Orbital Angular Momentum at Small x

Brandon Manley

Based on:
Kovchegov, BM (2310.18404)
BM (2401.05508)
Kovchegov, BM (2410.21260)

CFNS/SURGE school, June 2025

Why OAM at small x?

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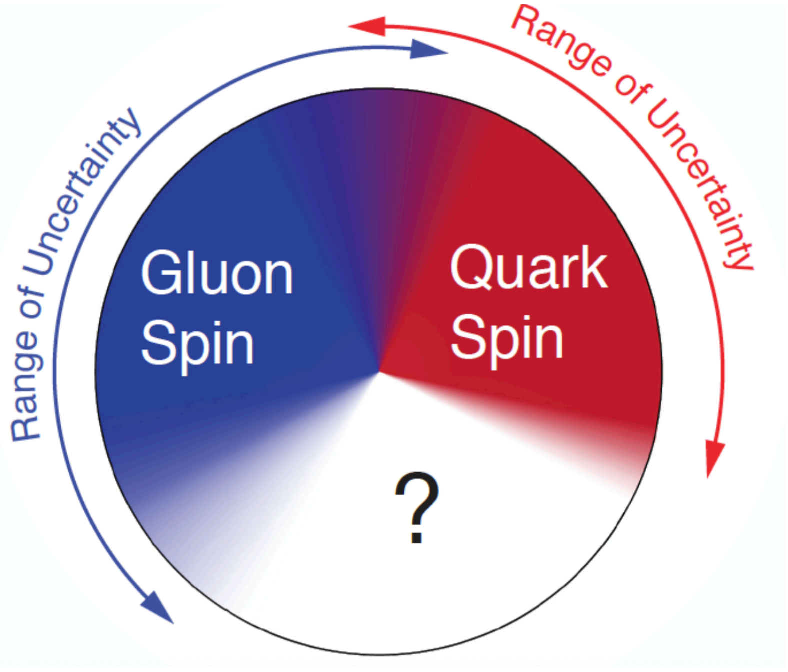
(see Bjoern's talk)

- Current proton spin budget does not include OAM
  $\rightarrow$ **No experimental data related to OAM!**

$$\int_0^1 dx \left[\frac{1}{2}\Delta\Sigma(x) + \Delta G(x) + L_{q+\bar{q}}(x) + L_G(x)\right] = \frac{1}{2}$$



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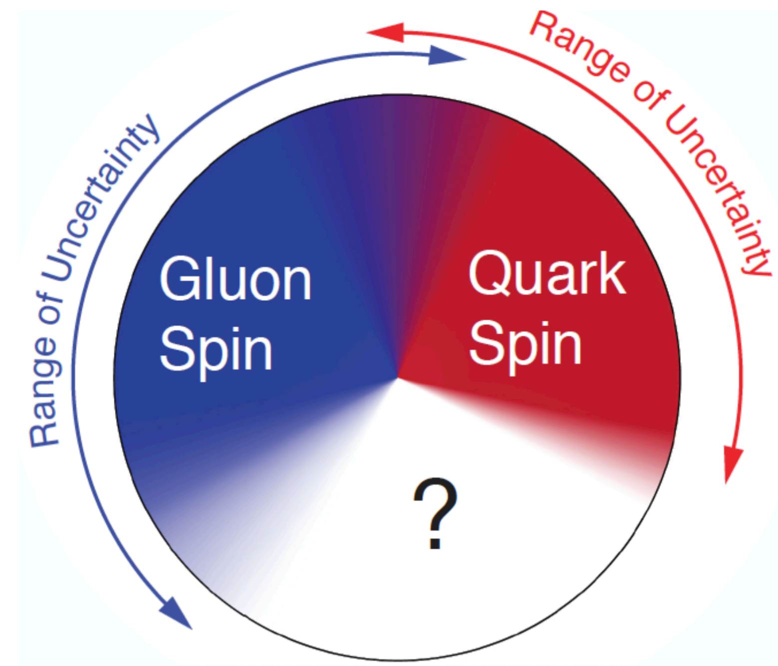
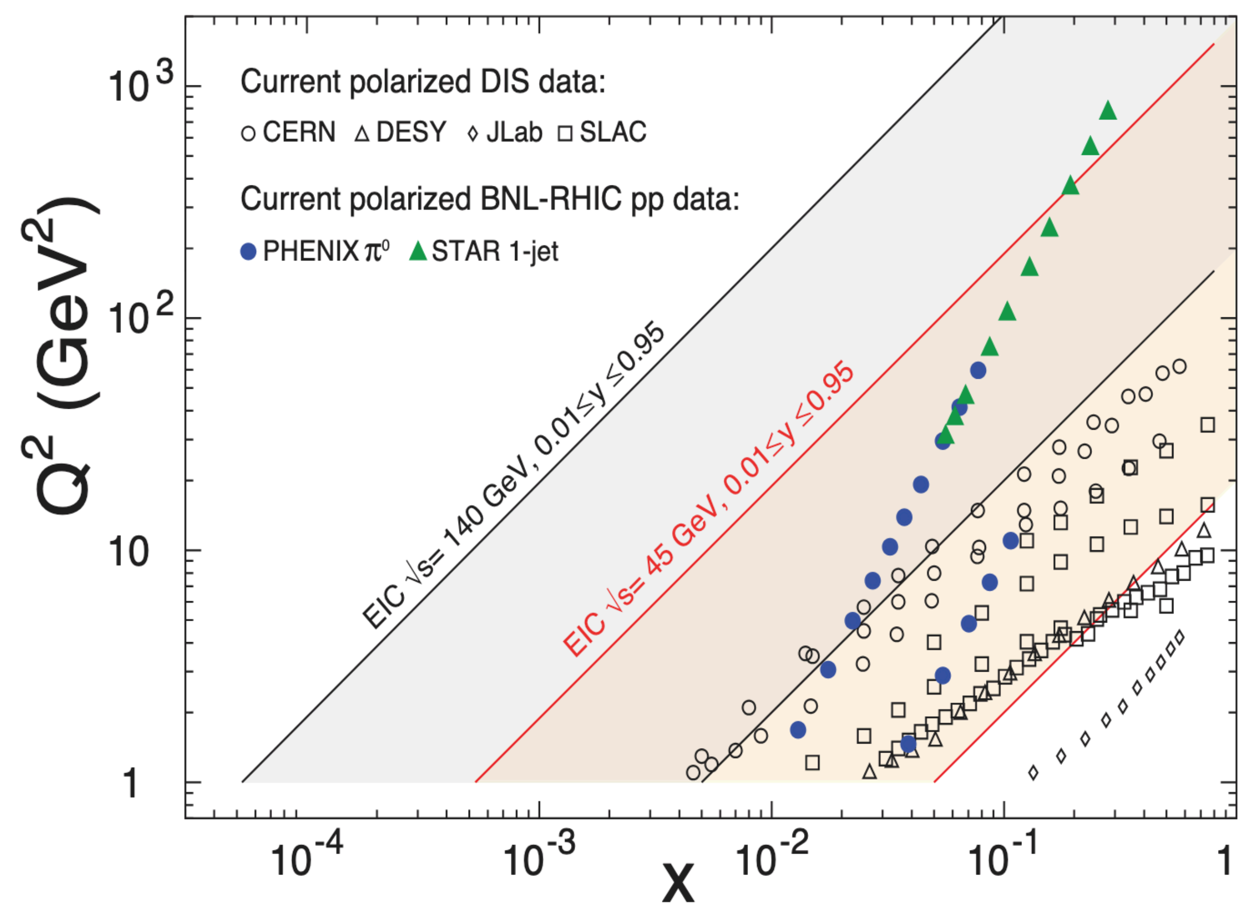


