

2025 CFNS-SURGE Summer Workshop on the **Physics of the Electron-Ion Collider** 

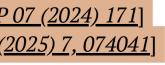
# **Electroweak gauge boson production in** hadronic collisions at forward rapidity

Based on: Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [JHEP 07 (2024) 171] Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [Phys.Rev.D 111 (2025) 7, 074041]



YAN B. BANDEIRA 1'2 PROF. DR. VICTOR P. GONÇALVES <sup>1</sup> PROF. DR. WOLFGANG SCHÄFER<sup>2</sup>

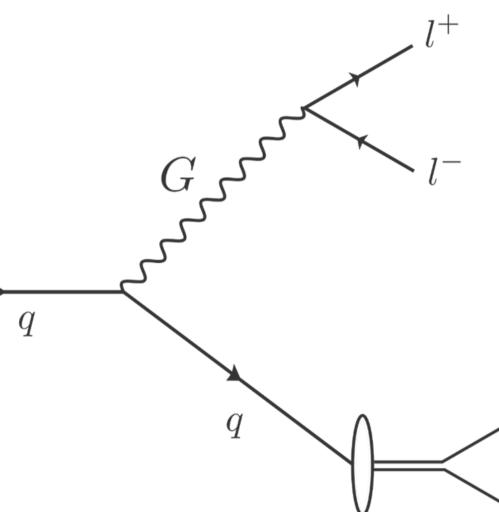
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 $d\sigma(h_Ah_B o H_1H_2X) \propto f_{a/A}(x_1) \otimes d\sigma(aB o bc) \otimes D_{H_1/b} \otimes D_{H_2/c}$ **Forward rapidity** parton - target cross-section projectile proton PDF **Hybrid factorization**  $P_1$  $x_1 \propto e^{\eta}$  $x_2 \propto e^{-\eta}$  $f_i(x_1)$ Ρ  $(q, x_1P_1)$ q $P_2$  $(g, x_2P_2)$  $f_j(x_2)$ qΡ

In the forward rapidity region, one expect the violation of collinear factorization





# **Color-dipole S-matrix framework**

The master dijet production in the color - dipole S - matrix framework is given by

$$egin{aligned} rac{d\sigma(a o b(p_b)c(p_c))}{dz d^2 oldsymbol{p}_b d^2 oldsymbol{p}_c} &= rac{1}{(2\pi)^4} \, \int d^2 oldsymbol{b}_b d^2 oldsymbol{b}_c d^2 oldsymbol{b}'_c \expigin{aligned} i oldsymbol{p}_b d^2 oldsymbol{b}'_c \expigin[i oldsymbol{p}_c \Phioldsymbol{p}'_c \Phioldsymbol{b}'_c \Phio$$

N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [J. Exp. Theor. Phys. 97, 441-465 (2003)] N. N. Nikolaev and W. Schäfer [Phys. Rev. D 71, 014023 (2005)] N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [Phys. Rev. D 72, 034033 (2005)] N. N. Nikolaev, W. Schäfer and B. G. Zakharov, [Phys. Rev. D 72, 114018 (2005)] N. N. Nikolaev, W. Schäfer, B. G. Zakharov and V. R. Zoller [JETP Lett. 82, 325-334 (2005)] N. N. Nikolaev, W. Schäfer and B. G. Zakharov [Phys. Rev. Lett. 95, 221803 (2005)]

### Our work was evaluate the case: $\, q_f ightarrow Gq_k \,$

# $a \rightarrow hc$

# $\cdot (\boldsymbol{b}_b - \boldsymbol{b}_b') + i \boldsymbol{p}_c \cdot (\boldsymbol{b}_c - \boldsymbol{b}_c')$ $m{b}) - S^{(3)}_{ar{b}ar{c}a}(m{b},m{b}_b',m{b}_c') - S^{(3)}_{ar{a}bc}(m{b}',m{b}_b,m{b}_c)\Big\},$



# **Gauge boson production**

Our work was use this formalism to an electroweak gauge boson production, for this case the cross-section expression simplifies as:

$$egin{aligned} rac{d\sigma^f_{T,L}(q_fN o G(p_G)q_k(p_q))}{dzd^2m{p}d^2m{\Delta}} &= rac{1}{2(2\pi)^4} \, \int\!d^2m{r}d^2m{r}d^2m{r}' \exp[-im{p}\cdot(m{r}-m{r}')] \overline{\sum_{ ext{pol.}}} \, \Psi_{T,L}(z,m{r}) \Psi^*_{T,L}(z,m{r}') \ & imes \int\!d^2s \expig[-im{\Delta}\cdotm{s}ig] igg\{\sigma_{qar{q}}(m{s}-zm{r},x) + \sigma_{qar{q}}(m{s}+zm{r}',x) - \sigma_{qar{q}}(m{s}-z(m{r}-m{r}'),x) - \sigma_{qar{q}}m{s}igg\} \end{aligned}$$

Proj.

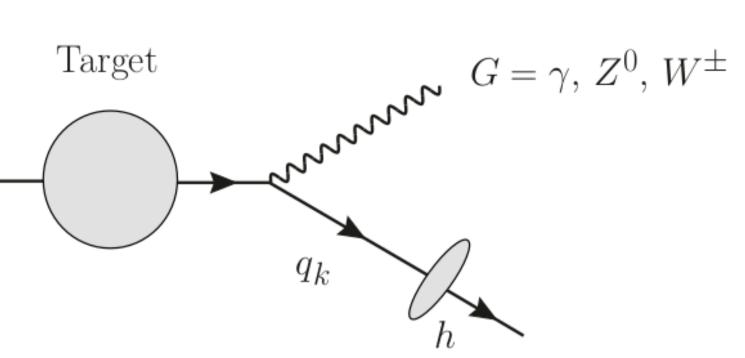
 $q_f$ 

where, we connect the S-matrix with dipole cross-section by

$$\sigma(oldsymbol{r})=2\int d^2oldsymbol{B}\left[1-S^{(2)}_{q\overline{q}}\Big(oldsymbol{B}+rac{oldsymbol{r}}{2},oldsymbol{B}-rac{oldsymbol{r}}{2}\Big)
ight]$$

the dipole cross-section is model dependent. Therefore,

### The unknown ingredient is the <u>Wave Function</u>!



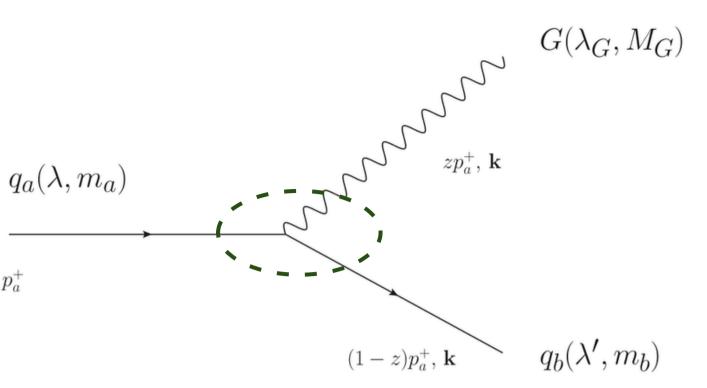
Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [JHEP 07 (2024) 171]

# **Light Front Wave Function**

$$\begin{split} \Gamma_{V} &= E_{\mu}^{*}(k,\lambda)\overline{u}(p_{b},\lambda',m_{b})\bigg\{\gamma^{\mu} + (m_{b}-m_{a})\frac{k^{\mu}}{M^{2}}\bigg\}u(p_{a},\lambda,m_{a})\\ \Gamma_{A} &= E_{\mu}^{*}(k,\lambda)\overline{u}(p_{b},\lambda',m_{b})\bigg\{\bigg[\gamma^{\mu} + (m_{b}+m_{a})\frac{k^{\mu}}{M^{2}}\bigg]\gamma_{5}\bigg\}u(p_{a},\lambda,m_{a})\\ \mathbf{E}^{*}(\lambda_{G}) + i\lambda\left[\mathbf{k},\mathbf{E}^{*}(\lambda_{G})\right]\bigg)I - \lambda\left[m_{b} - (1-z)m_{a}\right]\boldsymbol{\sigma}\cdot\mathbf{E}^{*}(\lambda_{G})\bigg\}\chi_{\lambda},\\ \lambda_{G}) + i\lambda\left[\mathbf{k},\mathbf{E}^{*}(\lambda_{G})\right]\bigg]\lambda I - (m_{b} + (1-z)m_{a})\boldsymbol{\sigma}\cdot\mathbf{E}^{*}(\lambda_{G})\bigg\}\chi_{\lambda},\\ \frac{h_{a}) - z\left(m_{b}^{2} - m_{a}^{2}\right) - 2(1-z)M^{2}}{\mathbf{k}^{2} + \epsilon^{2}}\bigg] + \frac{\left[z(m_{b} - m_{a})\right]}{\mathbf{k}^{2} + \epsilon^{2}}\lambda\left(\boldsymbol{\sigma}\cdot\mathbf{k}\right)\bigg\}\chi_{\lambda},\\ \frac{h_{b} + m_{a}) + z\left(m_{b}^{2} - m_{a}^{2}\right) + 2(1-z)M^{2}}{\mathbf{k}^{2} + \epsilon^{2}}\bigg] - z\left(m_{b} + m_{a}\right)\frac{\left(\boldsymbol{\sigma}\cdot\mathbf{k}\right)}{\mathbf{k}^{2} + \epsilon^{2}}\bigg\}\chi_{\lambda}. \end{split}$$

$$\begin{split} \Psi_{A}(z,\boldsymbol{k}) &= C_{f}^{C} g_{A,f}^{C} \sqrt{z(1-z)} \frac{k^{2} + \epsilon^{2}}{k^{2} + \epsilon^{2}} \qquad \Gamma_{V} = E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \gamma^{\mu} + (m_{b} - m_{a}) \frac{k^{\mu}}{M^{2}} \right\} u(p_{a},\lambda,m_{a}) \\ \Gamma_{A} &= E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \left[ \gamma^{\mu} + (m_{b} + m_{a}) \frac{k^{\mu}}{M^{2}} \right] \gamma_{5} \right\} u(p_{a},\lambda,m_{a}) \\ \Psi_{V}^{T}(z,\boldsymbol{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{\sqrt{z}}{k^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left( \left( \frac{2-z}{z} \right) \boldsymbol{k} \cdot \boldsymbol{E}^{*}(\lambda_{G}) + i\lambda \left[ \boldsymbol{k}, \boldsymbol{E}^{*}(\lambda_{G}) \right] \right) I - \lambda \left[ m_{b} - (1-z)m_{a} \right] \boldsymbol{\sigma} \cdot \boldsymbol{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{A}^{T}(z,\boldsymbol{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{\sqrt{z}}{k^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left[ \frac{2-z}{z} \boldsymbol{k} \cdot \boldsymbol{E}^{*}(\lambda_{G}) + i\lambda \left[ \boldsymbol{k}, \boldsymbol{E}^{*}(\lambda_{G}) \right] \right] \lambda I - (m_{b} + (1-z)m_{a})\boldsymbol{\sigma} \cdot \boldsymbol{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{V}^{L}(z,\boldsymbol{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{1}{\sqrt{zM}} \chi_{\lambda'}^{\dagger} \left\{ I \left[ \frac{2^{2}m_{a} \left( m_{b} - m_{a} \right) - z \left( m_{b}^{2} - m_{a}^{2} \right) - 2(1-z)M^{2}}{k^{2} + \epsilon^{2}} \right] + \frac{\left[ z(m_{b} - m_{a}) \right]}{k^{2} + \epsilon^{2}} \lambda \left( \boldsymbol{\sigma} \cdot \boldsymbol{k} \right) \right\} \chi_{\lambda}, \\ \Psi_{L}^{A}(z,\boldsymbol{k}) &= C_{f}^{G} g_{A,f}^{G} \frac{1}{\sqrt{zM}} \chi_{\lambda'}^{\dagger} \left\{ \lambda I \left[ - \frac{z^{2}m_{a} \left( m_{b} + m_{a} \right) + z \left( m_{b}^{2} - m_{a}^{2} \right) + 2(1-z)M^{2}}{k^{2} + \epsilon^{2}} \right] - z \left( m_{b} + m_{a} \right) \frac{\left( \boldsymbol{\sigma} \cdot \boldsymbol{k} \right)}{k^{2} + \epsilon^{2}} \right\} \chi_{\lambda}. \end{split}$$

