Key processes to access parton Orbital Angular Momentum at the EIC

ePIC/EIC Early Science Workshop

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University of Connecticut
April 24, 2025

In Collaboration with:
Duxin Zheng, Jian Zhou **(PRL 133, 051901)**

Renaud Boussarie, Yoshitaka Hatta
(PRL 128, 182002/ PRD 111, 034019)

Outline

- Generalized TMDs & connection to spin physics
- Observable(s) for quark/gluon OAM & spin-orbit correlations
- Summary

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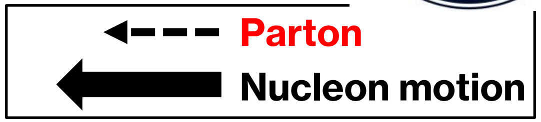
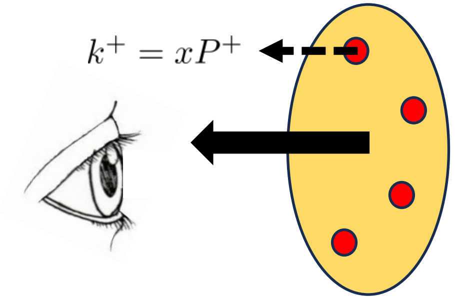
I will look at exclusive ep processes unless otherwise stated
(EIC early science matrix: Years 2-4)

Wigner function - The "mother function"



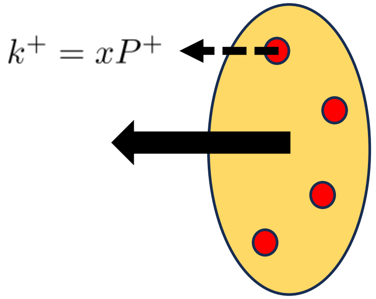


Parton Distribution Functions

PDFs (x)

Wigner function - The “mother function”

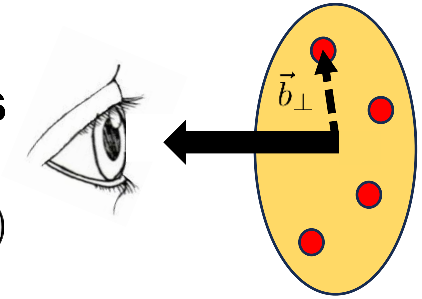
Parton
Nucleon motion



Form Factors

PDFs (x)

FFs (Δ)

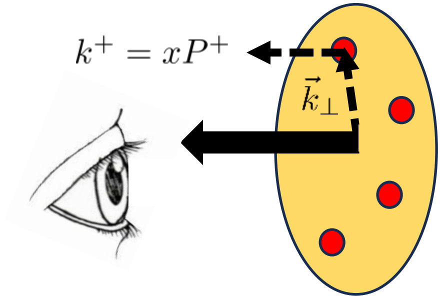


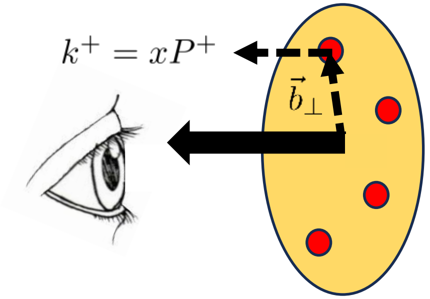
Wigner function - The “mother function”

Parton
Nucleon motion

Transverse Momentum-dependent Distributions

Generalized Parton Distributions

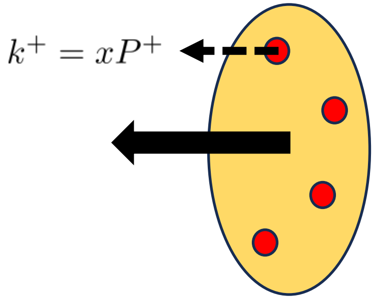


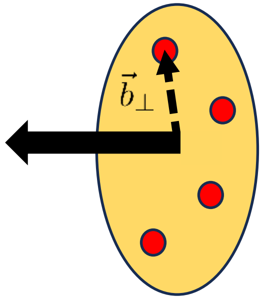


TMDs $(x, \vec{k}_\perp)$ **GPDs** (x, Δ)

$\int d^2\vec{k}_\perp$ $\Delta = 0$ $\int dx$

PDFs (x) **FFs** (Δ)





Wigner function - The “mother function”

