Jets in Heavy Ion collisions

Lecture 2

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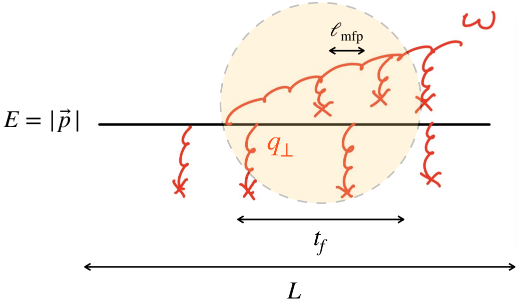
Elementary processes: medium-induced gluon radiation

- Multiple scattering can coherently induce radiation

$$\ell_{\mathrm{mfp}} \quad \ll \quad t_f(\omega) \quad \lesssim L$$

- t_f is the quantum mechanical gluon formation time

$$t_f \sim \frac{\omega}{k_\perp^2} \sim \frac{\omega}{\hat{q}\, t_f} \sim \sqrt{\frac{\omega}{\hat{q}}}$$



Elementary processes: medium-induced gluon radiation

- Characteristic scales in the Landau-Pomerantchuk-Migdal effect: Suppression of radiation due to coherent scattering: multiple scattering centers act coherently as a single scattering.
- Maximum suppression achieved when $t_f \sim L$ corresponding to the characteristic frequency

$$\omega_c = \hat{q}L^2$$

- Other characteristic scales, Transverse momentum and radiation angle:

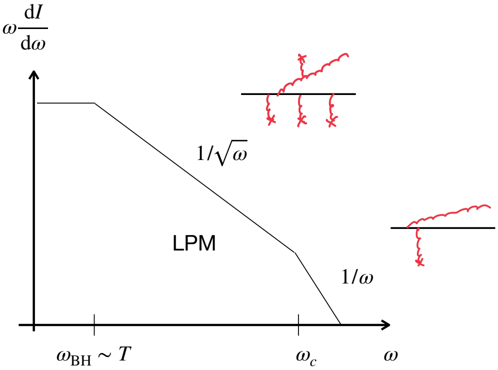
$$k_f^2 = \sqrt{\hat{q}\,\omega} < \hat{q}L \qquad \text{and} \qquad \theta_f = \left(\frac{\hat{q}}{\omega}\right)^{1/4} > \theta_c = \frac{1}{\sqrt{\hat{q}L^3}}$$

- Ex: $\hat{q} = 1\ \mathrm{GeV}^2/\mathrm{fm}$, $L = 5$ fm, $\omega_c = 125$ GeV.

Elementary processes: medium-induced gluon radiation

Three distinct regimes for medium induced gluon radiation:

1. Hard single scattering regime: $\omega > \omega_c$ and $t_f > L$ (long formation time)
2. Multiple soft scattering: $T < \omega < \omega_c$ and $\ell_{mfp} < t_f < L$ (long formation time)
3. Bethe-Heitler regime: $\omega \sim T$



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Energy loss distribution

- At leading order the jet spectrum in the medium can be written as

$$\frac{d\sigma_{med}}{dp_T} = \int_0^{\infty} d\epsilon P(\epsilon)\ \delta(E - p_T - \epsilon)\ \frac{d\sigma_{vac}}{dp_T}$$

- where the jet cross-section in vacuum is a steep power spectrum with $n \gg 1$ (typically $n \sim 5 - 6$)

$$\frac{d\sigma_{vac}}{dp_T} = \frac{1}{p_T^n}$$

- $P(\epsilon)$ is the probability for a parent parton of energy E loses ϵ of its energy to the QGP