$$\begin{split} \Psi_{A}(z,\mathbf{k}) &= C_{f} g_{A,f}^{G} \sqrt{z(1-z)} \frac{k^{2} + \epsilon^{2}}{\mathbf{k}^{2} + \epsilon^{2}} \qquad \Gamma_{V} = E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \gamma^{\mu} + (m_{b} - m_{a}) \frac{k^{\mu}}{M^{2}} \right\} u(p_{a},\lambda,m_{a}) \\ \Gamma_{A} &= E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \left[ \gamma^{\mu} + (m_{b} + m_{a}) \frac{k^{\mu}}{M^{2}} \right] \gamma_{5} \right\} u(p_{a},\lambda,m_{a}) \\ \Psi_{V}^{T}(z,\mathbf{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{\sqrt{z}}{\mathbf{k}^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left( \left( \frac{2-z}{z} \right) \mathbf{k} \cdot \mathbf{E}^{*}(\lambda_{G}) + i\lambda \left[ \mathbf{k}, \mathbf{E}^{*}(\lambda_{G}) \right] \right) I - \lambda \left[ m_{b} - (1-z)m_{a} \right] \boldsymbol{\sigma} \cdot \mathbf{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{A}^{T}(z,\mathbf{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{\sqrt{z}}{\mathbf{k}^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left[ \frac{2-z}{z} \mathbf{k} \cdot \mathbf{E}^{*}(\lambda_{G}) + i\lambda \left[ \mathbf{k}, \mathbf{E}^{*}(\lambda_{G}) \right] \right] \lambda I - (m_{b} + (1-z)m_{a}) \boldsymbol{\sigma} \cdot \mathbf{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{V}^{L}(z,\mathbf{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{1}{\sqrt{z}M} \chi_{\lambda'}^{\dagger} \left\{ I \left[ \frac{2^{2}m_{a} (m_{b} - m_{a}) - z \left( m_{b}^{2} - m_{a}^{2} \right) - 2(1-z)M^{2}}{\mathbf{k}^{2} + \epsilon^{2}} \right] + \frac{\left[ z(m_{b} - m_{a}) \right]}{\mathbf{k}^{2} + \epsilon^{2}} \lambda \left( \boldsymbol{\sigma} \cdot \mathbf{k} \right) \right\} \chi_{\lambda}, \\ \Psi_{L}^{A}(z,\mathbf{k}) &= C_{f}^{G} g_{A,f}^{G} \frac{1}{\sqrt{z}M} \chi_{\lambda'}^{\dagger} \left\{ \lambda I \left[ - \frac{z^{2}m_{a} (m_{b} + m_{a}) + z \left( m_{b}^{2} - m_{a}^{2} \right) + 2(1-z)M^{2}}{\mathbf{k}^{2} + \epsilon^{2}} \right] - z (m_{b} + m_{a}) \frac{(\boldsymbol{\sigma} \cdot \mathbf{k})}{\mathbf{k}^{2} + \epsilon^{2}} \right\} \chi_{\lambda}. \end{split}$$



$$\frac{d\sigma_{T,L}^{f}}{dz \, d^{2} \boldsymbol{p}}\Big|_{V,A} = \left.\frac{1}{2(2\pi)^{2}} \int d^{2} \boldsymbol{r} \int d^{2} \boldsymbol{r}' e^{i \boldsymbol{p} \cdot (\boldsymbol{r}-\boldsymbol{r}')} \boldsymbol{\rho}_{V,A}^{T,L} \left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}')\right] \right] + \sigma_{q\bar{q}}(z|\boldsymbol{r},x) + \sigma_{q\bar{q}}(z$$

$$zrac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2oldsymbol{p}}|_V = rac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{2\pi^2}\int\mathrm{d}^2oldsymbol{k}\,f(x,oldsymbol{k})\left\{z^2[(m_b-m_a)+zm_a]^2\,oldsymbol{\mathcal{E}}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)+[1+(1-z)^2]oldsymbol{\mathcal{E}}_2(oldsymbol{p},oldsymbol{k},\epsilon,z)
ight\}$$

$$zrac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2oldsymbol{p}}|_A = rac{(\mathcal{C}_f^G)^2(g^G_{v,f})^2}{2\pi^2}\int\mathrm{d}^2oldsymbol{k}\,f(x,oldsymbol{k})\left\{z^2[(m_b+m_a)-zm_a]^2\,\mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)+[1+(1-z)^2]\mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z)
ight\}$$

$$z \frac{\mathrm{d}\sigma_L^f}{\mathrm{d}z \mathrm{d}^2 \boldsymbol{p}}|_V = \frac{(\mathcal{C}_f^G)^2 (g_{v,f}^G)^2}{4\pi^2} \int \mathrm{d}^2 \boldsymbol{k} \, f(x, \boldsymbol{k}) \left\{ \frac{(z^2 m_a (m_b - m_a) - z(m_b^2 - m_a^2) - 2(1 - z)M_G^2)^2}{M_G^2} \mathcal{E}_1(\boldsymbol{p}, \boldsymbol{k}, \epsilon, z) + \frac{z^2 (m_b - m_a)^2}{M_G^2} \mathcal{E}_2(\boldsymbol{p}, \boldsymbol{k}, \epsilon, z) \right\}$$

$$z rac{\mathrm{d}\sigma_L^f}{\mathrm{d}z \mathrm{d}^2 oldsymbol{p}}|_A = rac{(\mathcal{C}_f^G)^2 (g_{v,f}^G)^2}{4\pi^2} \int \mathrm{d}^2 oldsymbol{k} f(x,oldsymbol{k}) \left\{ rac{(z^2 m_a (m_b + m_a) + z (m_b^2 - m_a^2) - 2(1 - z) M_G^2)^2}{M_G^2} \mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z) + rac{z^2 (m_b - m_a)^2}{M_G^2} \mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z) 
ight\}$$

### A generalized description for all electroweak gauge boson

$$egin{aligned} eta_{V,A}^{T,L} &= rac{1}{2} \sum_{\lambda\lambda'\lambda_G} \psi_{V,A}^{T,L}(z,m{r}) \psi_{V,A}^{T,L,*}(z,m{r}') \ & \sigma_{q\overline{q}}(m{r}) &= \int \mathrm{d}^2m{k} \, f(x,m{k}) \left(1-e^{im{k}\cdotm{r}}
ight) \ & \mathcal{E}_1(m{p},m{k},\epsilon,z) &\equiv rac{1}{2} \Big[rac{1}{p^2+\epsilon^2} - rac{1}{(m{p}-zm{k})^2+\epsilon^2}\Big]^2 \ & \mathcal{E}_2(m{p},m{k},\epsilon,z) &\equiv rac{1}{2} \Big[rac{m{p}}{p^2+\epsilon^2} - rac{m{p}-zm{k}}{(m{p}-m{k})^2+\epsilon^2}\Big]^2 \end{aligned}$$

$$\frac{\mathrm{d}\sigma_{T,L}^{f}(q_{a} \rightarrow G(p_{G})q_{b}(p_{q}))}{\mathrm{d}z\mathrm{d}^{2}\boldsymbol{p}\mathrm{d}^{2}\boldsymbol{\Delta}} = \frac{1}{2(2\pi)^{4}} \int \mathrm{d}^{2}\boldsymbol{k} \int \mathrm{d}^{2}\boldsymbol{r} \mathrm{d}^{2}\boldsymbol{r}' \, e^{-i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \left[\rho_{V}^{T,L}(z,\boldsymbol{r},\boldsymbol{r}') + \rho_{A}^{T,L}(z,\boldsymbol{r},\boldsymbol{r}')\right] \int \mathrm{d}^{2}\boldsymbol{s} \, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{s}} \left\{ e^{i\boldsymbol{k}\cdot\boldsymbol{s}} + e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-z(\boldsymbol{r}-\boldsymbol{r}'))} - e^{i\boldsymbol{k}\cdot(\boldsymbol{s}+z\boldsymbol{r}')} - e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-z\boldsymbol{r})} \right\} f(x,\boldsymbol{k})$$

$$\left. z rac{\mathrm{d}\sigma_T^f(q_a o Gq_b)}{\mathrm{d}z \mathrm{d}^2 oldsymbol{p} \mathrm{d}^2 oldsymbol{\Delta}} 
ight|_V = rac{\left(C_f^G
ight)^2 \left(g_{V,f}^G
ight)^2}{2\pi^2} f(x, oldsymbol{\Delta}) \Biggl\{ z^2 \Bigl[ (m_b - m_a)^2 + z m_a \Bigr]^2 \mathcal{E}_1(oldsymbol{p}, oldsymbol{\Delta}, \epsilon, z) + \Bigl[ 1 + (1 - z)^2 \Bigr] \mathcal{E}_2(oldsymbol{p}, oldsymbol{\Delta}, \epsilon, z) \Biggr\}$$

$$\left. z rac{\mathrm{d}\sigma_T^f(q_a o Gq_b)}{\mathrm{d}z \mathrm{d}^2 oldsymbol{p} \mathrm{d}^2 \mathbf{\Delta}} 
ight|_A = rac{\left(C_f^G
ight)^2 \left(g_{A,f}^G
ight)^2}{2\pi^2} f(x, \mathbf{\Delta}) \Biggl\{ z^2 \Bigl[ (m_b + m_a) - z m_a \Bigr]^2 \mathcal{E}_1(oldsymbol{p}, \mathbf{\Delta}, \epsilon, z) + \Bigl[ 1 + (1 - z)^2 \Bigr] \mathcal{E}_2(oldsymbol{p}, \mathbf{\Delta}, \epsilon, z) \Biggr\}$$

