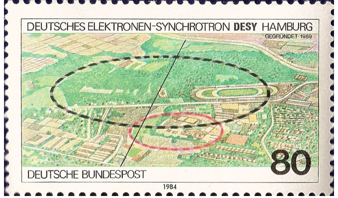
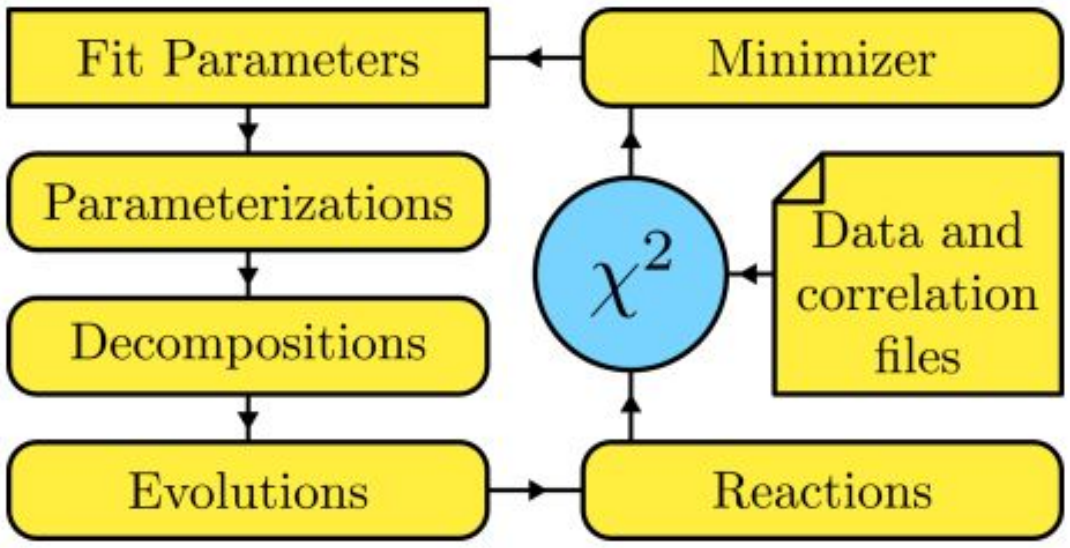


xFitter N^{3}LO fits

S. Glazov, Stony Brook University,
May 2025

What is xFitter



Quantitative comparison of the data and QCD predictions aimed to determine theory parameters involves a number of steps. **xFitter** binds them together.

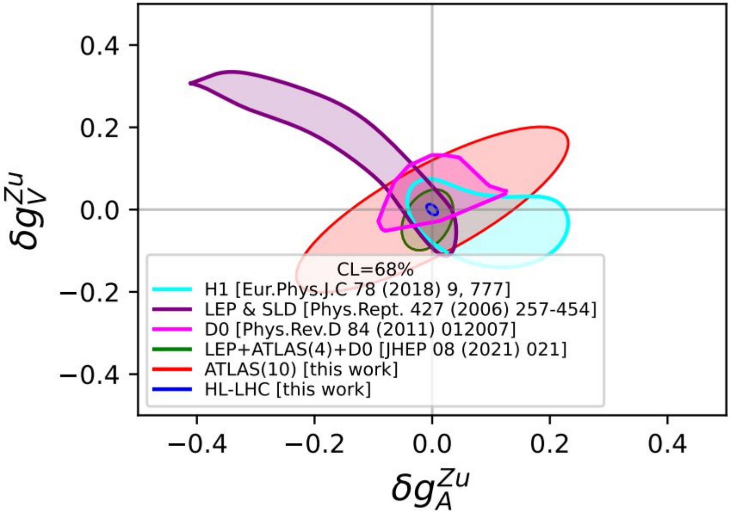
Status of xFitter project

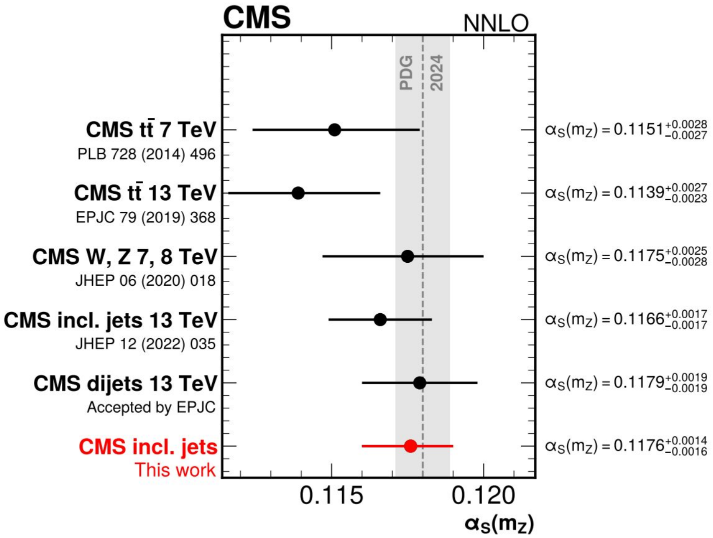
- Project is in active development on https://gitlab.cern.ch/fitters/xfitter
  - Stable release 2.2.0 (“future freeze”)
  - Nightly “master” branch built is validated vs several public results
  - Nightly docker image

```
docker run -it gitlab-registry.cern.ch/fitters/xfitter:master bash
```

- Many updates of the functionality and productivity
  - Added interfaces to ploughshare, hoppet, PineAPPL, CIJET
  - Added N^{3}LO FONLL for APFEL++, updates of DYTurbo interface
  - Numerical improvements: parallel derivatives in CERES, parallel profiling, parallel asymmetric Hessian uncertainties evaluation (based on J. Pumlin *Phys.Rev.D* 65 (2001) 014013)

Example analyses for the last year





Many analyses using xFitter in the last year. Examples are SMEFT analysis of Z boson couplings using forward-backward DY production at LHC (EPJC 84 (2024) 1277) and CMS analysis of inclusive jet production to extract α_s (arXiv:2412.16665)