Generalized Transverse Momentum-dependent Distributions

(Meissner, Metz, Schlegel, 2009)

$$
\begin{tikzcd}
 & \textbf{GTMDs}\ (x, \vec{k}_\perp, \Delta) \arrow[dl, "\Delta = 0"'] \arrow[dr, "\int d^2\vec{k}_\perp"] & & \\
\textbf{TMDs}\ (x, \vec{k}_\perp) \arrow[dr, "\int d^2\vec{k}_\perp"'] & & \textbf{GPDs}\ (x, \Delta) \arrow[dl, "\Delta = 0"'] \arrow[dr, "\int dx"] & \\
 & \textbf{PDFs}\ (x) & & \textbf{FFs}\ (\Delta)
\end{tikzcd}
$$

Wigner function - The “mother function”

Wigner functions $(x, \vec{k}_\perp, \vec{b}_\perp)$ (Belitsky, Ji, Yuan, 2003)

2-D Fourier Transform $(\vec{\Delta}_\perp)$

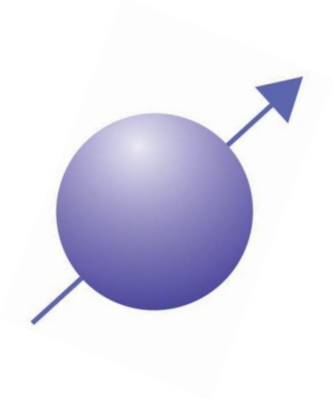
(Meissner, Metz, Schlegel, 2009) **GTMDs** $(x, \vec{k}_\perp, \Delta)$

$$
\begin{tikzcd}[column sep=small]
 & & \text{Wigner functions } (x, \vec{k}_\perp, \vec{b}_\perp) & & \\
 & & \text{GTMDs } (x, \vec{k}_\perp, \Delta) \arrow[u, "\text{2-D Fourier Transform } (\vec{\Delta}_\perp)"] \arrow[dl, "\Delta = 0"'] \arrow[dr, "\int d^2\vec{k}_\perp"] & & \\
 & \text{TMDs } (x, \vec{k}_\perp) \arrow[dr, "\int d^2\vec{k}_\perp"'] & & \text{GPDs } (x, \Delta) \arrow[dl, "\Delta = 0"'] \arrow[dr, "\int dx"] & \\
 & & \text{PDFs } (x) & & \text{FFs } (\Delta)
\end{tikzcd}
$$

Spin of proton

Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2}$$

Spin of proton

Jaffe-Manohar spin decomposition

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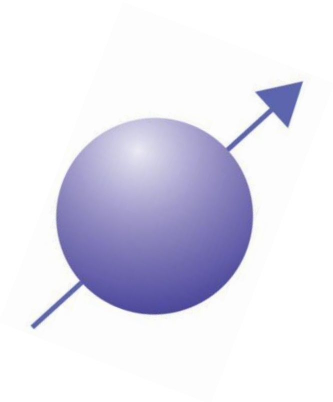

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma$$

Best known

Quark helicity $\sim 30\%$

Spin of proton

Jaffe-Manohar spin decomposition

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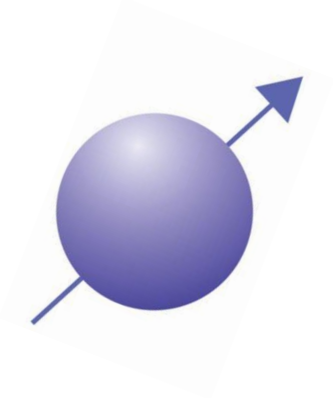


$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G$$

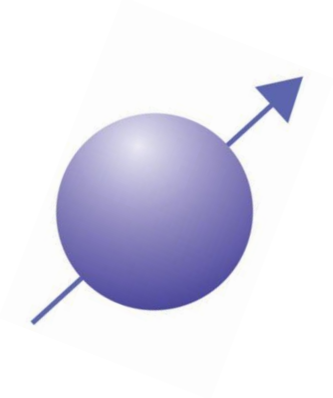
Best known **How well do we know?**

Quark helicity $\sim 30\%$ **Gluon helicity** $\sim 40\%$

Spin of proton

Jaffe-Manohar spin decomposition

An incomplete story:



$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Best known | **How well do we know?** | **???????**

Quark helicity $\sim 30\%$ | **Gluon helicity** $\sim 40\%$ | **OAM of quarks & gluons**

