Precision DIS thrust predictions for HERA and EIC

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arXiv:2504.05234, LA-UR-25-24323

Extracting the Strong Coupling at the
EIC and other Future Colliders
May 5, 2025

Outline

- Background and Motivation:
  Why and how we study τ_1^b
- Theoretical Formalism
- Results and comparison with HERA data
- Summary

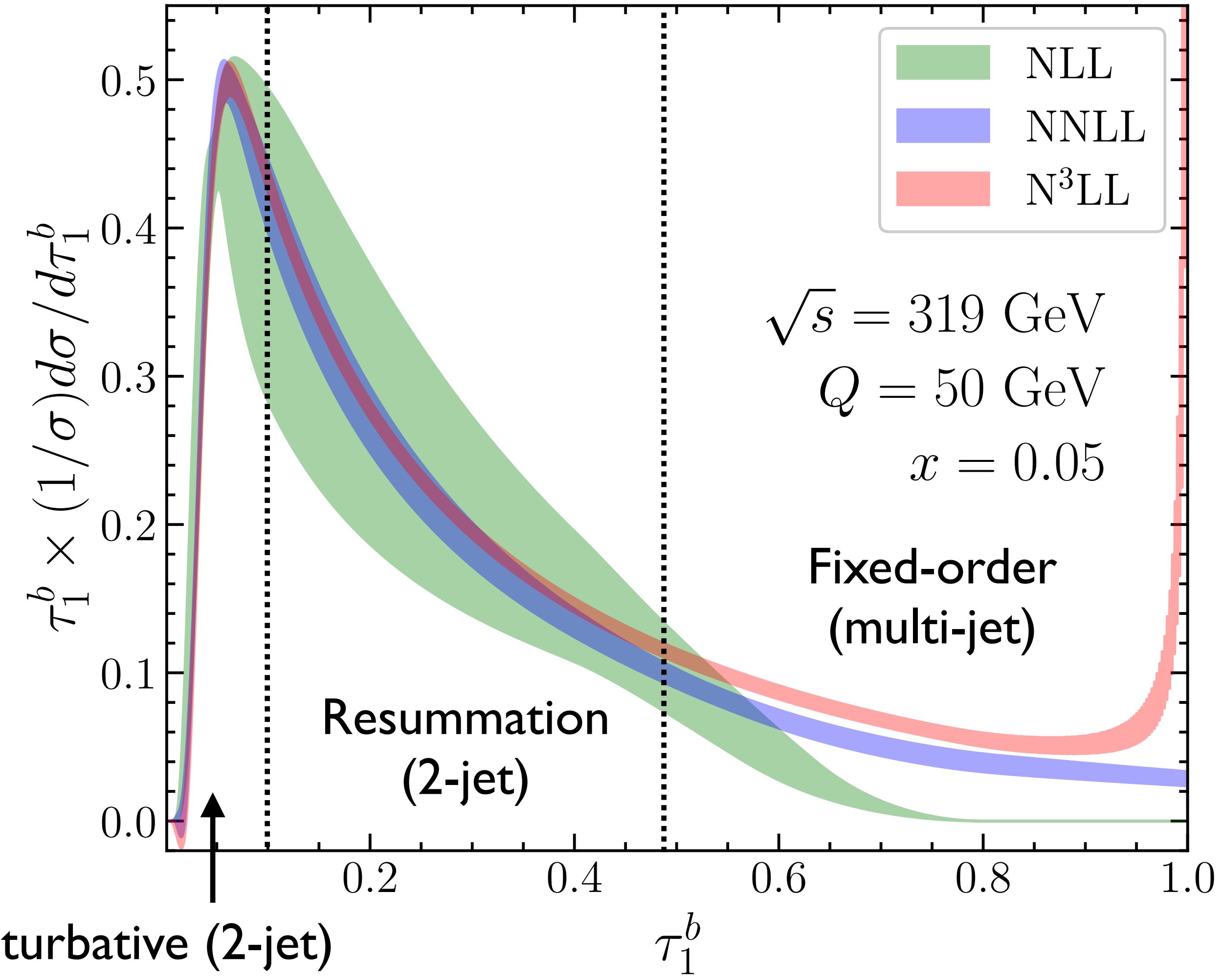
Background and Motivations:

Why do we study event shape τ_1^b?

Background and Motivation

- **Objective**: Accurate description of cross sections in DIS (ep) for jet production.
- **Observable**: DIS event shape τ_1^b, a special form of N-jettiness.
- **Method**: SCET-1 factorization theorem with N^{3}LL resummation, combined with two-loop fixed-order QCD corrections
- **Result**: Cross section expressed as a distribution in τ_1^b

This framework offers one of the most precise approaches to determining α_s and universal nonperturbative constant Ω_1 in DIS.

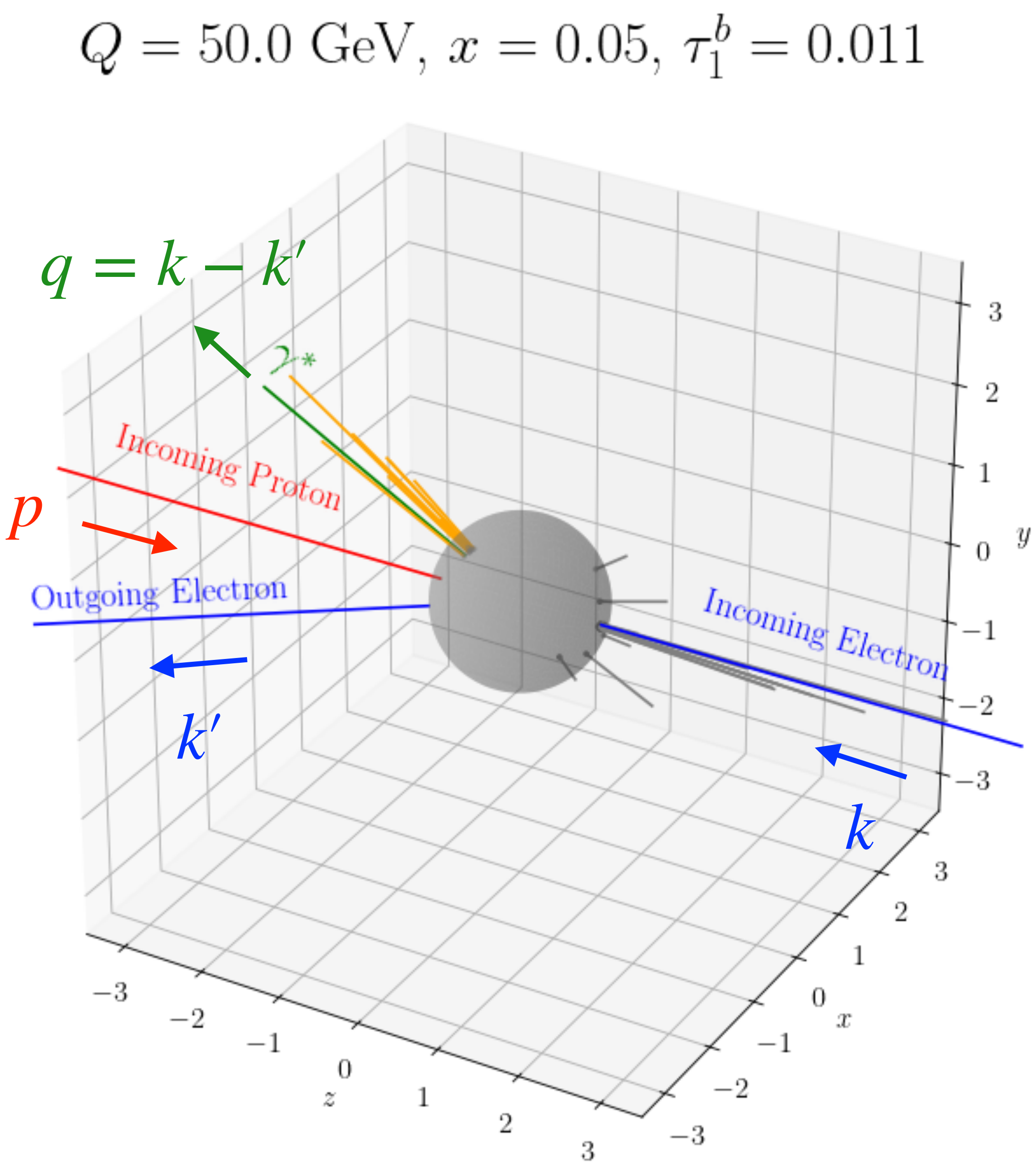


Non-perturbative (2-jet)

Definition of τ_1^b

- τ_1^b is defined using the momenta of the final states, p_i, projected onto one of the reference vectors q_B or q_J.

$$\tau_1^b = \frac{2}{Q^2}\sum_{i\in X}\min\left\{\underset{\substack{\downarrow\\ \mathscr{H}_B}}{q_B^b\cdot p_i},\ \underset{\substack{\downarrow\\ \mathscr{H}_J}}{q_J^b\cdot p_i}\right\}$$



$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 0.011$

- The reference vectors

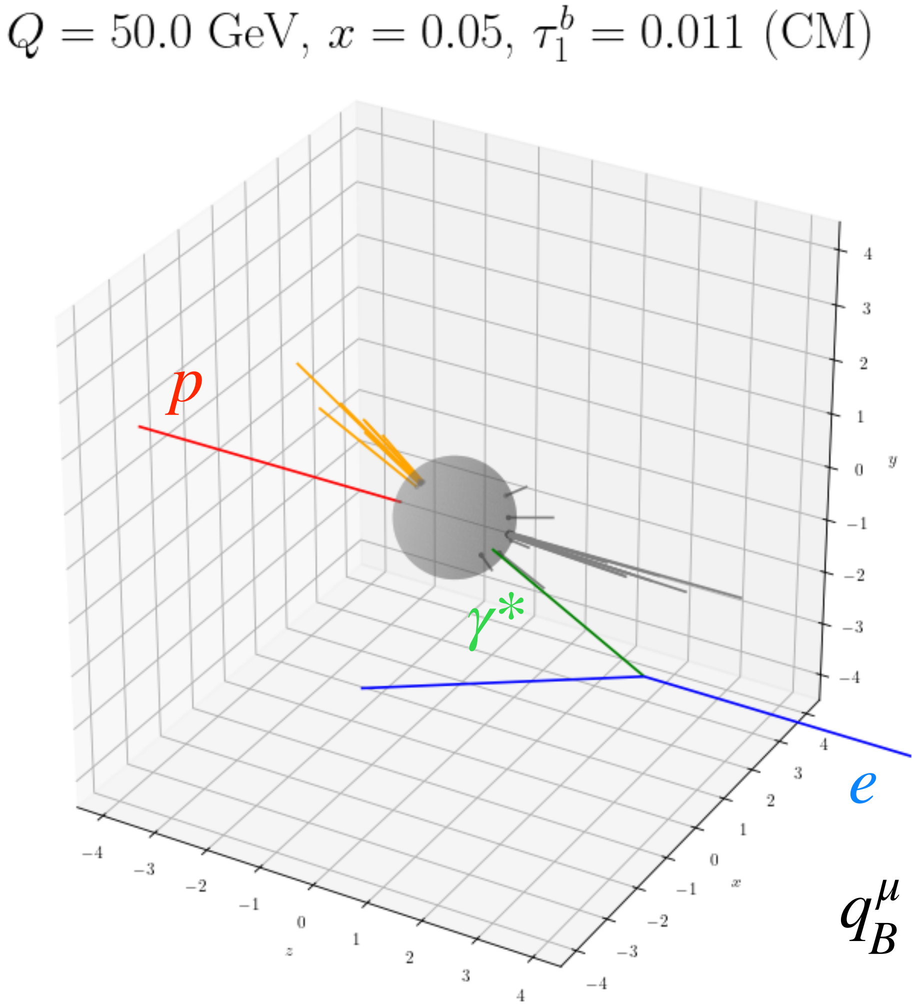
$$q_B^\mu = xP^\mu \qquad q_J^\mu = q^\mu + xP^\mu$$

- Once p_i is measured, τ_1^b can be computed straightforwardly.

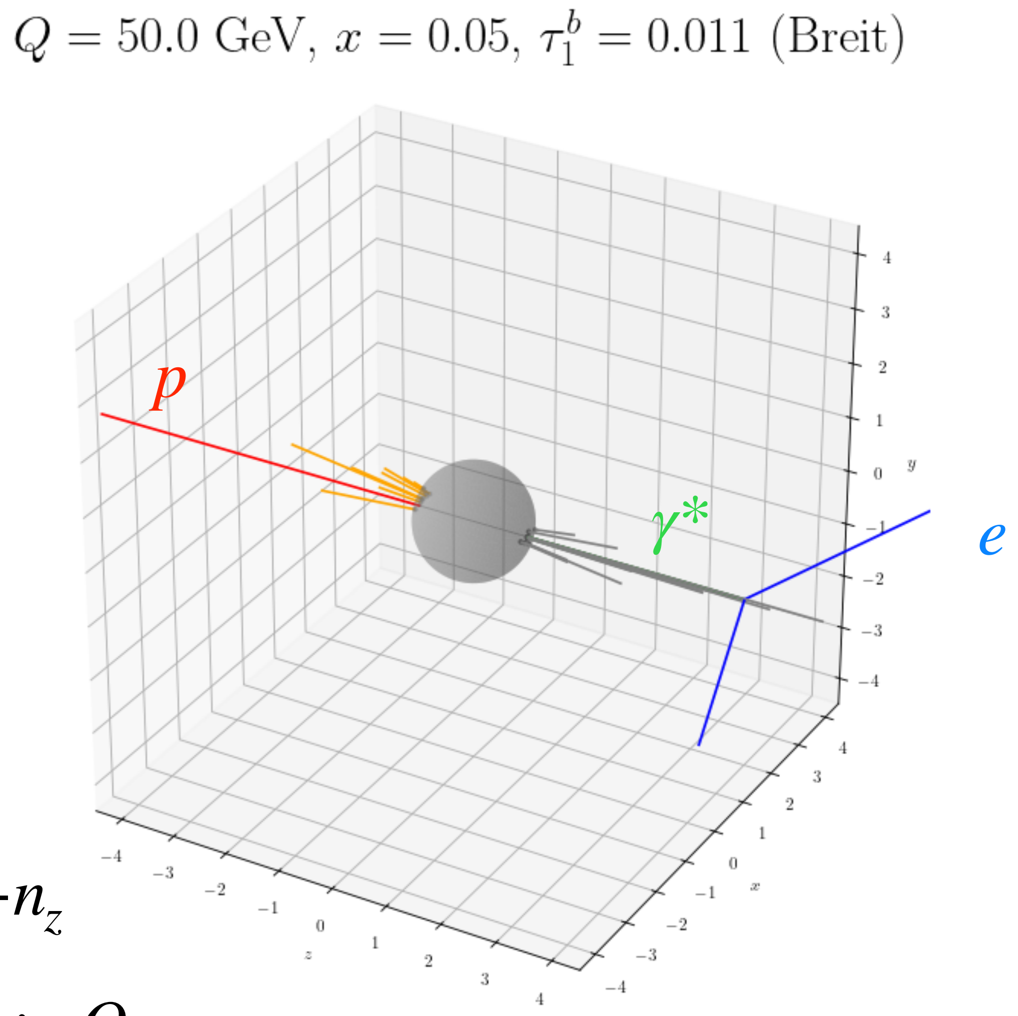
$$\boxed{Q^2 = -q^2 \quad x = \frac{Q^2}{2p\cdot q} \quad y = \frac{p\cdot q}{p\cdot k}}$$

τ_1^b in Breit frame

- τ_1^b has the most intuitive interpretation in Breit frame, where the off-shell photon and proton are aligned along z axis.



Breit frame $\longrightarrow$



$$q_B^\mu = xP^\mu \overset{\text{Breit}}{=} \frac{Q}{2} n_z$$

$$q_J^\mu = q^\mu + xP^\mu \overset{\text{Breit}}{=} \frac{Q}{2} \bar{n}_z$$

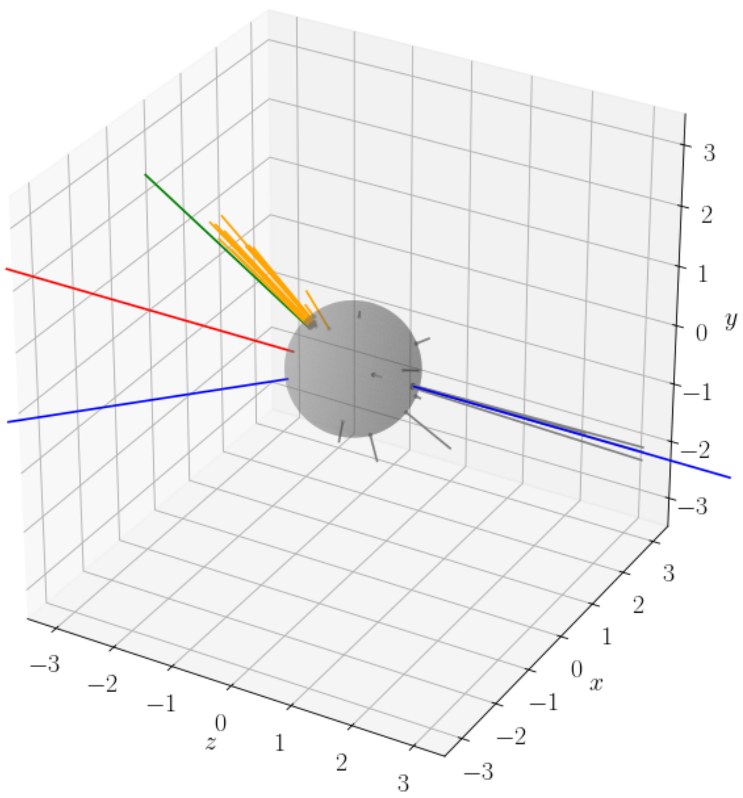
$$n_z = (1,0,0,1) \quad \bar{n}_z = (1,0,0,-1)$$

- In this frame, τ_1^b separates the final-state particles based on the z component of their momenta p_i.

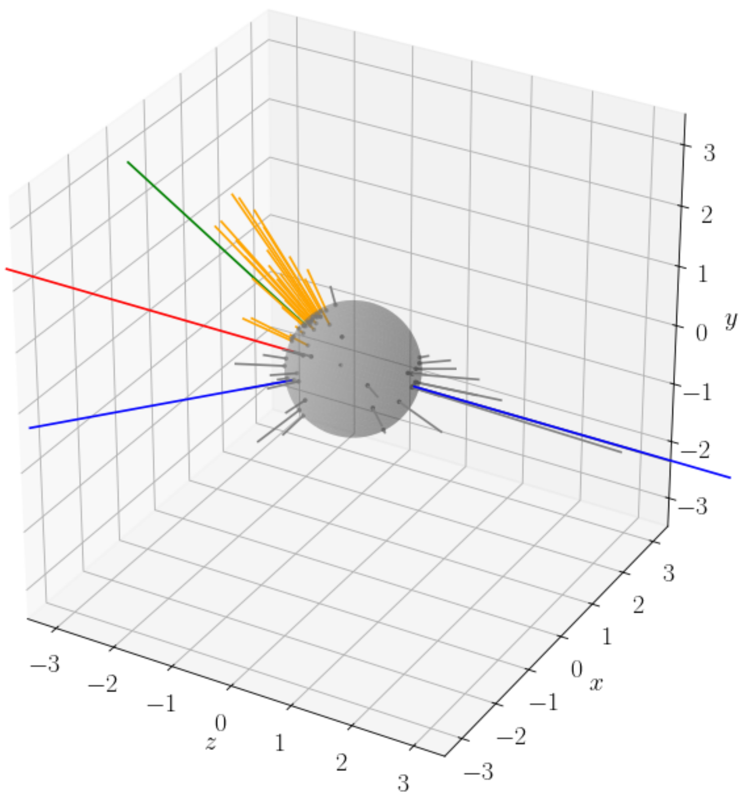
$$\tau_1^b \overset{\text{Breit}}{=} \frac{1}{Q} \sum_{i \in X} \min \left\{ n_z \cdot p_i, \bar{n}_z \cdot p_i \right\}$$

Event shape τ_1^b

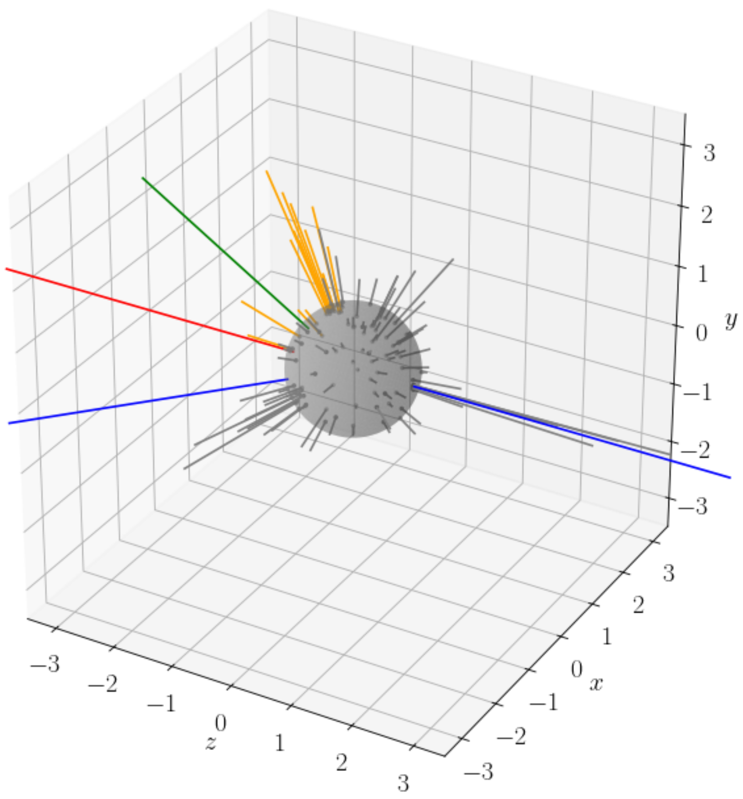
$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 0.038$



