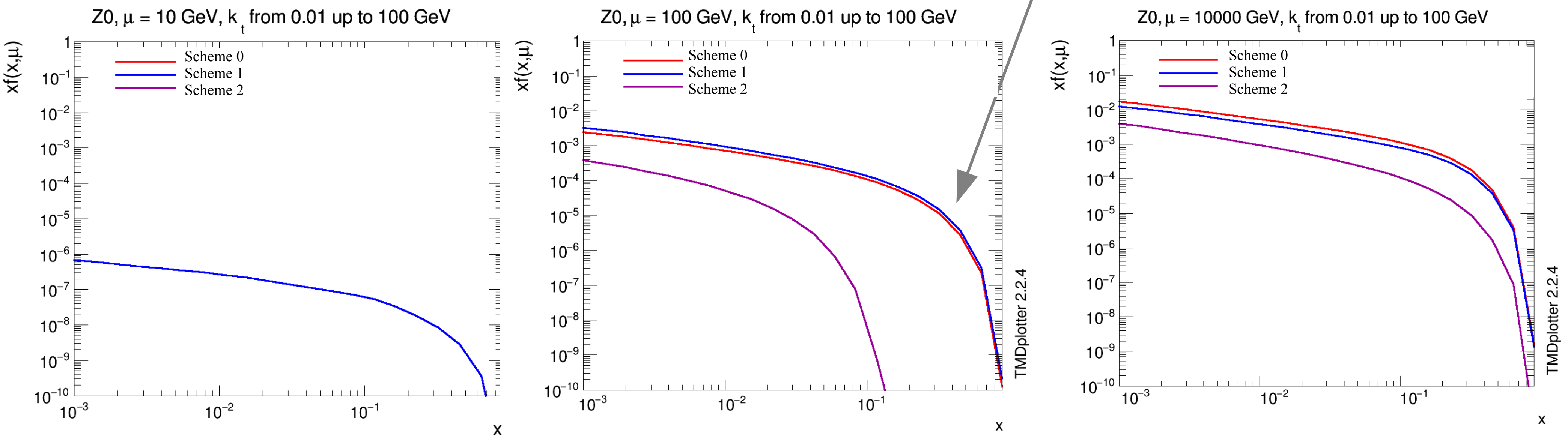
μ_0 is the starting scale
$\Delta_a(\mu^2) := \Delta a(\mu^2, \mu^2_0)$ is the Sudakov form factor (probability of an evolution without resolvable branching between two scales)

$$\Delta_a(z_M,\mu^2,\mu_0^2) = \exp\left(-\sum_b \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_0^{z_M} dz\, z\, P_{ba}^{(R)}(\alpha_s, z)\right)$$

Z boson PDF

At high momentum both schemes 0 and 1 approach each other since $Q^2 > M^2$

Z collinear densities



Z^0 collinear densities at μ = 10 GeV, μ = 100 GeV and μ = 10^4 GeV as a function of x

Validation

The photon and Z densities can be directly validated using measured DIS neutral current cross-sections.

$$\sigma^{\pm}_{r,\mathrm{NC}} = \frac{\mathrm{d}^2\sigma^{e^\pm p}_{\mathrm{NC}}}{\mathrm{d}x_{\mathrm{Bj}}\mathrm{d}Q^2} \cdot \frac{Q^4 x_{\mathrm{Bj}}}{2\pi\alpha^2 Y_+} = \tilde{F}_2 \mp \frac{Y_-}{Y_+} x\tilde{F}_3 - \frac{y^2}{Y_+}\tilde{F}_{\mathrm{L}}$$

Contribution of Z exchange is negligible at low Q^2, i.e. $Q^2 \ll M^2_Z$

$$\sigma^{\pm}_{r,\mathrm{NC}} = F_2 - \frac{y^2}{Y_+} F_{\mathrm{L}} \approx F_2$$

Photon:

$$\frac{d\sigma_{NC}}{dQ^2} = \frac{2\pi\alpha^2_{\mathrm{em}}}{Q^4} \int \frac{dx}{x} \left(1+(1-y)^2\right) \times \left(e_u^2 \left[xU(x,Q^2) + x\bar{U}(x,Q^2)\right] + e_d^2 \left[xD(x,Q^2) + x\bar{D}(x,Q^2)\right]\right)$$

Z boson:

$$\frac{d\sigma_{NC}}{dQ^2} = \frac{2\pi\alpha^2_{\mathrm{em}}}{Q^4} \left[\frac{Q^2}{Q^2+M_Z^2}\right]^2 \int \frac{dx}{x} \left(1+(1-y)^2\right) \times \sum_{u,d} \frac{\left((V_i^2+A_i^2)\left[xF_i(x,Q^2) + x\bar{F}_i(x,Q^2)\right]\right)}{4\sin^2\theta_{\mathrm{W}}\cos^2\theta_{\mathrm{W}}}$$