So far, <u>no</u> experimental constraints on OAM of quarks & gluons

Wigner functions & Orbital Angular Momentum

Wigner functions in Quantum Mechanics

(Wigner, 1932)

- Calculate from wave functions:

$$W(x,k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x+\frac{x'}{2})\, \psi^*(x-\frac{x'}{2})$$

- Expectation value of observables:

$$\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$$

Wigner functions & Orbital Angular Momentum

Wigner functions in Quantum Mechanics (Wigner, 1932)	**Wigner functions in parton physics (Belitsky, Ji, Yuan, 2003)**
• Calculate from wave functions: $W(x,k) = \int \frac{dx'}{2\pi} e^{-ikx'} \psi(x+\frac{x'}{2})\, \psi^*(x-\frac{x'}{2})$	• Calculate from fourier transform of GTMD correlator: $W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$
• Expectation value of observables: $\langle \mathcal{O} \rangle = \int dx \int dk \mathcal{O}(x,k) W(x,k)$	• Application: **O**rbital **A**ngular **M**omentum (**OAM**) $L_z^{q,g} = \int dx \int d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z\, W^{q,g}\ (x, \vec{b}_\perp, \vec{k}_\perp)$ (Lorcé, Pasquini, 2011 / Hatta, 2011)

Wigner functions & Orbital Angular Momentum

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Wigner functions in parton physics

(Belitsky, Ji, Yuan, 2003)

- Calculate from fourier transform of GTMD correlator:

$$W^{[\Gamma]}(x, \vec{k}_\perp, \vec{b}_\perp)$$

- Application: Relation between GTMD $\boldsymbol{F_{1,4}^{q,g}}$ & OAM

$$\boxed{L_z^{q,g} = -\int dx \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} \boldsymbol{F_{1,4}^{q,g}(x, k_\perp, \xi = 0, \Delta_\perp = 0)}}$$

(Lorcé, Pasquini, 2011 / Hatta, 2011)

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Wigner functions in Quantum Mechanics (Wigner, 1932)	Wigner functions in parton physics (Belitsky, Ji, Yuan, 2003)
• Calculate from wav	of GTMD correlator:

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• Expectation value of observables:

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Big question:
Experimental observable?

• Application: Relation between GTMD $F_{1,4}^{q,g}$ & OAM

$$\boxed{L_z^{q,g} = -\int dx \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, k_\perp, \xi = 0, \Delta_\perp = 0)}$$

(Lorcé, Pasquini, 2011 / Hatta, 2011)

Developments

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]

Developments

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Exclusive double quarkonium production and generalized TM

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Probing the Weizsäcker-Williams gluon Wigner distribution in *pp* collisions

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Probing the gluon Sivers function with an unpolarized target: GTMD distributions and the Odderons

Renaud Boussarie,[1] Yoshitaka Hatta,[1] Lech Szymanowski,[2] and Samuel Wallon[3,4]

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Exclusive double quarkonium production and generalized TM...

...tion in pp collisions

...d Feng Yuan[5]

Exclusive double Drell-Yan:

Until now, this has been the sole known process sensitive to quark GTMDs

...ction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

Renaud Boussarie,[1] Yoshitaka Hatta,[1] Lech Szymanowski,[2] and Sa...

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Probing quark OAM through double Drell-Yan

Main findings

Physics Letters B 771 (2017) 396–400

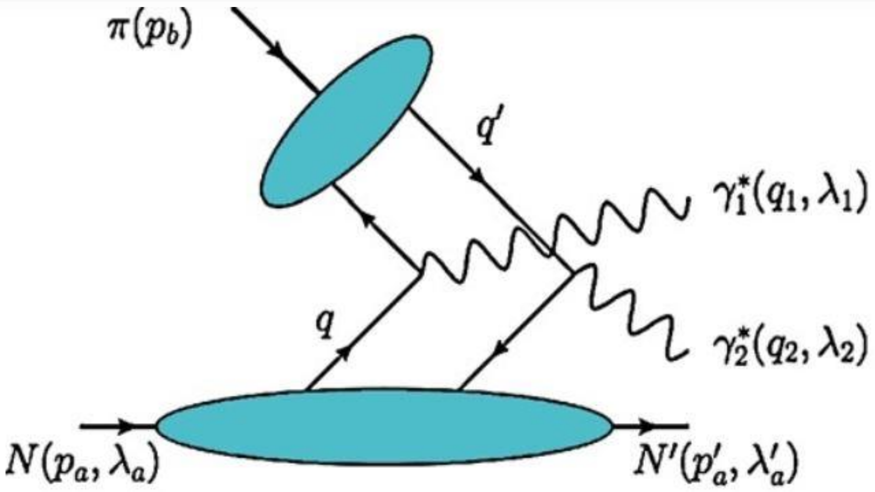
Contents lists available at ScienceDirect