- Experiments cannot access down to $x = 0$
  $\rightarrow$ **Need small x evolution!**



OAM in QCD

Classical

QFT

OAM in QCD

Kinematics

$$L = I\,\omega$$

Structure

Classical

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OAM in QCD

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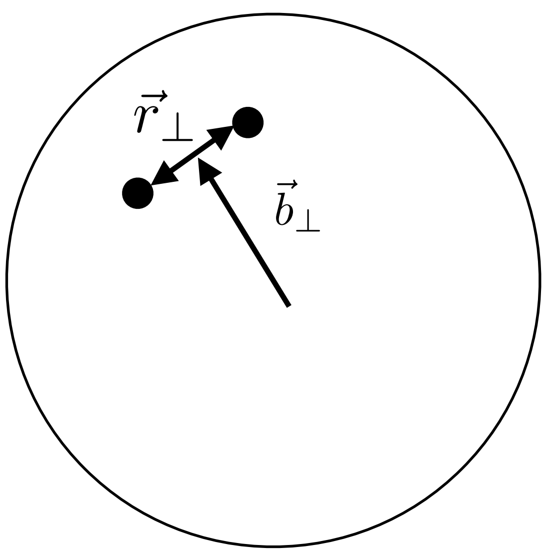

$$L \propto \int d^2 r_\perp \, I(\vec{r}_\perp, s)\, \mathcal{K}(\vec{r}_\perp)$$

"Moment amplitude"

$$I(\vec{r}_\perp, s) \propto \int d^2 b_\perp \, b_\perp \, G(\vec{r}_\perp, \vec{b}_\perp, s)$$

Polarized dipole amplitude (see Nick's talk)

Kovchegov (2019)
BM, Kovchegov (2023)

Small x evolution of OAM

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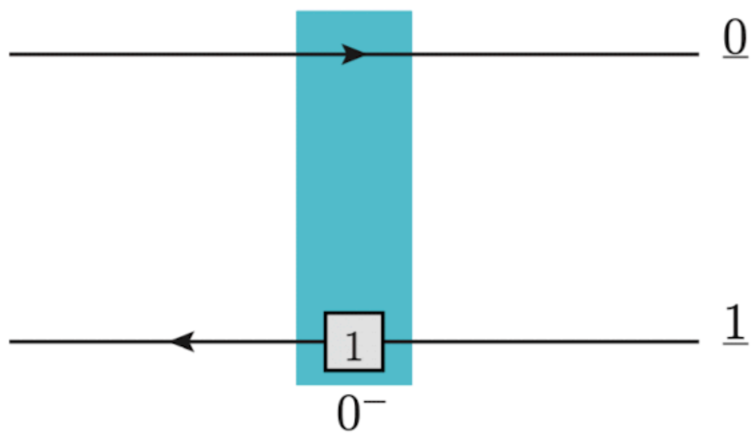
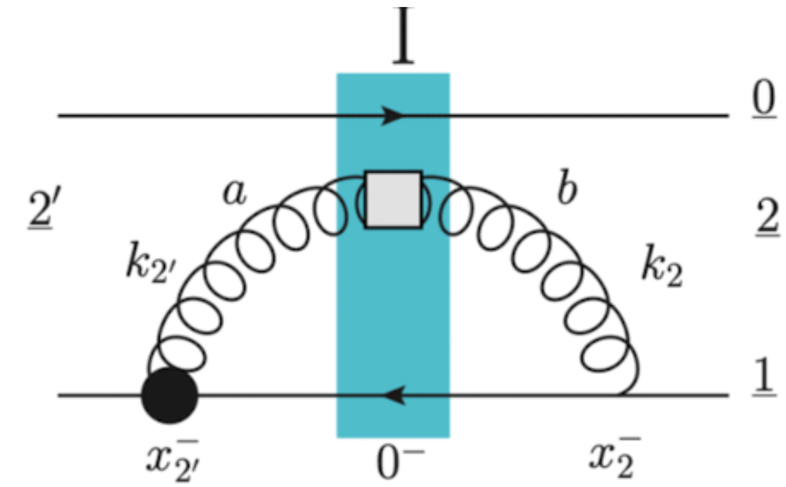
- Can evolve $I(\vec{r}_\perp, s)$ in energy s

