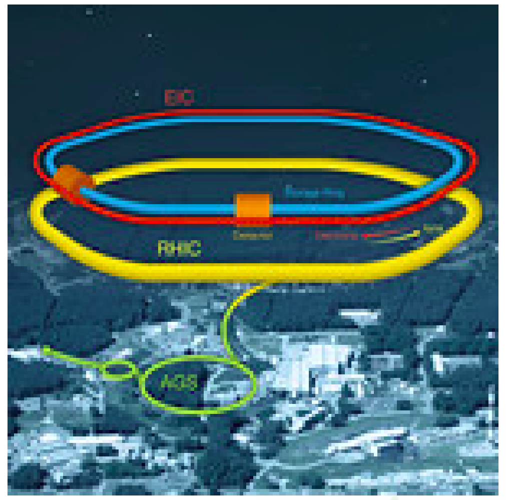

2025 CFNS-SURGE Summer Workshop on the
Physics of the Electron-Ion Collider

Electroweak gauge boson production in hadronic collisions at forward rapidity

YAN B. BANDEIRA [1,2]
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Based on: *Y. B. Bandeira, V. P. Gonçalves and W. Schäfer* [JHEP 07 (2024) 171]
Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [Phys.Rev.D 111 (2025) 7, 074041]

$$d\sigma(h_A h_B \to H_1 H_2 X) \propto f_{a/A}(x_1) \otimes d\sigma(aB \to bc) \otimes D_{H_1/b} \otimes D_{H_2/c}$$

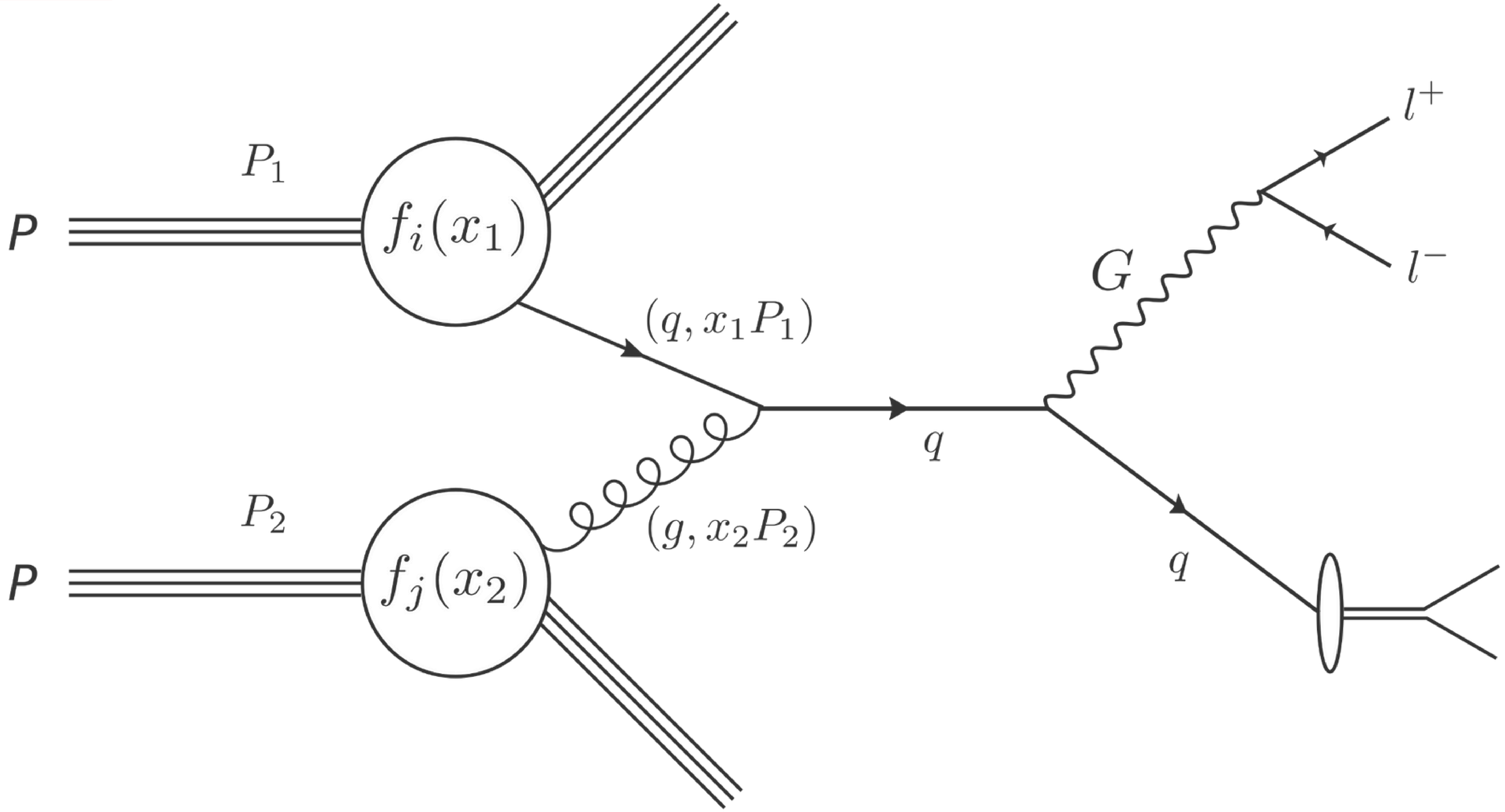
parton - target cross-section

projectile proton PDF

Forward rapidity

Hybrid factorization

$$x_1 \propto e^{\eta}$$
$$x_2 \propto e^{-\eta}$$



In the forward rapidity region, one expect the violation of collinear factorization

$$a \to bc$$

Color-dipole *S*-matrix framework

The master dijet production in the color - dipole *S* - matrix framework is given by

$$\frac{d\sigma(a\to b(p_b)c(p_c))}{dz d^2\boldsymbol{p}_b d^2\boldsymbol{p}_c} = \frac{1}{(2\pi)^4}\int d^2\boldsymbol{b}_b d^2\boldsymbol{b}_c d^2\boldsymbol{b}'_b d^2\boldsymbol{b}'_c \exp[i\boldsymbol{p}_b\cdot(\boldsymbol{b}_b-\boldsymbol{b}'_b)+i\boldsymbol{p}_c\cdot(\boldsymbol{b}_c-\boldsymbol{b}'_c)]$$

$$\times\Psi(z,\boldsymbol{b}_b-\boldsymbol{b}_c)\Psi^*(z,\boldsymbol{b}'_b-\boldsymbol{b}'_c)\left\{S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b,\boldsymbol{b}'_c,\boldsymbol{b}_b,\boldsymbol{b}_c)+S^{(2)}_{\bar{a}a}(\boldsymbol{b}',\boldsymbol{b})-S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b},\boldsymbol{b}'_b,\boldsymbol{b}'_c)-S^{(3)}_{\bar{a}bc}(\boldsymbol{b}',\boldsymbol{b}_b,\boldsymbol{b}_c)\right\}$$

N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [*J. Exp. Theor. Phys. 97, 441-465 (2003)*]

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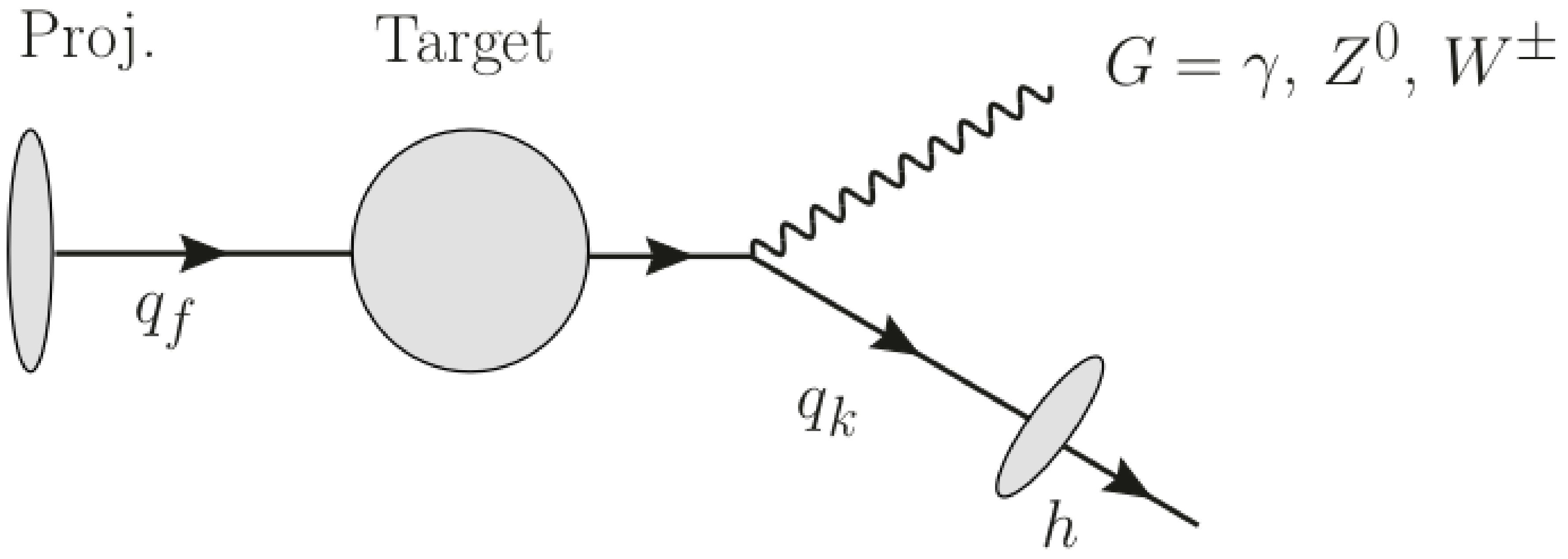
Our work was evaluate the case: $q_f \to G q_k$