Physics Letters B

www.elsevier.com/locate/physletb

Generalized TMDs and the exclusive double Drell–Yan process

Shohini Bhattacharya [a], Andreas Metz [a,*], Jian Zhou [b]



Probing quark OAM through double Drell-Yan

Main findings

Example of an observable sensitive to OAM

$$\frac{1}{2}\left(\tau_{XY}-\tau_{YX}\right) = \frac{4}{M_a^2}\left(\varepsilon_\perp^{ij}\Delta q_\perp^i\Delta_{a\perp}^j\right)\mathrm{Re.}\left\{C^{(-)}\left[F_{1,1}\,\phi_\pi\right]C^{(+)}\left[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\,\boldsymbol{F^*_{1,4}}\,\phi_\pi^*\right]\right.$$

Probing quark OAM through double Drell-Yan

Main findings

Example of an observable sensitive to OAM & spin-orbit correlation :

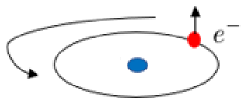
$$\frac{1}{2}(\tau_{XY}-\tau_{YX}) = \frac{4}{M_a^2}(\varepsilon_\perp^{ij}\Delta q_\perp^i \Delta_{a\perp}^j)\,\mathrm{Re.}\Big\{C^{(-)}\Big[F_{1,1}\,\phi_\pi\Big]C^{(+)}\Big[\vec{\beta}_\perp\cdot\vec{k}_{a\perp}\,\boldsymbol{F^*_{1,4}}\,\phi_\pi^*\Big] - C^{(+)}\Big[G_{1,4}\,\phi_\pi\Big]C^{(-)}\Big[\vec{\beta}_\perp\cdot\vec{p}_{a\perp}\,\boldsymbol{G^*_{1,1}}\,\phi_\pi^*\Big]\Big\}$$

Spin-orbit entanglement in the Color Glass Condensate

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[3,4,‡]

2404.04208

Recall Spin-Orbit coupling in H atom!



$$\boldsymbol{G_{1,1}^{q/g}} \to L^{q/g}\cdot S^{q/g}$$

See also:
Lorcé, Pasquini, 2011

Probing quark OAM through double Drell-Yan

Main findings

Challenges:

- Low count rate (Amplitude $\sim \alpha_{em}^2$)

Probing quark OAM through double Drell-Yan

Main findings

Challenges:

- Low count rate (Amplitude $\sim \alpha_{em}^2$)
- Sensitivity to GTMDs only in the ERBL region $-\xi < x < \xi$

OAM density: $$L^{q/g}(x, \xi) = -\int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^{q,g}(x, k_\perp, \xi, \Delta_\perp = 0)$$

OAM: $$L^{q/g} = \int dx\, L^{q/g}(x, \boldsymbol{\xi = 0})$$

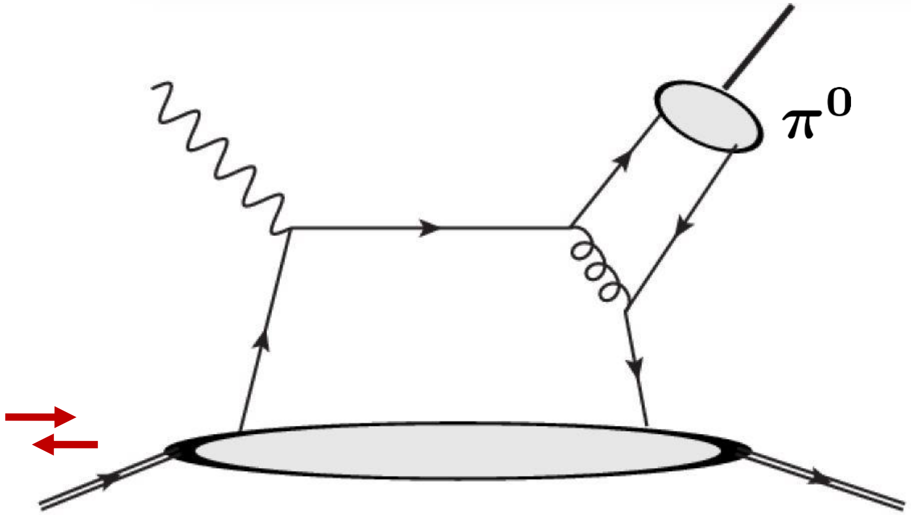
The challenge lies in extrapolating the distribution to the forward limit, where the OAM equation is applicable

