Nucleon spin, form factor and GPD

Yoshitaka Hatta

BNL/RIKEN BNL

About myself:

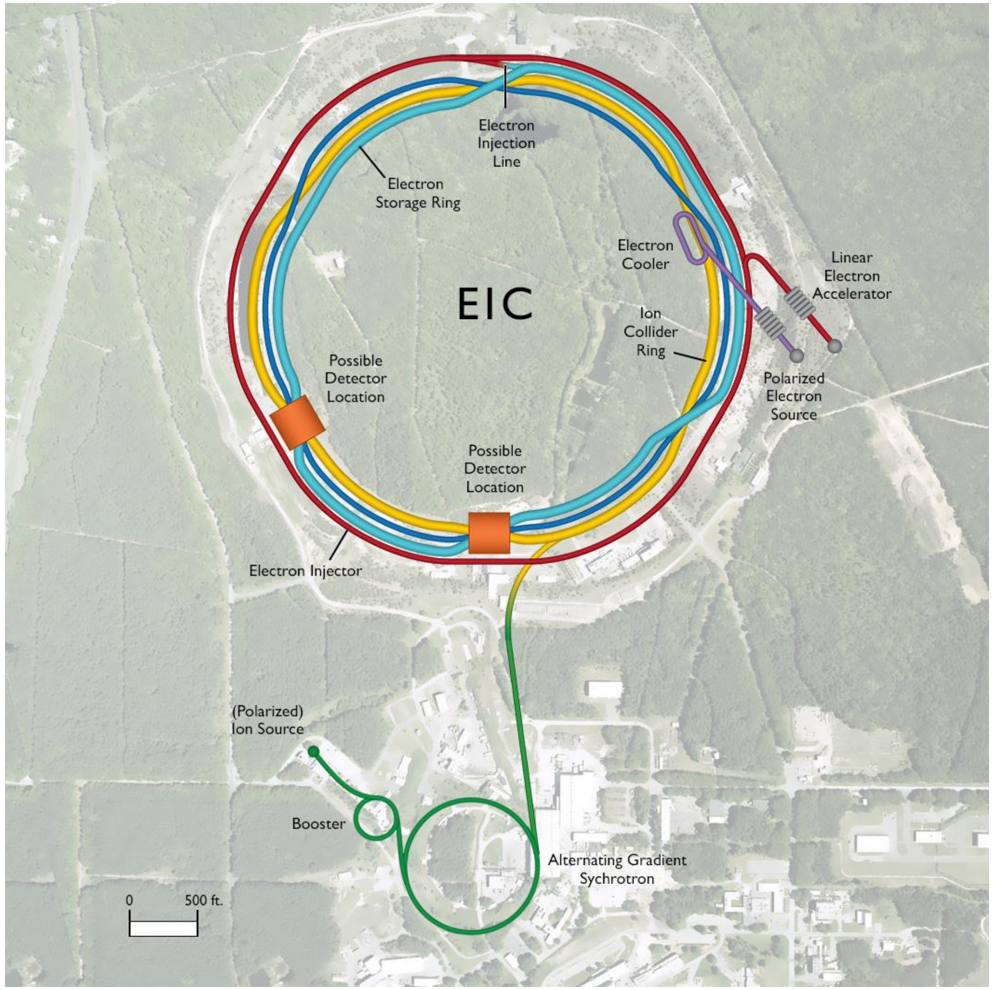
- PhD from Kyoto university (2004)
- Postdocs at RIKEN BNL and Saclay (France)
- Faculty positions at University of Tsukuba (2008~2013), Kyoto university (2013~2018)
- Moved to Brookhaven National Laboratory in 2018
- Research interest: Hadron structure from QCD: spin, mass, 3D&5D tomography, small-x.

The Electron-Ion Collider (EIC) project

Next-generation (your generation) nuclear physics facility
to be built at Brookhaven National Laboratory, New York

As of now, the only new high energy collider in the world officially approved for construction.

QCD first!
Unravel the structure of proton and nuclei

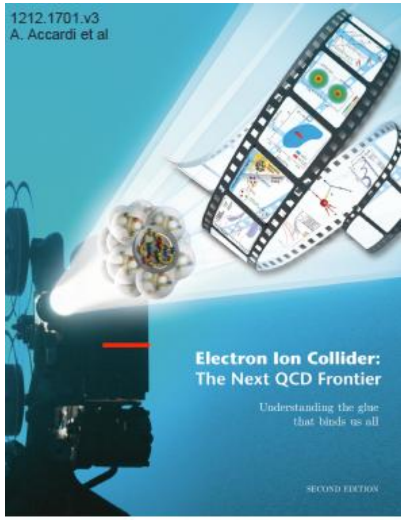


Scientific goals of EIC

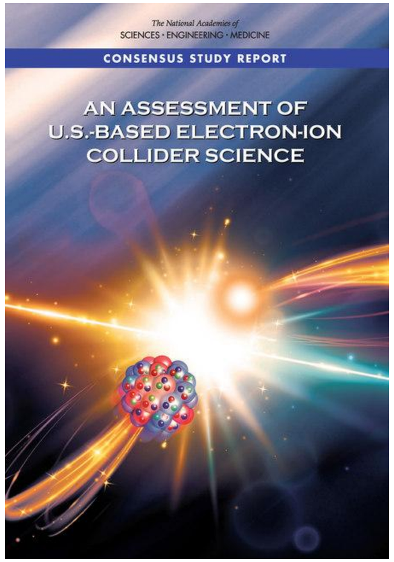
Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

- How does the mass of the nucleon arise?
- How does the spin of the nucleon arise?
- What are the emergent properties of dense systems of gluons?

