

The 2025 CFNS-SURGE Summer Workshop on the Physics  
of the Electron-Ion Collider

# Lattice QCD (selected topics)

## *Lecture 2*

**Martha Constantinou**



**Temple University**

**Center for Frontiers in Nuclear Science**

**Stony Brook University, USA**

**June 2 - 13, 2025**

# OUTLINE OF LECTURE 2

- ★ **Types of quantities we study in lattice QCD for hadron structure**
- ★ **Proton spin crisis**
- ★ **Accessing x-dependent distributions**
- ★ **Key points of Lectures 2**

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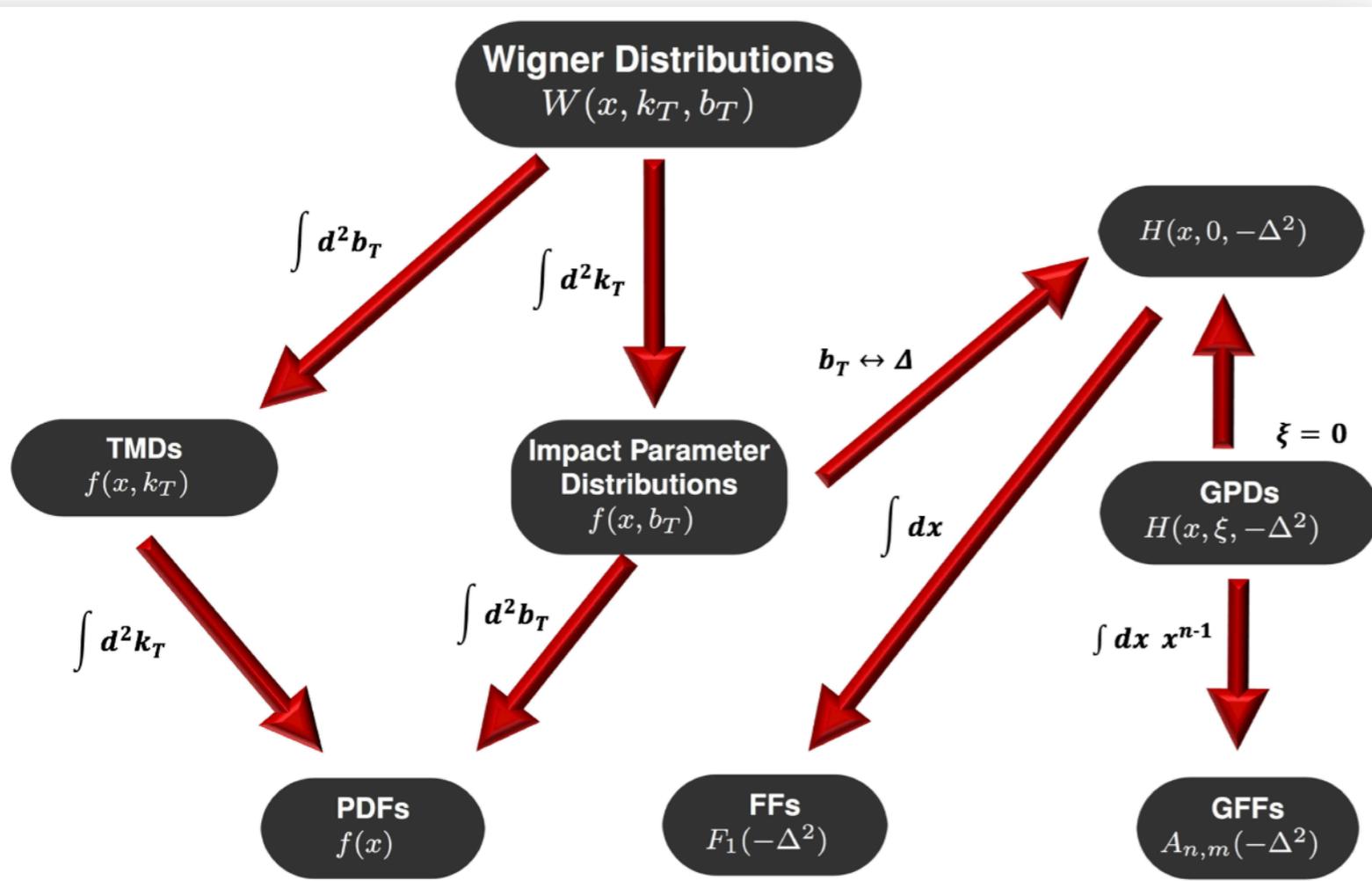
- ★ **Types of quantities we study in lattice QCD for hadron structure**
- ★ **Proton spin crisis** *But not really*
- ★ **Accessing x-dependent distributions**
- ★ **Key points of Lectures 2**

# How to study Hadron Structure

# Nucleon Characterization

## Wigner distributions

- ★ Fully characterize partonic structure of hadrons
- ★ Provide multi-dim images of the parton distributions in phase space

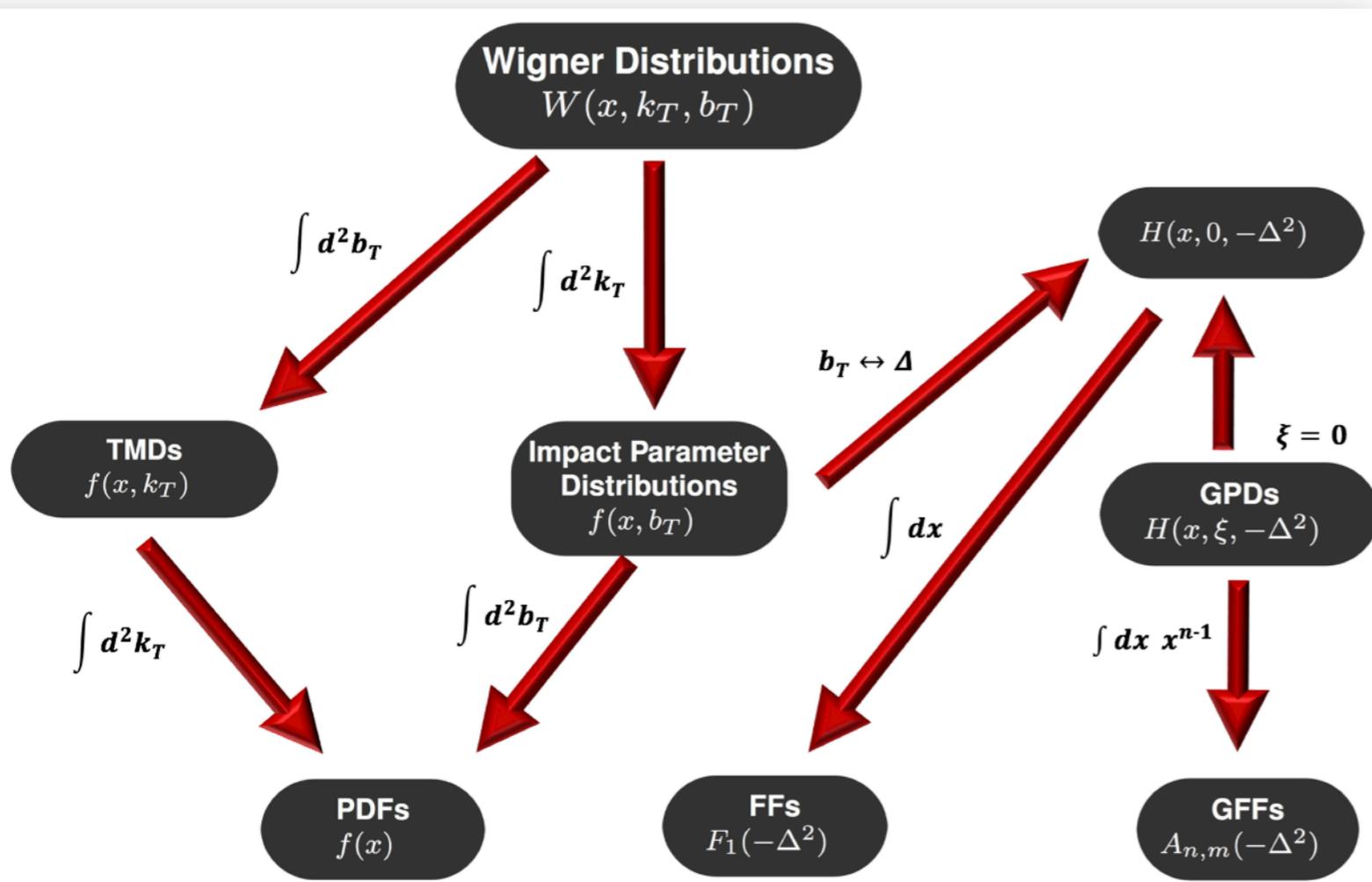


- ★ Correlations between momenta, positions, spins
- ★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

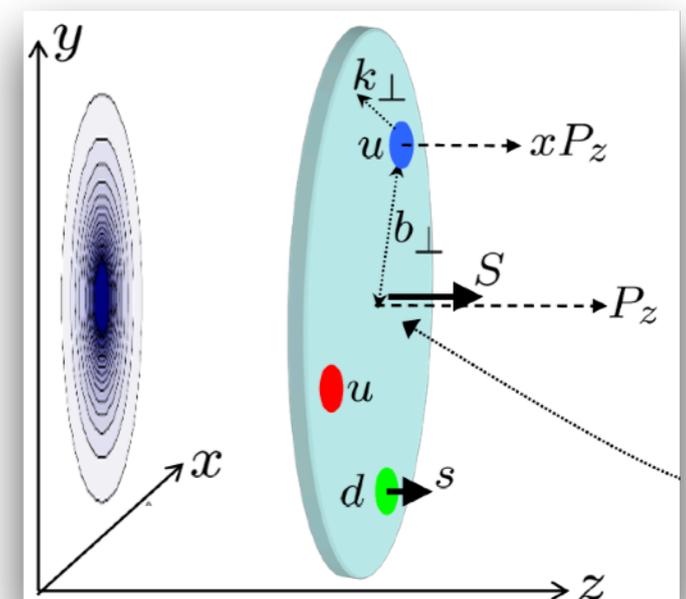
# Nucleon Characterization

## Wigner distributions

- ★ Fully characterize partonic structure of hadrons
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- ★ Partons contain information on
  - $x$ : longitudinal momentum fraction
  - $k_T$ : transverse momentum
  - $b_\perp$ : impact parameter



- ★ Correlations between momenta, positions, spins
- ★ Information on the hadron's mechanical properties (OAM, pressure, etc.)

# Accessing information on PDFs/GPDs

- ★ Parton model: physical picture valid for infinite momentum frame

[R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969)]

- ★ PDFs via matrix elements of nonlocal light-cone operators ( $-t^2 + \vec{r}^2 = 0$ )

$$f(x) = \frac{1}{4\pi} \int dy^- e^{-ixP^+y^-} \langle P, S | \bar{\psi}_f \gamma^+ \mathcal{W} \psi_f | P, S \rangle$$

- ★ Light-cone correlations inaccessible from Euclidean lattices ( $\tau^2 + \vec{r}^2 = 0$ )



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## A. Mellin moments (local OPE expansion)

local operators

$$\bar{q}\left(-\frac{1}{2}z\right) \gamma^\sigma W\left[-\frac{1}{2}z, \frac{1}{2}z\right] q\left(\frac{1}{2}z\right) = \sum_{n=0}^{\infty} \frac{1}{n!} z_{\alpha_1} \dots z_{\alpha_n} \left[ \bar{q} \gamma^\sigma \overleftrightarrow{D}^{\alpha_1} \dots \overleftrightarrow{D}^{\alpha_n} q \right]$$

$$\langle N(P') | \mathcal{O}_V^{\mu_1 \dots \mu_{n-1}} | N(P) \rangle \sim \sum_{\substack{i=0 \\ \text{even}}}^{n-1} \left\{ \gamma^{\mu_1 \dots \mu_i} \Delta^{\mu_1} \dots \Delta^{\mu_i} \bar{P}^{\mu_{i+1}} \dots \bar{P}^{\mu_{n-1}} A_{n,i}(t) - i \frac{\Delta_\alpha \sigma^{\alpha\mu}}{2m_N} \Delta^{\mu_1} \dots \Delta^{\mu_i} \bar{P}^{\mu_{i+1}} \dots \bar{P}^{\mu_{n-1}} B_{n,i}(t) \right\} + \frac{\Delta^\mu \Delta^{\mu_1} \dots \Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2) \Big|_{n \text{ even}} \left. \right\} + \frac{\Delta^\mu \Delta^{\mu_1} \dots \Delta^{\mu_{n-1}}}{m_N} C_{n,0}(\Delta^2) \Big|_{n \text{ even}} \Big] U(P)$$

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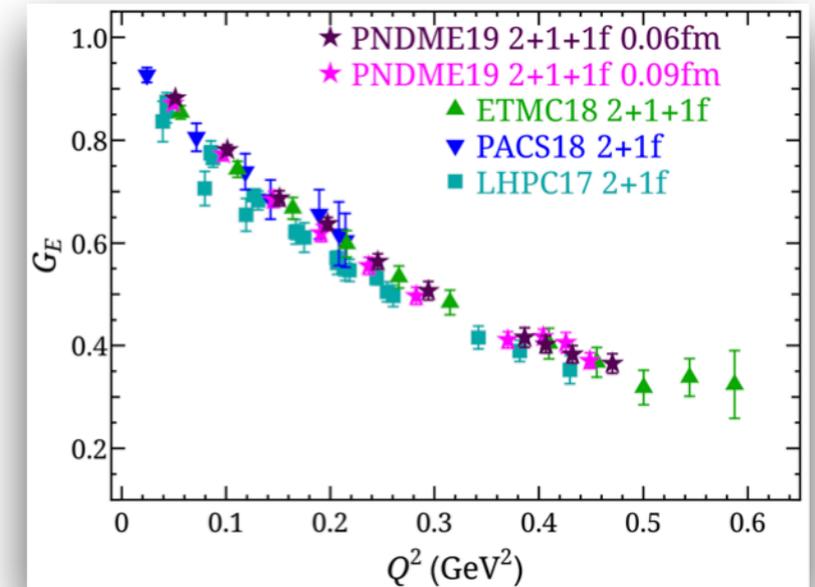
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- 👎 No direct access to  $x$       👎 skewness independent
- 👎 Power-divergent mixing for high Mellin moments (derivatives  $> 3$ )
- 👎 Signal-to-noise ratio decays with the addition of covariant derivatives
- 👎 Number of GFFs increases with order of Mellin moment

# Accessing information on PDFs/GPDs

Computationally efficient extraction of  $Q^2$  dependence



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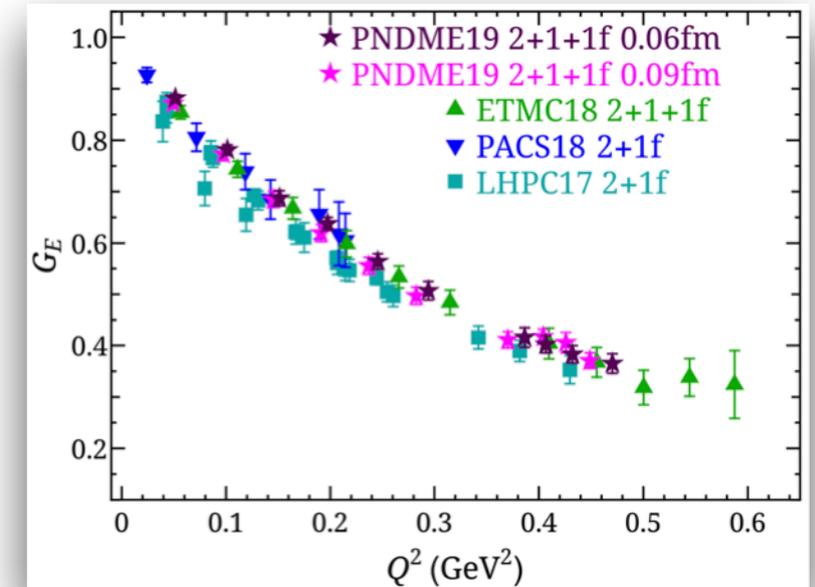
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Reconstruction of PDFs/GPDs very challenging

# Accessing information on PDFs/GPDs

## B. Matrix elements of nonlocal operators (quasi-GPDs, pseudo-GPDs)

$$\langle N(P_f) | \bar{\Psi}(z) \Gamma \mathcal{W}(z,0) \Psi(0) | N(P_i) \rangle_\mu$$

Nonlocal operator with Wilson line

$$\langle N(P') | O_V^\mu(x) | N(P) \rangle = \bar{U}(P') \left\{ \gamma^\mu H(x, \xi, t) + \frac{i\sigma^{\mu\nu} \Delta_\nu}{2m_N} E(x, \xi, t) \right\} U(P) + \text{ht},$$

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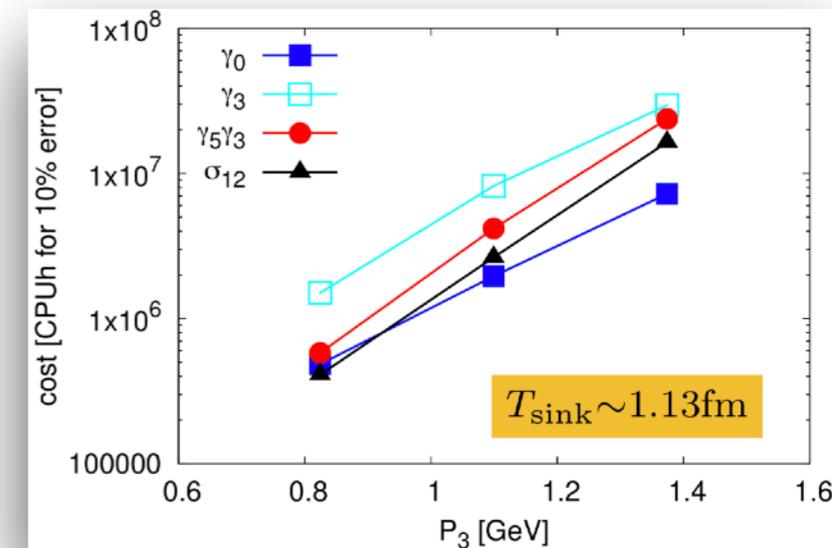
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## Calculation challenges

- ◆ Standard definition of GPDs in symmetric frame  
*separate calculations at each t*
- ◆ Statistical noise increases with  $P_3, t$   
**Projection:**  
billions of core-hours for physical point at  $P_3 = 3 \text{ GeV}$



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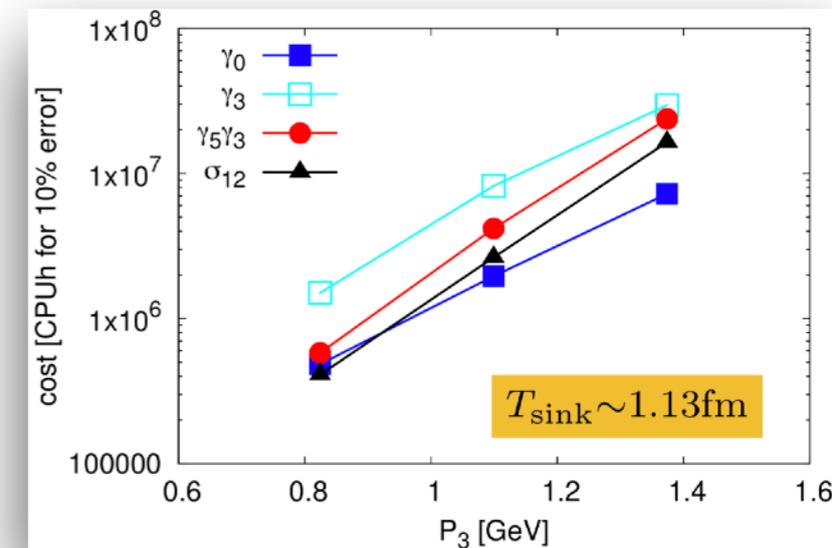
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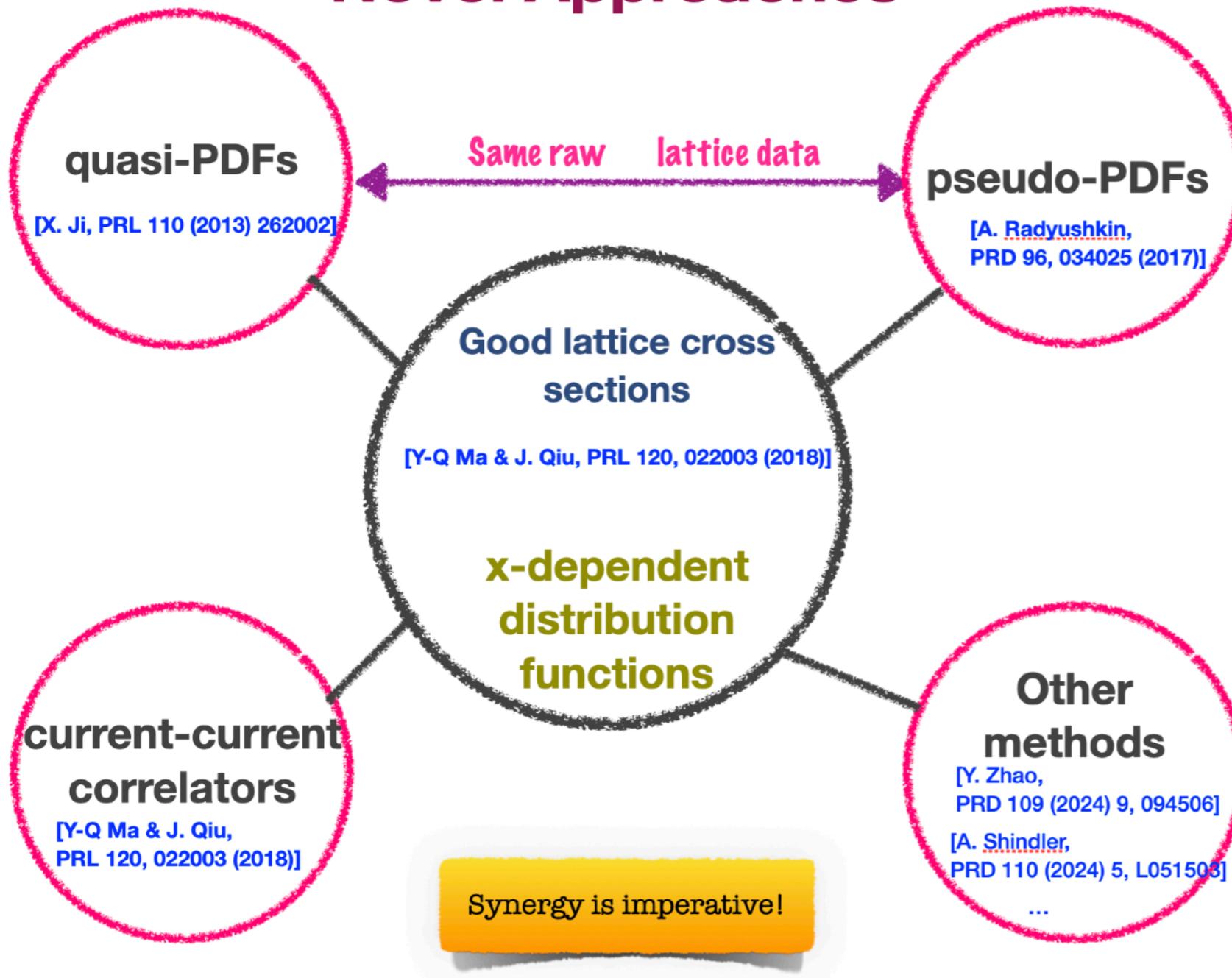


## C. Other methods

See next slide



# Novel Approaches



- Hadronic tensor [K.F. Liu, S.J. Dong, PRL 72 (1994) 1790, K.F. Liu, PoS(LATTICE 2015) 115]
- Auxiliary scalar quark [U. Aglietti et al., Phys. Lett. B441, 371 (1998), arXiv:hep-ph/9806277]
- Fictitious heavy quark [W. Detmold, C. J. D. Lin, Phys. Rev. D73, 014501 (2006) ]
- Auxiliary scalar quark [V. Braun & D. Mueller, Eur. Phys. J. C55, 349 (2008), arXiv:0709.1348]
- Higher moments [Z. Davoudi, M. Savage, Phys. Rev. D86, 054505 (2012) ]
- Quasi-distributions (LaMET) [X. Ji, PRL 110 (2013) 262002, arXiv:1305.1539; Sci. China PPMA. 57, 1407 (2014)]
- Compton amplitude and OPE [A. Chambers et al. (QCDSF), PRL 118, 242001 (2017), arXiv:1703.01153]
- Pseudo-distributions [A. Radyushkin, Phys. Rev. D 96, 034025 (2017), arXiv:1705.01488]
- Good lattice cross sections [Y-Q Ma & J. Qiu, Phys. Rev. Lett. 120, 022003 (2018), arXiv:1709.03018 ]
- PDFs without Wilson line [Y. Zhao Phys.Rev.D 109 (2024) 9, 094506, arXiv:2306.14960]
- Moments of PDFs of any order [A. Shindler, Phys.Rev.D 110 (2024) 5, L051503, arXiv:2311.18704 ]

## Reviews of methods and applications

- *A guide to light-cone PDFs from Lattice QCD: an overview of approaches, techniques and results*  
K. Cichy & M. Constantinou (invited review) Advances in HEP 2019, 3036904, arXiv:1811.07248
- *Large Momentum Effective Theory*  
X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543
- *The x-dependence of hadronic parton distributions: A review on the progress of lattice QCD*  
M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445



# Well-studied “novel” methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields)  
with **boosted hadrons**

$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f) | \bar{\Psi}(z) \Gamma \mathcal{W}(z,0) \Psi(0) | N(P_i) \rangle_\mu$$

Calculation very taxing!

