Is η_c meson production a golden channel for the study of the Odderon?

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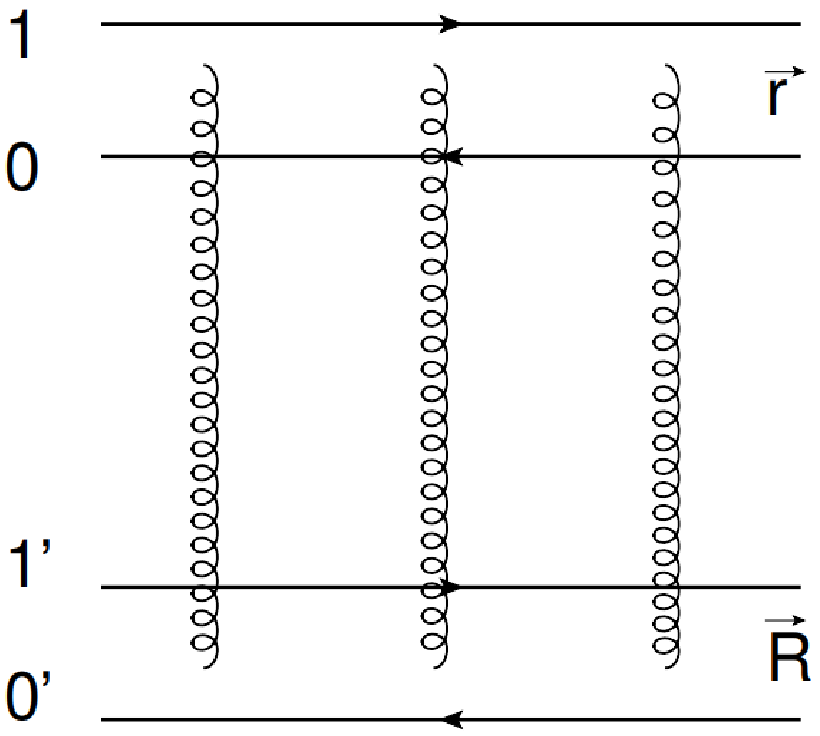
The 2025 CFNS-SURGE summer workshop on the Physics of the EIC.

Let's first see what the Odderon is:



- A **Regge trajectory** with **odd charge conjugation** parity: C=−1.
- It is the C-odd counterpart of the Pomeron (C=+1).
- Describes the **exchange of an odd number of gluons**, minimally **three** in a color-singlet configuration.
- Appears naturally in **high-energy scattering**, especially in **differences** between pp and $p\bar{p}$ cross sections.

$$C: \quad \vec{x}_{1\perp} \leftrightarrow \vec{x}_{0\perp} \longrightarrow \mathcal{O}(\vec{x}_{1\perp}, \vec{x}_{0\perp}, Y) = -\mathcal{O}(\vec{x}_{0\perp}, \vec{x}_{1\perp}, Y)$$