Gauge boson production



Our work was use this formalism to an electroweak gauge boson production, for this case the cross-section expression simplifies as:

$$\frac{d\sigma^f_{T,L}(q_f N \to G(p_G) q_k(p_q))}{dz d^2\boldsymbol{p} d^2\boldsymbol{\Delta}} = \frac{1}{2(2\pi)^4} \int d^2\boldsymbol{r} d^2\boldsymbol{r}' \exp[-i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')] \overline{\sum_{\text{pol.}}} \Psi_{T,L}(z,\boldsymbol{r}) \Psi^*_{T,L}(z,\boldsymbol{r}')$$

$$\times \int d^2 s \exp\left[-i\boldsymbol{\Delta}\cdot\boldsymbol{s}\right] \left\{ \sigma_{q\bar{q}}(\boldsymbol{s}-z\boldsymbol{r},x) + \sigma_{q\bar{q}}(\boldsymbol{s}+z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(\boldsymbol{s}-z(\boldsymbol{r}-\boldsymbol{r}'),x) - \sigma_{q\bar{q}}\boldsymbol{s} \right\}$$

where, we connect the S-matrix with dipole cross-section by

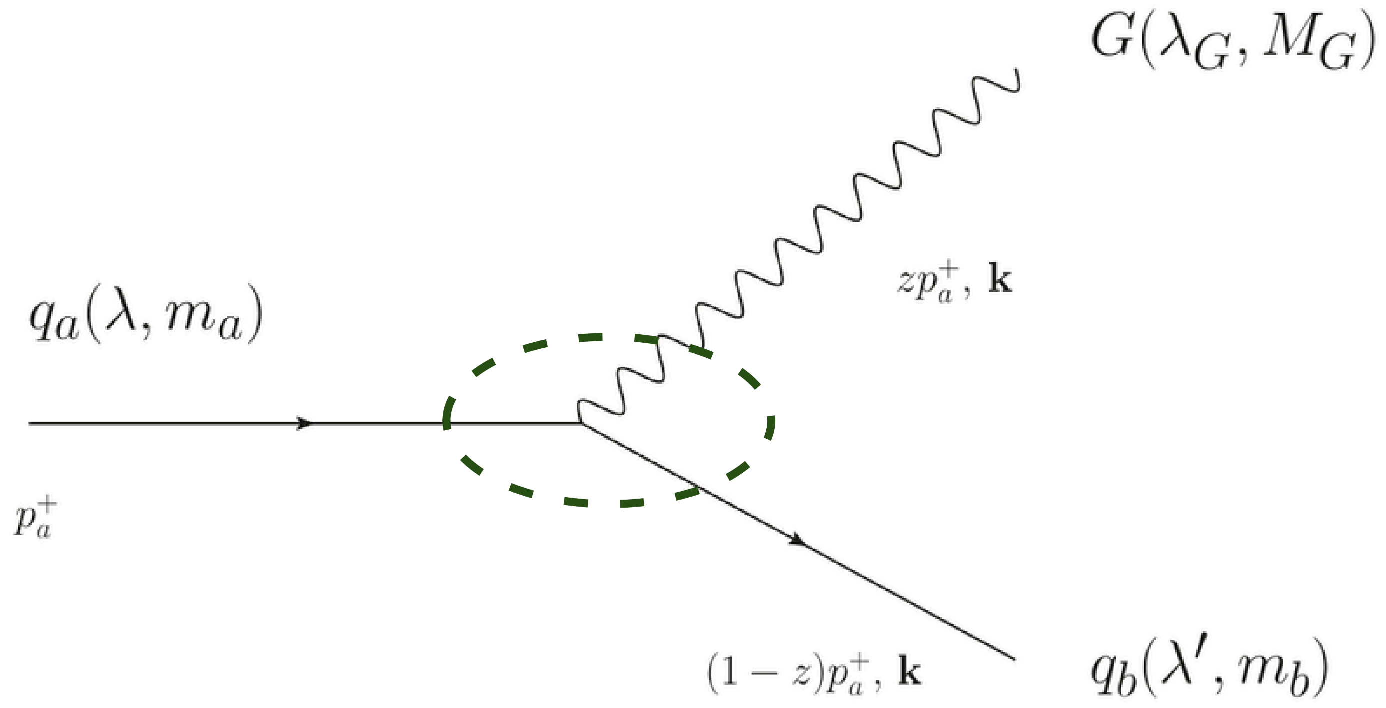
$$\sigma(\boldsymbol{r}) = 2\int d^2\boldsymbol{B} \left[1 - S^{(2)}_{q\bar{q}}\left(\boldsymbol{B}+\frac{\boldsymbol{r}}{2}, \boldsymbol{B}-\frac{\boldsymbol{r}}{2}\right)\right]$$

the dipole cross-section is model dependent. Therefore,

The unknown ingredient is the Wave Function!

Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [JHEP 07 (2024) 171]

Light Front Wave Function



$$\Psi_V(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)}\,\frac{\Gamma_V}{\boldsymbol{k}^2+\epsilon^2}$$

$$\Psi_A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \sqrt{z(1-z)}\,\frac{\Gamma_A}{\boldsymbol{k}^2+\epsilon^2}$$

$$\Gamma_V = E_\mu^*(k,\lambda)\overline{u}(p_b,\lambda',m_b)\left\{\gamma^\mu + (m_b-m_a)\frac{k^\mu}{M^2}\right\}u(p_a,\lambda,m_a)$$

$$\Gamma_A = E_\mu^*(k,\lambda)\overline{u}(p_b,\lambda',m_b)\left\{\left[\gamma^\mu + (m_b+m_a)\frac{k^\mu}{M^2}\right]\gamma_5\right\}u(p_a,\lambda,m_a)$$

$$\Psi_V^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left(\left(\frac{2-z}{z}\right)\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G) + i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right)I - \lambda\left[m_b-(1-z)m_a\right]\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_A^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left[\frac{2-z}{z}\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G) + i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right]\lambda I - (m_b+(1-z)m_a)\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_V^L(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{I\left[\frac{z^2 m_a(m_b-m_a) - z\left(m_b^2-m_a^2\right) - 2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right] + \frac{[z(m_b-m_a)]}{\boldsymbol{k}^2+\epsilon^2}\lambda(\boldsymbol{\sigma}\cdot\boldsymbol{k})\right\}\chi_\lambda,$$

$$\Psi_L^A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{\lambda I\left[-\frac{z^2 m_a(m_b+m_a) + z\left(m_b^2-m_a^2\right) + 2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right] - z(m_b+m_a)\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k})}{\boldsymbol{k}^2+\epsilon^2}\right\}\chi_\lambda.$$

Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [JHEP 07 (2024) 171]

Parton - level cross-section

Isolated production

$$\rho^{T,L}_{V,A} = \frac{1}{2}\sum_{\lambda\lambda'\lambda_G} \psi^{T,L}_{V,A}(z,\boldsymbol{r})\psi^{T,L,*}_{V,A}(z,\boldsymbol{r}')$$

$$\sigma_{q\bar{q}}(\boldsymbol{r}) = \int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)$$

$$\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) \equiv \frac{1}{2}\left[\frac{1}{p^2+\epsilon^2} - \frac{1}{(\boldsymbol{p}-z\boldsymbol{k})^2+\epsilon^2}\right]^2$$

$$\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z) \equiv \frac{1}{2}\left[\frac{\boldsymbol{p}}{p^2+\epsilon^2} - \frac{\boldsymbol{p}-z\boldsymbol{k}}{(\boldsymbol{p}-\boldsymbol{k})^2+\epsilon^2}\right]^2$$

$$\left.\frac{d\sigma^f_{T,L}}{dz\, d^2\boldsymbol{p}}\right|_{V,A} = \frac{1}{2(2\pi)^2}\int d^2\boldsymbol{r}\int d^2\boldsymbol{r}' e^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{T,L}_{V,A}\left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

↓

$$z\frac{\mathrm{d}\sigma^f_T}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{z^2[(m_b-m_a)+zm_a]^2\,\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + [1+(1-z)^2]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_T}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_A = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{z^2[(m_b+m_a)-zm_a]^2\,\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + [1+(1-z)^2]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_L}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{\frac{(z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_L}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_A = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{\frac{(z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)-2(1-z)M_G^2)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

A generalized description for all electroweak gauge boson

Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [Phys.Rev.D 111 (2025) 7, 074041]

Parton - level cross-section

Associated production

$$\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z) \equiv \frac{1}{2}\left[\frac{1}{p^2+\epsilon^2} - \frac{1}{(\boldsymbol{p}-z\boldsymbol{\Delta})^2+\epsilon^2}\right]^2$$

$$\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z) \equiv \frac{1}{2}\left[\frac{\boldsymbol{p}}{p^2+\epsilon^2} - \frac{\boldsymbol{p}-z\boldsymbol{\Delta}}{(\boldsymbol{p}-z\boldsymbol{\Delta})^2+\epsilon^2}\right]^2$$

$$\frac{\mathrm{d}\sigma^f_{T,L}(q_a\to G(p_G)q_b(p_q))}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}} = \frac{1}{2(2\pi)^4}\int \mathrm{d}^2\boldsymbol{k}\int \mathrm{d}^2\boldsymbol{r}\mathrm{d}^2\boldsymbol{r}'\, e^{-i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\left[\rho_V^{T,L}(z,\boldsymbol{r},\boldsymbol{r}') + \rho_A^{T,L}(z,\boldsymbol{r},\boldsymbol{r}')\right]\int \mathrm{d}^2\boldsymbol{s}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{s}}\left\{e^{i\boldsymbol{k}\cdot\boldsymbol{s}} + e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-z(\boldsymbol{r}-\boldsymbol{r}'))} - e^{i\boldsymbol{k}\cdot(\boldsymbol{s}+z\boldsymbol{r}')} - e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-z\boldsymbol{r})}\right\} f(x,\boldsymbol{k})$$

↓

$$z\frac{\mathrm{d}\sigma^f_T(q_a\to Gq_b)}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_V = \frac{(C_f^G)^2(g^G_{V,f})^2}{2\pi^2} f(x,\boldsymbol{\Delta})\left\{z^2\left[(m_b-m_a)^2+zm_a\right]^2\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z) + \left[1+(1-z)^2\right]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)\right\}$$

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$$z\frac{\mathrm{d}\sigma^f_L(q_a\to Gq_b)}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_V = \frac{(C_f^G)^2(g^G_{V,f})^2}{(2\pi)^2} f(x,\boldsymbol{\Delta})\left\{\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)\right\}$$

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We presented for the first time the expresion for the $W^{\pm}$ production

Summary

- We derived, for the *first time*, the generic expressions for the **LFWF's**.
  - We have estimated the *vector* and *axial* contributions for the description of the longitudinal and transverse spectra associated with the isolated gauge boson production in the impact parameter and transverse momentum spaces.e

- We demonstrated that our results reduce to expressions previously used in the literature for the description of the *real photon production* and *Drell - Yan* process at forward rapidities in some particular limits.

- As seen, the expressions obtained are the main ingredients for the calculation of the *pp* cross - sections, which can be compared with the current and forthcoming LHC data.