- length of the Wilson line ( $z$ )
  - nucleon momentum boost ( $P_3$ )
  - momentum transfer ( $t$ )
  - skewness ( $\xi$ )
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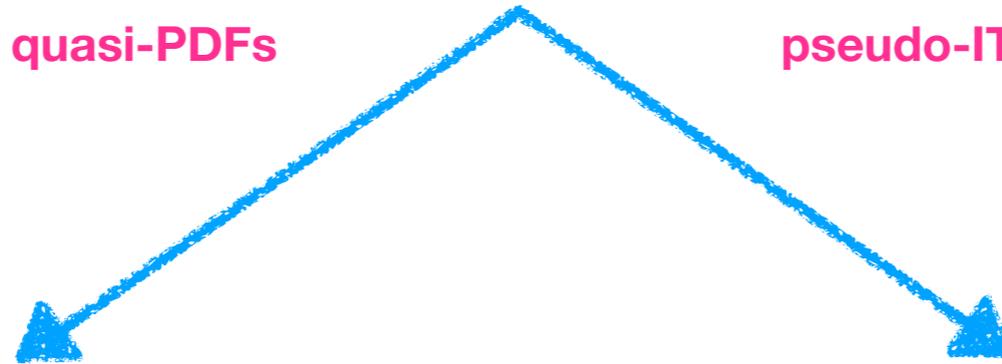
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[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

pseudo-ITD

[A. Radyushkin, PRD 96, 034025 (2017)]



$$\tilde{q}_\Gamma^{\text{GPD}}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-i x P_3 z} \mathcal{M}(P_f, P_i, z)$$

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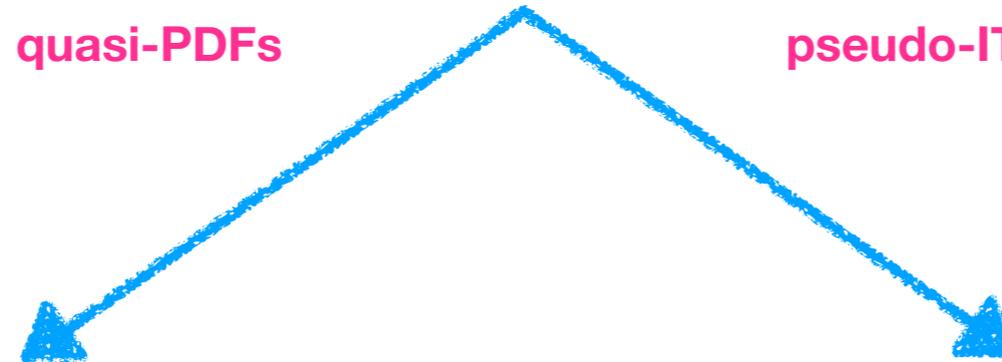
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Matching in momentum space  
(Large Momentum  
Effective Theory)

Matching in  $\nu$  space

$$Q(\nu, \mu^2) = \int_{-1}^1 dx e^{i\nu x} q(x, \mu^2)$$

Light-cone PDFs & GPDs

Calculation very taxing!

- length of the Wilson line ( $z$ )
  - nucleon momentum boost ( $P_3$ )
  - momentum transfer ( $t$ )
  - skewness ( $\xi$ )
- } PDFs, GPDs  
} GPDs

# Well-studied “novel” methods for PDFs/GPDs in LQCD

Matrix elements of non-local operators (space-like separated fields) with **boosted hadrons**

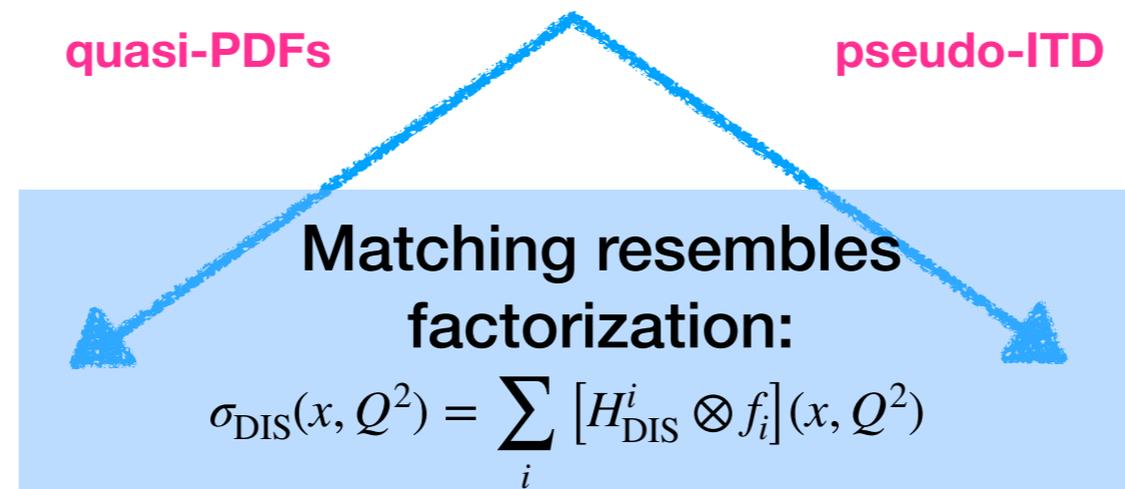
$$\mathcal{M}(P_f, P_i, z) = \langle N(P_f) | \bar{\Psi}(z) \Gamma \mathcal{W}(z, 0) \Psi(0) | N(P_i) \rangle_\mu$$

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002]  
[X. Ji, Sci. China Phys. M.A. 57 (2014) 1407]

quasi-PDFs

pseudo-ITD

[A. Radyushkin, PRD 96, 034025 (2017)]



$$\tilde{q}_\Gamma^{\text{GPD}}(x, t, \xi, P_3, \mu) = \int \frac{dz}{4\pi} e^{-ixP_3z} \mathcal{M}(P_f, P_i, z)$$

$$\mathfrak{M}(\nu, \xi, t; z_3^2) \equiv \frac{\mathcal{M}(\nu, \xi, t; z_3^2)}{\mathcal{M}(0, 0, 0; z_3^2)} \quad (\nu = z \cdot p)$$

Matching in momentum space  
(Large Momentum  
Effective Theory)

Matching in  $\nu$  space

$$Q(\nu, \mu^2) = \int_{-1}^1 dx e^{i\nu x} q(x, \mu^2)$$

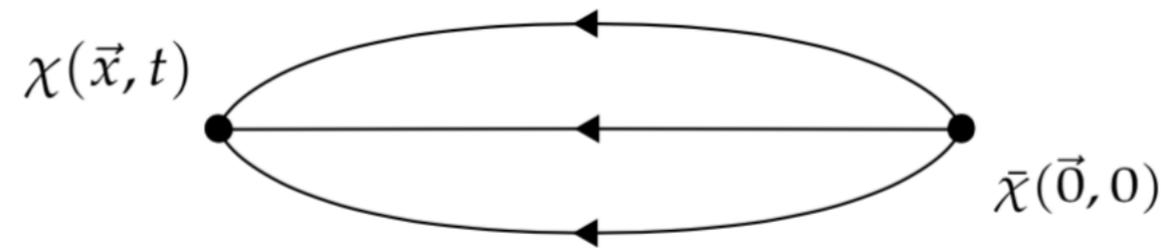
Light-cone PDFs & GPDs

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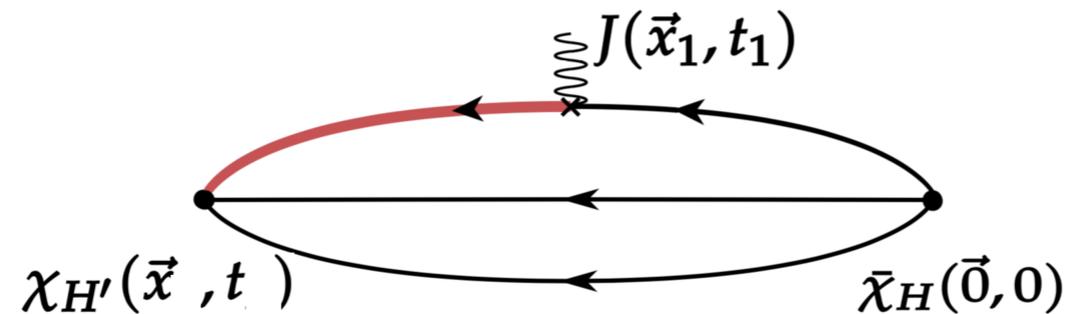
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# Correlation functions in lattice QCD

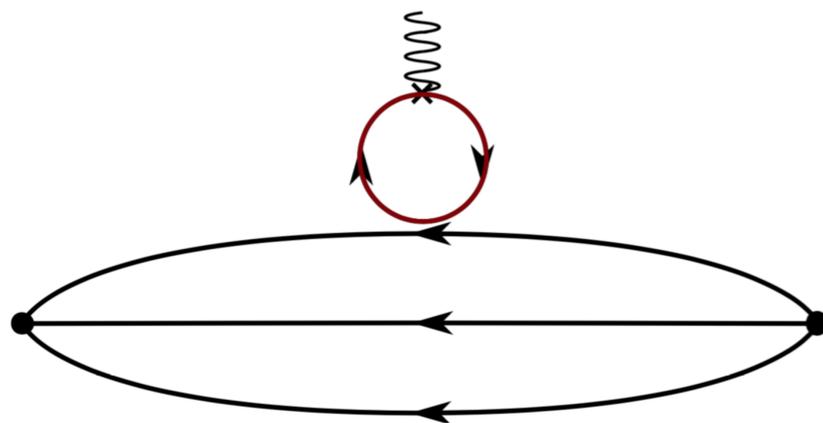
2pt-functions



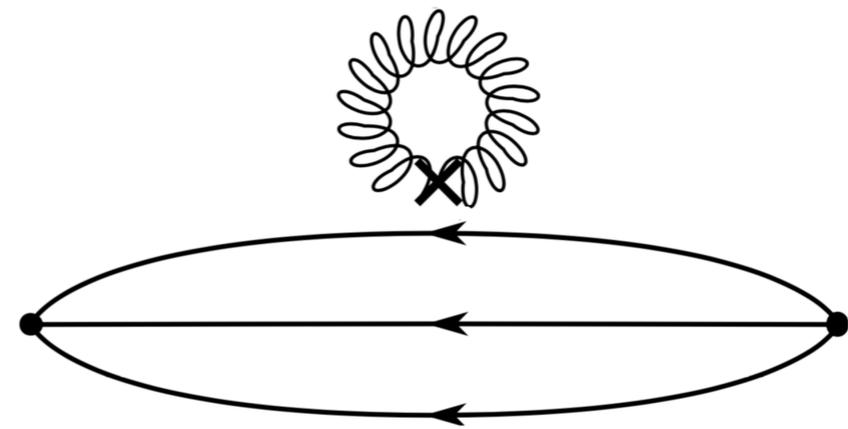
3pt-functions



**Connected**



**Disconnected Quark loop**

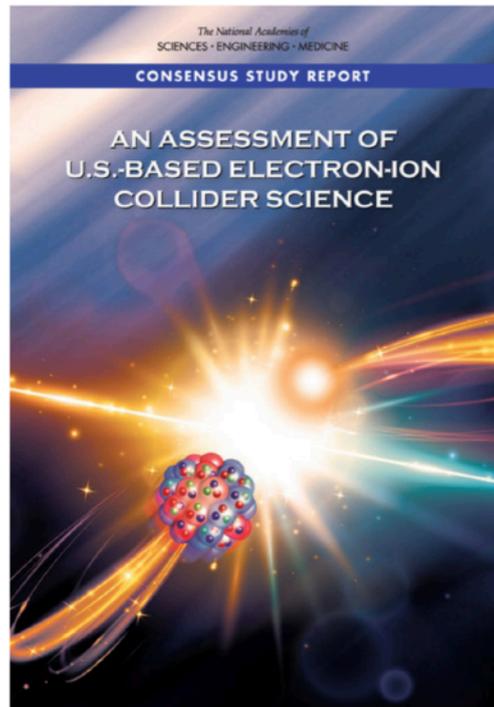


**Disconnected Gluon loop**

# Understanding how lattice calculations support EIC physics: the proton spin

# Hadron structure at core of nuclear physics

## Main Pillar of NAS: Assessment report for EIC

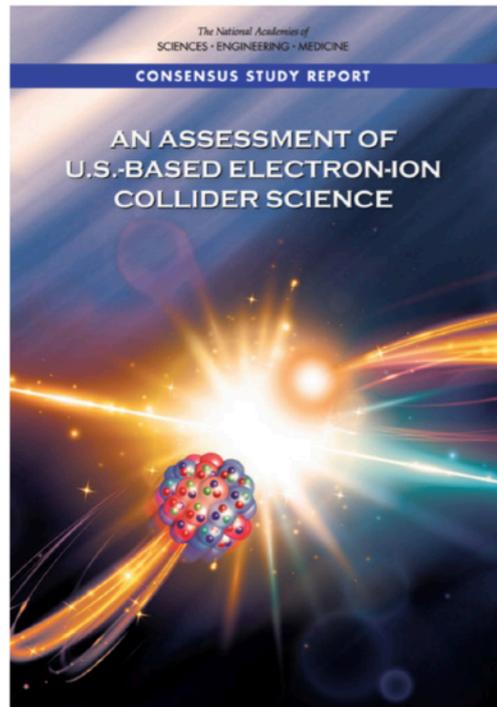


**Finding 1:** An EIC can uniquely address three profound questions about nucleons—neutrons and protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

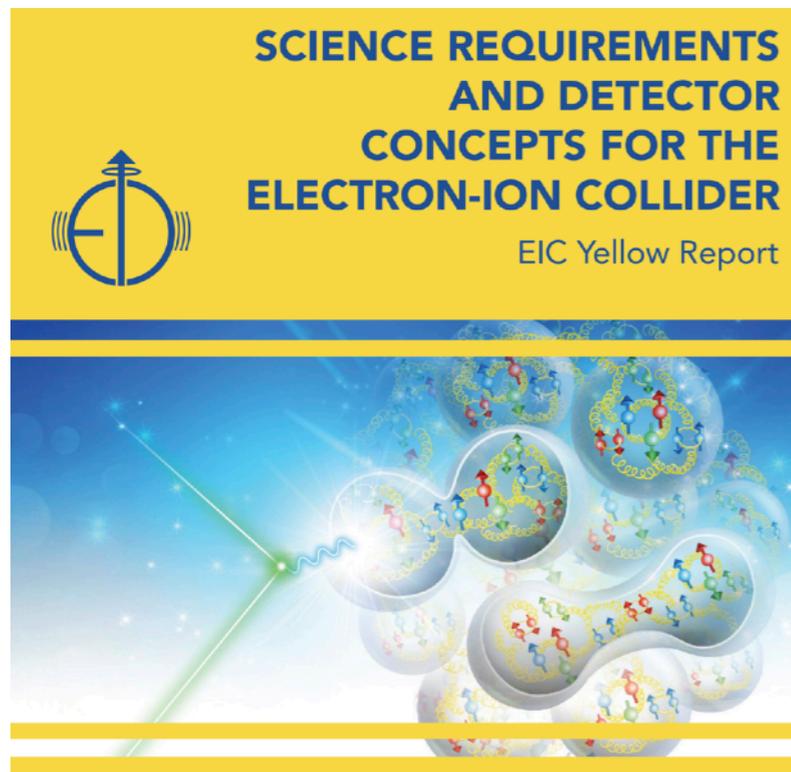
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## Advances of Lattice QCD are timely

Lattice QCD is featured in the EIC Yellow Report

- 900-page document
- scientist from 151 Institutions

Lattice QCD can provide input in the proton mass and spin decomposition from **first principles**

# Proton spin “puzzle”

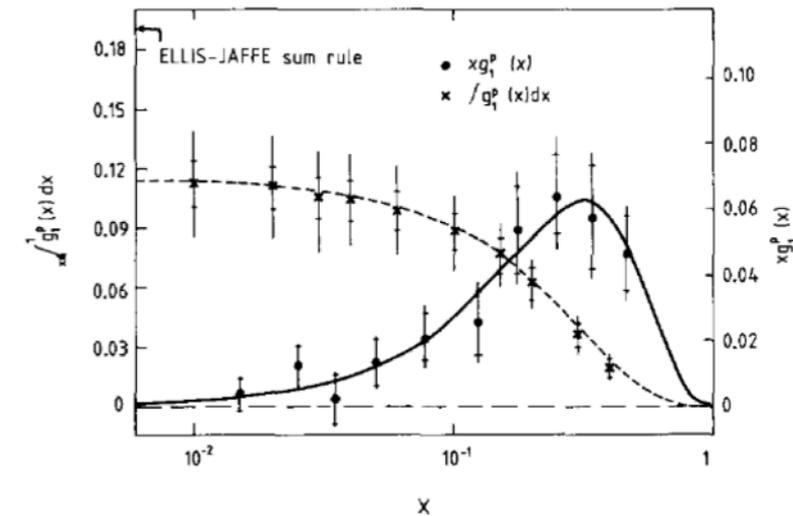
- ★ Fundamental degree of freedom (from space-time symmetry) **Proton spin:1/2**
- ★ Spin plays an important role in determining the structure of composite particles, like the proton
- ★ Simple models predict that the 3 quarks responsible for the proton’s quantum numbers carry 1/3 of its spin
- ★ DIS experiments (1988) show surprising results for proton spin

[J. Ashman et al., Phys. Lett., vol. B206 (1988) 364]

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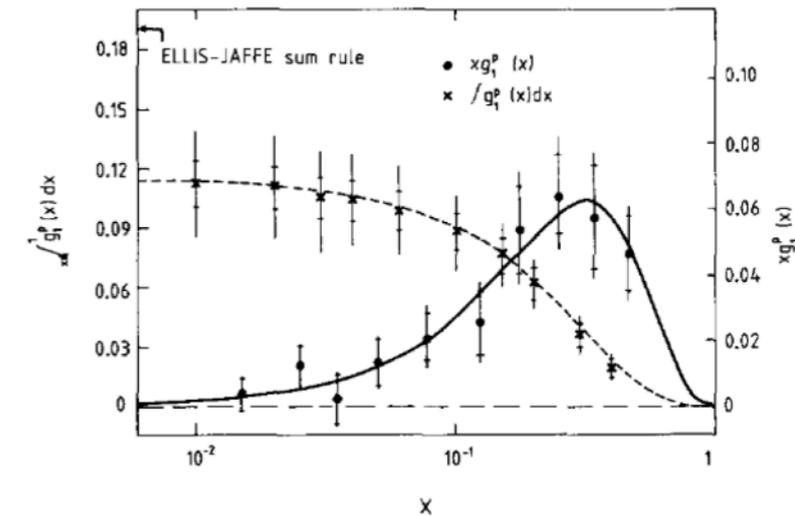


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**SPIN CRISIS !**



# Proton spin “puzzle”

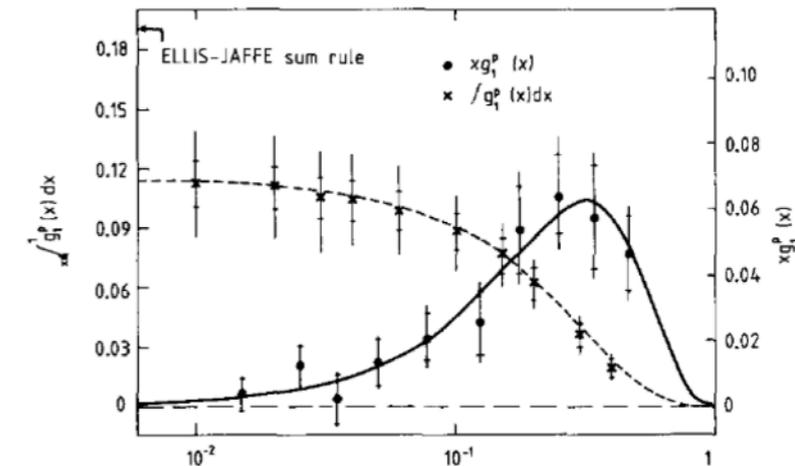
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## SPIN CRISIS !

### Still open questions:

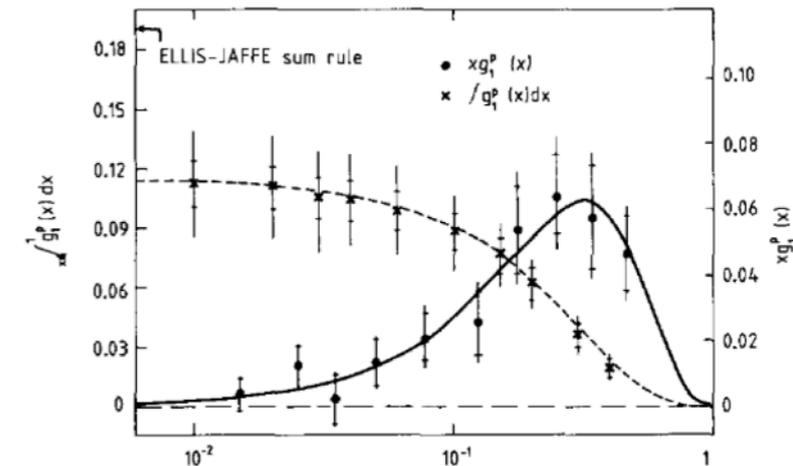
- Sea quark and gluon contributions
- Parton orbital angular momentum



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**SPIN CRISIS !**

## Still open questions:

- Sea quark and gluon contributions
- Parton orbital angular momentum

**We must quantify the proton spin decomposition**



# Spin structure from first principles

Lattice QCD can provide important information on the spin

## Ji's Spin Decomposition

$$\frac{1}{2} = \sum_q J^q + J^G = \sum_q \left( L^q + \frac{1}{2} \Delta\Sigma^q \right) + J^G$$

$L_q$ : Quark orbital angular momentum

$\Delta\Sigma_q$ : Intrinsic spin

$J_g$ : Gluon spin

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All these quantities can be computed within Lattice QCD

## Extraction from Lattice QCD:

$$J^q = \frac{1}{2} \left( A_{20}^q + B_{20}^q \right)$$

$$L^q = J^q - \Sigma^q$$

$$\Sigma^q = g_A^q$$

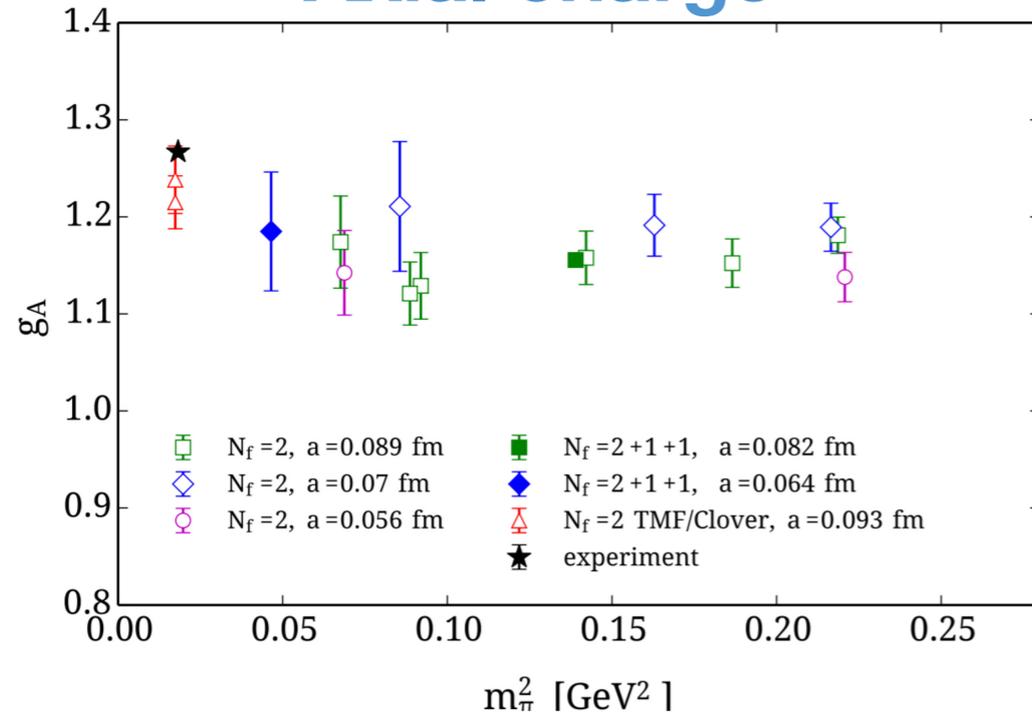
## Necessary computations:

- Axial Charge
- Quark momentum fraction
- Gluon momentum fraction

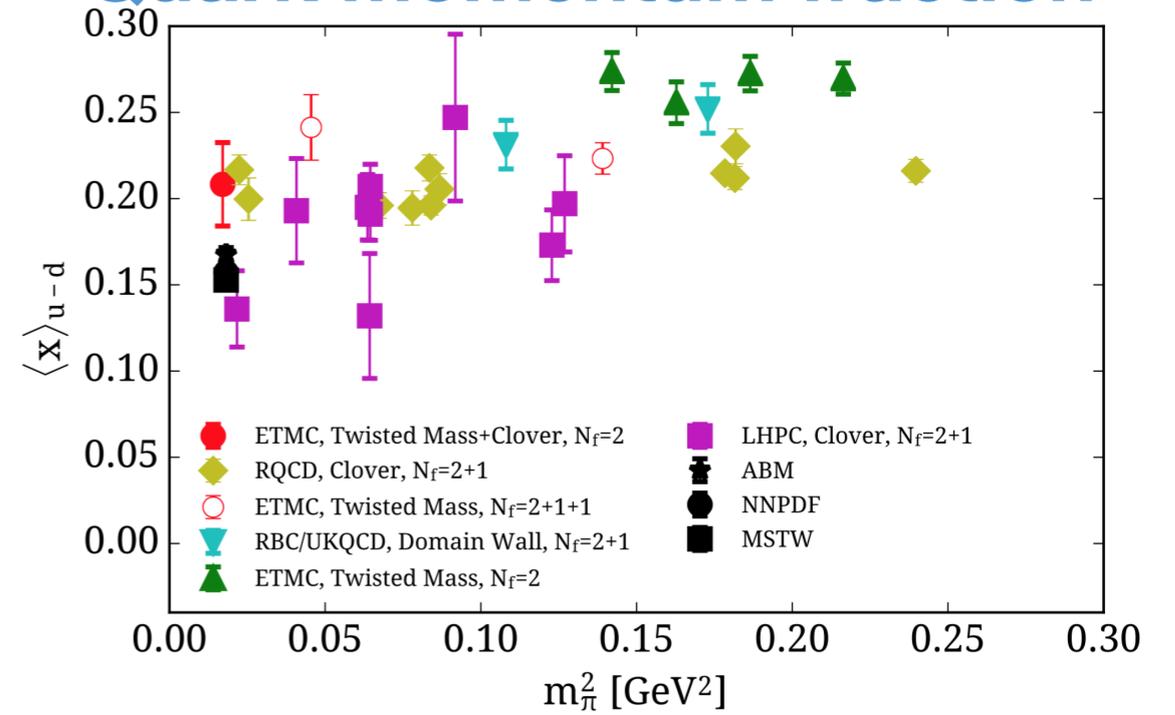
Quark Orbital Angular extracted indirectly