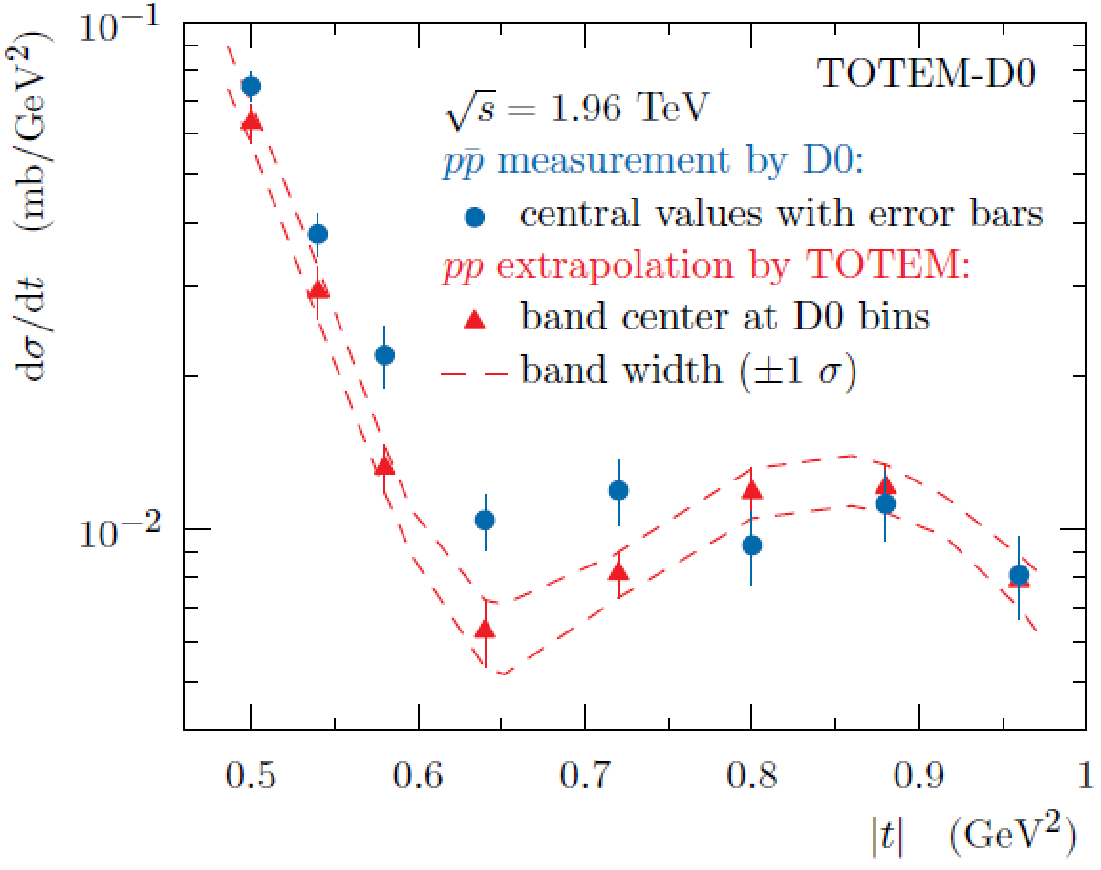
Cornille-Martin theorem

The ratio of the two differential croos section should approach 1 at high energies.

$$\lim_{s\to\infty} \frac{d\sigma^{p+\bar{p}}/dt}{d\sigma^{p+p}/dt} = 1$$

Does this mean that the two differential cross section are equal at high energies?

$$\lim_{s\to\infty} \left[\frac{d\sigma^{p+\bar{p}}}{dt} - \frac{d\sigma^{p+p}}{dt}\right] \neq 1$$



