Jets in Heavy Ion collisions

Lecture 1

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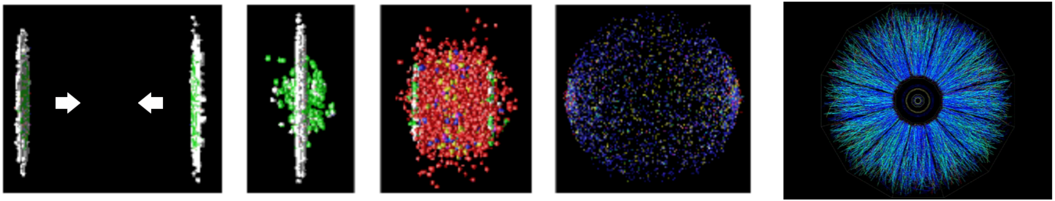
The little bang: creation of the quark-gluon-plasma (QGP)

- Discovery of the QGP – hot deconfined QCD matter– at RHIC (early 2000's)
- (i) Bulk collective elliptic flow (ii) High-pt adron suppression



Ultrarelativistic nuclei (Au or Pb) → Nuclei collide → QGP formation → System expands and cools → Detection

The little bang: creation of the quark-gluon-plasma (QGP)

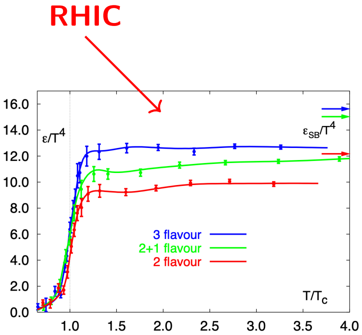
- QCD EOS from Lattice simulations:

- Phase transition $T_c \sim 170$ GeV:

$$\epsilon_c \sim 0.5\,\mathrm{GeV/fm^3}$$

- Energy densities at RHIC energies:

$$\epsilon_{\mathrm{RHIC}} \sim 1 - 5\,\mathrm{GeV/fm^3}$$

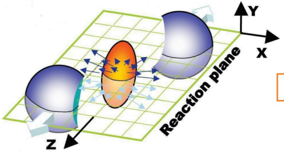


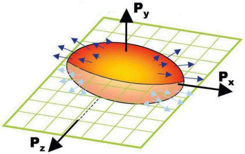
The QCD energy density
[Figure from Karch, NPA (2001)]

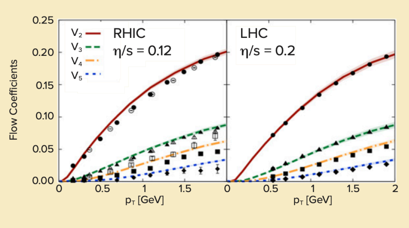
The QGP: a "perfect" liquid

- Collective behavior: Spacial $\rightarrow$ Momentum anisotropy
- Low shear viscosity to entropy ratio (from Viscous hydrodynamic) – near the Gauge/Gravity Duality: $\eta/s = 1/4\pi$ [Kovtun, Son, Starinet (2001)] .
- Flow harmonics:)

$$v_n = \left\langle \cos\left(n(\phi - \Psi_n)\right)\right\rangle$$

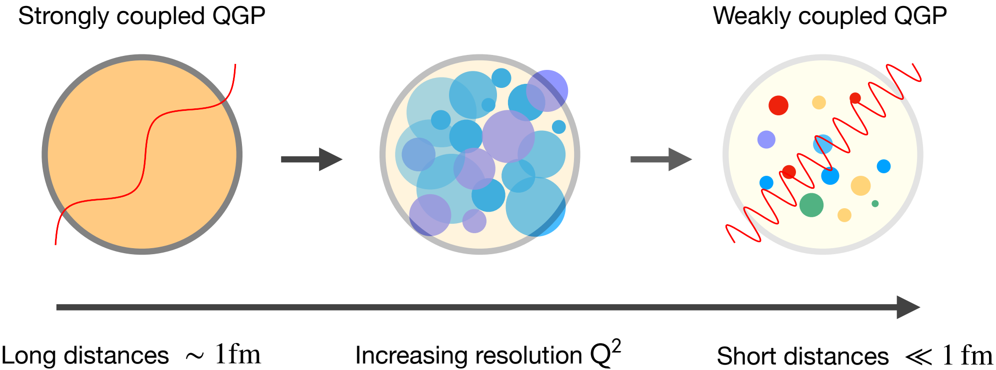






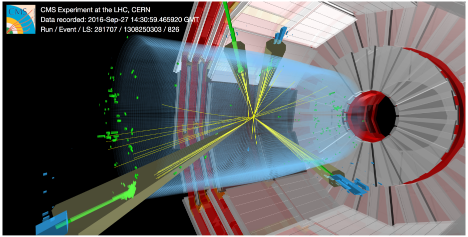
[2015 Long Range Plan for Nuclear Science]

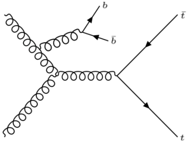
How does this behavior emerge from QCD?



QCD jets in proton-proton

- QCD Jets are the direct manifestation of high energy quarks and gluons and are well understood from first principles. Paramount for Higgs discovery and BSM searches.