# Spin components in Lattice QCD

## Axial charge

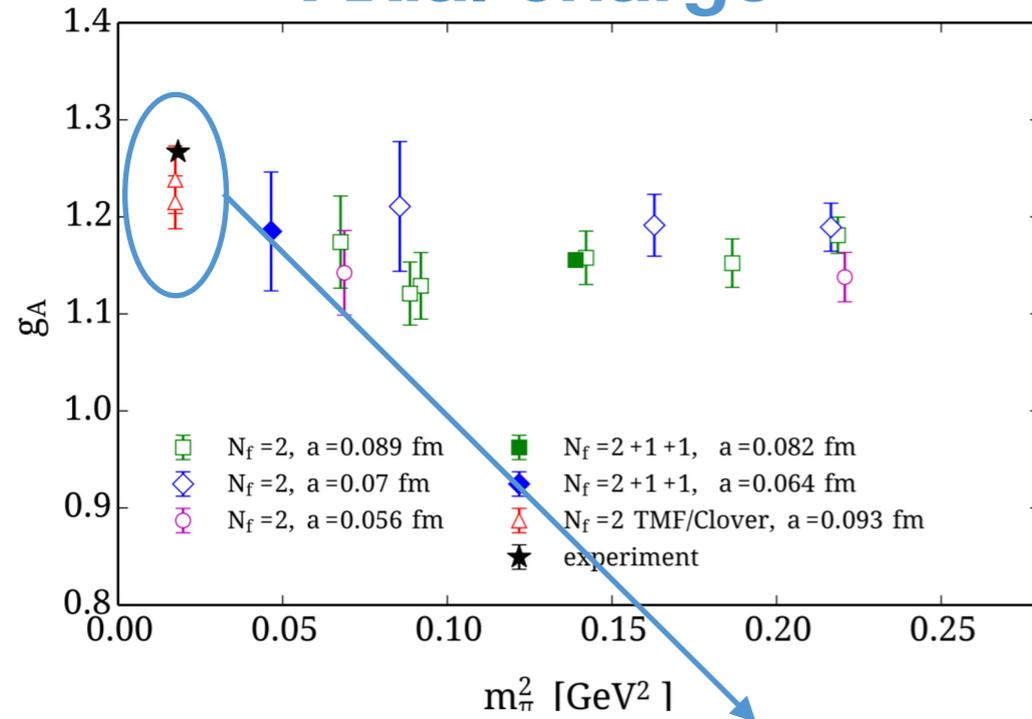


## Quark momentum fraction

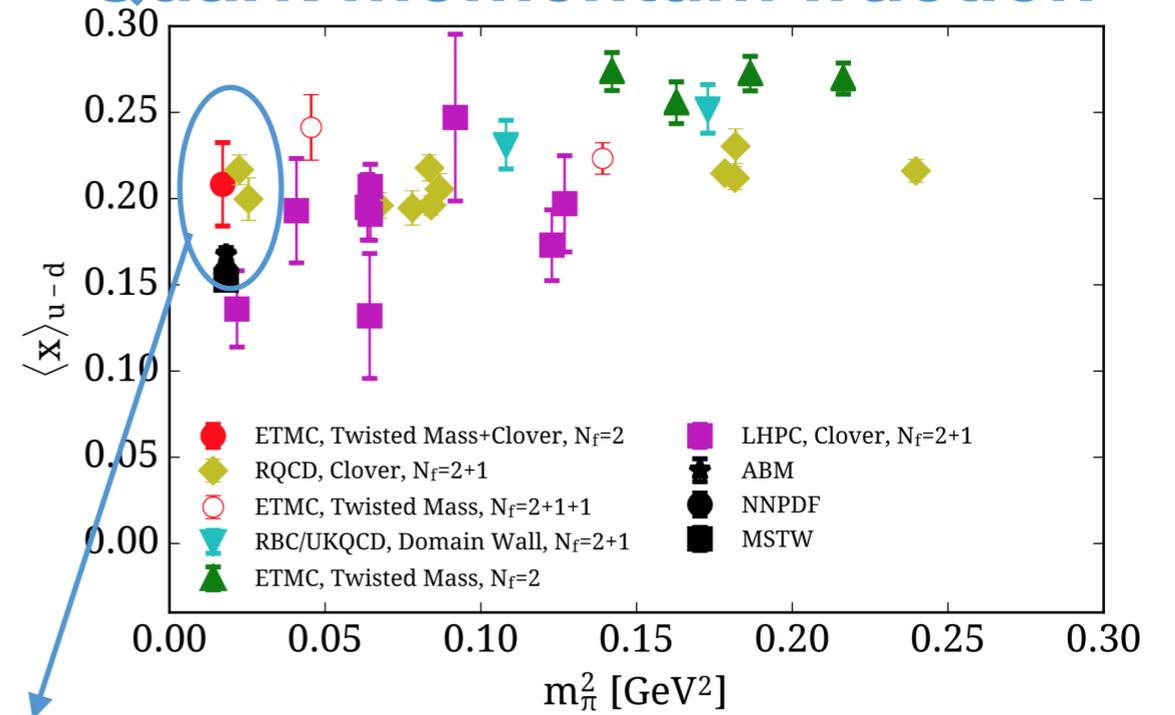


# Spin components in Lattice QCD

## Axial charge



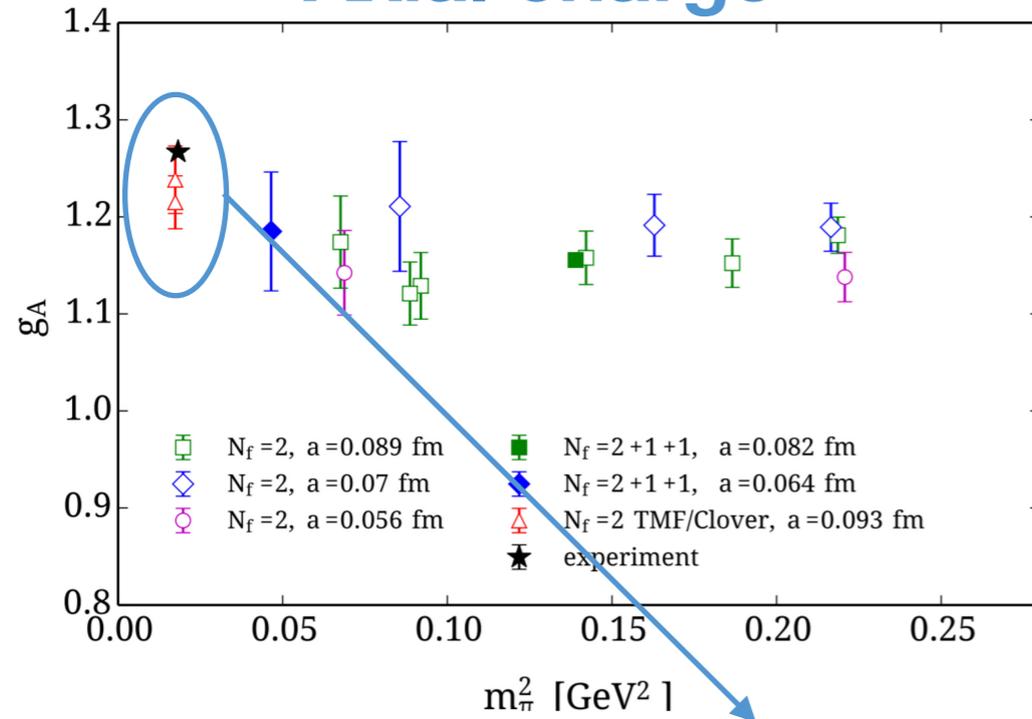
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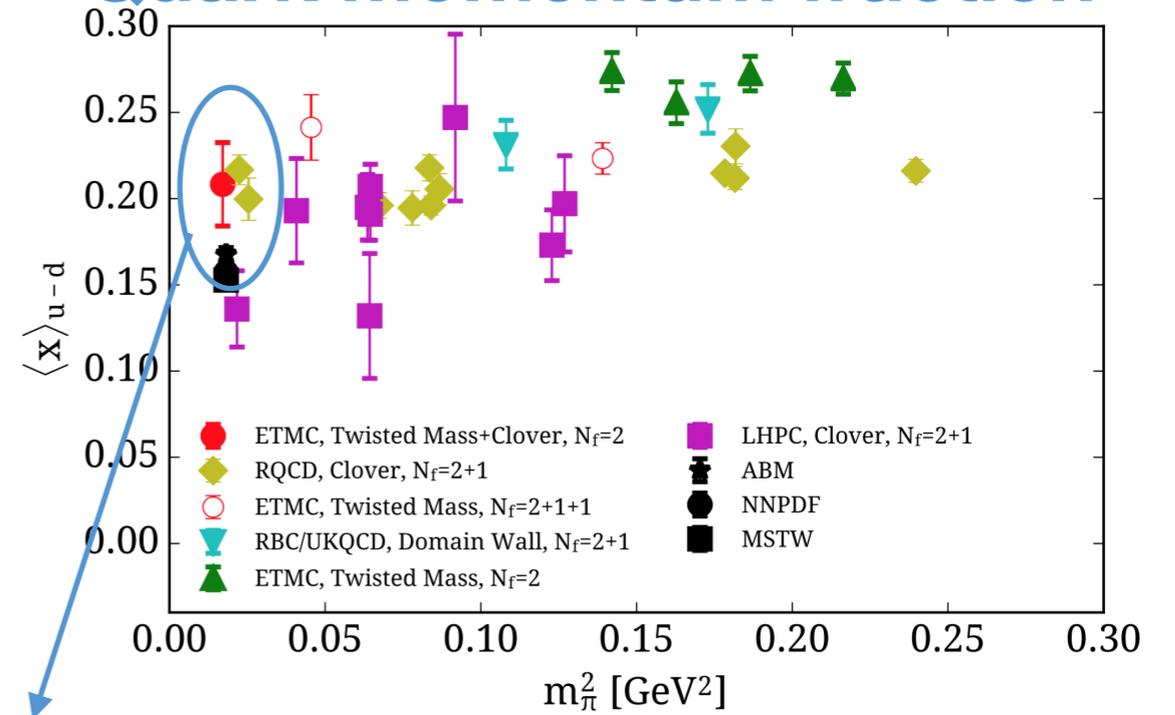
Simulations of nature enabled from new methods, algorithms, computer architecture

# Spin components in Lattice QCD

## Axial charge

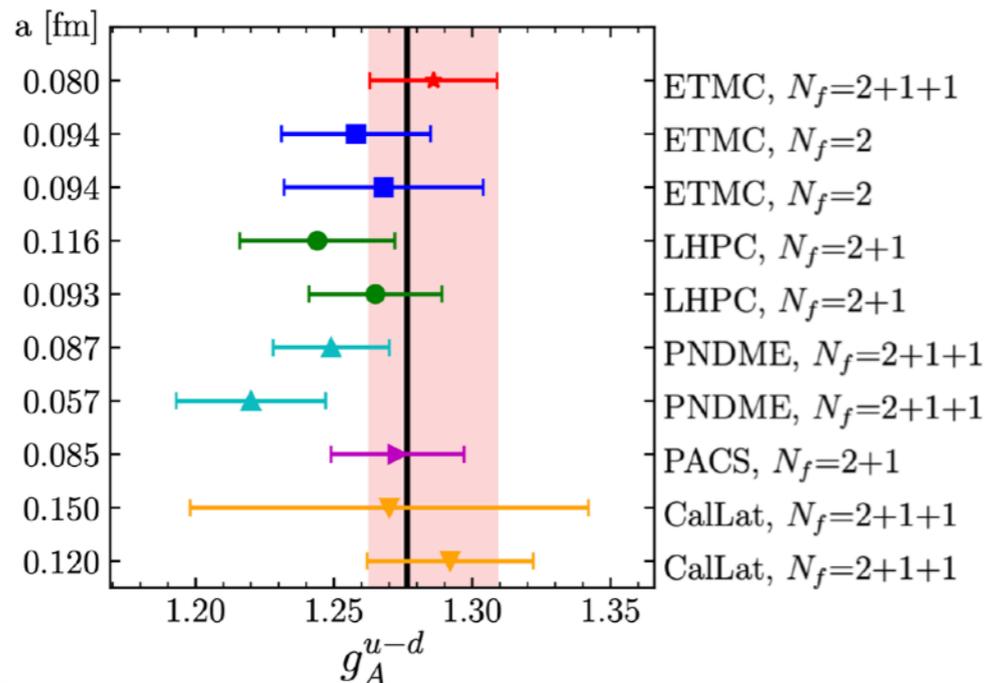


## Quark momentum fraction



Simulations of nature enabled from new methods, algorithms, computer architecture

## Summary of results



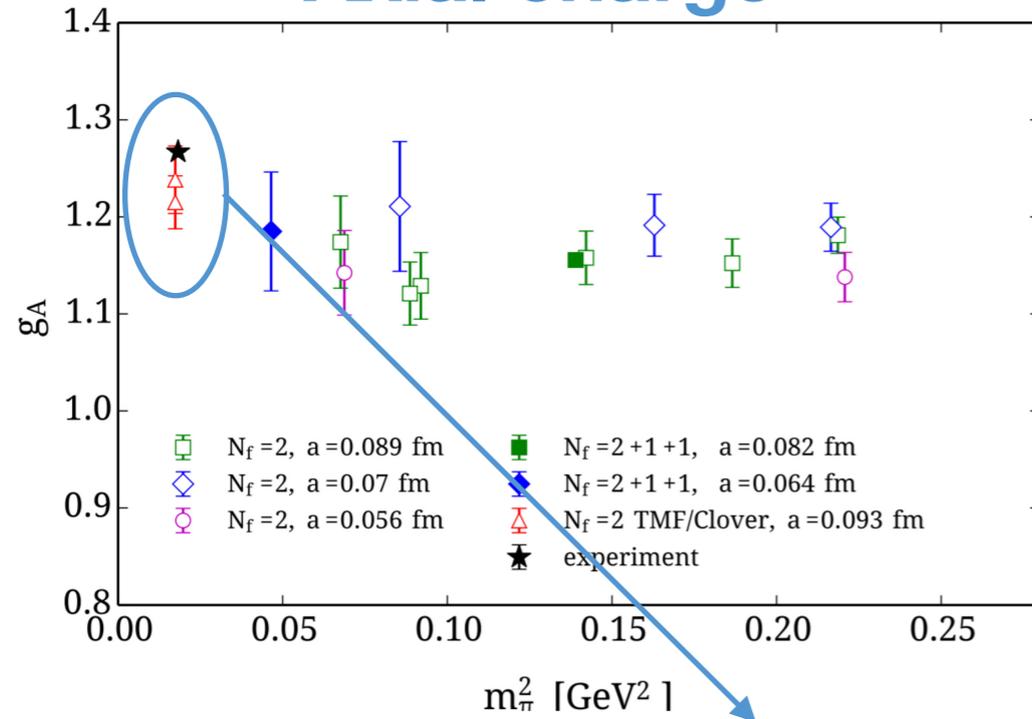
	$u$	$d$	$s$	$c$
$g_A$	0.862(17)	-0.424(16)	-0.0458(73)	-0.0098(34)

Taking into account the disconnected contributions is crucial for the spin

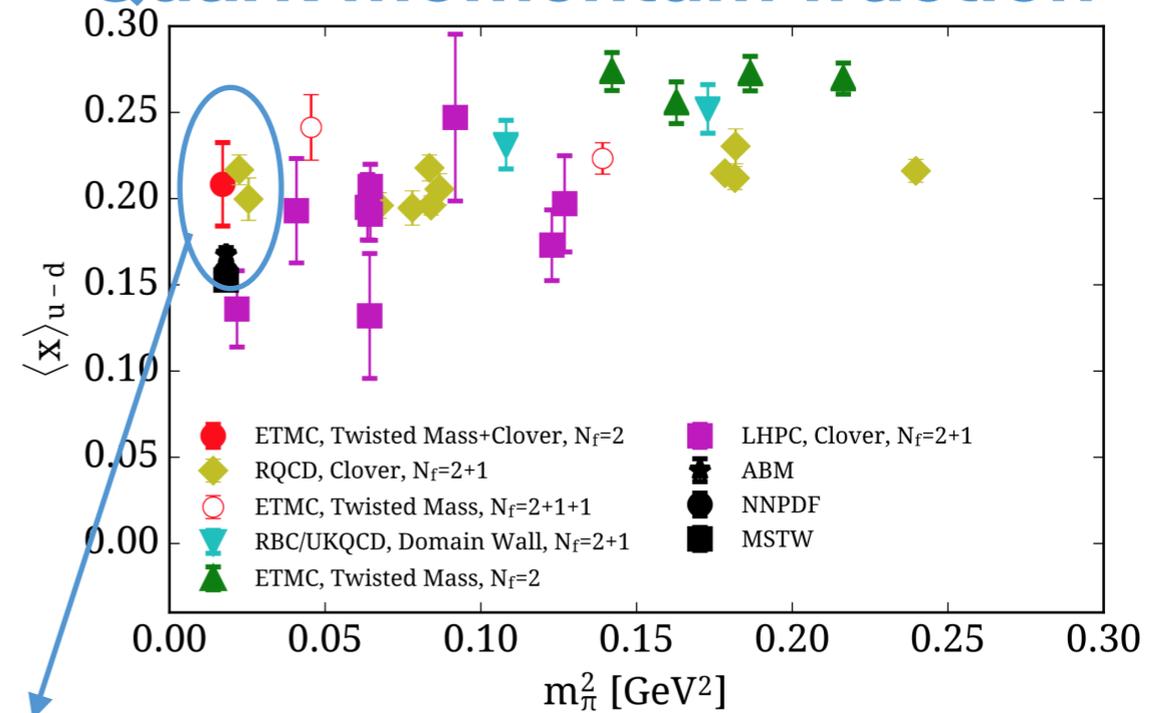


# Spin components in Lattice QCD

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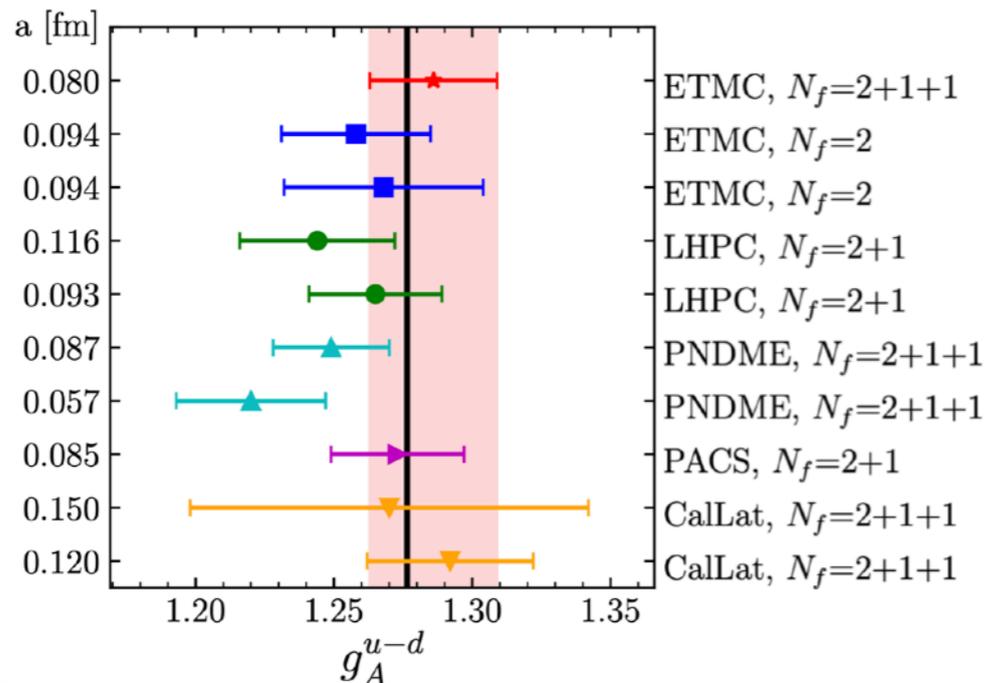


## Quark momentum fraction



Simulations of nature enabled from new methods, algorithms, computer architecture

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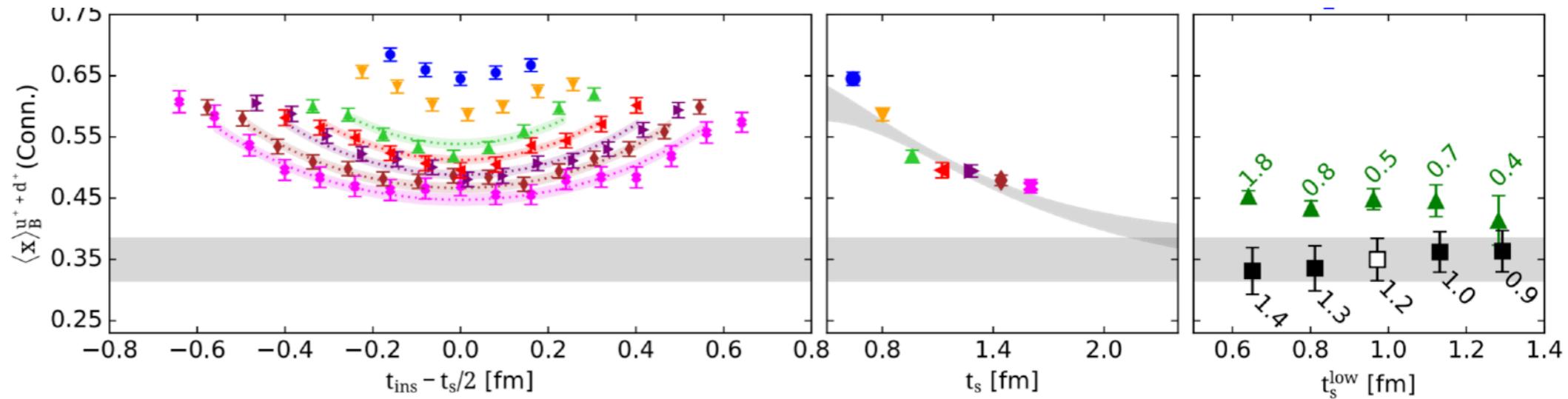
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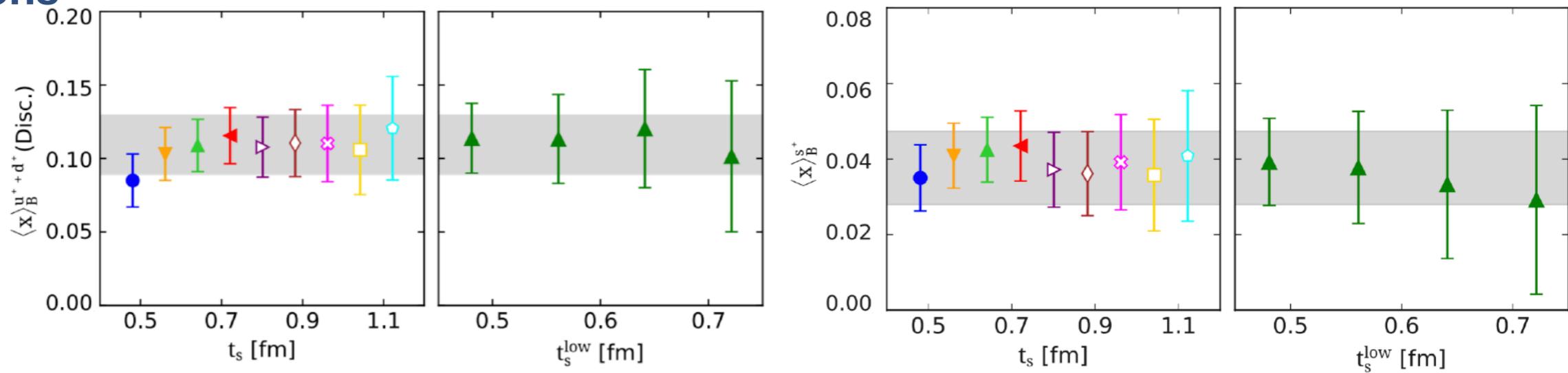