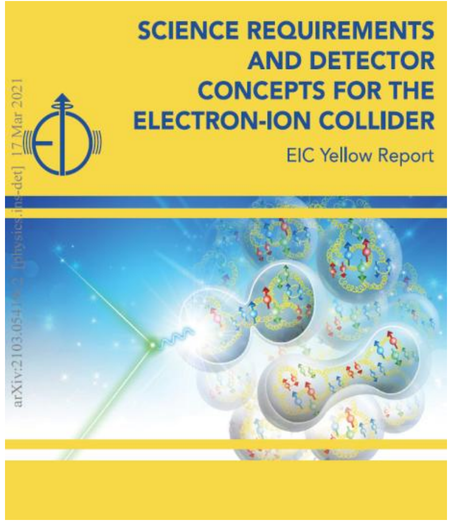
White paper (2012)



NAS report (2018)



Yellow report (2022)



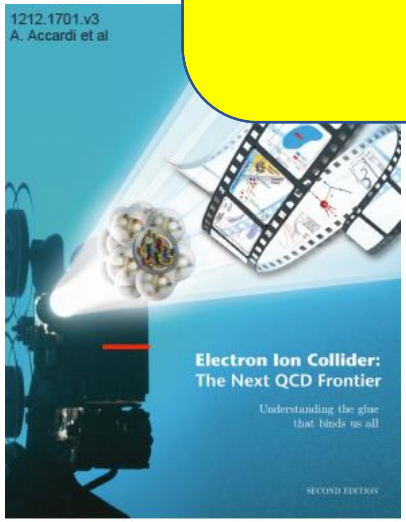
Scientific goals of EIC

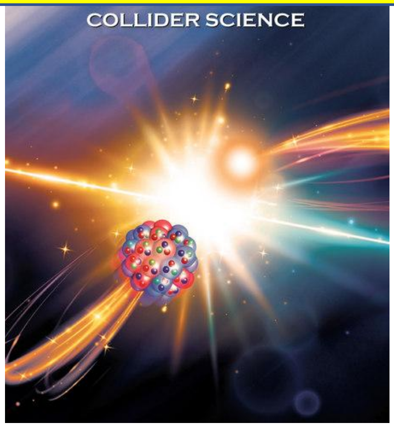
Finding 1: An EIC can uniquely address three profound questions about nucleons-protons—and how they are assembled to form the nuclei of atoms:

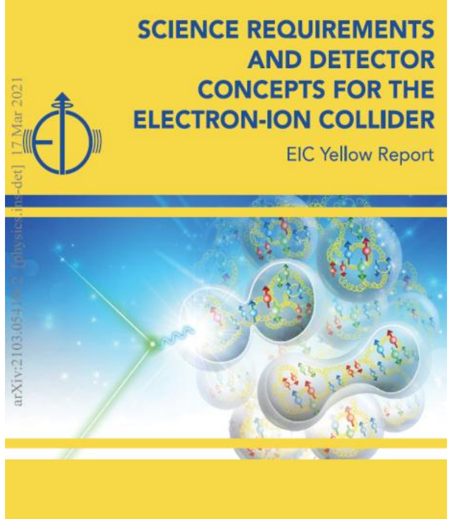
- How does the mass of the nucleon arise?
- How
- What ons?

The era of precision QCD study of nucleon and nuclear structures in the next 20-30 years!

White paper (2012)







Plan

- Proton spin decomposition
- Polarized Deep Inelastic scattering
- Orbital angular momentum
- Generalized parton distribution
- Deeply Virtual Compton Scattering
- Electromagnetic form factor
- Gravitational form factor

Lecture 1

Lecture 2

Lecture 3

Notations

Metric $g^{\mu\nu} = (+1, -1, -1, -1)$ $\mu, \nu = 0, 1, 2, 3$ $i, j = 1, 2$ (transverse)

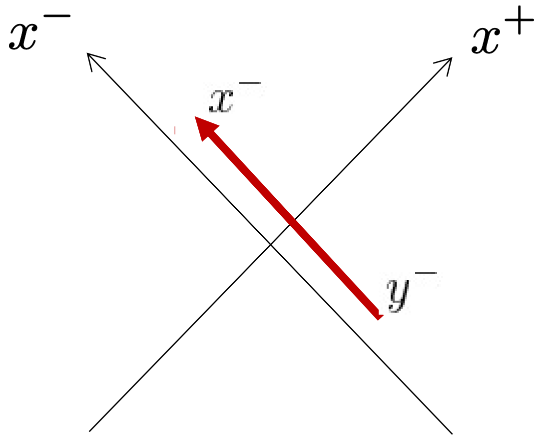
Light-cone coordinates $P^{\pm} = \frac{1}{\sqrt{2}}(P^0 \pm P^3)$ $g^{+-} = 1$ $P^+ = P_-$

