Few-body elastic scattering polarimetry

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1. Amplitudes and Analyzing Powers
2. Kinematic Region and Polarimetry

Polarized Ion Sources and Beams at EIC

Stony Brook University

Center for Frontiers of Nuclear Science

ANALYZING POWER for POLARIZED LIGHT IONS

Z and m: charge and mass of the ion with magnetic moment $\mu \cdot \frac{e}{2m_p}$

A_{N}: analyzing power for incident polarized ion (neglecting double-flip)

2 Im (non-flip* × spin-flip) / (|non-flip|^2 + |spin-flip|^2)

Re-scaled amplitudes by e.g. σ_{tot} do not alter A_{N} so that we write

non-flip: $i + \rho - \frac{t_C}{t} e^{i\,\delta_C - (b_h - b_e)\,t}$

$\frac{m}{\sqrt{-t}}$ spin-flip: $i\,I_S + R_S - \frac{t_C}{2t}\left(\frac{\mu}{Z} - \frac{m_p}{m}\right) e^{i\delta_C - (b_h - b_m)t}$

EM and hadronic amplitudes equal at: $-t_C = \frac{4hc\,Z\,\tilde{Z}}{137\,\sigma_{\mathrm{tot}}} \approx \frac{Z\,\tilde{Z}}{14\,\sigma_{\mathrm{tot}}}\ (\mathrm{GeV}/c)^2$

Hadronic slope: $2b_h$; EM form factor slopes: b_e , b_m; Target charge: $\tilde{Z}e$

Hadronic spin-flip: $i\,I_S + R_S$; Coulomb phase: $\delta_C = -Z\tilde{Z}\alpha\left(\ln|b_h t + b_e t| + \gamma\right)$

ANALYZING POWER AT LOW MOMENTUM TRANSFER

The analyzing power for an incident polarized hadron is approximately

$$\frac{m_p}{\sqrt{-t}} A_{\mathrm{N}} = \frac{\left(\frac{\mu}{Z} - \frac{m_p}{m} - 2\, I_S \right) \frac{t_C}{t} - 2\, R_S + 2\, \varrho\, I_S}{\left(\frac{t_C}{t} - 2\varrho - 2\delta_C \right) \frac{t_C}{t} + 1 + \varrho^2 + R_S^2 + I_S^2}$$

Further terms appear in A. Poblaguev et al., PRL 123 (2019) 16, 162001

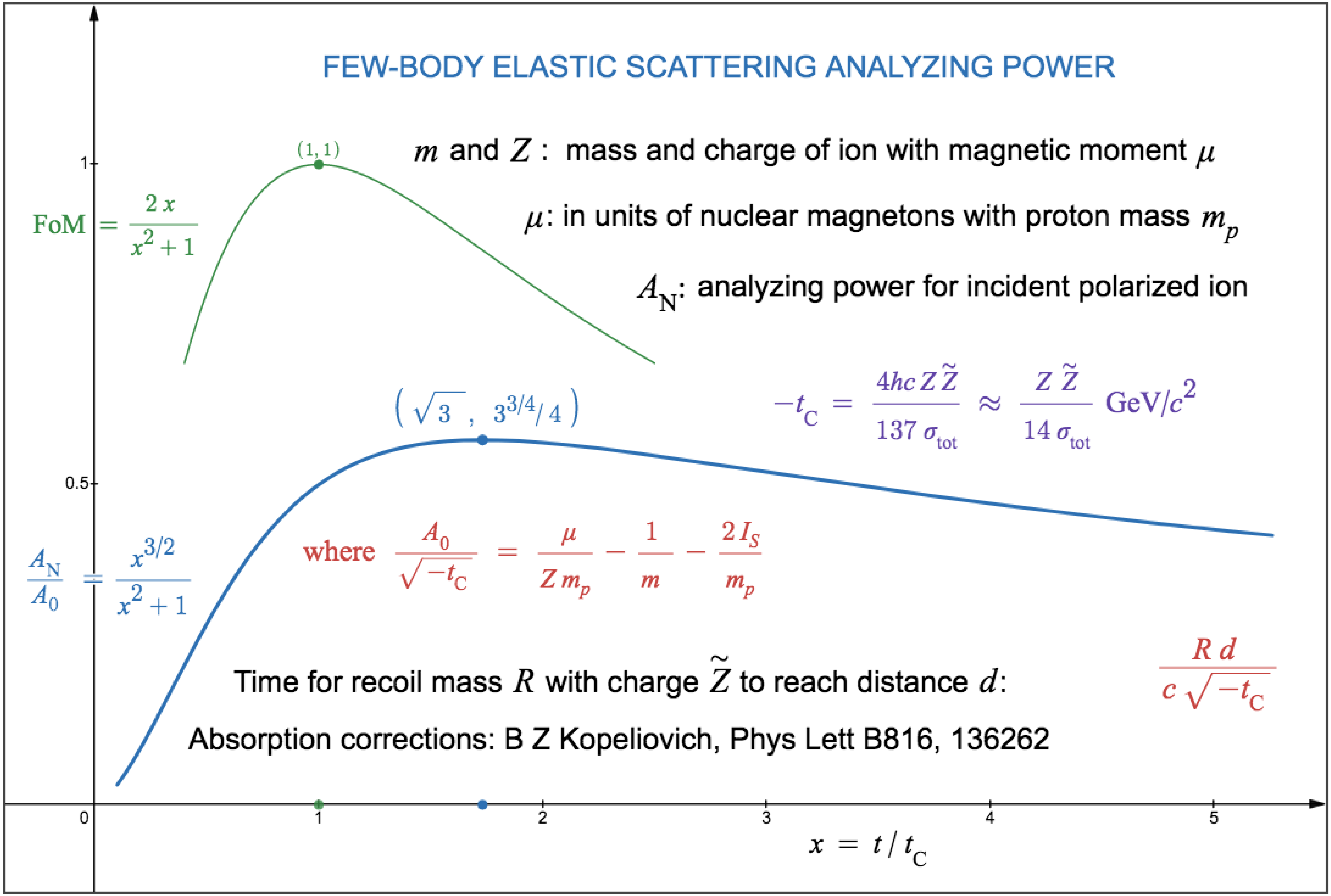
Spin $\frac{\mu}{m_p}$ and charge $\frac{Z}{m}$ terms arise from Gordon decompositiion (1928)