$$\left. z \frac{\mathrm{d}\sigma_{L}^{f}(q_{a} \to Gq_{b})}{\mathrm{d}z \mathrm{d}^{2} \boldsymbol{p} \mathrm{d}^{2} \boldsymbol{\Delta}} \right|_{V} = \frac{\left(C_{f}^{G}\right)^{2} \left(g_{V,f}^{G}\right)^{2}}{(2\pi)^{2}} f(x, \boldsymbol{\Delta}) \left\{ \frac{\left[z^{2} m_{a}(m_{b} - m_{a}) - z(m_{b}^{2} - m_{a}^{2}) - 2(1 - z)M_{G}^{2}\right]^{2}}{M_{G}^{2}} \mathcal{E}_{1}(\boldsymbol{p}, \boldsymbol{\Delta}, \epsilon, z) + \frac{z^{2} (m_{b} - m_{a})^{2}}{M_{G}^{2}} \mathcal{E}_{2}(\boldsymbol{p}, \boldsymbol{\Delta}, \epsilon, z) \right\}$$

$$\left. z \frac{\mathrm{d}\sigma_{L}^{f}(q_{a} \to Gq_{b})}{\mathrm{d}z \mathrm{d}^{2} \boldsymbol{p} \mathrm{d}^{2} \boldsymbol{\Delta}} \right|_{A} = \frac{\left(C_{f}^{G}\right)^{2} \left(g_{A,f}^{G}\right)^{2}}{(2\pi)^{2}} f(x, \boldsymbol{\Delta}) \left\{ \frac{\left[z^{2} m_{a}(m_{b} + m_{a}) + z(m_{b}^{2} - m_{a}^{2}) + 2(1-z)M_{G}^{2}\right]^{2}}{M_{G}^{2}} \mathcal{E}_{1}(\boldsymbol{p}, z\boldsymbol{\Delta}, \epsilon) + \frac{z^{2}(m_{b} + m_{a})^{2}}{M_{G}^{2}} \mathcal{E}_{2}(\boldsymbol{p}, z\boldsymbol{\Delta}, \epsilon) \right\}$$

### We presented for the first time the expresion for the $W^{\pm}$ production

$$egin{split} \mathcal{E}_1(oldsymbol{p},oldsymbol{\Delta},\epsilon,z) &\equiv rac{1}{2} iggl[ rac{1}{p^2+\epsilon^2} - rac{1}{(oldsymbol{p}-zoldsymbol{\Delta})^2+\epsilon^2} iggr]^2 \ \mathcal{E}_2(oldsymbol{p},oldsymbol{\Delta},\epsilon,z) &\equiv rac{1}{2} iggl[ rac{oldsymbol{p}}{p^2+\epsilon^2} - rac{oldsymbol{p}-zoldsymbol{\Delta}}{(oldsymbol{p}-zoldsymbol{\Delta})^2+\epsilon^2} iggr]^2 \end{split}$$



- We <u>derived</u>, for the *first time*, the <u>generic expressions</u> for the **LFWF's**.
  - We have estimated the *vector* and *axial* contributions for the description of the <u>longitudinal</u> and transverse spectra associated with the isolated gauge boson production in the impact parameter and transverse momentum spaces.e

• We demonstrated that our results reduce to expressions previously used in the literature for the description of the real photon production and Drell - Yan process at forward rapidities in some particular limits.

• As seen, the expressions obtained are the main ingredients for the calculation of the pp cross - sections, which can be compared with the current and forthcoming LHC data.