# Quark momentum fraction

★ connected contributions

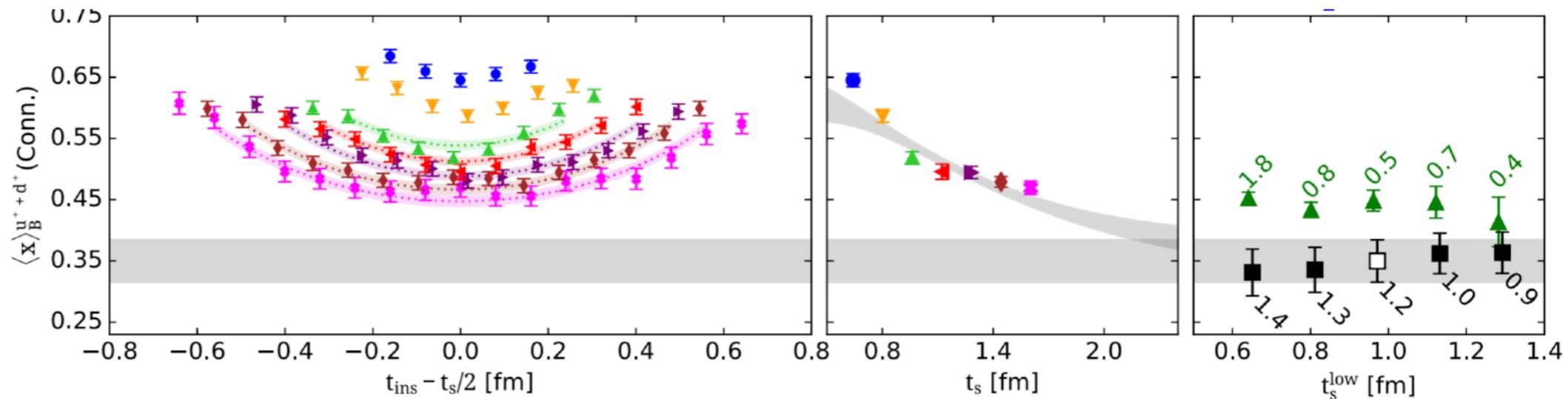


★ disconnected contributions

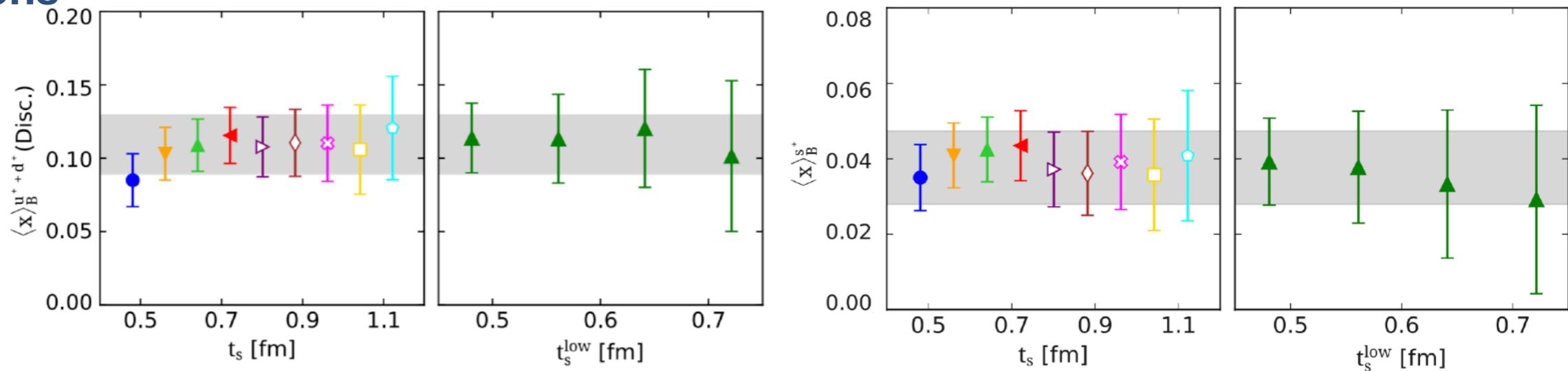


# Quark momentum fraction

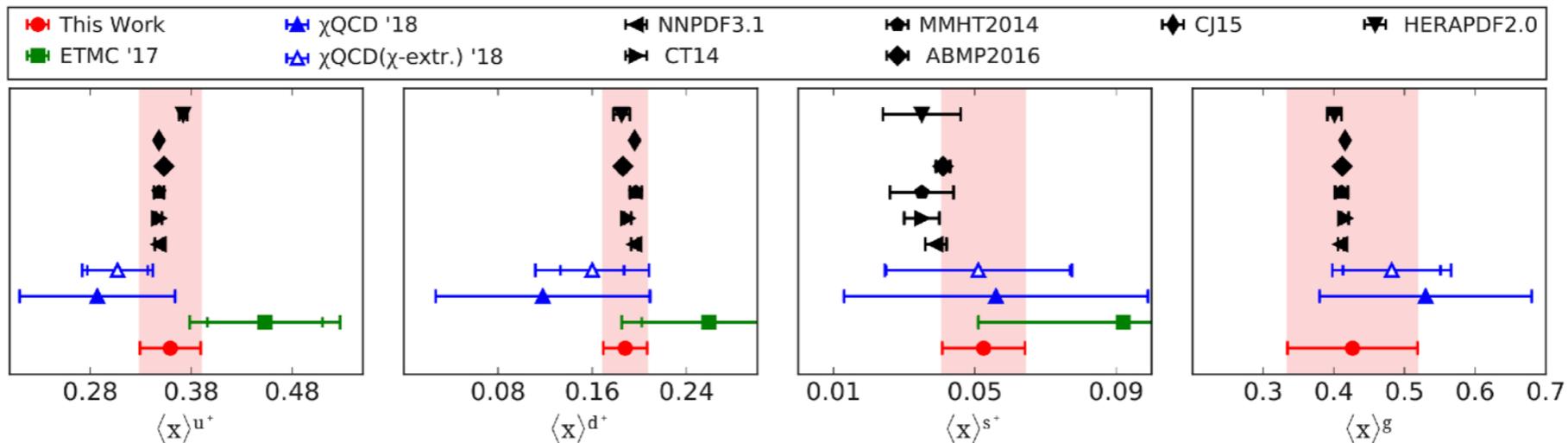
★ connected contributions



★ disconnected contributions

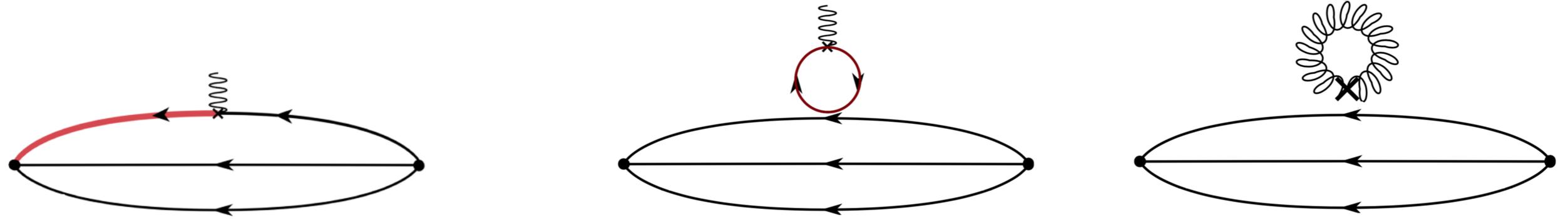


## Collection of various results



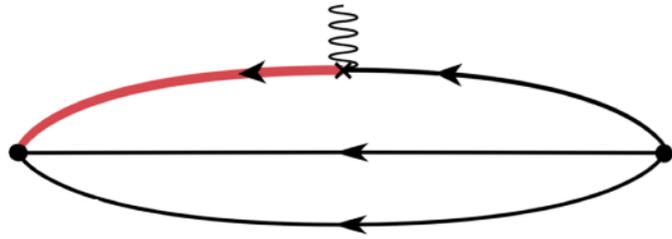
# Results @ physical pion mass

$\overline{MS}(2\text{GeV})$

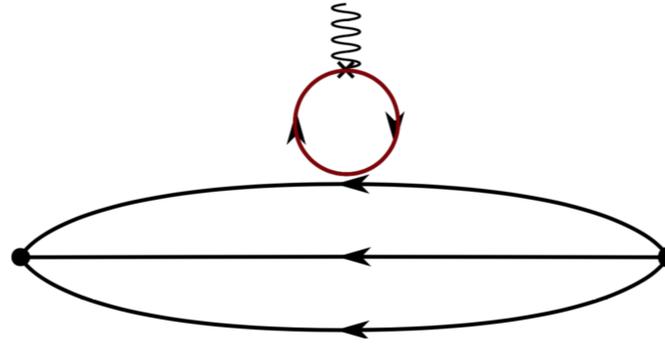


# Results @ physical pion mass

$\overline{MS}(2\text{GeV})$



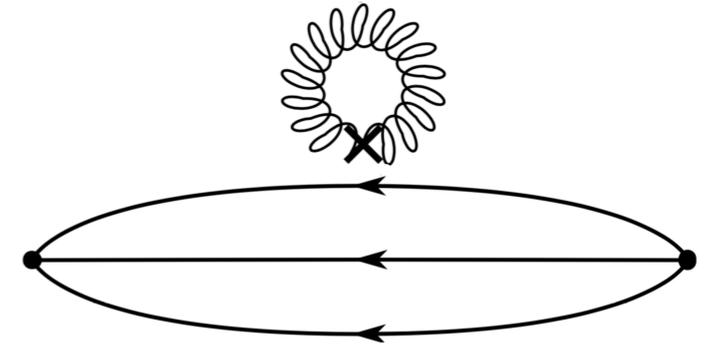
$$\langle x \rangle_{u+d}^B = 0.350(35)$$



$$\langle x \rangle_{u+d}^B = 0.109(20)$$

$$\langle x \rangle_s^B = 0.038(10)$$

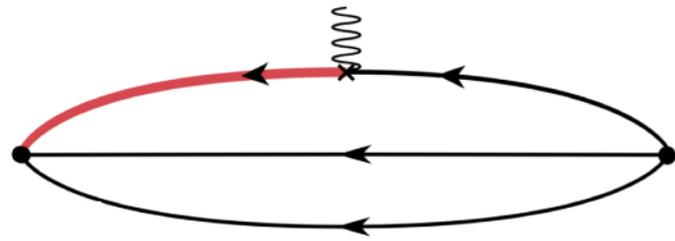
$$\langle x \rangle_c^B = 0.008(8)$$



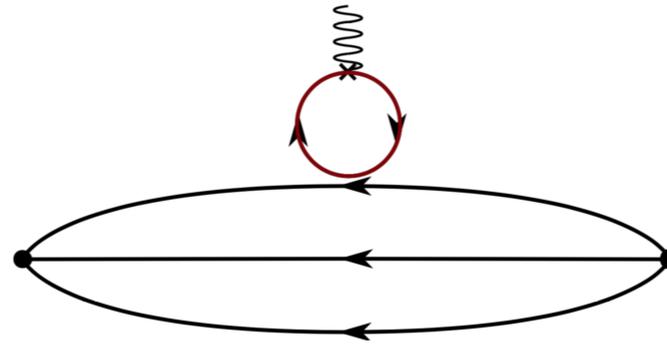
$$\langle x \rangle_g^B = 0.407(54)$$

# Results @ physical pion mass

## $\overline{MS}(2\text{GeV})$



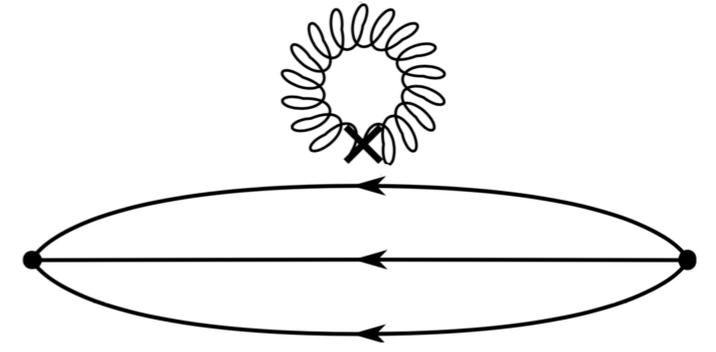
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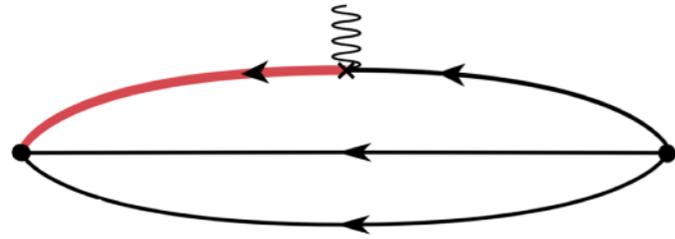
### ★ Mixing between quark and gluon contributions to $\langle x \rangle$

$$\sum_q \langle x \rangle_q^R = Z_{qq} \sum_q \langle x \rangle_q^B + Z_{qg} \langle x \rangle_g^B$$

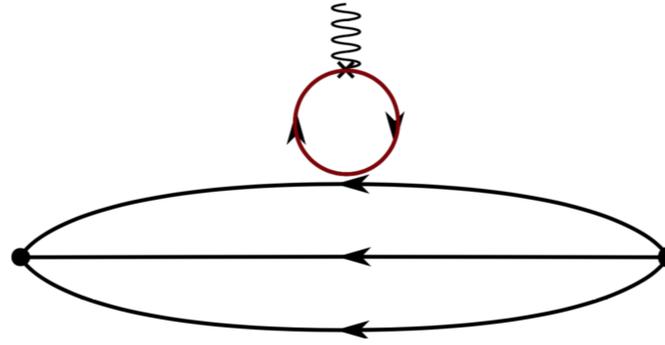
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# Results @ physical pion mass

## $\overline{MS}(2\text{GeV})$



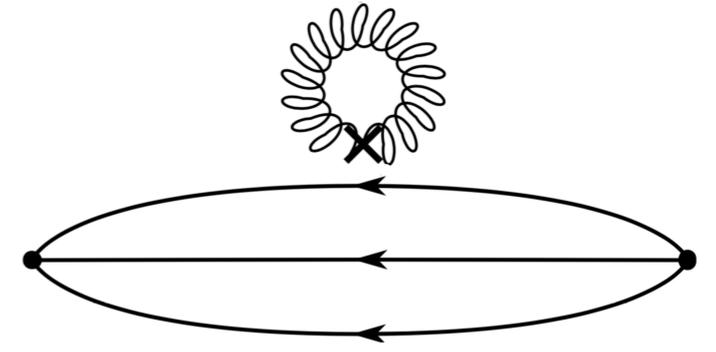
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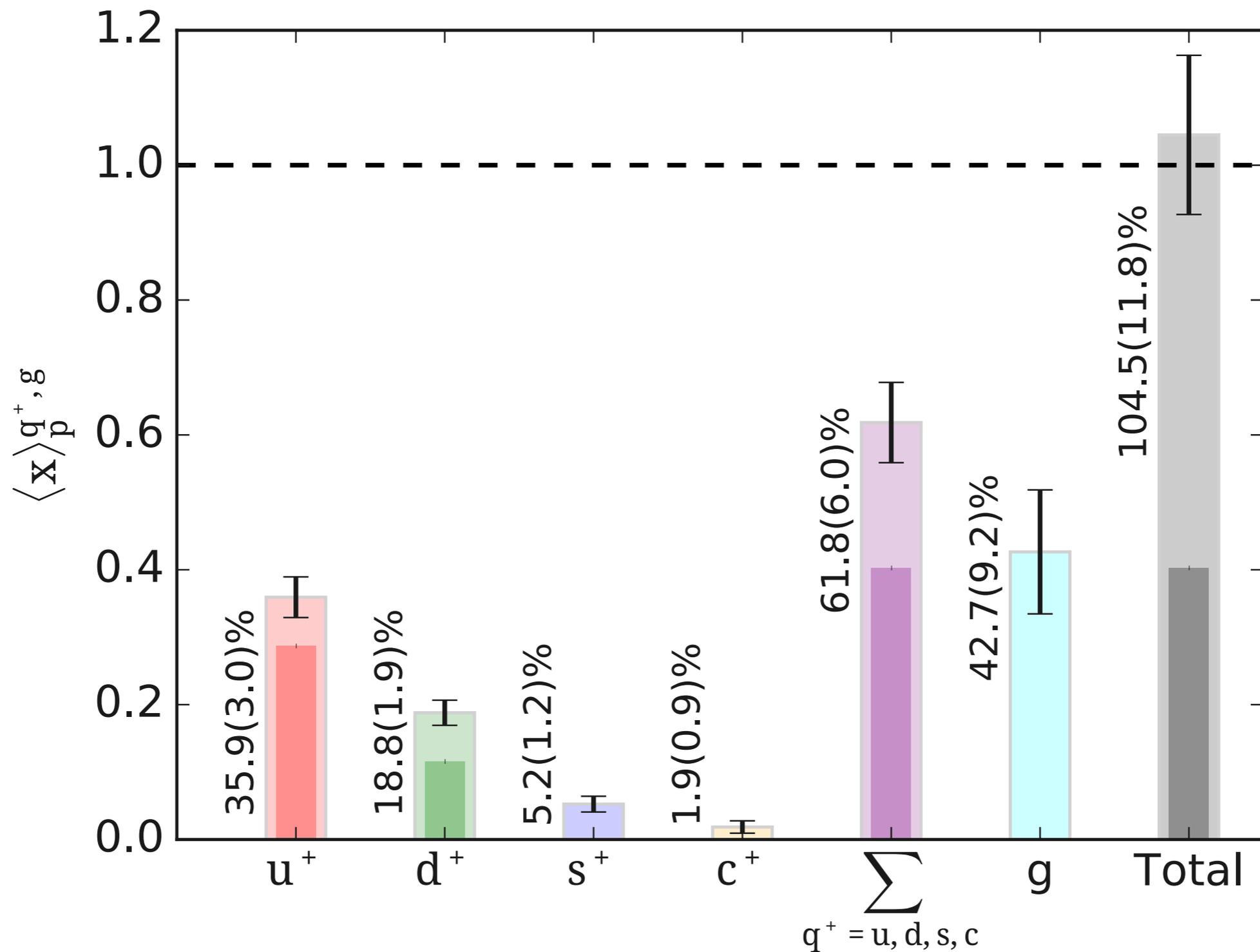
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$$\langle x \rangle_u = 0.359(30) \quad \langle x \rangle_d = 0.188(19) \quad \langle x \rangle_s = 0.052(12) \quad \langle x \rangle_c = 0.019(9) \quad \langle x \rangle_g = 0.427(92)$$

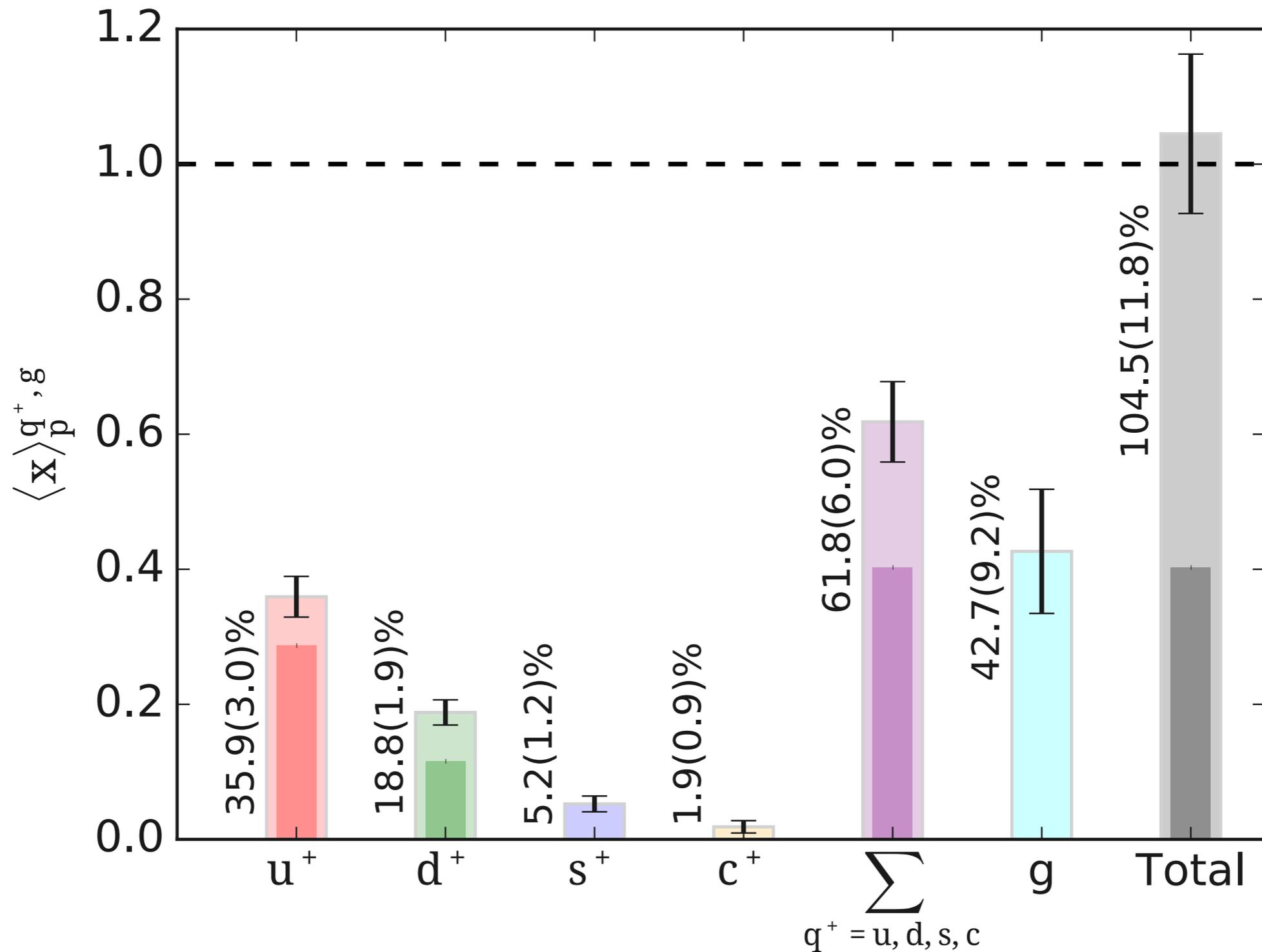
# Percentage of momentum



**Momentum sum rule satisfied!**

# Percentage of momentum

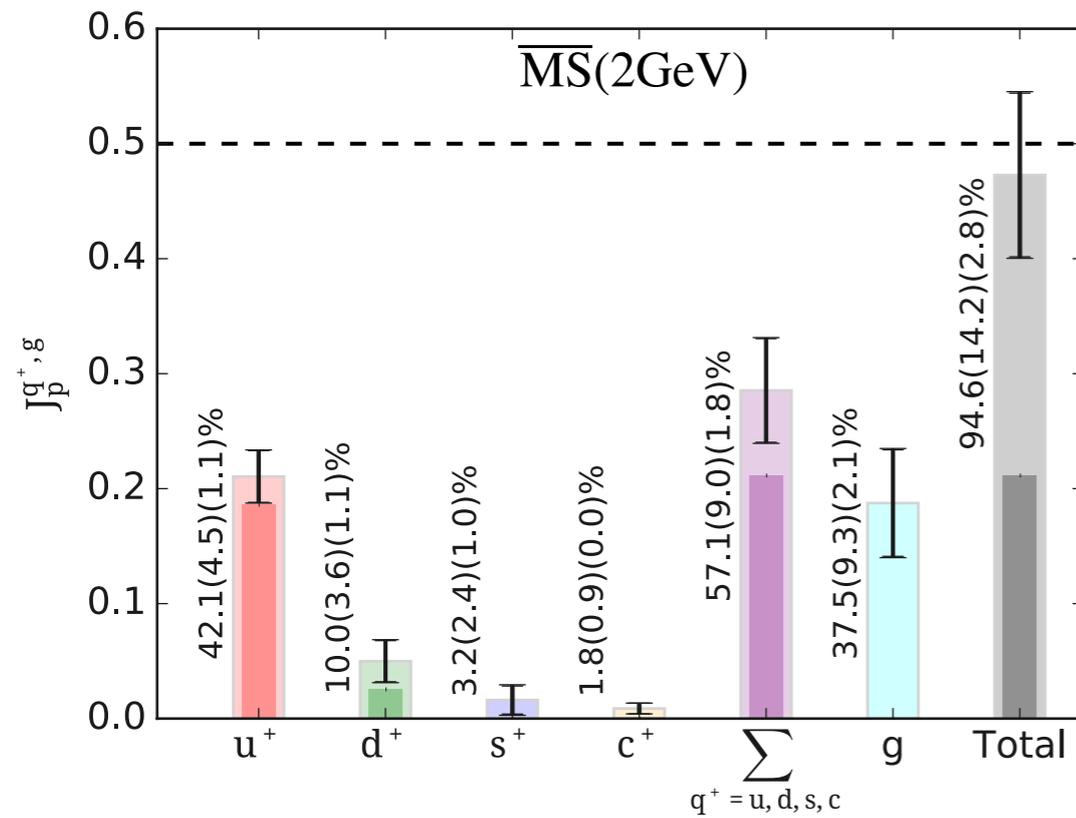
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**Momentum sum rule satisfied!**

# Spin decomposition

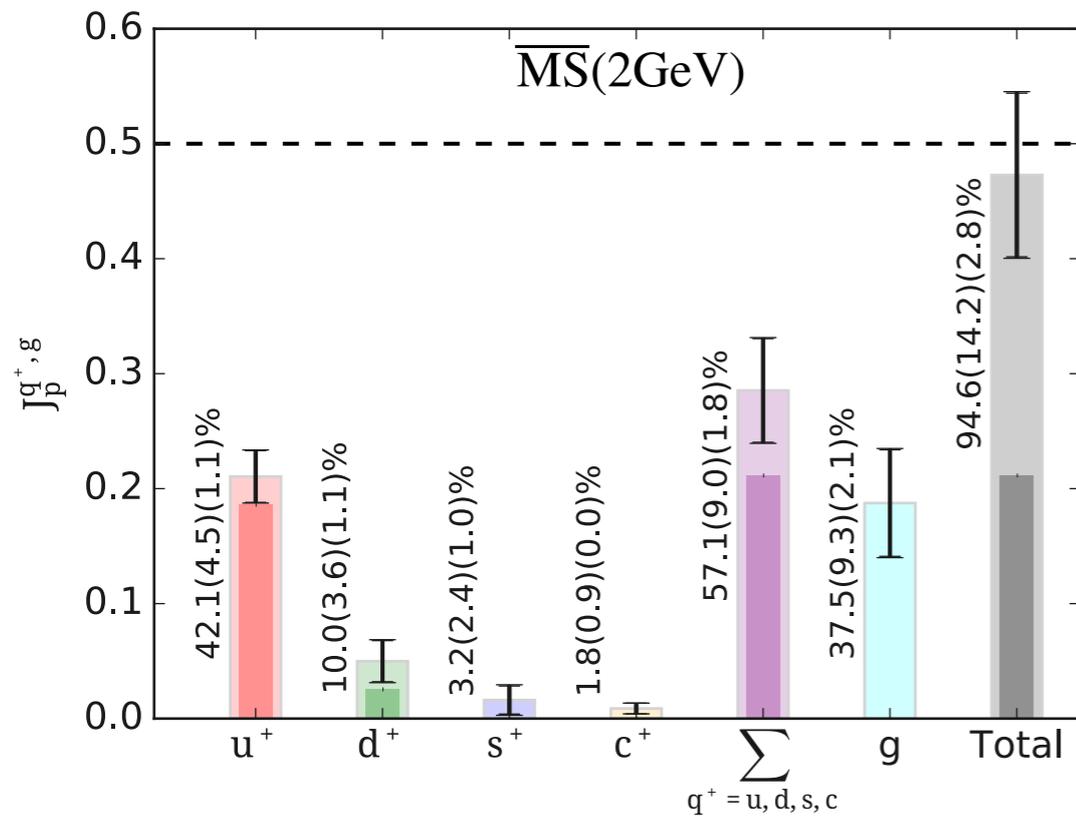
[C. Alexandrou et al., Phys. Rev. D 101, 094513 (2020), arXiv:2003.08486]