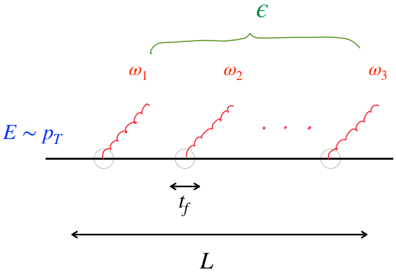
Poisson distribution

- In the soft radiation regime: $t_f \ll L$ (leading power in L), multiple emissions are frequent and uncorrelated $\to$ Poisson distribution – quasi-instantaneous radiation

Length enhancement of the rad. spect.

$$\omega \frac{dI}{d\omega} = \bar{\alpha}_s \sqrt{\frac{\omega_c}{\omega}} \propto L$$

Require resummation of all orders in $\bar{\alpha}_s$



$$\boxed{P(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} e^{-\langle n \rangle} \prod_{i=1}^{n} \frac{dI}{d\omega_i} \delta(\epsilon - \omega_1 - \omega_2 - \ldots - \omega_n)}$$

- The hard regime, $\omega \sim \omega_c$, treated order by order in $\bar{\alpha}_s$ (no length enhancement)

Energy loss distribution

- Multiple emission factorize and exponentiate in Laplace space

$$\tilde{P}(\nu) = \int d\epsilon P(\epsilon) e^{-\nu\epsilon} = \exp\left[-\int_0^\infty d\omega \frac{dI}{d\omega}\left(1 - e^{-\nu\omega}\right)\right]$$

- Using the standard integral

$$\int_0^\infty \frac{dx}{x^{1/2}}(1 - e^{-x}) = \Gamma(-1/2) = \sqrt{\pi}$$

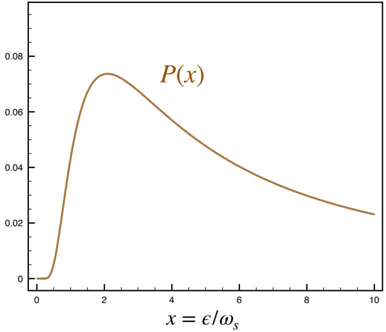
- which yields

$$\tilde{P}(\nu) = e^{-\sqrt{\pi\bar{\alpha}_s^2\omega_c\nu}} \quad \to \quad P(\epsilon) = \sqrt{\frac{\bar{\alpha}_s^2\omega_c}{\epsilon^3}} e^{-\frac{\pi\bar{\alpha}_s^2\omega_c}{\epsilon}}$$

Poisson distribution

- $P(\epsilon)$ heavy tailed distribution: mean energy loss sensitive to the hard sector $\epsilon \sim p_T$ $(x = \bar{\alpha}_s^2 \omega_c / p_T \gg 1,$ $\omega_c = \hat{q} L^2)$

$$\langle \epsilon \rangle \simeq \bar{\alpha}_s \omega_c \ln(p_T/\omega_c) \gg \bar{\alpha}_s^2 \omega_c$$



Poisson distribution

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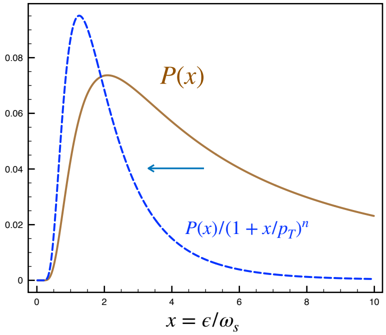
$$\langle\epsilon\rangle \simeq \bar{\alpha}_s\omega_c \ln(p_T/\omega_c) \gg \bar{\alpha}_s^2\omega_c$$

- However, when multiplied by the initial jet spectrum $1/(p_T+\epsilon)^n$ the distribution is skewed towards the soft sector:

$$\epsilon < \frac{p_T}{n} \ll p_T$$

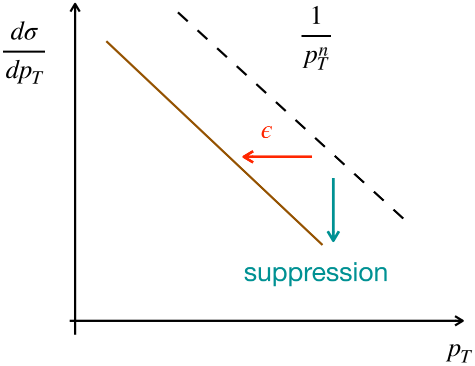
- Figure: $n = 5,\ p_T = 5\omega_s = 5\bar{\alpha}_s^2\omega_c$.