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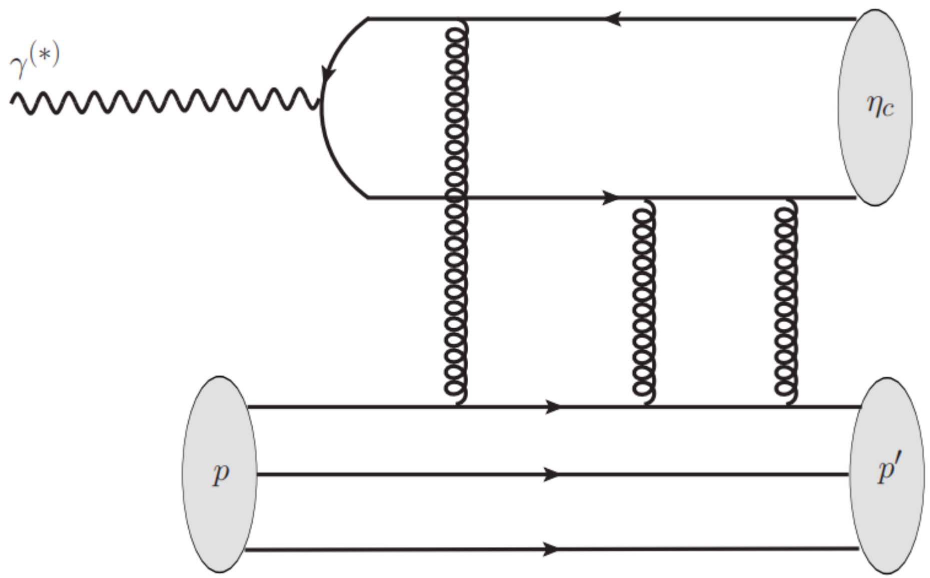
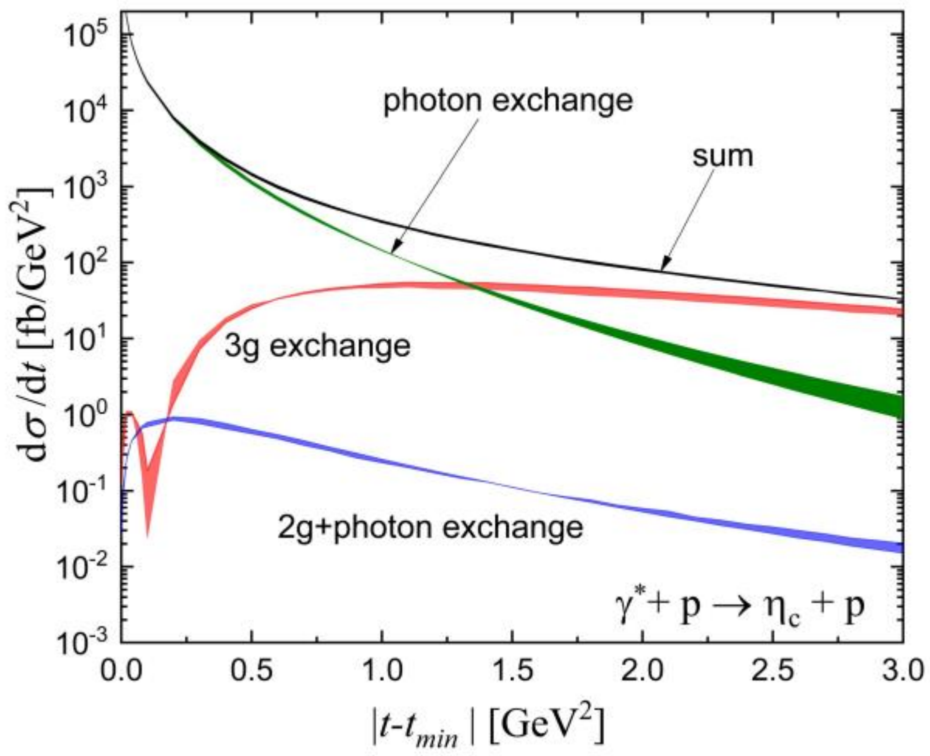
Let's examine briefly the literature