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ACKNOWLEDGEMENTS

CAPES

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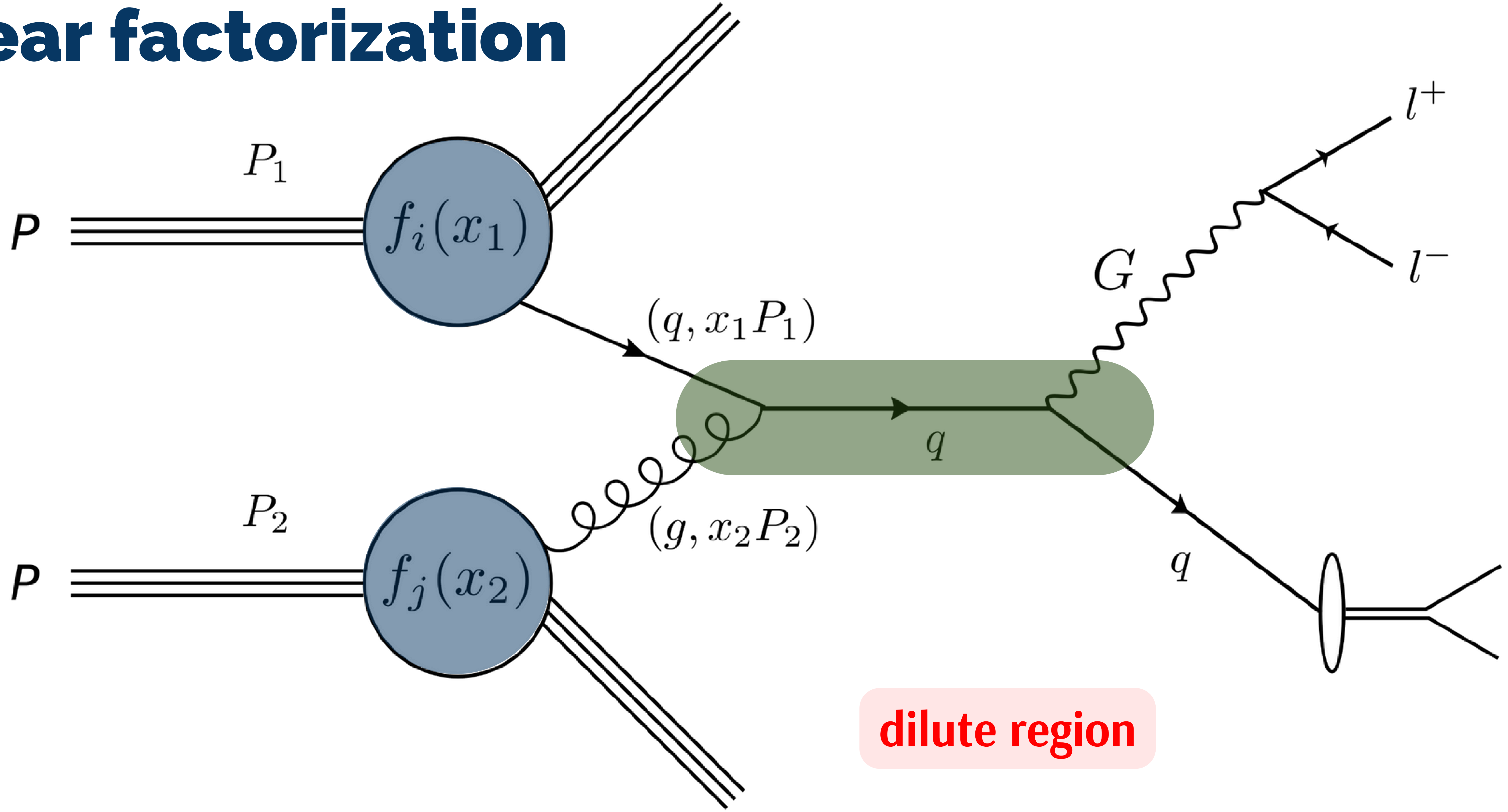
RS-BRASIL