Nuclear modification factor

- The leading order jet spectrum

$$\frac{d\sigma_{med}}{dp_T} = \int_0^\infty d\epsilon P(\epsilon)\ \delta(E - p_T - \epsilon)\ \frac{d\sigma_{vac}}{dp_T}$$



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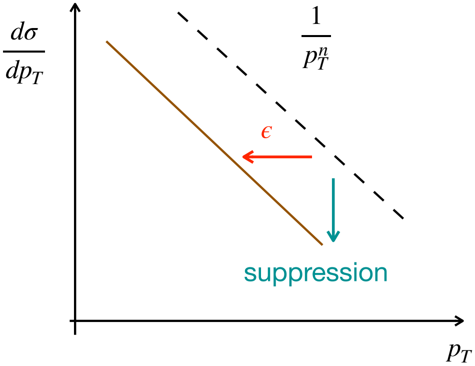
$$\frac{d\sigma_{med}}{dp_T} = \int_0^\infty d\epsilon P(\epsilon) \; \frac{1}{(p_T+\epsilon)^n}$$



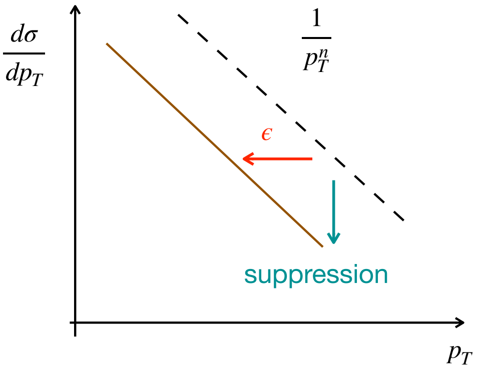
Nuclear modification factor

- The leading order jet spectrum

$$\frac{d\sigma_{med}}{dp_T} = \int_0^\infty d\epsilon P(\epsilon) \; \frac{1}{(p_T+\epsilon)^n}$$

- The Nuclear Modification factor reads

$$R_{AA} = \frac{d\sigma_{med}/dp_T}{d\sigma_{vac}/dp_T}$$



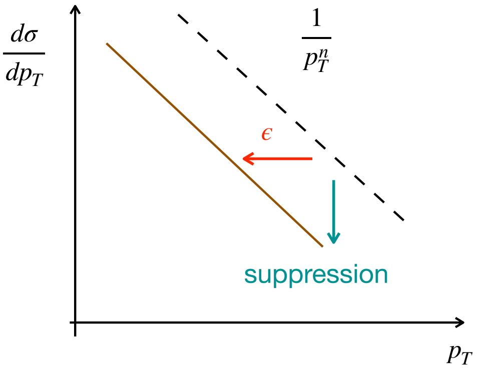
Nuclear modification factor

- The leading order jet spectrum

$$\frac{d\sigma_{med}}{dp_T} = \int_0^\infty d\epsilon P(\epsilon) \, \frac{1}{(p_T + \epsilon)^n}$$

- The Nuclear Modification factor reads

$$R_{AA} = \int_0^\infty d\epsilon P(\epsilon) \, \frac{1}{(1 + \epsilon/p_T)^n}$$