# **ACKNOWLEDGEMENTS**

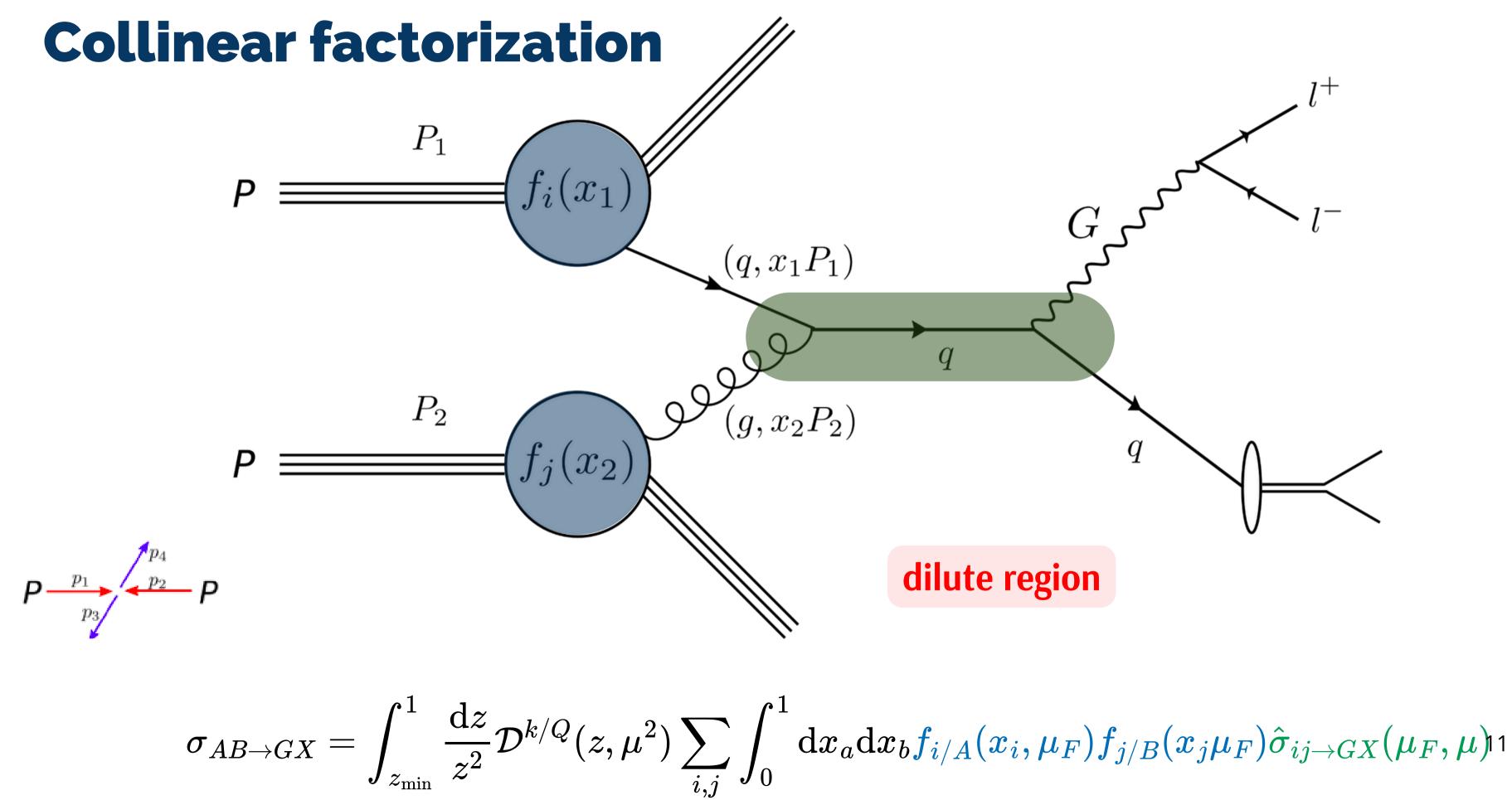




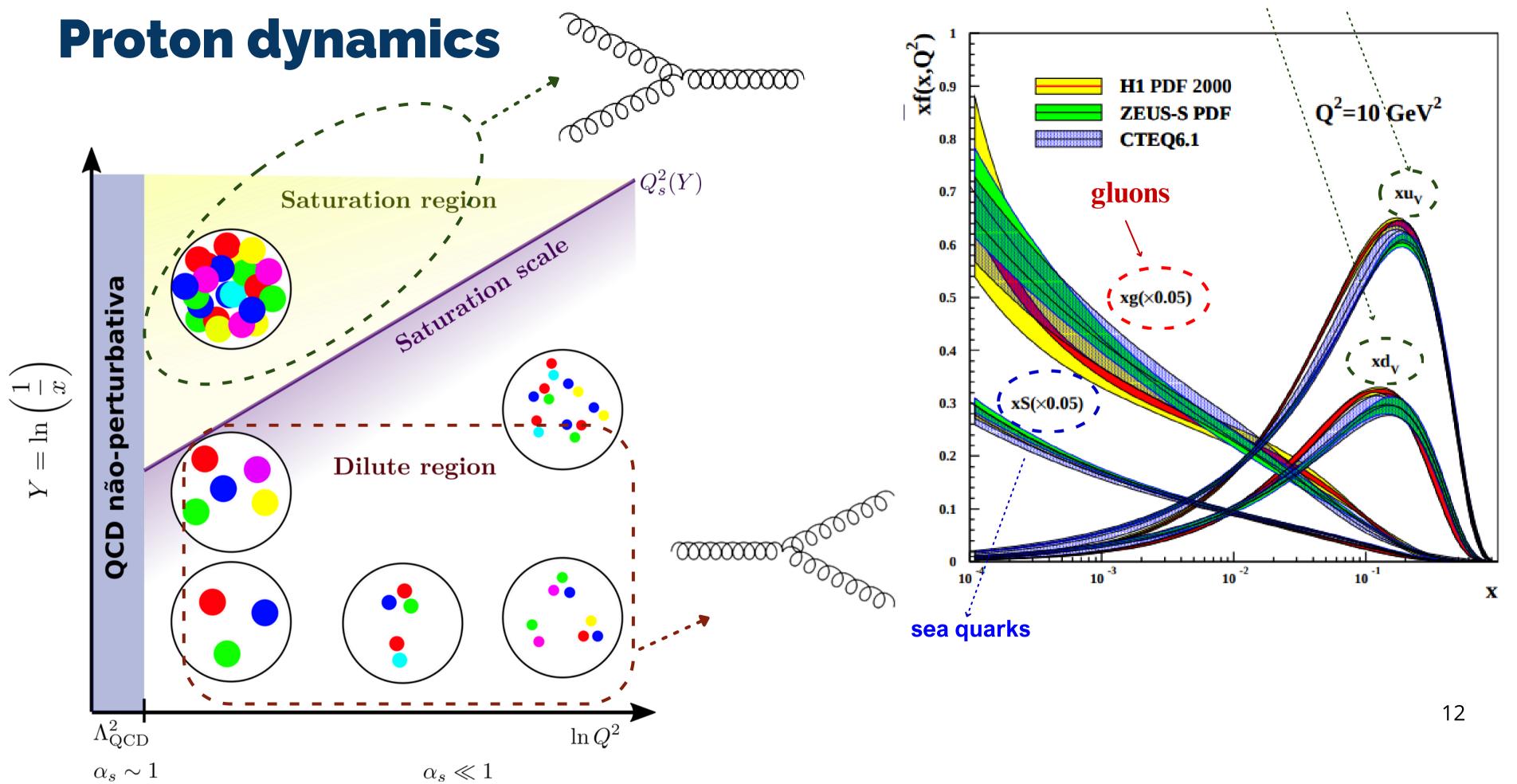


# **Extra slides**





 $G=\gamma,\,Z^0,\,W^\pm$ 

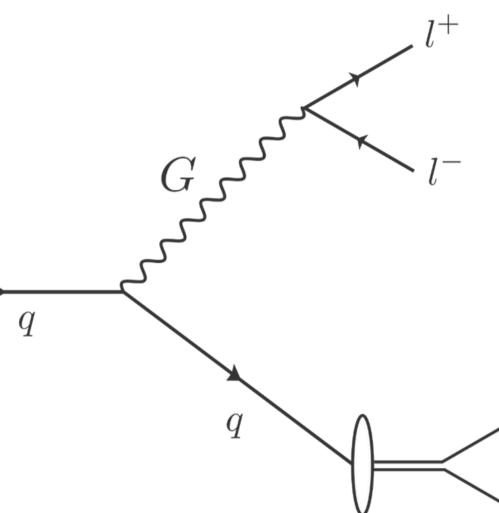


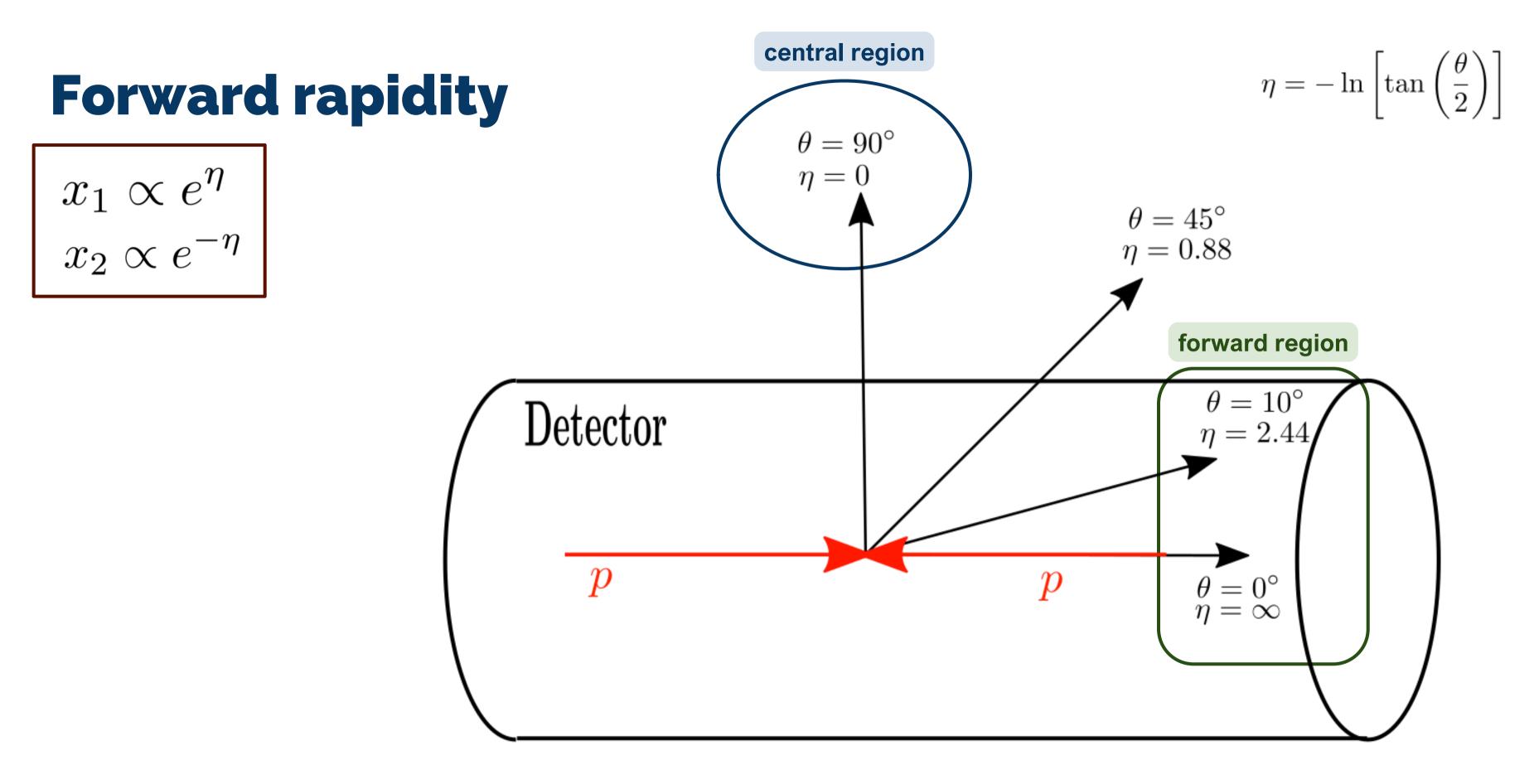
#### valence quarks

 $d\sigma(h_Ah_B o H_1H_2X) \propto f_{a/A}(x_1) \otimes d\sigma(aB o bc) \otimes D_{H_1/b} \otimes D_{H_2/c}$ **Forward rapidity** parton - target cross-section projectile proton PDF **Hybrid factorization**  $P_1$  $x_1 \propto e^{\eta}$  $x_2 \propto e^{-\eta}$  $f_i(x_1)$ Ρ  $(q, x_1P_1)$ q $P_2$  $(g, x_2P_2)$  $f_j(x_2)$ qΡ

In the forward rapidity region, one expect the violation of collinear factorization







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# **Color-dipole S-matrix framework**

The master dijet production in the color - dipole S - matrix framework is given by

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### Our work was evaluate the case: qq ightarrow Gq

# $ag \rightarrow bc$

# $\cdot (\boldsymbol{b}_b - \boldsymbol{b}_b') + i \boldsymbol{p}_c \cdot (\boldsymbol{b}_c - \boldsymbol{b}_c')$ $m{b}) - S^{(3)}_{ar{b}ar{c}a}(m{b},m{b}_b',m{b}_c') - S^{(3)}_{ar{a}bc}(m{b}',m{b}_b,m{b}_c)\Big\},$

# **Gauge boson production**

Our work was use this formalism to an electroweak gauge boson production, for this case the cross-section expression simplifies as:

$$egin{aligned} rac{d\sigma^f_{T,L}(q_fN o G(p_G)q_k(p_q))}{dzd^2m{p}d^2m{\Delta}} &= rac{1}{2(2\pi)^4} \int\!d^2m{r}d^2m{r}d^2m{r}' \exp[-im{p}\cdot(m{r}-m{r}')] \overline{\sum_{ ext{pol.}}} \,\Psi_{T,L}(z,m{r}) \Psi^*_{T,L}(z,m{r}') \ & imes \int\!d^2s \expigg[-im{\Delta}\cdotm{s}igg] igg\{\sigma_{qar{q}}(m{s}-zm{r},x) + \sigma_{qar{q}}(m{s}+zm{r}',x) - \sigma_{qar{q}}(m{s}-z(m{r}-m{r}'),x) - \sigma_{qar{q}}m{s}igg\} \end{aligned}$$

Proj.

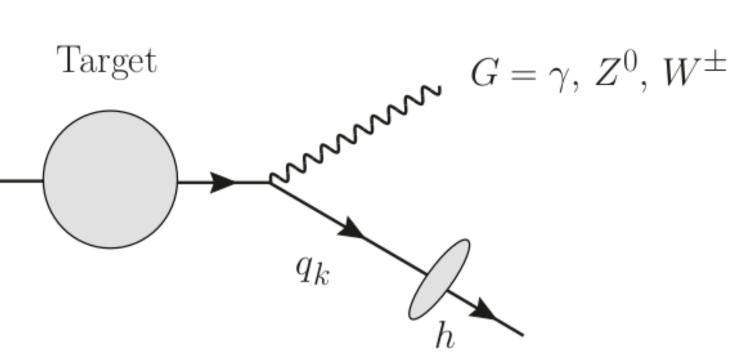
 $q_f$ 

where, we connect the S-matrix with dipole cross-section by

$$\sigma(oldsymbol{r})=2\int d^2oldsymbol{B}\left[1-S^{(2)}_{q\overline{q}}\Big(oldsymbol{B}+rac{oldsymbol{r}}{2},oldsymbol{B}-rac{oldsymbol{r}}{2}\Big)
ight]$$

the dipole cross-section is model dependent. Therefore,

### The unknown ingredient is the <u>Wave Function</u>!

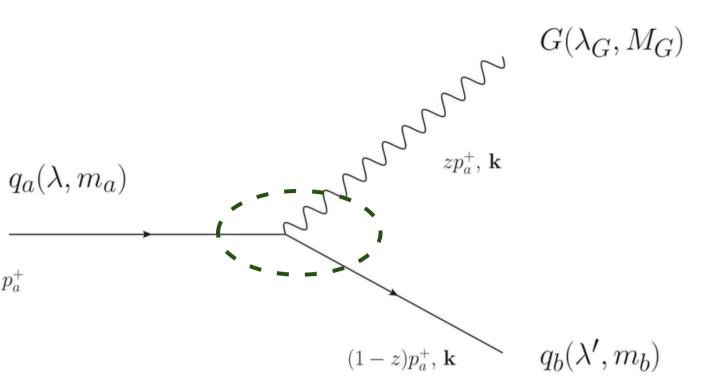


# **Light Front Wave Function**

$$\Psi_V(z,oldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)} rac{\Gamma_V}{oldsymbol{k}^2+\epsilon^2}$$

$$\Psi_A(z,oldsymbol{k}) = C_f^G g^G_{A,f} \sqrt{z(1-z)} rac{\Gamma_A}{oldsymbol{k}^2+\epsilon^2}$$

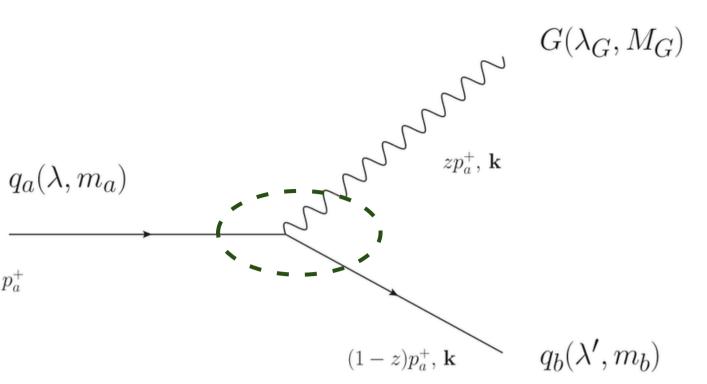
 $p_a^+$ 



# **Light Front Wave Function**

$$\Psi_V(z,oldsymbol{k}) = C_f^G g_{V,f}^G \sqrt{z(1-z)} rac{\Gamma_V}{oldsymbol{k}^2 + \epsilon^2}$$

$$egin{aligned} \Gamma_V &= E^*_\mu(k,\lambda)\overline{u}(p_b,\lambda',m_b)iggl\{\gamma^\mu+(m_b-m_a)rac{k^\mu}{M^2}iggr\}u(p_a,\lambda,m_a)\ \Gamma_A &= E^*_\mu(k,\lambda)\overline{u}(p_b,\lambda',m_b)iggl\{iggl[\gamma^\mu+(m_b+m_a)rac{k^\mu}{M^2}iggr]\gamma_5iggr\}u(p_a,\lambda,m_a) \end{aligned}$$