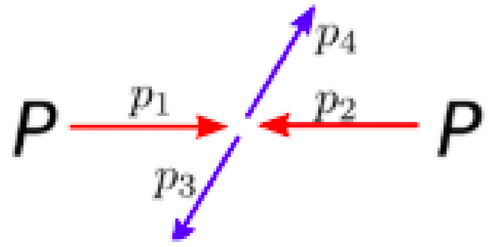
PPGFís

Extra slides

$$G = \gamma,\ Z^0,\ W^{\pm}$$

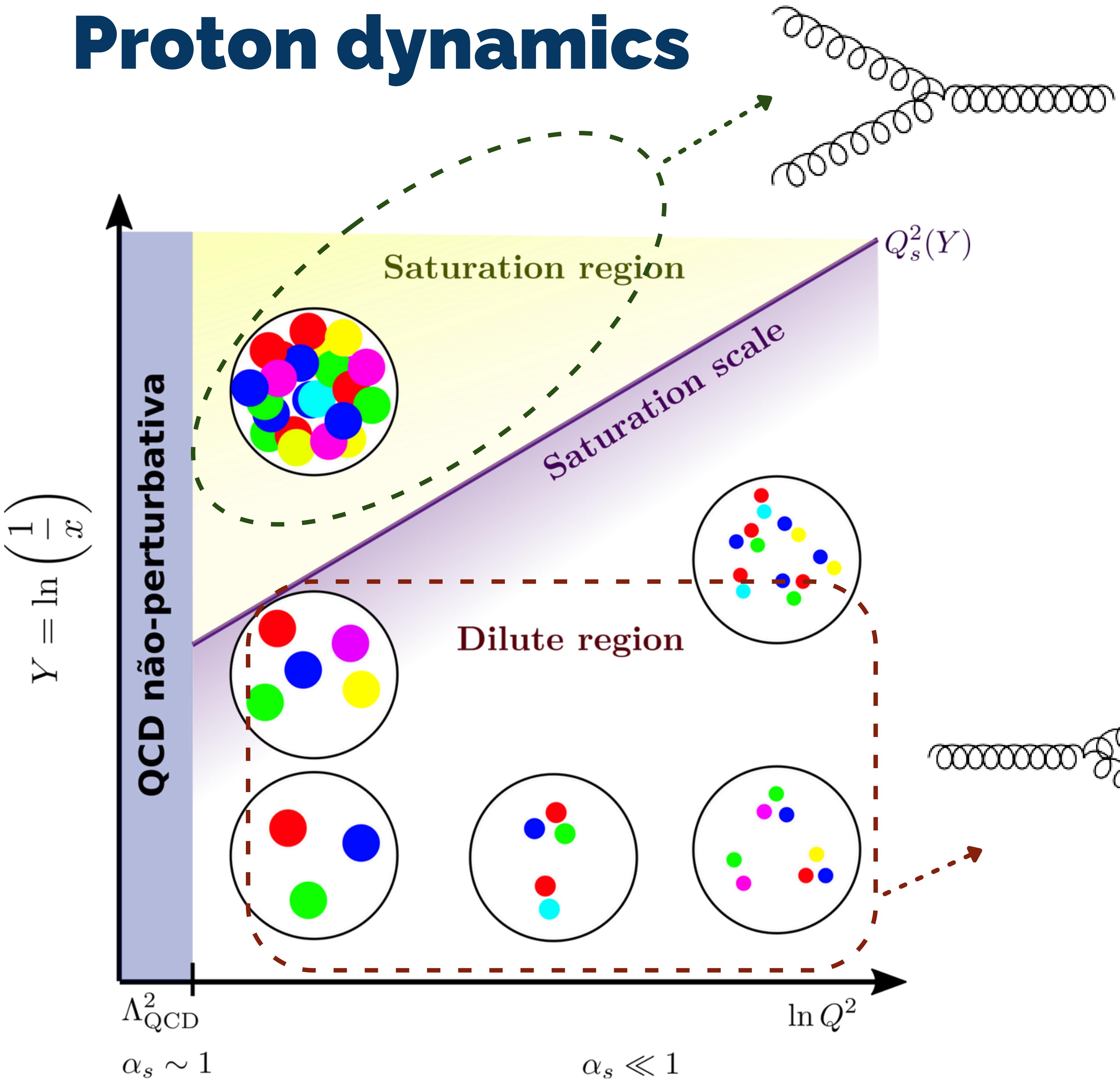
Collinear factorization

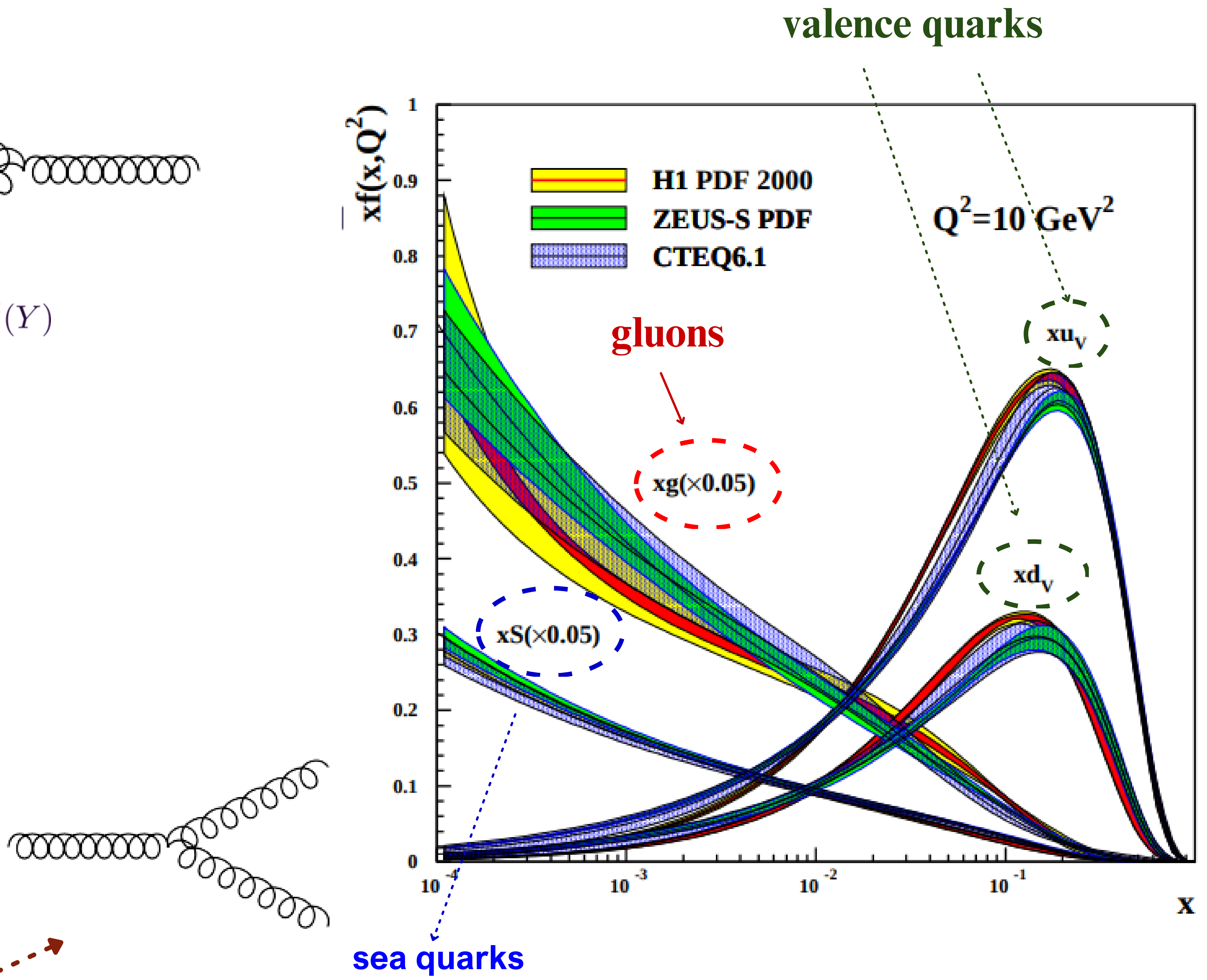




$$\sigma_{AB\to GX} = \int_{z_{\min}}^{1} \frac{\mathrm{d}z}{z^2} \mathcal{D}^{k/Q}(z,\mu^2) \sum_{i,j} \int_0^1 \mathrm{d}x_a \mathrm{d}x_b f_{i/A}(x_i,\mu_F) f_{j/B}(x_j \mu_F) \hat{\sigma}_{ij\to GX}(\mu_F,\mu)$$

Proton dynamics





$$d\sigma(h_A h_B \to H_1 H_2 X) \propto f_{a/A}(x_1) \otimes d\sigma(aB \to bc) \otimes D_{H_1/b} \otimes D_{H_2/c}$$

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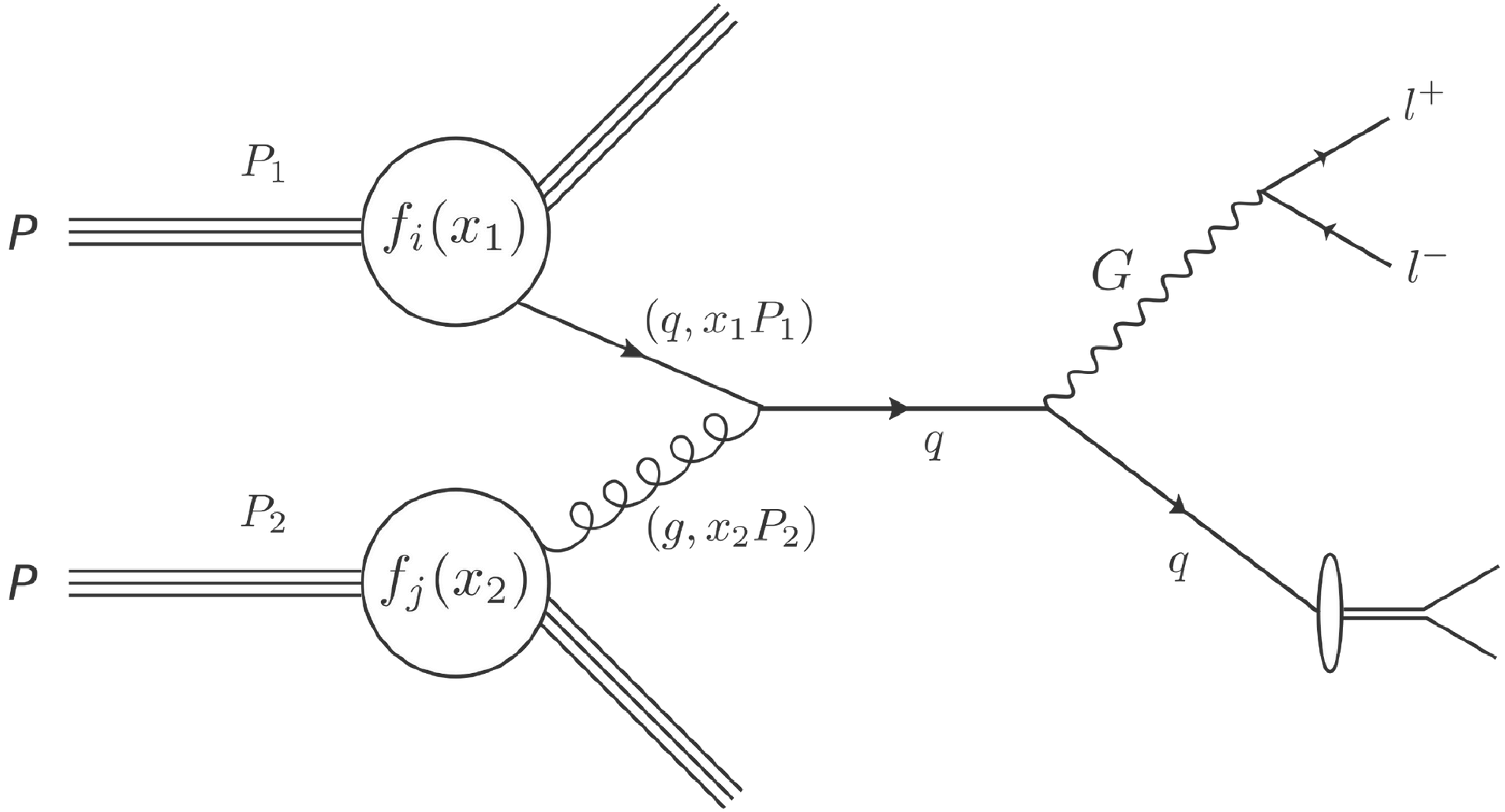
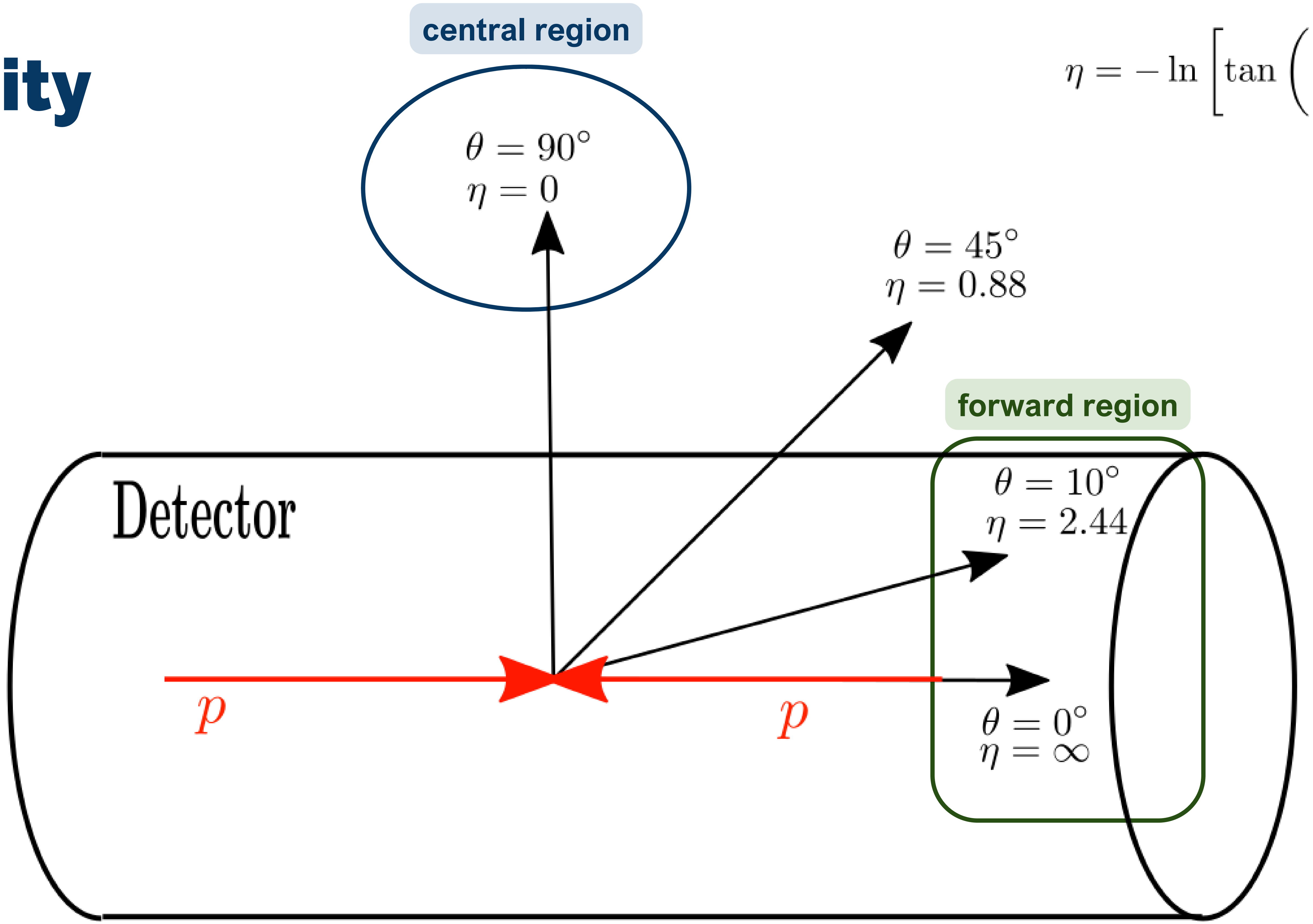


In the forward rapidity region, one expect the violation of collinear factorization

Forward rapidity

$$x_1 \propto e^{\eta}$$
$$x_2 \propto e^{-\eta}$$

$$\eta = -\ln\left[\tan\left(\frac{\theta}{2}\right)\right]$$



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$$\times \Psi(z, \boldsymbol{b}_b - \boldsymbol{b}_c)\Psi^*(z, \boldsymbol{b}'_b - \boldsymbol{b}'_c) \left\{ S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b, \boldsymbol{b}'_c, \boldsymbol{b}_b, \boldsymbol{b}_c) + S^{(2)}_{\bar{a}a}(\boldsymbol{b}', \boldsymbol{b}) - S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b}, \boldsymbol{b}'_b, \boldsymbol{b}'_c) - S^{(3)}_{\bar{a}bc}(\boldsymbol{b}', \boldsymbol{b}_b, \boldsymbol{b}_c) \right\}$$