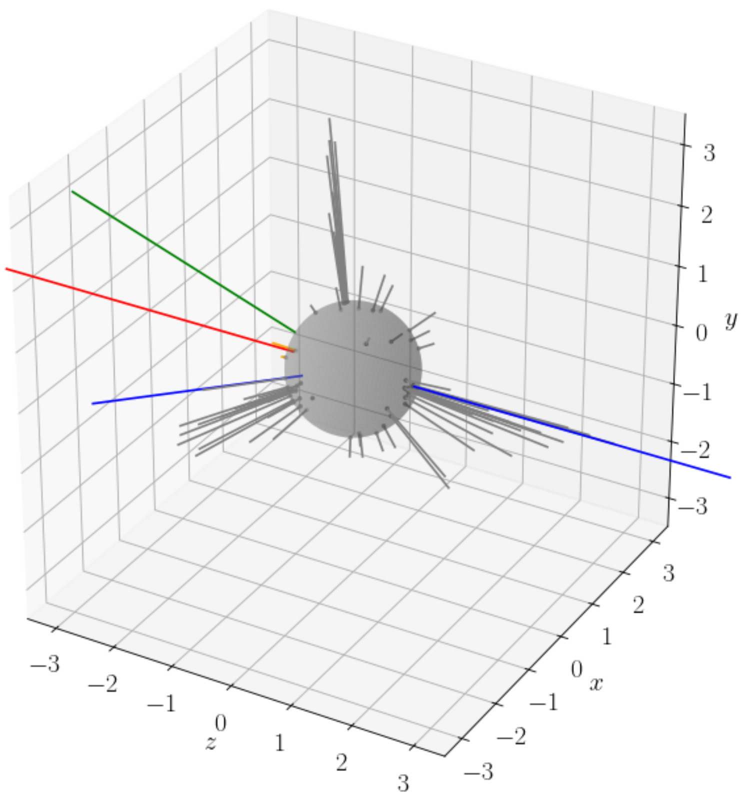
$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 0.195$



$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 0.715$



$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 0.998$



Advantages of τ_1^b

- τ_1^b agrees with the classical DIS thrust τ_Q:

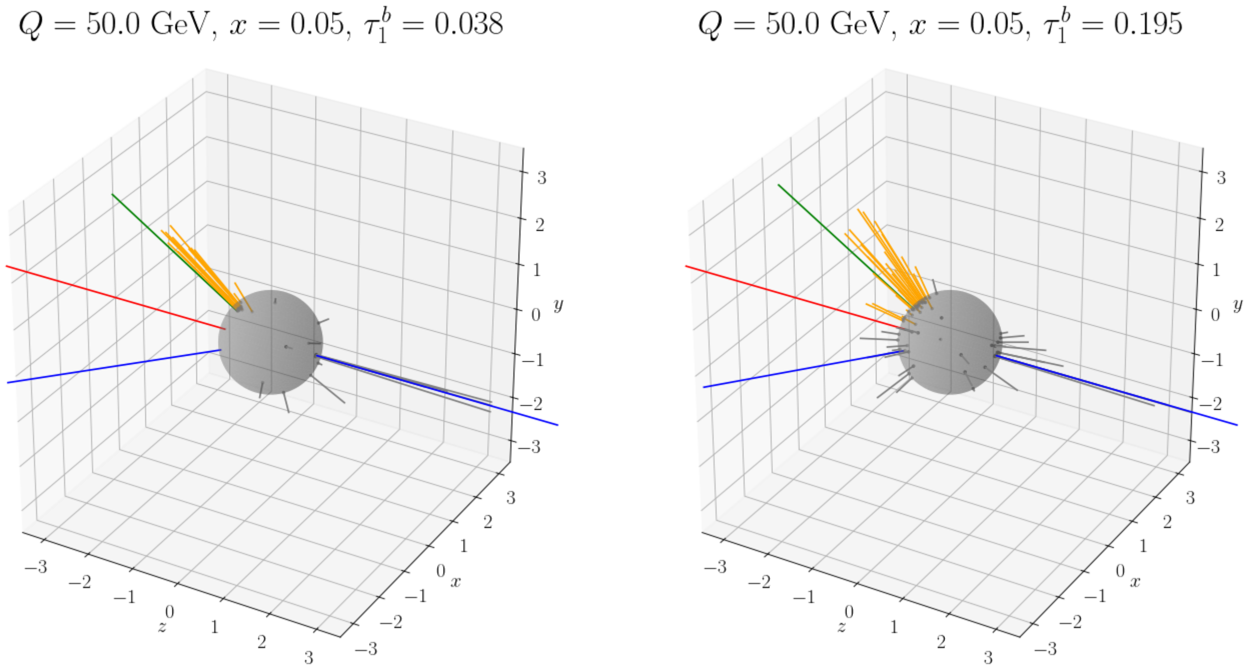
Energy-momentum conservation

classical DIS thrust variable τ_Q

arXiv:hep-ph/9912488
Antonelli, Dasgupta, Salam

$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q} \sum_{i \in \mathscr{H}_J} (p_i)_z = \tau_Q$$

- Reduces contamination from remnant fragmentation, making it highly **suitable for experimental studies.**


$Q = 50.0\ \mathrm{GeV},\ x = 0.05,\ \tau_1^b = 0.038$

$Q = 50.0\ \mathrm{GeV},\ x = 0.05,\ \tau_1^b = 0.195$

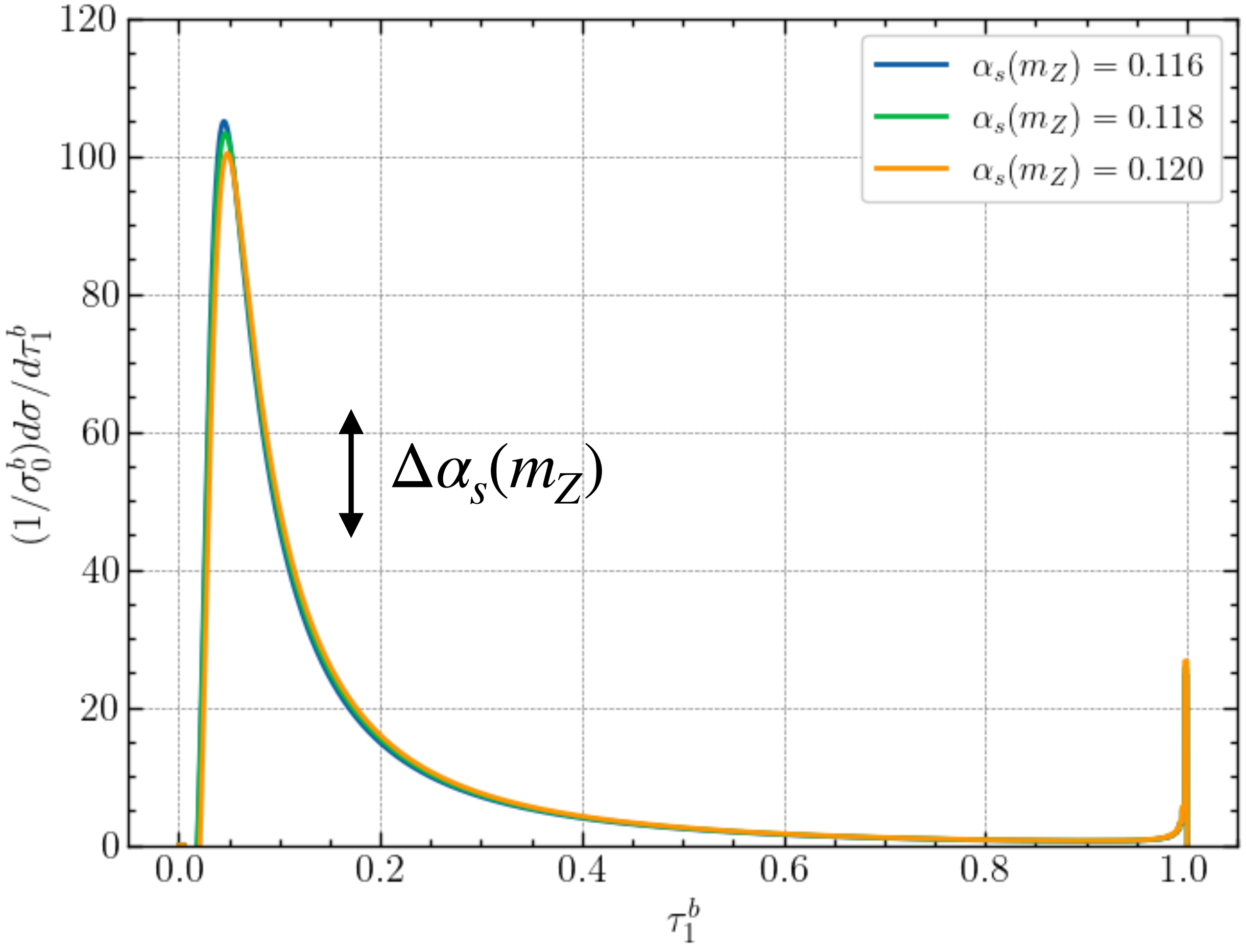
Can be determined by measuring only the momenta in $\mathscr{H}_J$ (orange)

- Lorentz invariant, and global observable, eliminating non-global logarithms (NGLs).
→ **Allows for precise theoretical predictions**

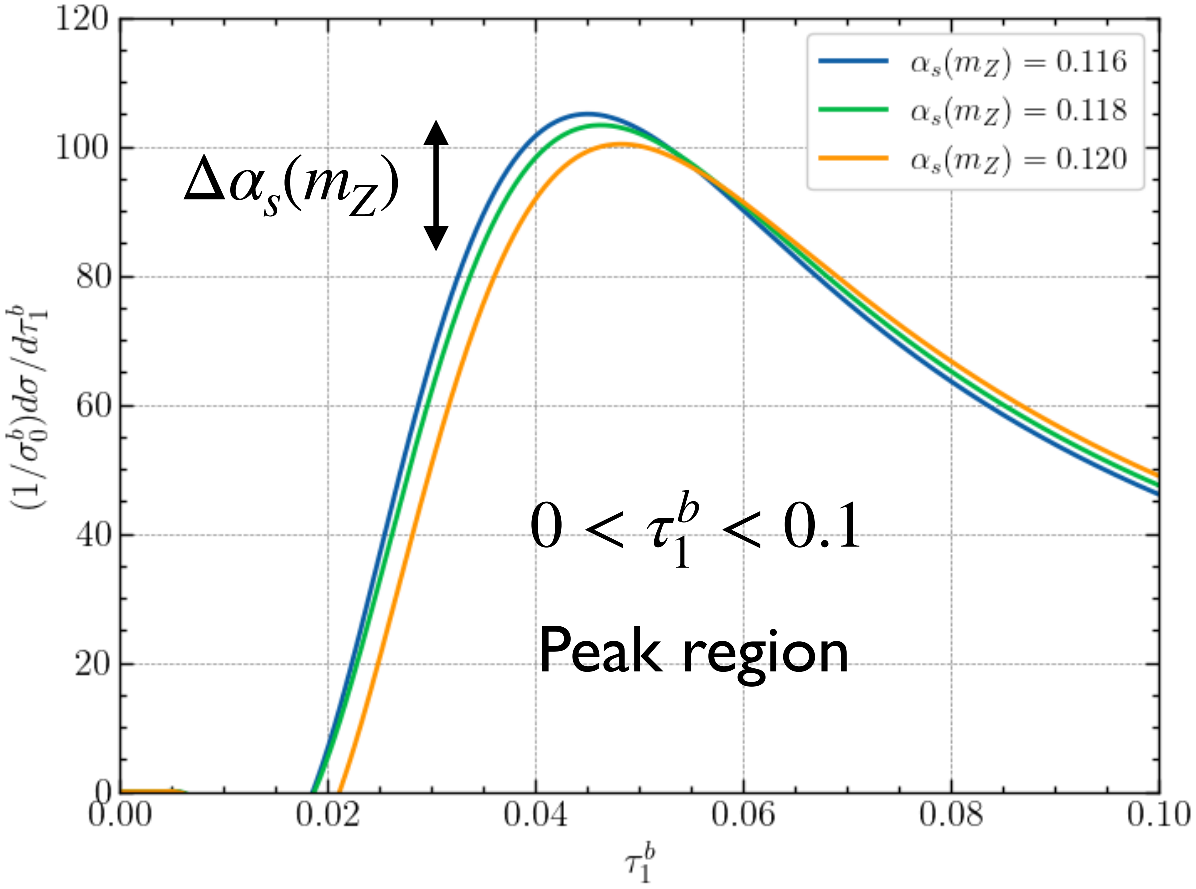
Event shapes and α_s, Ω_1

- Our ultimate goal is to determine the fundamental constants of QCD, $\alpha_s(m_Z)$ and universal nonperturbative constant Ω_1 (1st moment of the shape function):

- The shape of the τ_1^b distributions depend on the values of $\alpha_s(m_Z)$ and Ω_1:



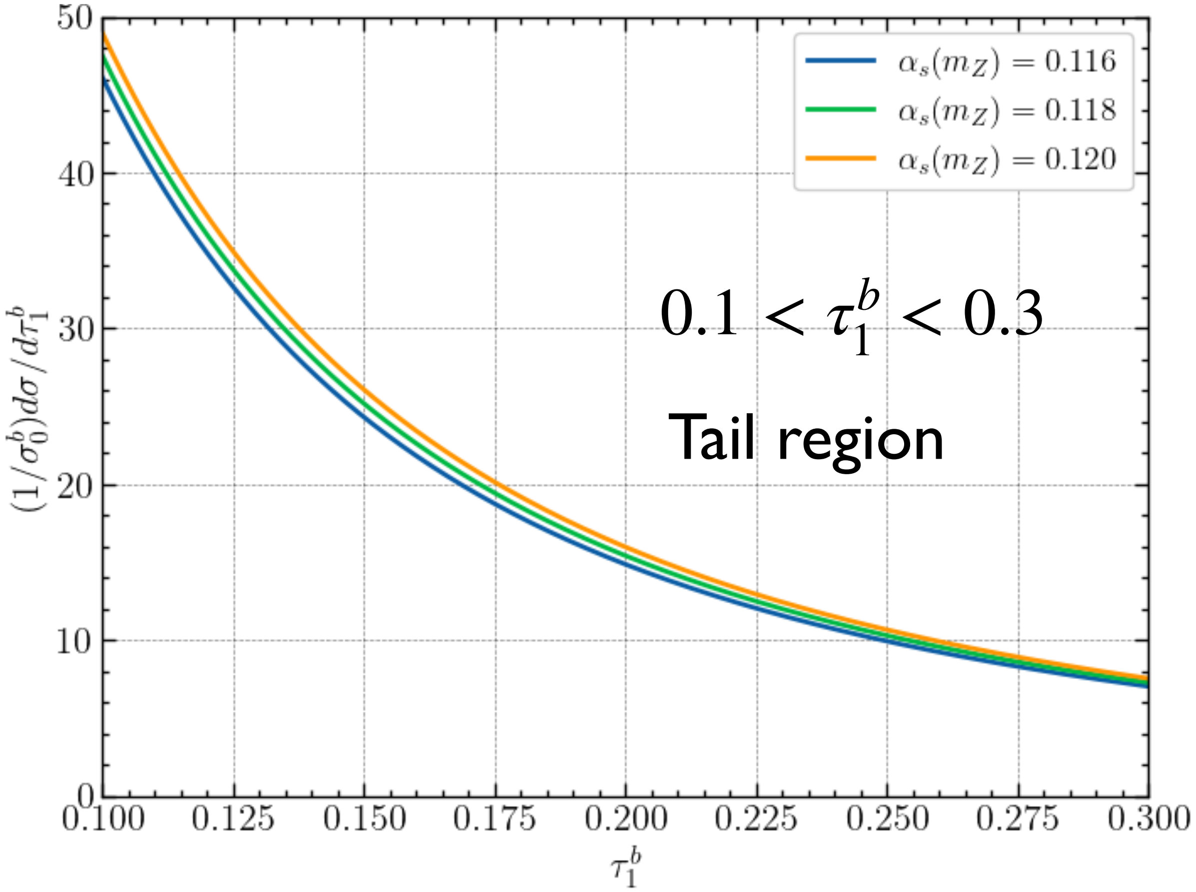
$\Delta\alpha_s(m_Z)$



$\Delta\alpha_s(m_Z)$

$0 < \tau_1^b < 0.1$

Peak region

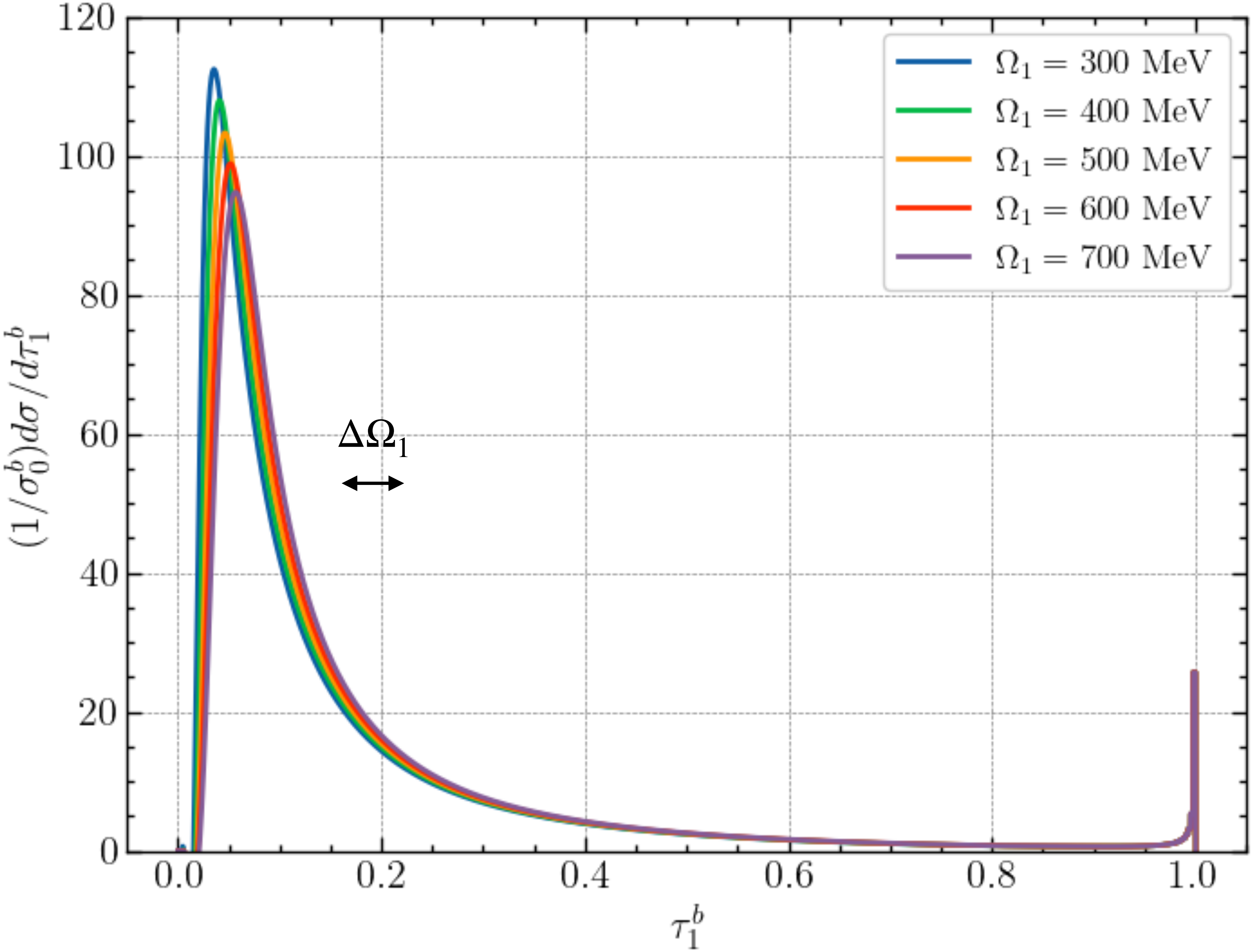


$0.1 < \tau_1^b < 0.3$

Tail region

Event shapes and α_s, Ω_1

- The shape of the τ_1^b distributions depends also on Ω_1:



Other DIS 1-jettiness work

- *$\tau_1^{a,b,c}$ and τ_1^b analytic1-loop correction*

arXiv:1303.6952, 1407.6706,
PoS(DIS2015)142
Kang, Lee, Stewart

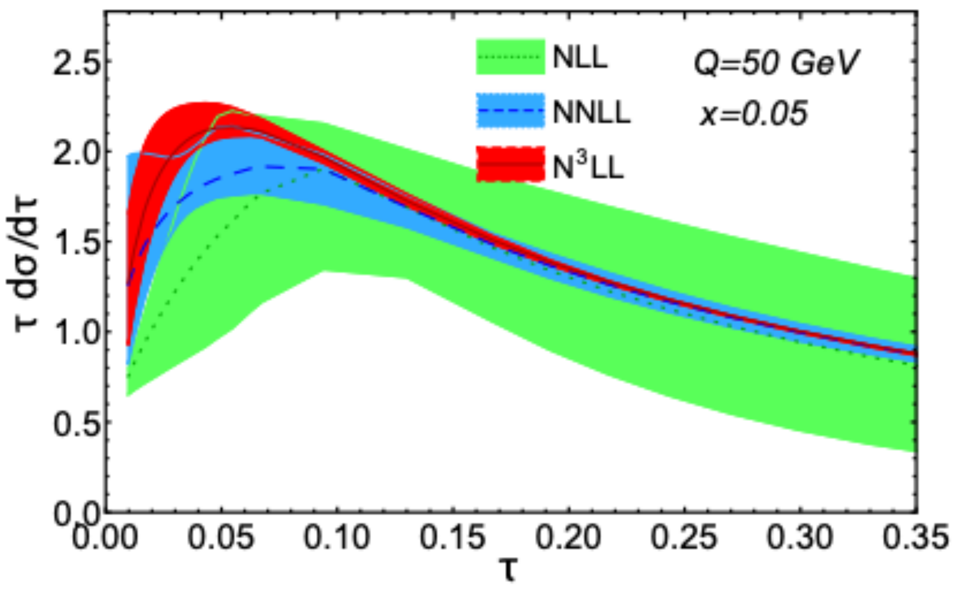
$$\tau_1^a = \frac{2}{Q^2} \sum_{i \in X} \min\{q_B^a \cdot p_i, q_J^a \cdot p_i\}$$

$$q_B^{a\,\mu} = xP^\mu\,, \qquad q_J^{a\,\mu} = q^\mu + xP^\mu + q_J^{\perp\,\mu}$$

$$q_B^{b\,\mu} = xP^\mu\,, \quad q_J^{b\,\mu} = q^\mu + xP^\mu$$

$$q_B^{c\,\mu} = P^\mu\,, \qquad q_J^{c\,\mu} = k^\mu$$

- Up to N3LL resummation + 1-loop corrections



- *The DIS 1-jettiness Event Shape at N³LL + $\mathcal{O}(\alpha_s^2)$*

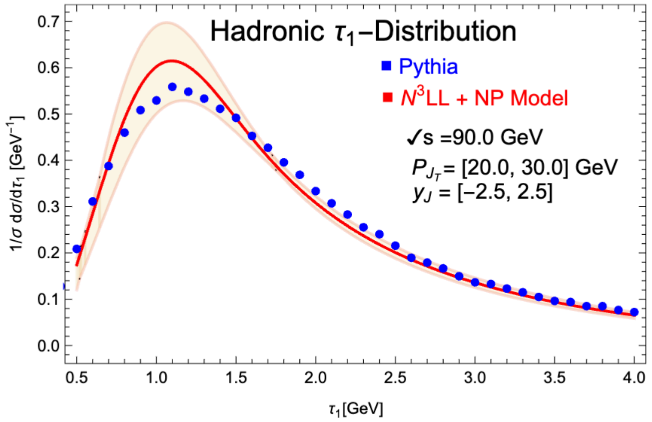
arXiv:2401.01941
Cao, Z. Kang, Liu, Mantry