Analysis of DIS data at N^{3}LO: settings

Model variations:

q^2_0	1.9±0.2
q^2_{min}	3.5±1.0
m_c	1.47±0.06
m_b	4.50±0.25
f_s	0.40±0.08

HERAPDF2.0 parameterisation:

$$xg(x,\mu_0^2) = A_g\, x^{B_g}(1-x)^{C_g} - A'_g\, x^{B'_g}(1-x)^{C'_g}$$

$$xu_v(x,\mu_0^2) = A_{u_v}\, x^{B_{uv}}(1-x)^{C_{uv}}\left[1+E_{u_v}x^2\right]$$

$$xd_v(x,\mu_0^2) = A_{d_v}\, x^{B_{dv}}(1-x)^{C_{dv}}$$

$$x\bar{u}(x,\mu_0^2) = A_{\bar{u}}\, x^{B_{\bar{u}}}(1-x)^{C_{\bar{u}}}\left[1+D_{\bar{u}}x\right]$$

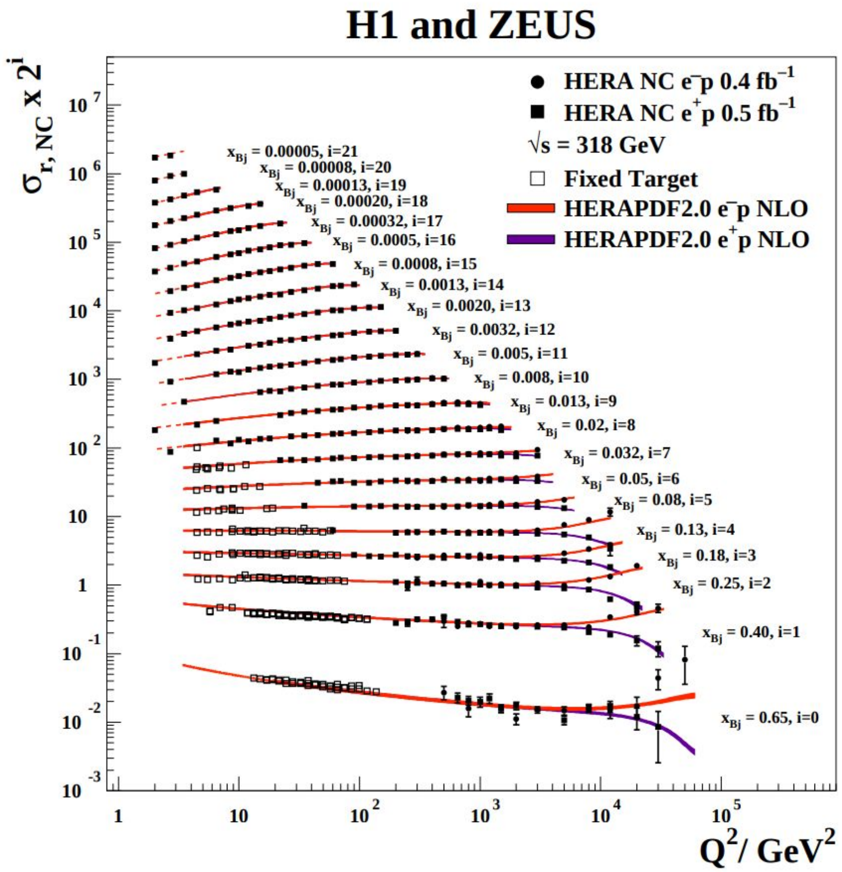
$$x\bar{d}(x,\mu_0^2) = A_{\bar{d}}\, x^{B_{\bar{d}}}(1-x)^{C_{\bar{d}}} \quad xs(x,\mu_0^2) = x\bar{s}(x,\mu_0^2) = r_s\, x\bar{d}(x,\mu_0^2) \qquad r_s = \frac{f_s}{1-f_s}$$

Alternative gluon parameterisation:

$$xf(x,\mu_0^2) = A\, x^{B}(1-x)^{C}\left[1+Dx+Ex^2\right]\left[1+F\log x + \cancel{G\log^2 x} + \cancel{H\log^3 x}\right]$$

- HERAI+II inclusive (arXiv:1506.06042) and charm+bottom data (arXiv:1804.01019)
- N^{3}LO evolution as implemented in APFEL (arXiv:1708.00911) (based on code in HOPPET, arxiv:0804.3755)
- **Exact** (DESY 24--037) and approximate N^{3}LO matching conditions provided to HOPPET and implemented in APFEL (benchmark: arXiv:2404.15711)
- FONLL VFNS implemented in xFitter, checked with fortran APFEL for N^{2}LO

Determination of the PDFs using HERA data



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$$\frac{d^2\sigma^{NC}_{e^\mp p}}{dx dQ^2} = \frac{2\pi\alpha^2 Y_+}{xQ^4}\left(F_2 - \frac{y^2}{Y_+}F_L \pm \frac{Y_-}{Y_+}xF_3\right)$$

$$Y_\pm = 1 \pm (1-y)^2$$

$$F_2 = x\sum e_q^2(q(x)+\bar{q}(x))$$

$$xF_3 = x\sum 2e_q a_q(q(x)-\bar{q}(x))$$

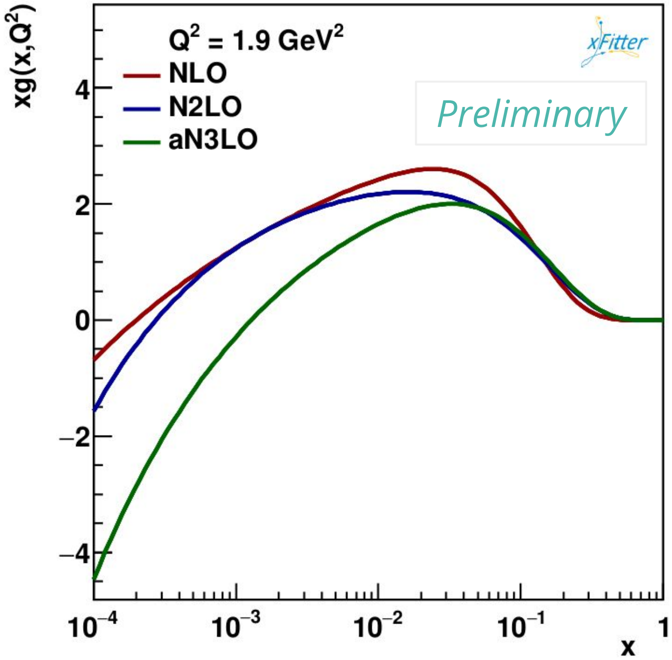
At HERA photon exchange dominates, the best PDF determined is ***u(x)***.