Nuclear modification factor

- The leading order jet spectrum

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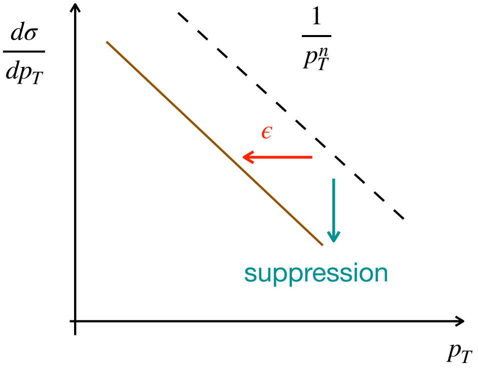
$$R_{AA} = \int_0^\infty d\epsilon P(\epsilon)\ \frac{1}{(1 + \epsilon/p_T)^n}$$

- for large $n \gg 1$ we can approximate

$$\left(1 + \frac{\epsilon}{p_T}\right)^n = e^{-\frac{n\epsilon}{p_T}} + \mathcal{O}((\epsilon/p_T)^2)$$



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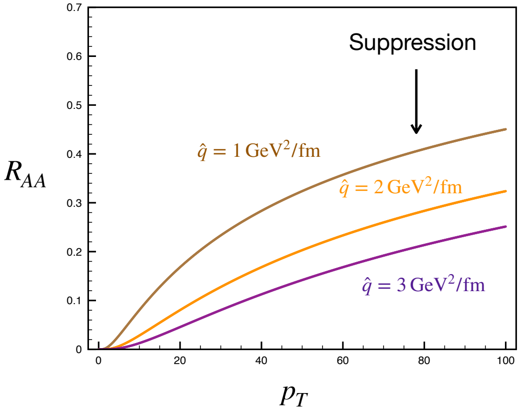
$$R_{AA} = \int_0^\infty d\epsilon P(\epsilon) \; \frac{1}{(1 + \epsilon/p_T)^n}$$

- Connecting with Laplace transform:

$$\boxed{R_{AA} \simeq \tilde{P}(\nu = n/p_T) = \exp\left(-\sqrt{\frac{\pi\, n\, \bar{\alpha}_s^2 \omega_c}{p_T}}\right)}$$



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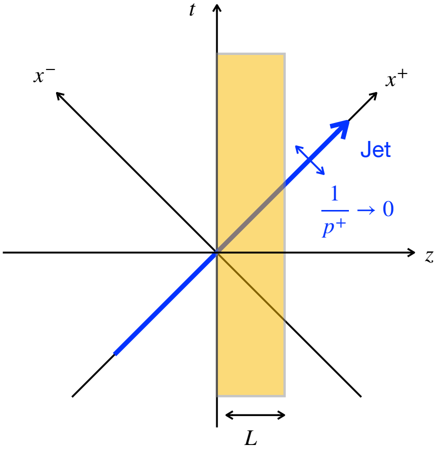
Eikonal interaction with A^{μ}_{bkg}

- High energy approximation $E \gg k_{\perp} \sim T$
- Light-cone variables: $p^{+} = \frac{1}{2}(E + p_z), \qquad p^{-} = E - p_z, \qquad p_{\perp}$



Eikonal interaction with A^μ_{bkg}

- Eikonal interaction of the collinear jet particles with the background field (QGP). Performing a multipole expansion as $p^+ \to \infty$:

$$g \int_0^{+\infty} dk^+ \bar{u}(-p) \not{A}_{bkg}(k) u(p-k) \simeq g \int_0^{+\infty} dk^+ p_\mu A^\mu_{bkg}(k) \Big|_{k^+=0} + \mathcal{O}(k^+/p^+)\,,$$

enables us to apply the integral over k^+ solely on the gauge field leading to

$$\int_0^{+\infty} dk^+ A^\mu_{bkg}(k) = A^\mu_{bkg}(x^- = 0)\,.$$

Eikonal interaction with A^{μ}_{bkg}

- Semi-Eikonal Dirac propagator: (i) neglect $powersofk^+/p^+$ and spin flip in the interaction vertex (ii) keep track of the quantum-phase : $p_\perp^2/p^+L \sim 1$

$$\frac{1}{p^2 + i0p^+} \sim \frac{1}{p^+}\int_0^{+\infty} dx^+ e^{-i\frac{p_\perp^2}{2p^+}x^+}$$