$$P \cdot x = P^+ x^- + P^- x^+ - P_\perp^i x_\perp^i$$

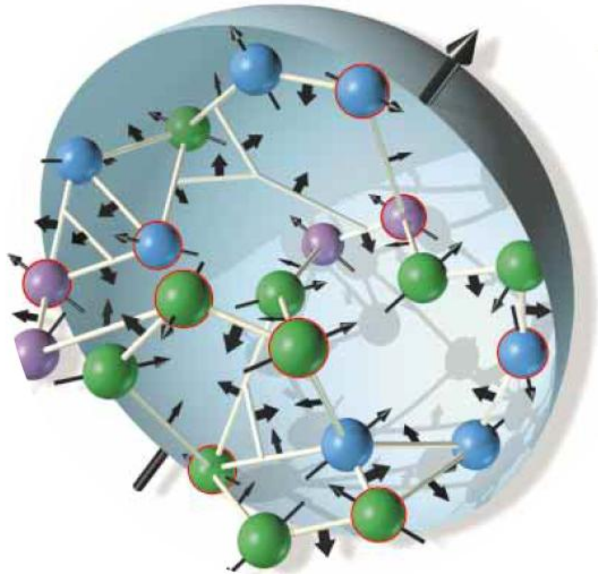
γ_5, antisymmetric tensor $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ $\epsilon^{0123} = +1$ $\epsilon^{12} = \epsilon_{12} = +1$

Coupling constant $D^\mu = \partial^\mu + igA^\mu$

Wilson line $W[x^-, y^-, x_\perp] = P\exp\left(-ig\int_{y^-}^{x^-} dz^- A^+(z^-, x_\perp)\right)$



The proton spin problem



The proton has spin ½.

The proton is not an elementary particle.

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Quarks' helicity

Gluons' helicity

Orbital angular Momentum (OAM)

QCD angular momentum tensor

QCD Lagrangian → Lorentz invariant $x^\mu \to x^\mu + \omega^{\mu\nu} x_\nu$

→ Noether current $\partial_\mu M^{\mu\nu\lambda}_{can} = 0$

QCD angular momentum tensor

$$M^{\mu\nu\lambda}_{can} = x^\nu T^{\mu\lambda}_{can} - x^\lambda T^{\mu\nu}_{can} - \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}\bar{\psi}\gamma_\rho\gamma_5\psi + F^{\mu\lambda}A^\nu - F^{\mu\nu}A^\lambda$$

quark helicity gluon helicity

canonical energy momentum tensor

$$T^{\mu\nu}_{can} = \bar{\psi} i\gamma^\mu \overleftrightarrow{\partial}^\nu \psi - F^{\mu\alpha}\partial^\nu A^\alpha - g^{\mu\nu}\mathcal{L}$$

→ Quark OAM → Gluon OAM

Exercise: Derive the canonical angular momentum tensor $M_{can}^{\mu\nu\lambda}$

Hint: Under an infinitesimal Lorentz transformation

$$\delta\psi = -\omega^{\mu\nu}\left(\frac{1}{2}(x_\nu\partial_\mu - x_\mu\partial_\nu)\psi - \frac{1}{8}[\gamma_\mu,\gamma_\nu]\psi\right)$$

$$\delta A^\alpha = -w^{\mu\nu}\left(x_\nu\partial_\mu A^\alpha - \frac{1}{2}(\delta^\alpha_\mu g_{\nu\beta} - g_{\mu\beta}\delta^\alpha_\nu)A^\beta\right)$$

$$\delta\mathcal{L} = -w^{\mu\nu}x_\nu\partial_\mu\mathcal{L}$$

Problems

$T_{can}^{\mu\nu}$ is not symmetric, not gauge invariant

$T_{can}^{\mu\nu}$ is conserved wrt the first index $\partial_\mu T_{can}^{\mu\nu} = 0$ but not the second $\partial_\nu T_{can}^{\mu\nu} \neq 0$

Jaffe-Manohar decomposition

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q_{can} + L^g_{can}$$

$\mu\nu\lambda = +12$ component of the canonical angular momentum tensor $M^{\mu\nu\lambda}_{can}$

Operators NOT gauge invariant except the quark helicity $\Delta\Sigma \sim \bar{\psi}\gamma^+\gamma_5\psi$

$$\Delta G \sim \epsilon^{ij} F^{+i} A^j \qquad L^q_{can} \sim \bar{\psi} x \times i\partial\psi \qquad L^g_{can} \sim F x \times \partial A$$

To be understood in the light-cone gauge $A^+ = 0$

Naïve replacement $\partial^\mu \to D^\mu$ does not solve the problem.