At low x, contribution of ***c(x)*** is sizable too (up to 30% of F_2)

Gluon is measured from q^2 dependence of the structure function F_2

Lower accuracy at high x, ***$d_v(x)$*** and ***$u_v(x)$*** separated using charged-current data

N^{3}LO with approx. matching vs N^{2}LO and N^{1}LO

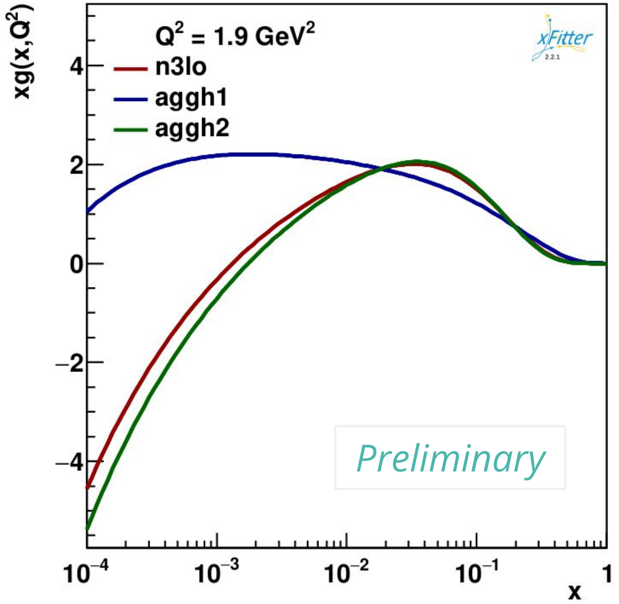


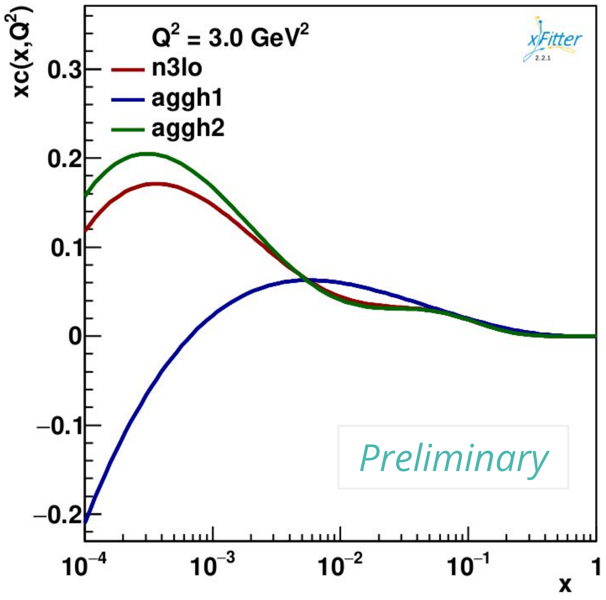
Preliminary

Dataset	NLO	N2LO	aN3LO
HERA1+2 NCep 820	68 / 70	67 / 70	68 / 70
HERA1+2 NCep 460	217 / 204	218 / 204	215 / 204
HERA1+2 CCep	43 / 39	44 / 39	48 / 39
HERA1+2 NCem	218 / 159	216 / 159	223 / 159
HERA1+2 CCem	53 / 42	56 / 42	61 / 42
HERA1+2 NCep 575	216 / 254	220 / 254	215 / 254
HERA1+2 NCep 920	424 / 377	452 / 377	476 / 377
Correlated χ^2	77	98	103
Log penalty χ^2	+7.7	+14	+14
Total χ^2 / dof	1325 / 1131	1385 / 1131	1422 / 1131

- Implement approximate N^{3}LO based on arXiv:2207.04739
- Similar pattern as observed in the past: N^{3}LO gluon is more negative at low *x* for the starting scale, χ^2/dof is the worst.

N^3LO with approximate matching uncertainties

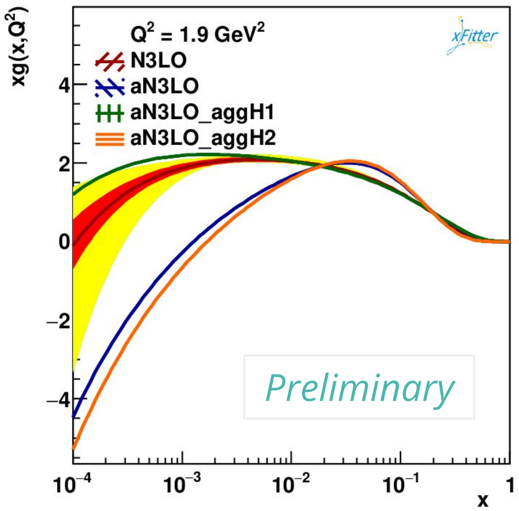


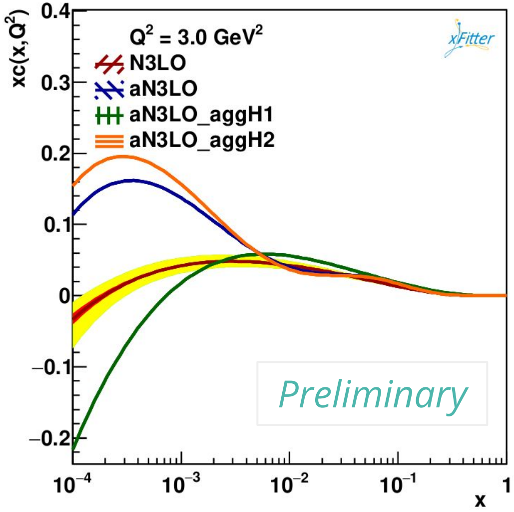


Approximate matching conditions have uncertainties for transition matrix elements. Large impact on gluon/charm due to *$-2000 < a_{gg,H} < -700$*

$$A^{(3)}_{gg,H} = A_1 \ln^2(1-x) + A_2 \ln(1-x) + A_3 x^2 + A_4 \ln x + A_5 x + a_{gg,H} \frac{\ln x}{x}$$