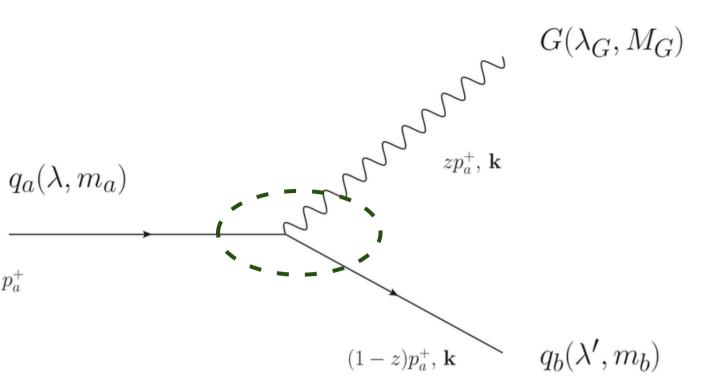
Y. B. Bandeira, V. P. Gonçalves and W. Schäfer [JHEP 07 (2024) 171]

# **Light Front Wave Function**

$$egin{aligned} \Gamma_V &= E^*_\mu(k,\lambda)\overline{u}(p_b,\lambda',m_b)igg\{\gamma^\mu+(m_b-m_a)rac{k^\mu}{M^2}igg\}u(p_a,\lambda,m_a)\ \Gamma_A &= E^*_\mu(k,\lambda)\overline{u}(p_b,\lambda',m_b)igg\{\left[\gamma^\mu+(m_b+m_a)rac{k^\mu}{M^2}
ight]\gamma_5igg\}u(p_a,\lambda,m_a)\ E^*(\lambda_G)+i\lambda\left[m k,m E^*(\lambda_G)
ight]igg)I-\lambda\left[m_b-(1-z)m_a
igh]m \sigma\cdotm E^*(\lambda_G)igg\}\chi_\lambda,\ \lambda_G)+i\lambda\left[m k,m E^*(\lambda_G)
ight]igg]\lambda I-(m_b+(1-z)m_a)m \sigma\cdotm E^*(\lambda_G)igg\}\chi_\lambda,\ rac{n_a)-z\left(m_b^2-m_a^2\right)-2(1-z)M^2}{m k^2+\epsilon^2}igg]+rac{\left[z(m_b-m_a)\right]}{m k^2+\epsilon^2}\lambda\left(m \sigma\cdotm k
ight)igg\}\chi_\lambda,\ rac{h+m_a)+z\left(m_b^2-m_a^2
ight)+2(1-z)M^2}{m k^2+\epsilon^2}igg]-z\left(m_b+m_a)rac{\left(m \sigma\cdotm k
ight)}{m k^2+\epsilon^2}igg\}\chi_\lambda. \end{aligned}$$

$$\begin{split} \Psi_{A}(z,\boldsymbol{k}) &= C_{f}^{-} g_{A,f}^{-} \sqrt{z(1-z)} \frac{\boldsymbol{k}^{2} + \epsilon^{2}}{\boldsymbol{k}^{2} + \epsilon^{2}} \qquad \Gamma_{V} = E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \gamma^{\mu} + (m_{b} - m_{a}) \frac{\boldsymbol{k}^{\mu}}{M^{2}} \right\} u(p_{a},\lambda,m_{a}) \\ \Gamma_{A} &= E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \left[ \gamma^{\mu} + (m_{b} + m_{a}) \frac{\boldsymbol{k}^{\mu}}{M^{2}} \right] \gamma_{5} \right\} u(p_{a},\lambda,m_{a}) \\ \Psi_{V}^{T}(z,\boldsymbol{k}) &= C_{f}^{C} g_{V,f}^{C} \frac{\sqrt{z}}{\boldsymbol{k}^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left( \left( \frac{2-z}{z} \right) \boldsymbol{k} \cdot \boldsymbol{E}^{*}(\lambda_{G}) + i\lambda [\boldsymbol{k}, \boldsymbol{E}^{*}(\lambda_{G})] \right) I - \lambda [m_{b} - (1-z)m_{a}] \boldsymbol{\sigma} \cdot \boldsymbol{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{A}^{T}(z,\boldsymbol{k}) &= C_{f}^{C} g_{V,f}^{C} \frac{\sqrt{z}}{\boldsymbol{k}^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left[ \frac{2-z}{z} \boldsymbol{k} \cdot \boldsymbol{E}^{*}(\lambda_{G}) + i\lambda [\boldsymbol{k}, \boldsymbol{E}^{*}(\lambda_{G})] \right] \lambda I - (m_{b} + (1-z)m_{a}) \boldsymbol{\sigma} \cdot \boldsymbol{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{V}^{L}(z,\boldsymbol{k}) &= C_{f}^{C} g_{V,f}^{C} \frac{1}{\sqrt{zM}} \chi_{\lambda'}^{\dagger} \left\{ I \left[ \frac{2^{2}m_{a}(m_{b} - m_{a}) - z(m_{b}^{2} - m_{a}^{2}) - 2(1-z)M^{2}}{\boldsymbol{k}^{2} + \epsilon^{2}} \right] + \frac{[z(m_{b} - m_{a})]}{\boldsymbol{k}^{2} + \epsilon^{2}} \lambda (\boldsymbol{\sigma} \cdot \boldsymbol{k}) \right\} \chi_{\lambda}, \\ \Psi_{L}^{A}(z,\boldsymbol{k}) &= C_{f}^{C} g_{A,f}^{C} \frac{1}{\sqrt{zM}} \chi_{\lambda'}^{\dagger} \left\{ \lambda I \left[ -\frac{z^{2}m_{a}(m_{b} + m_{a}) + z(m_{b}^{2} - m_{a}^{2}) + 2(1-z)M^{2}}{\boldsymbol{k}^{2} + \epsilon^{2}} \right] - z(m_{b} + m_{a}) \frac{(\boldsymbol{\sigma} \cdot \boldsymbol{k})}{\boldsymbol{k}^{2} + \epsilon^{2}} \right\} \chi_{\lambda}. \end{split}$$

$$\begin{split} \Psi_{A}(z,\boldsymbol{k}) &= C_{f}^{-} g_{A,f}^{-} \sqrt{z(1-z)} \frac{\boldsymbol{k}^{2} + \epsilon^{2}}{\boldsymbol{k}^{2} + \epsilon^{2}} \qquad \Gamma_{V} = E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \gamma^{\mu} + (m_{b} - m_{a}) \frac{\boldsymbol{k}^{\mu}}{M^{2}} \right\} u(p_{a},\lambda,m_{a}) \\ \Gamma_{A} &= E_{\mu}^{*}(k,\lambda) \overline{u}(p_{b},\lambda',m_{b}) \left\{ \left[ \gamma^{\mu} + (m_{b} + m_{a}) \frac{\boldsymbol{k}^{\mu}}{M^{2}} \right] \gamma_{5} \right\} u(p_{a},\lambda,m_{a}) \\ \Psi_{V}^{T}(z,\boldsymbol{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{\sqrt{z}}{\boldsymbol{k}^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left( \left( \frac{2-z}{z} \right) \boldsymbol{k} \cdot \boldsymbol{E}^{*}(\lambda_{G}) + i\lambda [\boldsymbol{k}, \boldsymbol{E}^{*}(\lambda_{G})] \right) I - \lambda [m_{b} - (1-z)m_{a}] \boldsymbol{\sigma} \cdot \boldsymbol{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{A}^{T}(z,\boldsymbol{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{\sqrt{z}}{\boldsymbol{k}^{2} + \epsilon^{2}} \chi_{\lambda'}^{\dagger} \left\{ \left[ \frac{2-z}{z} \boldsymbol{k} \cdot \boldsymbol{E}^{*}(\lambda_{G}) + i\lambda [\boldsymbol{k}, \boldsymbol{E}^{*}(\lambda_{G})] \right] \lambda I - (m_{b} + (1-z)m_{a}) \boldsymbol{\sigma} \cdot \boldsymbol{E}^{*}(\lambda_{G}) \right\} \chi_{\lambda}, \\ \Psi_{V}^{L}(z,\boldsymbol{k}) &= C_{f}^{G} g_{V,f}^{G} \frac{1}{\sqrt{z}M} \chi_{\lambda'}^{\dagger} \left\{ I \left[ \frac{2^{2}m_{a} (m_{b} - m_{a}) - z \left(m_{b}^{2} - m_{a}^{2}\right) - 2(1-z)M^{2}}{\boldsymbol{k}^{2} + \epsilon^{2}} \right] + \frac{[z(m_{b} - m_{a})]}{\boldsymbol{k}^{2} + \epsilon^{2}} \lambda (\boldsymbol{\sigma} \cdot \boldsymbol{k}) \right\} \chi_{\lambda}, \\ \Psi_{L}^{A}(z,\boldsymbol{k}) &= C_{f}^{G} g_{A,f}^{G} \frac{1}{\sqrt{z}M} \chi_{\lambda'}^{\dagger} \left\{ \lambda I \left[ -\frac{z^{2}m_{a} (m_{b} + m_{a}) + z \left(m_{b}^{2} - m_{a}^{2}\right) + 2(1-z)M^{2}}{\boldsymbol{k}^{2} + \epsilon^{2}} \right] - z (m_{b} + m_{a}) \frac{(\boldsymbol{\sigma} \cdot \boldsymbol{k})}{\boldsymbol{k}^{2} + \epsilon^{2}} \right\} \chi_{\lambda}. \end{split}$$