Quark helicity: definition

proton single-particle state,

$$2\Delta\Sigma S^\mu = \sum_f \langle PS|\bar{\psi}_f \gamma^\mu \gamma_5 \psi_f|PS\rangle$$

spin 4-vector $$2S^\mu = \bar{u}(PS)\gamma^\mu\gamma_5 u(PS)$$

proton Dirac spinor

Exercise : show that

$$P^\mu S_\mu = 0$$

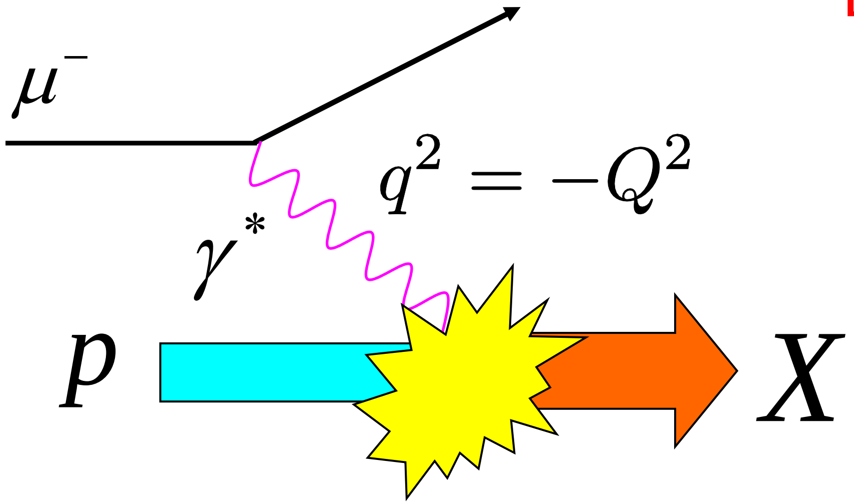
$$S^2 = -M^2$$

$$u(PS)\bar{u}(PS) = \frac{\not{P}+M}{2}\left(1+\frac{\gamma_5 \not{S}}{M}\right)$$

In the quark model, $$\frac{1}{2} = \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \quad \Rightarrow \quad \Delta\Sigma = 1$$

With relativistic effects, $\Delta\Sigma \approx 0.7$

Deep inelastic scattering



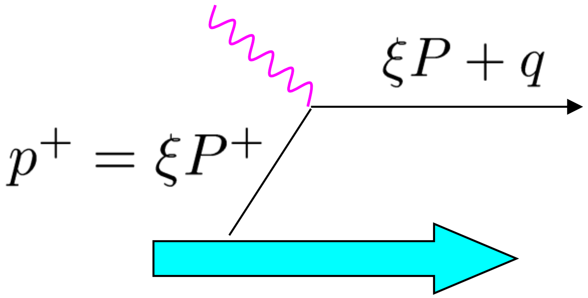
Bjorken variable

$$x = \frac{Q^2}{2P\cdot q} = \frac{Q^2}{(P+q)^2 + Q^2 - m_p^2}$$

$$= \frac{Q^2}{Q^2 + m_X^2 - m_p^2}$$

$$\sim \frac{Q^2}{s} \quad (x \ll 1)$$

Physical meaning of x : momentum fraction carried by the struck parton



$$(\xi P + q)^2 = \xi^2 m_p^2 + 2\xi P\cdot q - Q^2 = 0$$

$$\xi \approx \frac{Q^2}{2Pq} = x$$

DIS structure functions

Unpolarized

$$\mathrm{Im}\frac{i}{2\pi}\int d^4y e^{iqy}\langle PS|T\{J^\mu(y)J^\nu(0)\}|PS\rangle\Big|_{sym}$$

$$= \left(-\eta^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{P\cdot q}$$

Polarized

$$\mathrm{Im}\frac{1}{2\pi}\int d^4y e^{iqy}\langle PS|T\{J^\mu(y)J^\nu(0)\}|PS\rangle\Big|_{asym}$$

$$= -\epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha}{P\cdot q}\left[S_\beta g_1 + \left(S_\beta - \frac{S\cdot q}{P\cdot q}P_\beta\right) g_2\right]$$

Exercise

Forward Compton amplitude $q^0 > 0$

$$T^{\mu\nu} = \frac{i}{2\pi} \int d^4x e^{iq\cdot x} \langle PS | \mathrm{T}\{J^\mu(x) J^\nu(0)\} | PS \rangle = T_S^{\mu\nu} + i T_A^{\mu\nu}$$

Not to be confused with real/imaginary decomposition

symmetric antisymmetric

Hadronic tensor

$$W^{\mu\nu} = \frac{1}{2\pi} \int d^4x e^{iq\cdot x} \langle PS | [J^\mu(x), J^\nu(0)] | PS \rangle = W_S^{\mu\nu} + i W_A^{\mu\nu}$$

Show that

$$2\mathrm{Im} T_S^{\mu\nu} = W_S^{\mu\nu}$$

$$2\mathrm{Im} T_A^{\mu\nu} = W_A^{\mu\nu}$$

Light-cone dominance

Want to study the correlator $$\int d^4y e^{iqy}\langle P|T\{J^\mu(y)J^\nu(0)|P\rangle$$

in the Bjorken limit $Q^2 \to \infty, \quad P\cdot q \to \infty, \quad x = \frac{Q^2}{2P\cdot q} = const.$

Naively the integral is dominated by $|y^\mu| \sim \frac{1}{|q^\mu|} \to 0 \quad ?$

Proton rest frame (photon in the minus direction) $$x = \frac{(q^3-q^0)(q^3+q^0)}{2m_p q^0} \simeq \frac{q^3+q^0}{m_p}$$

$$y^+ \sim \frac{1}{q^-} \sim \frac{m_p x}{Q^2} \to 0, \qquad y^- \sim \frac{1}{q^+} \sim \frac{1}{m_p x} \qquad y^2 \sim \frac{1}{Q^2} \to 0$$

finite !