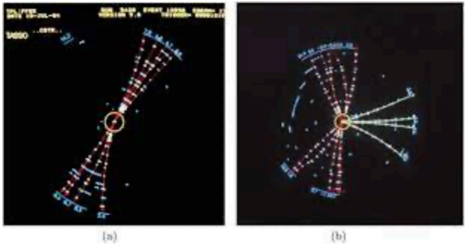


- Multijet event with transverse momentum of order 1 TeV each, produced in proton-proton collisions at a centre-of-mass energy of 13 TeV at the LHC.

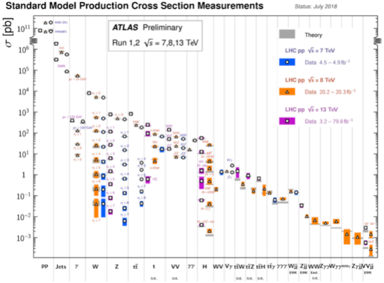
QCD jets in proton-proton

- Extensively studied: from the discovery of the gluon to precision tests of QCD

DESY 1979 - electron positron collisions



Early tests of QCD

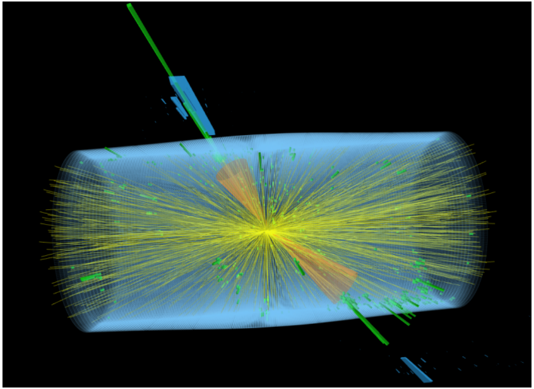


Today's theory accuracy $\gtrsim 5\,\%$

- Paramount for Higgs discovery and new physics searches

QCD jets in Heavy Ion Collisions

- In Heavy Ion Collisions QCD Jets are embedded in a background of 1000s of soft particles



Dijet event in Pb-Pb collisions at 5.02 TeV

Evidence of the QGP from jet quenching

- Bjorken (1982) predicted the phenomenon of jet quenching in high energy hadronic collisions as a consequence of elastic energy loss in the quark-gluon plasma

Energy Loss of Energetic Partons in Quark-Gluon Plasma:
Possible Extinction of High p_T Jets in Hadron-Hadron Collisions.

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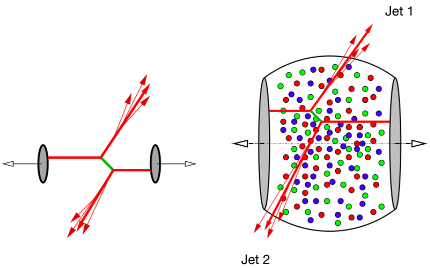
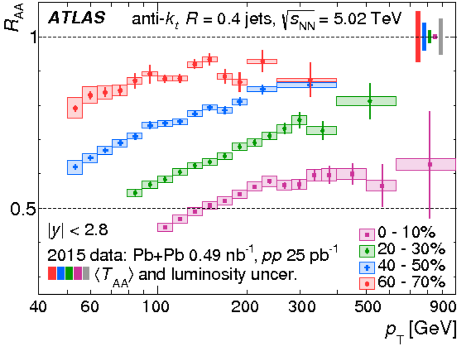
Abstract

High energy quarks and gluons propagating through quark-gluon plasma suffer differential energy loss via elastic scattering from quanta in the plasma. This mechanism is very similar in structure to ionization loss of charged particles in ordinary matter. The dE/dx is roughly proportional to the square of the plasma temperature. For

Evidence of the QGP from jet quenching

- Substantial final state interactions: jets lose energy to the QGP constituents
- Strong suppression and modification of jets observed at RHIC and LHC