# **Light Front Wave Function**

Gauge Boson	$\mathcal{C}_{f}^{G}$	$g^G_{v,f}$
$Z^0$	$\mathcal{C}_f^Z = \frac{\sqrt{\alpha_{em}}}{\sin 2\theta_W}$	$g_{v,f_u}^Z = \frac{1}{2} - \frac{4}{3}\sin^2\theta$
		$g_{v,f_d}^Z = -\frac{1}{2} + \frac{2}{3}\sin^2$
$W^{\pm}$	$\mathcal{C}_f^{W^+} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_W} V_{f_u f_d}$	$g_{v,f}^W = 1$
	$\mathcal{C}_f^{W^-} = \frac{\sqrt{\alpha_{em}}}{2\sqrt{2}\sin\theta_W} V_{f_d f_u}$	
Photon	$C_f^{\gamma} = \sqrt{\alpha_{em}} e_f$	$g_{v,f}^{\gamma} = 1$

$$\begin{split} \Psi_{V}^{T}(z,\boldsymbol{k}) &= \mathcal{O}_{f}^{G}g_{V,f}^{G}\frac{\sqrt{z}}{\boldsymbol{k}^{2}+\epsilon^{2}}\chi_{\lambda'}^{\dagger} \bigg\{ \left( \left(\frac{2-z}{z}\right)\boldsymbol{k}\cdot\boldsymbol{E}^{*}(\lambda_{G})+i\lambda\left[\boldsymbol{k},\boldsymbol{E}^{*}(\lambda_{G})\right] \right)I -\lambda\left[m_{b}-(1-z)m_{a}\right]\boldsymbol{\sigma}\cdot\boldsymbol{E}^{*}(\lambda_{G}) \bigg\}\chi_{\lambda}, \\ \Psi_{A}^{T}(z,\boldsymbol{k}) &= \mathcal{O}_{f}^{G}g_{A,f}^{G}\frac{\sqrt{z}}{\boldsymbol{k}^{2}+\epsilon^{2}}\chi_{\lambda'}^{\dagger} \bigg\{ \left[ \frac{2-z}{z}\boldsymbol{k}\cdot\boldsymbol{E}^{*}(\lambda_{G})+i\lambda\left[\boldsymbol{k},\boldsymbol{E}^{*}(\lambda_{G})\right] \right]\lambda I - (m_{b}+(1-z)m_{a})\boldsymbol{\sigma}\cdot\boldsymbol{E}^{*}(\lambda_{G}) \bigg\}\chi_{\lambda}, \\ \Psi_{V}^{L}(z,\boldsymbol{k}) &= \mathcal{O}_{f}^{G}g_{V,f}^{G}\frac{1}{\sqrt{z}M}\chi_{\lambda'}^{\dagger} \bigg\{ I \bigg[ \frac{z^{2}m_{a}\left(m_{b}-m_{a}\right)-z\left(m_{b}^{2}-m_{a}^{2}\right)-2(1-z)M^{2}}{\boldsymbol{k}^{2}+\epsilon^{2}} \bigg] + \frac{[z(m_{b}-m_{a})]}{\boldsymbol{k}^{2}+\epsilon^{2}}\lambda\left(\boldsymbol{\sigma}\cdot\boldsymbol{k}\right) \bigg\}\chi_{\lambda}, \\ \Psi_{L}^{A}(z,\boldsymbol{k}) &= \mathcal{O}_{f}^{G}g_{A,f}^{G}\frac{1}{\sqrt{z}M}\chi_{\lambda'}^{\dagger} \bigg\{\lambda I \bigg[ -\frac{z^{2}m_{a}\left(m_{b}+m_{a}\right)+z\left(m_{b}^{2}-m_{a}^{2}\right)+2(1-z)M^{2}}{\boldsymbol{k}^{2}+\epsilon^{2}} \bigg] - z\left(m_{b}+m_{a}\right)\frac{(\boldsymbol{\sigma}\cdot\boldsymbol{k})}{\boldsymbol{k}^{2}+\epsilon^{2}} \bigg\}\chi_{\lambda}. \end{split}$$

$$\epsilon^2 = (1-z)M_G^2 + zm_b^2 - z(1-z)m_a^2 = (1-z)M_G^2 + z(m_b^2 - m_a^2) + z^2m_a^2$$

$$egin{aligned} rac{d\sigma^f_{T,L}(q_fN o G(p_G)q_k(p_q))}{dz d^2 oldsymbol{p} d^2 oldsymbol{\Delta}} &= rac{1}{2(2\pi)^4} \int d^2 oldsymbol{r} d^2 oldsymbol{r} d^2 oldsymbol{r} \exp[-ioldsymbol{p}\cdot(oldsymbol{r}-oldsymbol{r}')] \overline{\sum_{ ext{pol}.}} \Psi_{T,L}(z,oldsymbol{r}) \Psi_{T,L}^*(z,oldsymbol{r}') 
onumber \ imes \int d^2 oldsymbol{s} \exp\left[-ioldsymbol{\Delta}\cdotoldsymbol{s}
ight] \left\{ \sigma_{qar{q}}(oldsymbol{s}-zoldsymbol{r},x) + \sigma_{qar{q}}(oldsymbol{s}+zoldsymbol{r}',x) - \sigma_{qar{q}}(oldsymbol{s}-z(oldsymbol{r}-oldsymbol{r}'),x) - \sigma_{qar{q}}oldsymbol{s}
ight\} \end{aligned}$$

integrating over  $\Delta$ 

$$\left. rac{d\sigma^f_{T,L}}{dz\,d^2oldsymbol{p}} 
ight|_{V,A} = \left. rac{1}{2(2\pi)^2} \int d^2oldsymbol{r} \int d^2oldsymbol{r}' e^{ioldsymbol{p}\cdot(oldsymbol{r}-oldsymbol{r}')} 
ho^{T,L}_{V,A} \left[ \sigma_{q\overline{q}}(zoldsymbol{r},x) + \sigma_{q\overline{q}}(zoldsymbol{r}',x) - \sigma_{q\overline{q}}(z|oldsymbol{r}-oldsymbol{r}'|,x) 
ight]$$

$$\frac{d\sigma_{T,L}^{f}}{dz \, d^{2} \boldsymbol{p}}\Big|_{V,A} = \left.\frac{1}{2(2\pi)^{2}} \int d^{2} \boldsymbol{r} \int d^{2} \boldsymbol{r}' e^{i \boldsymbol{p} \cdot (\boldsymbol{r}-\boldsymbol{r}')} \rho_{V,A}^{T,L} \left[\sigma_{q\overline{q}}(z\boldsymbol{r},x) + \sigma_{q\overline{q}}(z\boldsymbol{r}',x) - \sigma_{q\overline{q}}(z|\boldsymbol{r}-\boldsymbol{r}')\right]\right]$$

$$zrac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2oldsymbol{p}}|_V = rac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{2\pi^2}\int\mathrm{d}^2oldsymbol{k}\,f(x,oldsymbol{k})\left\{z^2[(m_b-m_a)+zm_a]^2rac{\mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)}{\mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)}+[1+(1-z)^2]rac{\mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z)}{2\pi^2}
ight\}$$

$$zrac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2oldsymbol{p}}|_A = rac{(\mathcal{C}_f^G)^2(g^G_{v,f})^2}{2\pi^2}\int\mathrm{d}^2oldsymbol{k}\,f(x,oldsymbol{k})\left\{z^2[(m_b+m_a)-zm_a]^2\,\mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)+[1+(1-z)^2]\mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z)
ight\}$$

$$z \frac{\mathrm{d}\sigma_L^f}{\mathrm{d}z\mathrm{d}^2\boldsymbol{p}}|_V = \frac{(\mathcal{C}_f^G)^2 (g_{v,f}^G)^2}{4\pi^2} \int \mathrm{d}^2\boldsymbol{k} \, f(x,\boldsymbol{k}) \left\{ \frac{(z^2 m_a (m_b - m_a) - z(m_b^2 - m_a^2) - 2(1-z)M_G^2)^2}{M_G^2} \right\}$$

$$z rac{\mathrm{d}\sigma_L^f}{\mathrm{d}z\mathrm{d}^2 oldsymbol{p}}|_A = rac{(\mathcal{C}_f^G)^2 (g_{v,f}^G)^2}{4\pi^2} \int \mathrm{d}^2 oldsymbol{k} f(x,oldsymbol{k}) \left\{ rac{(z^2 m_a (m_b + m_a) + z(m_b^2 - m_a^2) - 2(1-z)M_G^2)^2}{M_G^2} \mathcal{E}_{M_G^2} 
ight\}$$

$$egin{aligned} eta_{V,A}^{T,L} &= rac{1}{2} \sum_{\lambda\lambda'\lambda_G} \psi_{V,A}^{T,L}(z,m{r}) \psi_{V,A}^{T,L,*}(z,m{r}') \ & \sigma_{q\overline{q}}(m{r}) &= \int \mathrm{d}^2m{k} \, f(x,m{k}) \left(1-e^{im{k}\cdotm{r}}
ight) \ & \mathcal{E}_1(m{p},m{k},\epsilon,z) &\equiv rac{1}{2} \Big[rac{1}{p^2+\epsilon^2} - rac{1}{(m{p}-zm{k})^2+\epsilon^2}\Big]^2 \ & \mathcal{E}_2(m{p},m{k},\epsilon,z) &\equiv rac{1}{2} \Big[rac{m{p}}{p^2+\epsilon^2} - rac{m{p}-zm{k}}{(m{p}-m{k})^2+\epsilon^2}\Big]^2 \end{aligned}$$

$$egin{aligned} & + \mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z) + rac{z^2(m_b-m_a)^2}{M_G^2} \mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z) 
ight\} \ & + rac{z^2(m_b-m_a)^2}{M_G^2} \mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z) 
ight\} \end{aligned}$$

$$\frac{d\sigma_{T,L}^{f}}{dz \, d^{2} \boldsymbol{p}}\Big|_{V,A} = \left.\frac{1}{2(2\pi)^{2}} \int d^{2} \boldsymbol{r} \int d^{2} \boldsymbol{r}' e^{i \boldsymbol{p} \cdot (\boldsymbol{r}-\boldsymbol{r}')} \boldsymbol{\rho}_{V,A}^{T,L} \left[\sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}')\right] \right] + \sigma_{q\bar{q}}(z|\boldsymbol{r},x) + \sigma_{q\bar{q}}(z$$

$$zrac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2oldsymbol{p}}|_V = rac{(\mathcal{C}_f^G)^2(g_{v,f}^G)^2}{2\pi^2}\int\mathrm{d}^2oldsymbol{k}\,f(x,oldsymbol{k})\left\{z^2[(m_b-m_a)+zm_a]^2\,oldsymbol{\mathcal{E}}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)+[1+(1-z)^2]oldsymbol{\mathcal{E}}_2(oldsymbol{p},oldsymbol{k},\epsilon,z)
ight\}$$

$$zrac{\mathrm{d}\sigma_T^f}{\mathrm{d}z\mathrm{d}^2oldsymbol{p}}|_A = rac{(\mathcal{C}_f^G)^2(g^G_{v,f})^2}{2\pi^2}\int\mathrm{d}^2oldsymbol{k}\,f(x,oldsymbol{k})\left\{z^2[(m_b+m_a)-zm_a]^2\,\mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z)+[1+(1-z)^2]\mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z)
ight\}$$

$$z\frac{\mathrm{d}\sigma_{L}^{f}}{\mathrm{d}z\mathrm{d}^{2}\boldsymbol{p}}|_{V} = \frac{(\mathcal{C}_{f}^{G})^{2}(g_{v,f}^{G})^{2}}{4\pi^{2}}\int\mathrm{d}^{2}\boldsymbol{k}\,f(x,\boldsymbol{k})\left\{\frac{(z^{2}m_{a}(m_{b}-m_{a})-z(m_{b}^{2}-m_{a}^{2})-2(1-z)M_{G}^{2})^{2}}{M_{G}^{2}}\mathcal{E}_{1}(\boldsymbol{p},\boldsymbol{k},\epsilon,z) + \frac{z^{2}(m_{b}-m_{a})^{2}}{M_{G}^{2}}\mathcal{E}_{2}(\boldsymbol{p},\boldsymbol{k},\epsilon,z)\right\}$$

$$z rac{\mathrm{d}\sigma_L^f}{\mathrm{d}z \mathrm{d}^2 oldsymbol{p}}|_A = rac{(\mathcal{C}_f^G)^2 (g_{v,f}^G)^2}{4\pi^2} \int \mathrm{d}^2 oldsymbol{k} f(x,oldsymbol{k}) \left\{ rac{(z^2 m_a (m_b + m_a) + z (m_b^2 - m_a^2) - 2(1 - z) M_G^2)^2}{M_G^2} \mathcal{E}_1(oldsymbol{p},oldsymbol{k},\epsilon,z) + rac{z^2 (m_b - m_a)^2}{M_G^2} \mathcal{E}_2(oldsymbol{p},oldsymbol{k},\epsilon,z) 
ight\}$$