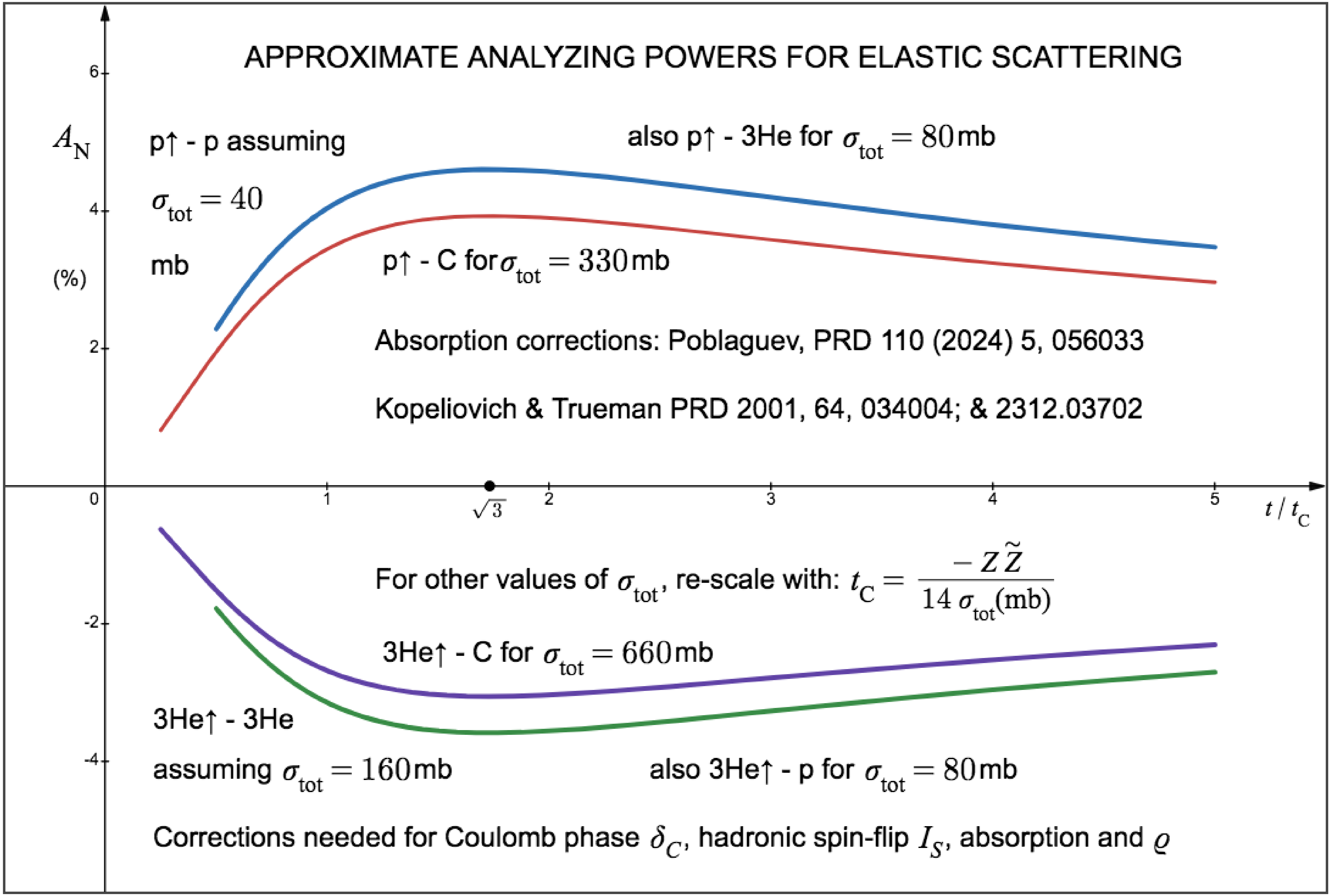
The A_N peak is related linearly to the unknown hadronic spin-flip term I_S