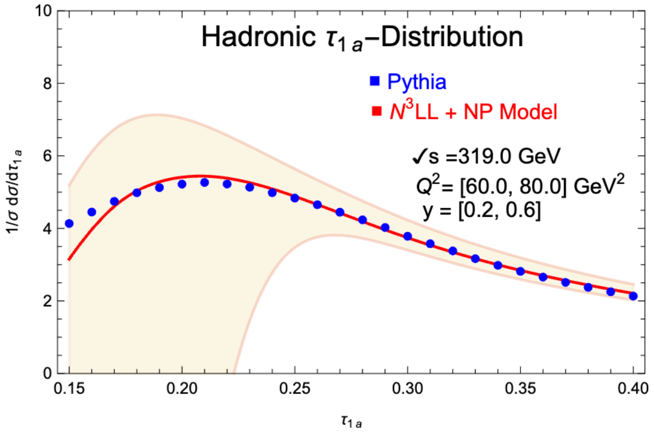
- Same theoretical accuracy, but different type of DIS 1-jettiness (depends on the jet algorithm)

$$\tau_1 = \sum_k \min\left\{\frac{2q_B \cdot p_k}{Q_B}, \frac{2q_J \cdot p_k}{Q_J}\right\}$$

$$q_B = xP, \quad q_J = (K_{J_T} \cosh y_K, \vec{K}_{J_T}, K_{J_T} \sinh y_K).$$

$$Q_B = x\sqrt{s}. \qquad Q_J = 2K_{J_T} \cosh y_K$$





Theoretical Formalism

Theoretical Formalism

- In this work, we compute the τ_1^b distribution as follows:

$$\sigma(\tau_1^b) = \int dk \left[\sigma_{\mathrm{PT}}^{\mathrm{s}} + \sigma_{\mathrm{PT}}^{\mathrm{ns}}\right]\left(\tau_1^b - \frac{k}{Q}\right)\left[e^{-2\delta(R,\mu_S)(d/dk)} F\left(k - 2\Delta(R,\mu_S)\right)\right]$$

- $\sigma_{\mathrm{PT}}^{\mathrm{s}}$: Singular contribution (Leading Power in SCET)

  Represents two-jet events, including all-order log resummation at N^{3}LL level

- $\sigma_{\mathrm{PT}}^{\mathrm{ns}}$: Nonsingular contribution (Power Suppressions)

  Represents multi-jet events, estimated using full-QCD fixed-order up to $\mathcal{O}(\alpha_s^2)$

- $e^{-2\delta(R,\mu_S)(d/dk)} F\left(k - 2\Delta(R,\mu_S)\right)$: Nonperturbative hadronization corrections

  Incorporates the nonperturbative shape function F, and employs R-gap scheme to subtract $\mathcal{O}(\Lambda_{\mathrm{QCD}})$ renormalon ambiguity.

$\sigma_{\mathrm{PT}}^{\mathrm{S}}$: Singular contribution

- The SCET factorization formula for τ_1^b distribution is given by

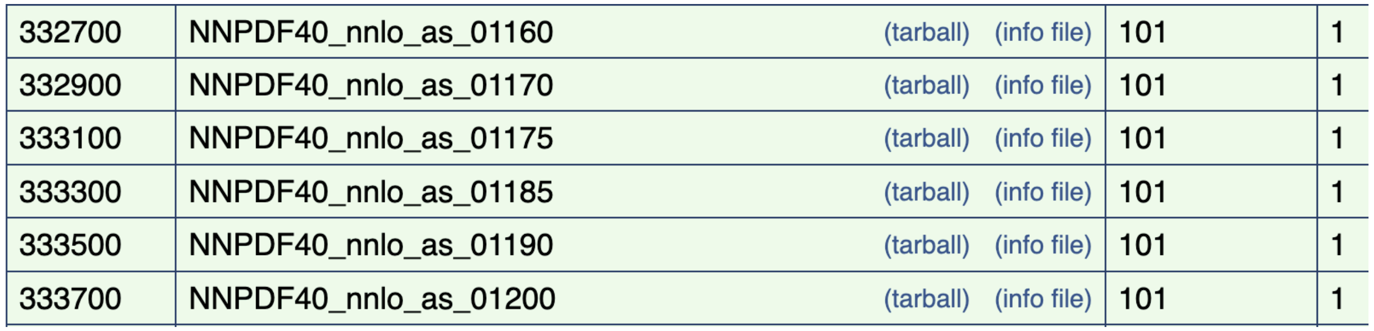
arXiv:1303.6952
Kang, Lee, Stewart

$$\frac{d\sigma}{dx dQ^2 d\tau_1^b} = \frac{d\sigma_0^b}{dx dQ^2} \int dt_J dt_B dk_S \, \overbrace{\delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right)}^{\text{Measurement function for } \tau_1^b} \overbrace{S(k_S, \mu)}^{\text{Single variable soft function}}$$

$$\times \int d^2\mathbf{p}_\perp \underbrace{J_q(t_J - \mathbf{p}_\perp^2, \mu)}_{\text{Quark jet function}} \Big[\underbrace{H_q^b(y, Q^2, \mu)}_{\text{Hard function}} \underbrace{\mathscr{B}_q(t_B, x, \mathbf{p}_\perp^2, \mu)}_{\text{Quark beam function}} + (q \to \bar{q}) \Big],$$

where Born-level cross section $\frac{d\sigma_0^b}{dx dQ^2} = \frac{2\pi\alpha_{\text{em}}^2}{Q^4}\left[(1-y)^2 + 1\right]$ (Note that $Q^2 = sxy$)

- τ_1^b **quark beam function**: $\mathscr{B}_i(t, x, \mathbf{k}_\perp^2, \mu) = \mathscr{I}_{ij}(t, x/\xi, \mathbf{k}_\perp^2, \mu) \otimes_\xi \underbrace{f_j(\xi, \mu)}_{\text{PDF for parton } j}$



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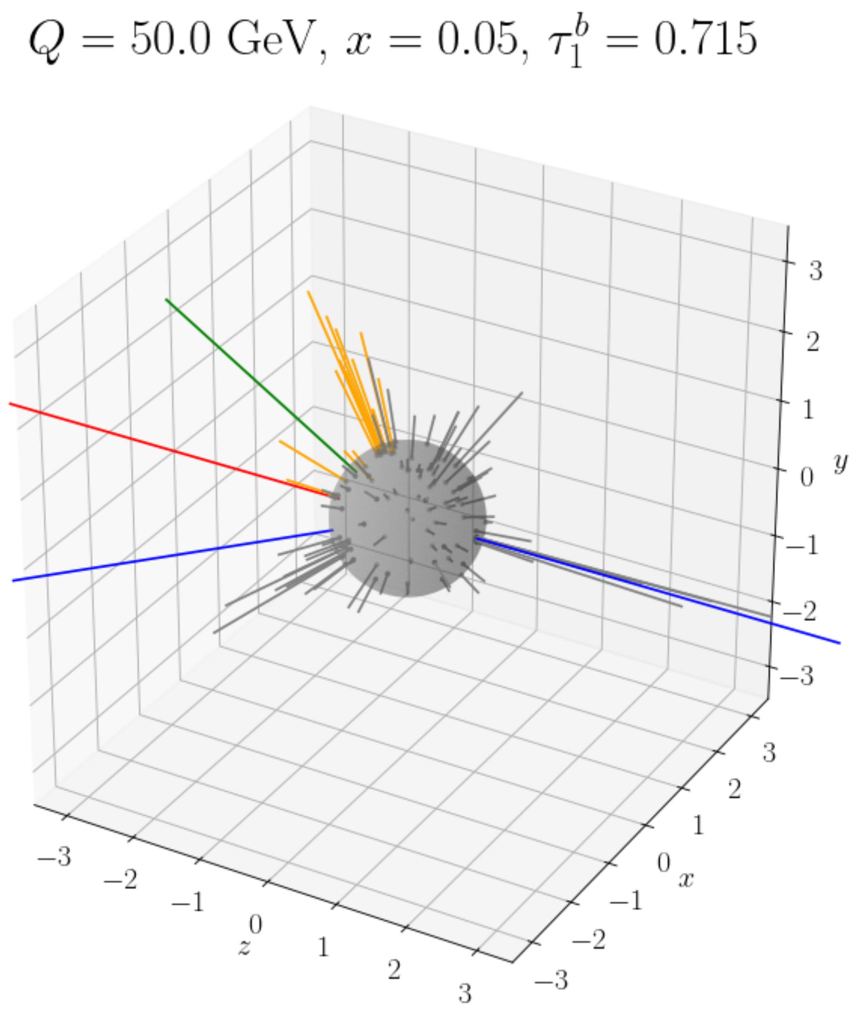
333700	NNPDF40_nnlo_as_01200	(tarball) (info file)	101	1

$\sigma_{\mathrm{PT}}^{\mathrm{ns}}$: Nonsingular contribution



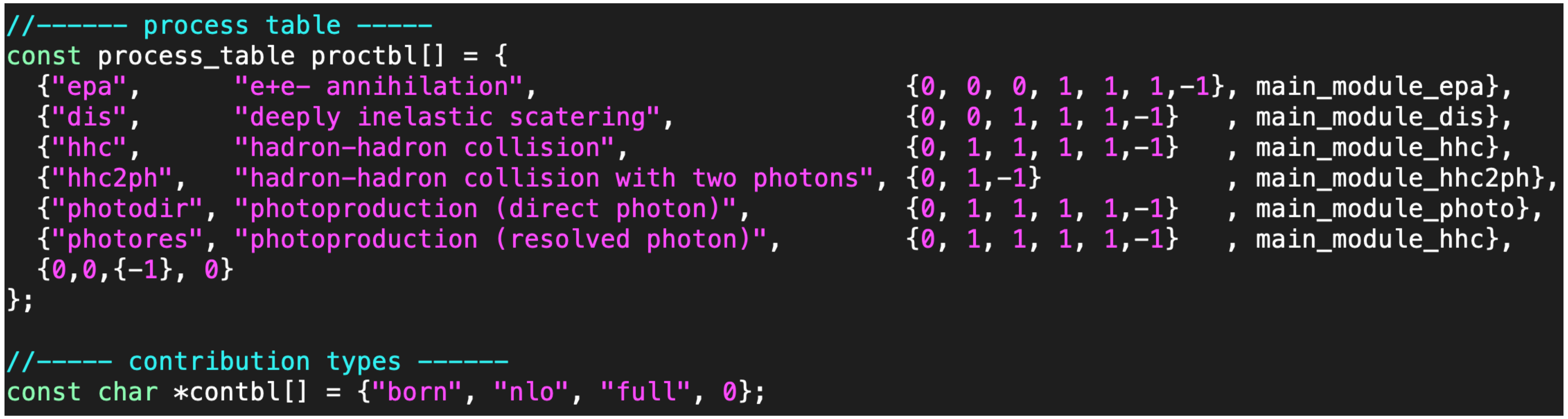
- Nonsingular contributions from fixed-order full QCD calculations:

$$\frac{d\sigma_{\mathrm{ns}}}{d\tau_1^b} = \frac{d\sigma_{\mathrm{QCD}}}{d\tau_1^b} - \frac{d\sigma_{\mathrm{s}}}{d\tau_1^b}$$

LO nonsingular:
arXiv:1407.6706
Kang, Lee, Stewart

- NLOJet++ is the C++ program for calculating LO and NLO QCD jet cross sections based on Catani-Seymour dipole subtraction method. (Author: Zoltan Nagy at DESY)

arXiv:hep-ph/0307268
Nagy



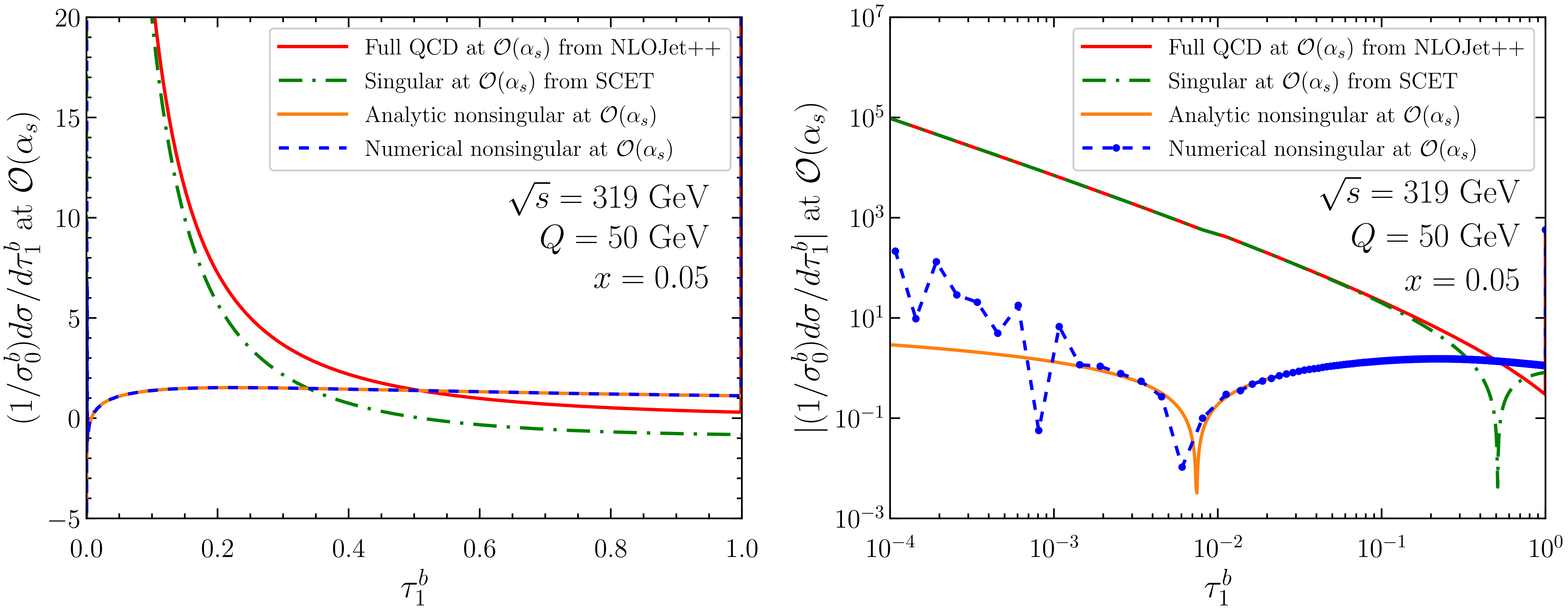
```
//------ process table -----
const process_table proctbl[] = {
  {"epa",      "e+e- annihilation",                        {0, 0, 0, 1, 1, 1,-1}, main_module_epa},
  {"dis",      "deeply inelastic scatering",               {0, 0, 1, 1, 1,-1}   , main_module_dis},
  {"hhc",      "hadron-hadron collision",                  {0, 1, 1, 1, 1,-1}   , main_module_hhc},
  {"hhc2ph",   "hadron-hadron collision with two photons", {0, 1,-1}            , main_module_hhc2ph},
  {"photodir", "photoproduction (direct photon)",          {0, 1, 1, 1, 1,-1}   , main_module_photo},
  {"photores", "photoproduction (resolved photon)",        {0, 1, 1, 1, 1,-1}   , main_module_hhc},
  {0,0,{-1}, 0}
};

//----- contribution types ------
const char *contbl[] = {"born", "nlo", "full", 0};
```

- e^+e^-, ep, pp and photo production processes.

$\sigma^{\mathrm{ns}}_{\mathrm{PT}}$: Nonsingular at LO

Analytic 1-loop
nonsingular

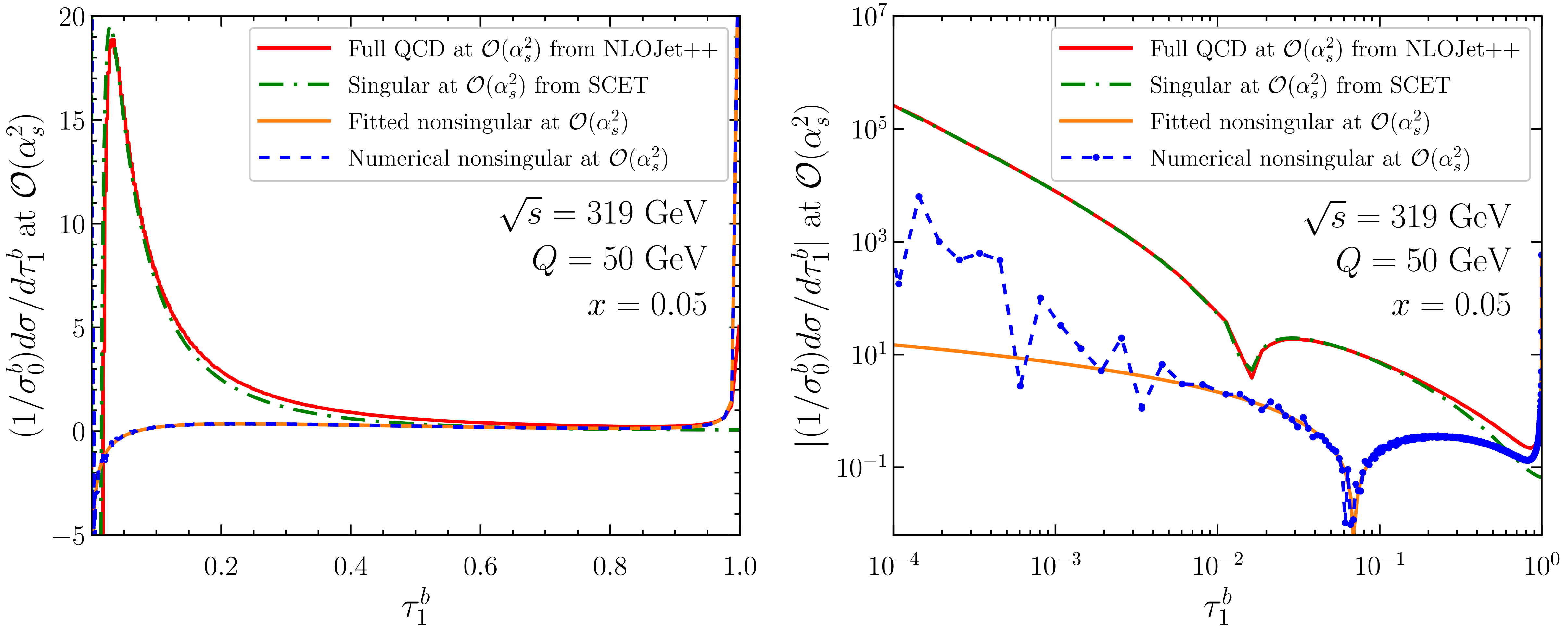
arXiv:1407.6706
Kang, Lee, Stewart



- From NLOJet++, we obtain the fixed-order 1-loop full QCD cross section (red) and we subtract the SCET singular contribution (green) to obtain the nonsingular contribution (orange). (Remove double counting)

- The analytic results are available at 1-loop (orange) and the numerical results from NLOJet++ agree well with it before the numerical noise kicks in for very small τ_1^b.