$$\frac{\partial}{\partial s}$$



$$=$$



$$+ \cdots$$

BM, Kovchegov (2023)
BM (2024)

Small x evolution of OAM

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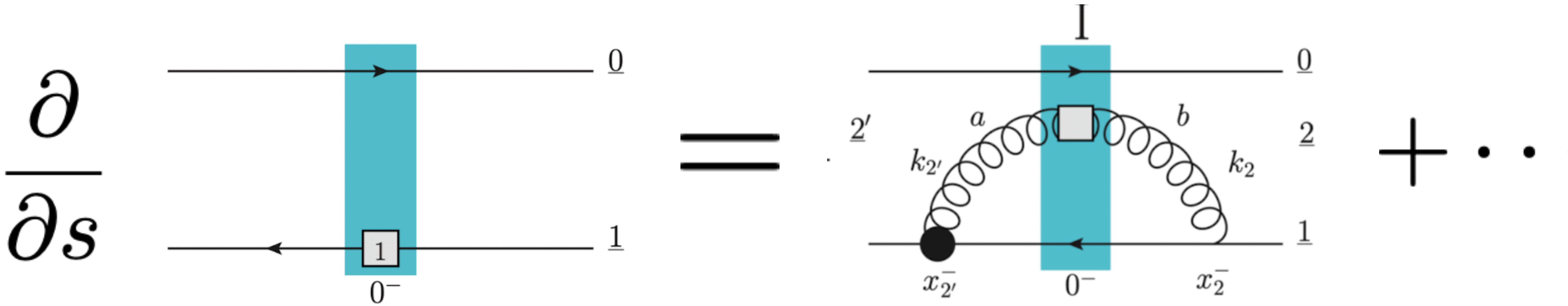

- Result: OAM **just as important as helicity PDFs!!**

$$L_{q+\bar{q}}(x) \sim L_G(x) \sim \Delta\Sigma(x) \sim \Delta G(x) \sim \left(\frac{1}{x}\right)^{3.66\sqrt{\frac{\alpha_s N_c}{2\pi}}}$$

BM, Kovchegov (2023)
BM (2024)

Measuring OAM

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(see Yoshitaka's lectures)

- Need two transverse momenta for OAM $\rightarrow$ Dijets!

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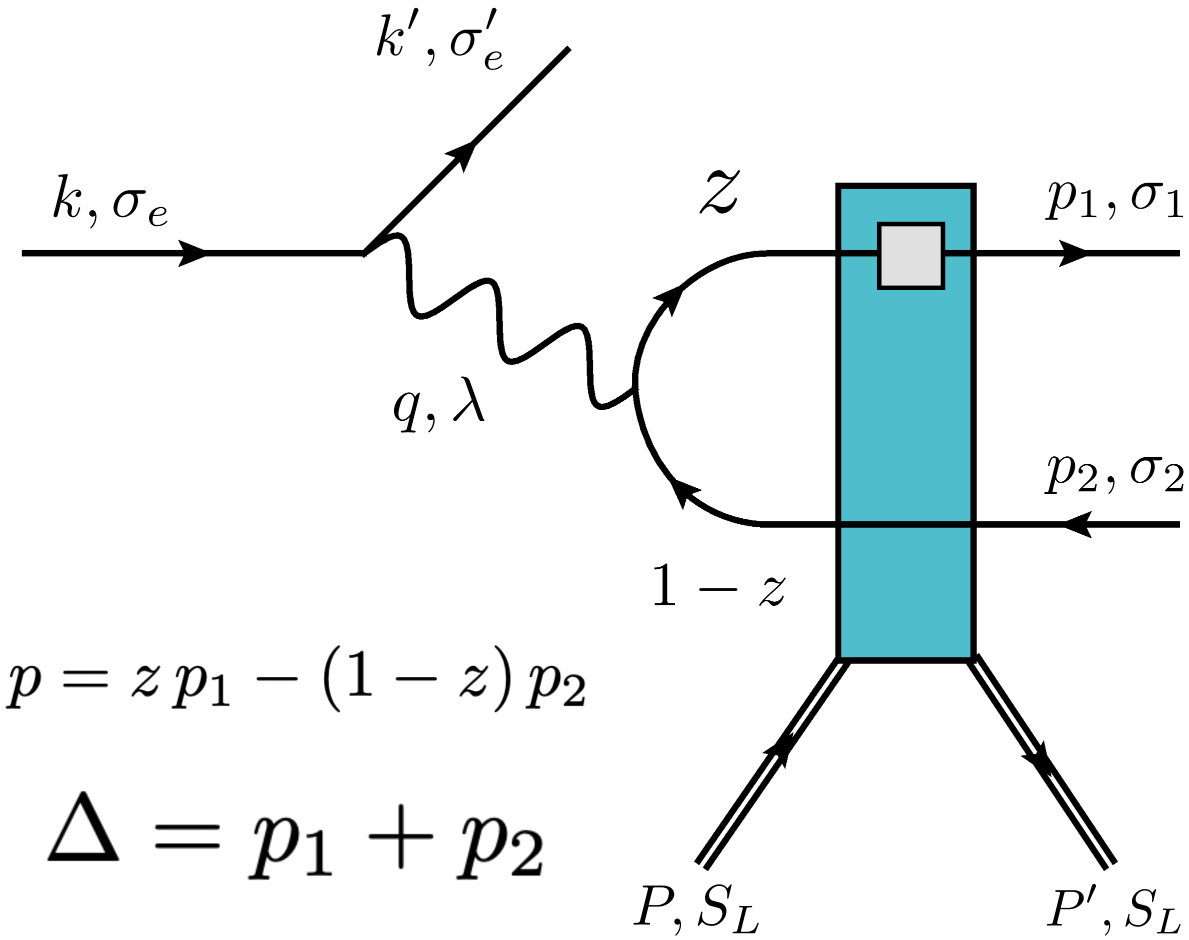