### A generalized description for all electroweak gauge boson

$$egin{aligned} eta_{V,A}^{T,L} &= rac{1}{2} \sum_{\lambda\lambda'\lambda_G} \psi_{V,A}^{T,L}(z,m{r}) \psi_{V,A}^{T,L,*}(z,m{r}') \ & \sigma_{q\overline{q}}(m{r}) &= \int \mathrm{d}^2m{k} \, f(x,m{k}) \left(1-e^{im{k}\cdotm{r}}
ight) \ & \mathcal{E}_1(m{p},m{k},\epsilon,z) &\equiv rac{1}{2} \Big[rac{1}{p^2+\epsilon^2} - rac{1}{(m{p}-zm{k})^2+\epsilon^2}\Big]^2 \ & \mathcal{E}_2(m{p},m{k},\epsilon,z) &\equiv rac{1}{2} \Big[rac{m{p}}{p^2+\epsilon^2} - rac{m{p}-zm{k}}{(m{p}-m{k})^2+\epsilon^2}\Big]^2 \end{aligned}$$

**Real photon production** 

$$\left.rac{d\sigma^f_T}{d\ln z\,d^2oldsymbol{p}}
ight|_{qp
ightarrow\gamma X} = rac{lpha_{em}e_f^2}{2\pi^2}ig\{m_f^2 z^4\, {\cal D}_1(z,p,\epsilon)+\,[1+(1-z)^2]\epsilon^2\, {\cal D}_2(z)ig\}$$

J.Jalilian-Marian and A.H.Rezaeian [Phys.Rev.D 86 (2012) 034016] V. P. Goncalves, Y.Lima, R.Pasechnik and M.Šumbera [Phys.Rev.D 101 (2020) 9, 094019] B.Ducloué, T.Lappi and H.Mäntysaari [Phys.Rev.D 97 (2018) 5, 054023]

**Drell-Yan process** 

$$rac{d\sigma(qp 
ightarrow [G 
ightarrow lar{l}]X)}{dz d^2 oldsymbol{p} dM^2} = \mathcal{F}_G(M) \, rac{d\sigma(qp 
ightarrow G)}{dz d^2 oldsymbol{p}}$$

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#### Our general formalism cover particular case present in the literature.

### Particular cases

 $(z, p, \epsilon) \}$ 

GX)

$$\frac{\mathrm{d}\sigma_{T,L}^{f}(q_{a} \rightarrow G(p_{G})q_{b}(p_{q}))}{\mathrm{d}z\mathrm{d}^{2}\boldsymbol{p}\mathrm{d}^{2}\boldsymbol{\Delta}} = \frac{1}{2(2\pi)^{4}} \int \mathrm{d}^{2}\boldsymbol{k} \int \mathrm{d}^{2}\boldsymbol{r}\mathrm{d}^{2}\boldsymbol{r}' \, e^{-i\boldsymbol{p}\cdot(\boldsymbol{r}-\boldsymbol{r}')} \left[\rho_{V}^{T,L}(z,\boldsymbol{r},\boldsymbol{r}') + \rho_{A}^{T,L}(z,\boldsymbol{r},\boldsymbol{r}')\right] \int \mathrm{d}^{2}\boldsymbol{s} \, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{s}} \left\{ e^{i\boldsymbol{k}\cdot\boldsymbol{s}} + e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{z}(\boldsymbol{r}-\boldsymbol{r}'))} - e^{i\boldsymbol{k}\cdot(\boldsymbol{s}+\boldsymbol{z}\boldsymbol{r}')} - e^{i\boldsymbol{k}\cdot(\boldsymbol{s}-\boldsymbol{z}\boldsymbol{r})} \right\} f(\boldsymbol{x},\boldsymbol{k})$$

$$\left. z \frac{\mathrm{d}\sigma_T^f(q_a \to Gq_b)}{\mathrm{d}z \mathrm{d}^2 \boldsymbol{p} \mathrm{d}^2 \boldsymbol{\Delta}} \right|_V = \frac{\left(C_f^G\right)^2 \left(g_{V,f}^G\right)^2}{2\pi^2} f(x, \boldsymbol{\Delta}) \left\{ z^2 \Big[ (m_b - m_a)^2 + z m_a \Big]^2 \boldsymbol{\mathcal{E}}_1(\boldsymbol{p}, \boldsymbol{\Delta}, \epsilon, z) + \Big[ 1 + (1-z)^2 \Big] \boldsymbol{\mathcal{E}}_2(\boldsymbol{p}, \boldsymbol{\Delta}, \epsilon, z) \right\}$$

$$\left.z rac{\mathrm{d}\sigma_T^f(q_a o Gq_b)}{\mathrm{d}z \mathrm{d}^2 oldsymbol{p} \mathrm{d}^2 \mathbf{\Delta}}
ight|_A = rac{\left(C_f^G
ight)^2 \left(g_{A,f}^G
ight)^2}{2\pi^2} f(x, \mathbf{\Delta}) \Biggl\{ z^2 \Bigl[(m_b + m_a) - zm_a \Bigr]^2 \mathcal{E}_1(oldsymbol{p}, \mathbf{\Delta}, \epsilon, z) + \Bigl[1 + (1 - z)^2 \Bigr] \mathcal{E}_2(oldsymbol{p}, \mathbf{\Delta}, \epsilon, z) \Biggr\}$$

$$\left. z \frac{\mathrm{d}\sigma_{L}^{f}(q_{a} \to Gq_{b})}{\mathrm{d}z \mathrm{d}^{2} \boldsymbol{p} \mathrm{d}^{2} \boldsymbol{\Delta}} \right|_{V} = \frac{\left(C_{f}^{G}\right)^{2} \left(g_{V,f}^{G}\right)^{2}}{(2\pi)^{2}} f(x, \boldsymbol{\Delta}) \left\{ \frac{\left[z^{2} m_{a}(m_{b} - m_{a}) - z(m_{b}^{2} - m_{a}^{2}) - 2(1 - z)M_{G}^{2}\right]^{2}}{M_{G}^{2}} \mathcal{E}_{1}(\boldsymbol{p}, \boldsymbol{\Delta}, \epsilon, z) + \frac{z^{2} (m_{b} - m_{a})^{2}}{M_{G}^{2}} \mathcal{E}_{2}(\boldsymbol{p}, \boldsymbol{\Delta}, \epsilon, z) \right\}$$

$$z \frac{\mathrm{d}\sigma_{L}^{f}(q_{a} \to Gq_{b})}{\mathrm{d}z \mathrm{d}^{2} \boldsymbol{p} \mathrm{d}^{2} \boldsymbol{\Delta}}\Big|_{A} = \frac{\left(C_{f}^{G}\right)^{2} \left(g_{A,f}^{G}\right)^{2}}{(2\pi)^{2}} f(x, \boldsymbol{\Delta}) \left\{ \frac{\left[z^{2} m_{a}(m_{b} + m_{a}) + z(m_{b}^{2} - m_{a}^{2}) + 2(1-z)M_{G}^{2}\right]^{2}}{M_{G}^{2}} \mathcal{E}_{1}(\boldsymbol{p}, z\boldsymbol{\Delta}, \epsilon) + \frac{z^{2}(m_{b} + m_{a})^{2}}{M_{G}^{2}} \mathcal{E}_{2}(\boldsymbol{p}, z\boldsymbol{\Delta}, \epsilon) \right\}$$

### We presented for the first time the expresion for the $W^{\pm}$ production

$$egin{split} \mathcal{E}_1(oldsymbol{p},oldsymbol{\Delta},\epsilon,z) &\equiv rac{1}{2} iggl[ rac{1}{p^2+\epsilon^2} - rac{1}{(oldsymbol{p}-zoldsymbol{\Delta})^2+\epsilon^2} iggr]^2 \ \mathcal{E}_2(oldsymbol{p},oldsymbol{\Delta},\epsilon,z) &\equiv rac{1}{2} iggl[ rac{oldsymbol{p}}{p^2+\epsilon^2} - rac{oldsymbol{p}-zoldsymbol{\Delta}}{(oldsymbol{p}-zoldsymbol{\Delta})^2+\epsilon^2} iggr]^2 \end{split}$$

 $\gamma + jet$  production

$$\left. z \, rac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^2 oldsymbol{p}\mathrm{d}^2 oldsymbol{\Delta}} 
ight|_{qp o \gamma X} = rac{lpha_{em} e_f^2}{(2\pi)^2} f(x,oldsymbol{\Delta}) \left[ 1 + (1-z)^2 
ight] rac{z^2 oldsymbol{\Delta}^2}{(oldsymbol{p}-zoldsymbol{\Delta})^2 oldsymbol{p}^2}$$

F. Dominguez, C. Marquet, B. W. Xiao and F. Yuan, Phys. Rev. D 83, 105005 (2011) A. Stasto, B. W. Xiao and D. Zaslavsky, Phys. Rev. D 86, 014009 (2012) P. Taels, JHEP 01, 005 (2024)

 $Z^0 + \text{jet produciton}$ 

$$\frac{\mathrm{d}\sigma}{\mathrm{d}z\mathrm{d}^{2}\boldsymbol{p}\mathrm{d}^{2}\boldsymbol{\Delta}}\Big|_{qp \to Z^{0}X}^{m_{f}=0} = \frac{(C_{f}^{Z})^{2}}{(2\pi)^{2}}\Big[(g_{V,f}^{Z})^{2} + (g_{A,f}^{Z})^{2}\Big]f(x,\boldsymbol{\Delta})\left\{\frac{1 + (1-z)^{2}}{z}\Big[\frac{\boldsymbol{p}-z\boldsymbol{\Delta}}{z}\Big[\frac{\boldsymbol{p}-z\boldsymbol{\Delta}}{(p^{2}+\bar{\epsilon}^{2})} - \frac{\boldsymbol{p}}{(p^{2}+\bar{\epsilon}^{2})}\Big]^{2} + 2\frac{(1-z)^{2}}{z}M_{Z}^{2}\Big[\frac{1}{[(\boldsymbol{p}-z\boldsymbol{\Delta})+\bar{\epsilon}^{2}]} - \frac{1}{(p^{2}+\bar{\epsilon}^{2})}\Big]^{2}\right\}$$

for  $\bar{\epsilon}^2 = (1-z)M_z^2$ 

E. Basso, V. P. Goncalves, M. Krelina, J. Nemchik and R. Pasechnik, Phys. Rev. D 93, no.9, 094027 (2016) E. Basso, V. P. Goncalves, J. Nemchik, R. Pasechnik and M. Sumbera, Phys. Rev. D 93, no.3, 034023 (2016)

#### **Once more, our general formalism cover particular case present in the literature.**

**Particular cases** 



- We <u>derived</u>, for the *first time*, the <u>generic expressions</u> for the **LFWF's**.
  - We have estimated the *vector* and *axial* contributions for the description of the <u>longitudinal</u> and transverse spectra associated with the isolated gauge boson production in the impact parameter and transverse momentum spaces.