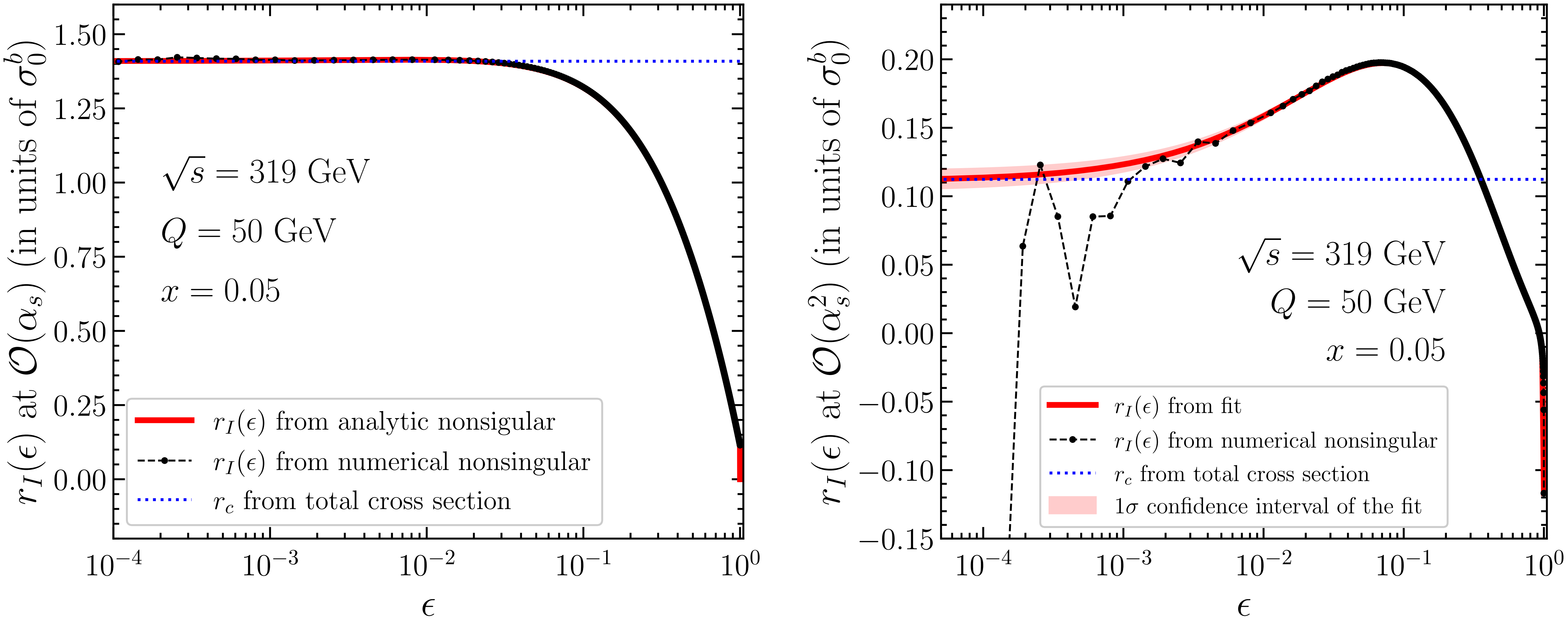
$\sigma^{\mathrm{ns}}_{\mathrm{PT}}$: Nonsingular at NLO



- At 2-loop, there are no analytic results, so we should use NLOJet++ results (blue).
- The numerical results are not reliable below $\tau_1^b \sim 10^{-2}$, so instead of using the numerical results directly, we use the weighted fit of the numerical results using the following fit function (orange):

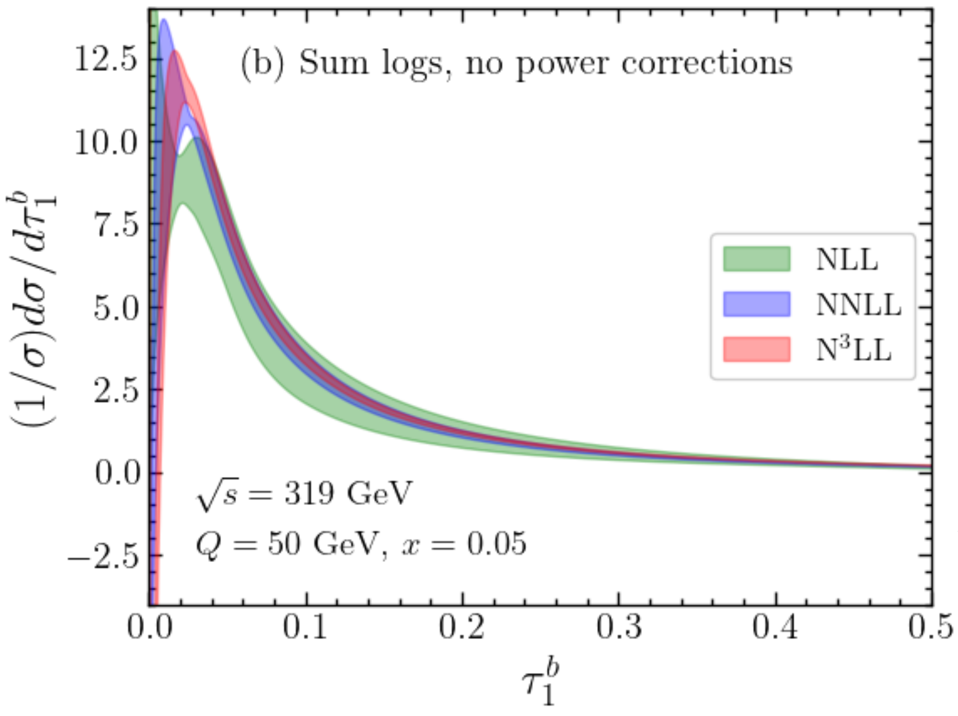
$$\left.\frac{d\sigma^{\mathrm{ns}}}{d\tau_1^b}\right|_{\mathrm{fit}} = a_0 + a_1 \log\tau_1^b + a_2 \log^2\tau_1^b + a_3 \log^3\tau_1^b + b_3 \tau_1^b \log^3\tau_1^b \qquad \chi^2/\text{d.o.f.} = 1.02$$

$\sigma^{\mathrm{ns}}_{\mathrm{PT}}$: Check of nonsingular



NP corr.: Shape function

$$S(k,\mu) = \int dk'\, S_{\text{pert}}(k-k',\mu)F(k') \quad \longrightarrow \quad \frac{d\sigma}{d\tau_1^b}(\tau_1^b) = \int dk\, \frac{d\sigma_{\text{pert}}}{d\tau_1^b}\left(\tau_1^b - \frac{k}{Q}\right) F(k)$$

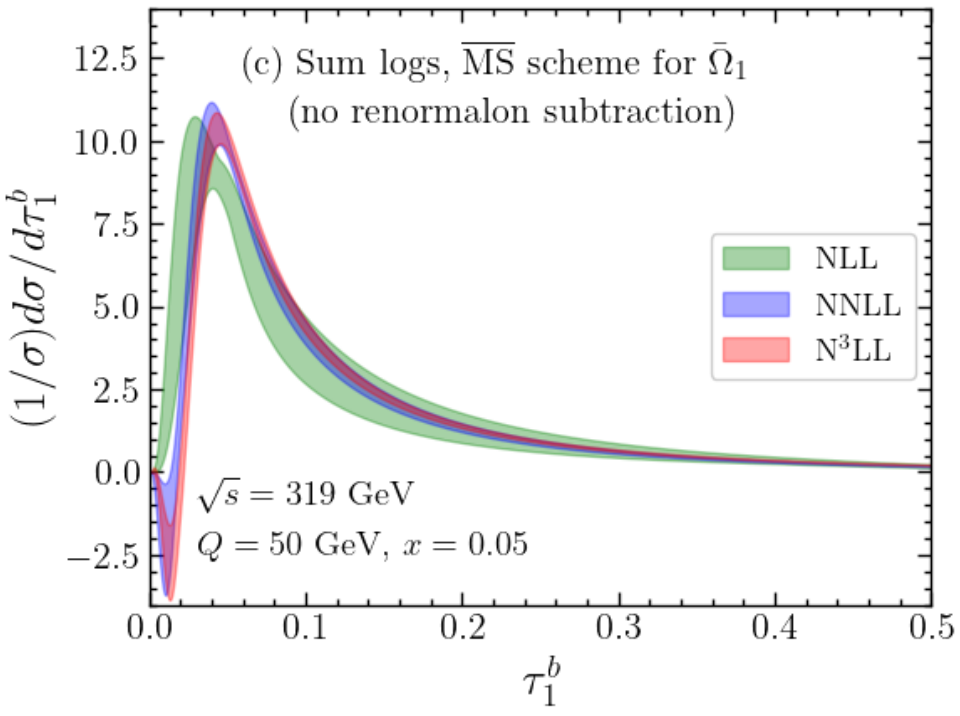


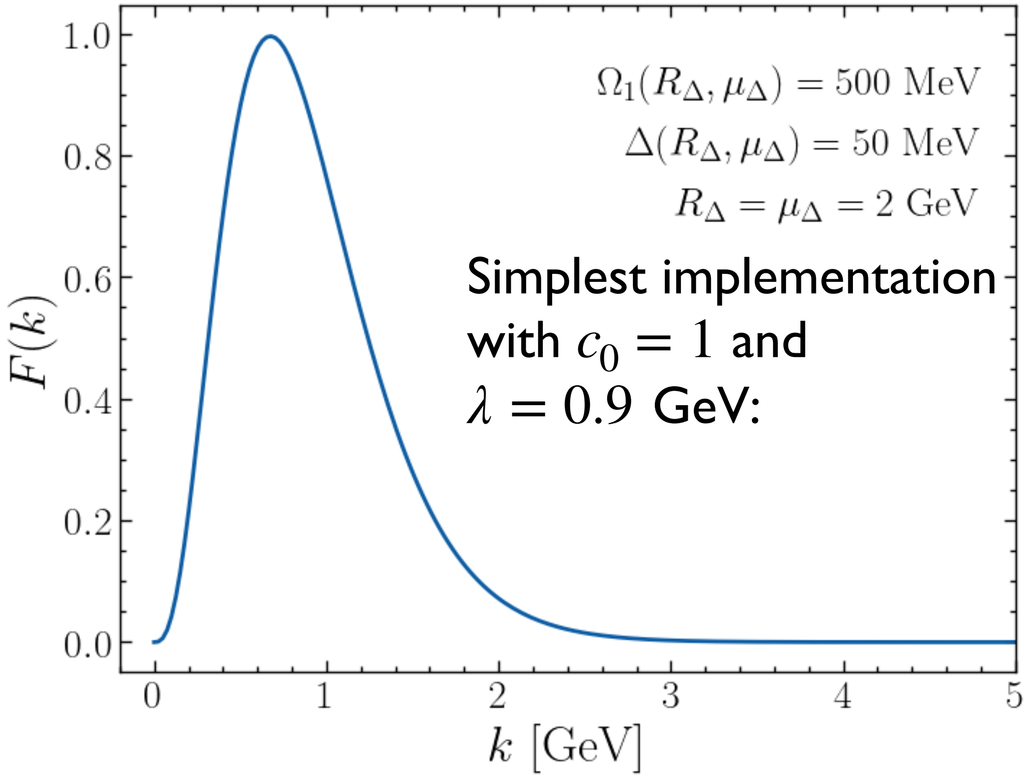
- According to OPE,

$$\frac{d\sigma}{d\tau_1^b}(\tau_1^b) = \left\{ \frac{d\sigma_{\text{pert}}(\tau_1^b)}{d\tau_1^b} - \frac{2\Omega_1}{Q}\frac{d\sigma^2_{\text{pert}}(\tau_1^b)}{d\tau_1^{b2}} \right\}\left[1 + \mathcal{O}(\Lambda_{\text{QCD}}/(\tau_1^b Q))\right]$$

$2\Omega_1$ is the **first moment** of $F(k)$: $2\Omega_1 = \int dk\, kF(k)$

In the tail region,
$\tau_1^b \to \tau_1^b - 2\Omega_1/Q$
(translation!)



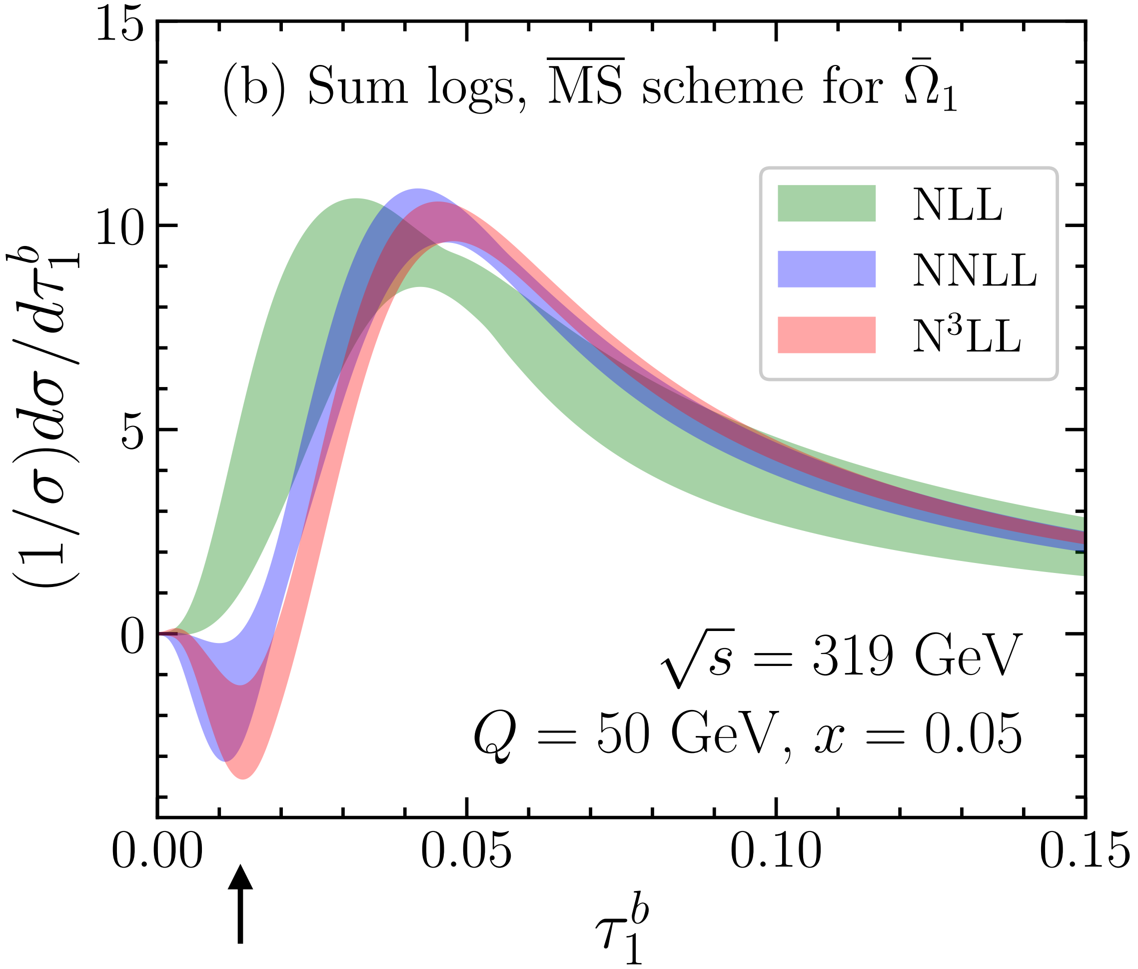


Simplest implementation
with $c_0 = 1$ and
$\lambda = 0.9$ GeV:

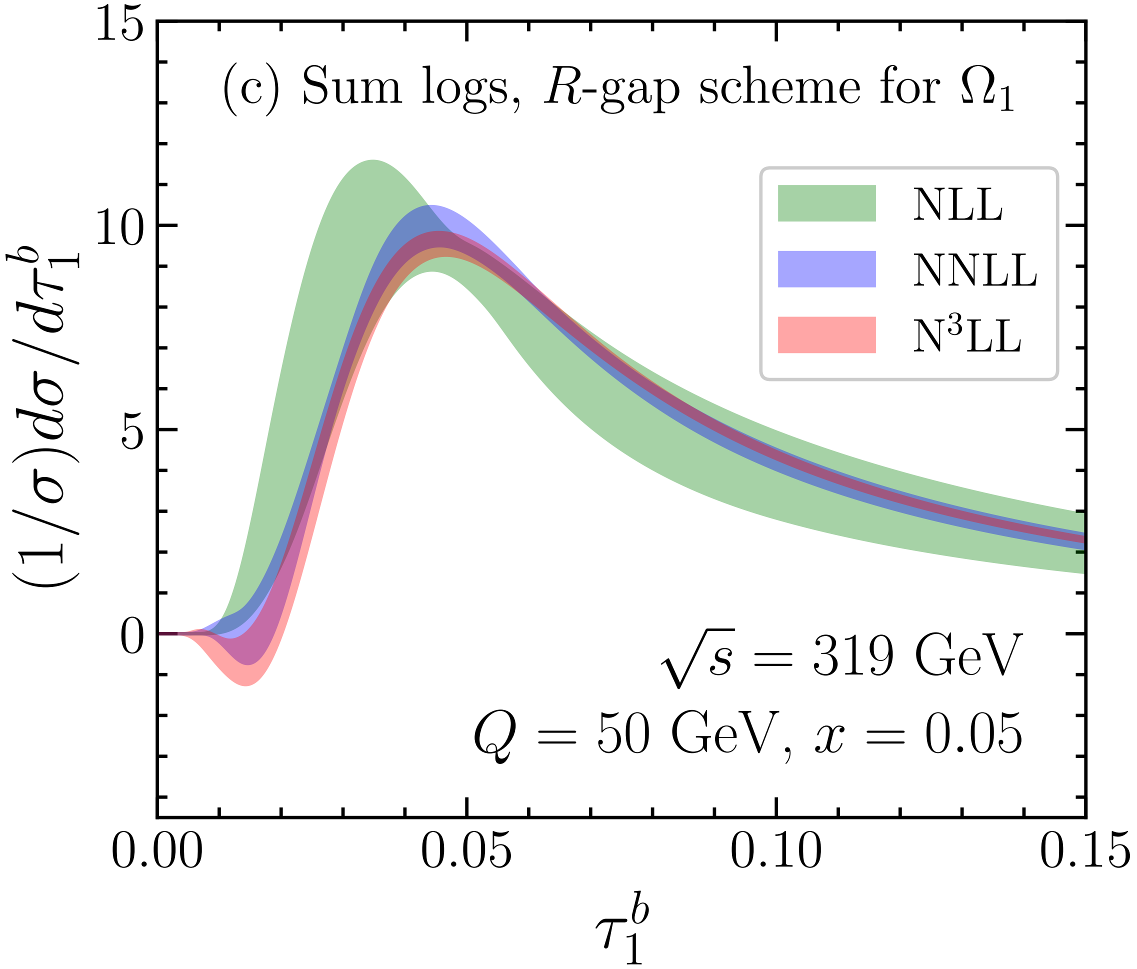
NP corr.: Renormalon ambiguity

- We employ the R-gap scheme introduced in [arXiv:0806.3852, Hoang, Kluth].

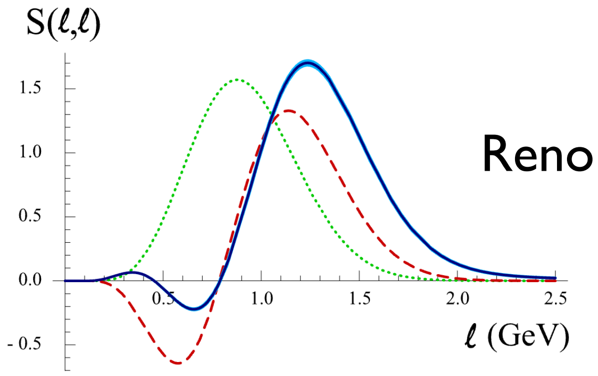
Before renormalon subtraction



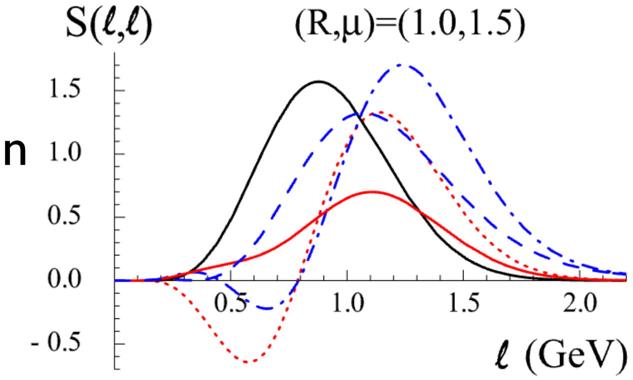
After renormalon subtraction
(R-gap scheme)