Operator product expansion

$$\int d^4y e^{iqy} \bar{\psi}\gamma^\mu \psi(y) \bar{\psi}\gamma^\nu \psi(0) = \bar{\psi} i (i\partial_\alpha + q_\alpha)\gamma^\mu\gamma^\alpha\gamma^\nu \frac{-1}{Q^2} \sum_n \left(\frac{2iq\cdot\partial}{Q^2} \right)^n \psi(0) + \cdots$$

$$+(\mu \to \nu, q \to -q)$$

c.f., Peskin (18.125)

Pick up the antisymmetric part

$$\gamma^\mu\gamma^\alpha\gamma^\nu = g^{\mu\alpha}\gamma^\nu - g^{\mu\nu}\gamma^\alpha + g^{\alpha\nu}\gamma^\mu + i\epsilon^{\mu\alpha\nu\rho}\gamma_\rho\gamma_5$$

$$\int d^4y e^{iqy} \bar{\psi}\gamma^\mu \psi(y) \bar{\psi}\gamma^\nu \psi(0) = 2\epsilon^{\mu\nu\lambda\alpha} q_\alpha \sum_n^{even} \frac{2q_{\mu_1}\cdots 2q_{\mu_n}}{Q^{2(n+1)}} \bar{\psi}\gamma_\lambda\gamma_5 i\partial^{\mu_1}\cdots i\partial^{\mu_n}\psi(0)$$

When $Q^2 \to \infty$, naively, the most important operators are those with smallest dimensions (smallest n)

Twist expansion

However, in the proton matrix element, $i\partial^\mu \to P^\mu$, and is not small in the Bjorken limit $Q^2 \to \infty,\ x = const.$

$$\frac{2P\cdot q}{Q^2} = \frac{1}{x}$$

The most important operators are those with lowest twist

(twist) = (dimension) – (spin)

↑

In practice, the number of plus Lorentz indices

$\bar{\psi}\gamma^+\psi$ twist-2 $\bar{\psi}\gamma_\perp\psi$ twist-3

Twist-2 polarized quark operators
(symmetrized in all Lorentz indices and trace subtracted)

$$\bar{\psi}\gamma_5\gamma^{(\lambda}iD^{\mu_1}iD^{\mu_2}\dots iD^{\mu_n)}\psi\ -(\mathrm{traces})$$

Totally symmetric in all indices → twist-2

$$\langle PS|\frac{1}{n+1}\bar{\psi}\Big(\gamma_\lambda\gamma_5 i\partial_{(\mu_1}\cdots i\partial_{\mu_n)} + \sum_{i=1}^{n}\gamma_{\mu_i}\gamma_5 i\partial_{(\mu_1}\cdots i\partial_\lambda\cdots i\partial_{\mu_n)}\Big)\psi|PS\rangle\frac{2q^{\mu_1}\cdots 2q^{\mu_n}}{Q^{2n}}$$

$$\equiv \frac{a_n}{n+1}\Big(S_\lambda P_{\mu_1}\cdots P_{\mu_n} + \sum_{i=1}^{n} S_{\mu_i}P_{\mu_1}\cdots P_\lambda\cdots P_{\mu_n}\Big)\frac{2q^{\mu_1}\cdots 2q^{\mu_n}}{Q^{2n}}$$

$$= \frac{a_n}{n+1}\frac{1}{x^n}\left(S_\lambda + n\frac{S\cdot q}{P\cdot q}P_\lambda\right)$$

→ longitudinal/transverse polarization $g_1(x), g_2(x)$

Anti-symmetric in λ and $\mu_1, \mu_2, \cdots$ → One twist higher (twist-3)

$$\langle PS|\frac{1}{n+1}\sum_{i=1}^{n}\bar{\psi}\Big(\gamma_\lambda\gamma_5 i\partial_{(\mu_1}\cdots i\partial_{\mu_n)} - \gamma_{\mu_i}\gamma_5 i\partial_{(\mu_1}\cdots i\partial_\lambda\cdots i\partial_{\mu_n)}\Big)\psi|PS\rangle\frac{2q^{\mu_1}\cdots 2q^{\mu_n}}{Q^{2n}}$$

$$\equiv \frac{d_n}{n+1}\sum_{i=1}^{n}\Big(S_\lambda P_{\mu_1}\cdots P_{\mu_n} - S_{\mu_i}P_{\mu_1}\cdots P_\lambda\cdots P_{\mu_n}\Big)\frac{2q^{\mu_1}\cdots 2q^{\mu_n}}{Q^{2n}}$$

$$= \frac{nd_n}{n+1}\frac{1}{x^n}\left(S_\lambda - \frac{S\cdot q}{P\cdot q}P_\lambda\right)$$

→ transverse polarization $g_2(x)$

g_1 structure function

$$g_1(x) = \frac{1}{2\pi}\mathrm{Im}\sum_{n=0}^{even}\frac{a_n}{x^{n+1}} = \frac{1}{2\pi S^+}\mathrm{Im}\sum_{n=0}^{even}\frac{1}{(P^+)^n x^{n+1}}\langle PS|\bar{\psi}\gamma_5\gamma^+(iD^+)^n\psi|PS\rangle + \cdots$$

Convergent only when $|x| > 1$!