Exact N^{3}LO vs approximate N^{3}LO



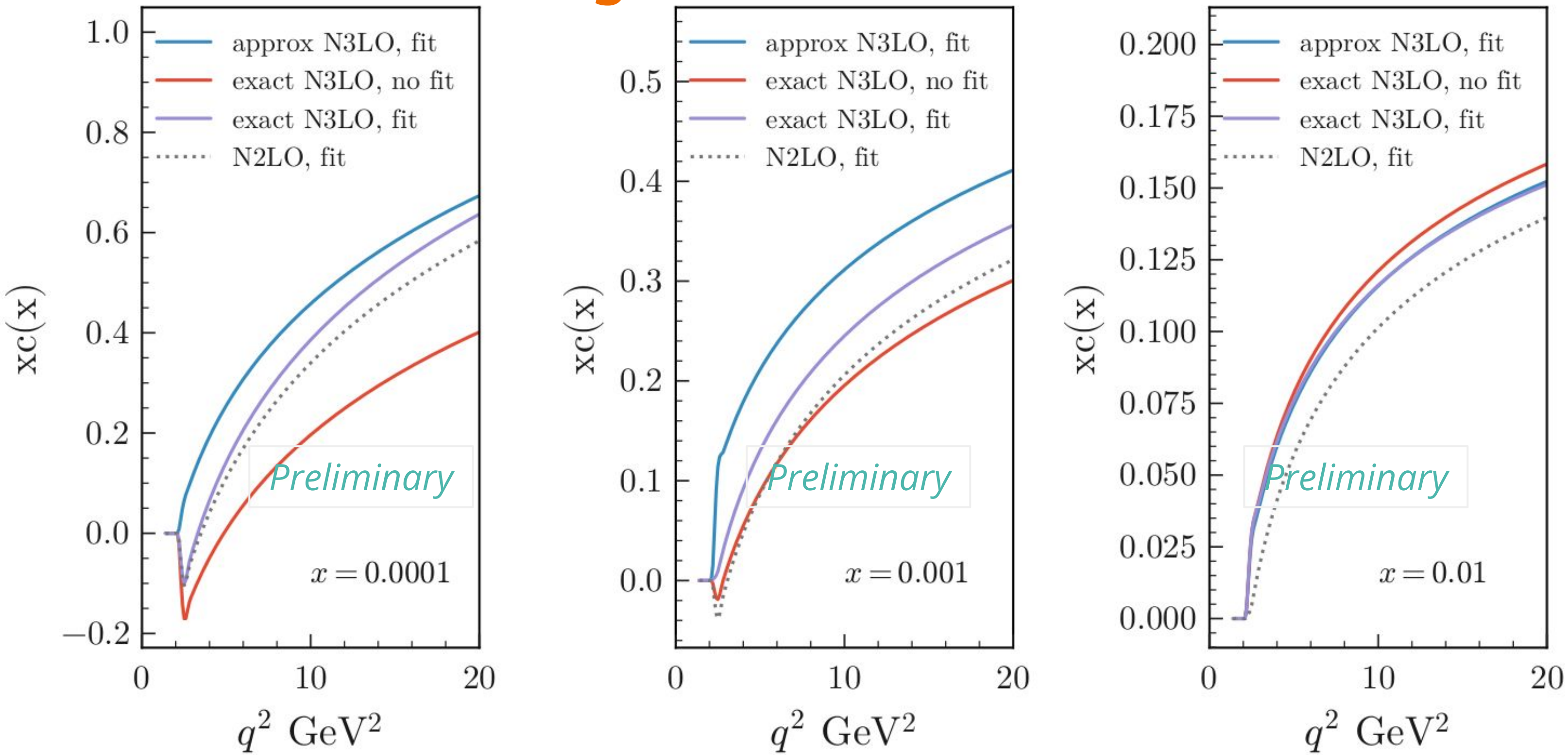


Preliminary

Dataset	N3LO	aN3LO
HERA1+2 NCep 820	66 / 70	68 / 70
HERA1+2 NCep 460	222 / 204	215 / 204
HERA1+2 CCep	48 / 39	48 / 39
HERA1+2 NCem	224 / 159	223 / 159
HERA1+2 CCem	59 / 42	61 / 42
HERA1+2 NCep 575	215 / 254	215 / 254
HERA1+2 NCep 920	471 / 377	476 / 377
Correlated χ^2	106	103
Log penalty χ^2	+9.9	+14
Total χ^2 / dof	1422 / 1131	1422 / 1131

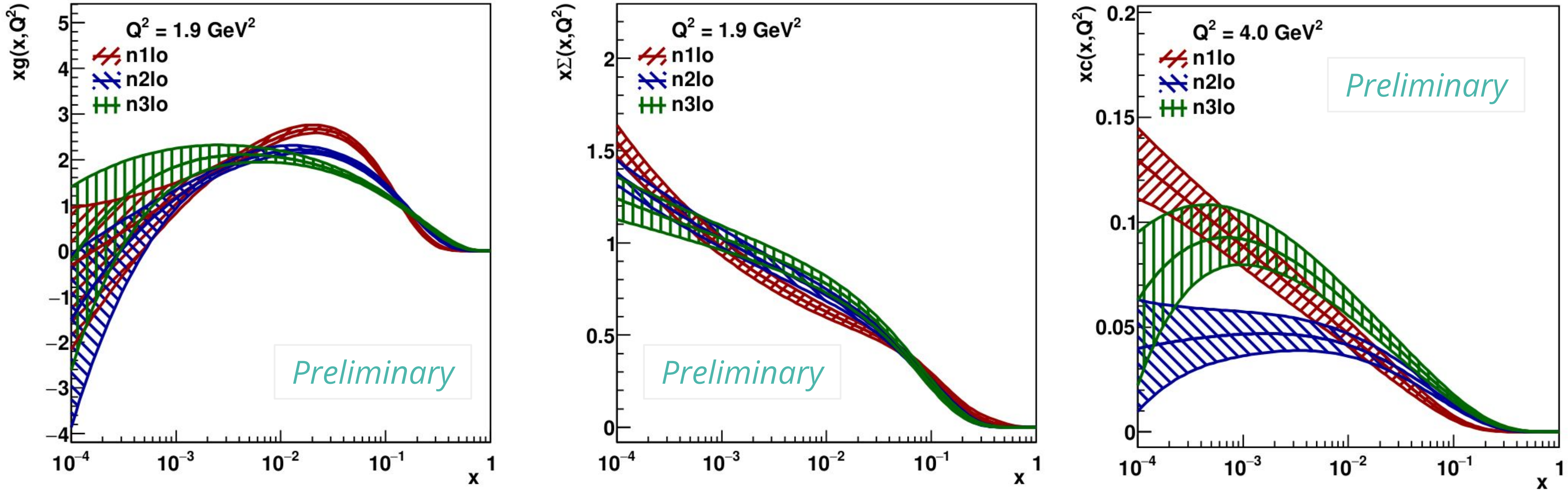
Fit using exact N^{3}LO matching conditions, shown with full experimental plus model uncertainties. Large difference vs aN^{3}LO central fit for the gluon and charm, however within uncertainties of aN^{3}LO. Fit quality to the data is about the same