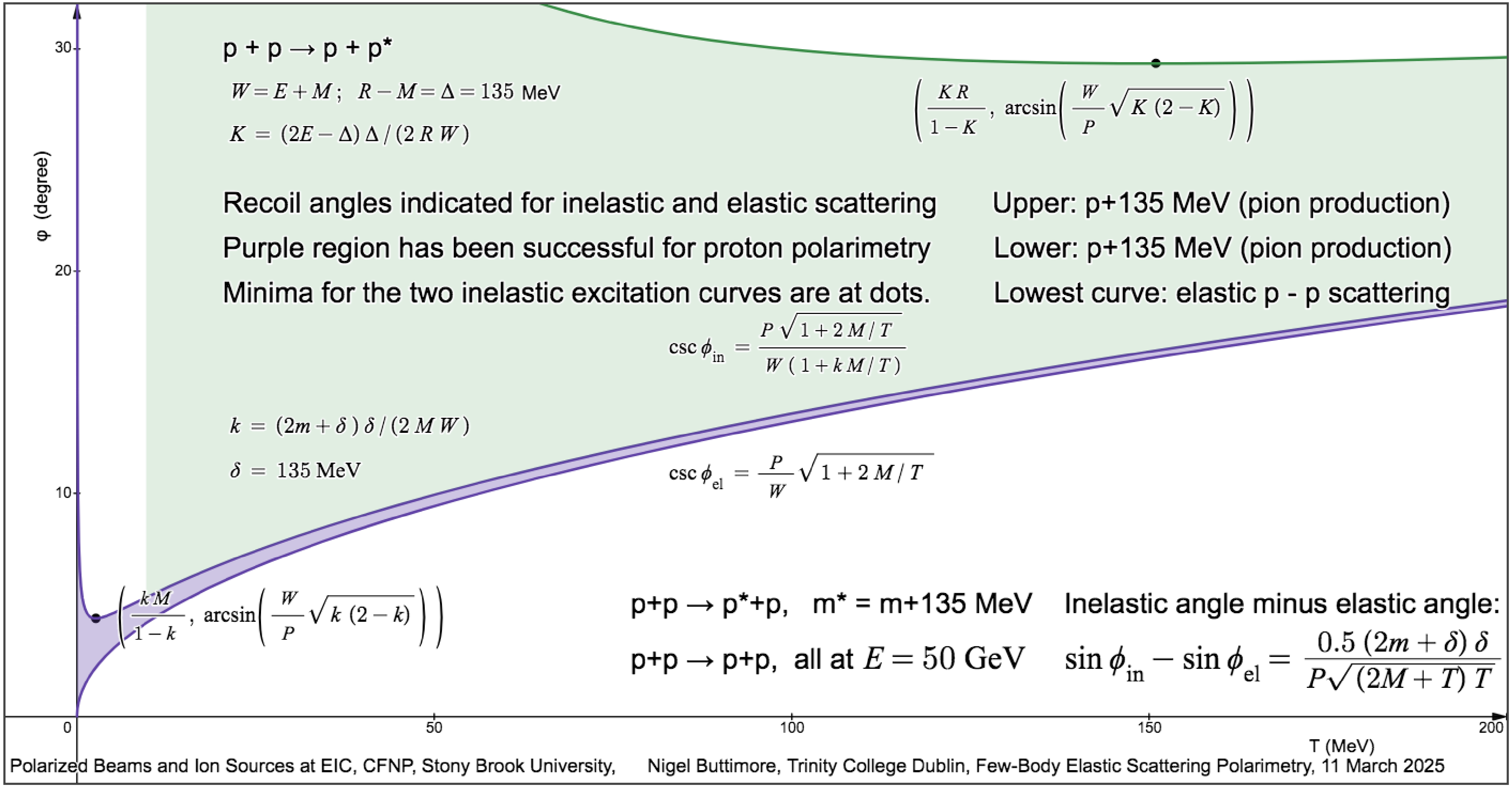
Multiplying by the square of $x = t\,/t_C$ the analyzing power approximates

$$\frac{m_p}{\sqrt{-t_C}} \frac{A_{\mathrm{N}}}{\sqrt{x}} = \frac{\left(\frac{\mu}{Z} - \frac{m_p}{m} - 2\, I_S \right) x}{1 - 2(\varrho + \delta_C)\, x + x^2}$$