Our work

PHYSICAL REVIEW LETTERS **133**, 051901 (2024)

Probing the Quark Orbital Angular Momentum at Electron-Ion Colliders Using Exclusive π^0 Production

Shohini Bhattacharya,[1] Duxin Zheng,[2] and Jian Zhou[3]

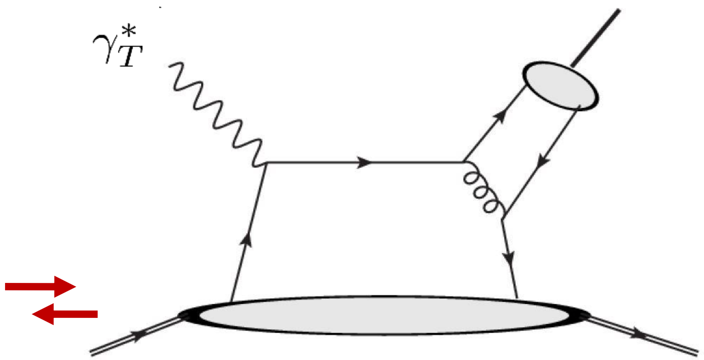


Main Observable:

Longitudinal single-target spin asymmetry

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



4 leading-order Feynman diagrams

Probing quark OAM through π^0 production in ep collisions

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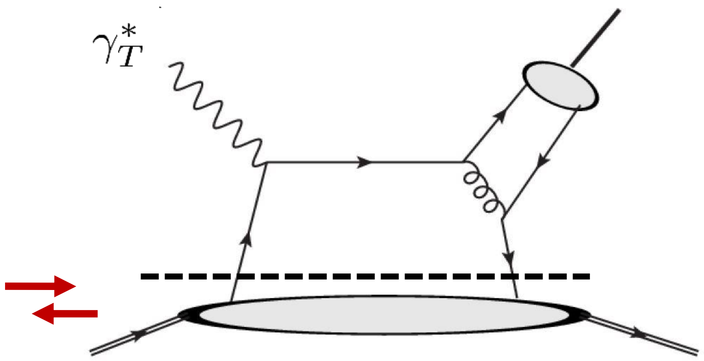
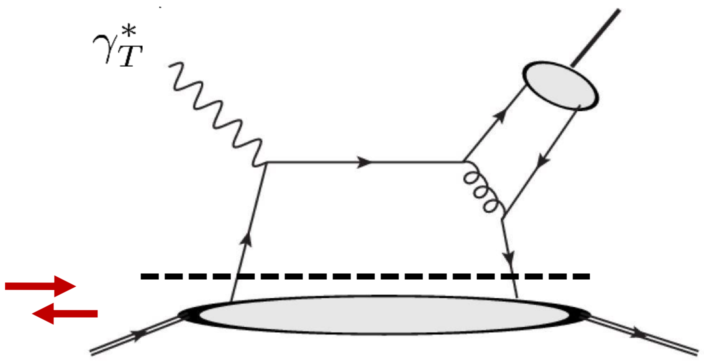


Scattering amplitude:

$$\boxed{A \propto \int dx \int d^2k_\perp \, \underbrace{H(x,\xi,z,k_\perp,\Delta_\perp)}_{\text{Hard part}} \, \underbrace{f^q(x,\xi,k_\perp,\Delta_\perp)}_{\text{Soft part from proton}} \int dz \, \underbrace{\phi_\pi(z)}_{\text{Pion Distribution Amplitude}}}$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