Effect of the matching conditions on charm



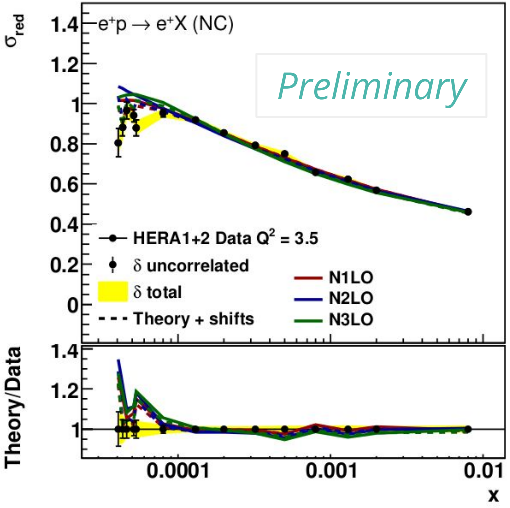
Compare *xc(x)* vs *q^2* between approximate and exact N^{3}LO (for the same PDFs), exact N^{3}LO fit and N^{2}LO fit. Significant differences at low *x*

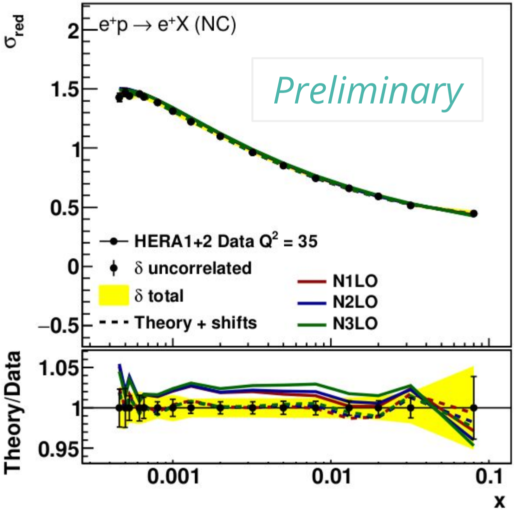
N^1LO, N^2LO vs N^3LO with exact matching



Fits with alternative gluon parameterisation, consistent results to HERAPDF style.
N^3LO gluon is the hardest, also largest at small *x*. For *x=0.01*, all orders agree.

Comparison to most accurate √s=920 GeV data



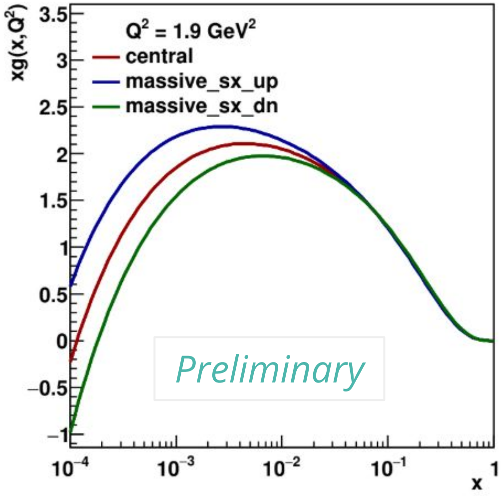


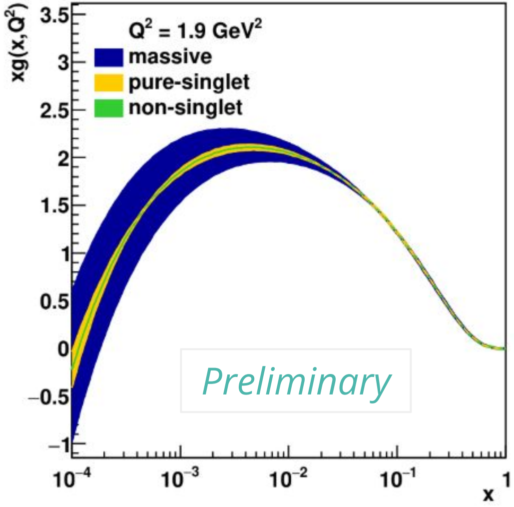
Preliminary

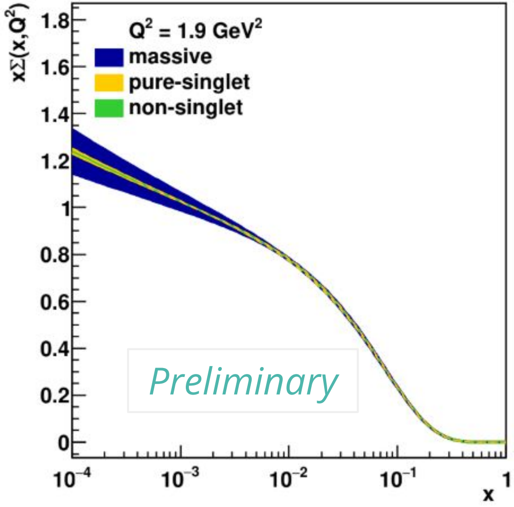
Dataset	N1LO	N2LO	N3LO
HERA1+2 NCep 820	69 / 70	68 / 70	66 / 70
HERA1+2 NCep 460	217 / 204	218 / 204	222 / 204
HERA1+2 CCep	44 / 39	43 / 39	45 / 39
HERA1+2 NCem	213 / 159	217 / 159	226 / 159
HERA1+2 CCem	63 / 42	58 / 42	57 / 42
HERA1+2 NCep 575	215 / 254	221 / 254	216 / 254
HERA1+2 NCep 920	427 / 377	456 / 377	472 / 377
Correlated χ^2	76	95	104
Log penalty χ^2	+8.7	+9.7	+6.2
Total χ^2 / dof	1333 / 1132	1385 / 1132	1414 / 1132

All fixed-order fits do not describe data very well at lowest x/highest inelasticity y, with N^{3}LO improving slightly vs N^{2}LO. For intermediate Q^2, N^{3}LO is further above the data, driving increase in correlated χ^2