- ★ Inner bars: connected contributions
- ★ Outer - Inner bars: disconnected contributions

# Spin decomposition

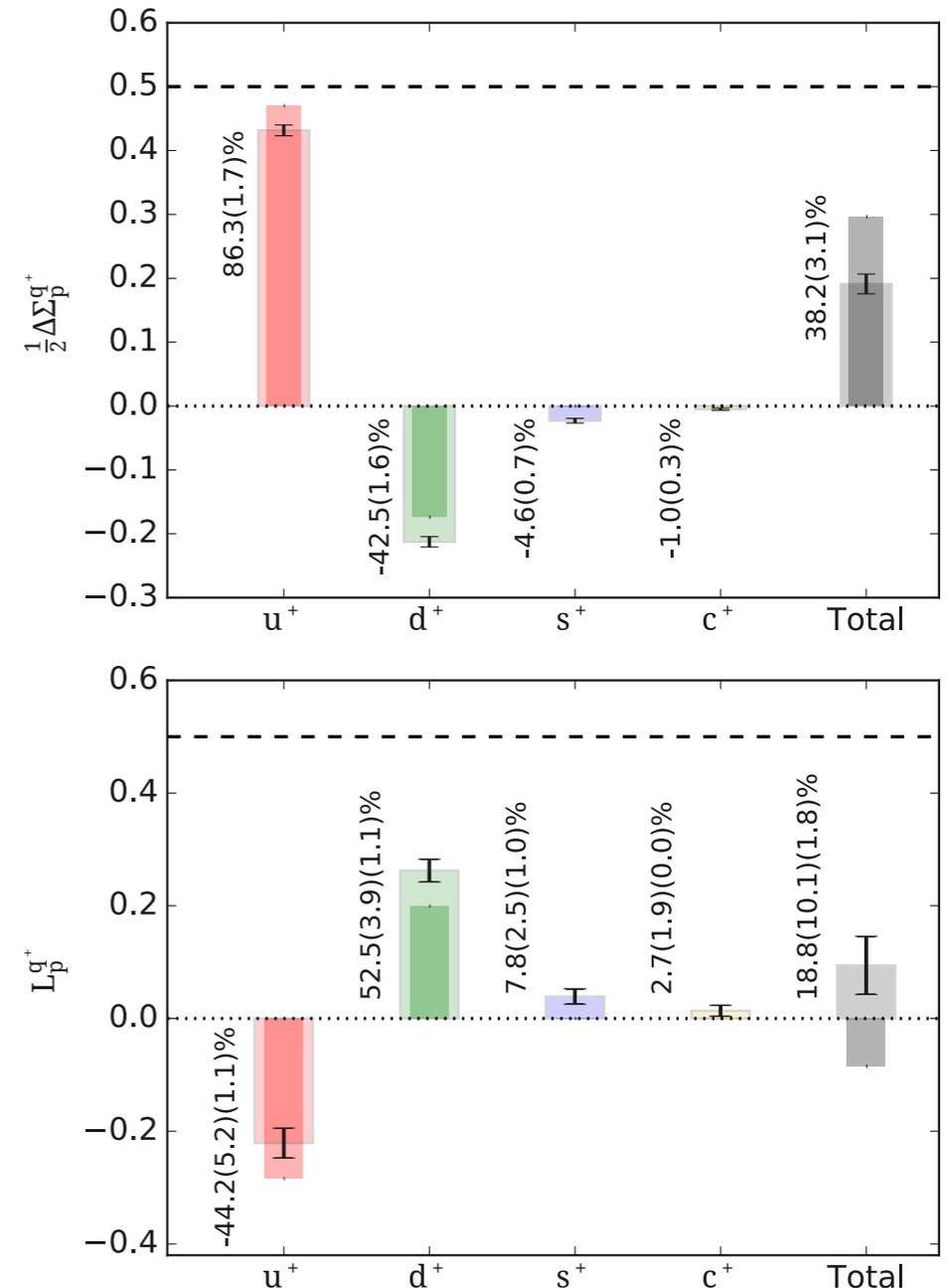
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★ Inner bars: connected contributions

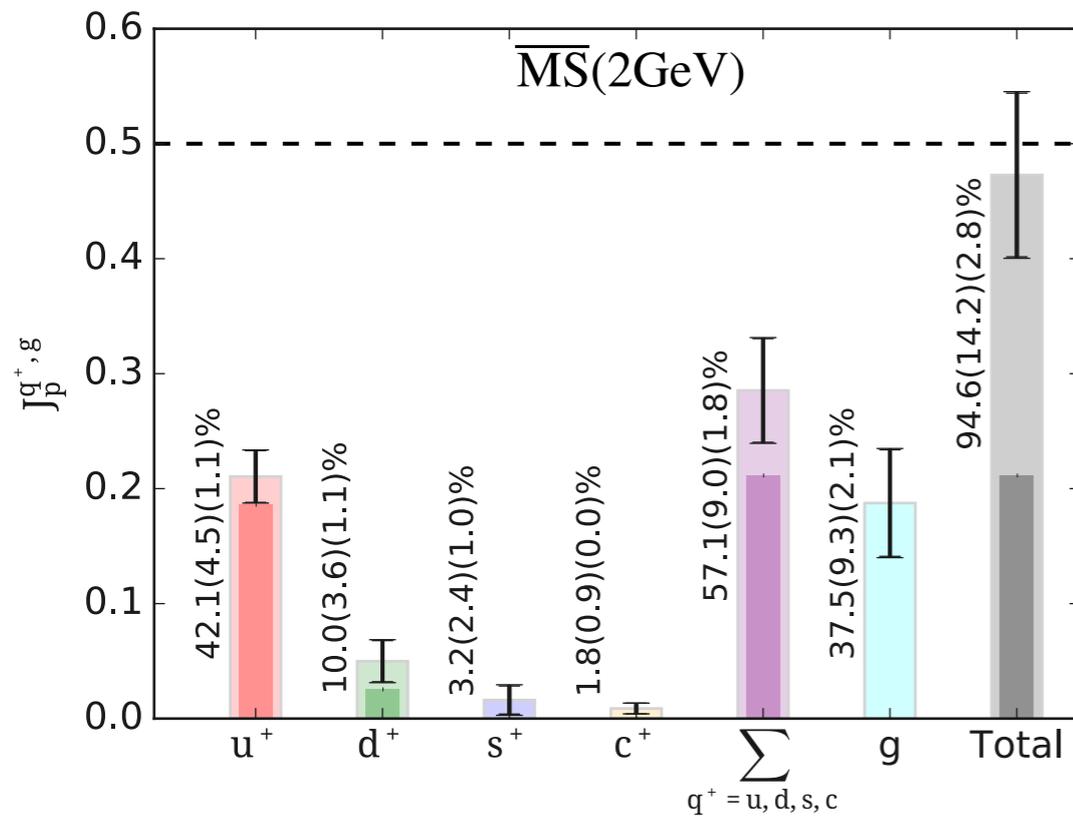
★ Outer - Inner bars: disconnected contributions

Quark orbital angular momentum extracted indirectly ( $L_q = J_q - \Sigma_q$ )



# Spin decomposition

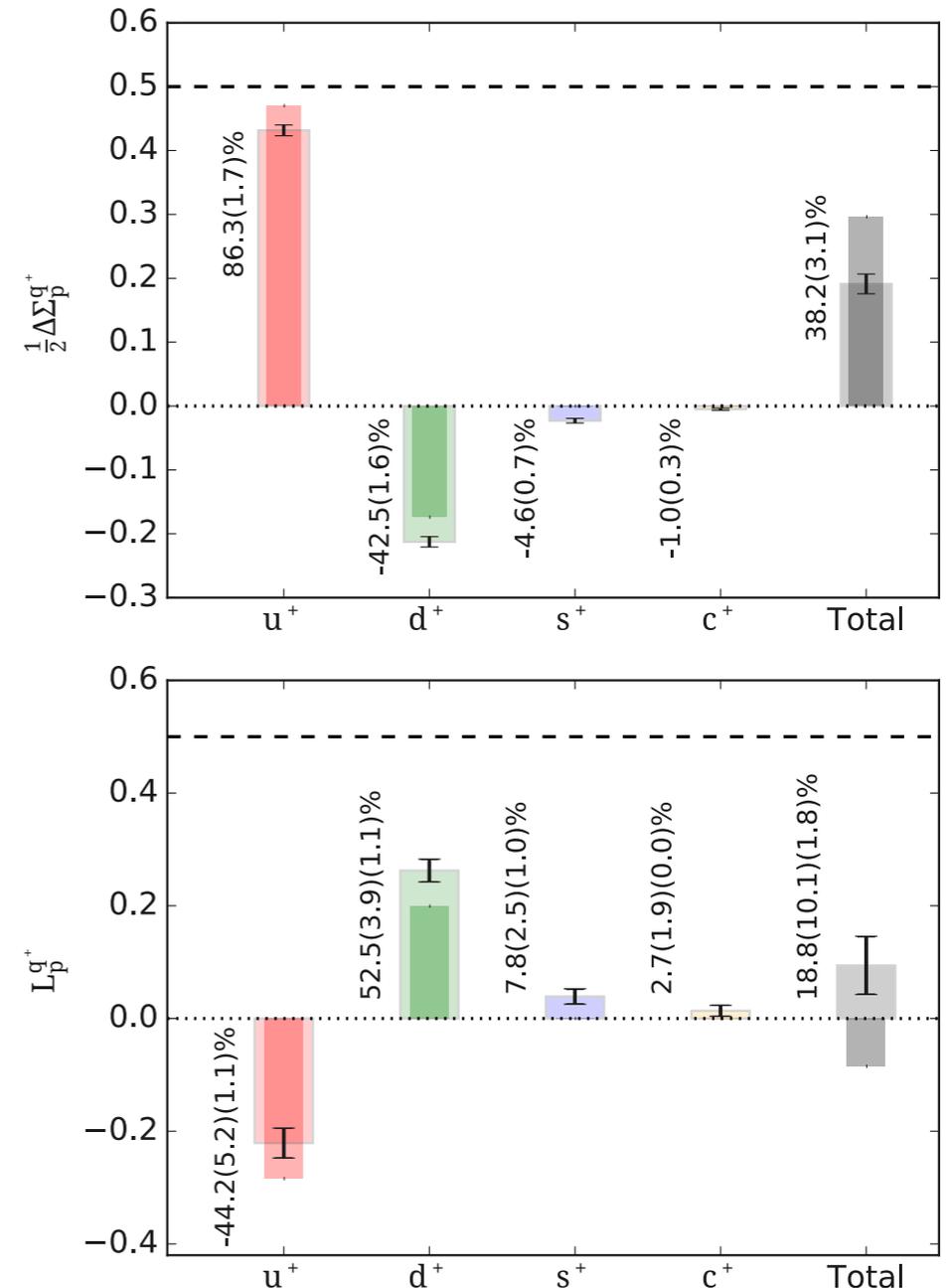
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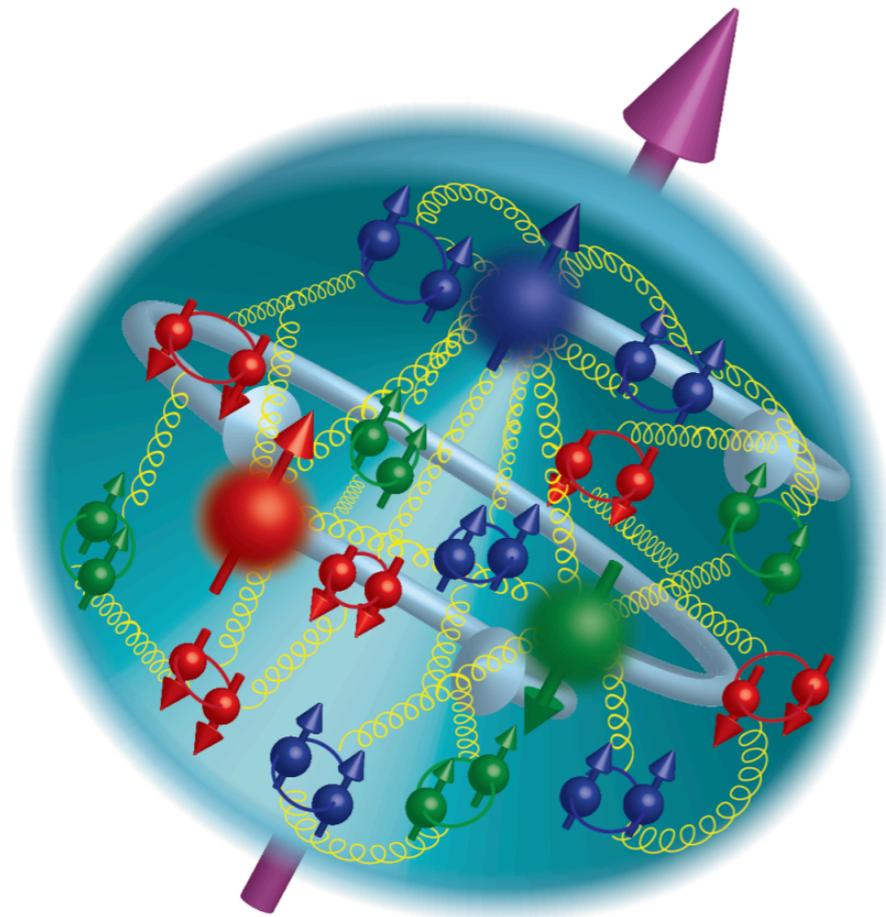


Satisfaction of spin and momentum sum rule is not forced

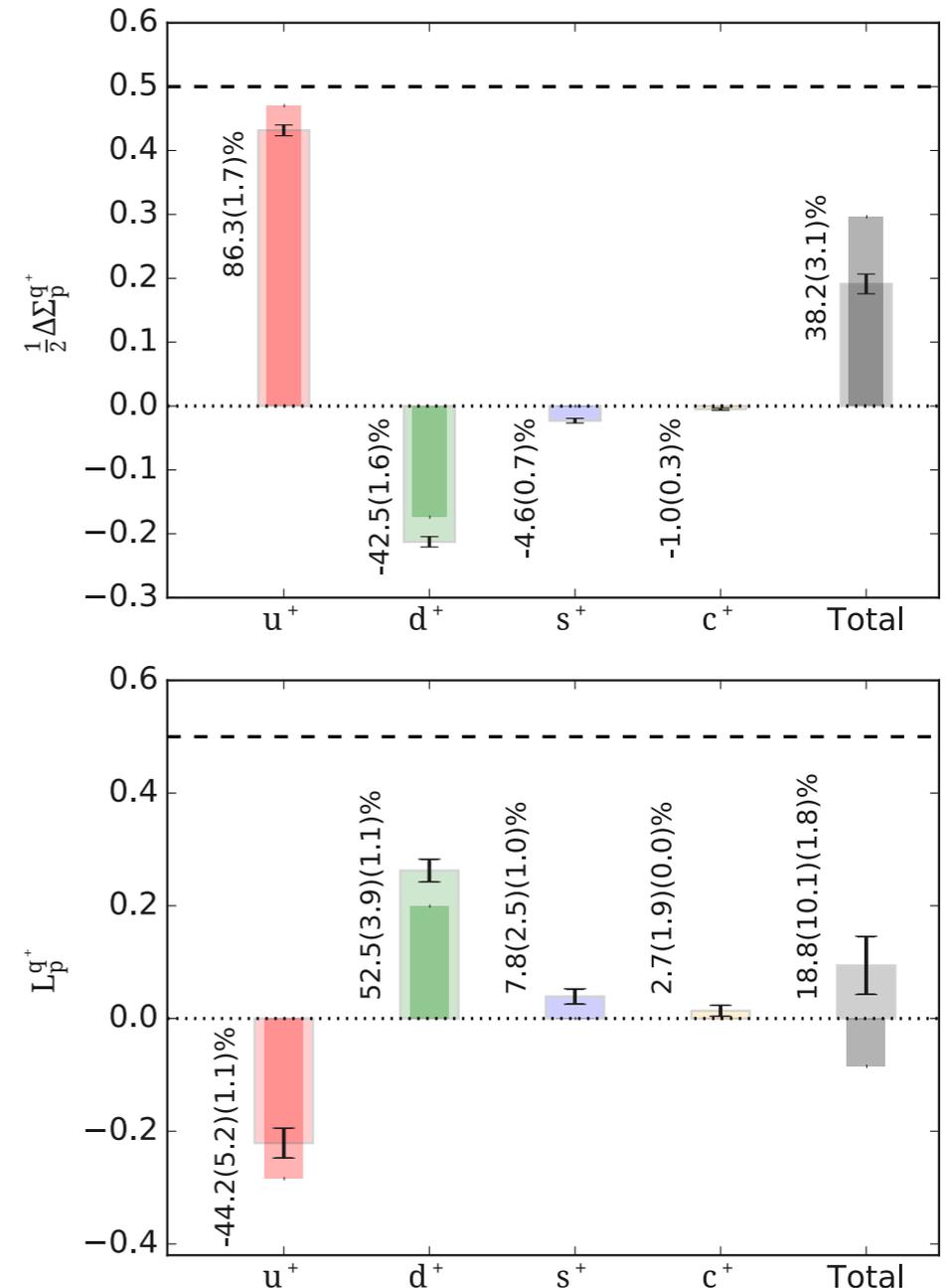
# Spin decomposition

[C. Alexandrou et al., Phys. Rev. D 101, 094513 (2020), arXiv:2003.08486]

Better understanding of  
the spin distribution



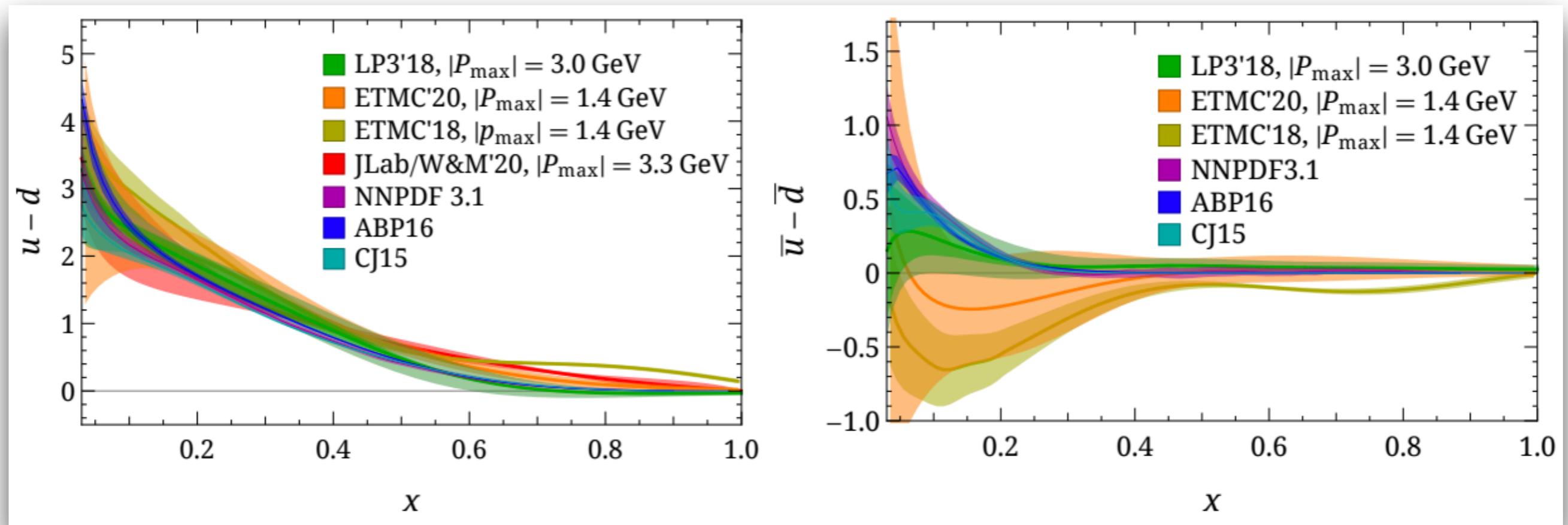
Designed by Z.-E. Meziani



Satisfaction of spin and momentum sum rule is not forced

# Accessing PDFs/GPDs from lattice QCD

# Collection of results for unpolarized PDF

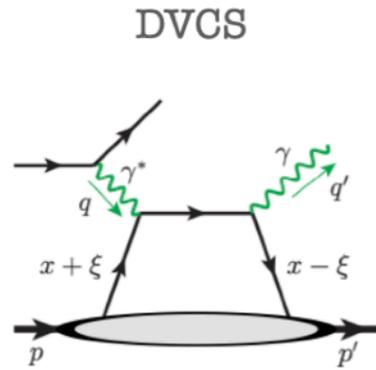


[M. Constantinou et al. (2020 PDFLattice Report), Prog.Part.Nucl.Phys. 121 (2021) 103908]

- ★ Several improvements:
  - More calculations at physical quark masses
  - Ensembles at various lattice spacings
  - Addressing systematic uncertainties due to methodologies
- ★ Progress extended to gluon PDFs, GPDs, TMDs

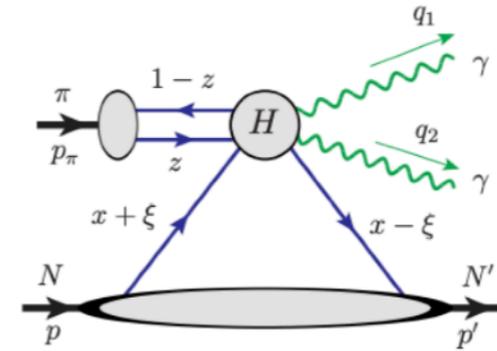
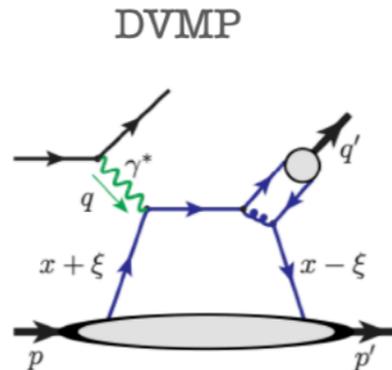
# Generalized Parton Distributions

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)



[X.-D. Ji, PRD 55, 7114 (1997)]

★ exclusive pion-nucleon diffractive production of a  $\gamma$  pair of high  $p_{\perp}$



[J. Qiu et al, arXiv:2205.07846]

★ GPDs are not well-constrained experimentally:

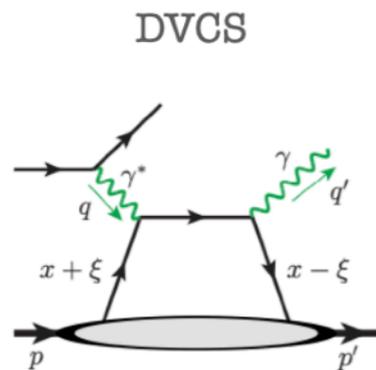
- **x-dependence extraction is not direct. DVCS amplitude:**  $\mathcal{H} = \int_{-1}^{+1} \frac{H(x, \xi, t)}{x - \xi + i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges ...

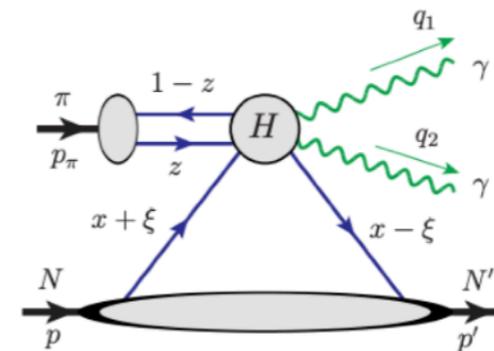
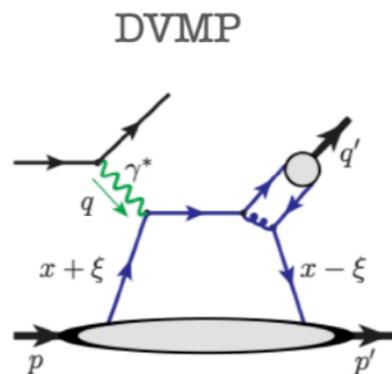
# Generalized Parton Distributions

★ GPDs may be accessed via exclusive reactions (DVCS, DVMP)



[X.-D. Ji, PRD 55, 7114 (1997)]

★ exclusive pion-nucleon diffractive production of a  $\gamma$  pair of high  $p_{\perp}$



[J. Qiu et al, arXiv:2205.07846]

★ GPDs are not well-constrained experimentally:

- **x-dependence extraction is not direct. DVCS amplitude:**  $\mathcal{H} = \int_{-1}^{+1} \frac{H(x, \xi, t)}{x - \xi + i\epsilon} dx$

(SDHEP [J. Qiu et al, arXiv:2205.07846] gives access to x)

- independent measurements to disentangle GPDs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)
- and more challenges ...