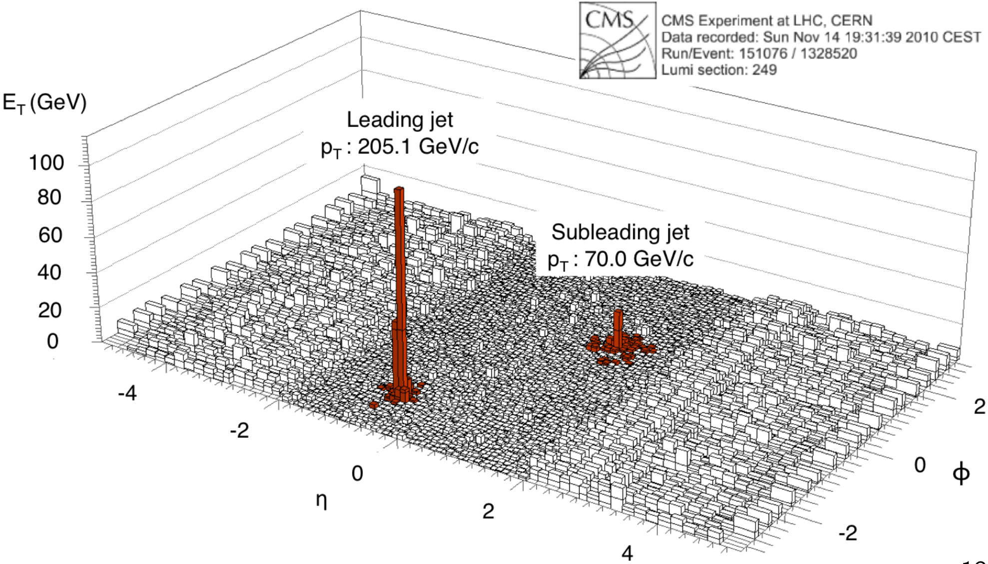




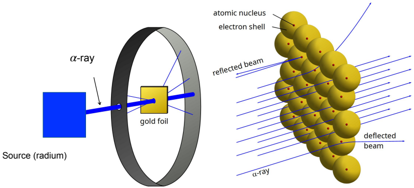
Nuclear modification factor

$$R_{AA} \equiv \frac{dN_{AA}/dp_T}{N_{coll}\, dN_{pp}/dp_T}$$

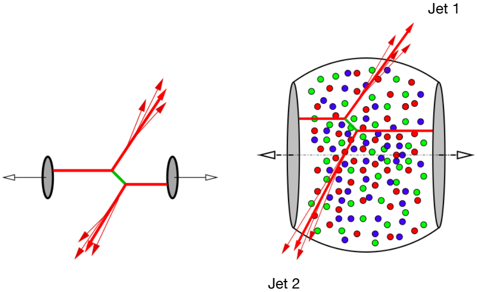
Evidence of the QGP from jet quenching



A Rutherford-like experiment



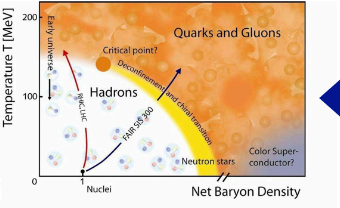
Discovery of the atomic nucleus



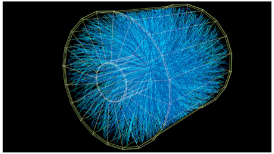
Probing the microscopic properties of the QGP with jets

Jet quenching: multiscale dynamics

Thermal equilibrium $(T \neq 0)$

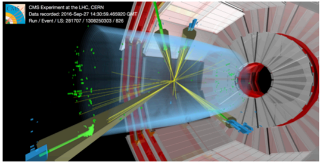


Non-equilibrium

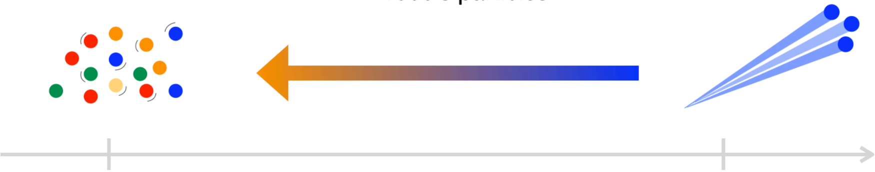


Heavy ion event
~ 1000's particles

$T=0$



Jets in pp



Energy

$T \sim 1$ GeV

$p_T \sim 1$ TeV

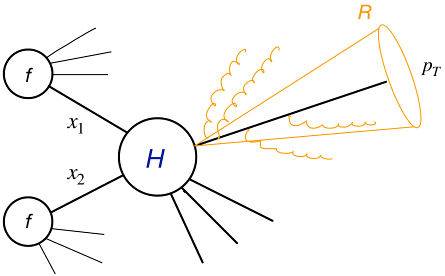
QCD factorization

The factorized cross section for inclusive jet production in proton–proton collisions, differential in transverse momentum p_T and rapidity y, is given by:

$$\frac{d\sigma}{dp_T\, dy} = \sum_{a,b,c} \int dx_a\, dx_b\, f_a(x_a, \mu)\, f_b(x_b, \mu)\, H_{ab\to cX}(\hat{s}, \hat{t}, \mu)\, J_c(p_T, R, \mu)$$

where:

- $f_{a,b}(x, \mu)$ are PDF's
- $H_{ab\to cX}$ is the hard function
- $J(p_T, R, \mu)$ is the inclusive jet function
- R is the jet radius parameter



[Dasgupta, Salam, Soyez (2015), Kang, Ringer, Vitev (2016)]

QCD factorization

- In the presence of the QGP the jet interacts elastically and inelastically with the QGP scattering centers
- In addition to p_T and R the jet function is function of intrinsic medium scales: temperature T and medium size L, etc

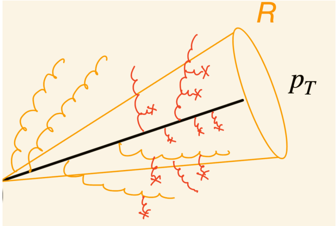
$$J(p_T, R) \to J(p_T, R, T, L)$$

- Are medium effects Perturbative $\mathcal{O}(\alpha_s)$? Sizable/computable power corrections?

$$\frac{Q^2_{med}}{Q^2} \sim \frac{\hat{q}L}{(p_T R)^2}$$

where $\hat{q} = g^4 T^3$ is the jet quenching parameter.

- $\rightarrow$ It depends on the observable.