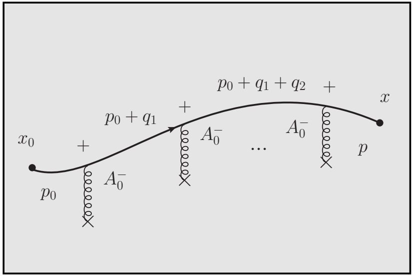
- The non-eikonal phase can be neglected for the jet but is responsible for the LPM suppression of gluon radiation for $k^+ \equiv \omega \ll \omega_c$

$$\frac{p_\perp^2}{p^+}L \sim \frac{\hat{q}L}{p^+}L = \frac{\omega_c}{k^+} \gg 1$$

Scalar propagator and 2+1D dynamcis

- Dirac propagator proportional to scalar propagator in the presence of $A^-_{bkg}(x^+, x_\perp)$

$$D(p, p_0) = (2\pi)^4\delta^{(4)}(p - p_0)D_0(p) + \frac{\not{p}\gamma^+\not{p}_0}{2p^+}\left[G_{scal}(p, p_0) - G^0_{scal}(p)\delta(p - p_0)\right],$$



Scalar propagator and 2+1D dynamics

- The dynamics id that of a non-relativistic 2+1D quantum system

$$(\boldsymbol{x}|\mathcal{G}(t,t')|\boldsymbol{x}') = \frac{i}{2E}\int \frac{dx^-}{2\pi}\mathrm{e}^{-iE(x-x')^-}\, G_{scal}(x,x'),$$

- where the propagator obeys the Schrödinger equation ($t = x^+$)

$$\left[i\frac{\partial}{\partial t} + \frac{\partial_\perp^2}{2E} + gA(t,\boldsymbol{x})\right](\boldsymbol{x}|\mathcal{G}(t-t')|\boldsymbol{x}') = i\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}'),$$

- $A^- \equiv A^{a,-}t^a_{ij}$ and $\mathcal{G}_{ij}(t-t')$ are color matrices in the fundamental representation.

Medium statistics

- Observables are computed for each fixed medium configuration and subsequently averaged over an ensemble [McLerran-Venugopalan model (1994)])

$$\langle med | O[A_{bkg}] | med \rangle$$

- Independent multiple scattering approximation yields Gaussian statistics (at leading order)

$$\langle med | A_a^-(x^+, \boldsymbol{q}) A_b^-(y^+, \boldsymbol{q}') | med \rangle = \delta_{ab} \delta(x^+ - y^+) \delta(\boldsymbol{q} - \boldsymbol{q}') \; \rho \frac{d\sigma_{el}}{d\boldsymbol{q}}$$

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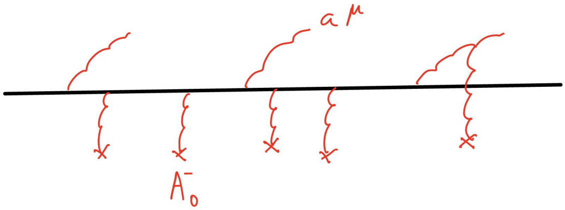
Operator definition of energy loss

- We are now equipped to provide a field theoretical definition for the energy loss probability distribution
- Soft interactions are encoded in semi infinite Wilson-lines

$$U(n) \equiv P \exp\left[ig\int_0^\infty \mathrm{d}s\, n\cdot A(ns)\right]$$

  where $\bar{n} \sim p^\mu/E \equiv (1,0,0,1)$
- The gauge field $A^\mu = A^\mu_{bkg} + a^\mu$ describes both radiative and elastic processes