Heavy flavour uncertainties



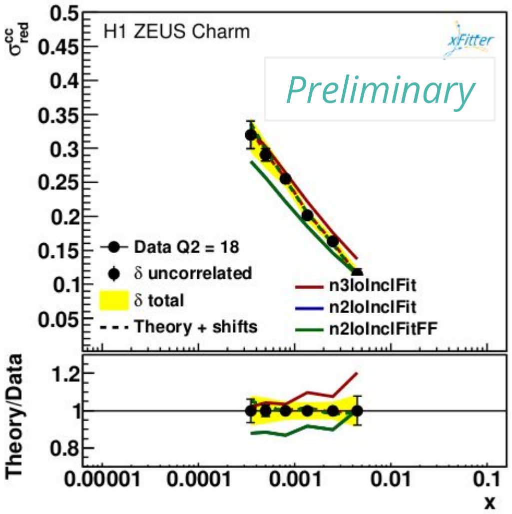




Additional uncertainties arise from approximate treatment of massive coefficient functions. Implemented based on arXiv:2401.12139. In addition, extra uncertainties from numerical approximations of splitting functions (small).

→significant uncertainty due to small-x parameter variation, dominant compared to other sources.

F_2 charm predictions (N2LO and N3LO)

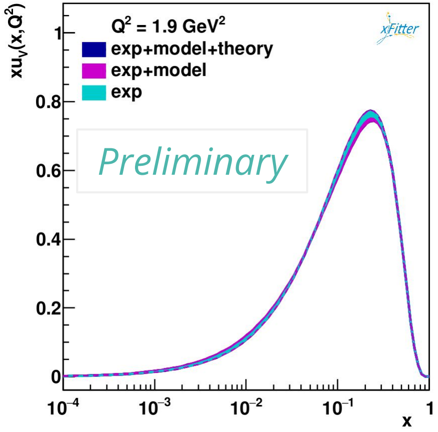


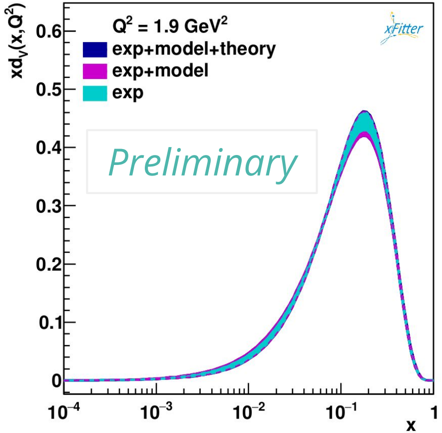
Preliminary

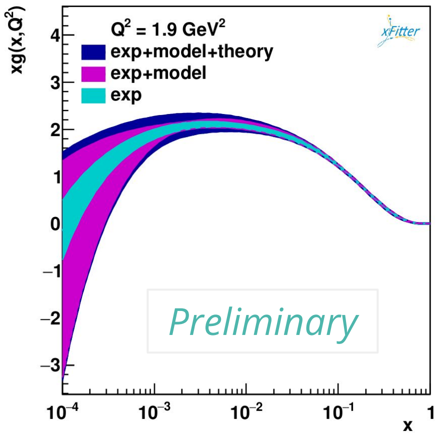
Dataset	N^3LO	N^2LO	N^2LO(fortranAPFEL)
HERA c	41 / 47	50 / 47	50 / 47
HERA b	15 / 26	20 / 26	20 / 26
Correlated χ^2	43	61	61
Log penalty χ^2	+8.6	-5.35	-5.34
Total χ^2 / dof	108 / 59	125 / 59	125 / 59
χ^2 p-value	0.00	0.00	0.00

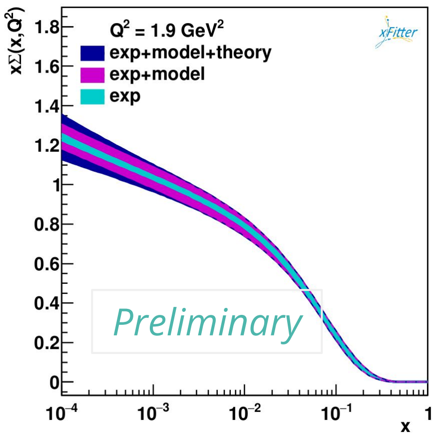
- Compare predictions of the N2LO and N3LO fit to the inclusive data vs charm from combined H1ZEUS analysis arXiv:1804.01019
- Better χ^2/dof for the N^3LO fit