All terms are real!

$$= \frac{1}{2\pi S^+}\mathrm{Im}\sum_{n=0}^{even}\frac{1}{x^{n+1}}\int\frac{dk^+}{2\pi}\left(\frac{k^+}{P^+}\right)^n\int dx^- e^{ik^+x^-}\langle PS\,\bar{\psi}(0)\gamma^+\gamma_5 W[0,x^-]\psi(x^-)|PS\rangle$$

Wilson line

Exercise: show this

$$= \frac{P^+}{4\pi S^+}\mathrm{Im}\int\frac{dk^+}{2\pi}\left(\frac{1}{xP^+ + k^+} + \frac{1}{xP^+ - k^+}\right)\int dx^- e^{ik^+x^-}\langle PS|\,\bar{\psi}(0)\gamma^+\gamma_5 W[0,x^-]\psi(x^-)|PS\rangle$$

Analytic continuation from

$|x| > 1$ to $1 > x > 0$ $\quad x \to x - i\epsilon$

(cf. $s \to s + i\epsilon$)

$$= \frac{P^+}{8\pi S^+} \int dx^- e^{ixP^+x^-} \langle PS|\bar{\psi}(0)\gamma_5\gamma^+ W[0,x^-]\psi(x^-)|PS\rangle + (x \to -x)$$

$$= \frac{1}{2}(\Delta q(x) + \Delta \bar{q}(x))$$

Polarized quark and antiquark distributions

Note the sign difference

$$q(-x) = -\bar{q}(x) \qquad \Delta q(-x) = \Delta \bar{q}(x)$$

unpolarized quark PDF — polarized quark PDF

Exercise: Show that for n even, $\int_0^1 dx x^n g_1(x) = \frac{a_n}{4}$

$g_2(x)$ structure function

Similarly,

$$g_2(x) = \frac{1}{2\pi}\mathrm{Im}\sum_n \frac{n(d_n - a_n)}{n+1}\frac{1}{x^{n+1}} \qquad \int_0^1 dx x^n g_2(x) = \frac{n(d_n - a_n)}{4(n+1)}$$

Invert these relations and get

$$g_2(x) = \underline{-g_1(x) + \int_x^1 \frac{dz}{x} g_1(z)} + \underline{\bar{g}_2(x)}$$

Wandzura, Wilczek (1977)

Wandzura-Wilczek part
related to twist-2 PDF

\`genuine twist-3' part
$q\bar{q}g$ correlation functions

$$\int_0^1 dx x^2 \bar{g}_2(x) = \frac{d_2}{6} \qquad \langle PS|\bar{\psi}\gamma^+ gF^{+i}\psi|PS\rangle = 2d_2(P^+)^2\epsilon^{ij}S_j$$

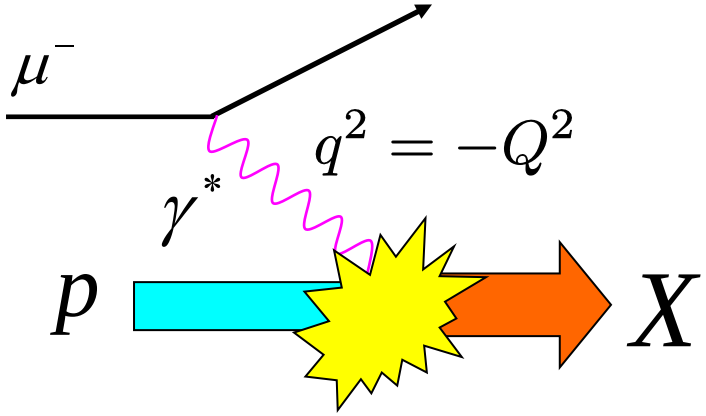
Shuryak, Vainshtein (1982)

$\Delta\Sigma$ from polarized DIS

Longitudinal double spin asymmetry in polarized DIS

$$A_{LL} = \frac{\mu^{\uparrow}p^{\downarrow} - \mu^{\uparrow}p^{\uparrow}}{\mu^{\uparrow}p^{\uparrow} + \mu^{\uparrow}p^{\downarrow}}$$

$$\sim \left(1 + \frac{\sigma_L}{\sigma_T}\right)\frac{2xg_1}{F_2}$$



$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{2}\sum_f e_f^2 \int_0^1 dx(\Delta q_f(x, Q^2) + \Delta\bar{q}_f(x, Q^2)) + \cdots$$

Flavor SU(3) decomposition

$$\sum_f e_f^2 = \begin{pmatrix} \frac{4}{9} & & \\ & \frac{1}{9} & \\ & & \frac{1}{9} \end{pmatrix} = \frac{2}{9} + \frac{1}{6}\begin{pmatrix} 1 & & \\ & -1 & \\ & & 0 \end{pmatrix} + \frac{1}{18}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix}$$

$$\Delta\Sigma$$

$$\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)$$

nucleon isovector axial charge

$$\langle p|\bar{q}\gamma^\mu\gamma_5 t^3 q|p\rangle \sim g_A^{(3)}$$

→ from neutron beta decay

$$n(udd) \to p(uud) + e^- + \bar{\nu}_e$$

$$\langle p|\bar{u}\gamma^\mu\gamma_5 d|n\rangle \sim g_A^{(3)}$$

octet axial charge $\langle p|\bar{q}\gamma^\mu\gamma_5 t^8 q|p\rangle \sim g_A^{(8)}$