Particle	C-parity
η_c	$+1$
J/ψ	-1
Photon γ	-1
Pomeron	$+1$
Odderon	-1

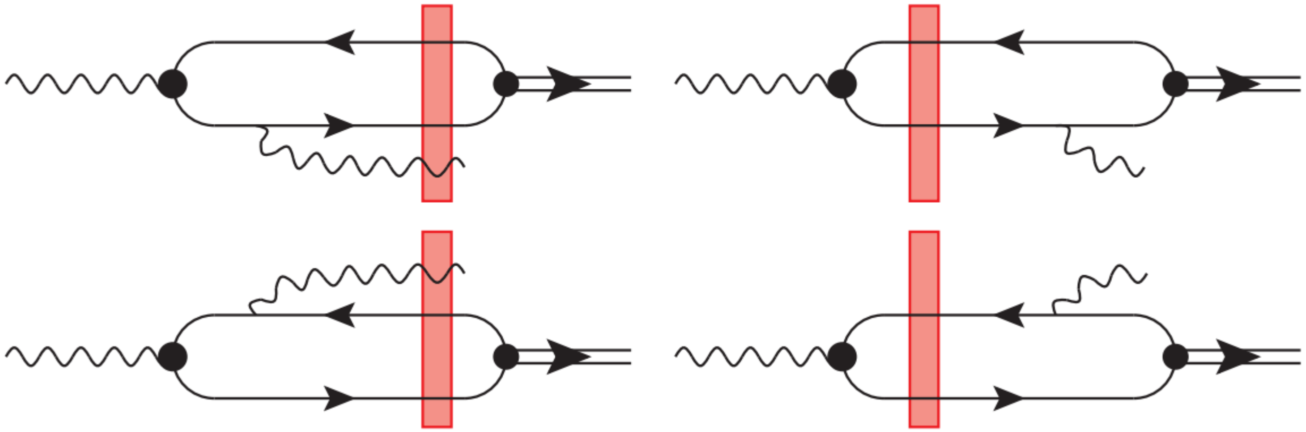


A. Dumitru and T. Stebel 2019

- The discovery of the three gluon QCD exchange in this process requires measurements at fairly large momentum transfer.

Our proposal: Pomeron exchange with an undetected photon.

$$\mathcal{A}_{\gamma p\to\eta_c\gamma p} = \mathcal{A}_1 + \mathcal{A}_2$$



$$\frac{d\sigma_{\gamma p\to\eta_c\gamma p}}{dt\,dt'\,dM_{\gamma\eta_c}} \approx \frac{\left|\mathcal{A}^{(\lambda,\sigma)}_{\gamma p\to\eta_c\gamma p}\right|^2}{128\pi^3 M_{\gamma\eta_c}}$$

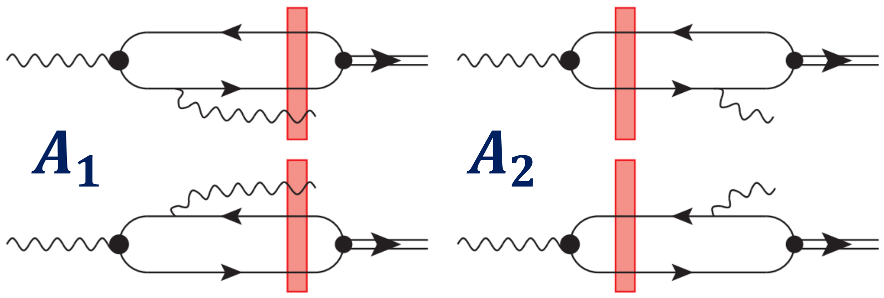
$$\left|\mathcal{A}^{(\text{unpolarized})}_{\gamma p\to\eta_c\gamma p}\right|^2 \equiv \frac{1}{2}\sum_{\lambda=\pm1}\sum_{\sigma=\pm1}\left|\mathcal{A}^{(\sigma,\lambda)}_{\gamma p\to\eta_c\gamma p}\right|^2 = \left|\mathcal{A}^{(+,+)}_{\gamma p\to\eta_c\gamma p}\right|^2 + \left|\mathcal{A}^{(+,-)}_{\gamma p\to\eta_c\gamma p}\right|^2.$$

For the unpolarized cross-section, summation over spins of photons in the final state and averaging over helicities in the initial state.