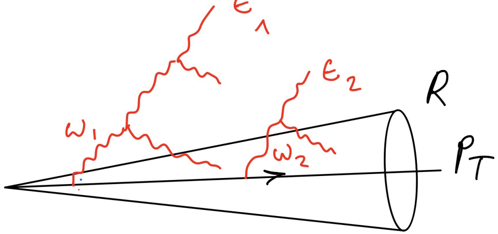


Operator definition of energy loss

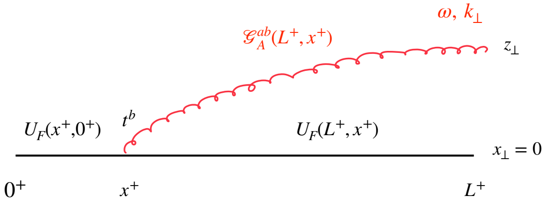
- The single parton energy loss probability distribution is defined as [YMT., Ringer, Singh, Vaidya, 2409.05957 [hep-ph]]

$$P(\epsilon) = \frac{1}{d_R} \sum \delta(\epsilon - \bar{n} \cdot k_{\text{loss}})\, \text{tr}_c \left[\langle \text{med} | U(n) | X \rangle \langle X | U^\dagger(n) | \text{med} \rangle \right],$$

- Energy loss k_{loss} Measured on final state X (Includes the jet algorithm). $U(n)$ and $U^\dagger(n)$ are Wilson-lines in the amplitude and c.c.

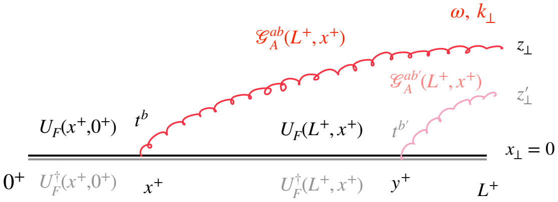


Amplitude



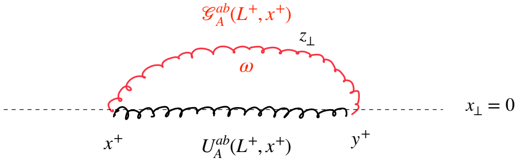
- The amplitude involves quark Wilson-lines at $x_\perp = 0$ and non-eikonal gluon propagator $\mathcal{G}$.

Amplitude squared



- The amplitude involves quark Wilson-lines at $x_\perp = 0$ and non-eikonal gluon propagator $\mathcal{G}$. N.B.: interactions with the background field are implicit.

Amplitude squared



- Thy color singlet in the intervals $[0^+, x^+]$ and $[y^+, L^+]$ do not contribute $UU^\dagger = 1$. In the interval $[x^+, y^+]$, we have a color octet state $U_F t^a U_F^\dagger = U_A^{ba} t^a$ (Fierz).
- Upon integrating over $k_\perp$, the gluon propagators after y^+ cancel out: gluon radiation rate is determined only by the dynamics during the interval $\Delta x^+ = y^+ - x^+$.

Radiative spectrum and medium averaging

- We are left with the evaluation of the expectation value of the Green's function

$$\mathcal{K}(\boldsymbol{z}_2, y^+, \boldsymbol{z}_1, x^+) = \frac{1}{N_c^2-1} \langle med | \mathrm{Tr}_c \left[\mathcal{U}^\dagger_{0_\perp}(y^+, x^+) (\boldsymbol{z}_2 | \mathcal{G}^\dagger(y^+, x^+) | \boldsymbol{z}_1) \right] | med \rangle \,.$$

[Wiedemann (2000) Blaizot, Dominguez, Iancu, MT (2013)]

- that obeys the Schödinger equation

$$\left[i \frac{\partial}{\partial x^+} + \frac{\boldsymbol{\partial}_{\boldsymbol{x}}^2}{2\omega} + i \frac{N_c \rho}{2} \sigma(\boldsymbol{x}) \right] \mathcal{K}(\boldsymbol{x}, x^+; \boldsymbol{y}, y^+) = i \delta^{(2)}(\boldsymbol{x} - \boldsymbol{y}) \delta(x^+ - y^+) \,, \qquad (1)$$