• We demonstrated that our results reduce to expressions previously used in the literature for the description of the real photon production and Drell - Yan process at forward rapidities in some particular limits.

• As seen, the expressions obtained are the main ingredients for the calculation of the pp cross - sections, which can be compared with the current and forthcoming LHC data.

### Thank you for your attention!

# The Color Dipole S - matrix formalism

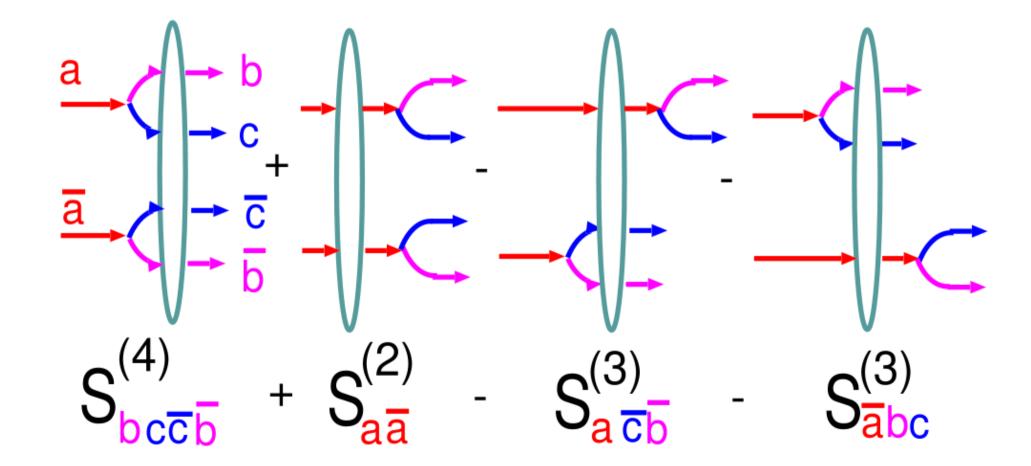
$$egin{aligned} rac{d\sigma(a o b(p_b) c(p_c))}{dz d^2 oldsymbol{p}_b d^2 oldsymbol{p}_c} &= rac{1}{(2\pi)^4} \, \int d^2 oldsymbol{b}_b d^2 oldsymbol{b}_c d^2 oldsymbol{b}_b' d^2 oldsymbol{b}_c' \expig[ioldsymbol{p}_b \cdotig(oldsymbol{b}_b + oldsymbol{b}_c) + oldsymbol{b}_c' oldsymbol{p}_c' oldsymbol{b}_c' o$$

$$S_{a\bar{a}}^{(2)}(\mathbf{b}', \mathbf{b}) = S_{a}^{\dagger}(\mathbf{b}')S_{a}(\mathbf{b}) ,$$
  

$$S_{\bar{a}bc}^{(3)}(\mathbf{b}', \mathbf{b}_{b}, \mathbf{b}_{c}) = S_{a}^{\dagger}(\mathbf{b}')S_{b}(\mathbf{b}_{b})S_{c}(\mathbf{b}_{c}) ,$$
  

$$S_{\bar{b}\bar{c}a}^{(3)}(\mathbf{b}, \mathbf{b}'_{b}, \mathbf{b}'_{c}) = S_{b}^{\dagger}(\mathbf{b}'_{b})S_{c}^{\dagger}(\mathbf{b}'_{c})S_{a}(\mathbf{b}) ,$$
  

$$S_{\bar{b}\bar{c}cb}^{(4)}(\mathbf{b}'_{b}, \mathbf{b}'_{c}, \mathbf{b}_{b}, \mathbf{b}_{c}) = S_{b}^{\dagger}(\mathbf{b}'_{b})S_{c}^{\dagger}(\mathbf{b}'_{c})S_{c}(\mathbf{b}_{c})S_{b}(\mathbf{b}_{b})$$





 $(-oldsymbol{b}_b)+ioldsymbol{p}_c\cdot(oldsymbol{b}_c-oldsymbol{b}_c')]$  $\left. {}^{(3)}_{ar{b}ar{c}a}(m{b},m{b}_b',m{b}_c') - S^{(3)}_{ar{a}bc}(m{b}',m{b}_b,m{b}_c) 
ight\}$ 

$$\begin{split} \frac{d\sigma_{T,L}^{f}}{dz \, d^{2} \boldsymbol{p}} \Big|_{V,A} &= \frac{1}{2(2\pi)^{2}} \int d^{2} \boldsymbol{r} \int d^{2} \boldsymbol{r}' e^{i \boldsymbol{p} \cdot (\boldsymbol{r}-\boldsymbol{r}')} \boldsymbol{\rho}_{V,A}^{T,L} \left[ \sigma_{q\bar{q}}(z\boldsymbol{r},x) + \sigma_{q\bar{q}}(z\boldsymbol{r}',x) - \sigma_{q\bar{q}}(z|\boldsymbol{r}-\boldsymbol{r}'|,x) \right] \\ & \downarrow \\ \frac{d\sigma_{T}^{f}}{dz d^{2} \boldsymbol{p}} \Big|_{V} &= \frac{(C_{f}^{G})^{2} (g_{f,V}^{G})^{2}}{(2\pi)^{2}} \left\{ z[(m_{b}-m_{a})+zm_{a}]^{2} \mathcal{D}_{1}(z,p,\epsilon) + \frac{\left[1+(1-z)^{2}\right]}{z} \epsilon^{2} \mathcal{D}_{2}(z,p,\epsilon) \right\}, \\ \frac{d\sigma_{T}^{f}}{dz d^{2} \boldsymbol{p}} \Big|_{A} &= \frac{(C_{f}^{G})^{2} (g_{f,A}^{G})^{2}}{2\pi^{2}} \left\{ z[(m_{b}+m_{a})-zm_{a}]^{2} \mathcal{D}_{1}(z,p,\epsilon) + \frac{1+(1-z)^{2}}{z} \epsilon^{2} \mathcal{D}_{2}(z,p,\epsilon) \right\}. \\ \frac{d\sigma_{L}^{f}}{dz d^{2} \boldsymbol{p}} \Big|_{V} &= \frac{(C_{f}^{G})^{2} (g_{f,V}^{G})^{2}}{(2\pi)^{2}} \left\{ \frac{\left[z^{2} m_{a}(m_{b}-m_{a})-z(m_{b}^{2}-m_{a}^{2})-2(1-z)M_{G}^{2}\right]^{2}}{zM_{C}^{2}} \mathcal{D}_{1}(z,p,\epsilon) + \frac{z(m_{b}-m_{a})^{2}}{M_{G}^{2}} \epsilon^{2} \mathcal{D}_{2}(z,p,\epsilon) \right\}. \end{split}$$

 $ho_{V,A}^{T,L} = rac{1}{2} \sum_{\lambda\lambda'\lambda_G} \psi_{V,A}^{T,L}(z,oldsymbol{r}) \psi_{V,A}^{T,L,*}(z,oldsymbol{r}')$ 

$$\left. rac{d\sigma^f_T}{dz d^2 oldsymbol{p}} 
ight|_V = \; rac{(C^G_f)^2 (g^G_{f,V})^2}{(2\pi)^2} \Biggl\{ z[(m_b-m_a)+zm_a]^2 \mathcal{D}_1(z,p,\epsilon) + rac{ig[1+(1-z)^2ig]}{z} \Biggr\}$$

$$\left. rac{d\sigma_T^f}{dz d^2 oldsymbol{p}} 
ight|_A = \; rac{(C_f^G)^2 (g_{f,A}^G)^2}{2\pi^2} iggl\{ z [(m_b+m_a)-zm_a]^2 \mathcal{D}_1 \left(z,p,\epsilon
ight) \; + \; rac{1+(1-z)^2}{z} \epsilon^2 \left(z - \frac{1+(1-z)^2}{z} + \frac$$

$$egin{aligned} & rac{d\sigma_L^f}{dzd^2m{p}} \Big|_V = \; rac{(C_f^G)^2(g_{f,V}^G)^2}{(2\pi)^2} iggl\{ rac{\left[z^2m_a(m_b-m_a)-z(m_b^2-m_a^2)-2(1-z)M_G^2
ight]^2}{zM_G^2} \mathcal{D}_1\left(z,p,\epsilon
ight) + rac{z(m_b-m_a)^2}{M_G^2}\epsilon^2 \mathcal{D}_2\left(z,p,\epsilon
ight) iggr\}. \ & rac{d\sigma_L^f}{dzd^2m{p}} \Big|_A = \; rac{(C_f^G)^2(g_{f,A}^G)^2}{(2\pi)^2} iggl\{ rac{\left[z^2m_a(m_b+m_a)+z(m_b^2-m_a^2)+2(1-z)M_G^2
ight]^2}{zM_G^2} \mathcal{D}_1\left(z,p,\epsilon
ight) + rac{z(m_b+m_a)^2}{M_G^2}\epsilon^2 \mathcal{D}_2\left(z,p,\epsilon
ight) iggr\}. \end{aligned}$$

$$\left. rac{d\sigma_L^f}{dz d^2 oldsymbol{p}} 
ight|_A = \; rac{(C_f^G)^2 (g_{f,A}^G)^2}{\left(2\pi
ight)^2} \Biggl\{ rac{\left[z^2 m_a (m_b+m_a)+z(m_b^2-m_a^2)+2(1-z)M_G^2
ight]^2}{zM_G^2} \mathcal{I}$$

for

$$egin{aligned} \mathcal{D}_1(z,p,\epsilon)&=rac{1}{p^2+\epsilon^2}I_1(z,p)-rac{1}{4\epsilon}I_2(z,p)\ \mathcal{D}_2(z,p,\epsilon)&=rac{1}{\epsilon}rac{p}{(p^2+\epsilon^2)}I_3(z,p)-rac{1}{2\epsilon^2}I_1(z,p)+rac{1}{4\epsilon}I_2(z,p). \end{aligned}$$

$$\left\{ \epsilon^{2} \mathcal{D}_{2}(z,p,\epsilon) \right\},$$
  
 $\left\{ \epsilon^{2} \mathcal{D}_{2}\left(z,p,\epsilon\right) \right\}.$ 

$$egin{aligned} &I_1(z,p)=\int dr r \mathrm{J}_0(pr) \mathrm{K}_0(\epsilon r) \sigma_{qar{q}}(zm{r}) \ &I_2(z,p)=\int dr r^2 \mathrm{J}_0(pr) \mathrm{K}_1(\epsilon r) \sigma_{qar{q}}(zm{r}) \ &I_3(z,p)=\int dr r \mathrm{J}_1(pr) \mathrm{K}_1(\epsilon r) \sigma_{qar{q}}(zm{r}) \end{aligned}$$