Difference between amplitudes: the wave functions.

$$\boldsymbol{A_1} = \int_0^{\alpha_{\eta_c}} dz_0 \prod_{k=1}^{3} \left(d^2\boldsymbol{r}_k\right)\; \Phi_{\eta_c}^{(h,\bar h)\dagger}\left(\frac{z_0}{z_0+z_1},\, \boldsymbol{r}_{10}\right) \Psi_{\gamma\to\gamma\bar{Q}Q}^{(\lambda,\sigma,h,\bar h)}\left(z_0,\, z_1=\alpha_{\eta_c}-z_0,\, z_2\equiv\bar\alpha_{\eta_c},\, \boldsymbol{r}_0,\, \boldsymbol{r}_1,\, \boldsymbol{r}_2\right)\times$$
$$\times\mathcal{N}\left(x,\, \boldsymbol{r}_{10},\; \boldsymbol{b}_{10}\right)\exp\left[-i\boldsymbol{p}_\perp^{\eta_c}\cdot\left(\boldsymbol{b}_{10}-\frac{\bar\alpha_{\eta_c}}{\alpha_{\eta_c}}\boldsymbol{r}_\gamma\right)-i\boldsymbol{k}_\gamma^\perp\cdot\left(\boldsymbol{r}_\gamma+\boldsymbol{b}_{10}\right)\right],$$

