Spin-flip gluon generalised transverse momentum distribution $F_{1,2}$ at small-x

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Generalized Transverse Momentum Dependent Distributions:

$p' - p = \Delta$

The off-forward bilocal correlator of two gluon field strength tensor is given by the expression,

$$W^{[i,j]}_{\lambda,\lambda'} = \int \frac{d^2 z_\perp dz^-}{(2\pi)^3 P^+} e^{ixP^+z^- - ik_\perp . z_\perp} \langle p', \lambda' | F^{+i}_a \left(-\frac{z}{2}\right) \mathcal{U}^{[+]}_{\frac{z}{2},-\frac{z}{2}} F^{+j}_a \left(+\frac{z}{2}\right) \mathcal{U}^{[-]}_{-\frac{z}{2},\frac{z}{2}} | p, \lambda \rangle \Big|_{z^+=0} .$$

The parametrization of the correlator gives various GTMDs. Here, we will talk about F-type GTMDs,

$$\delta^{ij} W^{[i,j]}_{\lambda,\lambda'} = \frac{1}{2M} \bar{u}\left(p', \lambda'\right) \left[F^g_{1,1} + i \frac{\sigma^{j+} k^j_\perp}{P^+} F^g_{1,2} + i \frac{\sigma^{j+} \Delta^j_\perp}{P^+} F^g_{1,3} + i \frac{\sigma^{ij} k^i_\perp \Delta^j_\perp}{M^2} F^g_{1,4} \right] u(p, \lambda) .$$

All GTMDs, in the above expression, are functions of $(x,\ k^2_\perp,\ \Delta^2_\perp,\ k_\perp \cdot \Delta_\perp,\ \xi)$ and are in general complex functions.

Until now, the most studied TMD is the spin-independent Re$F_{1,1}$ type and its evolution can be studied through the standard BKFL evolution equation.

Solutions to Evolution Equations :

The evolution of the TMD, Re$F_{1,2}$ or $\mathscr{F}_{1,2}$ is done by the following evolution equation,

$$\partial_Y \mathscr{F}_{1,2}\left(k_\perp\right) = \frac{\bar{\alpha}}{\pi}\int \frac{d^2 k'_\perp}{\left(k_\perp - k'_\perp\right)^2}\left[-\frac{k_\perp^2}{2k'^2_\perp}\mathscr{F}_{1,2}\left(k_\perp\right) + \left(\frac{2(k_\perp . k'_\perp)^2 - k_\perp^2 k'^2_\perp}{(k_\perp^2)^2}\right)\mathscr{F}_{1,2}\left(k'_\perp\right)\right].$$

In our work, we analytically solved the evolution equation for $\mathscr{F}_{1,2}$. We showed that all the IR singularities mutually cancel each other, making the equation consistent.

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We assume the solution for the evolution equation for the TMD $\mathscr{F}_{1,2}$ to be the Fourier Series in the azimuthal angle as,

$\mathscr{F}_{1,2}^{(n)}(x, k_\perp^2)$ are functions of x and $k_\perp$ only.

$$\mathscr{F}_{1,2}\left(|k_\perp|, \phi_k; Y\right) = \sum_{n=-\infty}^{\infty} \mathscr{F}_{1,2}^{(n)}(x, k_\perp^2)\, e^{in\phi_k}$$