$\mathcal{O}(\Lambda_{\text{QCD}})$ renormalon ambiguity



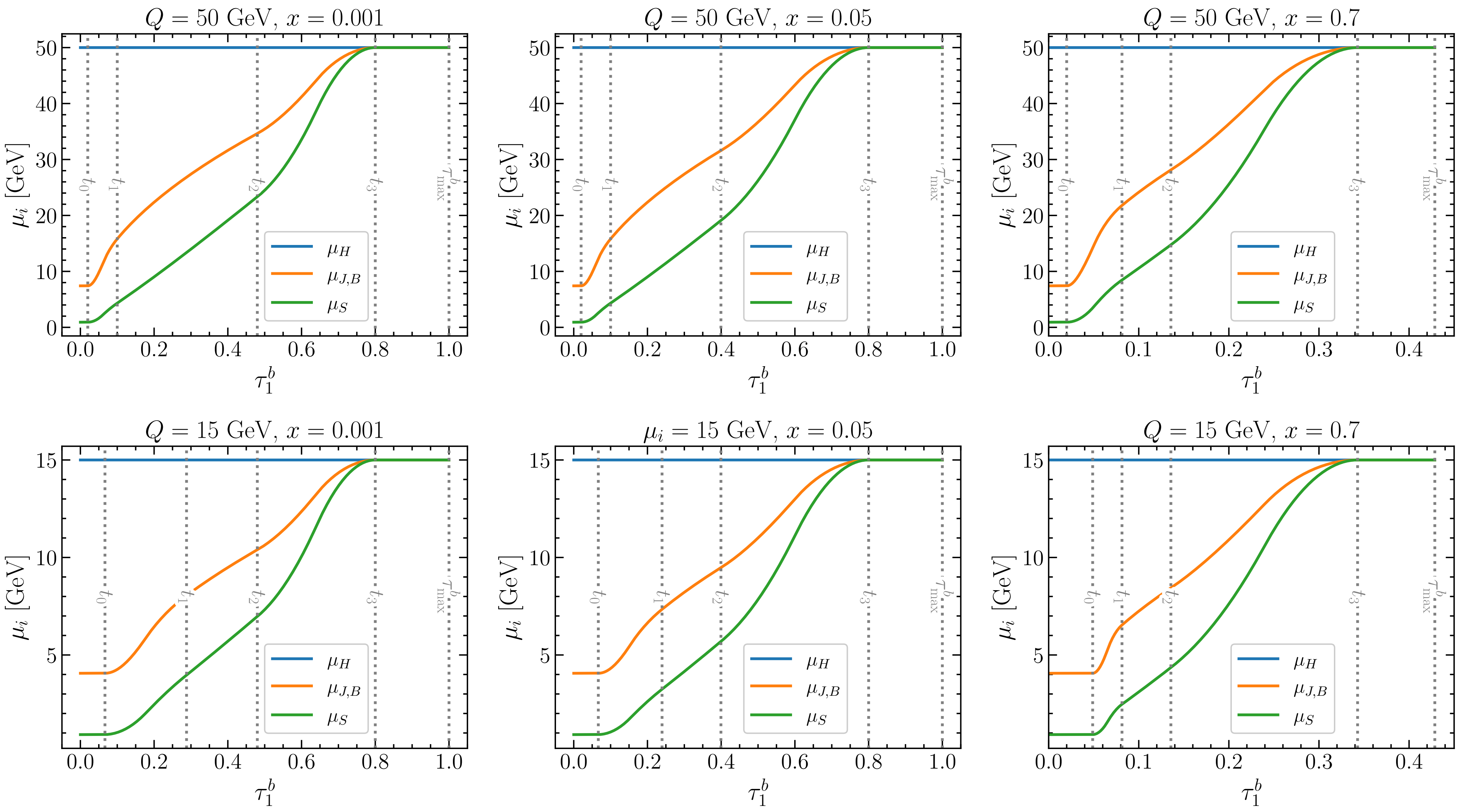
Renormalon subtraction



Profile functions for $\sigma^{\mathrm{S}}_{\mathrm{PT}}$

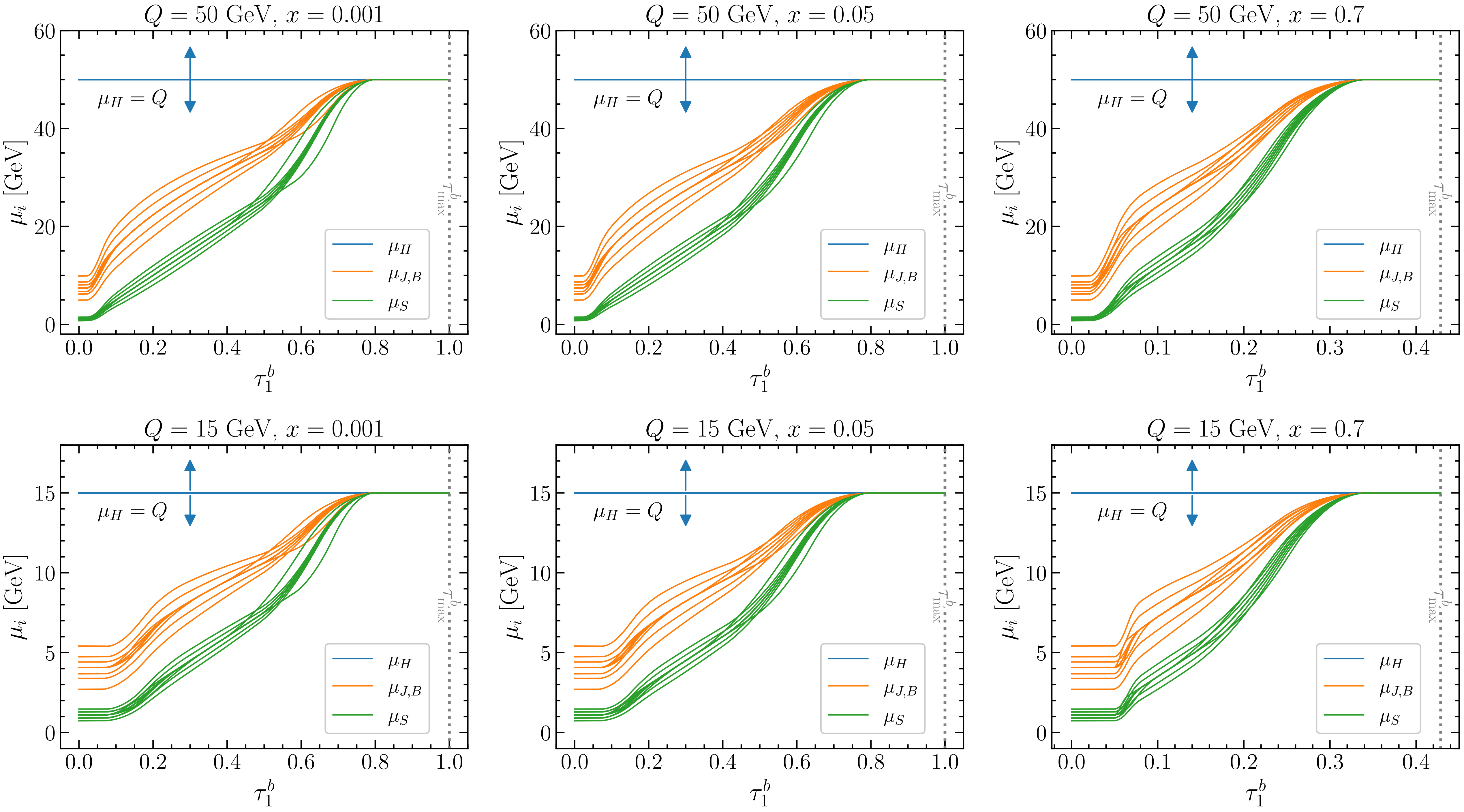
$$\frac{d\sigma}{dx\,dQ^2\,d\tau_1^b} = \frac{d\sigma_0^b}{dx\,dQ^2}\int dt_J\,dt_B\,dk_S\,\delta\left(\tau_1^b - \frac{t_J}{Q^2} - \frac{t_B}{Q^2} - \frac{k_S}{Q}\right) S(k_S,\mu)$$

$$\times\int d^2\mathbf{p}_\perp J_q(t_J - \mathbf{p}_\perp^2,\mu)\Big[H_q^b(y,Q^2,\mu)\mathscr{B}_q(t_B,x,\mathbf{p}_\perp^2,\mu) + (q\to\bar{q})\Big],$$



Dynamically changes w.r.t x, Q

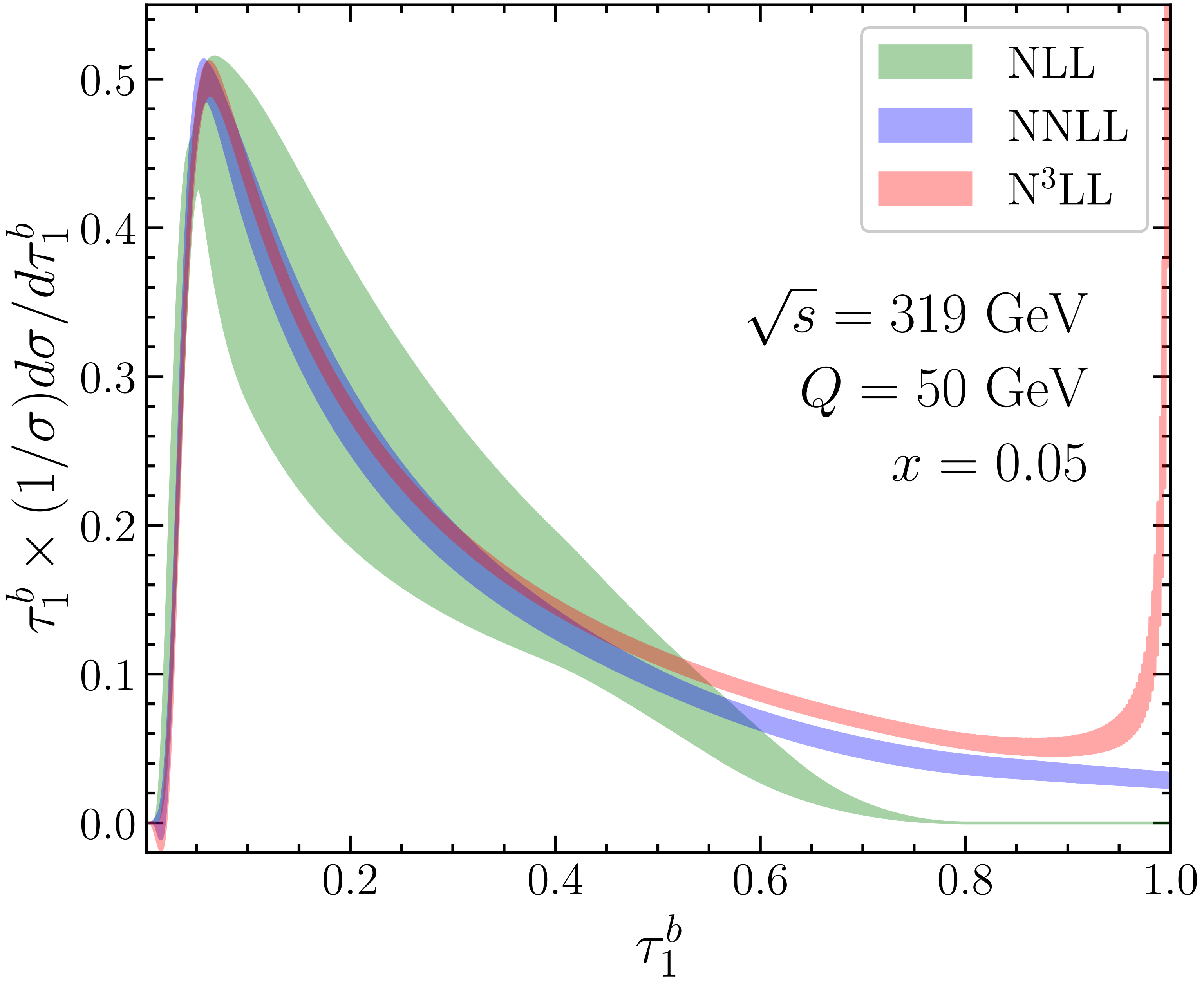
Uncertainties in σ^{s}_{PT}



Central + 16 scale variations estimate the perturbative uncertainties.

Results and comparison with HERA data

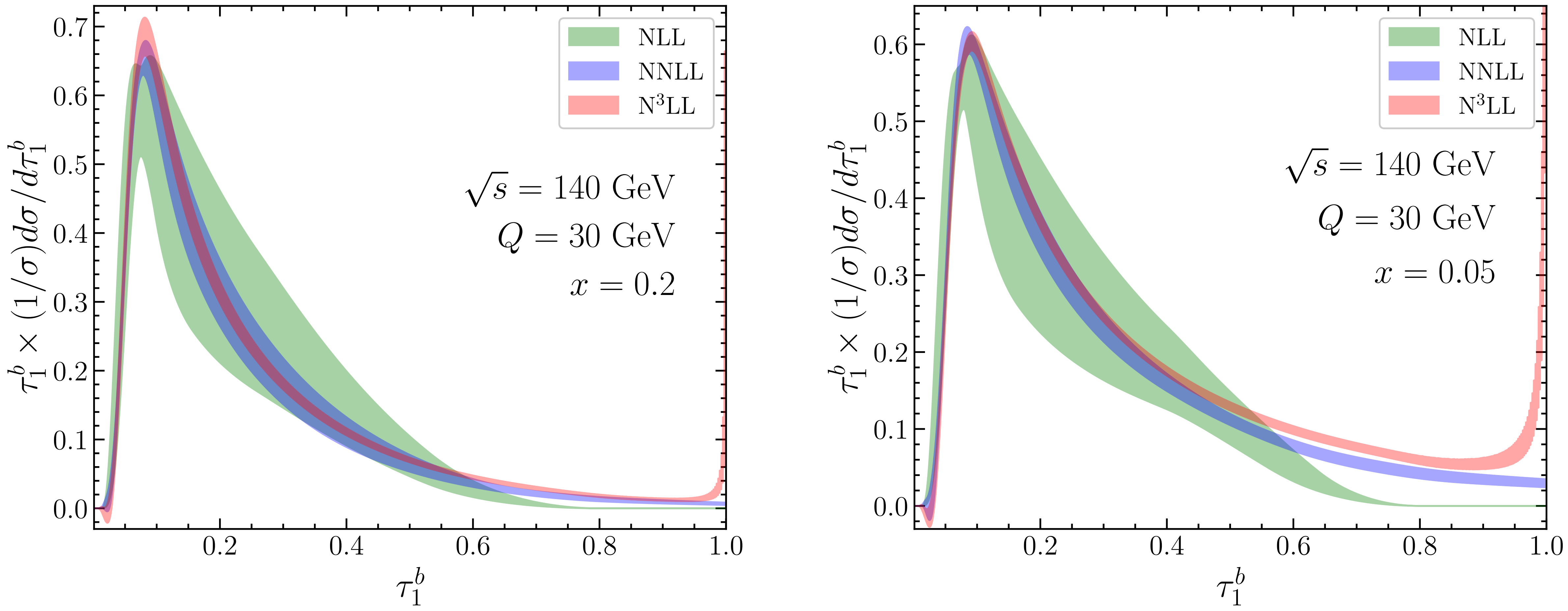
Final N³LL + $\mathcal{O}(\alpha_s^2)$ prediction



- Relevant to HERA setup
- Good perturbative convergence of the distributions, especially in the tail region.
- Can observe a peak as $\tau_1^b \to 1$, which characterizes the events with nearly empty jet hemisphere.

$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q} \sum_{i \in \mathcal{H}_J} (p_i)_z = \tau_Q$$

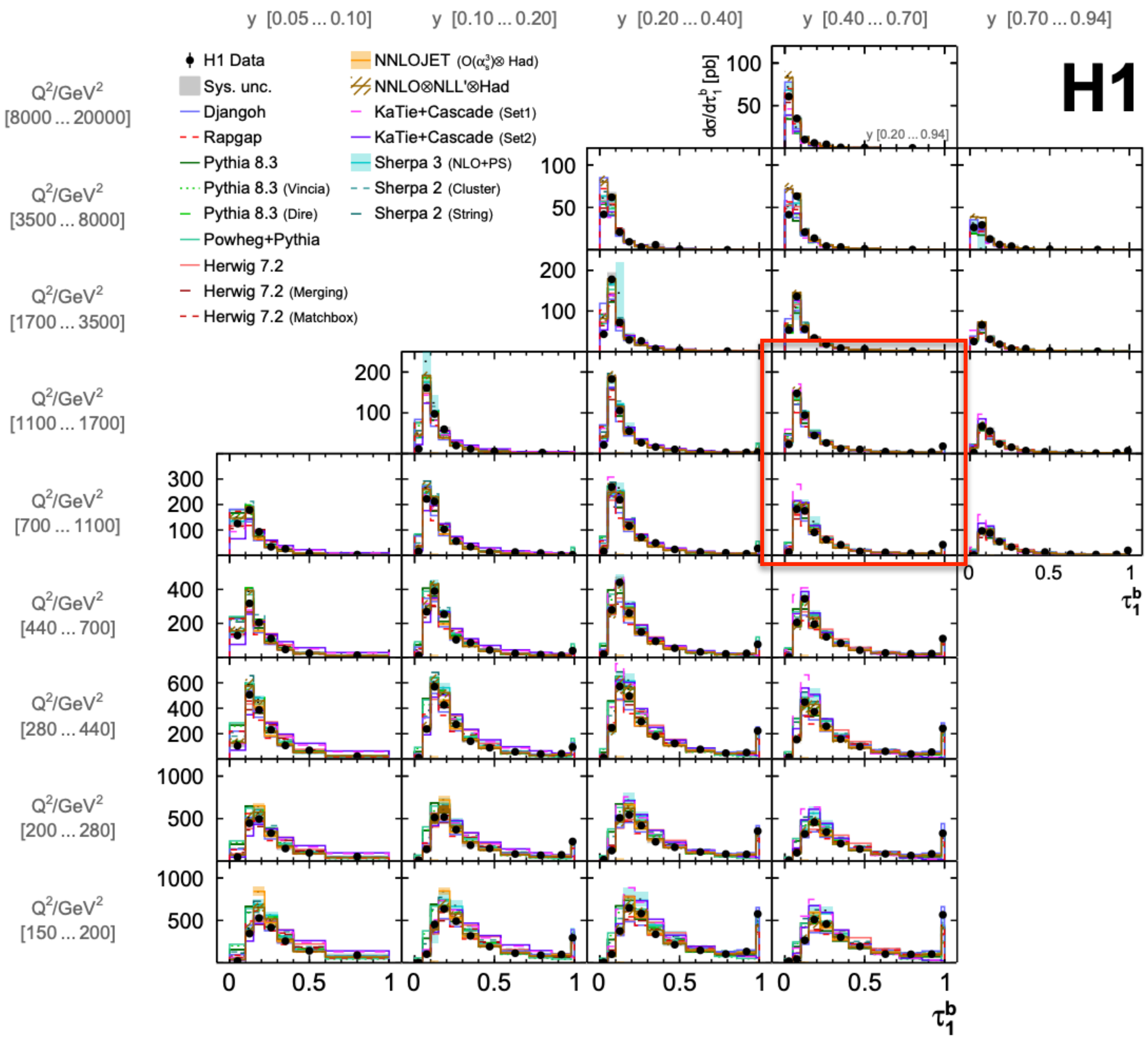
Final N³LL + $\mathcal{O}(\alpha_s^2)$ prediction



- Relevant to EIC setup
- Good perturbative convergence
- Can observe the peak as $\tau_1^b \to 1$, and this feature is more pronounced at smaller x.

HERA H1 measurement

- The HERA H1 collaboration reported measurements of τ_1^b in DIS using data collected between 2003-2007 $\sqrt{s} = 319$ GeV, with integrated luminosity of $\mathscr{L} = 351.1$ pb^{-1}.



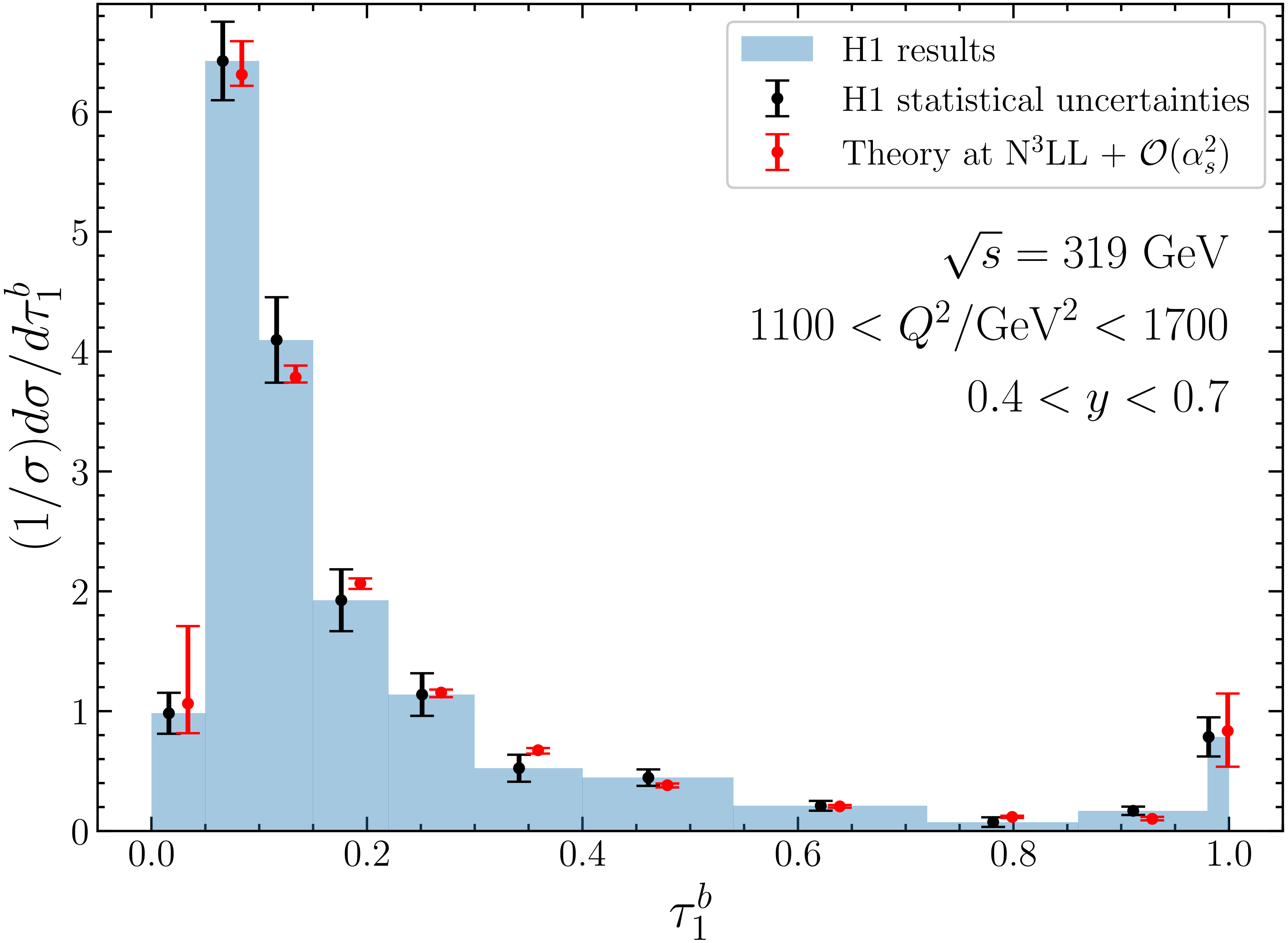
arXiv:2403.10109
H1 Collaboration

- The distribution in τ_1^b given by

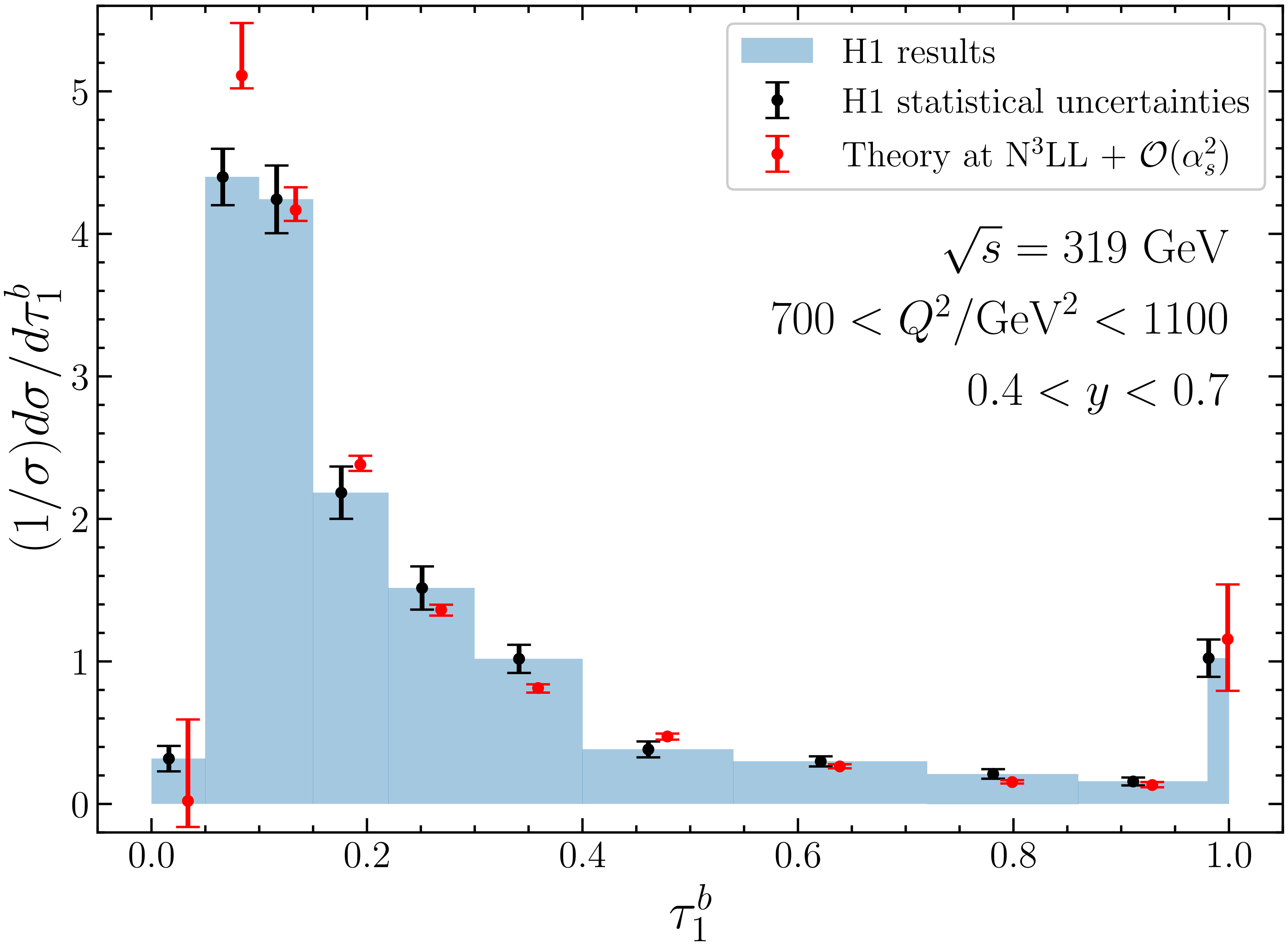
$$\int_{\Delta y} dy \int_{\Delta Q^2} dQ^2 \frac{d\sigma}{dy dQ^2 d\tau_1^b}$$

- We compare our theoretical predictions with these measurements (highlighted in the red box).