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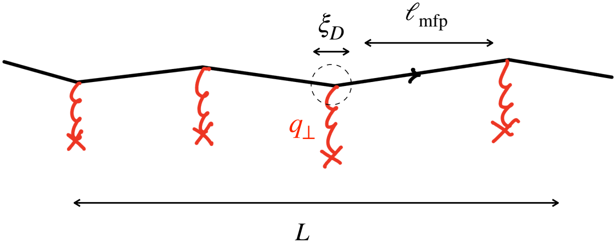
Elementary processes: transverse momentum broadening

- Independent scattering approximation:

$$\xi_D \sim \frac{1}{gT} \quad \ll \quad \ell_{\mathrm{mfp}} \sim \frac{1}{g^2 T}$$

- ξ_D is the Debye screening length and ℓ_{mfp} the mean-free-path.
- High energy approximation: $q \ll E$



Elementary processes: transverse momentum broadening

- Jet partons undergo Brownian motion in transverse momentum

$$\langle k_\perp^2 \rangle \sim \hat{q} L$$

- $\hat{q} \sim m_D^2/\ell_{\mathrm{mfp}}$ is the jet quenching transport coefficient:

$$\hat{q} \sim \rho \int d^2 q_\perp q_\perp^2 \frac{d\sigma_{el}}{d^2 q_\perp} \sim g^4 T$$

The LO the elastic cross-section is given by: $d\sigma_{el}/d^2 q_\perp \sim \frac{1}{s}|M_{2\to 2}|^2 \sim g^4/q_\perp^2$

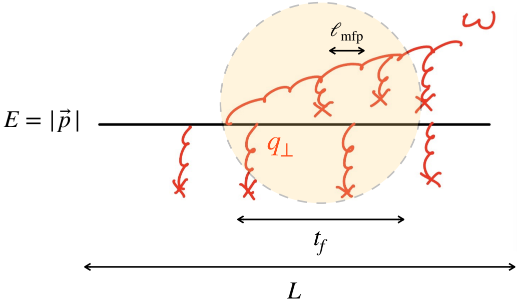
Elementary processes: medium-induced gluon radiation

- Multiple scattering can coherently induce radiation

$$\ell_{\rm mfp} \quad \ll \quad t_f(\omega) \quad \lesssim L$$

- t_f is the quantum mechanical gluon formation time

$$t_f \sim \frac{\omega}{k_\perp^2} \sim \frac{\omega}{\hat{q}\, t_f} \sim \sqrt{\frac{\omega}{\hat{q}}}$$



Elementary processes: medium-induced gluon radiation

- Characteristic scales in the Landau-Pomerantchuk-Migdal effect: Suppression of radiation due to coherent scattering: multiple scattering centers act coherently as a single scattering.
- Maximum suppression achieved when $t_f \sim L$ corresponding to the characteristic frequency

$$\omega_c = \hat{q} L^2$$

- Other characteristic scales, Transverse momentum and radiation angle:

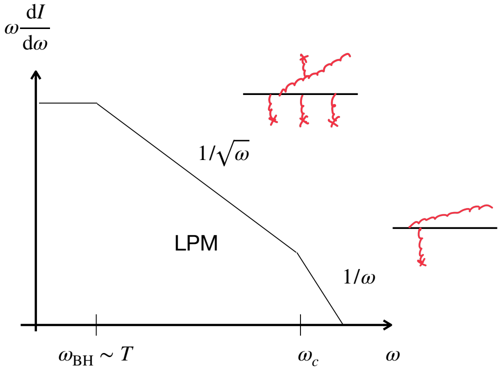
$$k_f^2 = \sqrt{\hat{q}\,\omega} < \hat{q} L \qquad \text{and} \qquad \theta_f = \left(\frac{\hat{q}}{\omega}\right)^{1/4} > \theta_c = \frac{1}{\sqrt{\hat{q} L^3}}$$

- Ex: $\hat{q} = 1$ GeV^2/fm, $L = 5$ fm, $\omega_c = 125$ GeV.

Elementary processes: medium-induced gluon radiation

Three distinct regimes for medium induced gluon radiation:

1. Hard single scattering regime: $\omega > \omega_c$ and $t_f > L$ (long formation time)
2. Multiple soft scattering: $T < \omega < \omega_c$ and $\ell_{mfp} < t_f < L$ (long formation time)
3. Bethe-Heitler regime: $\omega \sim T$



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Energy loss distribution

- At leading order the jet spectrum in the medium can be written as

$$\frac{d\sigma_{med}}{dp_T} = \int_0^{\infty} d\epsilon P(\epsilon)\ \delta(E - p_T - \epsilon)\ \frac{d\sigma_{vac}}{dp_T}$$

- where the jet cross-section in vacuum is a steep power spectrum with $n \gg 1$ (typically $n \sim 5 - 6$)

$$\frac{d\sigma_{vac}}{dp_T} = \frac{1}{p_T^n}$$

- $P(\epsilon)$ is the probability for a parent parton of energy E loses ϵ of its energy to the QGP

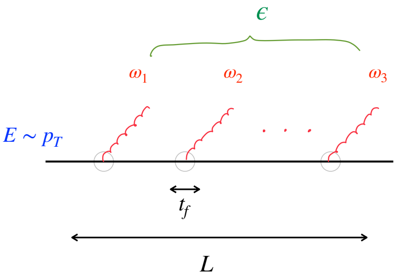
Poisson distribution

- In the soft radiation regime: $t_f \ll L$ (leading power in L), multiple emissions are frequent and uncorrelated $\to$ Poisson distribution – quasi-instantaneous radiation