Squared hadronic terms and R_S are relatively tiny for $1 < -1000\, t < 10$


FEW-BODY ELASTIC SCATTERING ANALYZING POWER
m and Z : mass and charge of ion with magnetic moment μ
μ: in units of nuclear magnetons with proton mass m_p
A_N: analyzing power for incident polarized ion
(1, 1)
$\mathrm{FoM} = \frac{2x}{x^2+1}$
$\left(\sqrt{3}\ ,\ 3^{3/4}/4\right)$
$-t_C = \frac{4hc\,Z\tilde{Z}}{137\,\sigma_{tot}} \approx \frac{Z\tilde{Z}}{14\,\sigma_{tot}}\ \mathrm{GeV}/c^2$
$\frac{A_N}{A_0} = \frac{x^{3/2}}{x^2+1}$
where $\frac{A_0}{\sqrt{-t_C}} = \frac{\mu}{Z\,m_p} - \frac{1}{m} - \frac{2\,I_S}{m_p}$
Time for recoil mass R with charge $\tilde{Z}$ to reach distance d:
$\frac{R\,d}{c\,\sqrt{-t_C}}$
Absorption corrections: B Z Kopeliovich, Phys Lett B816, 136262
0
0.5
1
2
3
4
5
$x = t / t_C$


APPROXIMATE ANALYZING POWERS FOR ELASTIC SCATTERING
A_N (%)
p↑ - p assuming $\sigma_{tot} = 40$ mb
also p↑ - 3He for $\sigma_{tot} = 80$ mb
p↑ - C for $\sigma_{tot} = 330$ mb
Absorption corrections: Poblaguev, PRD 110 (2024) 5, 056033
Kopeliovich & Trueman PRD 2001, 64, 034004; & 2312.03702
$\sqrt{3}$
t / t_C
For other values of σ_{tot}, re-scale with: $t_C = \frac{-Z\tilde{Z}}{14\,\sigma_{tot}(\text{mb})}$
3He↑ - C for $\sigma_{tot} = 660$ mb
3He↑ - 3He assuming $\sigma_{tot} = 160$ mb
also 3He↑ - p for $\sigma_{tot} = 80$ mb
Corrections needed for Coulomb phase δ_C, hadronic spin-flip I_S, absorption and ϱ