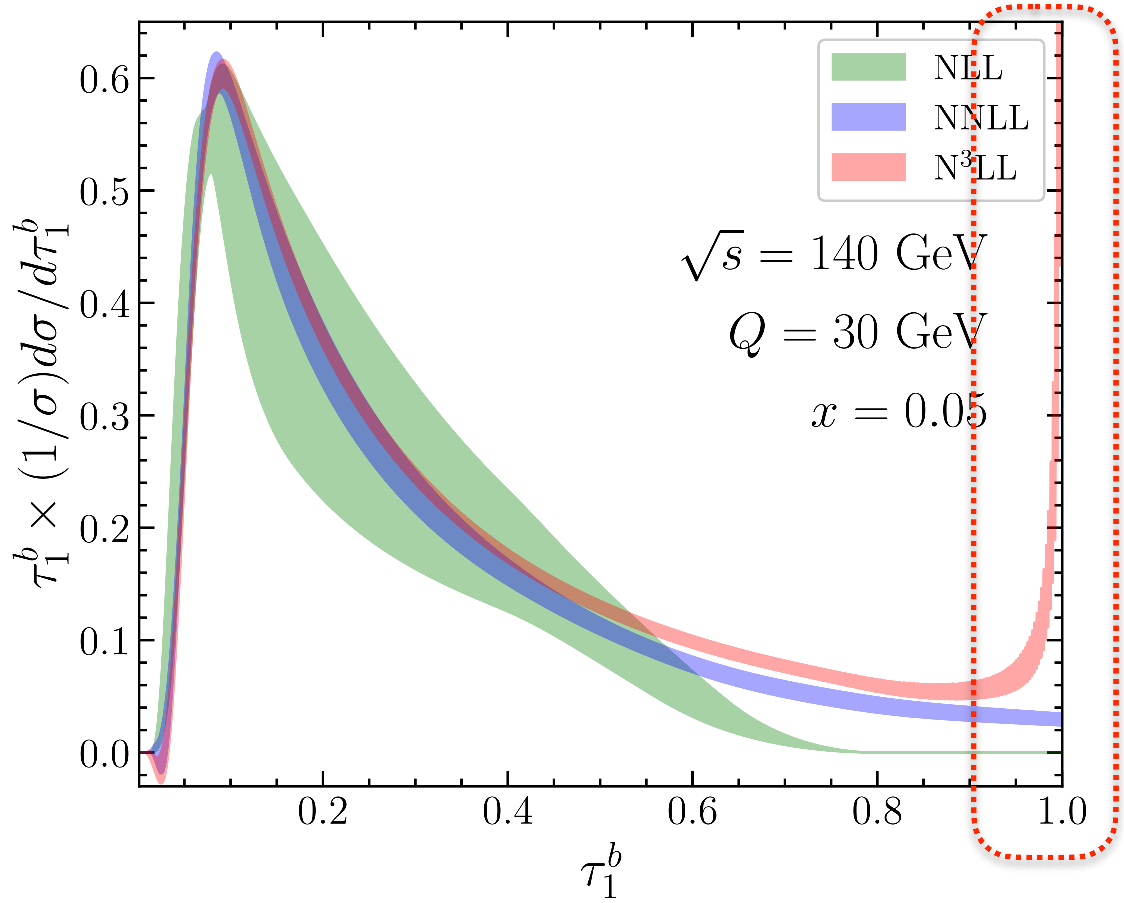
HERA H1 measurement

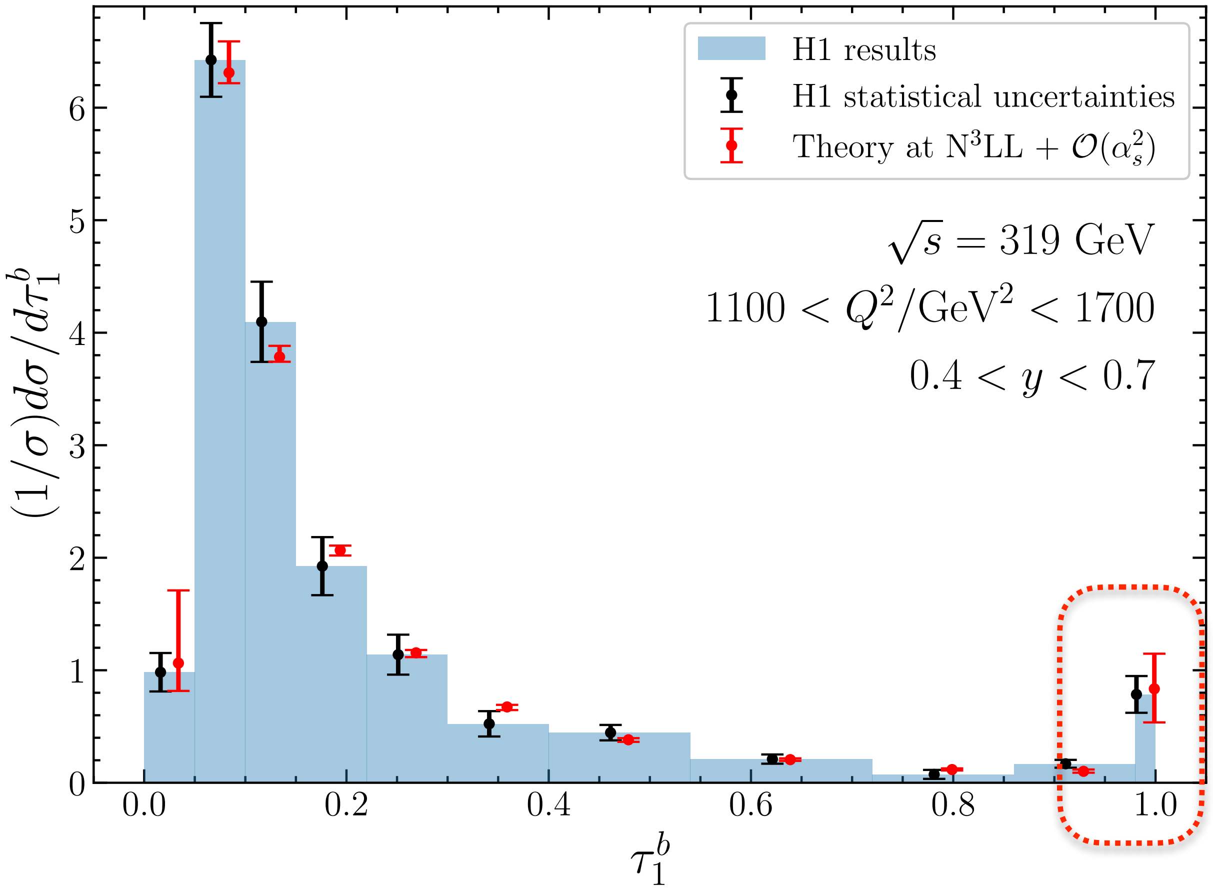


HERA H1 measurement



Peak as $\tau_1^b \to 1$

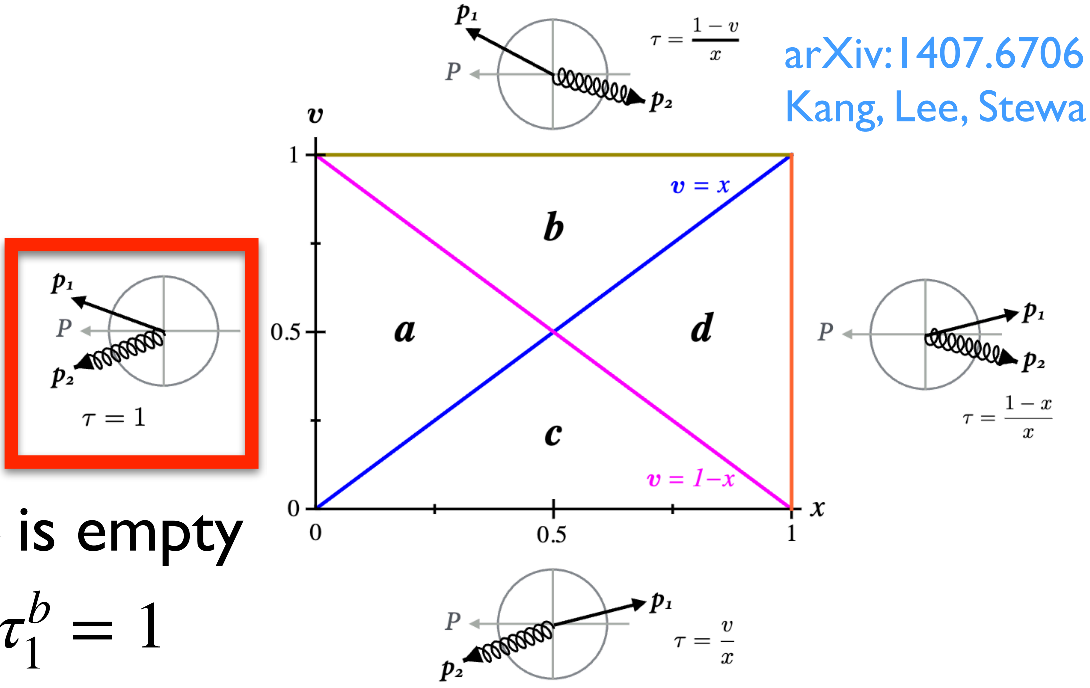




- $\tau_1^b \to 1$ characterizes events where nearly all final-state particles are confined to the beam hemisphere. (Empty jet hemisphere)

$$\tau_1^b \overset{\text{Breit}}{=} 1 - \frac{2}{Q} \sum_{i \in \mathscr{H}_J} (p_i)_z = \tau_Q$$

- This contribution becomes increasingly significant as $x \to 0$.



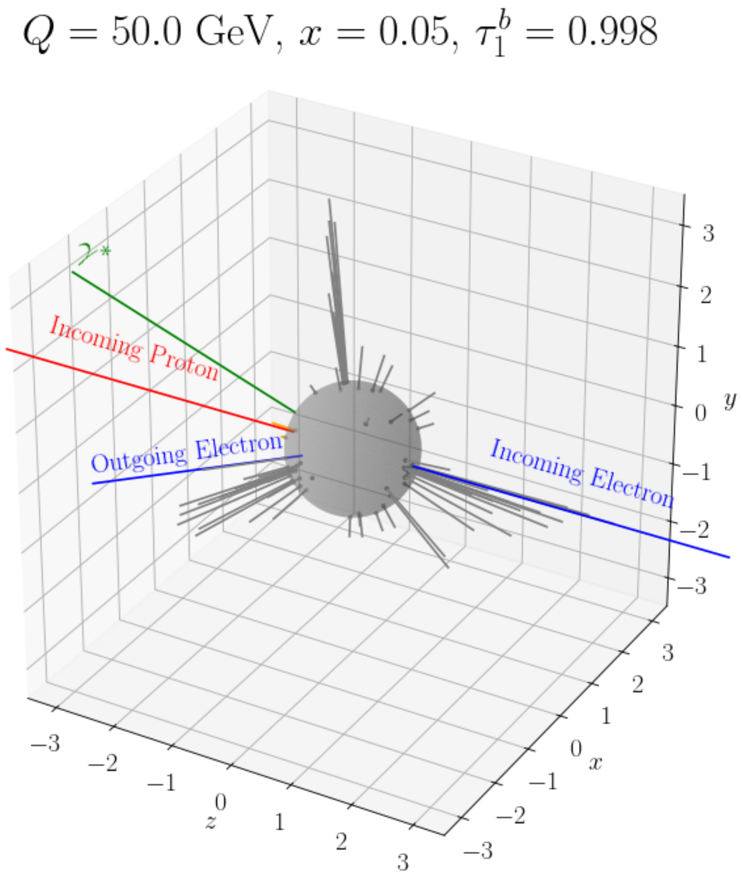
$\mathscr{H}_J$ is empty

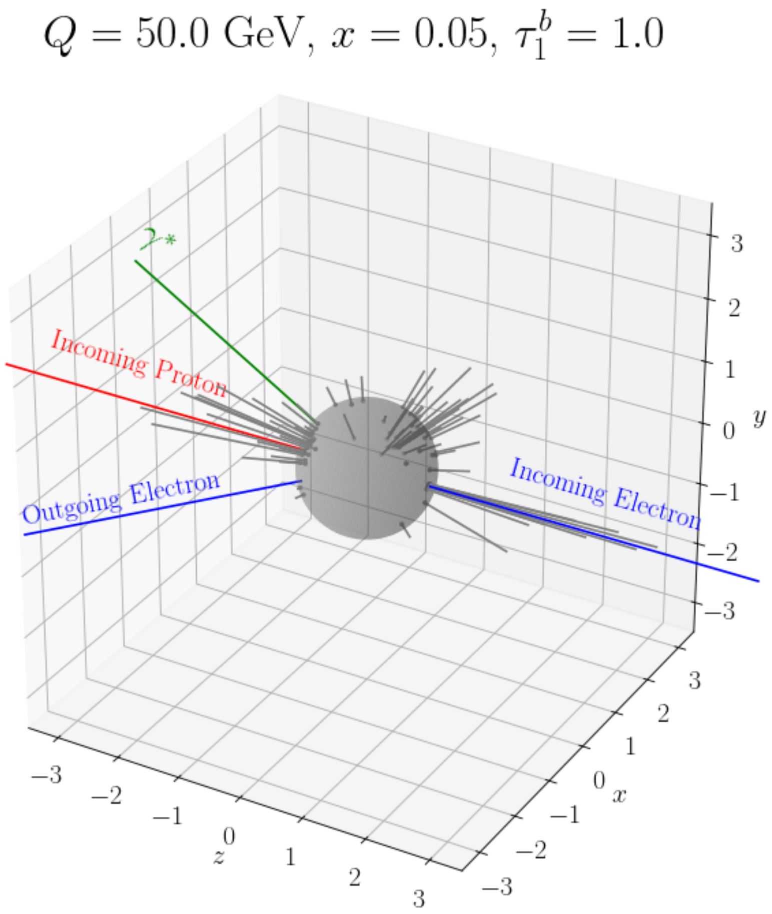
$\tau_1^b = 1$

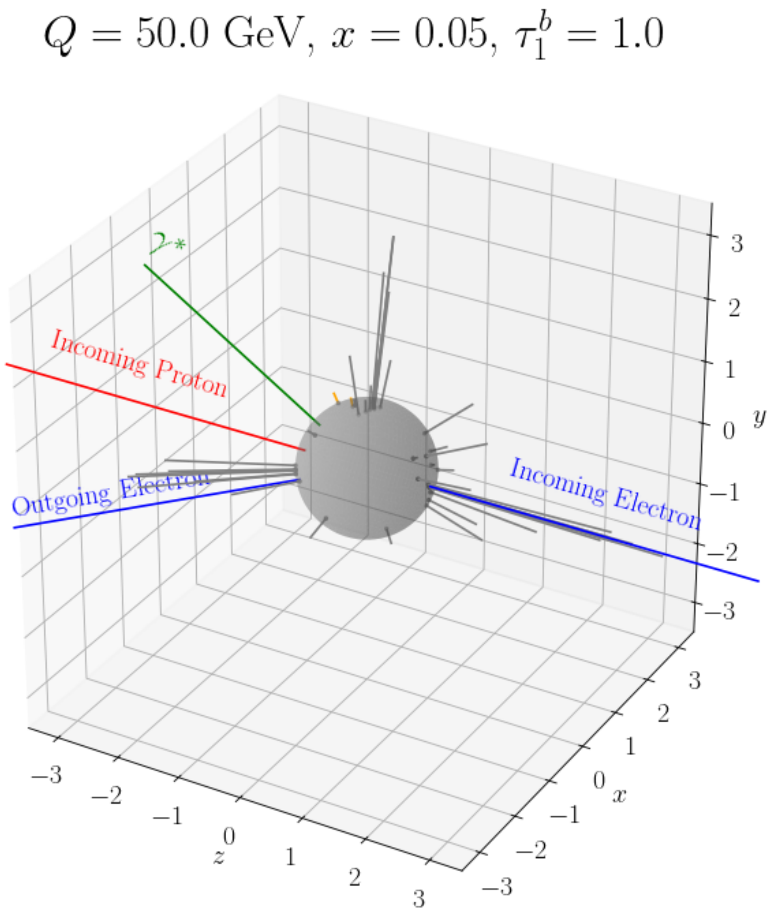
arXiv:1407.6706
Kang, Lee, Stewart

Peak as $\tau_1^b \to 1$

- In general, events with $\tau_1^b \sim 1$ describe the multi-jet events (more than 2 jets):