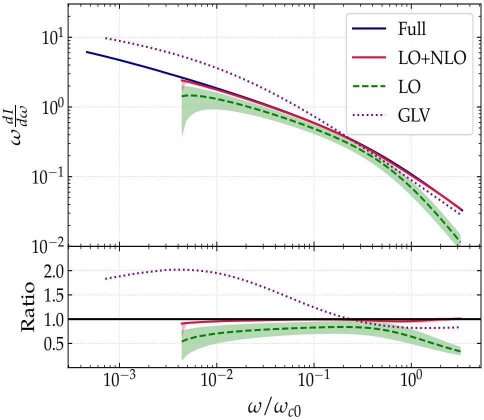
- with the imaginary potential (stochastic collisions)

$$\sigma(\boldsymbol{x}) \sim g^4 \rho \int \frac{d^2 q_\perp}{q_\perp^4} \left(1 - e^{-i \boldsymbol{x} \cdot \boldsymbol{q}} \right) \approx g^4 T^3 \boldsymbol{x}^2 \ln \frac{1}{\boldsymbol{x}^2 m_D^2} \sim \hat{q}\, \boldsymbol{x}^2$$

- Solutions: (1) order by order in opacity (powers of the density ρ) (2) to all orders in opacity in the Harmonic-oscillator approximation $\sigma \sim \hat{q}\, \boldsymbol{x}^2$.

Analytic solutions vs. numerics

- Full: exact numerical solutions [Andres, Dominguez and Gonzalez Martinez (2021)]
- LO: Harmonic oscillator approximation [Baier et al (1996) Zakharov, Wiedemann (2001)]
- NLO: Includes first Coulomb log corrections [MT (2019) Barata, MT, Soto-Ontoso, Tywoniuk (2021)]
- GLV: leading order in opacity [Gyulassy, Levai, Vitev (2001)] .



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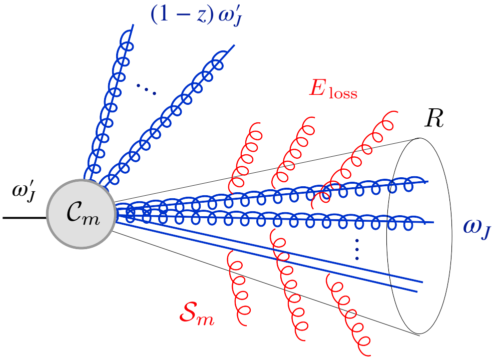
Re-factorization of the jet function

- DGLAP-like evolution (hard collinear modes $Q \sim p_T R$)
- Soft radiation encoded in Wilson-line correlators ($Q \sim Q_{med}$)



Re-factorization of the jet function

$$\boxed{J(z,E) = \int dz' \int d\epsilon\, \delta(\omega_J - \omega'_J - \epsilon) \sum_m C_m(\{n_i\}, z', \omega'_J, \mu) \otimes S_m(\{n_i\}, \epsilon, \mu)}$$

where C_m's are matching coefficients (hard-collinear modes at the scale $Q = p_T R$) and S_m's are Wilson line correlators that encode soft interactions with the medium at the scale Q_{med}

$$S(\{n_i\}, \epsilon) \equiv \sum_X \Theta_{\text{alg}}\, \delta\left(\epsilon - \sum \bar{n} \cdot p_{\text{loss}}\right) \langle \text{med}| U_m^\dagger \cdots U_1^\dagger \bar{U}_0^\dagger |X\rangle\langle X| \bar{U}_0 U_1 \cdots U_m |\text{med}\rangle$$

[MT, Ringer, Singh, Vaidya, (2024)]