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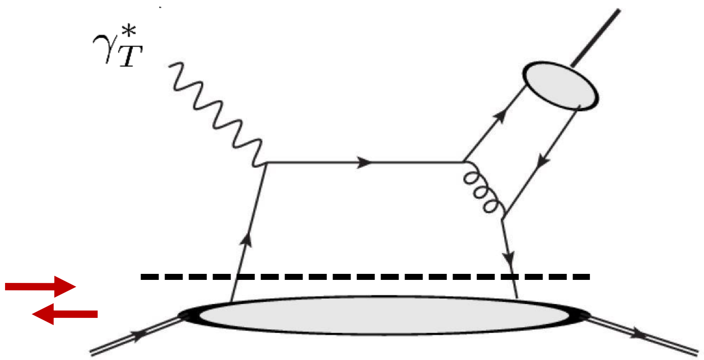
Collinear twist-expansion of hard part:

$$H(k_\perp,\Delta_\perp) = H(k_\perp=0,\Delta_\perp=0) + \frac{\partial H(k_\perp,\Delta_\perp=0)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} k_\perp^\mu + \frac{\partial H(k_\perp=0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\Big|_{\Delta_\perp=0} \Delta_\perp^\mu + \dots$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

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Collinear twist-expansion of hard part:

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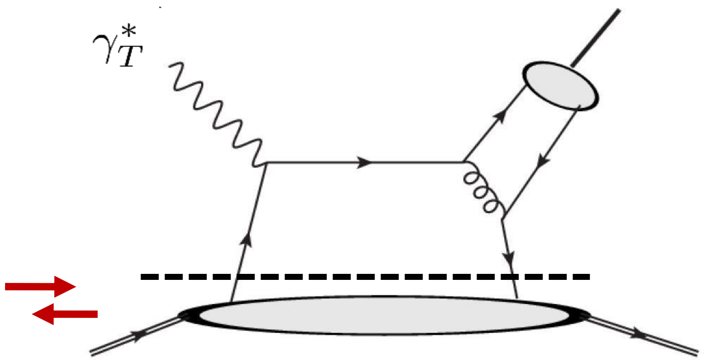
Twist 2 term vanishes

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



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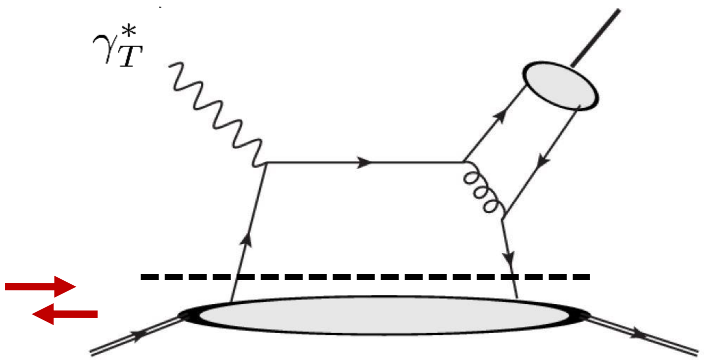
$$A \propto \int dx \int d^2k_\perp \, H(x,\xi,z,k_\perp,\Delta_\perp)\, f^q(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)$$

Collinear twist-expansion of hard part:

$$H(k_\perp,\Delta_\perp) = H(k_\perp = 0, \Delta_\perp = 0) + \underbrace{\left.\frac{\partial H(k_\perp,\Delta_\perp = 0)}{\partial k_\perp^\mu}\right|_{k_\perp=0} k_\perp^\mu + \left.\frac{\partial H(k_\perp = 0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\right|_{\Delta_\perp=0} \Delta_\perp^\mu}_{\text{Twist 3 term}} + \ldots$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$\boxed{A \propto \int dx \int \boldsymbol{d^2 k_\perp}\, H(x,\xi,z,k_\perp,\Delta_\perp)\, \boldsymbol{f^q}(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)}$$

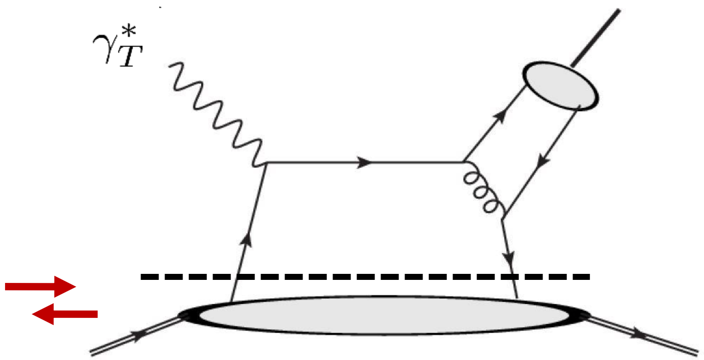
Collinear twist-expansion of hard part:

$$H(k_\perp,\Delta_\perp) = H(k_\perp=0,\Delta_\perp=0) + \frac{\partial H(k_\perp,\Delta_\perp=0)}{\partial k_\perp^\mu}\Big|_{k_\perp=0} \boldsymbol{k_\perp^\mu} + \frac{\partial H(k_\perp=0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\Big|_{\Delta_\perp=0} \Delta_\perp^\mu + \ldots$$

$$\boxed{A \propto \int \boldsymbol{d^2 k_\perp k_\perp^2}\ \mathbf{GTMD}}$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int d^2k_\perp\, H(x,\xi,z,k_\perp,\Delta_\perp)\, f^q(x,\xi,k_\perp,\Delta_\perp) \int dz \phi_\pi(z)$$

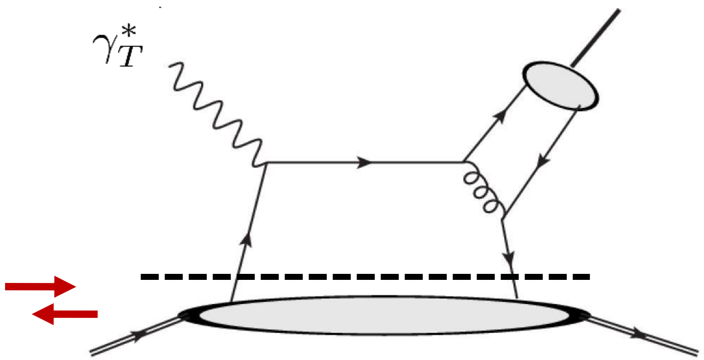
Collinear twist-expansion of hard part:

$$H(k_\perp,\Delta_\perp) = H(k_\perp=0,\Delta_\perp=0) + \frac{\partial H(k_\perp,\Delta_\perp=0)}{\partial k_\perp^\mu}\bigg|_{k_\perp=0} k_\perp^\mu + \frac{\partial H(k_\perp=0,\Delta_\perp)}{\partial \Delta_\perp^\mu}\bigg|_{\Delta_\perp=0} \Delta_\perp^\mu + \ldots$$

$$\boxed{A \propto \mathrm{GPD}}$$

Probing quark OAM through π^0 production in ep collisions

Scattering amplitude



Scattering amplitude:

$$A \propto \int dx \int d^2k_\perp \, H(x, \xi, z, k_\perp, \Delta_\perp) \, f^q(x, \xi, k_\perp, \Delta_\perp) \int dz \phi_\pi(z)$$

Collinear twist-expansion of hard part:

Consequently, the scattering amplitudes are a convolution of moments of GTMDs and GPDs and are of twist-3 nature

Probing quark OAM through π^0 production in ep collisions

Angular correlations

Scattering amplitudes depend on different angular correlations:

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \times \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda,-\lambda'} \frac{M\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{S}_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\} \qquad S_\perp^\mu = (0^+, 0^-, -i, \lambda)$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\boldsymbol{\epsilon}_\perp \cdot \boldsymbol{\Delta}_\perp}{Q^2} \{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}$$

Probing quark OAM through π^0 production

Compton Form Factors:

$$\mathcal{F}_{1,1} = \int_{-1}^{1} dx \frac{x^2 \int d^2k_\perp F_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^{1} dx x \frac{\xi(1-\xi^2)\int d^2k_\perp k_\perp^2 F_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (10)$$

$$\mathcal{G}_{1,2} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)(1-\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (11)$$

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (12)$$

$$\mathcal{G}_{1,4} = \int_{-1}^{1} dx \int_0^1 dz \frac{x(4\xi^2 z+\xi^2-2x^2z+x^2)}{z^2\xi(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\phi_\pi(z) \times \int d^2k_\perp G_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp). \quad (13)$$

Angular correlations

Scattering amplitudes depend on different angular correlations:

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda\lambda'} \frac{\epsilon_\perp \times \Delta_\perp}{Q^2} \{\mathcal{F}_{1,1}+\mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda,-\lambda'} \frac{M\epsilon_\perp \cdot S_\perp}{Q^2} \{\mathcal{F}_{1,2}+\mathcal{G}_{1,2}\}$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{Q^2} \{\mathcal{F}_{1,4}+\mathcal{G}_{1,4}\}$$