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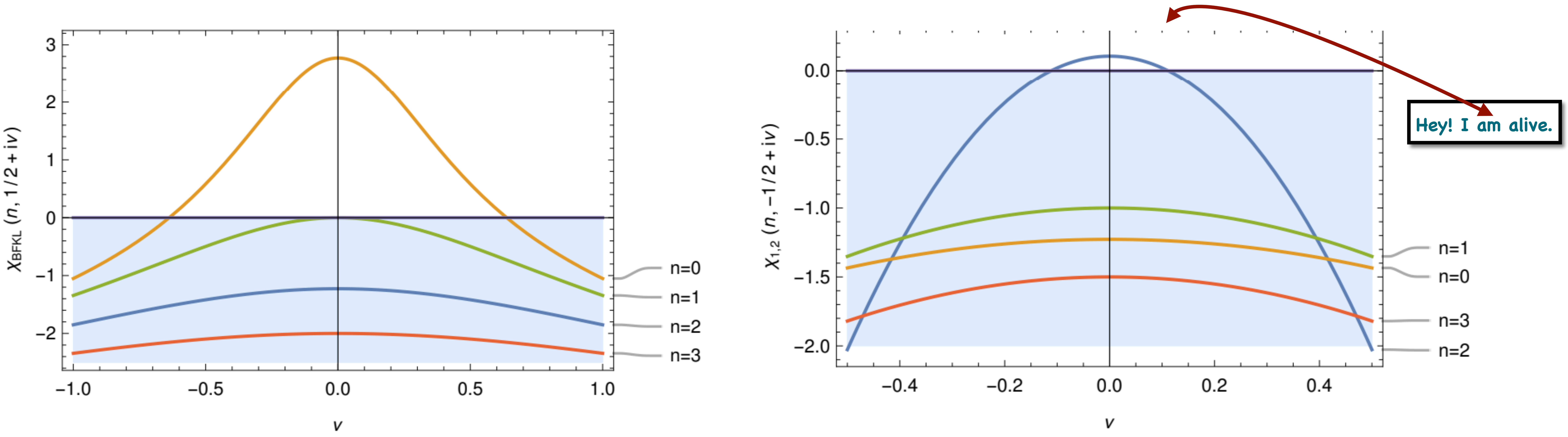
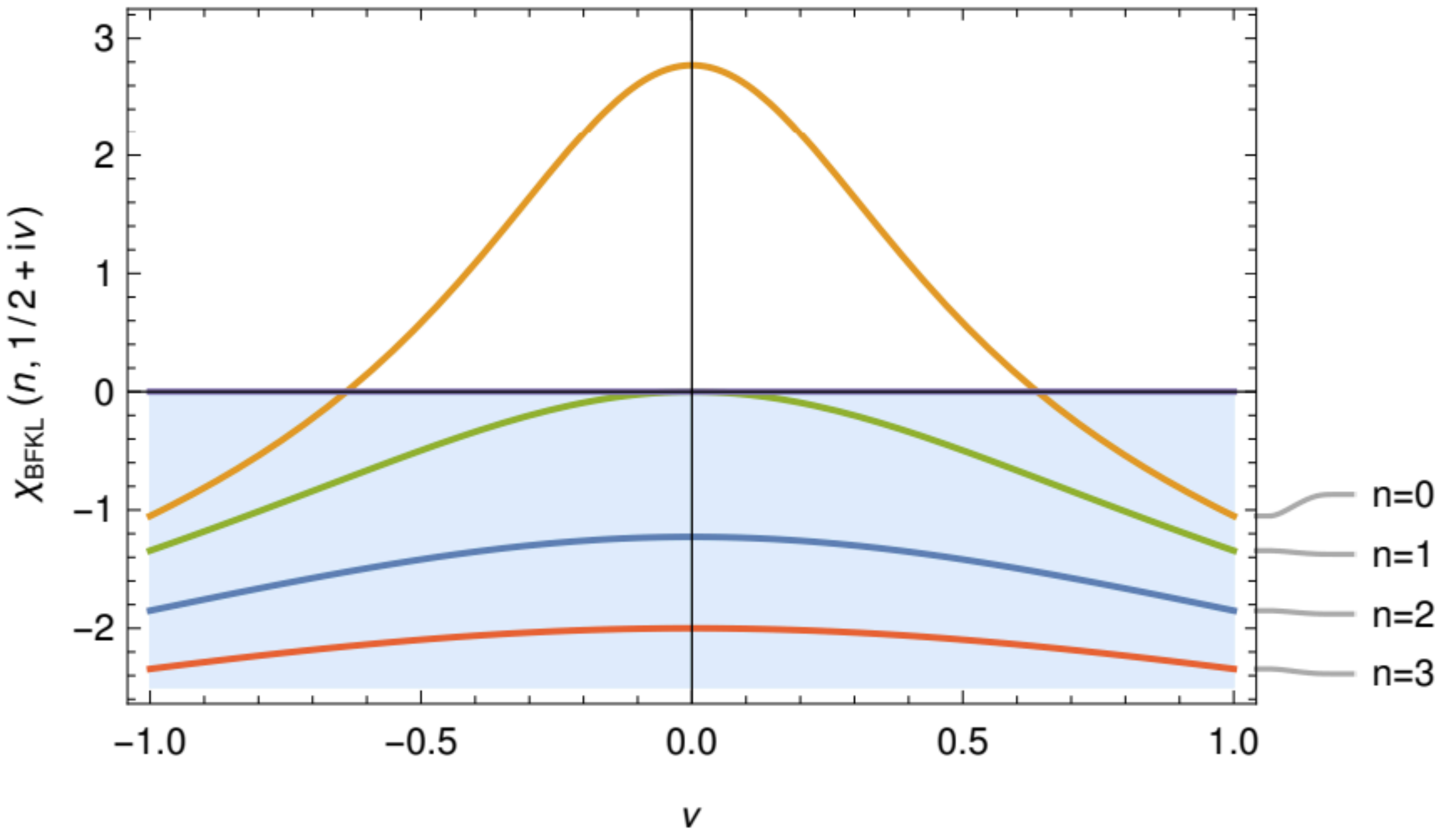
$$\mathscr{F}^{(n)}_{1,2}(x, k_\perp^2) = \int \frac{d\gamma}{2\pi i}\left(\frac{1}{x}\right)^{\bar{\alpha}_s \chi_{1,2}(n,\gamma)} \frac{k_\perp^{2\gamma}}{k_\perp^2}$$