$$p = z\, p_1 - (1 - z)\, p_2$$

$$\Delta = p_1 + p_2$$

Measuring OAM

(see Yoshitaka's lectures)

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  → Dijets!

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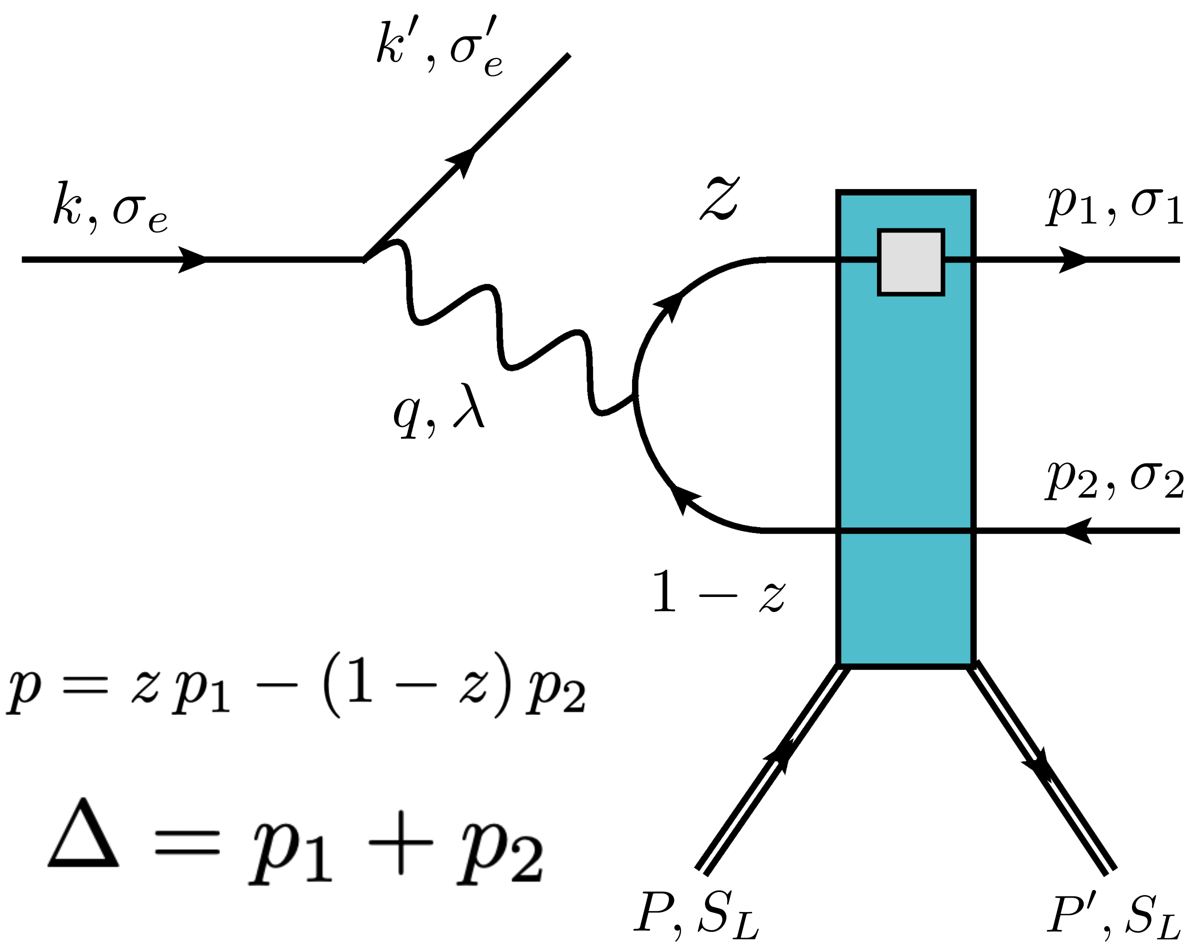


$$p = z\, p_1 - (1 - z)\, p_2$$

$$\Delta = p_1 + p_2$$

$$d\sigma^{\text{DSA}} = c_0 + \left(\vec{\Delta}_\perp \cdot \vec{p}_\perp\right) c_1 + \mathcal{O}(\Delta_\perp^2)$$

$$c_1 \propto \int d^2 r_\perp\, I(\vec{r}_\perp, s) N(\vec{r}_\perp, s) \mathcal{K}(\vec{r}_\perp)$$

Angular correlations
can access OAM!!