p + p → p + p*
$W = E + M; \quad R - M = \Delta = 135$ MeV
$K = (2E - \Delta)\,\Delta\,/\,(2\,R\,W)$
$\left(\frac{K R}{1 - K},\ \arcsin\left(\frac{W}{P}\sqrt{K\,(2 - K)}\right)\right)$
Recoil angles indicated for inelastic and elastic scattering
Purple region has been successful for proton polarimetry
Minima for the two inelastic excitation curves are at dots.
Upper: p+135 MeV (pion production)
Lower: p+135 MeV (pion production)
Lowest curve: elastic p - p scattering
$\csc\phi_{in} = \frac{P\sqrt{1 + 2\,M\,/\,T}}{W\,(1 + k\,M\,/\,T)}$
$k = (2m + \delta)\,\delta\,/\,(2\,M\,W)$
$\delta = 135$ MeV
$\csc\phi_{el} = \frac{P}{W}\sqrt{1 + 2\,M\,/\,T}$
$\left(\frac{k M}{1 - k},\ \arcsin\left(\frac{W}{P}\sqrt{k\,(2 - k)}\right)\right)$
p+p → p*+p, m* = m+135 MeV
p+p → p+p, all at $E = 50$ GeV
Inelastic angle minus elastic angle:
$\sin\phi_{in} - \sin\phi_{el} = \frac{0.5\,(2m + \delta)\,\delta}{P\sqrt{(2M + T)\,T}}$
φ (degree)
30
20
10
0
50
100
150
200
T (MeV)
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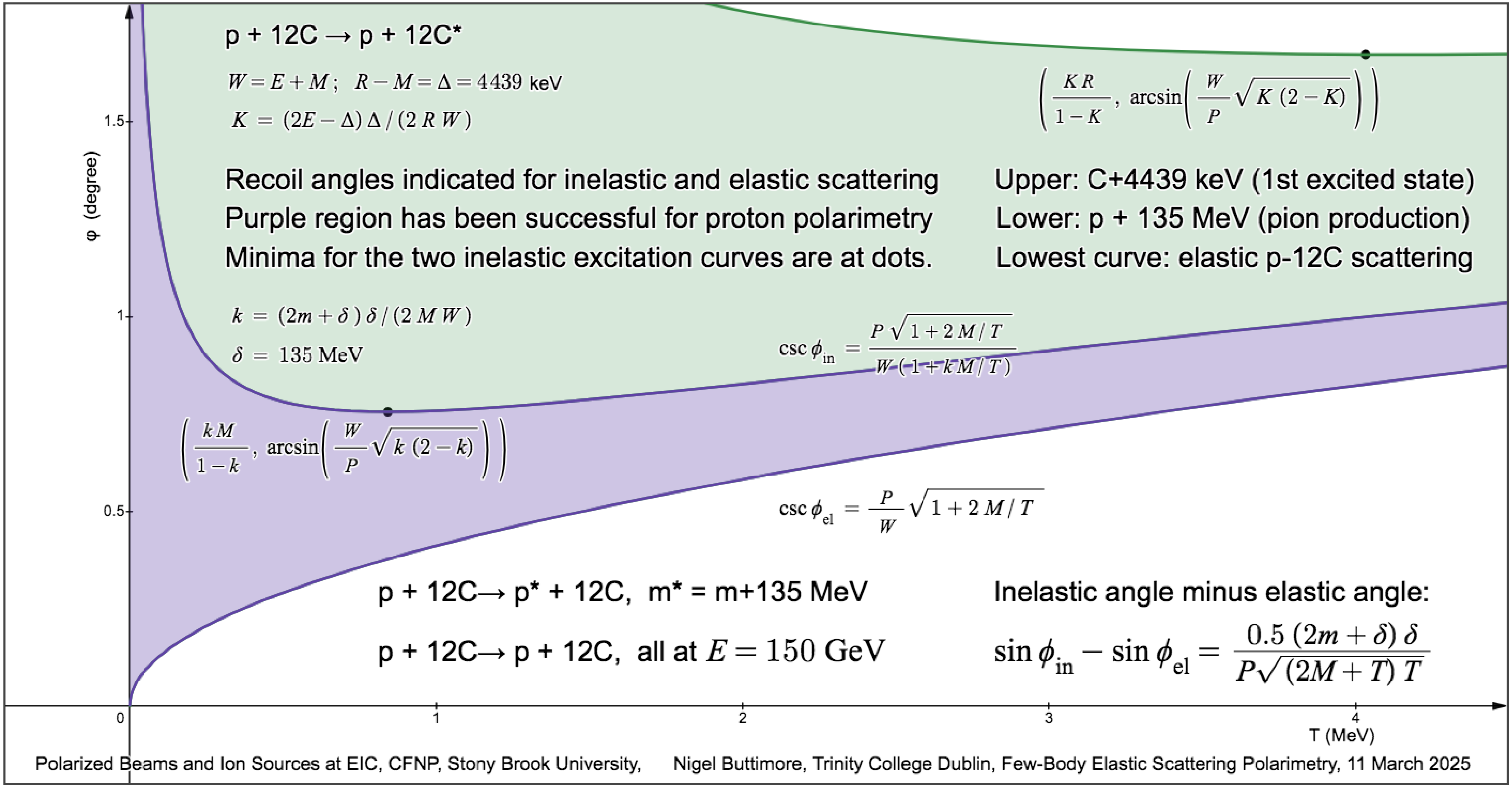
p + 12C → p + 12C*
$W = E + M; \quad R - M = \Delta = 4439$ keV
$K = (2E - \Delta)\,\Delta\,/\,(2\,R\,W)$
$\left(\frac{K\,R}{1-K},\ \arcsin\left(\frac{W}{P}\sqrt{K\,(2-K)}\right)\right)$
Recoil angles indicated for inelastic and elastic scattering
Purple region has been successful for proton polarimetry
Minima for the two inelastic excitation curves are at dots.
Upper: C+4439 keV (1st excited state)