(a) $\chi_{BFKL}(n,\gamma)$ at the saddle point $\gamma = 1/2 + i0$.

(b) $\chi_{1,2}(n,\gamma)$ at the saddle point $\gamma = -1/2 + i0$.

FIG 2: Eigen values of the two kernels, at their respective saddle points, as a function of ν.



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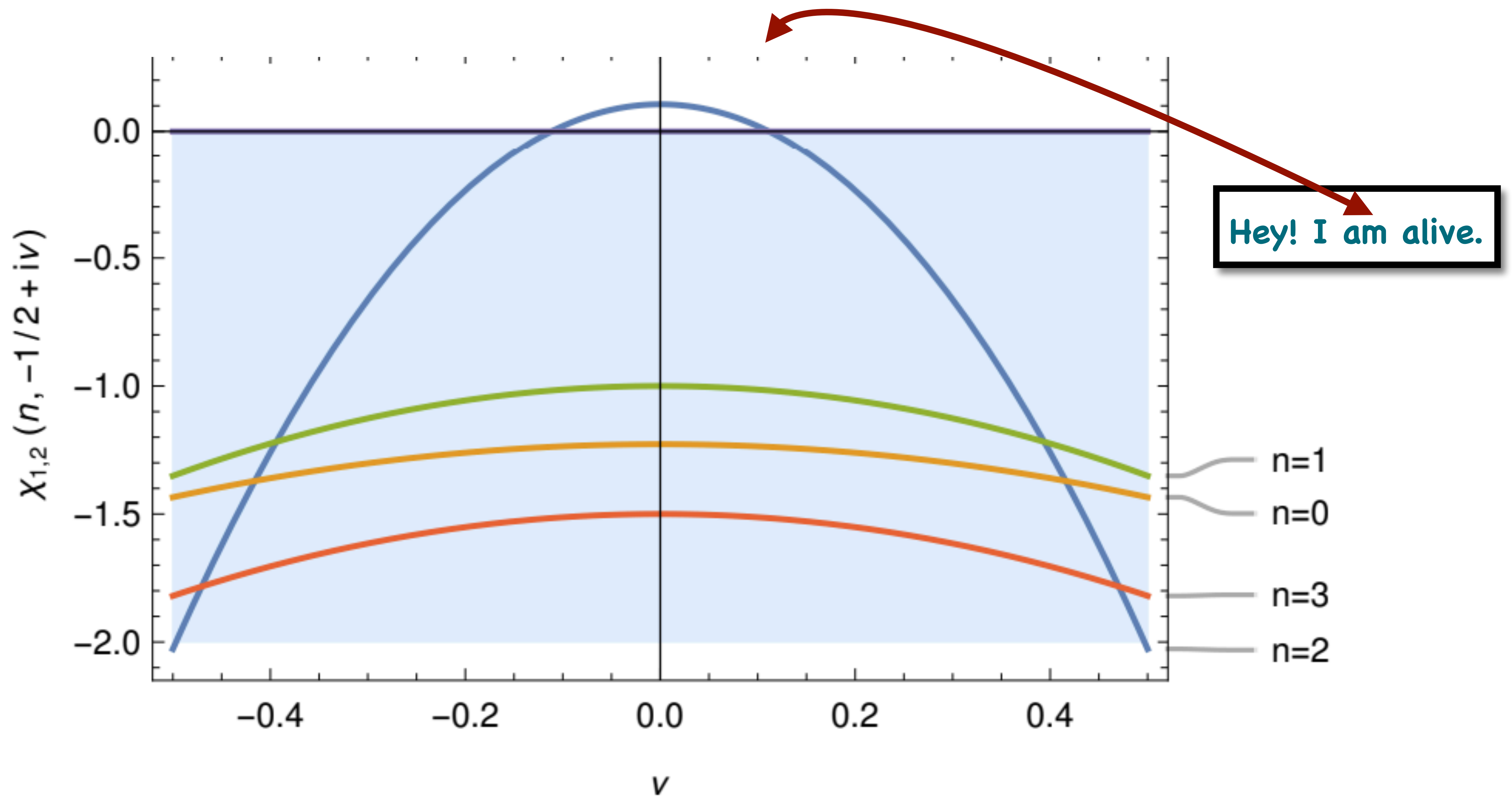

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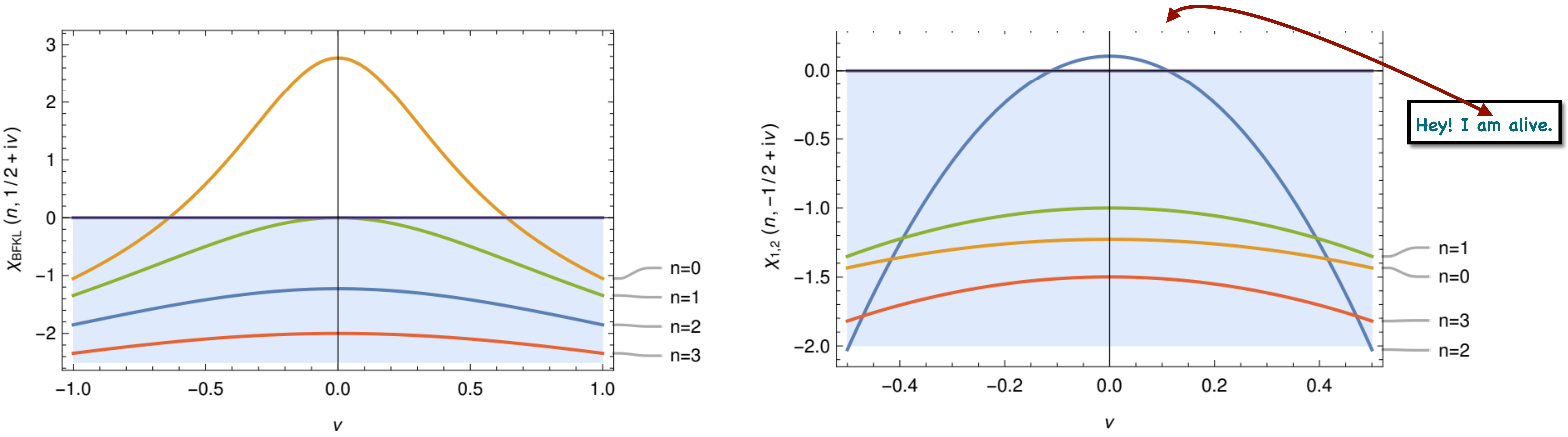