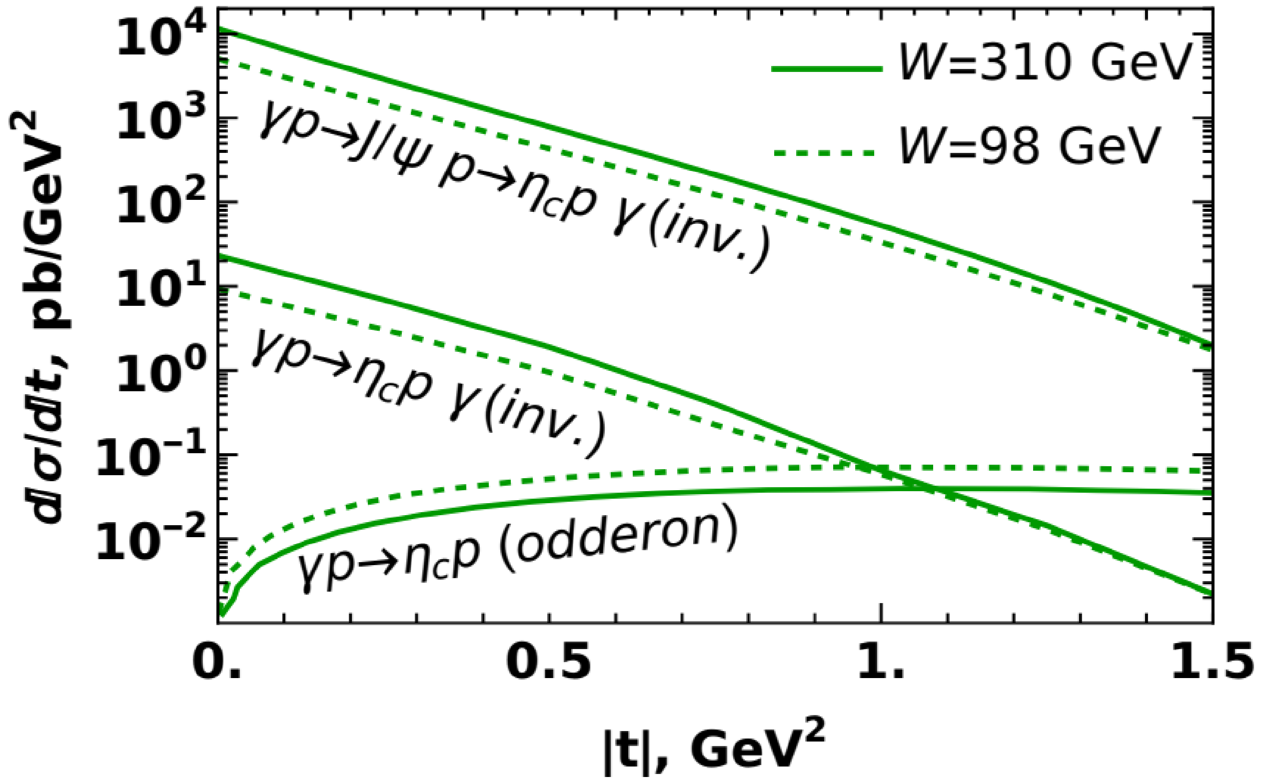
$$\begin{aligned}
&\Psi_{\gamma\to\gamma\bar{Q}Q}^{(\lambda,\sigma,h,\bar h)}\left(z_0,\, z_1,\, z_2,\, \boldsymbol{r}_0,\, \boldsymbol{r}_1,\, \boldsymbol{r}_2\right)=8\pi\alpha_{\rm em}e_f^2\sqrt{z_0z_1}\times \qquad (39)\\
&\left\{-\frac{1}{z_0+z_2}\left[\delta_{h,-\bar h}\left(\left(2z_0+z_2\right)\left(2z_1-1\right)+\lambda\sigma z_2\right)\varepsilon_\lambda^k\varepsilon_\sigma^{*i}-\epsilon_{h,\bar h}^\perp\varepsilon_\lambda^k\varepsilon_\sigma^{*i}\left(-\sigma z_2\left(2z_1-1\right)+\lambda\left(2z_0+z_2\right)\right)\right]\mathcal{I}_{(1)}^{ik}+\right.\\
&-\frac{1}{z_1+z_2}\left[\delta_{h,-\bar h}\left(\left(2z_1+z_2\right)\left(2z_0-1\right)-\lambda\sigma z_2\right)\varepsilon_\lambda^k\varepsilon_\sigma^{*i}+\epsilon_{h,\bar h}^\perp\varepsilon_\lambda^k\varepsilon_\sigma^{*i}\left(-\sigma z_2\left(2z_0-1\right)+\lambda\left(2z_1+z_2\right)\right)\right]\mathcal{I}_{(2)}^{ik}+\\
&+\frac{z_0z_2}{\left(z_0+z_2\right)^2}\left(\delta_{h,-\bar h}+\lambda\,\epsilon_{h,\bar h}^\perp\right)\delta_{\lambda,\sigma}\mathcal{J}_{(1)}-\frac{z_1z_2}{\left(z_1+z_2\right)^2}\left(\delta_{h,-\bar h}-\lambda\epsilon_{h,\bar h}^\perp\right)\delta_{\lambda,\sigma}\mathcal{J}_{(2)}+\\
&-\frac{m/q^+}{z_0+z_2}\left[\left(2z_0+z_2\right)\left(-\lambda\sqrt{2}\right)\varepsilon_\sigma^{*i}\delta_{2h,\lambda}\delta_{2\bar h,\lambda}+z_2\left(\sigma\sqrt{2}\varepsilon_\lambda^i\delta_{2h,-\sigma}\delta_{2\bar h,-\sigma}-\delta_{\lambda,\sigma}\left(\epsilon_{h,-\bar h}\delta_{i,1}-i\delta_{h,\bar h}\delta_{i,2}\right)\right)\right]\hat{\mathcal{I}}_{(1)}^i+\\
&+\frac{mz_2^2/q^+}{\left(z_0+z_2\right)^2}\left[\left(2z_1-1\right)\varepsilon_\lambda^k\left(-\sigma\sqrt{2}\delta_{2h,-\sigma}\delta_{2\bar h,-\sigma}\right)-\left(\varepsilon_\sigma^{*k}\left(\lambda\sqrt{2}\delta_{2h,\lambda}\delta_{2\bar h,\lambda}\right)-\delta_{\lambda,\sigma}\left(\epsilon_{h,-\bar h}\delta_{k,1}-i\delta_{h,\bar h}\delta_{k,2}\right)\right)\right]\hat{\mathcal{I}}_{(1)}^k+\\
&-\frac{m^2z_2^2/q^+}{\left(z_0+z_2\right)^2}\delta_{\lambda,\sigma}\left[\delta_{h,-\bar h}+\frac{\lambda}{q^+}\frac{2z_1-1}{z_1\left(1-z_1\right)}\left[m\epsilon_{h,\bar h}-\delta_{h,\bar h}\left(\partial_{r_{0x}}-ih\partial_{r_{0y}}\right)\right]\right]\mathcal{I}_{(1)}\\
&-\frac{m/q^+}{z_1+z_2}\left[\left(\left(2z_1+z_2\right)\varepsilon_\sigma^{*i}\lambda\sqrt{2}\delta_{2h,\lambda}\delta_{2\bar h,\lambda}-z_2\left(\varepsilon_\lambda^i\sigma\sqrt{2}\delta_{2h,-\sigma}\delta_{2\bar h,-\sigma}-\delta_{\lambda,\sigma}\varepsilon_\sigma^{*i}\lambda\sqrt{2}\delta_{2h,\lambda}\delta_{2\bar h,\lambda}\right)\right)\right]\hat{\mathcal{I}}_{(2)}^i+\\
&-\frac{z_2^2\,m/q^+}{\left(z_1+z_2\right)^2}\left[\left(\left(2z_0-1\right)\varepsilon_\lambda^k\sigma\sqrt{2}\delta_{2h,-\sigma}\delta_{2\bar h,-\sigma}+\left(\varepsilon_\sigma^{*k}\lambda\sqrt{2}\delta_{2h,\lambda}\delta_{2\bar h,\lambda}-\delta_{\lambda,\sigma}\left(\epsilon_{h,-\bar h}\delta_{k,1}-i\delta_{h,\bar h}\delta_{k,2}\right)\right)\right)\right]\hat{\mathcal{I}}_{(2)}^k+\\
&\left.+\frac{m^2z_2^2/q^+}{\left(z_1+z_2\right)^2}\delta_{\lambda,\sigma}\left[\delta_{h,-\bar h}-\frac{\lambda}{q^+}\frac{2z_1-1}{z_1\left(1-z_1\right)}\left[m\epsilon_{h,\bar h}-\delta_{h,\bar h}\left(\partial_{r_{1x}}-ih\partial_{r_{1y}}\right)\right]\right]\mathcal{I}_{(2)}\right\},
\end{aligned}$$