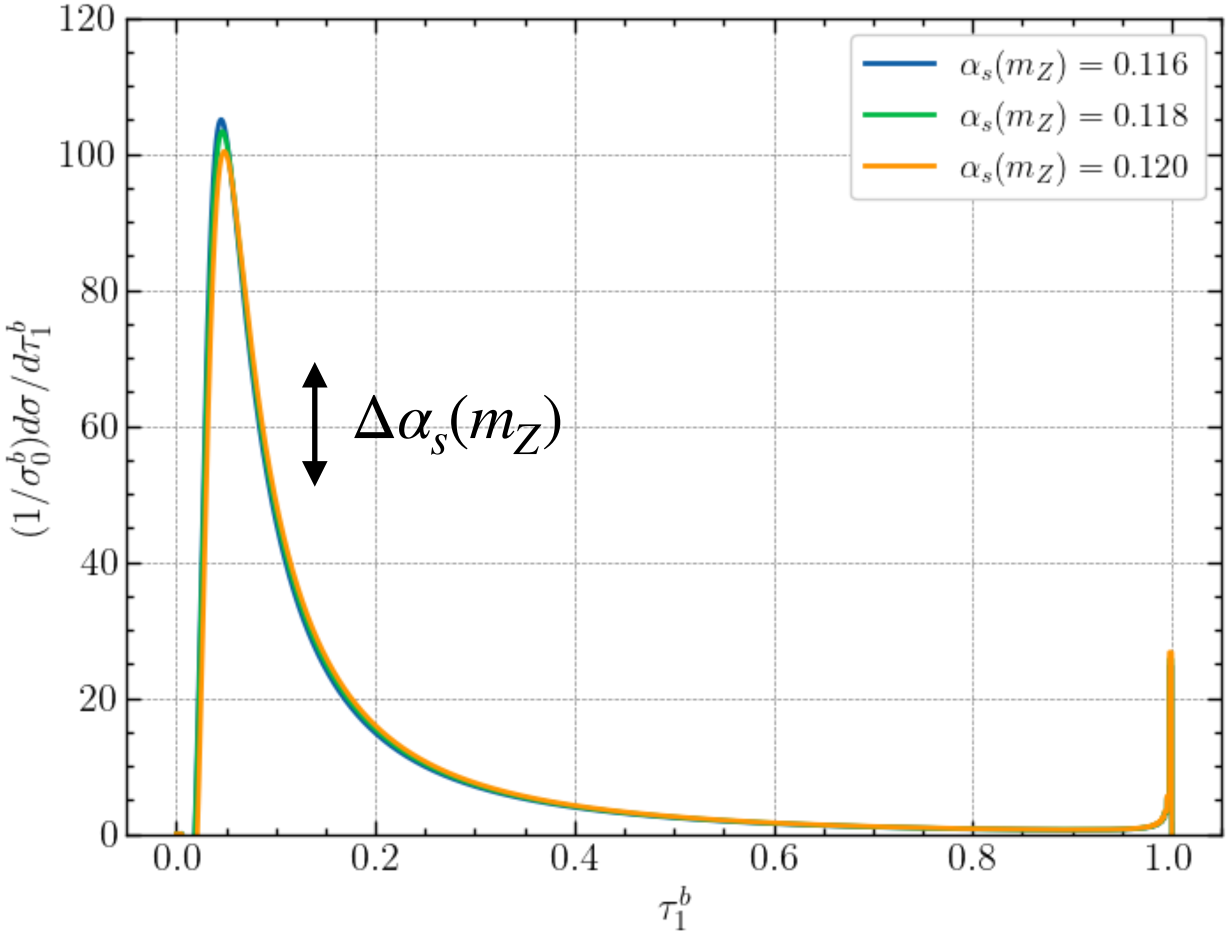

$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 0.998$

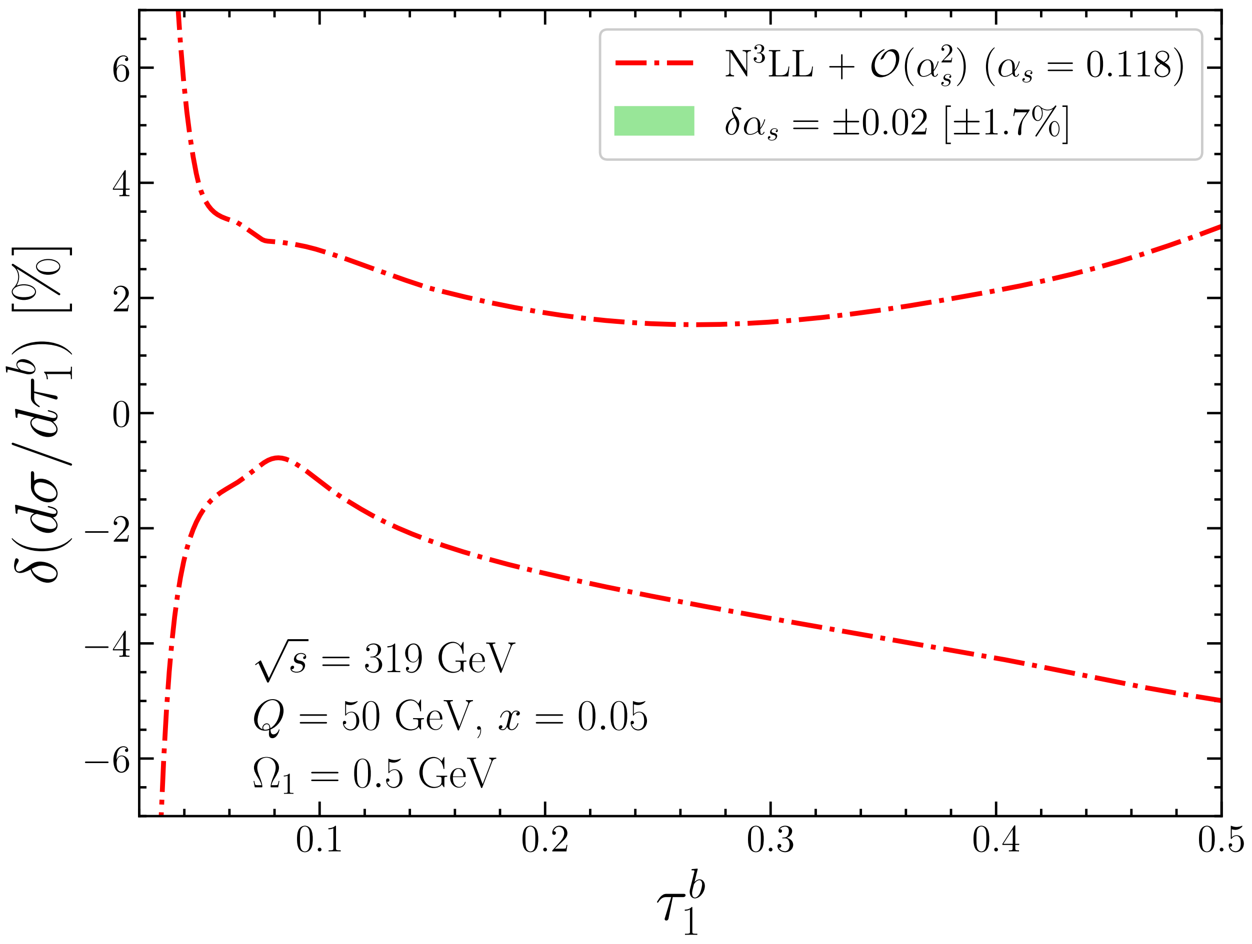

$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 1.0$


$Q = 50.0$ GeV, $x = 0.05$, $\tau_1^b = 1.0$

- The di-jet SCET factorization cannot describe these events, and we estimated these contributions using the full QCD 2-loop cross sections (using NLOJet++).

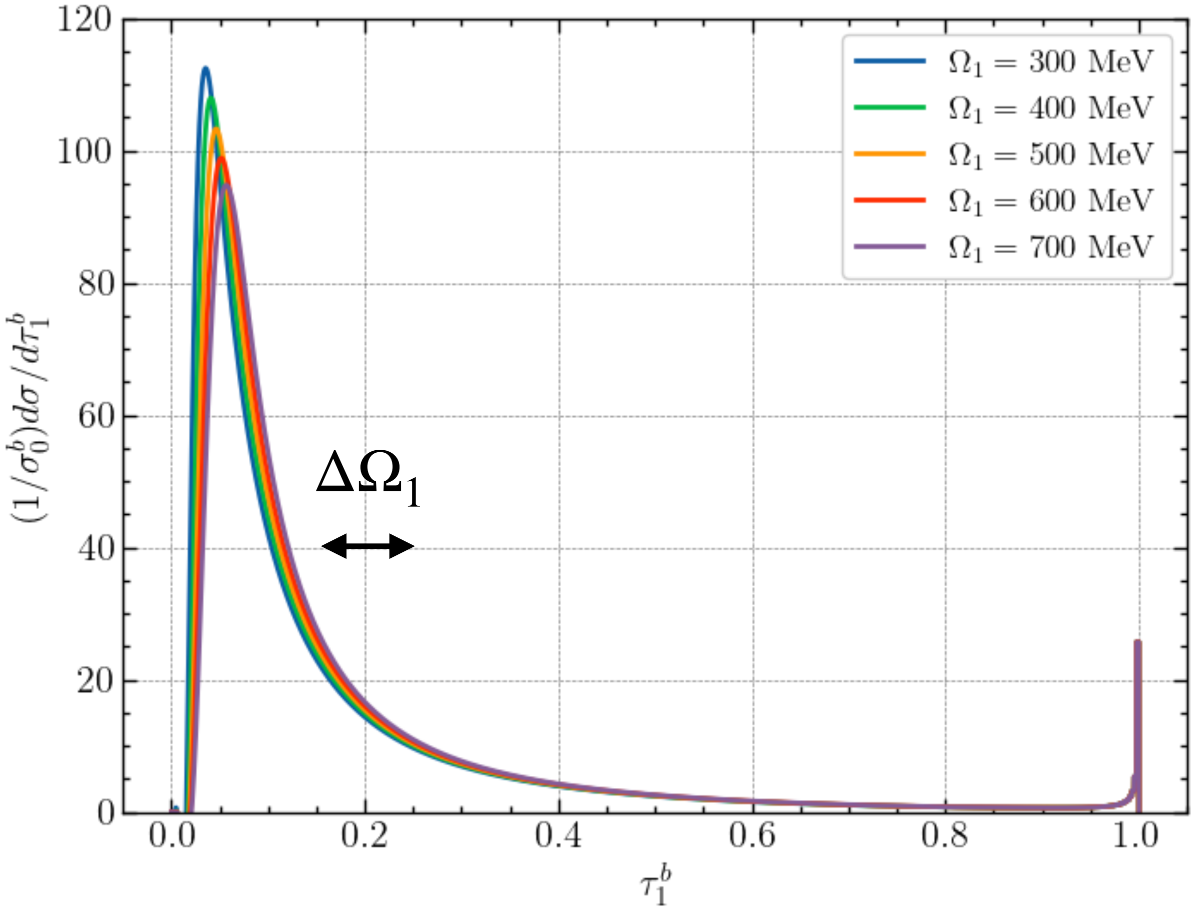
α_s and Ω_1 sensitivities

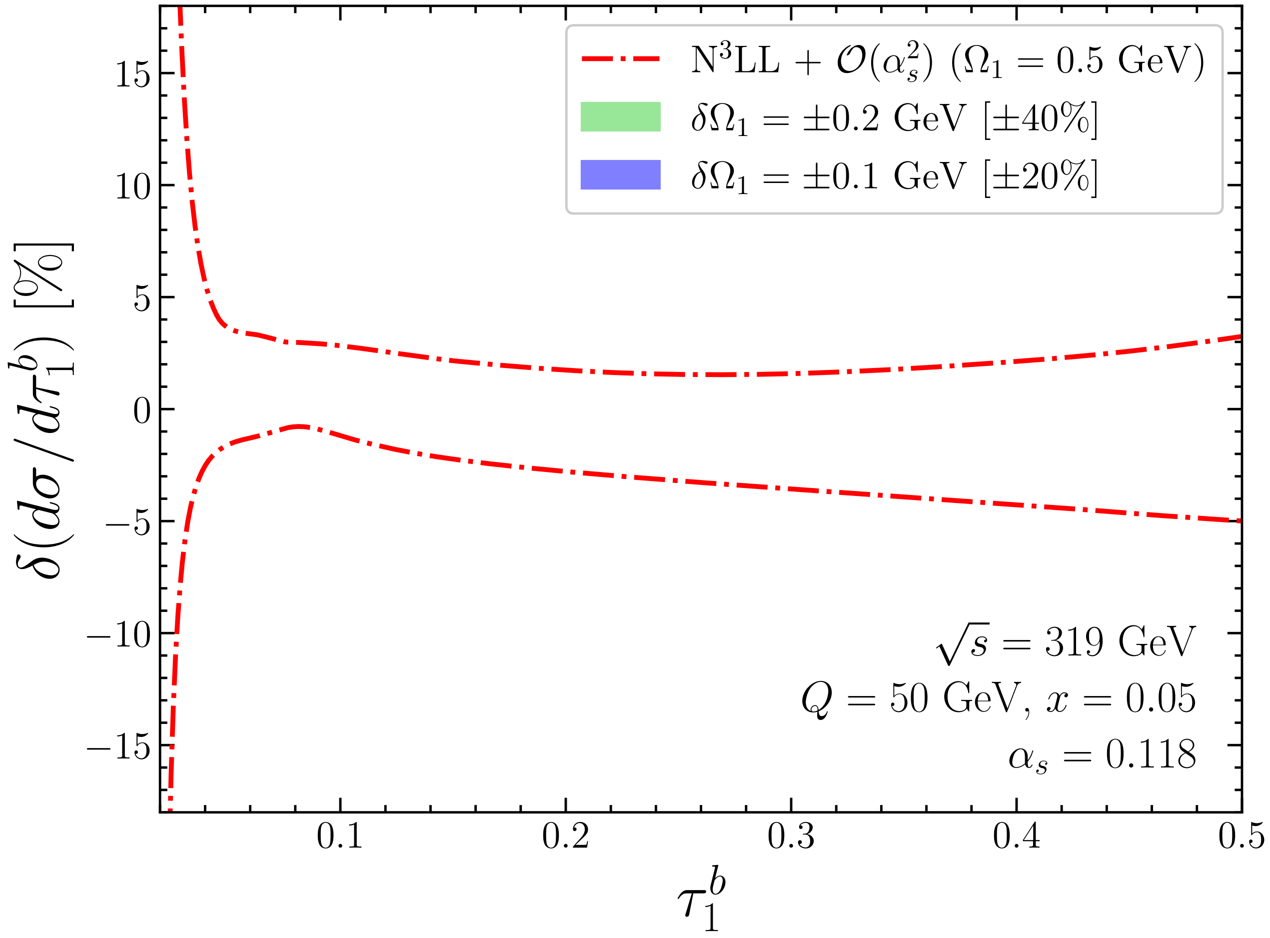




Requires uncertainties below 4% for $\delta\alpha_s = \pm 0.02$

α_s and Ω_1 sensitivities

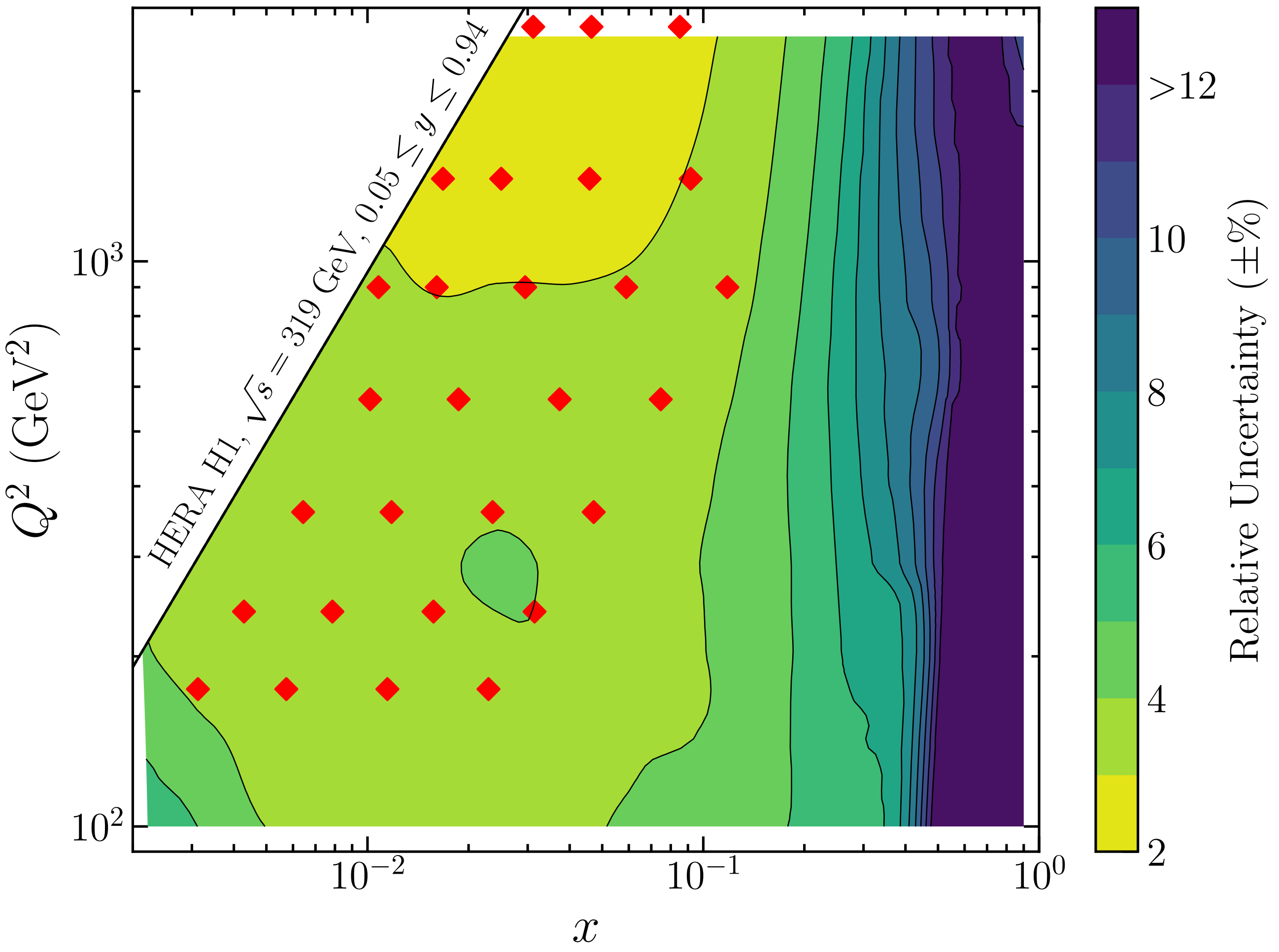




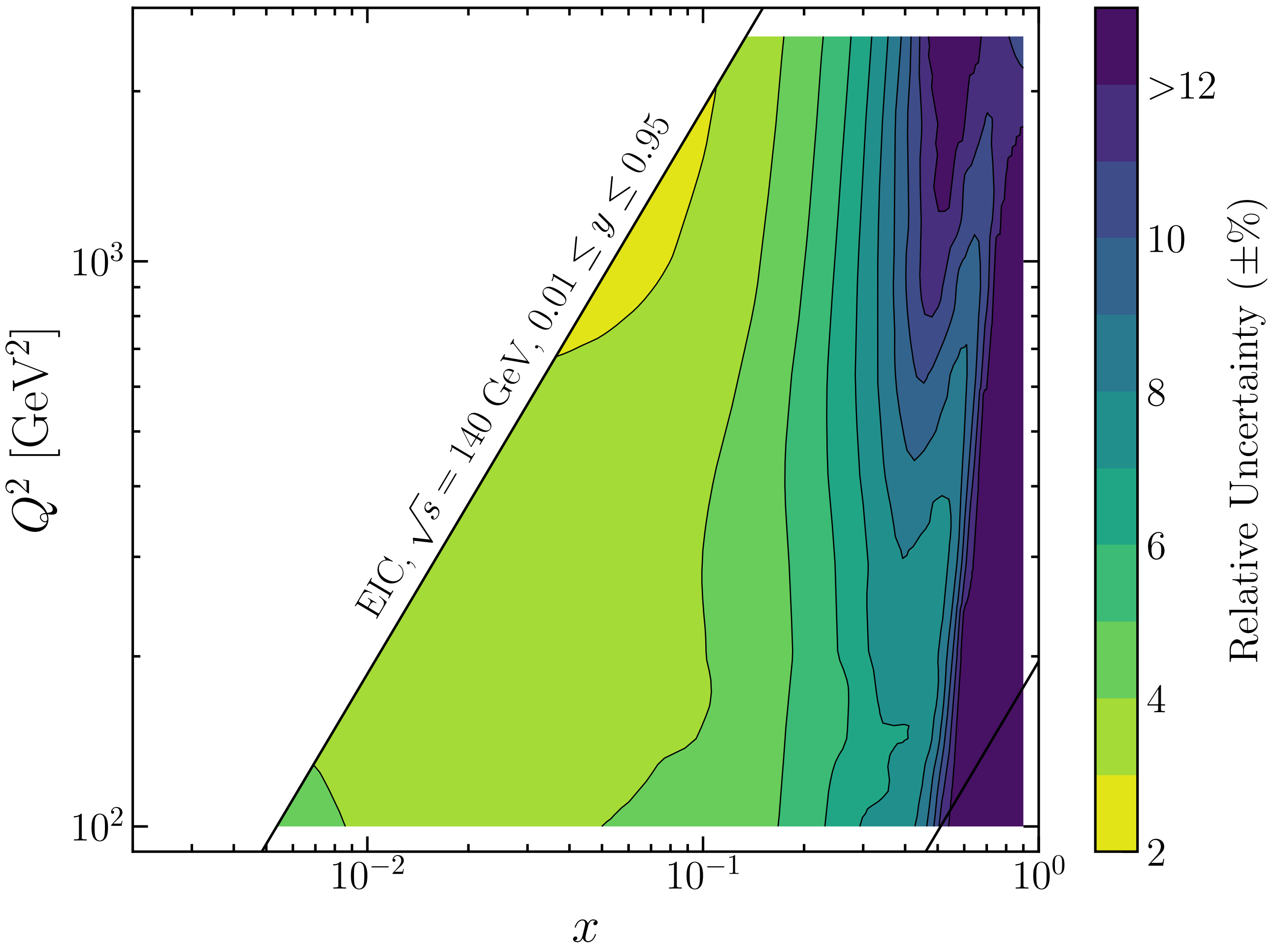
Requires uncertainties below 5% for $\delta\Omega_1 = \pm 100$ MeV

Uncertainties w.r.t. x and Q^2

HERA ($\sqrt{s} = 319$ GeV)



EIC ($\sqrt{s} = 140$ GeV)



- Our predictions exhibit uncertainties below 4% across large range of x and Q.

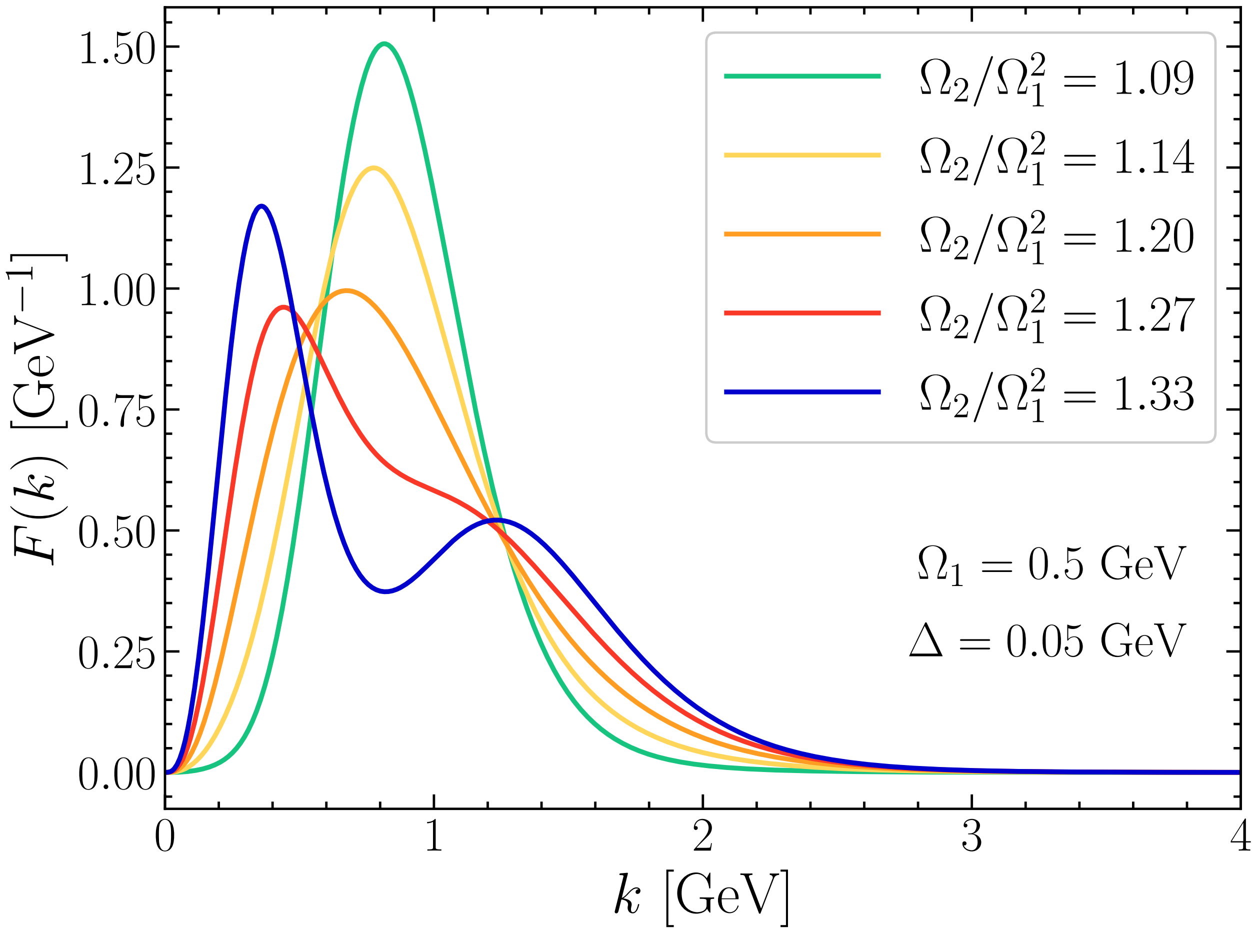
Summary

- τ_1^b is an DIS event shape with significant advantages for experimental measurements.
- As a global observable, it allows for high-precision theoretical predictions.
- We computed the τ_1^b distributions at N3LL + $\mathcal{O}(\alpha_s^2)$ accuracy, incorporating power corrections and renormalon subtractions for NP soft physics.
- With recent HERA measurements and future EIC results, τ_1^b provides an unique event shape method for the α_s and Ω_1 determination in DIS.