★ Essential to complement the knowledge on GPD from lattice QCD

★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of  $t$  and  $\xi$  dependence

# Disclaimer

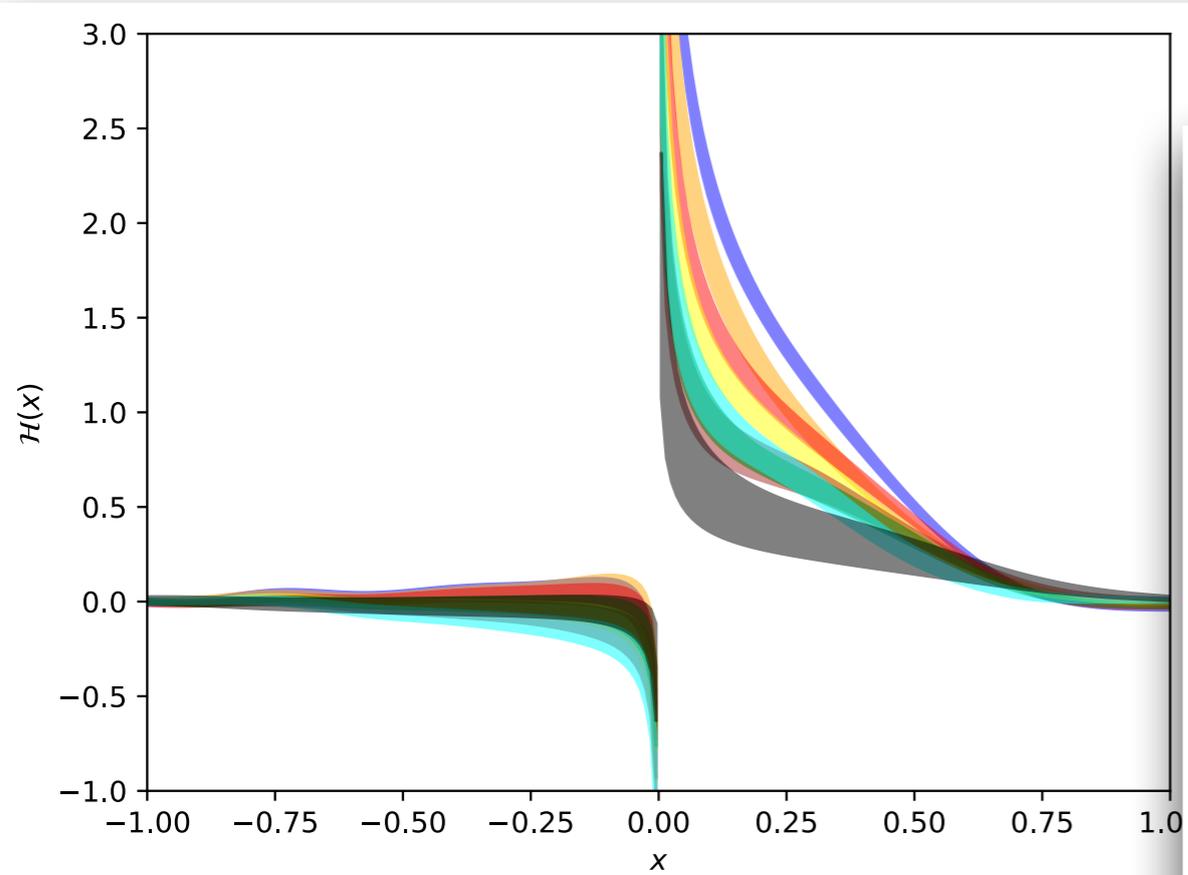
**The field of GPDs is still developing and sources of systematic uncertainties have not been fully addressed**

# Disclaimer

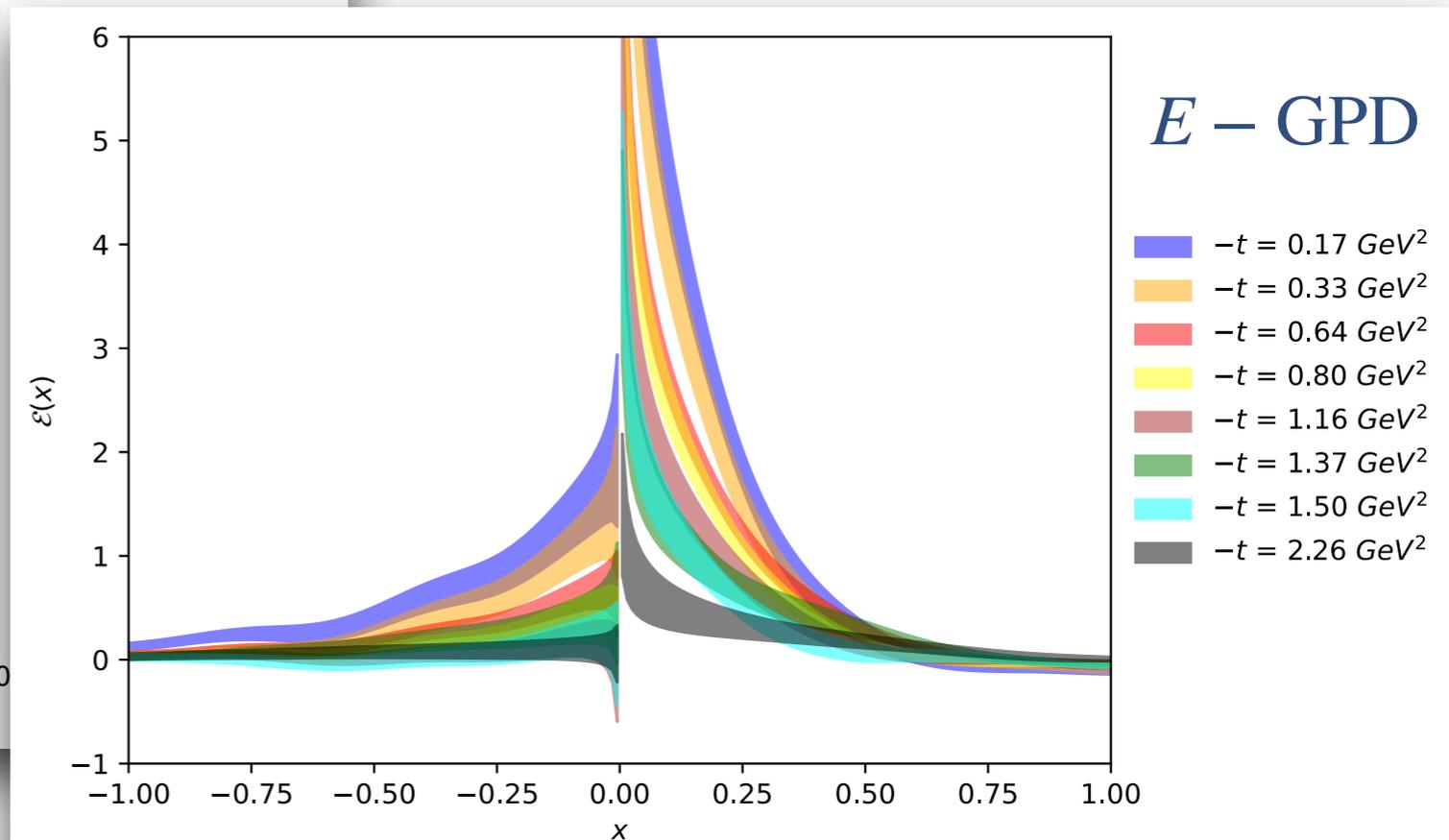
**The field of GPDs is still developing and sources of systematic uncertainties have not been fully addressed**

- Discretization effects
- physical pion mass
- volume effects
- inverse problem
- matching formalism
- connection to light-cone
- higher twist contaminations
- ...

# Light-cone GPDs



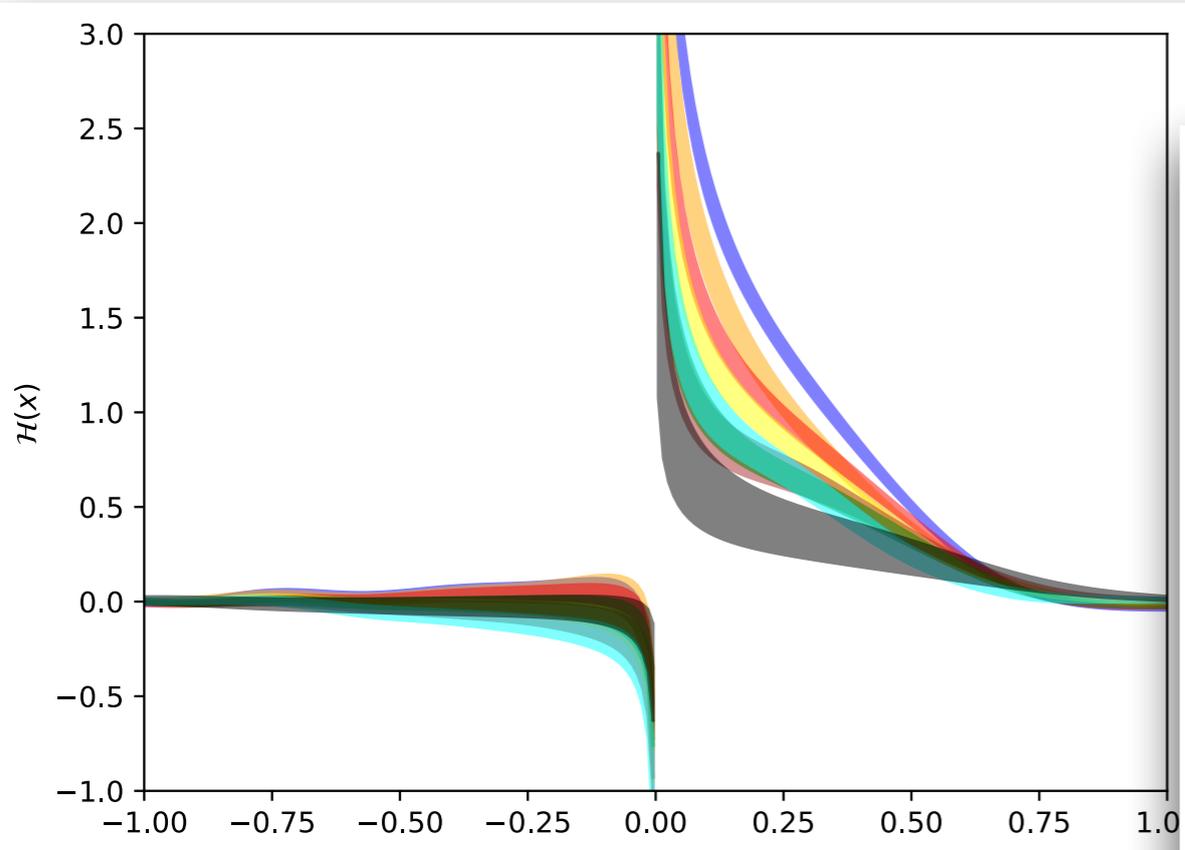
$H - \text{GPD}$



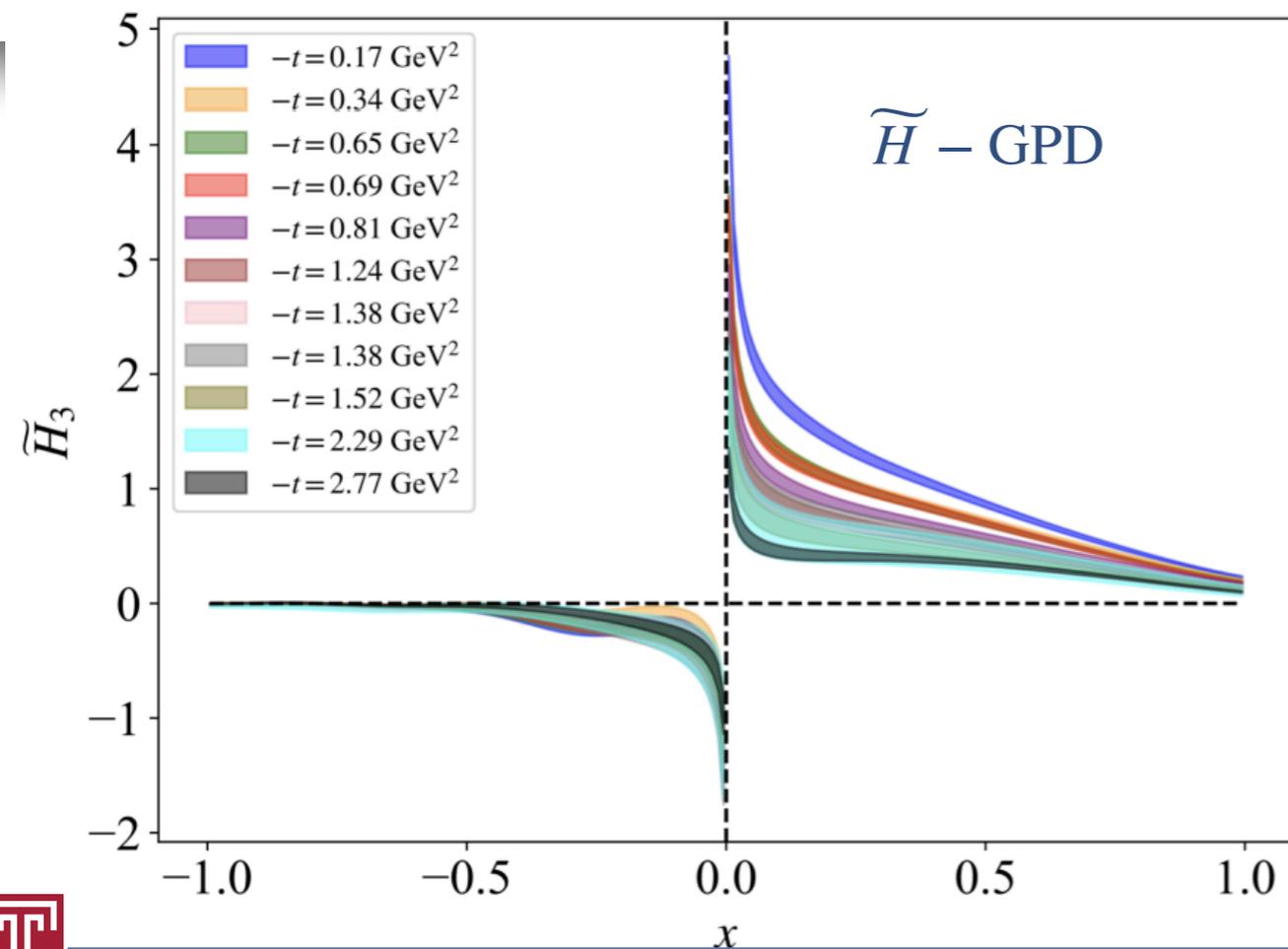
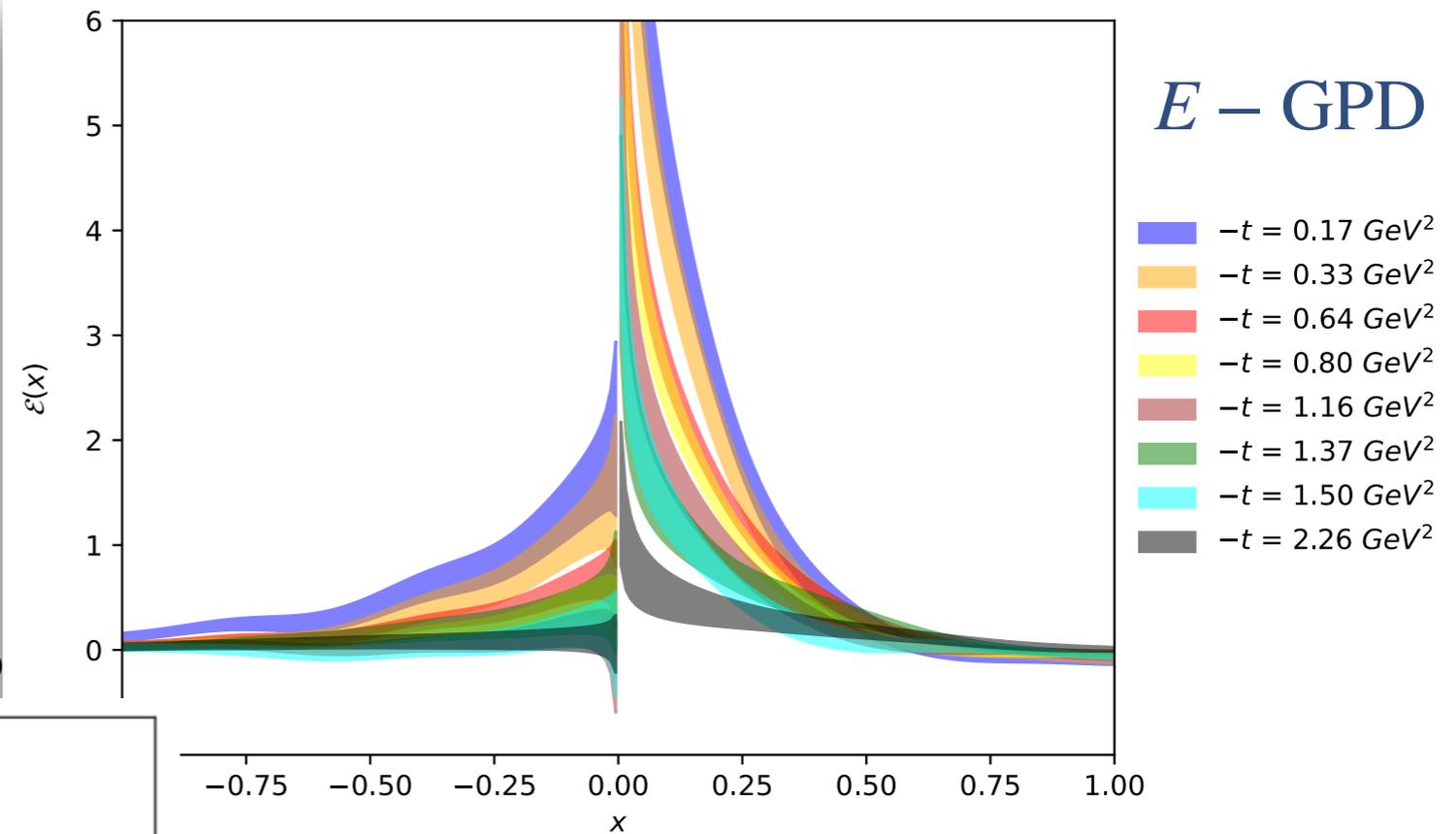
$E - \text{GPD}$

- $-t = 0.17 \text{ GeV}^2$
- $-t = 0.33 \text{ GeV}^2$
- $-t = 0.64 \text{ GeV}^2$
- $-t = 0.80 \text{ GeV}^2$
- $-t = 1.16 \text{ GeV}^2$
- $-t = 1.37 \text{ GeV}^2$
- $-t = 1.50 \text{ GeV}^2$
- $-t = 2.26 \text{ GeV}^2$