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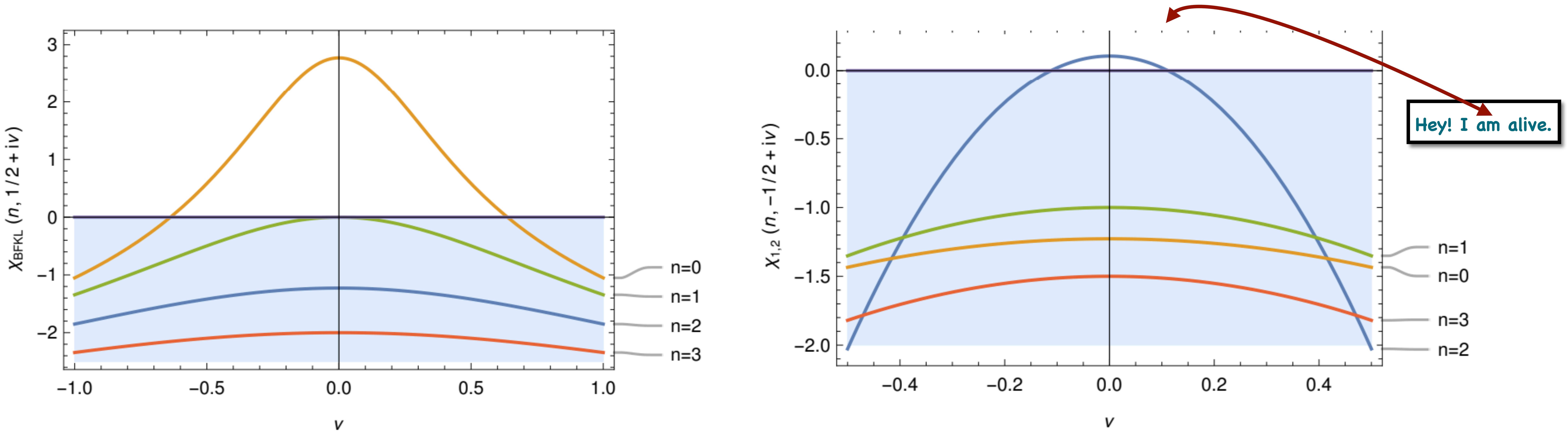