Length enhancement of the rad. spect.

$$\omega \frac{dI}{d\omega} = \bar{\alpha}_s \sqrt{\frac{\omega_c}{\omega}} \propto L$$

Require resummation of all orders in $\bar{\alpha}_s$



$$\boxed{P(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} e^{-\langle n \rangle} \prod_{i=1}^{n} \frac{dI}{d\omega_i} \delta(\epsilon - \omega_1 - \omega_2 - \ldots - \omega_n)}$$

- The hard regime, $\omega \sim \omega_c$, treated order by order in $\bar{\alpha}_s$ (no length enhancement)

Energy loss distribution

- Multiple emission factorize and exponentiate in Laplace space

$$\tilde{P}(\nu) = \int d\epsilon P(\epsilon) e^{-\nu\epsilon} = \exp\left[-\int_0^\infty d\omega \frac{dI}{d\omega}\left(1 - e^{-\nu\omega}\right)\right]$$

- Using the standard integral

$$\int_0^\infty \frac{dx}{x^{1/2}}(1 - e^{-x}) = \Gamma(-1/2) = \sqrt{\pi}$$

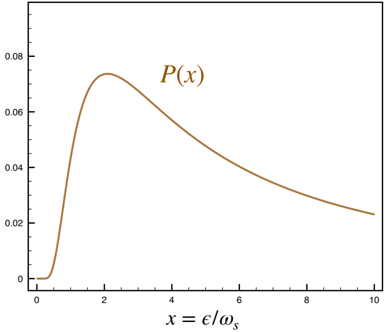
- which yields

$$\tilde{P}(\nu) = e^{-\sqrt{\pi\bar{\alpha}_s^2\omega_c\nu}} \quad \rightarrow \quad P(\epsilon) = \sqrt{\frac{\bar{\alpha}_s^2\omega_c}{\epsilon^3}} e^{-\frac{\pi\bar{\alpha}_s^2\omega_c}{\epsilon}}$$

Poisson distribution

- $P(\epsilon)$ heavy tailed distribution: mean energy loss sensitive to the hard sector $\epsilon \sim p_T$ ($x = \bar{\alpha}_s^2 \omega_c / p_T \gg 1$, $\omega_c = \hat{q} L^2$)

$$\langle \epsilon \rangle \simeq \bar{\alpha}_s \omega_c \ln(p_T/\omega_c) \gg \bar{\alpha}_s^2 \omega_c$$



Poisson distribution

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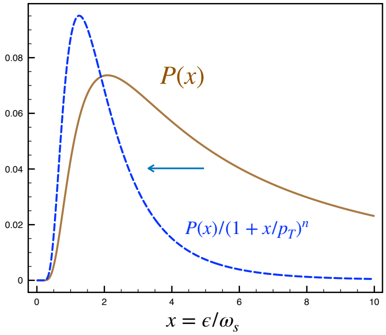
$$\langle \epsilon \rangle \simeq \bar{\alpha}_s \omega_c \ln(p_T/\omega_c) \gg \bar{\alpha}_s^2 \omega_c$$

- However, when multiplied by the initial jet spectrum $1/(p_T + \epsilon)^n$ the distribution is skewed towards the soft sector:

$$\epsilon < \frac{p_T}{n} \ll p_T$$

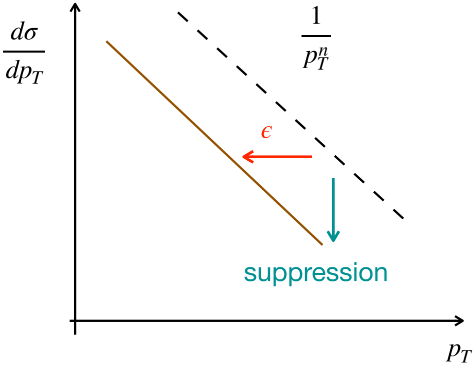
- Figure: $n = 5$, $p_T = 5\omega_s = 5\bar{\alpha}_s^2 \omega_c$.



Nuclear modification factor

- The leading order jet spectrum

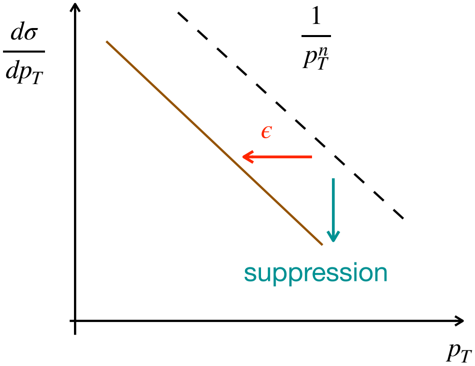
$$\frac{d\sigma_{med}}{dp_T} = \int_0^\infty d\epsilon P(\epsilon)\, \delta(E - p_T - \epsilon)\, \frac{d\sigma_{vac}}{dp_T}$$



Nuclear modification factor

- The leading order jet spectrum

$$\frac{d\sigma_{med}}{dp_T} = \int_0^{\infty} d\epsilon P(\epsilon) \; \frac{1}{(p_T + \epsilon)^n}$$