→ from hyperon semileptonic decay

$$\Xi^-(dss) \to \Lambda(uds) + e^- + \bar{\nu}_e$$

$$\langle \Lambda|\bar{u}\gamma^\mu\gamma_5 s|\Xi^-\rangle \sim 3F - D = g_A^{(8)}$$

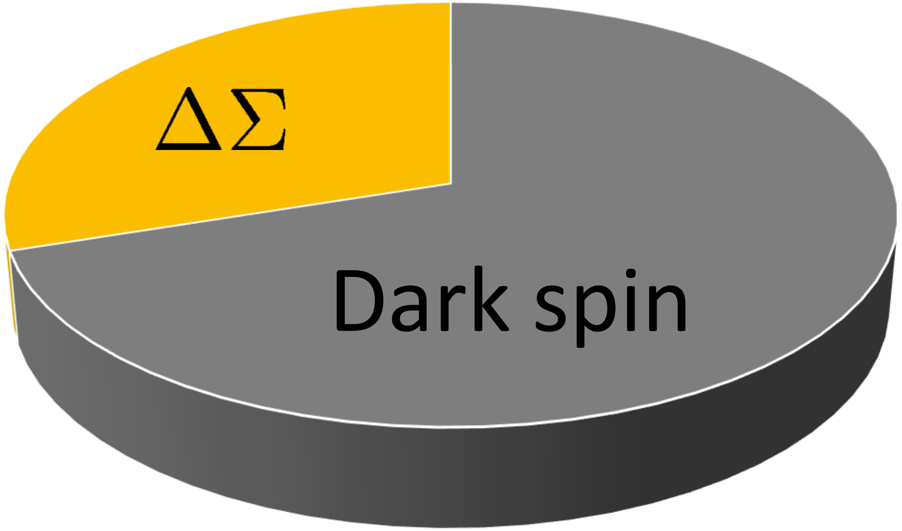
`Spin crisis'

In 1987, EMC (European Muon Collaboration) announced a very small value of the quark helicity contribution

$$\Delta\Sigma = 0.12 \pm 0.09 \pm 0.14 \text{ !?}$$

Recent value from NLO QCD global analysis

$$\Delta\Sigma = 0.25 \sim 0.3$$



Gluon polarization $\Delta G = \int_0^1 dx \Delta G(x)$

Polarized gluon distribution

$$\Delta G(x) = \frac{i}{xS^+} \int \frac{dy^-}{2\pi} e^{ixP^+y^-} \langle PS| F^{+\alpha}(0) \tilde{F}^+_{\alpha}(y^-) |PS\rangle$$

$$\epsilon^{\mu}_{R/L} = \frac{-1}{\sqrt{2}}(0, \pm 1, i, 0)$$

$$iF^{+i}\tilde{F}^+_i = (F^{+R})^\dagger F^{+R} - (F^{+L})^\dagger F^{+L}$$

Non-local, even after taking a moment.

$$\int_0^1 dx \Delta G(x) = -\frac{1}{2S^+} \int dy^- \theta(y^-) \langle PS| F^{+\alpha}(0) \tilde{F}^+_{\alpha}(y^-) |PS\rangle$$

Depends on the prescription of the pole $1/x$.
The value of ΔG independent of the prescription.

In the light-cone gauge $A^+ = 0$, it reduces to the local operator in the Jaffe-Manohar decomposition.