Hatta, Nakagawa, Yuan, Zhao, Xiao (2017)
Bhattacharya, Boussarie, Hatta (2022)
BM, Kovchegov (2024)

Backup

Elastic dijets

BM, Kovchegov '24

- Spin asymmetries

DSA: $$A_{LL}^{\gamma^*} = \frac{d\sigma(++) - d\sigma(+-)}{d\sigma(++) + d\sigma(+-)} \equiv \frac{d\sigma(++) - d\sigma(+-)}{2\, d\sigma_{unpol}} \equiv \frac{d\sigma^{DSA}}{d\sigma_{unpol}},$$

SSA: $$A_{UL}^{\gamma^*} = \frac{d\sigma(+) - d\sigma(-)}{d\sigma(+) + d\sigma(-)} = \frac{d\sigma(+) - d\sigma(-)}{2\, d\sigma_{unpol}} \equiv \frac{d\sigma^{SSA}}{d\sigma_{unpol}},$$

- PT symmetry determines which observable is more realistic

$$d\sigma = \Phi \otimes G \otimes S,$$

DSA contains leading amplitudes → $d\sigma^{\mathrm{DSA}} \sim N \otimes Q, N \otimes G_2, \ldots$ ✓

SSA contains "exotic" sub-leading amplitudes → $d\sigma^{\mathrm{SSA}} \sim \mathrm{Im} N \otimes Q, O \otimes Q^{NS}, \ldots$ ✗

Elastic dijets

- DSA in correlation limit: TT term

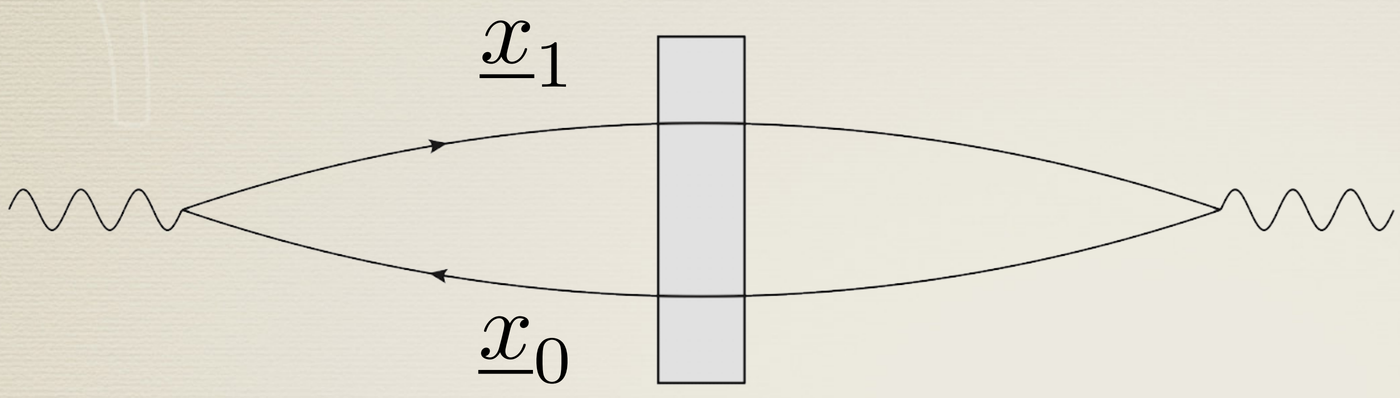
$$
\begin{aligned}
z(1-z)\,\frac{1}{2}\sum_{S_L,\lambda\pm1} S_L\,\lambda\,\frac{d\sigma^{\gamma^* p\to q\bar{q}p'}_{\text{symm.}\,\lambda\lambda}}{d^2p\,d^2\Delta\,dz} &= -\frac{2}{(2\pi)^5 z(1-z)s}\int d^2x_{12}\,d^2x_{1'2'}\,e^{-i\underline{p}\cdot(\underline{x}_{12}-\underline{x}_{1'2'})}\,N(x^2_{1'2'},s) \qquad (107\text{a})\\
&\times\Bigg\{\Bigg[\Bigg(1-2z+i\underline{\Delta}\cdot\underline{x}_{12}\,\left(z^2+(1-z)^2\right)-\frac{i}{2}\,\underline{\Delta}\cdot\underline{x}_{1'2'}\,(1-2z)^2\Bigg)\,Q(x^2_{12},s)-i\underline{\Delta}\cdot\underline{x}_{12}\,I_3(x^2_{12},s)\\
&\quad -i\underline{\Delta}\times\underline{x}_{12}\,J_3(x^2_{12},s)\Bigg]\,\Phi^{[1]}_{\mathrm{TT}}(\underline{x}_{12},\underline{x}_{1'2'},z)\\
&\quad+\Bigg[i(1-2z)\Big(\Delta^j\epsilon^{ji}x^2_{12}I_4(x^2_{12},s)+\underline{\Delta}\times\underline{x}_{12}\,x^i_{12}I_5(x^2_{12},s)+\Delta^i\,x^2_{12}J_4(x^2_{12},s)+\underline{\Delta}\cdot\underline{x}_{12}\,x^i_{12}J_5(x^2_{12},s)\Big)\\
&\quad -\left[1+i\,(1-2\,z)\,\underline{\Delta}\cdot\left(\underline{x}_{12}-\frac{\underline{x}_{1'2'}}{2}\right)\right]\left(\epsilon^{ik}x^k_{12}G_2(x^2_{12},s)+x^i_{12}G_1(x^2_{12},s)\right)\Bigg]\\
&\quad\times\left(\partial^i_{\underline{1}}-ip^i\right)\,\Phi^{[2]}_{\mathrm{TT}}(\underline{x}_{12},\underline{x}_{1'2'},z)\Bigg\}+\mathcal{O}(\Delta^2_\perp),
\end{aligned}
$$