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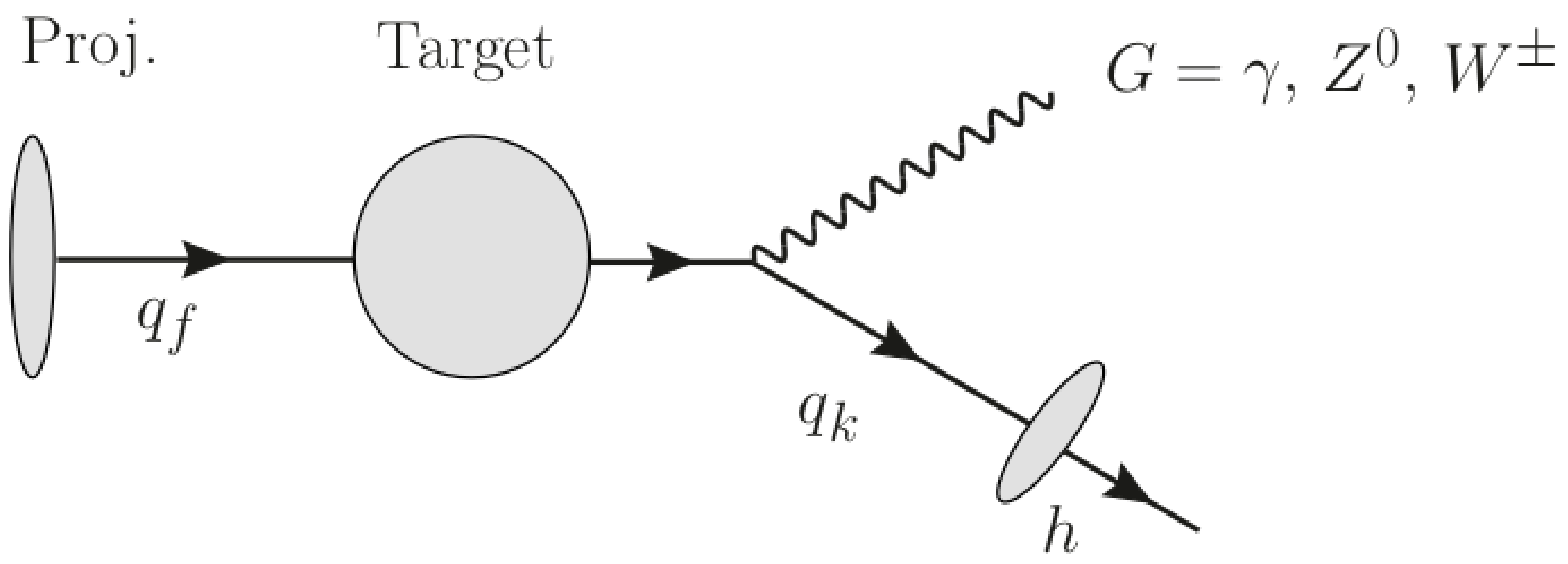

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$$\times \int d^2 s \exp\left[-i\boldsymbol{\Delta}\cdot\boldsymbol{s}\right] \left\{\sigma_{q\bar{q}}(\boldsymbol{s}-z\boldsymbol{r},x) + \sigma_{q\bar{q}}(\boldsymbol{s}+z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(\boldsymbol{s}-z(\boldsymbol{r}-\boldsymbol{r}'),x) - \sigma_{q\bar{q}}\boldsymbol{s}\right\}$$

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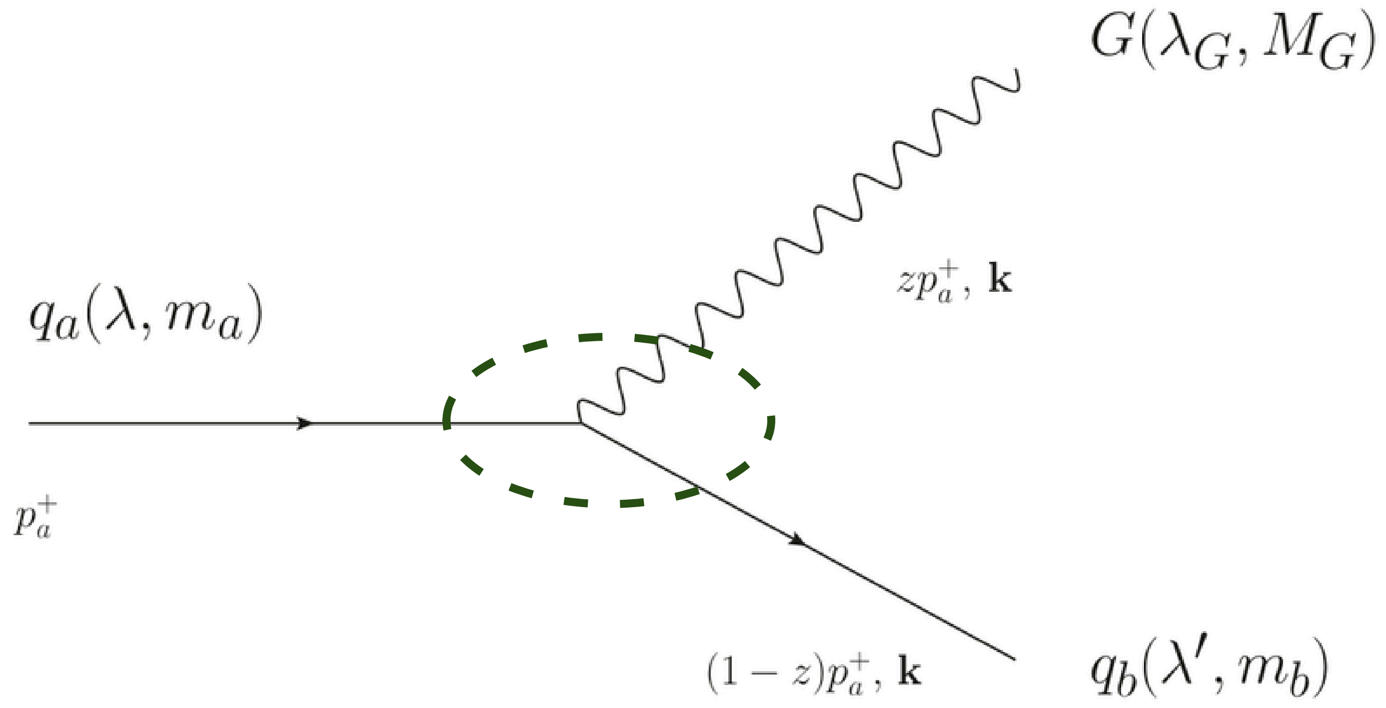
the dipole cross-section is model dependent. Therefore,

The unknown ingredient is the Wave Function!

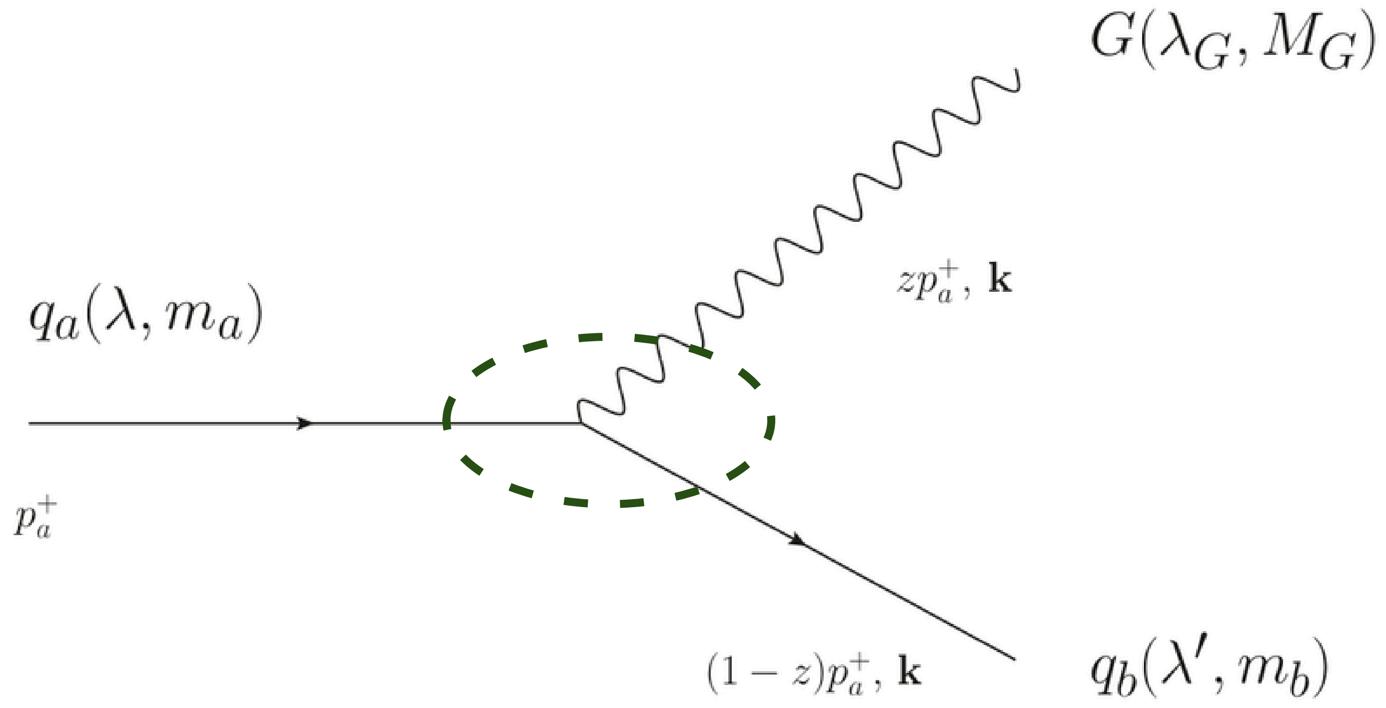
Light Front Wave Function

$$\Psi_V(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)}\frac{\Gamma_V}{\boldsymbol{k}^2+\epsilon^2}$$

$$\Psi_A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \sqrt{z(1-z)}\frac{\Gamma_A}{\boldsymbol{k}^2+\epsilon^2}$$



Light Front Wave Function



$$\Psi_V(z, \boldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)} \frac{\Gamma_V}{\boldsymbol{k}^2 + \epsilon^2}$$