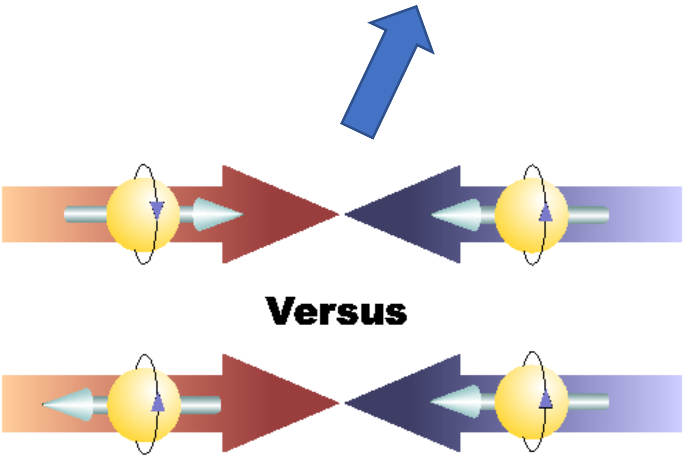
Determination of ΔG at RHIC

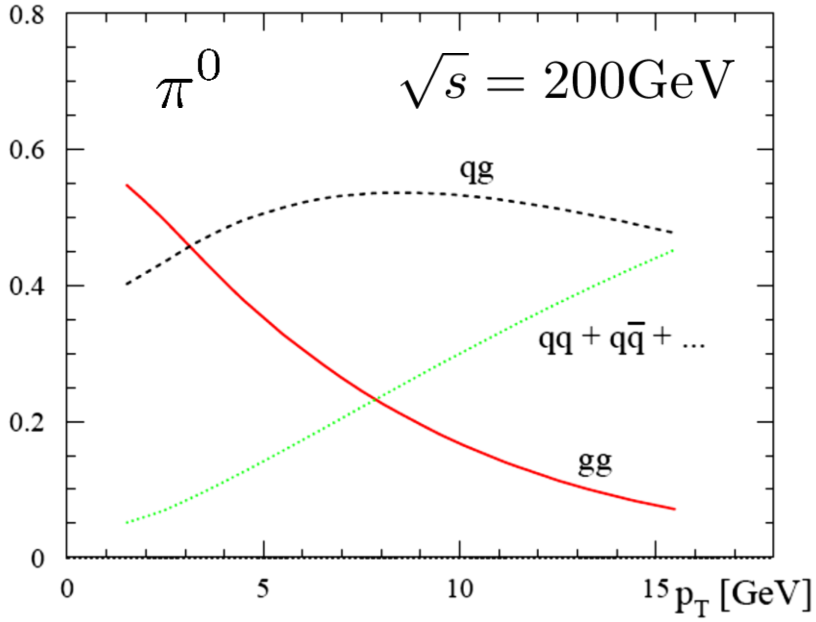
Double spin asymmetry of pions and photons in polarized pp.

$$A_{LL} = \frac{d\sigma^{++} - d\sigma^{+-}}{d\sigma^{++} + d\sigma^{+-}}$$

$$\propto \sum_{a,b} \Delta f_a \otimes \Delta f_b(x) \otimes \Delta\sigma_{ab}$$

pion, photon





Direct Photons Point to Positive Gluon Polarization

Results from 'golden measurement' at RHIC's PHENIX experiment show the spins of gluons align with the spin of the proton they're in

June 21, 2023

ınalysis of data from the PHENIX detector at the Relativistic Heavy Ion
· (RHIC) gives fresh insight into how gluons contribute to proton spin.

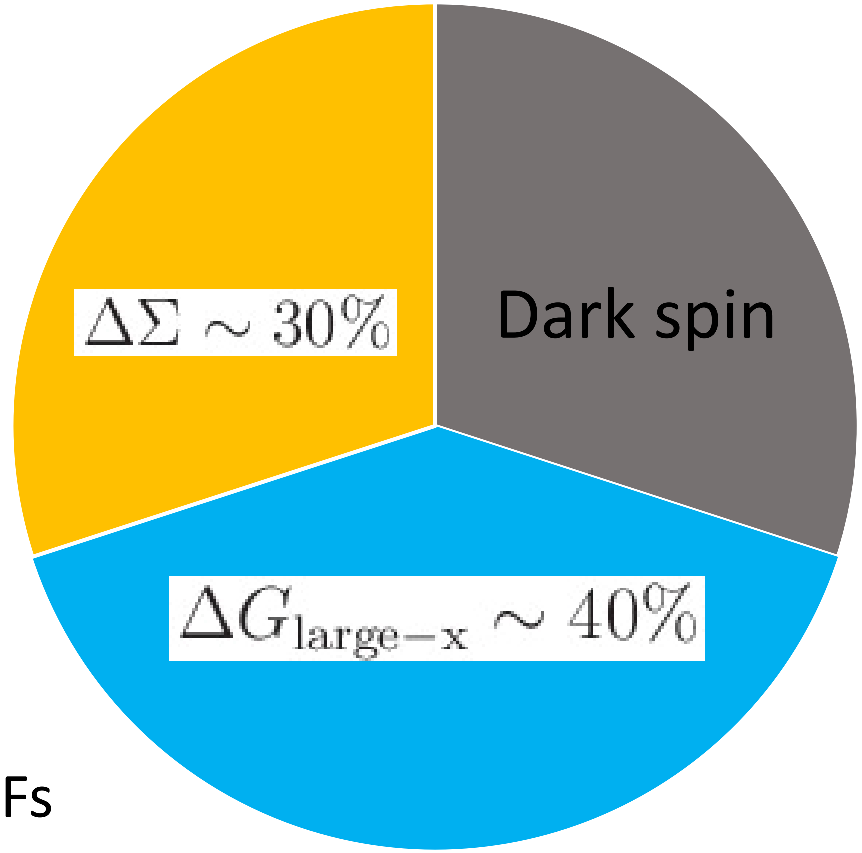
Evidence of nonzero gluon helicity $\Delta G = \int_0^1 dx \Delta G(x)$

A major achievement of the RHIC spin program!

$$\int_{0.05}^{1} dx \Delta G(x, Q^2 = 10\text{GeV}^2) = 0.20^{+.06}_{-.07}$$ DSSV

$$\int_{0.05}^{0.2} dx \Delta G(x, Q^2 = 10\text{GeV}^2) = 0.17 \pm 0.06$$ NNPDF

$$\int_{0.05}^{1} dx \Delta G(x, Q^2 = 10\text{GeV}^2) = 0.23 \pm 0.03$$ JAM



Huge uncertainty from the small-x region → EIC

Renewed interest in the small-x resummation of helicity PDFs

Kovchegov et al. 2015~

NNLO global analysis became available last year. Borsa et al. 2407.11635