Thanks!

Backup

Different Shape Function

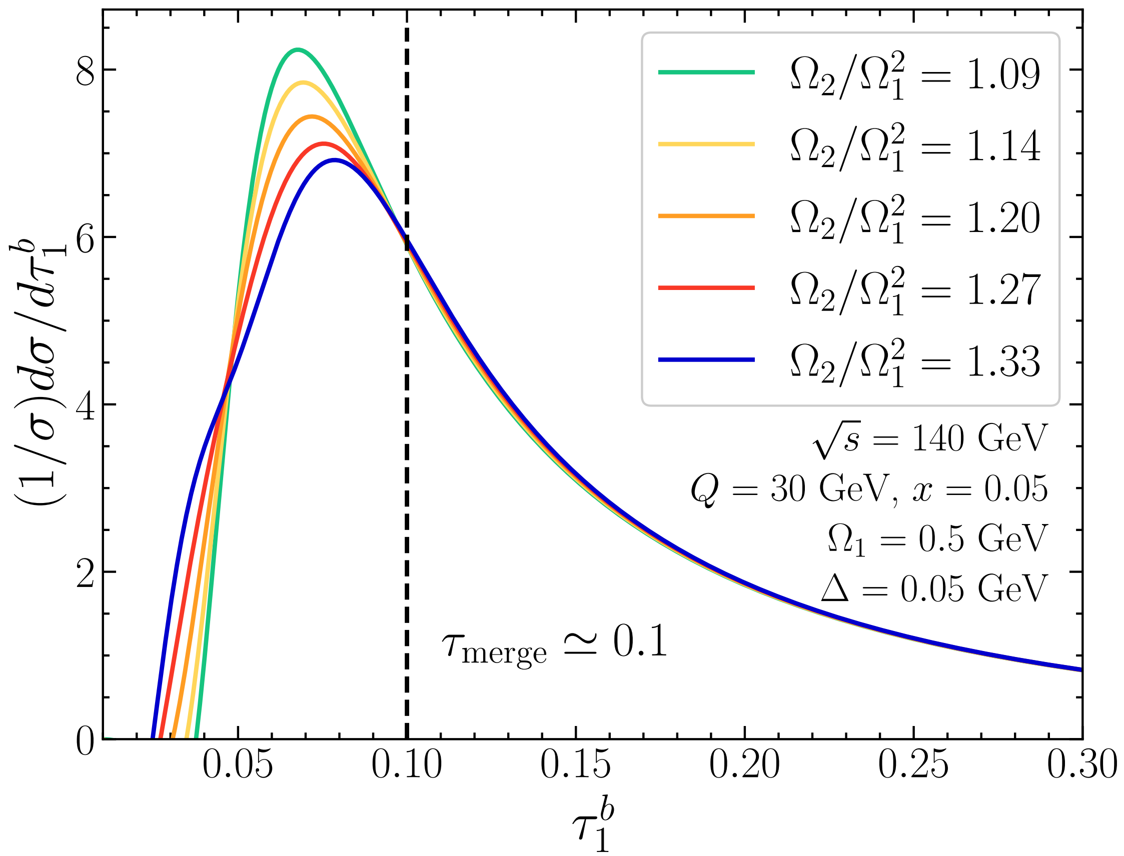


$$\frac{d\sigma}{d\tau_1^b}(\tau_1^b) = \int dk \frac{d\sigma_{\text{pert}}}{d\tau_1^b}\left(\tau_1^b - \frac{k}{Q}\right) F(k)$$

↓ Operator Product Expansion

$$\frac{d\sigma}{d\tau_1^b}(\tau_1^b) = \left\{\frac{d\sigma_{\text{pert}}(\tau_1^b)}{d\tau_1^b} - \frac{2\Omega_1}{Q}\frac{d\sigma^2_{\text{pert}}(\tau_1^b)}{d\tau_1^{b2}}\right\}\left[1 + \mathcal{O}(\Lambda_{\text{QCD}}/(\tau_1^b Q))\right]$$

Implementing different higher-moment modify the τ_1^b distributions in the peak region, $\tau_{\text{merge}} \sim \dfrac{3\text{ GeV}}{Q}$



First nontrivial extension of shape function

$$F(k) = \frac{1}{\lambda}\left[c_0 f_0\left(\frac{k}{\lambda}\right) + c_2 f_2\left(\frac{k}{\lambda}\right)\right]^2.$$

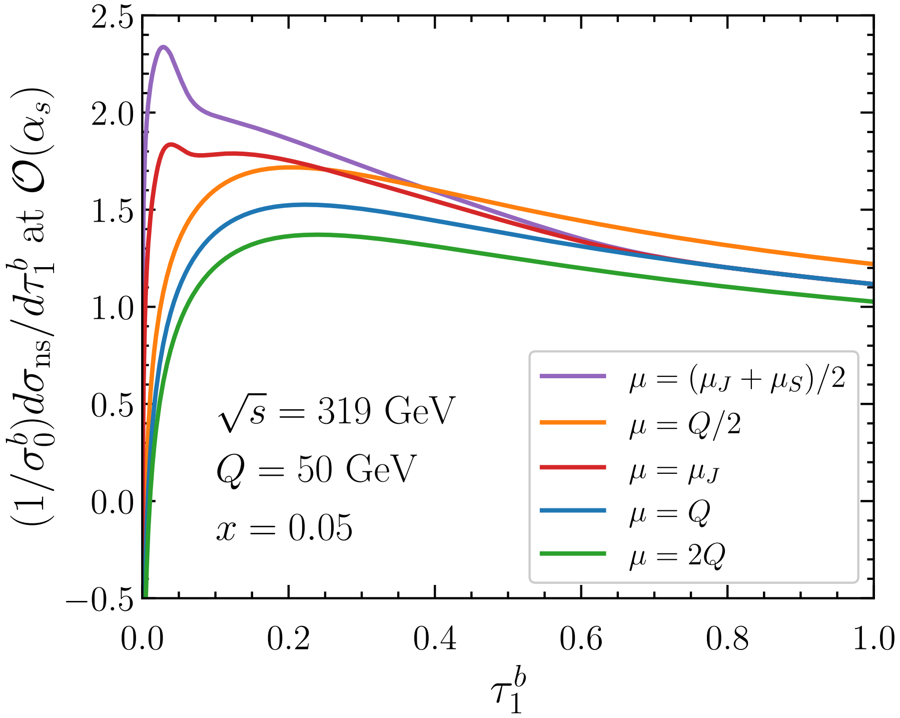
$$\int dk\, F(k - 2\Delta) = 1,$$

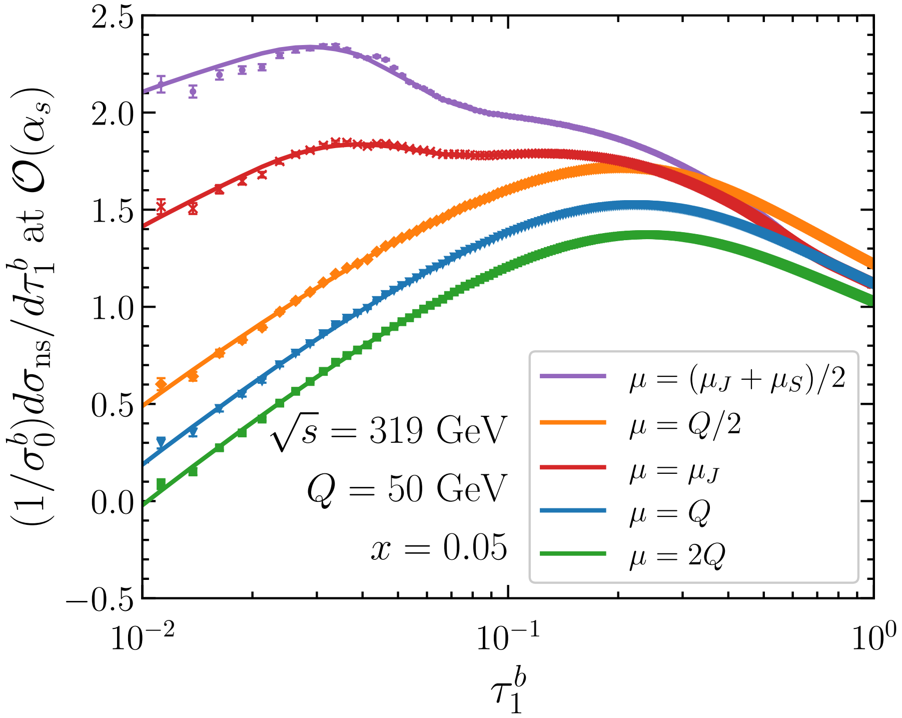
$$\int dk\, kF(k - 2\Delta) = 2\Omega_1.$$

Uncertainties in $\sigma_{\mathrm{PT}}^{\mathrm{ns}}$

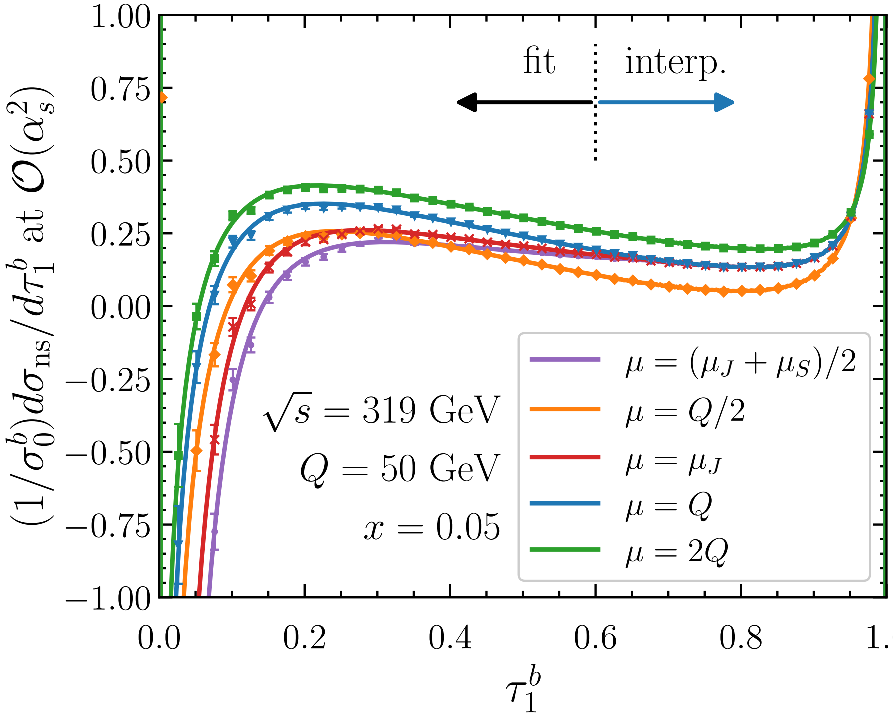
- We consider the 5 scale variations for the fixed-order calculations:

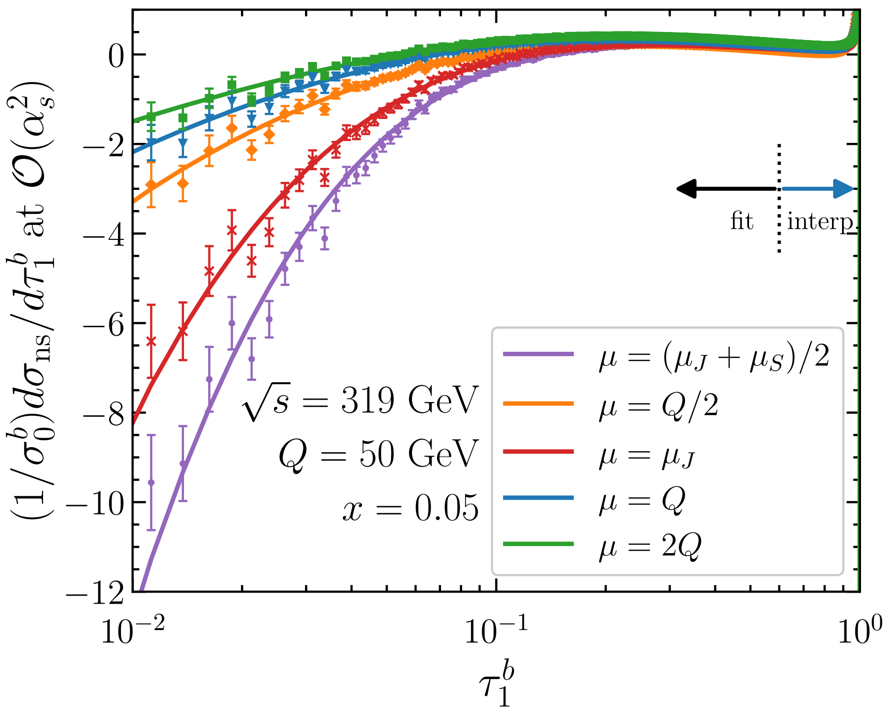
1-loop nonsingular



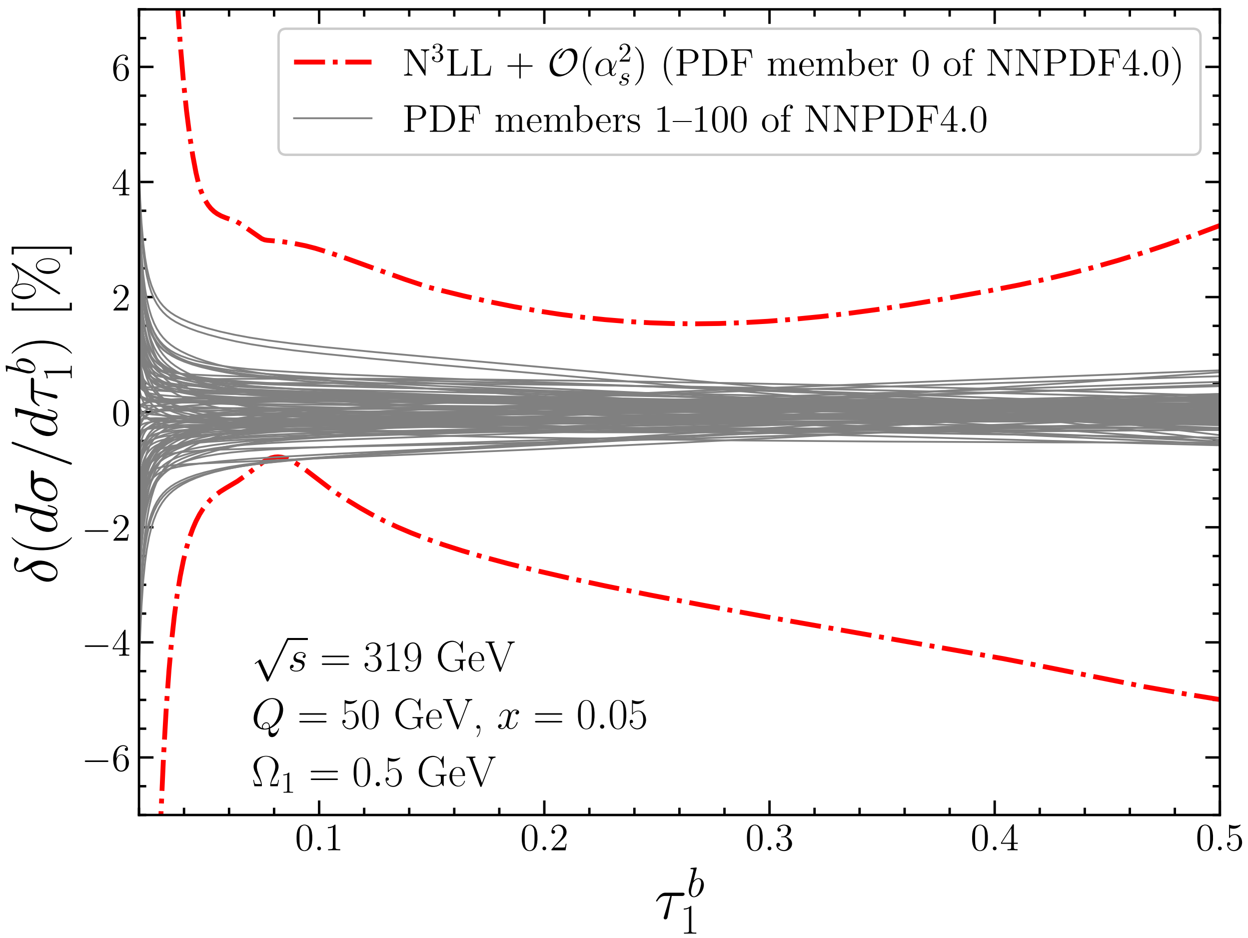


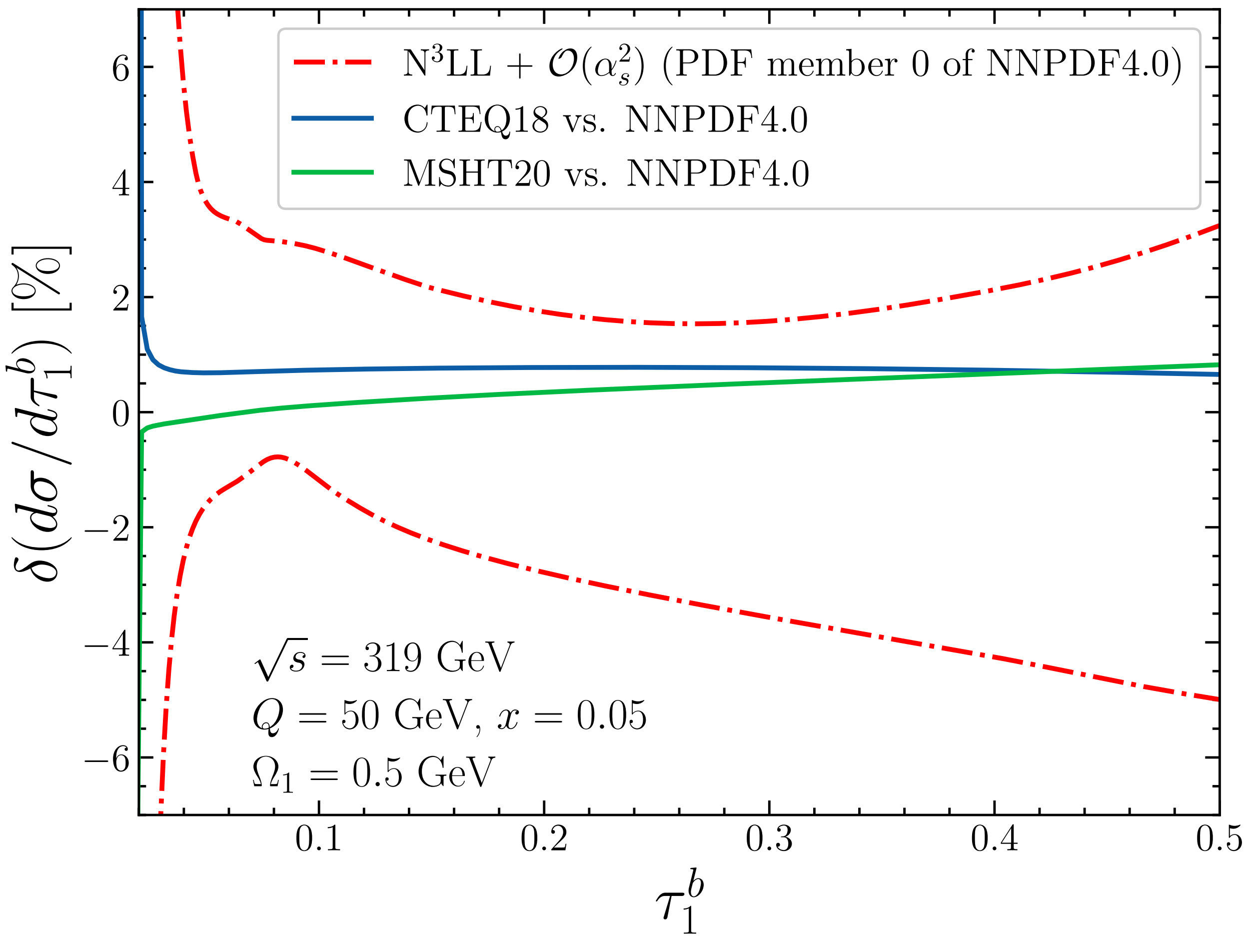
2-loop nonsingular





PDF uncertainties





- PDF uncertainties are well within our theoretical estimation of uncertainties at $\text{N}^3\text{LL} + \mathcal{O}(\alpha_s^2)$.