Lower: p + 135 MeV (pion production)
Lowest curve: elastic p-12C scattering
$k = (2m + \delta)\,\delta\,/\,(2\,M\,W)$
$\delta = 135$ MeV
$\csc\phi_{in} = \frac{P\sqrt{1 + 2\,M\,/\,T}}{W\,(1 + k\,M\,/\,T)}$
$\left(\frac{k\,M}{1-k},\ \arcsin\left(\frac{W}{P}\sqrt{k\,(2-k)}\right)\right)$
$\csc\phi_{el} = \frac{P}{W}\sqrt{1 + 2\,M\,/\,T}$
p + 12C→ p* + 12C, m* = m+135 MeV
p + 12C→ p + 12C, all at $E = 150$ GeV
Inelastic angle minus elastic angle:
$\sin\phi_{in} - \sin\phi_{el} = \frac{0.5\,(2m + \delta)\,\delta}{P\sqrt{(2M + T)\,T}}$
φ (degree)
1.5
1
0.5
0
1
2
3
4
T (MeV)
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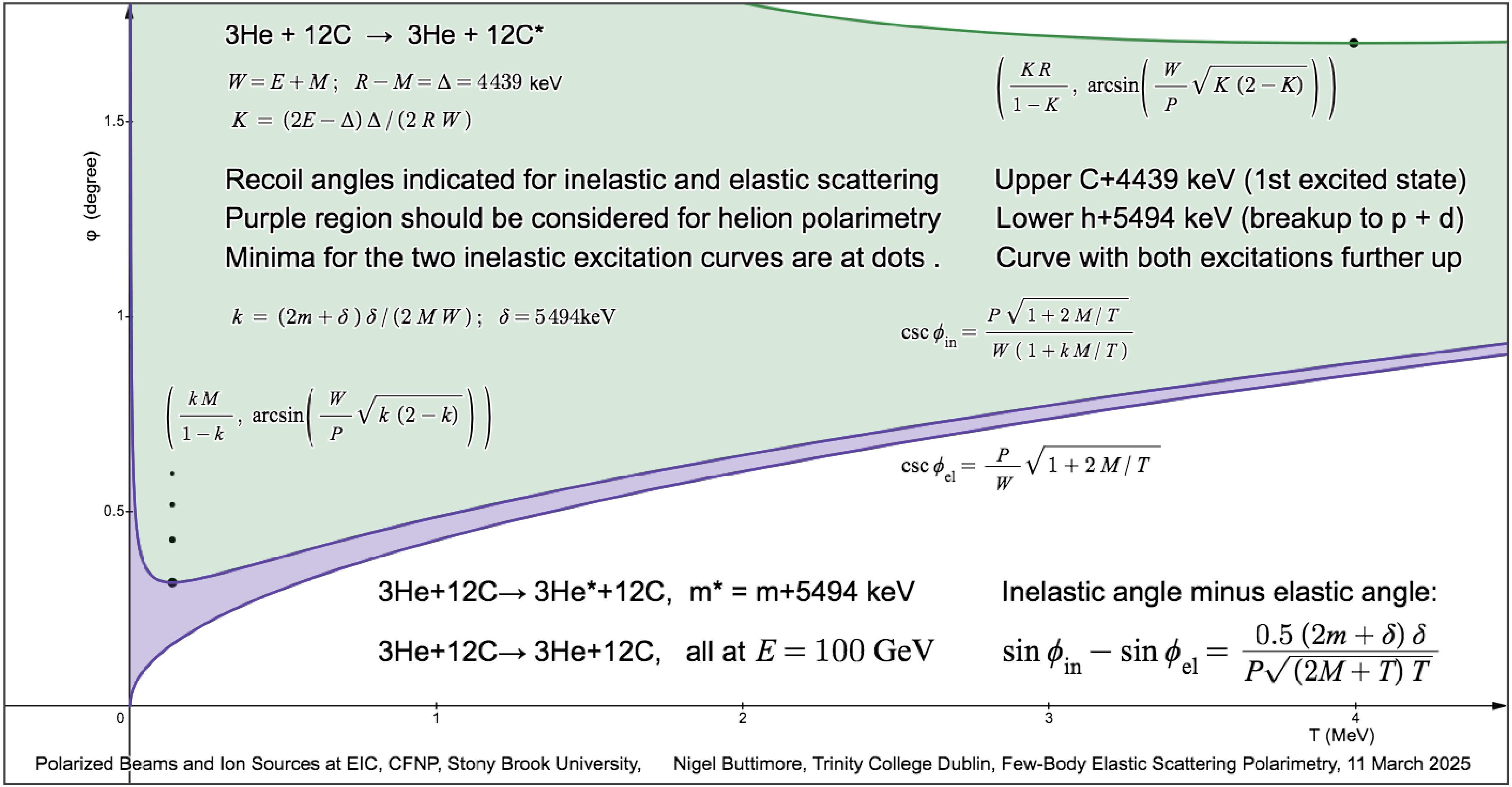
3He + 12C → 3He + 12C*
$W = E + M$; $R - M = \Delta = 4439$ keV
$K = (2E - \Delta)\,\Delta\,/\,(2\,R\,W)$
$\left(\frac{K R}{1-K},\ \arcsin\left(\frac{W}{P}\sqrt{K\,(2-K)}\right)\right)$
Recoil angles indicated for inelastic and elastic scattering
Purple region should be considered for helion polarimetry
Minima for the two inelastic excitation curves are at dots .
Upper C+4439 keV (1st excited state)
Lower h+5494 keV (breakup to p + d)
Curve with both excitations further up
$k = (2m + \delta)\,\delta\,/\,(2\,M\,W)$; $\delta = 5\,494\text{keV}$
$\csc\phi_{\text{in}} = \frac{P\sqrt{1 + 2\,M/T}}{W\,(1 + k\,M/T)}$
$\left(\frac{k M}{1-k},\ \arcsin\left(\frac{W}{P}\sqrt{k\,(2-k)}\right)\right)$
$\csc\phi_{\text{el}} = \frac{P}{W}\sqrt{1 + 2\,M/T}$