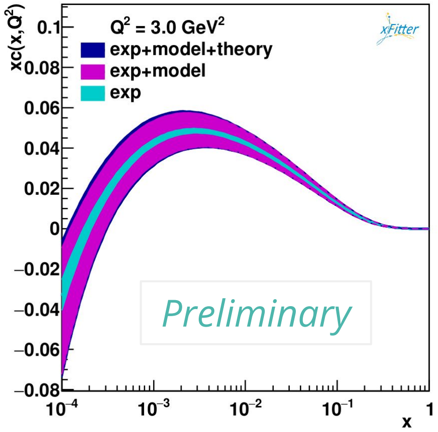
N^3LO pdfs with full uncertainties





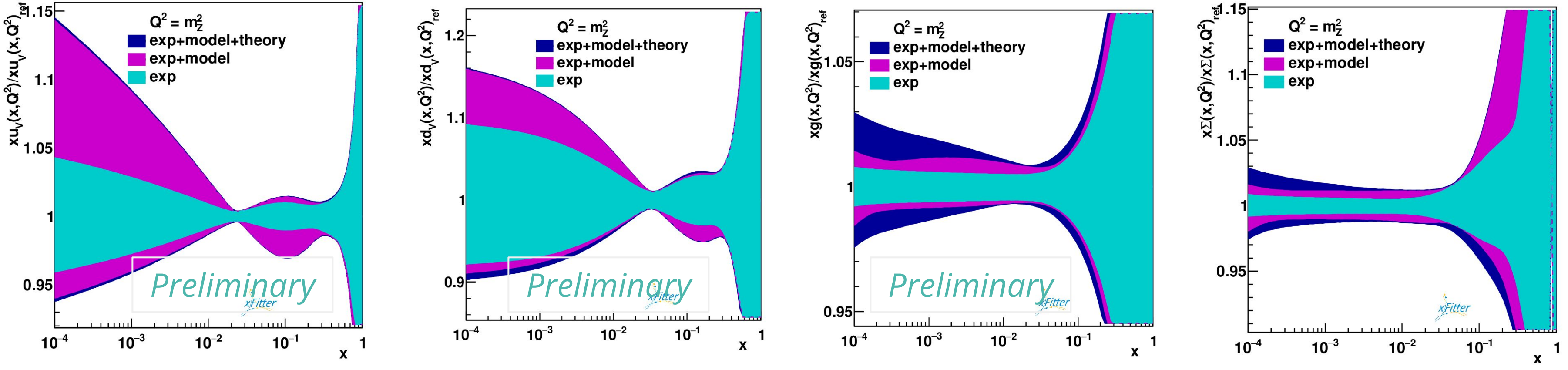


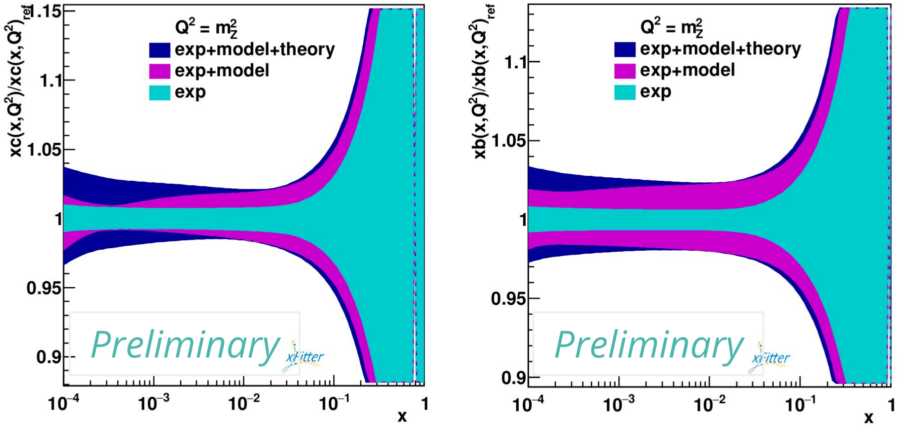




- PDF obtained for N^3LO fit to HERA inclusive plus charm and bottom data, including model and theory uncertainties.
- Leading model uncertainty: m_c variation

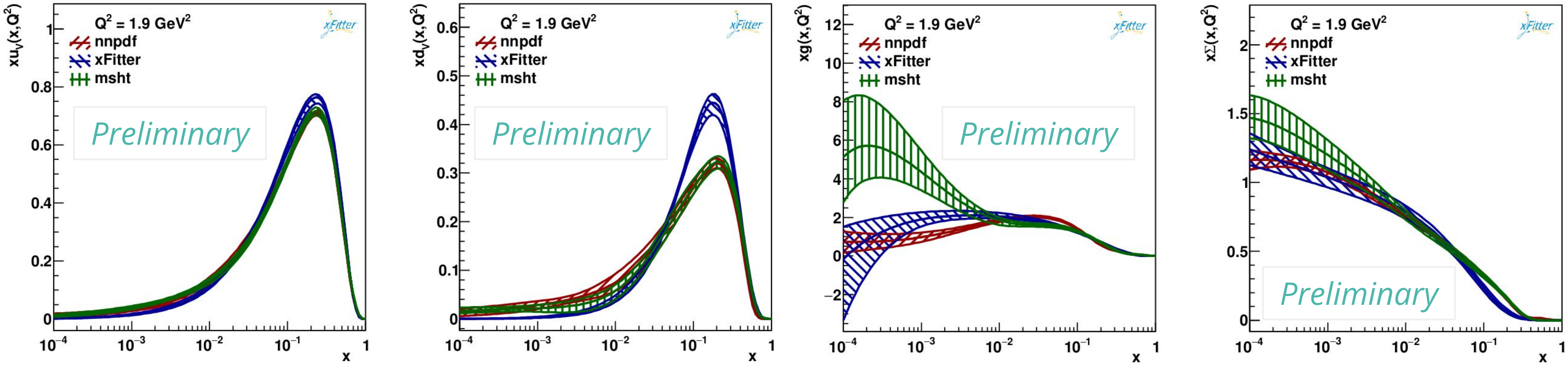
N^{3}LO uncertainties at $Q^2 = M_Z{}^2$

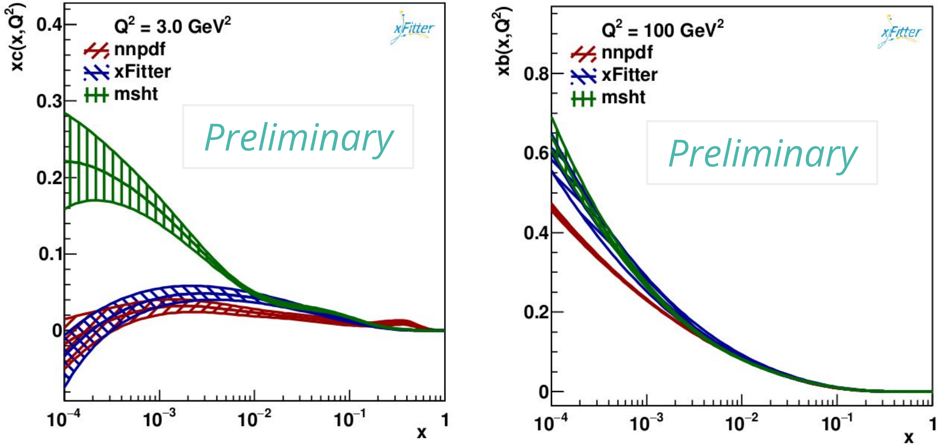




At higher scales, large uncertainties for valence quarks at low *x*. Significant theory uncertainty for gluon at *x<0.01*. Charm and bottom uncertainties stem from m_C and m_B variations