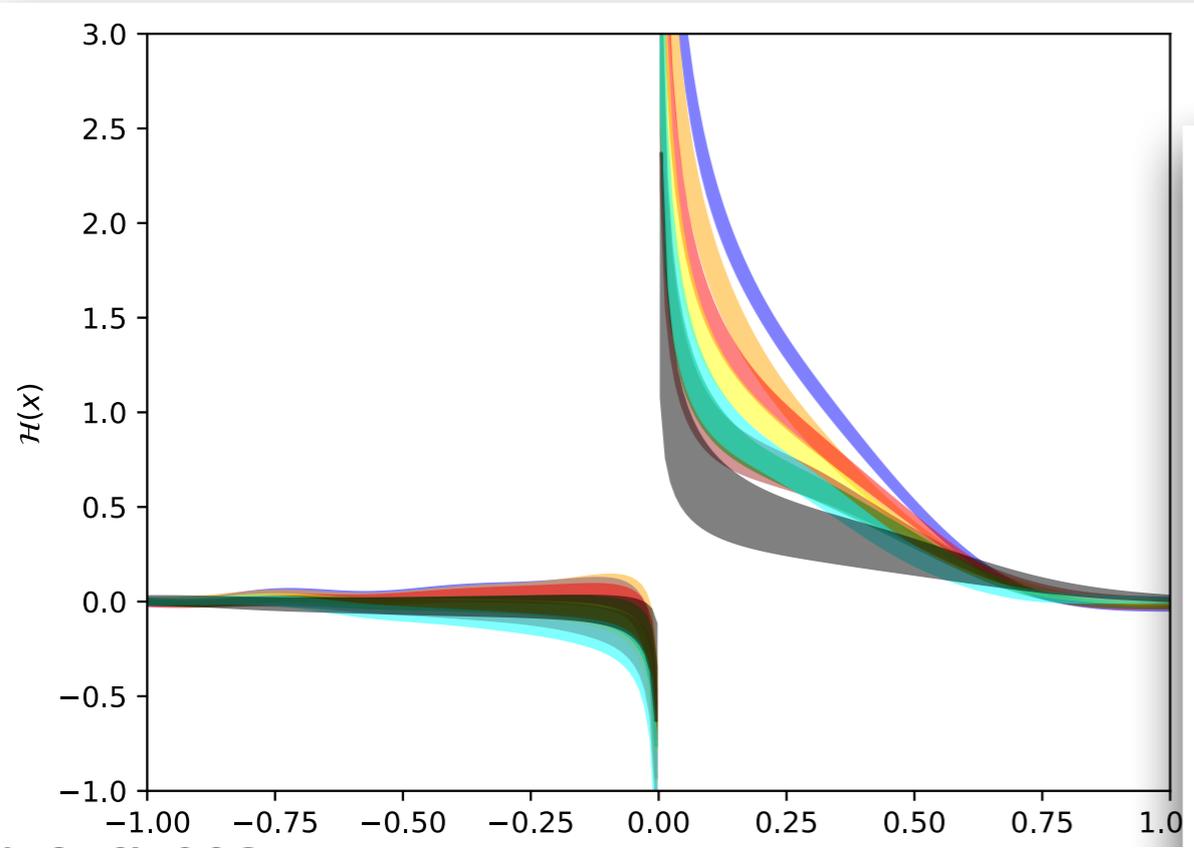
# Light-cone GPDs



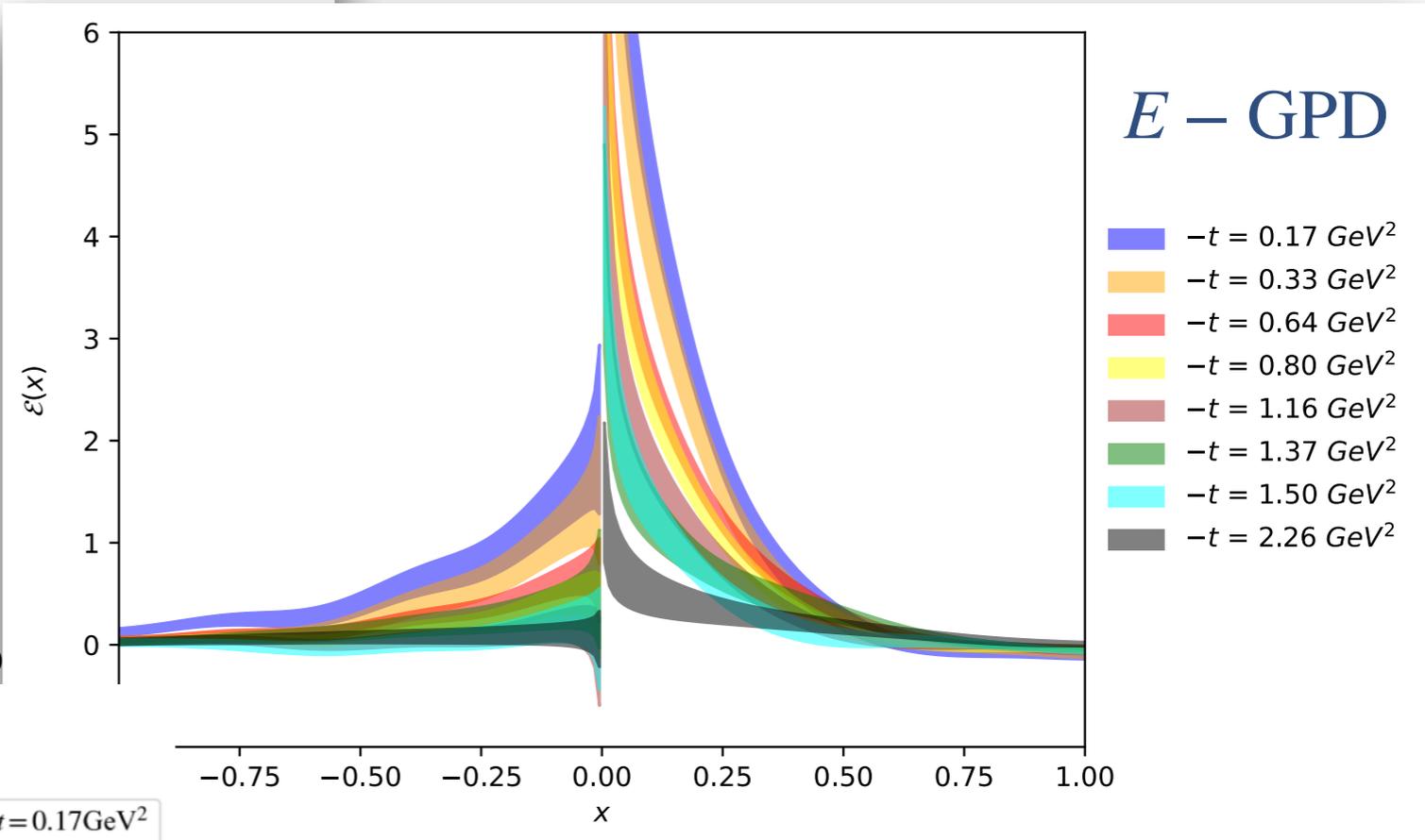
$H$  – GPD



# Light-cone GPDs

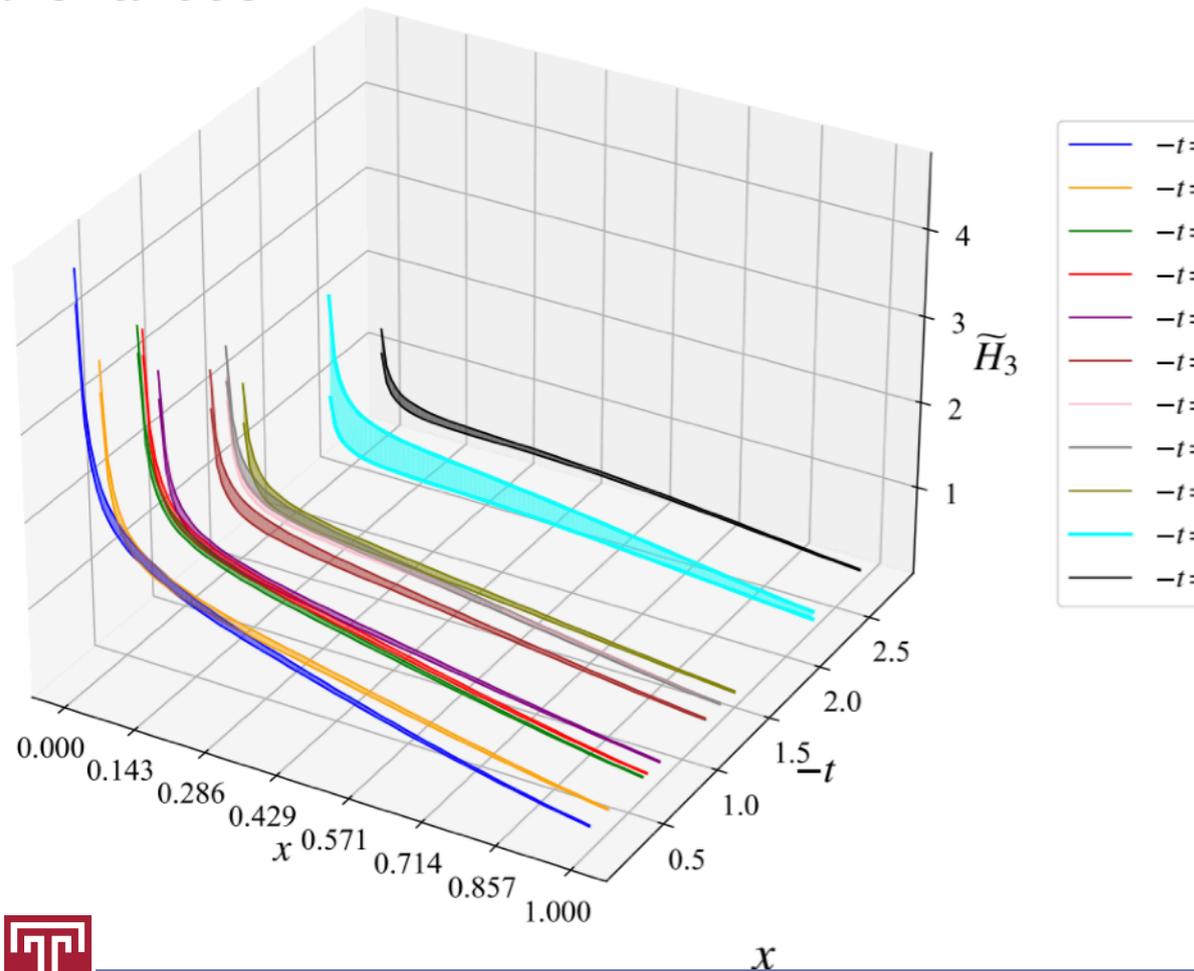


$H - \text{GPD}$



$E - \text{GPD}$

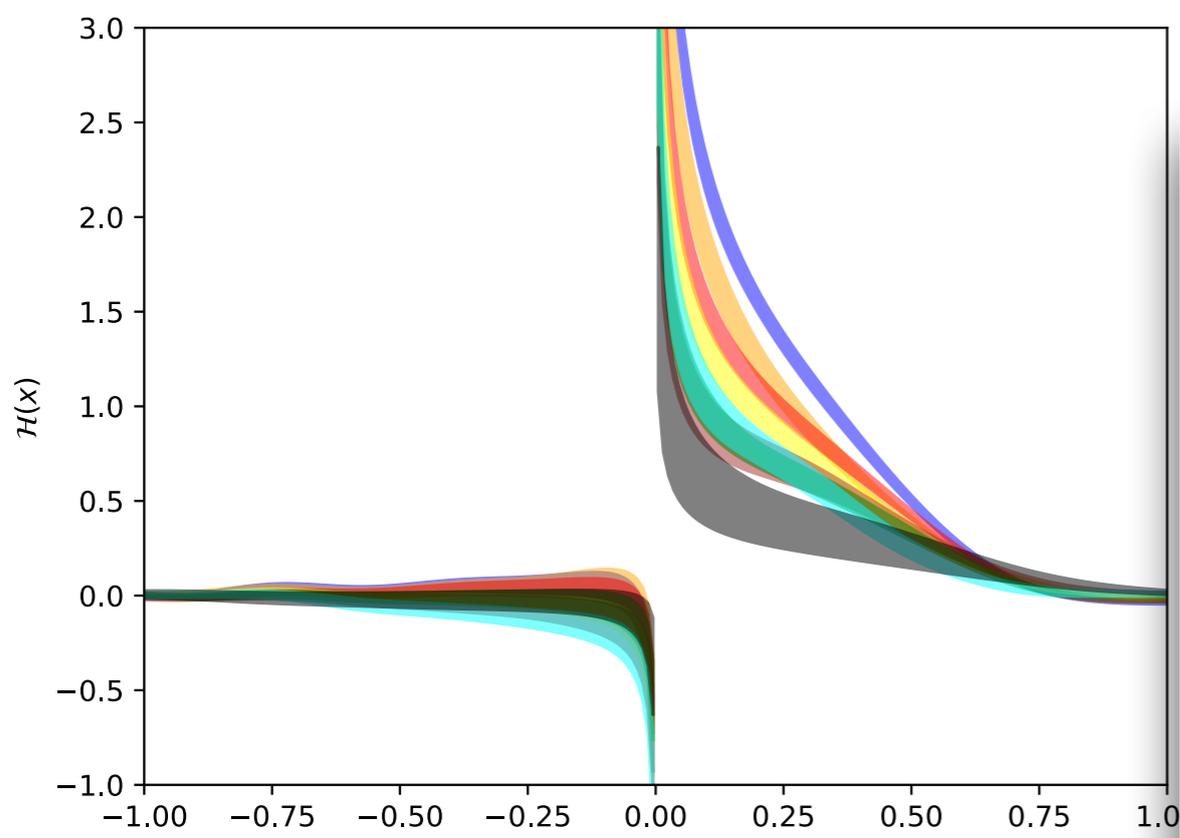
- $-t = 0.17 \text{ GeV}^2$
- $-t = 0.33 \text{ GeV}^2$
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- $-t = 0.80 \text{ GeV}^2$
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- $-t = 2.26 \text{ GeV}^2$



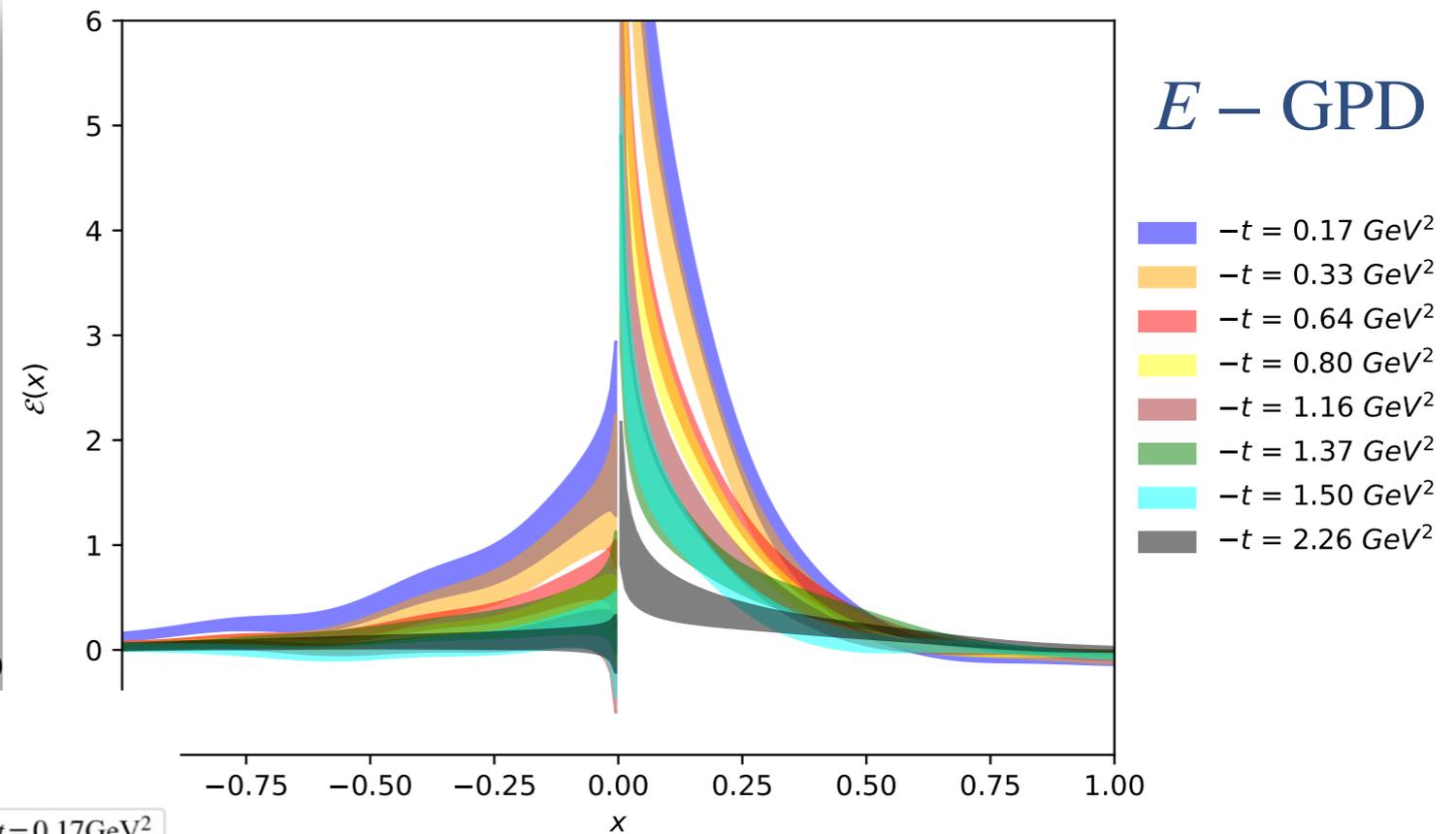
- $-t = 0.17 \text{ GeV}^2$
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- $-t = 1.52 \text{ GeV}^2$
- $-t = 2.29 \text{ GeV}^2$
- $-t = 2.77 \text{ GeV}^2$



# Light-cone GPDs

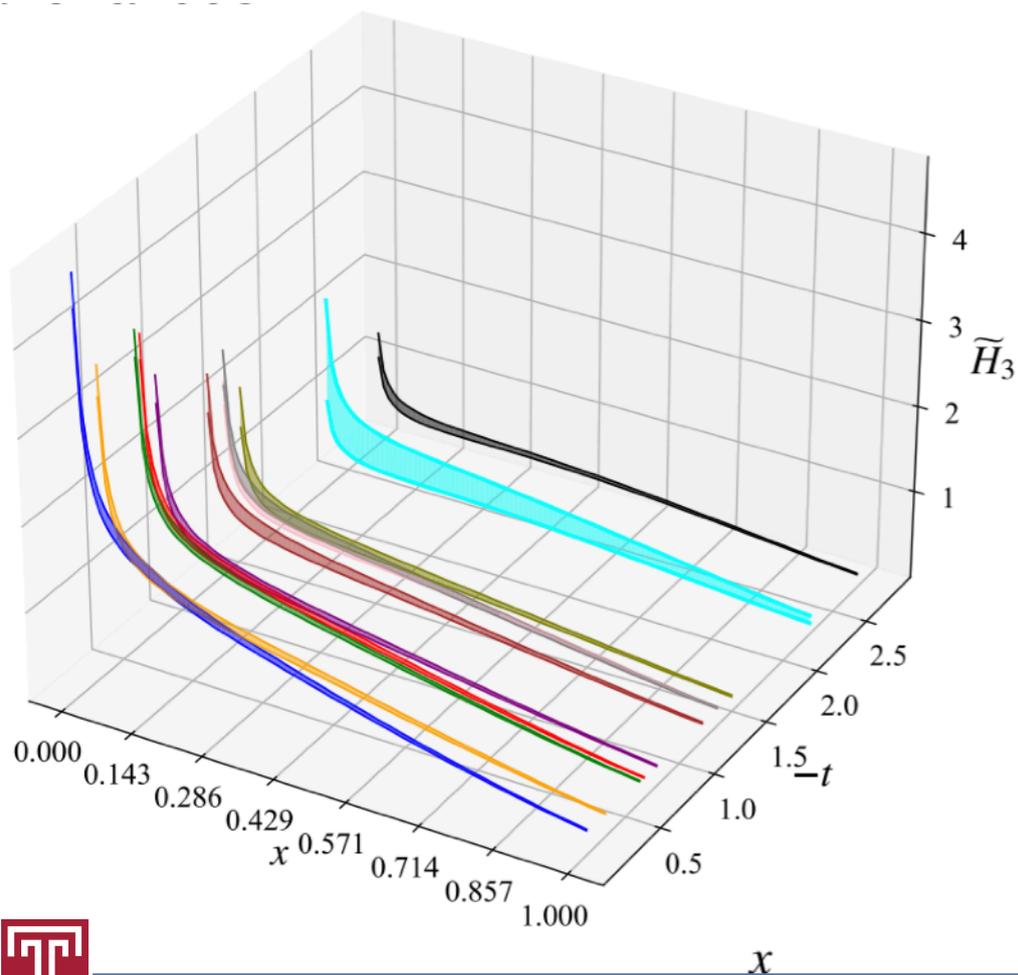


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- $-t = 0.17 \text{ GeV}^2$
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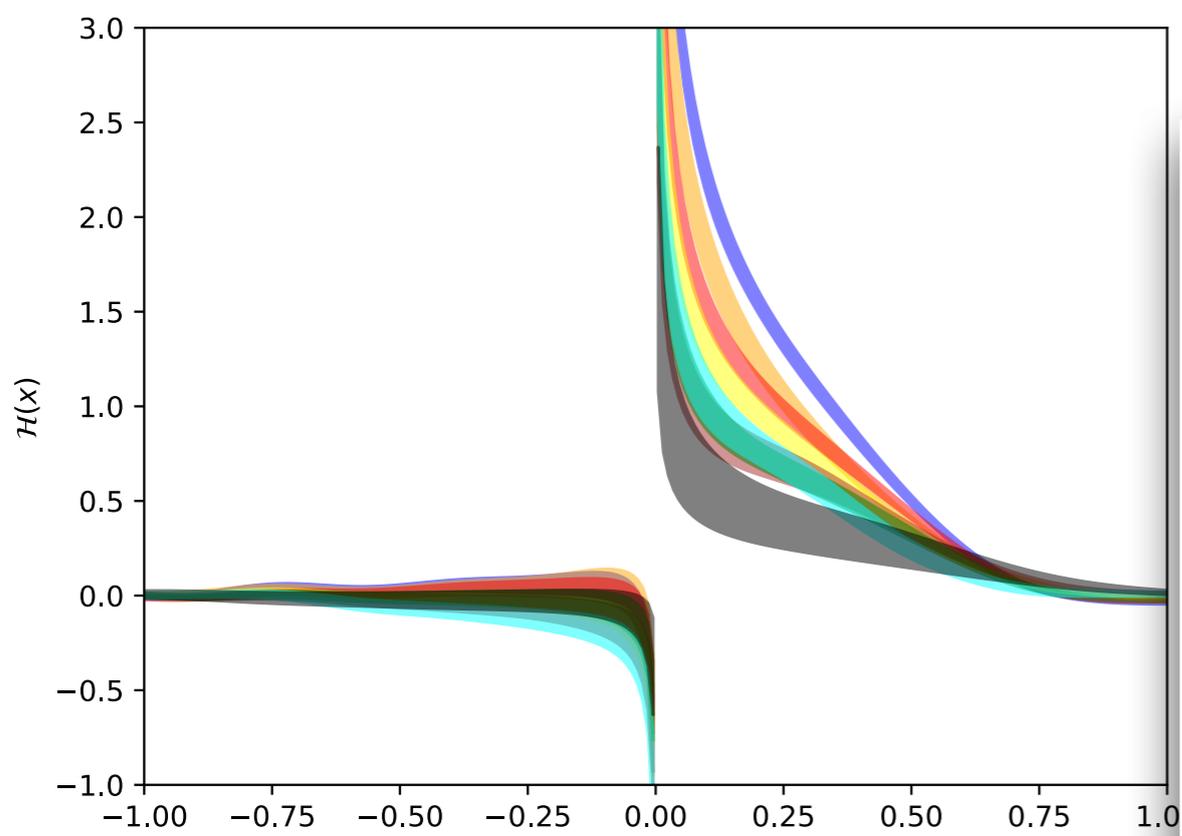


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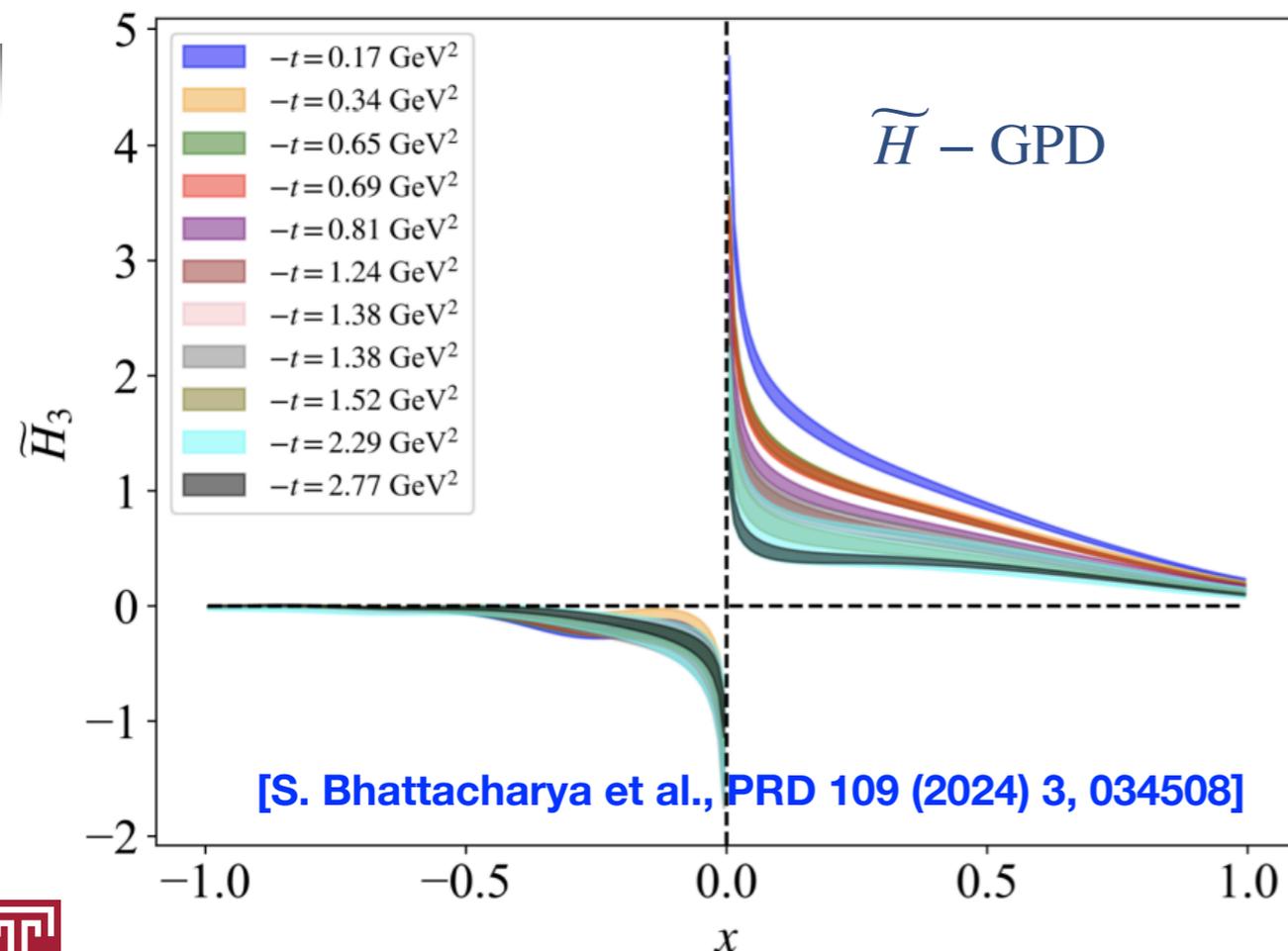
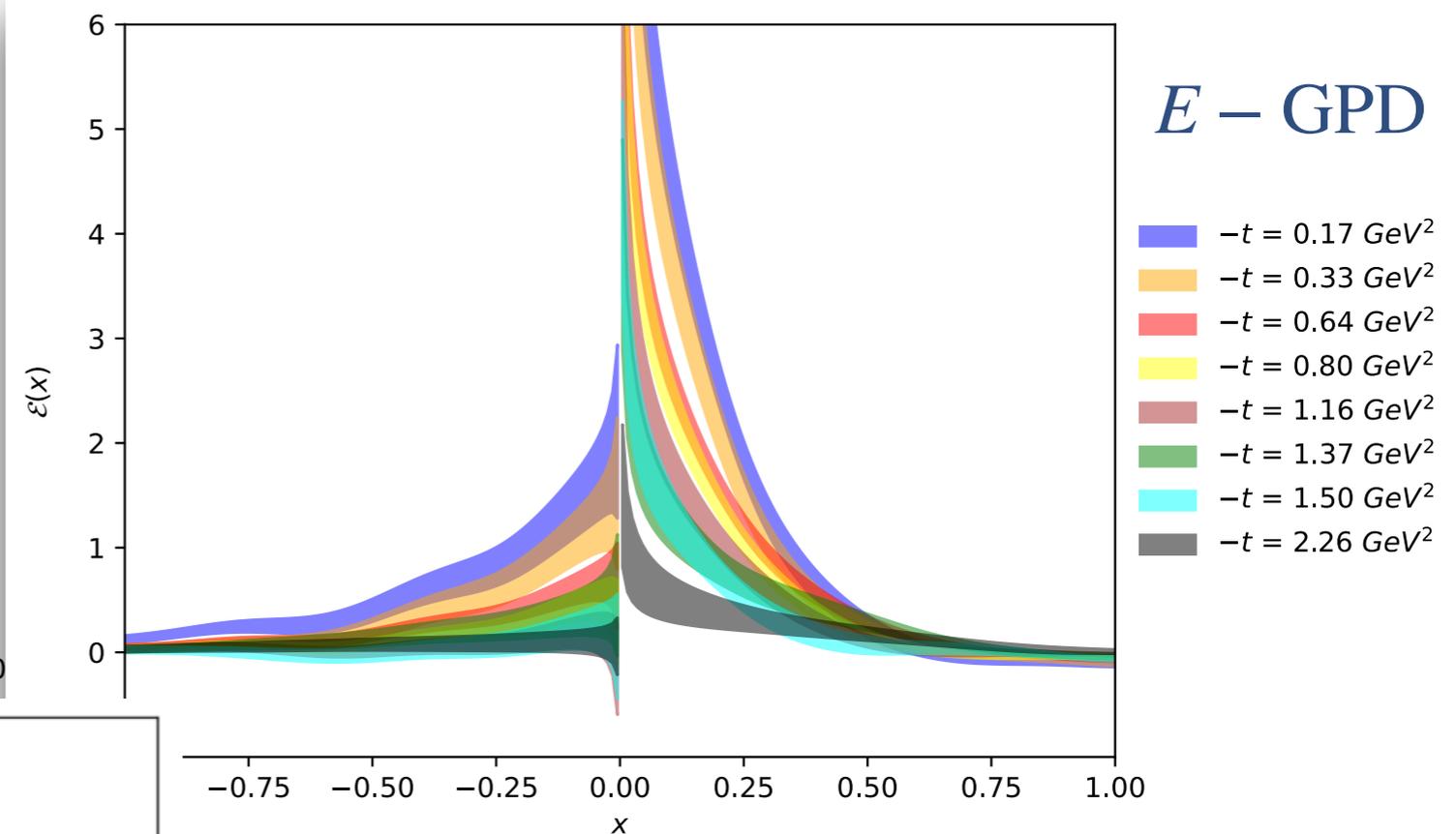
- ★ **+x region: quarks**
- ★ **-x region: anti-quarks**
- ★ **anti-quark region susceptible to more systematic uncertainties**
- ★ **small- and large- $x$  region not reliably extracted**
- ★ **large  $-t$  values unreliable but free**



# Reminder: Unpolarized & Helicity GPDs



[S. Bhattacharya et al., PRD 106 (2022) 11, 114512]