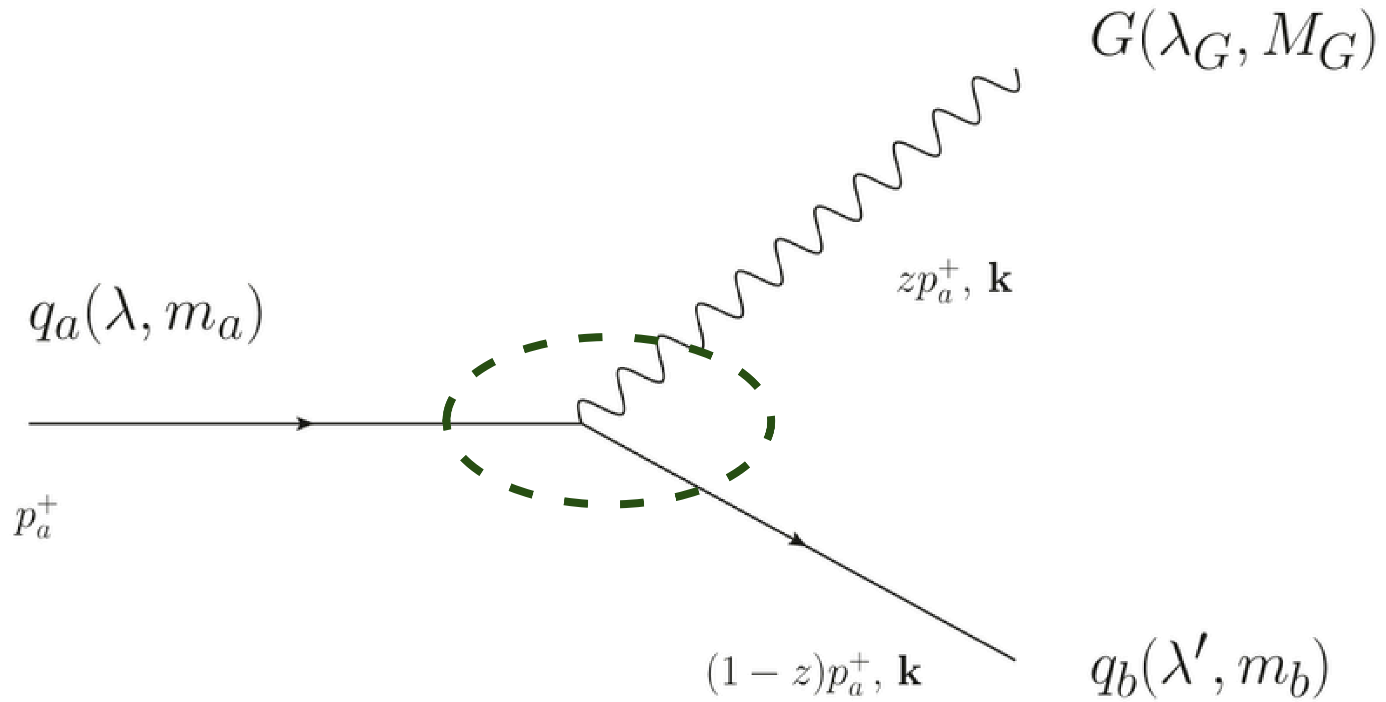
$$\Psi_A(z, \boldsymbol{k}) = C_f^G g_{A,f}^G \sqrt{z(1-z)} \frac{\Gamma_A}{\boldsymbol{k}^2 + \epsilon^2}$$

$$\Gamma_V = E_\mu^*(k, \lambda)\overline{u}(p_b, \lambda', m_b)\left\{\gamma^\mu + (m_b - m_a)\frac{k^\mu}{M^2}\right\}u(p_a, \lambda, m_a)$$

$$\Gamma_A = E_\mu^*(k, \lambda)\overline{u}(p_b, \lambda', m_b)\left\{\left[\gamma^\mu + (m_b + m_a)\frac{k^\mu}{M^2}\right]\gamma_5\right\}u(p_a, \lambda, m_a)$$

Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [JHEP 07 (2024) 171]

Light Front Wave Function



$$\Psi_V(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)}\frac{\Gamma_V}{\boldsymbol{k}^2+\epsilon^2}$$

$$\Psi_A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \sqrt{z(1-z)}\frac{\Gamma_A}{\boldsymbol{k}^2+\epsilon^2}$$

$$\Gamma_V = E_\mu^*(k,\lambda)\overline{u}(p_b,\lambda',m_b)\left\{\gamma^\mu + (m_b-m_a)\frac{k^\mu}{M^2}\right\}u(p_a,\lambda,m_a)$$

$$\Gamma_A = E_\mu^*(k,\lambda)\overline{u}(p_b,\lambda',m_b)\left\{\left[\gamma^\mu + (m_b+m_a)\frac{k^\mu}{M^2}\right]\gamma_5\right\}u(p_a,\lambda,m_a)$$

$$\Psi_V^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left(\left(\frac{2-z}{z}\right)\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G) + i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right)I - \lambda\left[m_b-(1-z)m_a\right]\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_A^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left[\frac{2-z}{z}\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G) + i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right]\lambda I - (m_b+(1-z)m_a)\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_V^L(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{I\left[\frac{z^2m_a(m_b-m_a) - z\left(m_b^2-m_a^2\right) - 2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right] + \frac{[z(m_b-m_a)]}{\boldsymbol{k}^2+\epsilon^2}\lambda(\boldsymbol{\sigma}\cdot\boldsymbol{k})\right\}\chi_\lambda,$$

$$\Psi_L^A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{\lambda I\left[-\frac{z^2m_a(m_b+m_a) + z\left(m_b^2-m_a^2\right) + 2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right] - z(m_b+m_a)\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k})}{\boldsymbol{k}^2+\epsilon^2}\right\}\chi_\lambda.$$

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$$\epsilon^2 = (1-z)M_G^2 + zm_b^2 - z(1-z)m_a^2 = (1-z)M_G^2 + z(m_b^2 - m_a^2) + z^2 m_a^2$$

Light Front Wave Function

Gauge Boson	$\mathcal{C}_f^G$	$g_{v,f}^G$	$g_{a,f}^G$
Z^0	$\mathcal{C}_f^Z = \frac{\sqrt{\alpha_{em}}}{\sin 2\theta_W}$	$g_{v,f_u}^Z = \frac{1}{2} - \frac{4}{3}\sin^2\theta_W$ $g_{v,f_d}^Z = -\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$g_{a,f_u}^Z = \frac{1}{2}$ $g_{a,f_d}^Z = -\frac{1}{2}$
$W^\pm$	$\mathcal{C}_f^{W^+} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_W} V_{f_u f_d}$ $\mathcal{C}_f^{W^-} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_W} V_{f_d f_u}$	$g_{v,f}^W = 1$	$g_{a,f}^W = 1$
Photon	$C_f^\gamma = \sqrt{\alpha_{em}} e_f$	$g_{v,f}^\gamma = 1$	$g_{a,f}^\gamma = 0$

$$\Psi_V^T(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left(\left(\frac{2-z}{z}\right)\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G) + i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right)I - \lambda\left[m_b - (1-z)m_a\right]\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_A^T(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \frac{\sqrt{z}}{\boldsymbol{k}^2+\epsilon^2}\chi_{\lambda'}^\dagger\left\{\left[\frac{2-z}{z}\boldsymbol{k}\cdot\boldsymbol{E}^*(\lambda_G) + i\lambda\left[\boldsymbol{k},\boldsymbol{E}^*(\lambda_G)\right]\right]\lambda I - (m_b + (1-z)m_a)\boldsymbol{\sigma}\cdot\boldsymbol{E}^*(\lambda_G)\right\}\chi_\lambda,$$

$$\Psi_V^L(z,\boldsymbol{k}) = C_f^G g_{V,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{I\left[\frac{z^2 m_a(m_b - m_a) - z(m_b^2 - m_a^2) - 2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right] + \frac{[z(m_b - m_a)]}{\boldsymbol{k}^2+\epsilon^2}\lambda(\boldsymbol{\sigma}\cdot\boldsymbol{k})\right\}\chi_\lambda,$$

$$\Psi_L^A(z,\boldsymbol{k}) = C_f^G g_{A,f}^G \frac{1}{\sqrt{z}M}\chi_{\lambda'}^\dagger\left\{\lambda I\left[-\frac{z^2 m_a(m_b + m_a) + z(m_b^2 - m_a^2) + 2(1-z)M^2}{\boldsymbol{k}^2+\epsilon^2}\right] - z(m_b + m_a)\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k})}{\boldsymbol{k}^2+\epsilon^2}\right\}\chi_\lambda.$$

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Parton - level cross-section

Isolated production

$$\frac{d\sigma^f_{T,L}(q_f N \to G(p_G) q_k(p_q))}{dz d^2\boldsymbol{p} d^2\boldsymbol{\Delta}} = \frac{1}{2(2\pi)^4} \int d^2\boldsymbol{r} d^2\boldsymbol{r}' \exp[-i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')] \overline{\sum_{\text{pol.}}} \Psi_{T,L}(z,\boldsymbol{r}) \Psi^*_{T,L}(z,\boldsymbol{r}')$$

$$\times \int d^2 s \exp\left[-i\boldsymbol{\Delta}\cdot\boldsymbol{s}\right] \left\{ \sigma_{q\bar{q}}(\boldsymbol{s}-z\boldsymbol{r},x) + \sigma_{q\bar{q}}(\boldsymbol{s}+z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(\boldsymbol{s}-z(\boldsymbol{r}-\boldsymbol{r}'),x) - \sigma_{q\bar{q}}\boldsymbol{s} \right\}$$

integrating over Δ ↓

$$\left.\frac{d\sigma^f_{T,L}}{dz\, d^2\boldsymbol{p}}\right|_{V,A} = \frac{1}{2(2\pi)^2} \int d^2\boldsymbol{r} \int d^2\boldsymbol{r}' e^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \rho^{T,L}_{V,A} \left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

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Parton - level cross-section

Isolated production

$$\rho^{T,L}_{V,A} = \frac{1}{2}\sum_{\lambda\lambda'\lambda_G} \psi^{T,L}_{V,A}(z,\boldsymbol{r})\psi^{T,L,*}_{V,A}(z,\boldsymbol{r}')$$

$$\sigma_{q\bar{q}}(\boldsymbol{r}) = \int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)$$

$$\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) \equiv \frac{1}{2}\left[\frac{1}{p^2+\epsilon^2} - \frac{1}{(\boldsymbol{p}-z\boldsymbol{k})^2+\epsilon^2}\right]^2$$

$$\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z) \equiv \frac{1}{2}\left[\frac{\boldsymbol{p}}{p^2+\epsilon^2} - \frac{\boldsymbol{p}-z\boldsymbol{k}}{(\boldsymbol{p}-\boldsymbol{k})^2+\epsilon^2}\right]^2$$

$$\left.\frac{d\sigma^f_{T,L}}{dz\, d^2\boldsymbol{p}}\right|_{V,A} = \frac{1}{2(2\pi)^2}\int d^2\boldsymbol{r}\int d^2\boldsymbol{r}' e^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{T,L}_{V,A}\left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