- Azimuthal correlations: $\underline{\Delta}\cdot\underline{p}$

BM, Kovchegov ‘24

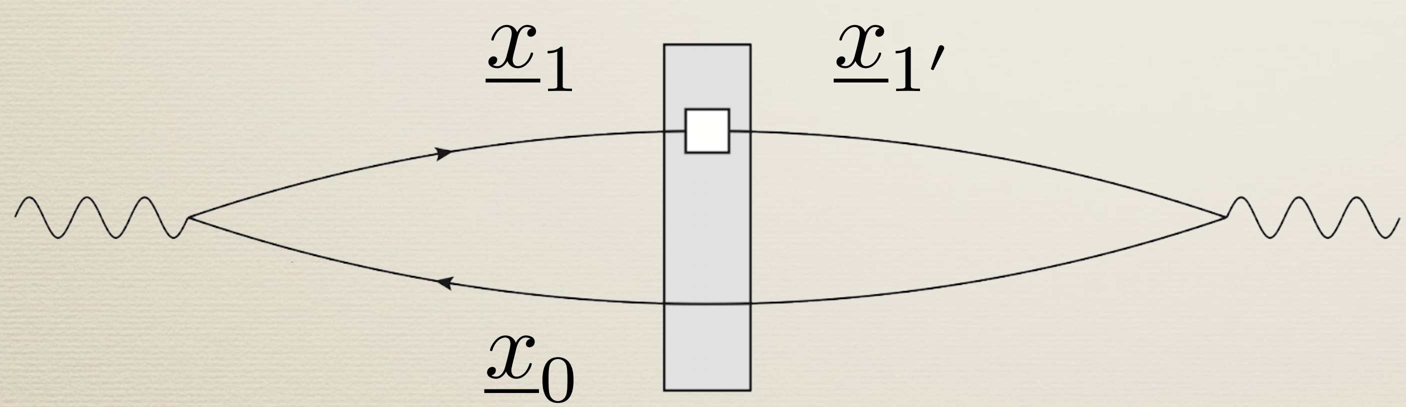
Polarized observables

* Unpolarized observables dominated by eikonal contributions



$$S \sim \mathrm{tr}\left[V_{\underline{x}_1} V^{\dagger}_{\underline{x}_0}\right]$$

* Polarized observables dominated by *sub*-eikonal contributions



$$S \sim \mathrm{tr}\left[V^{\mathrm{pol}}_{\underline{x}_1, \underline{x}_{1'}} V^{\dagger}_{\underline{x}_0}\right]$$

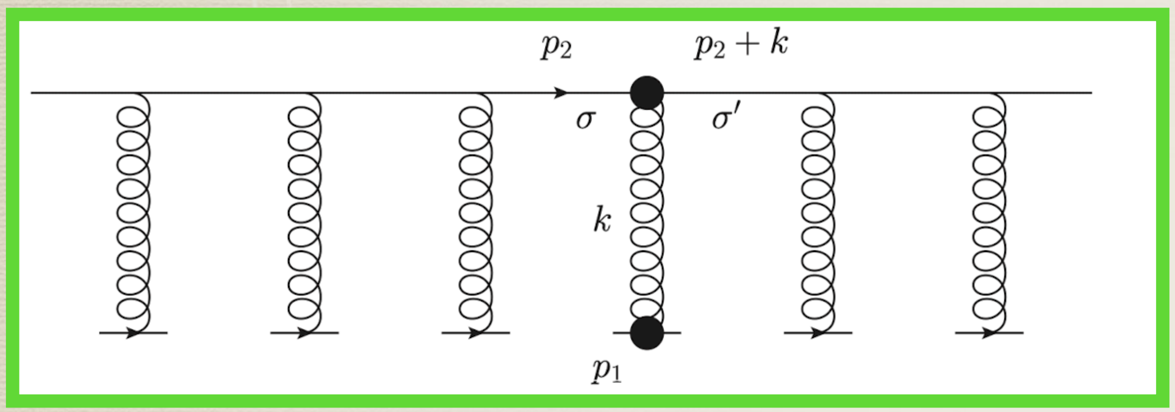
$$V_{\underline{x}}[x_f^-, x_i^-] \equiv \mathcal{P} \exp\left[ig \int\limits_{x_i^-}^{x_f^-} dx^- A^+(0^+, x^-, \underline{x}) \right] \qquad V_{\underline{x}} \equiv V_{\underline{x}}[\infty, -\infty]$$

Kovchegov, Pitonyak, Sievert '15 (1511.06737)