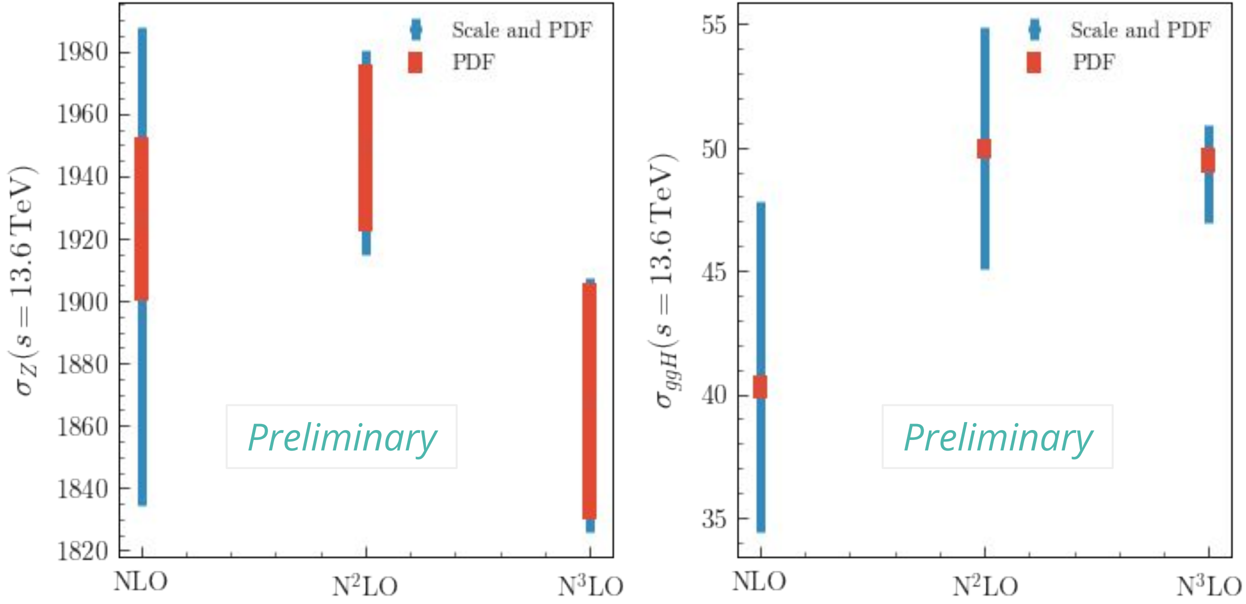
Comparison to aN^{3}LO sets





Comparison with NNPDF4.0 and MSHT20an3lo sets show large disagreements. The sets agree for u_v only. For d_v, xFitter set deviates, while for g and c, MSHT set is an outlier. For b, NNPDF deviates from the other sets.

Impact on predictions at the LHC



Predictions at NLO, N^{2}LO and N^{3}LO for *ggH, Z* computed using **n3loxs** (arXiv:2209.06138). PDF uncertainties for N^{3}LO include theory uncertainties

→good convergence for *H*, not so good for *Z*

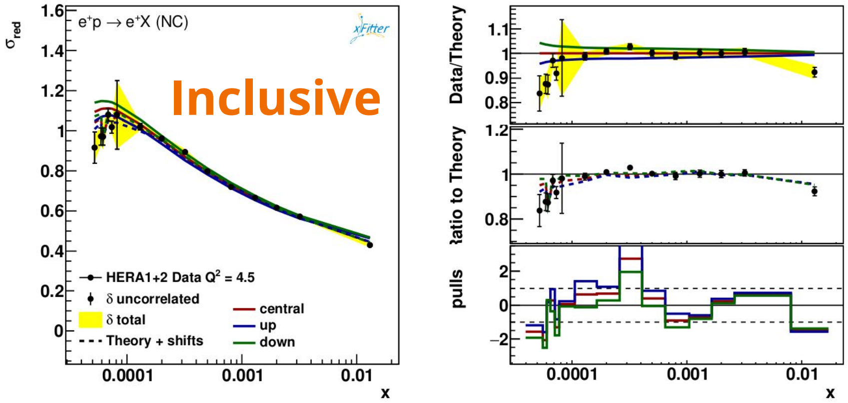
Summary and Outlook

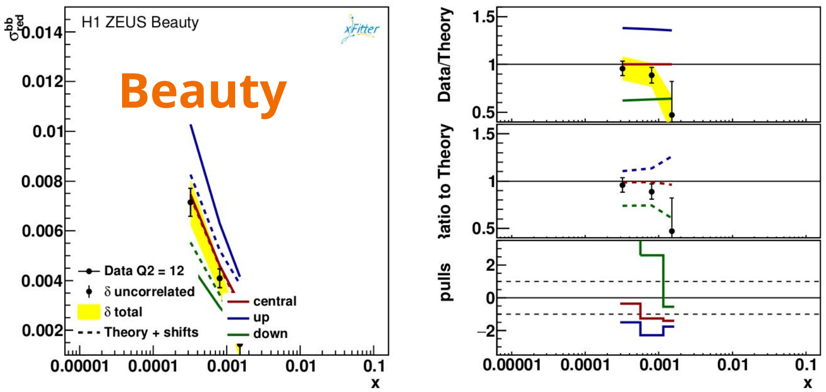
- DIS-only analysis with consistent N^{3}LO for the evolution and coefficient functions
- First analysis including exact matching conditions
- Residual uncertainties from heavy flavour coefficient functions
- Visible impact on PDFs and predictions for the LHC

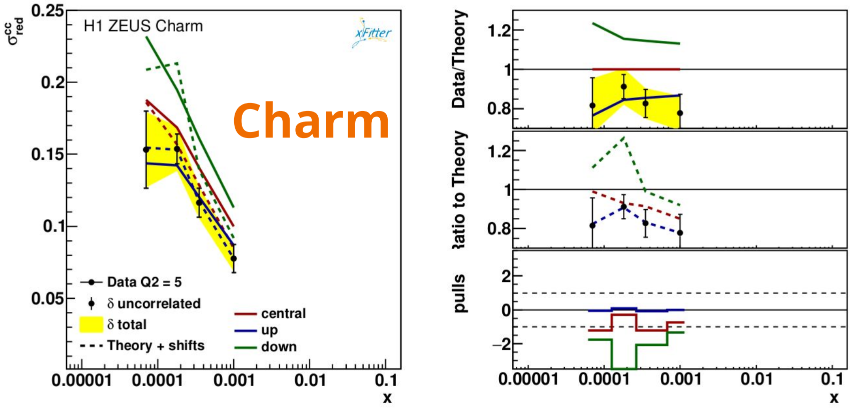
→further studies including effect of small *x* resummation

Backup

Uncertainties of small x masive coefficient function







Low x uncertainties in heavy flavour coefficient structure functions lead to sizable variation of F_2^{cc} and F_2^{bb} structure functions. In the FONLL prescription, important for sub-threshold regions.