$$\boldsymbol{A_2}=\int_0^{\alpha_{\eta_c}} dz_0 \prod_{k=1}^{3}\left(d^2\boldsymbol{r}_k\right)\; \Psi_{\bar{Q}Q\to\gamma\,\eta_c}^{(\sigma,h,\bar h)\dagger}\left(z_0,\, z_1=\alpha_{\eta_c}-z_0,\, z_2\equiv\bar\alpha_{\eta_c},\, \boldsymbol{r}_0,\, \boldsymbol{r}_1,\, \boldsymbol{r}_2\right)\psi_{\gamma\to\bar{Q}Q}^{(\lambda,\,h,\bar h)}\left(\frac{z_0}{z_0+z_1},\, \boldsymbol{r}_{10}\right)\times$$
$$\times\mathcal{N}\left(x,\, \boldsymbol{r}_{10},\; \boldsymbol{b}_{10}\right)\exp\left[-i\boldsymbol{p}_\perp^{\eta_c}\cdot\left(\boldsymbol{b}_{10}-\frac{\bar\alpha_{\eta_c}}{\alpha_{\eta_c}}\boldsymbol{r}_\gamma\right)-i\boldsymbol{k}_\gamma^\perp\cdot\left(\boldsymbol{r}_\gamma+\boldsymbol{b}_{10}\right)\right],$$