↓

$$z\frac{\mathrm{d}\sigma^f_T}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{z^2[(m_b-m_a)+zm_a]^2\,\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + [1+(1-z)^2]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_T}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_A = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{z^2[(m_b+m_a)-zm_a]^2\,\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + [1+(1-z)^2]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_L}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{\frac{(z^2m_a(m_b-m_a) - z(m_b^2-m_a^2) - 2(1-z)M_G^2)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_L}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_A = \frac{(\mathcal{C}^G_f)^2(g^G_{v,f})^2}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{\frac{(z^2m_a(m_b+m_a) + z(m_b^2-m_a^2) - 2(1-z)M_G^2)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

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Parton - level cross-section

Isolated production

$$\rho_{V,A}^{T,L} = \frac{1}{2}\sum_{\lambda\lambda'\lambda_G} \psi_{V,A}^{T,L}(z,\boldsymbol{r})\psi_{V,A}^{T,L,*}(z,\boldsymbol{r}')$$

$$\sigma_{q\bar{q}}(\boldsymbol{r}) = \int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left(1 - e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\right)$$

$$\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) \equiv \frac{1}{2}\left[\frac{1}{p^2+\epsilon^2} - \frac{1}{(\boldsymbol{p}-z\boldsymbol{k})^2+\epsilon^2}\right]^2$$

$$\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z) \equiv \frac{1}{2}\left[\frac{\boldsymbol{p}}{p^2+\epsilon^2} - \frac{\boldsymbol{p}-z\boldsymbol{k}}{(\boldsymbol{p}-\boldsymbol{k})^2+\epsilon^2}\right]^2$$

$$\left.\frac{d\sigma_{T,L}^f}{dz\, d^2\boldsymbol{p}}\right|_{V,A} = \frac{1}{2(2\pi)^2}\int d^2\boldsymbol{r}\int d^2\boldsymbol{r}' e^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho_{V,A}^{T,L}\left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

↓

$$z\frac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{z^2[(m_b-m_a)+zm_a]^2\,\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + [1+(1-z)^2]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_A = \frac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{2\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{z^2[(m_b+m_a)-zm_a]^2\,\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + [1+(1-z)^2]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma_L^f}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{\frac{(z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma_L^f}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_A = \frac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{4\pi^2}\int \mathrm{d}^2\boldsymbol{k}\, f(x,\boldsymbol{k})\left\{\frac{(z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)-2(1-z)M_G^2)^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

A generalized description for all electroweak gauge boson

Parton - level cross-section

Isolated production

Particular cases

Real photon production

$$\left.\frac{d\sigma_T^f}{d\ln z\, d^2\boldsymbol{p}}\right|_{qp\to\gamma X} = \frac{\alpha_{em}e_f^2}{2\pi^2}\{m_f^2 z^4\,\mathcal{D}_1(z,p,\epsilon) + [1+(1-z)^2]\epsilon^2\,\mathcal{D}_2(z,p,\epsilon)\}$$

J.Jalilian-Marian and A.H.Rezaeian [*Phys.Rev.D* 86 (2012) 034016]
V. P. Goncalves, Y.Lima, R.Pasechnik and M.Šumbera [*Phys.Rev.D* 101 (2020) 9, 094019]
B.Ducloué, T.Lappi and H.Mäntysaari [*Phys.Rev.D* 97 (2018) 5, 054023]

Drell-Yan process

$$\frac{d\sigma(qp\to[G\to l\bar{l}]X)}{dz d^2\boldsymbol{p} dM^2} = \mathcal{F}_G(M)\,\frac{d\sigma(qp\to GX)}{dz d^2\boldsymbol{p}}$$

F. Gelis and J. Jalilian-Marian, Phys. Rev. D 66, 094014 (2002)
R. Baier, A. H. Mueller and D. Schiff, Nucl. Phys. A 741, 358-380 (2004)
A. Stasto, B. W. Xiao and D. Zaslavsky, Phys. Rev. D 86, 014009 (2012)
Z. B. Kang and B. W. Xiao, Phys. Rev. D 87, no.3, 034038 (2013)
E. Basso, V. P. Goncalves, J. Nemchik, R. Pasechnik and M. Sumbera, Phys. Rev. D 93, no.3, 034023 (2016)

Our general formalism cover particular case present in the literature.

Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [arXiv: 2411.12675]

Parton - level cross-section

Associated production

$$\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z) \equiv \frac{1}{2}\left[\frac{1}{p^2+\epsilon^2} - \frac{1}{(\boldsymbol{p}-z\boldsymbol{\Delta})^2+\epsilon^2}\right]^2$$

$$\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z) \equiv \frac{1}{2}\left[\frac{\boldsymbol{p}}{p^2+\epsilon^2} - \frac{\boldsymbol{p}-z\boldsymbol{\Delta}}{(\boldsymbol{p}-z\boldsymbol{\Delta})^2+\epsilon^2}\right]^2$$

$$\frac{\mathrm{d}\sigma^f_{T,L}(q_a\to G(p_G)q_b(p_q))}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}} = \frac{1}{2(2\pi)^4}\int \mathrm{d}^2\boldsymbol{k}\int \mathrm{d}^2\boldsymbol{r}\mathrm{d}^2\boldsymbol{r}'\, e^{-i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\left[\rho_V^{T,L}(z,\boldsymbol{r},\boldsymbol{r}')+\rho_A^{T,L}(z,\boldsymbol{r},\boldsymbol{r}')\right]\int \mathrm{d}^2\boldsymbol{s}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{s}}\left\{e^{i\boldsymbol{k}\cdot\boldsymbol{s}}+e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-z(\boldsymbol{r}-\boldsymbol{r}'))}-e^{i\boldsymbol{k}\cdot(\boldsymbol{s}+z\boldsymbol{r}')}-e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-z\boldsymbol{r})}\right\}f(x,\boldsymbol{k})$$

↓

$$z\frac{\mathrm{d}\sigma^f_T(q_a\to Gq_b)}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_V = \frac{\left(C_f^G\right)^2\left(g_{V,f}^G\right)^2}{2\pi^2}f(x,\boldsymbol{\Delta})\left\{z^2\left[(m_b-m_a)^2+zm_a\right]^2\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)+\left[1+(1-z)^2\right]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_T(q_a\to Gq_b)}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_A = \frac{\left(C_f^G\right)^2\left(g_{A,f}^G\right)^2}{2\pi^2}f(x,\boldsymbol{\Delta})\left\{z^2\left[(m_b+m_a)-zm_a\right]^2\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)+\left[1+(1-z)^2\right]\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_L(q_a\to Gq_b)}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_V = \frac{\left(C_f^G\right)^2\left(g_{V,f}^G\right)^2}{(2\pi)^2}f(x,\boldsymbol{\Delta})\left\{\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)+\frac{z^2(m_b-m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},\boldsymbol{\Delta},\epsilon,z)\right\}$$

$$z\frac{\mathrm{d}\sigma^f_L(q_a\to Gq_b)}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_A = \frac{\left(C_f^G\right)^2\left(g_{A,f}^G\right)^2}{(2\pi)^2}f(x,\boldsymbol{\Delta})\left\{\frac{\left[z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)+2(1-z)M_G^2\right]^2}{M_G^2}\mathcal{E}_1(\boldsymbol{p},z\boldsymbol{\Delta},\epsilon)+\frac{z^2(m_b+m_a)^2}{M_G^2}\mathcal{E}_2(\boldsymbol{p},z\boldsymbol{\Delta},\epsilon)\right\}$$

We presented for the first time the expresion for the $W^\pm$ production

Parton - level cross-section

Associated production

Particular cases

γ + jet production

$$z\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_{qp\to\gamma X} = \frac{\alpha_{em}e_f^2}{(2\pi)^2}f(x,\boldsymbol{\Delta})\left[1+(1-z)^2\right]\frac{z^2\boldsymbol{\Delta}^2}{(\boldsymbol{p}-z\boldsymbol{\Delta})^2\boldsymbol{p}^2}$$

F. Dominguez, C. Marquet, B. W. Xiao and F. Yuan, Phys. Rev. D 83, 105005 (2011)
A. Stasto, B. W. Xiao and D. Zaslavsky, Phys. Rev. D 86, 014009 (2012)
P. Taels, JHEP 01, 005 (2024)

Z^0 + jet produciton

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}\mathrm{d}^2\boldsymbol{\Delta}}\bigg|_{qp\to Z^0X}^{m_f=0} = \frac{(C_f^Z)^2}{(2\pi)^2}\left[(g_{V,f}^Z)^2+(g_{A,f}^Z)^2\right]f(x,\boldsymbol{\Delta})\left\{\frac{1+(1-z)^2}{z}\left[\frac{\boldsymbol{p}-z\boldsymbol{\Delta}}{[(\boldsymbol{p}-z\boldsymbol{\Delta})^2+\bar{\epsilon}^2]}-\frac{\boldsymbol{p}}{(p^2+\bar{\epsilon}^2)}\right]^2+2\frac{(1-z)^2}{z}M_Z^2\left[\frac{1}{[(\boldsymbol{p}-z\boldsymbol{\Delta})+\bar{\epsilon}^2]}-\frac{1}{(p^2+\bar{\epsilon}^2)}\right]^2\right\}$$

for $\bar{\epsilon}^2 = (1-z)M_Z^2$

E. Basso, V. P. Goncalves, M. Krelina, J. Nemchik and R. Pasechnik, Phys. Rev. D 93, no.9, 094027 (2016)
E. Basso, V. P. Goncalves, J. Nemchik, R. Pasechnik and M. Sumbera, Phys. Rev. D 93, no.3, 034023 (2016)

Once more, our general formalism cover particular case present in the literature.

Summary

- We derived, for the *first time*, the generic expressions for the **LFWF's**.
  - We have estimated the *vector* and *axial* contributions for the description of the longitudinal and transverse spectra associated with the isolated gauge boson production in the impact parameter and transverse momentum spaces.

- We demonstrated that our results reduce to expressions previously used in the literature for the description of the *real photon production* and *Drell - Yan* process at forward rapidities in some particular limits.

- As seen, the expressions obtained are the main ingredients for the calculation of the *pp* cross - sections, which can be compared with the current and forthcoming LHC data.