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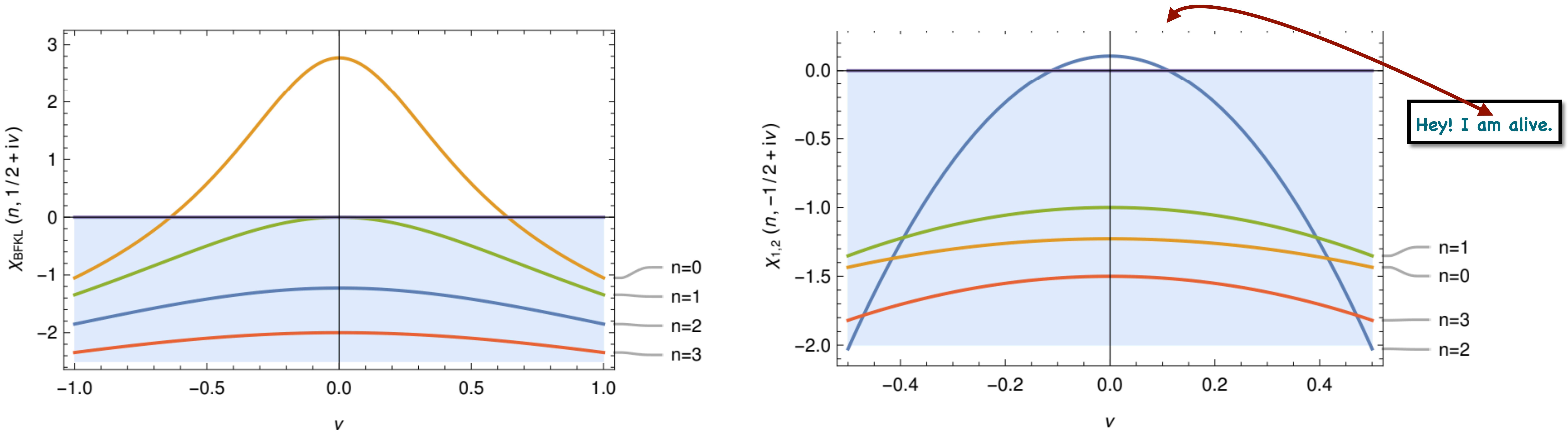

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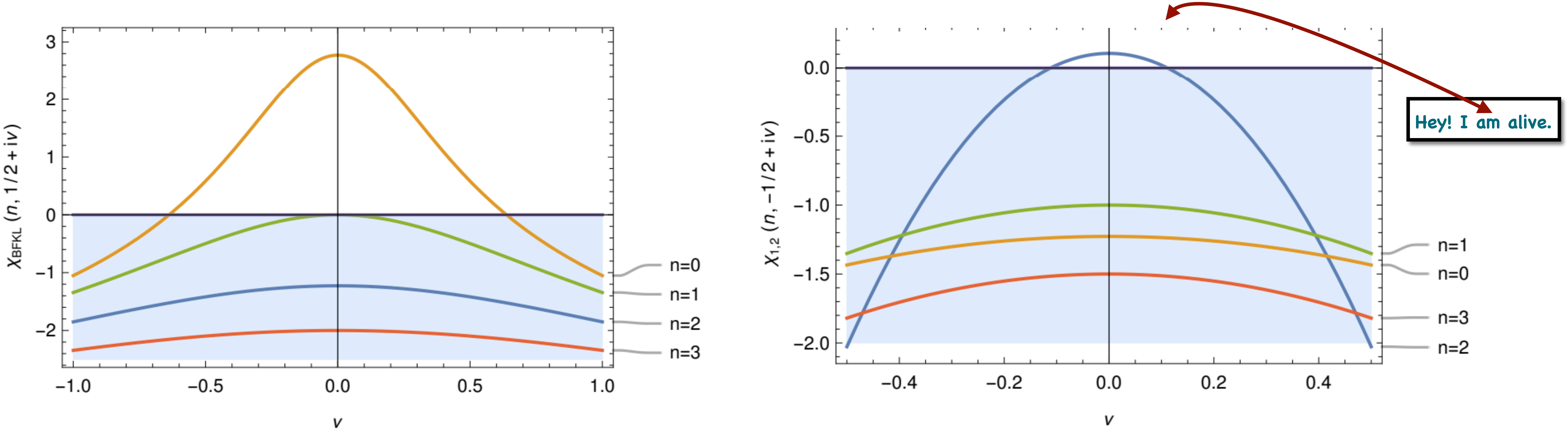
$\mathscr{F}_{1,2}$ will increase with energy and might not be suppressed at small-x.