Therefore, the cross-sections can be re-written as:

- Photon: $$\frac{d\sigma_{NC}}{dQ^2} = \frac{4\pi^2\alpha_{\mathrm{em}}}{Q^2} \frac{dx f_\gamma(x,Q^2)}{dQ^2}$$
- Z boson: $$\frac{d\sigma_{NC}}{dQ^2} = \frac{4\pi^2\alpha_{\mathrm{em}} Q^2 (V_e^2+A_e^2)}{(Q^2+M_Z^2)^2} \frac{dx f_Z(x,Q^2)}{dQ^2}$$

PDFs (remainder)

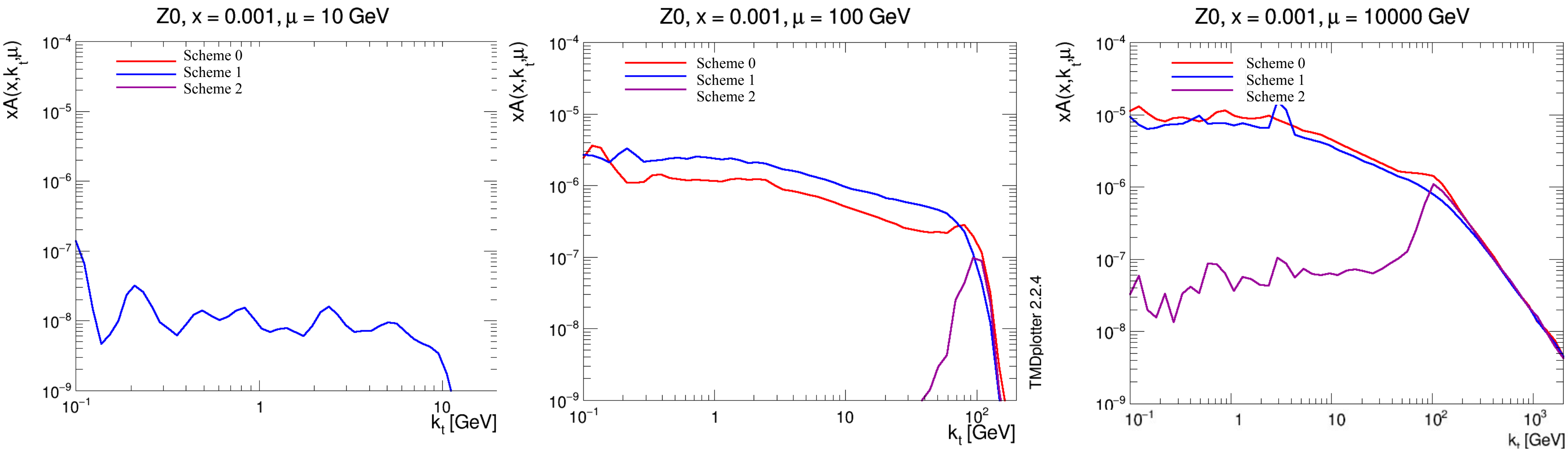
$$\mu^2 \frac{\partial x f_\gamma(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_{\mathrm{em}}}{2\pi} \int_x^1 \frac{dz}{z} \left(1+(1-z)^2\right) \sum_{u,d} e_i^2 \left[\frac{x}{z} F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

$$\mu^2 \frac{\partial x f_Z(x,\mu^2)}{\partial\mu^2} = \frac{\alpha_{\mathrm{em}}}{8\pi\sin^2\theta_{\mathrm{W}}\cos^2\theta_{\mathrm{W}}} \int_x^1 \frac{dz}{z} \left(1+(1-z)^2\right) \times \sum_{u,d} \left(V_i^2+A_i^2\right) \left[\frac{x}{z} F_i\left(\frac{x}{z},\mu^2\right) + \frac{x}{z}\bar{F}_i\left(\frac{x}{z},\mu^2\right)\right]$$

Z boson TMD

Z TMDs

- Finally, the same behaviour is shown in the PDFs
- The TMDs for different schemes get close to each other at high transverse momentum k_T
- Only Scheme 1 covers the whole transverse momentum range
- Scheme 2 shows a behaviour several orders of magnitude lower than the others



Z^0 TMD densities at μ = 10 GeV, μ = 100 GeV and $\mu = 10^4$ GeV as a function of k_T

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