Nuclear modification factor

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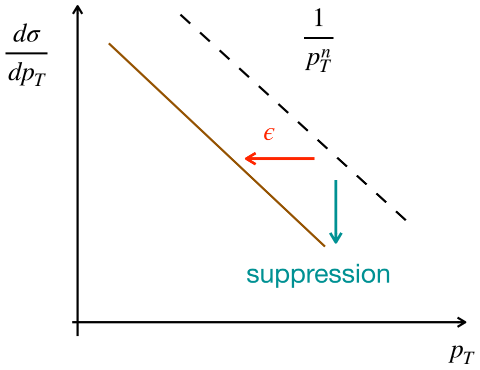
- The Nuclear Modification factor reads

$$R_{AA} = \frac{d\sigma_{med}/dp_T}{d\sigma_{vac}/dp_T}$$



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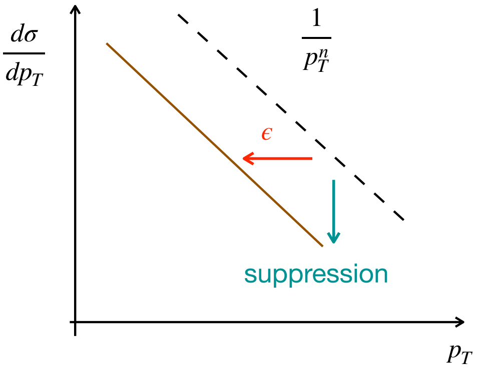

Nuclear modification factor

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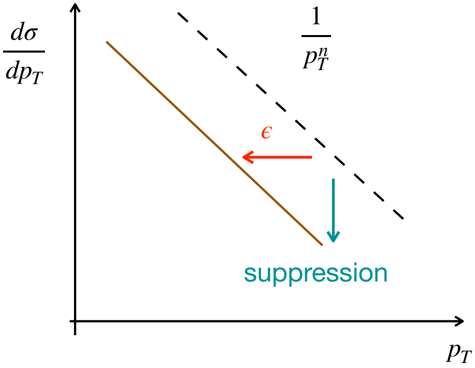
- The Nuclear Modification factor reads

$$R_{AA} = \int_0^\infty d\epsilon P(\epsilon) \; \frac{1}{(1 + \epsilon / p_T)^n}$$

- for large $n \gg 1$ we can approximate

$$\left(1 + \frac{\epsilon}{p_T}\right)^n = e^{-\frac{n\epsilon}{p_T}} + \mathcal{O}((\epsilon/p_T)^2)$$



Nuclear modification factor

- The leading order jet spectrum

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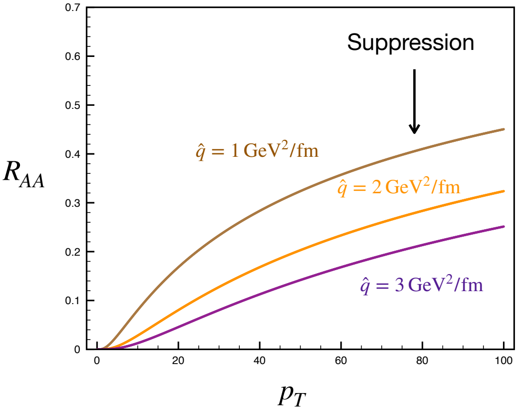
- The Nuclear Modification factor reads

$$R_{AA} = \int_0^\infty d\epsilon P(\epsilon) \; \frac{1}{(1 + \epsilon/p_T)^n}$$

- Connecting with Laplace transform:

$$\boxed{R_{AA} \simeq \tilde{P}(\nu = n/p_T) = \exp\left(-\sqrt{\frac{\pi \, n \, \bar{\alpha}_s^2 \omega_c}{p_T}} \right)}$$



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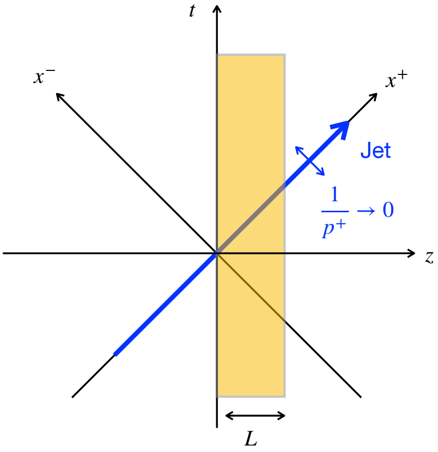
- High energy approximation $E \gg k_{\perp} \sim T$
- Light-cone variables: $p^{+} = \frac{1}{2}(E + p_z)$, $\quad p^{-} = E - p_z$, $\quad p_{\perp}$