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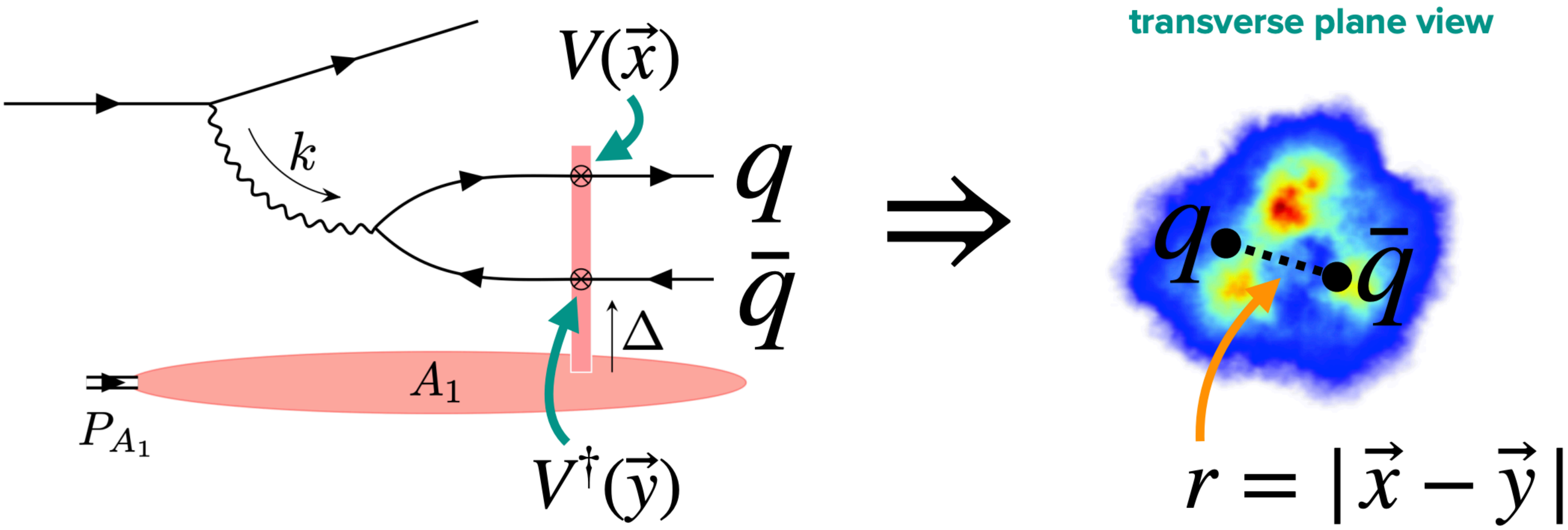
This leads to an explicit $\cos 3\phi_{k\Delta} + \cos \phi_{k\Delta}$ azimuthal dependence in the structure functions.

Dipole Picture



In position space, if there is translation symmetry (assumption of a large nucleus), the dipole cross section depends on the positions of quarks and anti-quarks only through their separation $r = x - y$, $i.e.$,

$$N\left(x, y\right) \approx N\left(x - y\right); \quad O(x, y) \approx O\left(x - y\right).$$

Forward-off forward Correspondence:

We studied an equivalent proposition in the momentum space.

Translation symmetry in the momentum space bifurcates the amplitudes into two translationally symmetric functions along the k line in the $k - \Delta$ plane.

$$\partial_Y \mathcal{N}(k_\perp, \Delta_\perp) \sim \frac{1}{2}\partial_Y \left[\mathcal{N}\left(k_\perp - \frac{\Delta_\perp}{2}, 0\right) + \mathcal{N}\left(k_\perp + \frac{\Delta_\perp}{2}, 0\right) \right],$$

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The same arguments has been extended to the odderon amplitude and it is shown that it's evolution can be studied through the evolution of the difference of the forward odderon amplitudes.

$$\partial_Y \mathcal{O}(k_\perp, \Delta_\perp) \sim \frac{1}{2}\partial_Y \left[\mathcal{O}\left(k_\perp - \frac{\Delta_\perp}{2}, 0\right) - \mathcal{O}\left(k_\perp + \frac{\Delta_\perp}{2}, 0\right) \right].$$

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odderons more likely to be found in the non-forward scatterings.