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Polarized propagators

- Sub-eikonal vertex sandwiched between semi-infinite Wilson lines (LCOT approach)



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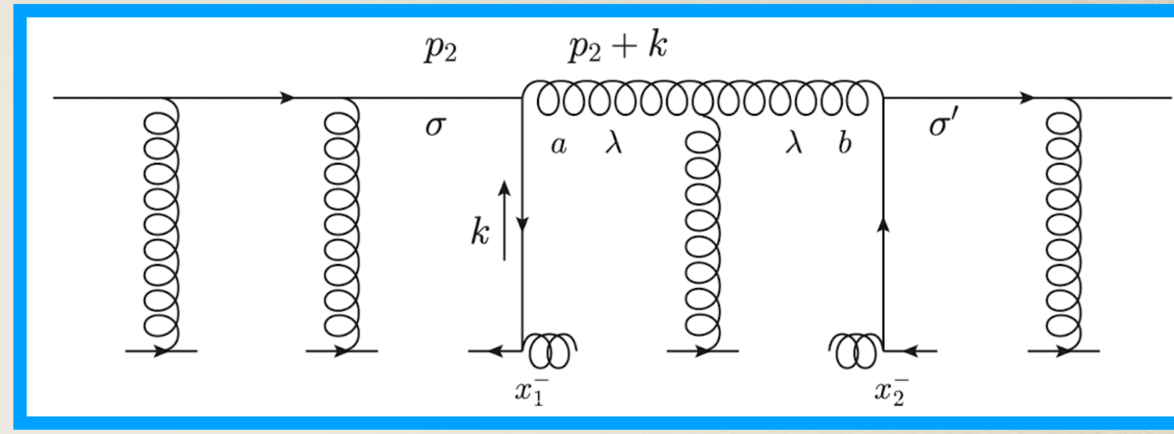


$$V^{\text{pol}}_{\underline{x},\underline{y};\sigma,\sigma'} = \sigma\delta_{\sigma,\sigma'}\left(V^{\text{q}[1]}_{\underline{x}} + V^{\text{G}[1]}_{\underline{x}}\right)_{\underline{y}=\underline{x}} + \delta_{\sigma,\sigma'}\left(V^{\text{q}[2]}_{\underline{x}}\Big|_{\underline{y}=\underline{x}} + V^{\text{G}[2]}_{\underline{x},\underline{y}}\right)$$

- *Polarized* dipole amplitudes

$$Q_{10}(zs) \propto \text{tr}\left[V_{\underline{x}_0}\left(V^{\text{pol}[1]}_{\underline{x}_1}\right)^\dagger + V^{\text{pol}[1]}_{\underline{x}_1}V^\dagger_{\underline{x}_0}\right]$$

$$G^i_{10}(zs) \propto \text{tr}\left[V^\dagger_{\underline{x}_0}V^{i\,\text{G}[2]}_{\underline{x}_1} + \left(V^{i\,\text{G}[2]}_{\underline{x}_1}\right)^\dagger V_{\underline{x}_0}\right]$$

$$V^i \propto \overleftarrow{D}^i - D^i$$

Kovchegov, Pitonyak, Sievert '15 (1511.06737)

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

OAM operator

Hatta '11 (1111.3547)
Ji, Xiong, Yuan '12 (1207.5221)

* Generic OAM operator

$$L_z\left(Q^2\right) = \int \frac{d^2 b_\perp d^2 k_\perp db^- dk^+}{(2\pi)^3} \left(\underline{b} \times \underline{k}\right) W(k, b)$$

where Wigner functions are

$$W^{q,\mathrm{SIDIS}}(k,b) = 2\sum_X \int d^2 r dr^- e^{ik\cdot r} \left\langle \bar{\psi}\left(b - \frac{1}{2}r\right) V_{\underline{b}-\frac{1}{2}\underline{r}}\left[b^- - \frac{1}{2}r^-, \infty\right] |X\rangle \frac{1}{2}\gamma^+ \right.$$
$$\left. \times \langle X| V_{\underline{b}+\frac{1}{2}\underline{r}}\left[\infty, b^- + \frac{1}{2}r^-\right] \psi\left(b + \frac{1}{2}r\right) \right\rangle$$

$$W^{G,\mathrm{dipole}}(k,b) = \frac{4}{xP^+} \int d^2 r dr^- e^{ixP^+ r^- - i\underline{k}\cdot\underline{r}} \left\langle \mathrm{tr}\left[F^{+i}\left(b - \frac{1}{2}r\right) \mathcal{U}^{[+]}(b,r) F^{+i}\left(b - \frac{1}{2}r\right) \mathcal{U}^{[-]}(b,r)\right] \right\rangle$$

Kovchegov '19 (1901.07453)