Eikonal interaction with A^{μ}_{bkg}

- Eikonal interaction of the collinear jet particles with the background field (QGP). Performing a multipole expansion as $p^+ \to \infty$:

$$g \int_0^{+\infty} dk^+ \bar{u}(-p) \not{A}_{bkg}(k) u(p-k) \simeq g \int_0^{+\infty} dk^+ p_\mu A^{\mu}_{bkg}(k) \Big|_{k^+=0} + \mathcal{O}(k^+/p^+)\,,$$

enables us to apply the integral over k^+ solely on the gauge field leading to

$$\int_0^{+\infty} dk^+ A^{\mu}_{bkg}(k) = A^{\mu}_{bkg}(x^- = 0)\,.$$

Eikonal interaction with A^{μ}_{bkg}

- Semi-Eikonal Dirac propagator: (i) neglect $powersofk^+/p^+$ and spin flip in the interaction vertex (ii) keep track of the quantum-phase : $p_\perp^2/p^+L \sim 1$

$$\frac{1}{p^2 + i0p^+} \sim \frac{1}{p^+}\int_0^{+\infty} dx^+ e^{-i\frac{p_\perp^2}{2p^+}x^+}$$

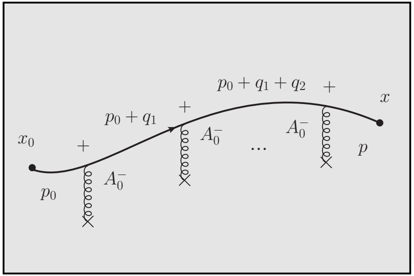
- The non-eikonal phase can be neglected for the jet but is responsible for the LPM suppression of gluon radiation for $k^+ \equiv \omega \ll \omega_c$

$$\frac{p_\perp^2}{p^+}L \sim \frac{\hat{q}L}{p^+}L = \frac{\omega_c}{k^+} \gg 1$$

Scalar propagator and 2+1D dynamcis

- Dirac propagator proportional to scalar propagator in the presence of $A^{-}_{bkg}(x^{+}, x_{\perp})$

$$D(p, p_0) = (2\pi)^4\delta^{(4)}(p - p_0)D_0(p) + \frac{\not{p}\gamma^{+}\not{p}_0}{2p^{+}}\left[G_{scal}(p, p_0) - G^{0}_{scal}(p)\delta(p - p_0)\right],$$



Scalar propagator and 2+1D dynamics

- The dynamics id that of a non-relativistic 2+1D quantum system

$$(\boldsymbol{x}|\mathcal{G}(t,t')|\boldsymbol{x}') = \frac{i}{2E}\int \frac{dx^-}{2\pi}\mathrm{e}^{-iE(x-x')^-}\,G_{scal}(x,x'),$$

- where the propagator obeys the Schrödinger equation ($t = x^+$)

$$\left[i\frac{\partial}{\partial t} + \frac{\partial_\perp^2}{2E} + gA(t,\boldsymbol{x})\right](\boldsymbol{x}|\mathcal{G}(t-t')|\boldsymbol{x}') = i\delta(t-t')\delta(\boldsymbol{x}-\boldsymbol{x}'),$$

- $A^- \equiv A^{a,-}t^a_{ij}$ and $\mathcal{G}_{ij}(t-t')$ are color matrices in the fundamental representation.

Medium statistics

- Observables are computed for each fixed medium configuration and subsequently averaged over an ensemble [McLerran-Venugopalan model (1994)])

$$\langle med|O[A_{bkg}]|med\rangle$$

- Independent multiple scattering approximation yields Gaussian statistics (at leading order)

$$\langle med|A_a^-(x^+,\boldsymbol{q})A_b^-(y^+,\boldsymbol{q}')|med\rangle = \delta_{ab}\delta(x^+-y^+)\delta(\boldsymbol{q}-\boldsymbol{q}')\;\rho\frac{d\sigma_{el}}{d\boldsymbol{q}}$$

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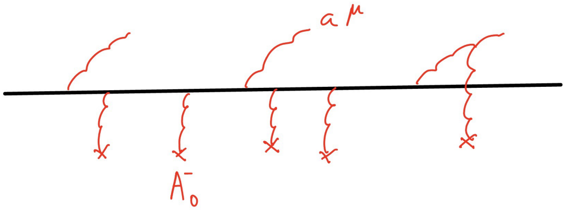
Operator definition of energy loss

- We are now equipped to provide a field theoretical definition for the energy loss probability distribution
- Soft interactions are encoded in semi infinite Wilson-lines

$$U(n) \equiv P \exp\left[ig \int_0^\infty \mathrm{d}s \, n \cdot A(ns)\right]$$

where $\bar{n} \sim p^\mu / E \equiv (1,0,0,1)$

- The gauge field $A^\mu = A^\mu_{bkg} + a^\mu$ describes both radiative and elastic processes

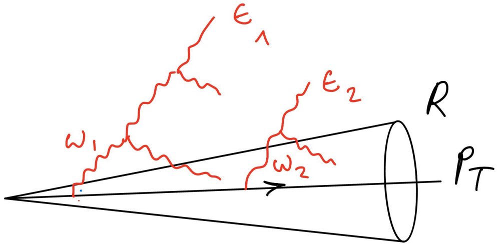


Operator definition of energy loss

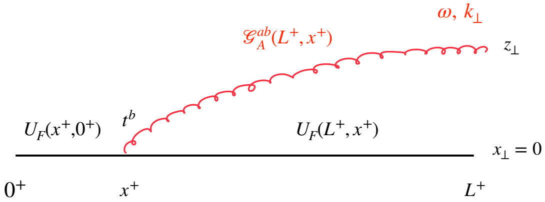
- The single parton energy loss probability distribution is defined as [YMT., Ringer, Singh, Vaidya, 2409.05957 [hep-ph]]

$$P(\epsilon) = \frac{1}{d_R} \sum \delta(\epsilon - \bar{n} \cdot k_{\mathrm{loss}})\, \mathrm{tr}_c \left[\langle \mathrm{med} | U(n) | X \rangle \langle X | U^\dagger(n) | \mathrm{med} \rangle \right],$$

- Energy loss k_{loss} Measured on final state X (Includes the jet algorithm). $U(n)$ and $U^\dagger(n)$ are Wilson-lines in the amplitude and c.c.