3He+12C→ 3He*+12C, m* = m+5494 keV
3He+12C→ 3He+12C, all at $E = 100$ GeV
Inelastic angle minus elastic angle:
$\sin\phi_{\text{in}} - \sin\phi_{\text{el}} = \frac{0.5\,(2m+\delta)\,\delta}{P\sqrt{(2M+T)\,T}}$
φ (degree)
1.5
1
0.5
0
1
2
3
4
T (MeV)
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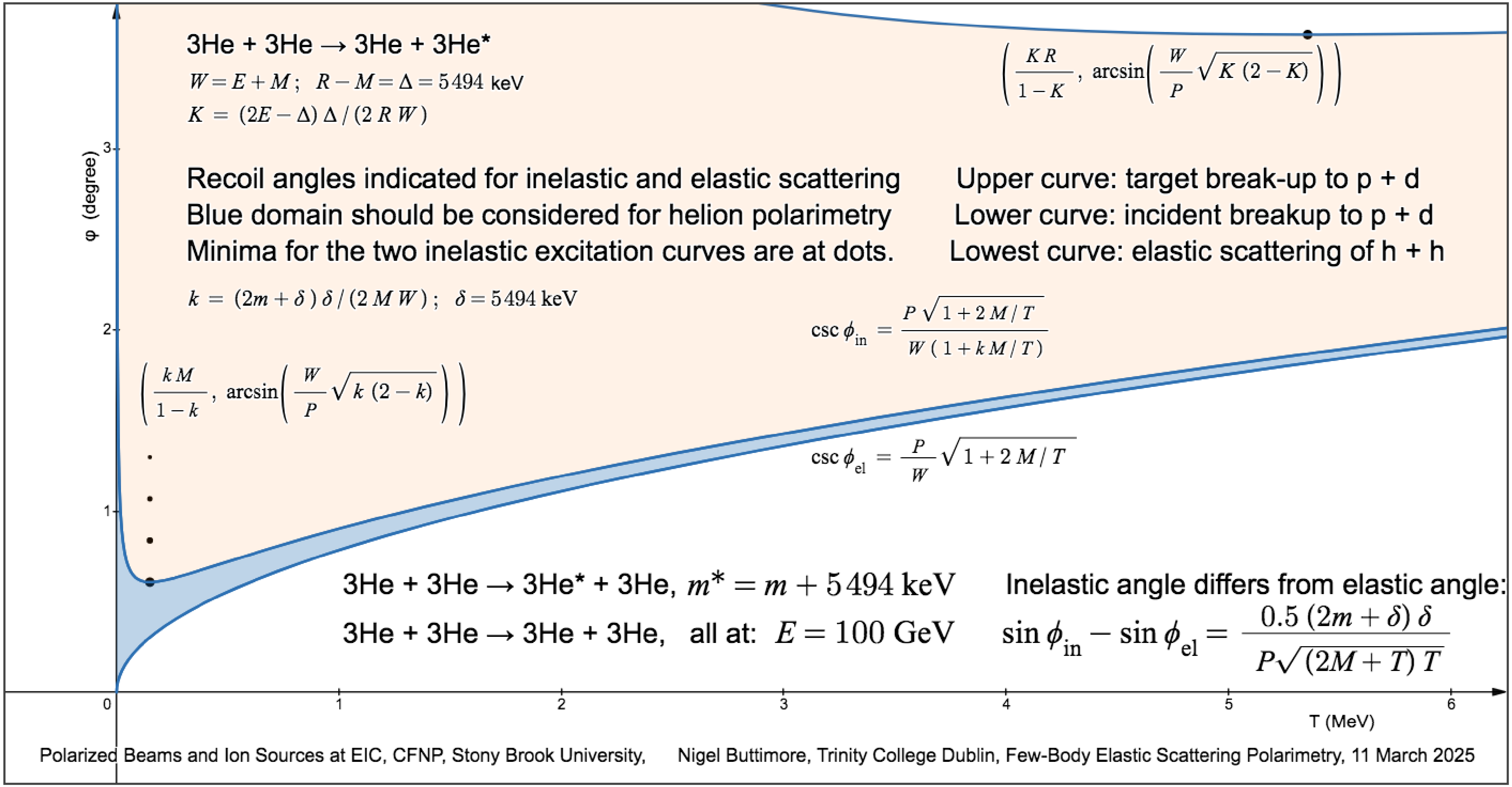
3He + 3He → 3He + 3He*
$W = E + M\,;\;\; R - M = \Delta = 5494$ keV
$K = (2E - \Delta)\,\Delta\,/\,(2\,R\,W)$
$\left(\frac{K\,R}{1-K},\ \arcsin\left(\frac{W}{P}\sqrt{K\,(2-K)}\right)\right)$
Recoil angles indicated for inelastic and elastic scattering
Blue domain should be considered for helion polarimetry
Minima for the two inelastic excitation curves are at dots.
Upper curve: target break-up to p + d
Lower curve: incident breakup to p + d
Lowest curve: elastic scattering of h + h
$k = (2m + \delta)\,\delta\,/\,(2\,M\,W)\,;\;\; \delta = 5494$ keV
$\csc\phi_{\rm in} = \frac{P\sqrt{1 + 2\,M/T}}{W\,(1 + k\,M/T)}$
$\left(\frac{k\,M}{1-k},\ \arcsin\left(\frac{W}{P}\sqrt{k\,(2-k)}\right)\right)$
$\csc\phi_{\rm el} = \frac{P}{W}\sqrt{1 + 2\,M/T}$
3He + 3He → 3He* + 3He, $m^* = m + 5\,494$ keV
3He + 3He → 3He + 3He, all at: $E = 100$ GeV
Inelastic angle differs from elastic angle:
$\sin\phi_{\rm in} - \sin\phi_{\rm el} = \frac{0.5\,(2m+\delta)\,\delta}{P\sqrt{(2M+T)\,T}}$
φ (degree)
T (MeV)
0
1
2
3
4
5
6
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Few-body elastic scattering polarimetry