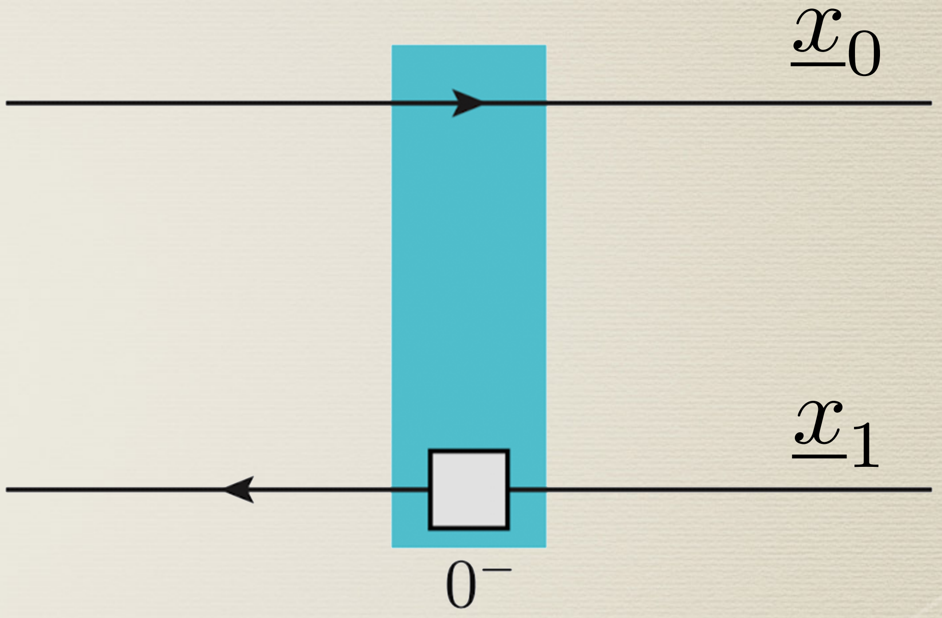
Future and past pointing staples

Polarized dipole amplitudes

- Impact-integrated polarized dipole amplitudes appear in calculations

$$Q(x_{10}^2, zs) \equiv \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) Q_{10}(zs)$$

$$G_2(x_{10}^2, zs) \equiv \frac{\epsilon^{ij} x_{10}^j}{x_{10}^2} \int d^2 \left(\frac{\underline{x}_0 + \underline{x}_1}{2} \right) G_{10}^i(zs)$$



- For OAM, need first x_1-moments (moment amplitudes)

$$I_3(x_{10}^2, zs) \equiv \frac{x_{10}^i}{x_{10}^2} \int d^2 x_1 \, x_1^i \, Q_{10}(zs)$$

$$\epsilon^{ij} \, I_4(x_{10}^2, zs) + \epsilon^{ik} x_{10}^k x_{10}^j \, I_5(x_{10}^2, zs) + \epsilon^{jk} x_{10}^k x_{10}^i \, I_6(x_{10}^2, zs) \equiv \int d^2 x_1 \, x_1^i \, G_{10}^j(zs)$$

OAM distributions at small x

* OAM distributions can be written in terms of the regular and moment dipole amplitudes

$$L_{q+\bar{q}}\left(x,Q^2\right) = \frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{1/zs}^{\min[1/zQ^2, 1/\Lambda^2]} \frac{dx_{10}^2}{x_{10}^2} \Big[Q - 3\,G_2 - I_3 - 2\,I_4 + I_5 + 3\,I_6\Big](x_{10}^2, zs)$$

$$L_G\left(x,Q^2\right) = \frac{2N_c}{\alpha_s \pi^2} \left[2\,I_4 + 3\,I_5 + I_6\right] \left(zs = \frac{Q^2}{x}, x_{10}^2 = \frac{1}{Q^2}\right)$$

* Compare to helicity PDFs at small x

$$\Delta\Sigma\left(x,Q^2\right) = -\frac{N_c N_f}{2\pi^3} \int\limits_{\Lambda^2/s}^{1} \frac{dz}{z} \int\limits_{1/zs}^{\min\left[1/zQ^2, 1/\Lambda^2\right]} \frac{dx_{10}^2}{x_{10}^2} \left[Q + 2\,G_2\right](x_{10}^2, zs)$$

$$\Delta G\left(x,Q^2\right) = \frac{2N_c}{\alpha_s \pi^2} G_2 \left(zs = \frac{Q^2}{x}, x_{10}^2 = \frac{1}{Q^2}\right)$$

Cougoulic, Kovchegov, Tarasov, Tawabutr '22 (2204.11898)

Moment amplitude evolution

* Large-N_c evolution equations

$$\begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix}(x_{10}^2, zs) = \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix}_0 (x_{10}^2, zs)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{1}{s x_{10}^2}}^{z} \frac{dz'}{z'} \int\limits_{\frac{1}{z' s}}^{x_{10}^2} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 2\,\Gamma_3 - 4\,\Gamma_4 + 2\,\Gamma_5 + 6\,\Gamma_6 - 2\,\Gamma_2 \\ 0 \\ 0 \\ 0 \end{pmatrix} (x_{10}^2, x_{21}^2, z' s)$$

$$+ \frac{\alpha_s N_c}{4\pi} \int\limits_{\frac{\Lambda^2}{s}}^{z} \frac{dz'}{z'} \int\limits_{\max[x_{10}^2, \frac{1}{z' s}]}^{\min[\frac{z}{z'} x_{10}^2, \frac{1}{\Lambda^2}]} \frac{dx_{21}^2}{x_{21}^2} \begin{pmatrix} 4 & -4 & 2 & 6 & -4 & -6 \\ 0 & 4 & 2 & -2 & 0 & 1 \\ -2 & 2 & -1 & -3 & 2 & 3 \\ 0 & 0 & 0 & 0 & 2 & 4 \end{pmatrix} \begin{pmatrix} I_3 \\ I_4 \\ I_5 \\ I_6 \\ G \\ G_2 \end{pmatrix} (x_{21}^2, z' s).$$

Mixing with "regular" polarized amplitudes!

(cf. polarized DGLAP)