$$\begin{aligned}
&\Psi_{\bar{Q}Q\to\gamma\,\eta_c}^{(\sigma,h,\bar h)\dagger}\,\psi_{\gamma\to\bar{Q}Q}^{(\lambda,\,h,\bar h)}\left(z_0,z_1,\boldsymbol{r}_0,r_1,r_2\right)=\\
&=\kappa m\delta_{\lambda,\sigma}\left\{iK_0\left(\varepsilon r\right)\left[\frac{2Z_0\bar Z_1\varepsilon_\lambda^k\varepsilon_\sigma^{*i}I_{(1)}^{ki}+m^2Z_2^2I_{(1)}}{\bar Z_1^2}-\frac{2Z_1\bar Z_0\varepsilon_\lambda^k\varepsilon_\sigma^{*i}I_{(1)}^{ki}-m^2Z_2^2I_{(2)}}{\bar Z_0^2}-\right.\right.\\
&\qquad\qquad\left.-\frac{Z_0Z_2}{\bar Z_1^2}J_{(1)}-\frac{Z_1Z_2}{\bar Z_0^2}J_{(2)}\right]\\
&\left.+\frac{\varepsilon K_1\left(\varepsilon r\right)}{Z_0+Z_1}e^{i\lambda\phi_r}\left[-I_{(1)}^{(-\lambda)}\left(\frac{Z_1\bar Z_1+Z_0^2}{\bar Z_1}\right)+I_{(2)}^{(-\lambda)}\left(\frac{Z_0\bar Z_0+Z_1^2}{\bar Z_0}\right)+Z_2^2\left(\frac{Z_0}{\bar Z_1^2}\hat I_{(1)}^{(-\lambda)}+\frac{Z_1}{\bar Z_0^2}\hat I_{(2)}^{(-\lambda)}\right)\right]\right\}\\
&+\kappa m\delta_{\lambda,-\sigma}\left\{e^{i\lambda\phi_r}\varepsilon K_1\left(\varepsilon r\right)\left[\frac{I_{(2)}^{(\lambda)}}{Z_0+Z_1}\frac{Z_1}{\bar Z_0}-\frac{I_{(1)}^{(\lambda)}}{Z_0+Z_1}\frac{Z_0}{\bar Z_1}+\frac{Z_2^2}{Z_0+Z_1}\left(\frac{Z_1}{\bar Z_1^2}\hat I_{(1)}^{(\lambda)}+\frac{Z_0}{\bar Z_0^2}\hat I_{(2)}^{(\lambda)}\right)\right]\right.\\
&\qquad\qquad\left.+\frac{i\lambda K_0\left(\varepsilon r\right)}{m}\left[I_{(2)}^{(\lambda,\lambda)}-I_{(1)}^{(\lambda,\lambda)}\right]\right\},
\end{aligned}$$

Results:



- For the typical EIC energy $\sqrt{s} \approx 100$ GeV

$$\sigma_{\rm tot}^{(\rm ep)}\left(\sqrt{s_{ep}} = 100\,\text{GeV},\ M_{\gamma\eta_c} \geq 3.5\,\text{GeV}\right) \approx 208\,\text{fb}$$

$$\mathcal{L} = 10^{34}\,\text{cm}^{-2}\text{s}^{-1} = 0.864\,\text{fb}^{-1}\text{day}^{-1}$$

- At future EIC, the cross section gives a production rate:

$$dN/dt \approx 180 \text{ events/day}$$

$$N = 2.1 \times 10^4 \text{ produced } \eta_c\gamma$$

η_c is not detected directly, we must know counting rate $dN_d/dt \approx 139$ events/month

$$\text{Br}_{\eta_c} = \text{Br}\left(\eta_c(1S) \to K_S^0 K^+\pi^-\right) = 2.6\%.$$

With N_d=540 detected events per 100 fb^{-1} of integrated luminosity

1903.05375

But:

- It is posible to separate different mechanisms, even when the final photon is not detected, imposing appropriate cuts of the variable $(q-\Delta)^2$.
- Where q : is the momentum of incoming photon, and Δ is the momentum transfer of the recoil proton.
- This variable corresponds to:
  - $M^2_{J/\psi}$, $M^2_{\gamma\eta_c}$ and $M^2_{\eta_c}$.

- However, separation of the odderon contribution using this method would require measurements of the momenta of recoil proton and scattered electron with **outstanding precision**.