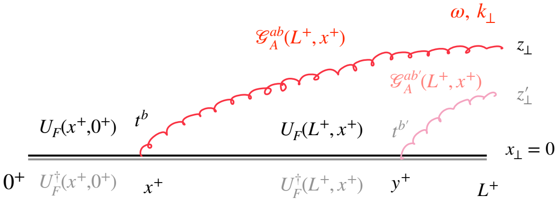


Amplitude



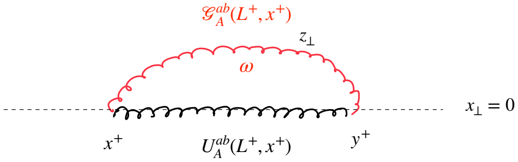
- The amplitude involves quark Wilson-lines at $x_\perp = 0$ and non-eikonal gluon propagator $\mathcal{G}$.

Amplitude squared



- The amplitude involves quark Wilson-lines at $x_\perp = 0$ and non-eikonal gluon propagator $\mathcal{G}$. N.B.: interactions with the background field are implicit.

Amplitude squared



- Thy color singlet in the intervals $[0^+, x^+]$ and $[y^+, L^+]$ do not contribute $UU^\dagger = 1$. In the interval $[x^+, y^+]$, we have a color octet state $U_F t^a U_F^\dagger = U_A^{ba} t^a$ (Fierz).
- Upon integrating over $k_\perp$, the gluon propagators after y^+ cancel out: gluon radiation rate is determined only by the dynamics during the interval $\Delta x^+ = y^+ - x^+$.

Radiative spectrum and medium averaging

- We are left with the evaluation of the expectation value of the Green's function

$$\mathcal{K}(\boldsymbol{z}_2, y^+, \boldsymbol{z}_1, x^+) = \frac{1}{N_c^2 - 1} \langle med | \mathrm{Tr}_c \left[\mathcal{U}^\dagger_{0_\perp}(y^+, x^+)(\boldsymbol{z}_2|\mathcal{G}^\dagger(y^+, x^+)|\boldsymbol{z}_1) \right] |med\rangle \,.$$

[Wiedemann (2000) Blaizot, Dominguez, Iancu, MT (2013)]

- that obeys the Schödinger equation

$$\left[i\frac{\partial}{\partial x^+} + \frac{\boldsymbol{\partial}_{\boldsymbol{x}}^2}{2\omega} + i\frac{N_c \rho}{2}\sigma(\boldsymbol{x}) \right] \mathcal{K}(\boldsymbol{x}, x^+; \boldsymbol{y}, y^+) = i\delta^{(2)}(\boldsymbol{x} - \boldsymbol{y})\delta(x^+ - y^+) \,, \qquad (1)$$

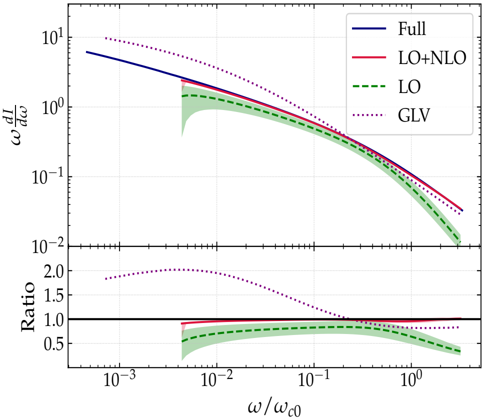
- with the imaginary potential (stochastic collisions)

$$\sigma(\boldsymbol{x}) \sim g^4 \rho \int \frac{d^2 q_\perp}{q_\perp^4} \left(1 - e^{-i\boldsymbol{x}\cdot\boldsymbol{q}}\right) \approx g^4 T^3 \boldsymbol{x}^2 \ln \frac{1}{\boldsymbol{x}^2 m_D^2} \sim \hat{q}\, \boldsymbol{x}^2$$

- Solutions: (1) order by order in opacity (powers of the density ρ) (2) to all orders in opacity in the Harmonic-oscillator approximation $\sigma \sim \hat{q}\, \boldsymbol{x}^2$.

Analytic solutions vs. numerics

- Full: exact numerical solutions [Andres, Dominguez and Gonzalez Martinez (2021)]
- LO: Harmonic oscillator approximation [Baier et al (1996) Zakharov, Wiedemann (2001)]
- NLO: Includes first Coulomb log corrections [MT (2019) Barata, MT, Soto-Ontoso, Tywoniuk (2021)]
- GLV: leading order in opacity [Gyulassy, Levai, Vitev (2001)] .



Lecture 2

- Energy dissipation in the QGP (Turbulent transport)
- Nonlinear dynamics of jet quenching
- Phenomenology