Conclusions

1. Absolute polarimetry required to assess hadronic spin contribution
2. Accurate total cross sections needed for precise analyzing powers
3. Polarization levels above nuclear excited states should be studied
4. A thorough understanding of the small corrections would be useful

Thank you

Polarized Ion Sources and Beams at EIC - Stony Brook University

(E, P) Projectile Energy and Momentum satisfy: $E^2 = P^2 + m^2$

EXTREMUM OF THE RECOIL ANGLE IN TERMS OF RECOIL KINETIC ENERGY

(QU, Q) Recoil Energy & Momentum, $U = c/v$, v recoil velocity

$(m, m+\delta)$ Projectile & Ejectile masses; $a = m^2$, $c = (m+\delta)^2$

δ and Δ: first excited states or break-up level of the projectile m and target M

$(M, M+\Delta = R)$ Target and Recoil masses; $b = M^2$, $d = R^2$

$4b\,P\,Q\sin\phi = (s-a-b)(b+d-t) + 2b(u-a-d)$

Recoil angle $\theta = 90^o - \phi$ in terms of Recoil Momentum Q and invariants s, t, u

$s - a - b = 2ME$, and $b + d - t = 2M\,QU = 2M(T+R)$

$u - a - d = (b+d-t) - (s-a-b) - (d+b) + (c-a)$

Invariants in terms of reciprocal Velocity U, Incident E, and Recoil Energy QU

$= 2M\,QU - 2ME - 2MR - \Delta^2 + (2m+\delta)\,\delta$

$PQ\sin\phi = (E+M)QU - ER - MR + (E - \Delta/2)\Delta + (m + \delta/2)\delta$

Recoil angle ϕ in terms of U, E, QU, and the excited state increments δ, Δ

$P\sin\phi = WU - [WR - (E - \Delta/2)\,\Delta - (m+\delta/2)\,\delta]\sqrt{UU-1}\,/\,R$

Define: $k = [\,(E-\Delta)\,\Delta - (m+\delta/2)\,\delta\,]\,/\,(\,WR\,)$, and hence

Define k to find an extremum with respect to U, and wrt recoil kinetic energy T

$P\sin\phi\,/\,W = U - (1-k)\sqrt{UU-1}$, extremum $d\sin\phi/dU = 0$

$1 = (1-k)\,U_e/\sqrt{U_eU_e - 1}$, leading to: $U_e = 1/\sqrt{k(2-k)}$

The extreme recoil kinetic energy is: $T_e = RU_e/\sqrt{U_eU_e-1} - R = R/k - R$

and hence: $T_e = kR/(1-k)$, with: $\sin\phi_e = W\sqrt{k\,(2-k)}\,/\,P$

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