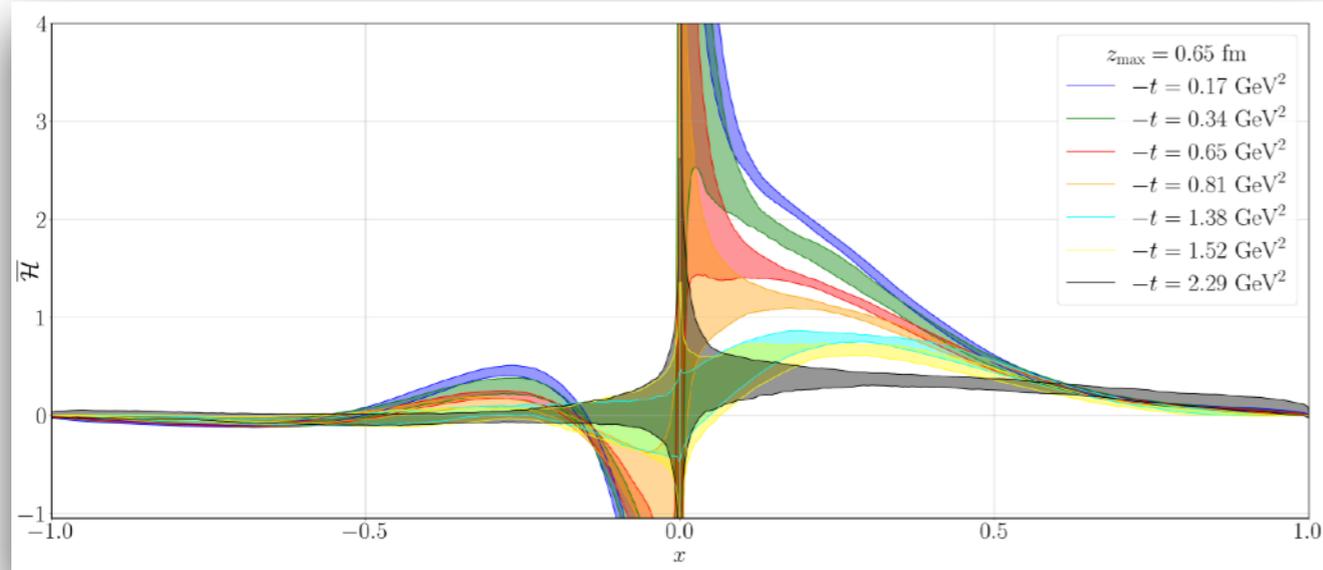


★ Signal for  $H_T$  comparable with  $H, \widetilde{H}$

★  $\widetilde{E}(\xi = 0) = 0$



# Alternative approach: pseudo-ITD

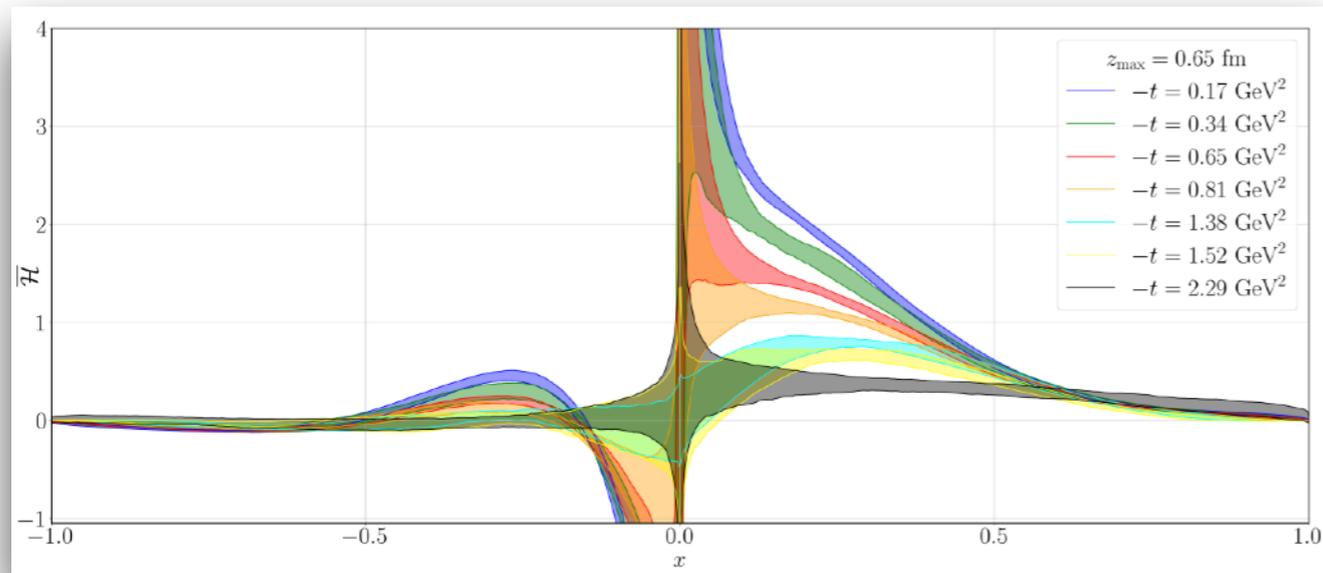


[Battacharya et al., PRD 110 (2024) 5, 054502]

*Different steps between approaches:*

- *renormalization*
- *x-dependence reconstruction*
- *matching formalism*

# Alternative approach: pseudo-ITD

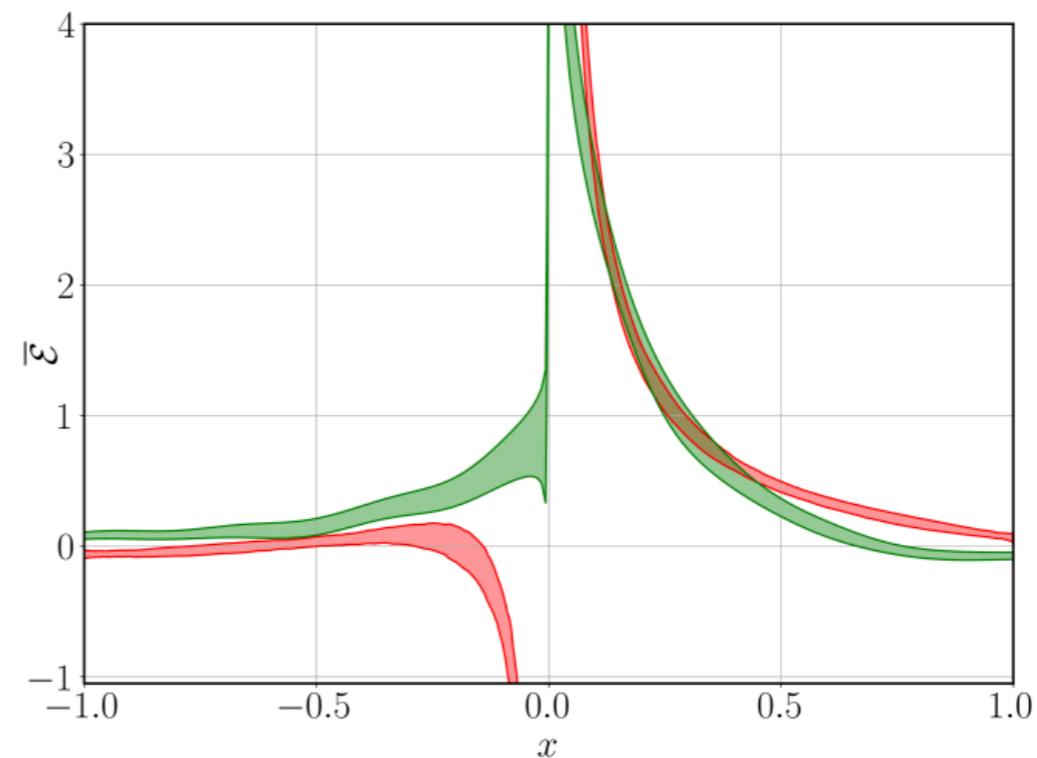
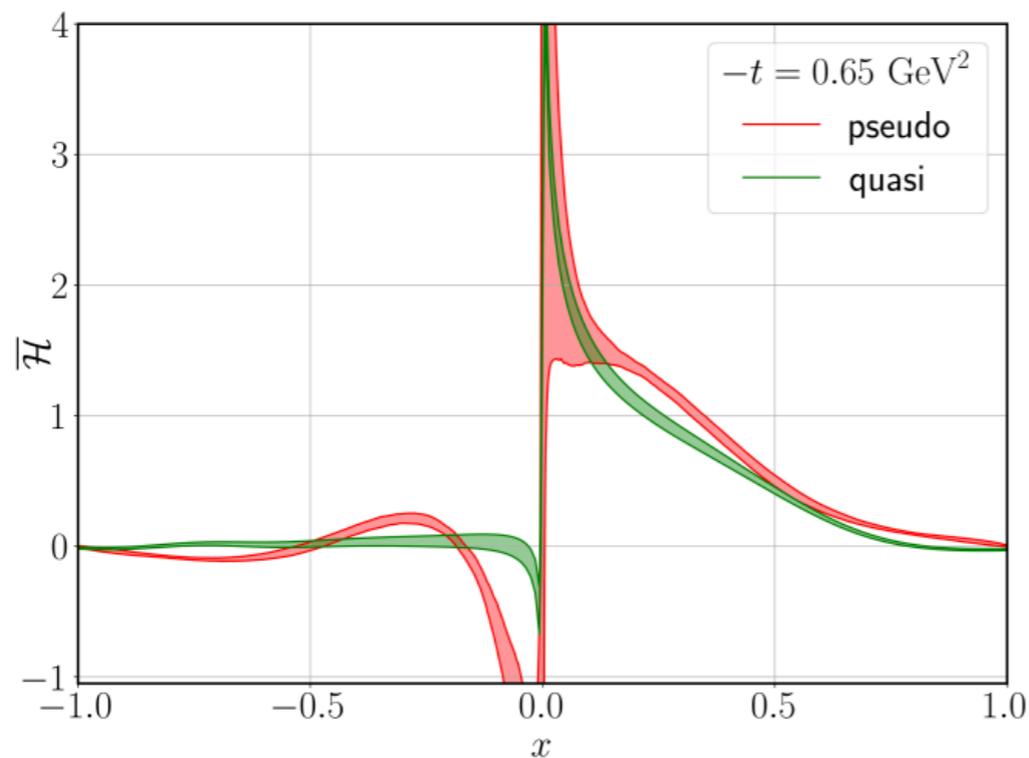


[Battacharya et al., PRD 110 (2024) 5, 054502]

*Different steps between approaches:*

- renormalization
- $x$ -dependence reconstruction
- matching formalism

★ Comparison between methods helps assess systematic effects

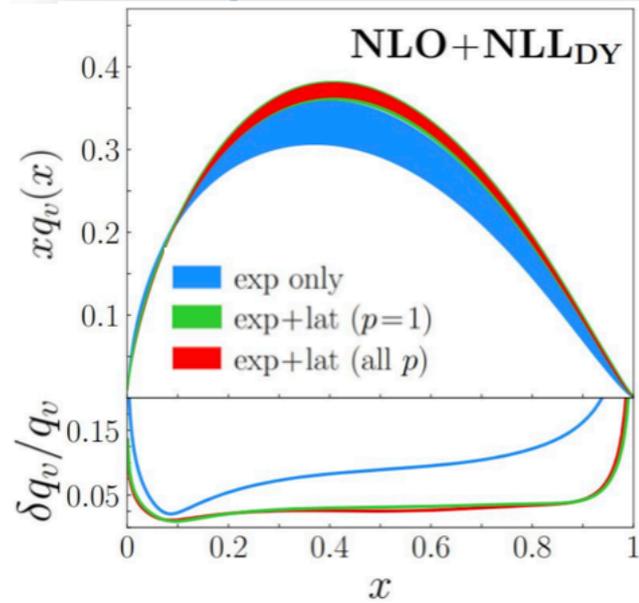


- ★  $x < 0$  and small- $x$  regions susceptible to systematic effects
- ★ Comparison only includes systematic uncertainties

# Synergy/Complementarity of lattice and phenomenology

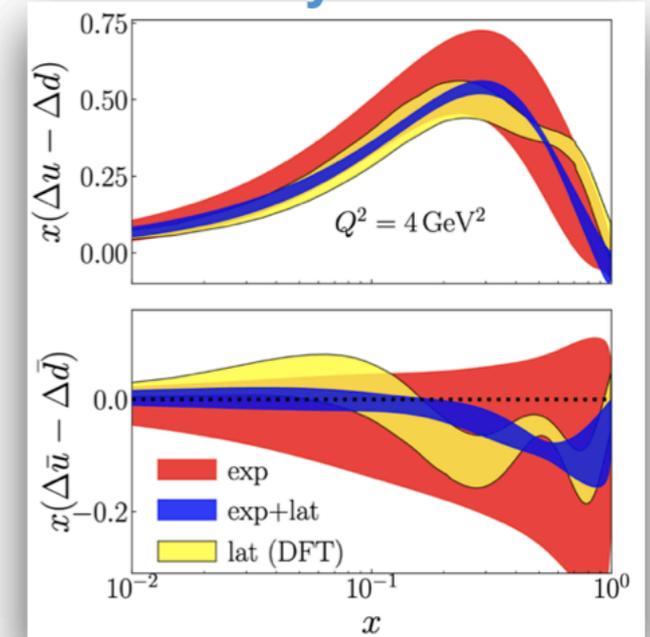
# Synergies: constraints & predictive power of lattice QCD

pion PDF

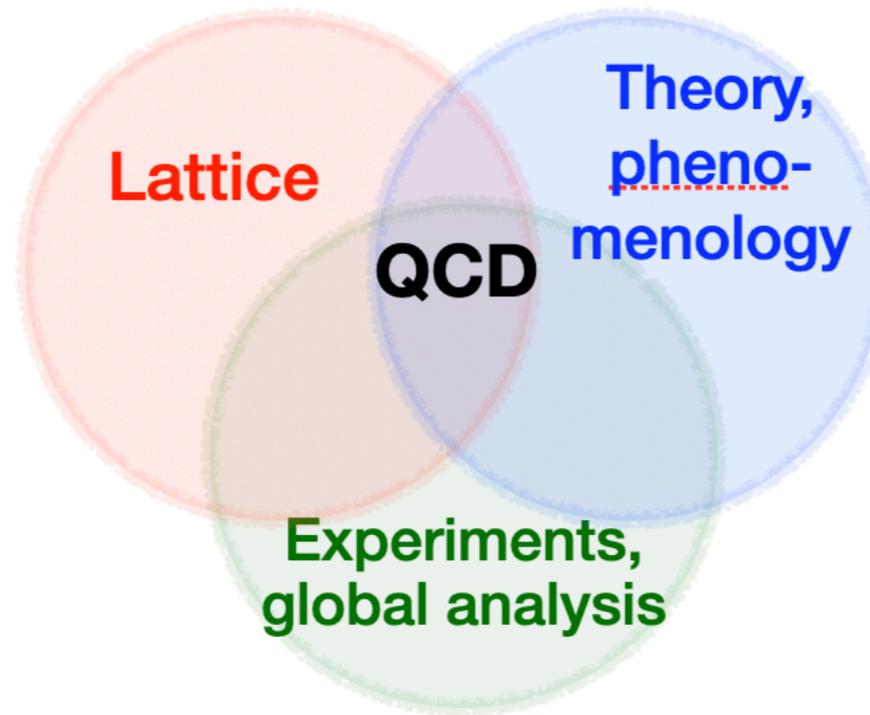


[JAM/HadStruc, PRD105 (2022) 114051]

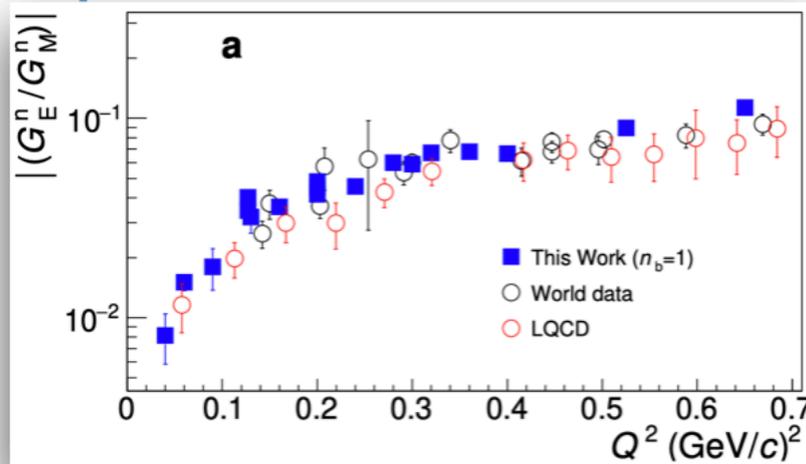
helicity PDF



[JAM & ETMC, PRD 103 (2021) 016003]

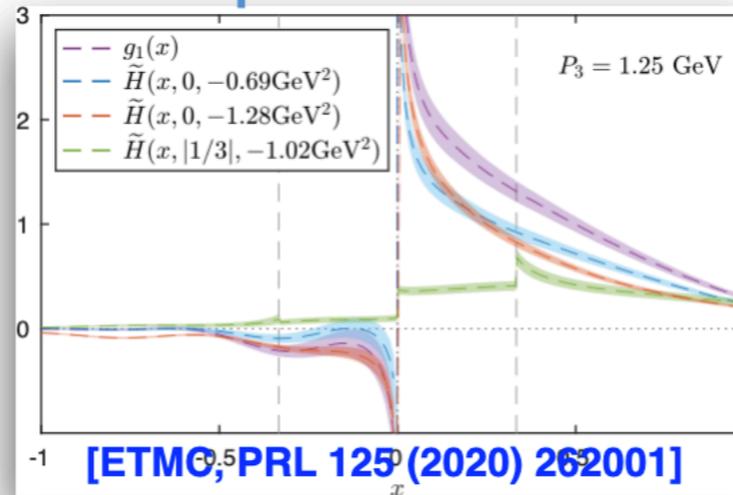


proton & neutron radius



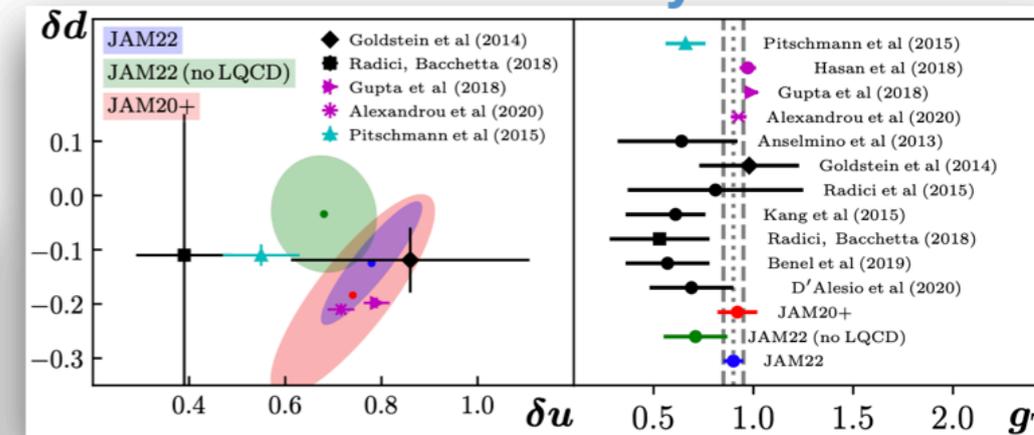
[Atac et al., Nature Comm. 12, 1759 (2021)]

proton GPDs



[ETMC, PRL 125 (2020) 262001]

transversity PDF



[JAM, PRD 106 (2022) 3, 034014]

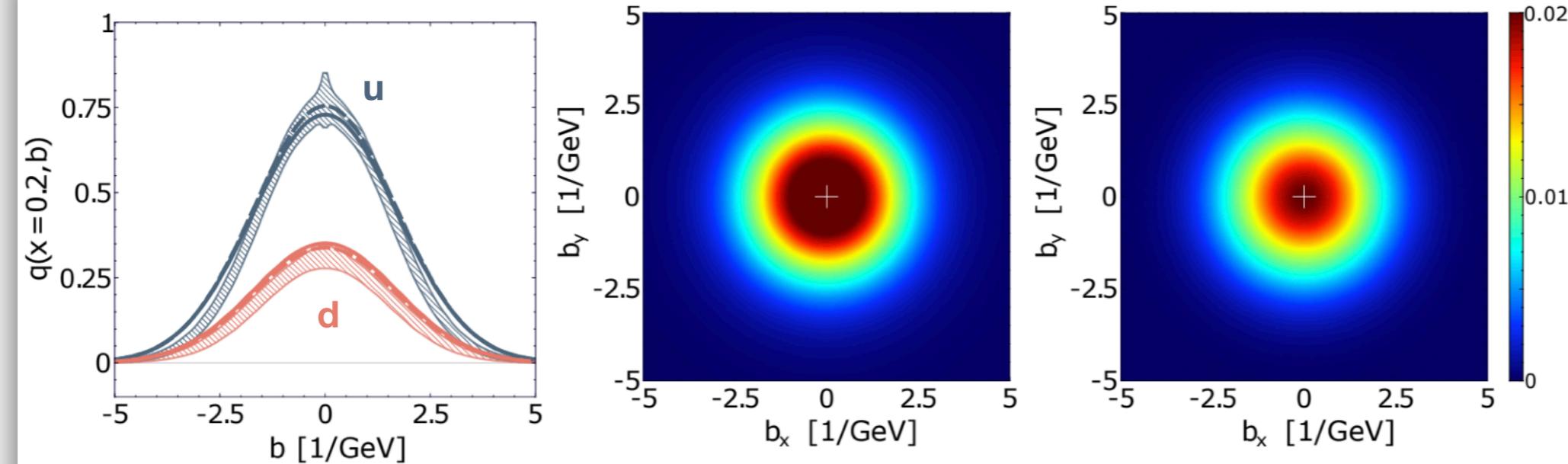
And many more!

# Toward synergy for GPDs

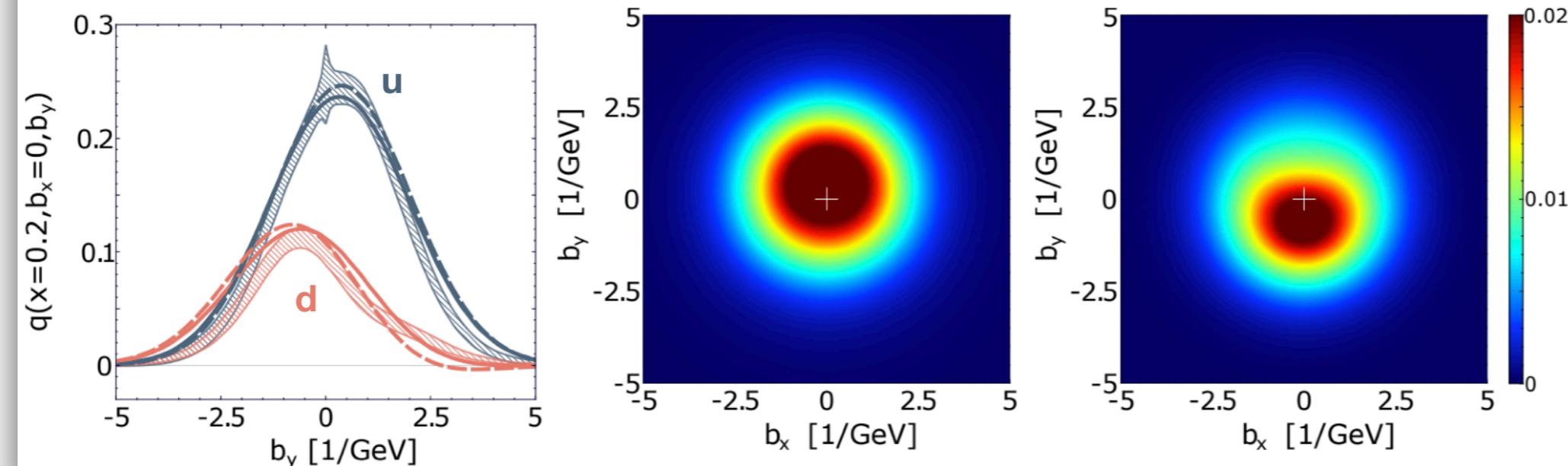
[K. Cichy et al., PRD 110 (2024) 11, 114025]

## Example: unpolarized GPDs

(a) Unpolarized proton for  $x = 0.2$



(b) Transversely polarized proton for  $x = 0.2$



- GK (solid line),
- VGG (dashed line)

- Good agreement for up quark; reasonable agreement for down quark
- Further study needed on how to combine lattice results with data

# *How to lattice QCD data fit into the overall effort for hadron tomography*

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- ★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of  $t$  and  $\xi$  dependence

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1. **Theoretical studies** of high-momentum transfer processes using perturbative QCD methods and study of GPDs properties
2. **Lattice QCD** calculations of GPDs and related structures
3. **Global analysis** of GPDs based on experimental data using modern data analysis techniques for inference and uncertainty quantification

# How to lattice QCD data fit into the overall effort for hadron tomography

- ★ Lattice data may be incorporated in global analysis of experimental data and may influence parametrization of  $t$  and  $\xi$  dependence



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## Other GPD global analysis efforts:

- Gepard [<https://gepard.phy.hr/>]
- PARTONS [<https://partons.cea.fr>]
- EXCLAIM [<https://exclaimcollab.github.io/web.github.io/#/>]

# Summary of Lecture 2

# Key points of Lecture 2

- ★ Hadron structure studies are critical for understanding the immensely rich and complex properties of the visible matter
- ★ The lattice formulation can provide first principle results for a very broad research program in Hot and Cold QCD, and beyond!
- ★ Many opportunities for synergies and complementarity.



DOE Early Career Award  
Grant No. DE-SC0020405 &  
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<https://2025.einnconference.org/>

28 October – 01 November, 2025

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Henry Klest (Argonne National Lab)  
Aleksandr Pustytsev (University of Mainz)  
Abhyuday Sharda (University of Tennessee)  
Natalie Wright (MIT)



Frontiers and Careers in Photonuclear Physics 2025  
26 - 27 October, 2025



16<sup>th</sup> European Research Conference on Electromagnetic Interactions with Nucleons and Nuclei  
● 28 October – 1 November 2025, Paphos, Cyprus ● Coral Beach Hotel & Resort

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- Nucleon form factors and low-energy hadron structure
- Partonic structure of nucleons and nuclei
- Precision electroweak physics and new physics searches
- Meson structure
- Baryon and light-meson spectroscopy
- Nuclear effects and few-body physics

**Workshops**

Non-perturbative approaches for hadron structure from low to high energy (Barbara Pasquini)

AI & ML in nuclear science: starting with design, optimization, and operation of the machine and detectors, to data analysis (Abhay Deshpande)

**Poster Session**

On Tuesday, October 28th, a poster session has been organized. The European Physical Society sponsors the poster prizes, and the three best posters will receive an "EPS Poster Prize," which will also be promoted for a plenary talk at the conference.

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**Workshops & Organizers**

Abhay Deshpande  
Barbara Pasquini

**Pre-conference**

Henry Klest (Argonne National Lab)  
Aleksandr Pustytsev (University of Mainz)  
Abhyuday Sharda (University of Tennessee)  
Natalie Wright (MIT)

**Important Dates – Deadlines**

Early registration deadline: 7 September, 2025  
Late registration: 8 September – 28 October, 2025  
Abstract submission for talks and posters: 31 August, 2025

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Coral Beach Hotel & Resort

## Abstract submission is Open!

Other topics relevant to EINN

Poster

Talk in workshop 1 "Non-perturbative approaches for hadron structure from low to high energy"

Talk in workshop 2: "AI & ML in nuclear science: starting with design, optimization, and data analysis"

*Thank you*