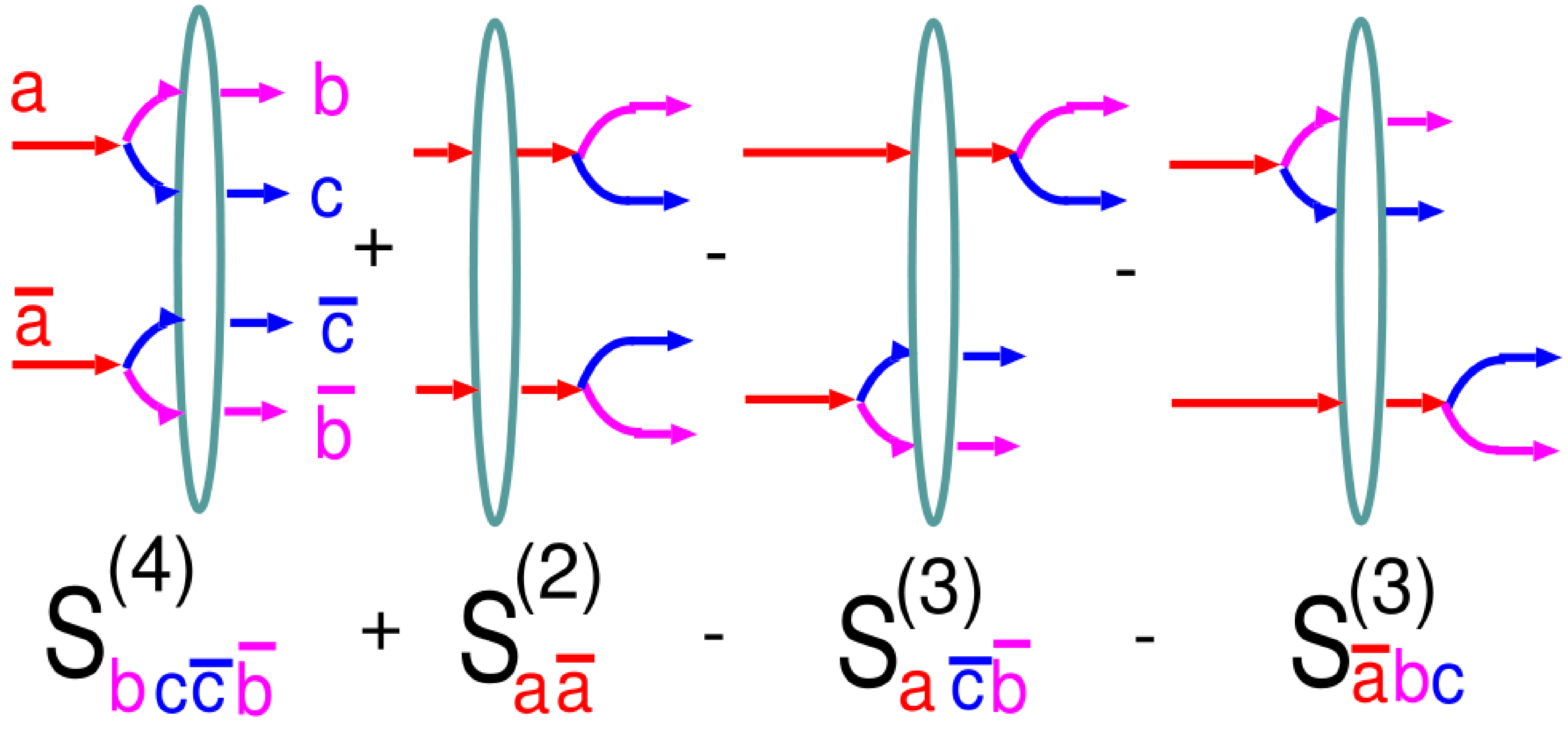
Thank you for your attention!

The Color Dipole S - matrix formalism

$$\frac{d\sigma(a\to b(p_b)c(p_c))}{dz d^2\boldsymbol{p}_b d^2\boldsymbol{p}_c} = \frac{1}{(2\pi)^4} \int d^2\boldsymbol{b}_b d^2\boldsymbol{b}_c d^2\boldsymbol{b}'_b d^2\boldsymbol{b}'_c \exp[i\boldsymbol{p}_b\cdot(\boldsymbol{b}_b-\boldsymbol{b}'_b)+i\boldsymbol{p}_c\cdot(\boldsymbol{b}_c-\boldsymbol{b}'_c)]$$

$$\times\Psi(z,\boldsymbol{b}_b-\boldsymbol{b}_c)\Psi^*(z,\boldsymbol{b}'_b-\boldsymbol{b}'_c)\left\{S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b,\boldsymbol{b}'_c,\boldsymbol{b}_b,\boldsymbol{b}_c)+S^{(2)}_{\bar{a}a}(\boldsymbol{b}',\boldsymbol{b})-S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b},\boldsymbol{b}'_b,\boldsymbol{b}'_c)-S^{(3)}_{\bar{a}bc}(\boldsymbol{b}',\boldsymbol{b}_b,\boldsymbol{b}_c)\right\}$$

$$\begin{aligned}
S^{(2)}_{a\bar{a}}(\boldsymbol{b}',\boldsymbol{b}) &= S^\dagger_a(\boldsymbol{b}')S_a(\boldsymbol{b})\ , \\
S^{(3)}_{\bar{a}bc}(\boldsymbol{b}',\boldsymbol{b}_b,\boldsymbol{b}_c) &= S^\dagger_a(\boldsymbol{b}')S_b(\boldsymbol{b}_b)S_c(\boldsymbol{b}_c)\ , \\
S^{(3)}_{\bar{b}\bar{c}a}(\boldsymbol{b},\boldsymbol{b}'_b,\boldsymbol{b}'_c) &= S^\dagger_b(\boldsymbol{b}'_b)S^\dagger_c(\boldsymbol{b}'_c)S_a(\boldsymbol{b})\ , \\
S^{(4)}_{\bar{b}\bar{c}cb}(\boldsymbol{b}'_b,\boldsymbol{b}'_c,\boldsymbol{b}_b,\boldsymbol{b}_c) &= S^\dagger_b(\boldsymbol{b}'_b)S^\dagger_c(\boldsymbol{b}'_c)S_c(\boldsymbol{b}_c)S_b(\boldsymbol{b}_b)\ .
\end{aligned}$$



$$\rho^{T,L}_{V,A} = \frac{1}{2}\sum_{\lambda\lambda'\lambda_G} \psi^{T,L}_{V,A}(z,\boldsymbol{r})\psi^{T,L,*}_{V,A}(z,\boldsymbol{r}')$$

Parton - level cross-section

Isolated production

$$\left.\frac{d\sigma^f_{T,L}}{dz\, d^2\boldsymbol{p}}\right|_{V,A} = \frac{1}{2(2\pi)^2}\int d^2\boldsymbol{r}\int d^2\boldsymbol{r}' e^{i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')}\rho^{T,L}_{V,A}\left[\sigma_{q\bar{q}}(z\boldsymbol{r},x)+\sigma_{q\bar{q}}(z\boldsymbol{r}',x)-\sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x)\right]$$

↓

$$\left.\frac{d\sigma^f_T}{dzd^2\mathbf{p}}\right|_V = \frac{(C^G_f)^2(g^G_{f,V})^2}{(2\pi)^2}\left\{z[(m_b-m_a)+zm_a]^2\mathcal{D}_1(z,p,\epsilon)+\frac{[1+(1-z)^2]}{z}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\},$$

$$\left.\frac{d\sigma^f_T}{dzd^2\mathbf{p}}\right|_A = \frac{(C^G_f)^2(g^G_{f,A})^2}{2\pi^2}\left\{z[(m_b+m_a)-zm_a]^2\mathcal{D}_1(z,p,\epsilon)+\frac{1+(1-z)^2}{z}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

$$\left.\frac{d\sigma^f_L}{dzd^2\mathbf{p}}\right|_V = \frac{(C^G_f)^2(g^G_{f,V})^2}{(2\pi)^2}\left\{\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{D}_1(z,p,\epsilon)+\frac{z(m_b-m_a)^2}{M_G^2}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

$$\left.\frac{d\sigma^f_L}{dzd^2\mathbf{p}}\right|_A = \frac{(C^G_f)^2(g^G_{f,A})^2}{(2\pi)^2}\left\{\frac{\left[z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)+2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{D}_1(z,p,\epsilon)+\frac{z(m_b+m_a)^2}{M_G^2}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

Parton - level cross-section

Isolated production

$$\frac{d\sigma_T^f}{dzd^2\mathbf{p}}\bigg|_V = \frac{(C_f^G)^2(g_{f,V}^G)^2}{(2\pi)^2}\left\{z[(m_b-m_a)+zm_a]^2\mathcal{D}_1(z,p,\epsilon)+\frac{[1+(1-z)^2]}{z}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\},$$

$$\frac{d\sigma_T^f}{dzd^2\mathbf{p}}\bigg|_A = \frac{(C_f^G)^2(g_{f,A}^G)^2}{2\pi^2}\left\{z[(m_b+m_a)-zm_a]^2\mathcal{D}_1(z,p,\epsilon)+\frac{1+(1-z)^2}{z}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

$$\frac{d\sigma_L^f}{dzd^2\mathbf{p}}\bigg|_V = \frac{(C_f^G)^2(g_{f,V}^G)^2}{(2\pi)^2}\left\{\frac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{D}_1(z,p,\epsilon)+\frac{z(m_b-m_a)^2}{M_G^2}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

$$\frac{d\sigma_L^f}{dzd^2\mathbf{p}}\bigg|_A = \frac{(C_f^G)^2(g_{f,A}^G)^2}{(2\pi)^2}\left\{\frac{\left[z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)+2(1-z)M_G^2\right]^2}{zM_G^2}\mathcal{D}_1(z,p,\epsilon)+\frac{z(m_b+m_a)^2}{M_G^2}\epsilon^2\mathcal{D}_2(z,p,\epsilon)\right\}.$$

for

$$\mathcal{D}_1(z,p,\epsilon)=\frac{1}{p^2+\epsilon^2}I_1(z,p)-\frac{1}{4\epsilon}I_2(z,p)$$

$$\mathcal{D}_2(z,p,\epsilon)=\frac{1}{\epsilon}\frac{p}{(p^2+\epsilon^2)}I_3(z,p)-\frac{1}{2\epsilon^2}I_1(z,p)+\frac{1}{4\epsilon}I_2(z,p).$$

$$I_1(z,p)=\int dr r \mathrm{J}_0(pr)\mathrm{K}_0(\epsilon r)\sigma_{q\bar{q}}(z\mathbf{r})$$

$$I_2(z,p)=\int dr r^2 \mathrm{J}_0(pr)\mathrm{K}_1(\epsilon r)\sigma_{q\bar{q}}(z\mathbf{r})$$

$$I_3(z,p)=\int dr r \mathrm{J}_1(pr)\mathrm{K}_1(\epsilon r)\sigma_{q\bar{q}}(z\mathbf{r})$$