Probing quark OAM through π^0 production in ep collisions

Angular correlations

Scattering amplitudes depend on different angular correlations:

$$\mathcal{M}_1 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda\lambda'} \frac{\epsilon_\perp \times \Delta_\perp}{Q^2} \{\mathcal{F}_{1,1} + \mathcal{G}_{1,1}\}$$

$$\mathcal{M}_2 = \frac{g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \delta_{\lambda,-\lambda'} \frac{M\epsilon_\perp \cdot S_\perp}{Q^2} \{\mathcal{F}_{1,2} + \mathcal{G}_{1,2}\}$$

$$\mathcal{M}_4 = \frac{i g_s^2 e f_\pi}{2\sqrt{2}} \frac{(N_c^2-1)2\xi}{N_c^2\sqrt{1-\xi^2}} \lambda\delta_{\lambda\lambda'} \frac{\epsilon_\perp \cdot \Delta_\perp}{Q^2} \boxed{\{\mathcal{F}_{1,4} + \mathcal{G}_{1,4}\}}$$

Sensitivity to quark OAM

$$\mathcal{F}_{1,1} = \int_{-1}^{1} dx \frac{x^2 \int d^2k_\perp F_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (8)$$

$$\mathcal{G}_{1,1} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,1}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (9)$$

$$\mathcal{F}_{1,2} = \int_{-1}^{1} dx x \frac{\xi(1-\xi^2) \int d^2k_\perp k_\perp^2 F_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (10)$$

$$\mathcal{G}_{1,2} = \int_{-1}^{1} dx \int_0^1 dz \frac{\phi_\pi(z)(x^2+2x^2z+\xi^2)(1-\xi^2)}{z^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int d^2k_\perp \frac{k_\perp^2}{M^2} G_{1,2}^{u+d}(x,\xi,\Delta_\perp,k_\perp), \quad (11)$$

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}, \quad (12)$$

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Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2\right] + \cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.$$

$$\left.+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Unpolarized

Single spin asymmetry

$$a=\frac{2(1-y)}{1+(1-y)^2}$$

Probing quark OAM through π^0 production in ep collisions

Cross section

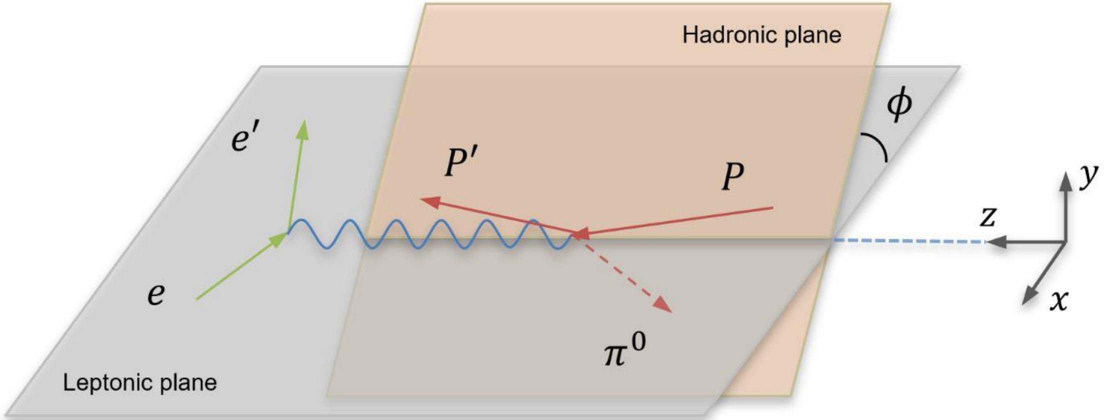
$$\frac{d\sigma}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2-1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4 (1-\xi^2) Q^{10} (1+\xi)} [1+(1-y)^2]$$

$$\times \left\{ \left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2} |\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 \right] \right.$$

$$\boxed{\left. + \lambda \sin(2\phi)\, 2a\, \mathrm{Re} \left[(i\mathcal{F}_{1,4} + i\mathcal{G}_{1,4}) \left(\mathcal{F}_{1,1}^* + \mathcal{G}_{1,1}^* \right) \right] \right\}}$$

Distinguished experimental signature of quark OAM

$$\phi = \phi_{l_\perp} - \phi_{\Delta_\perp}$$



Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dt\,dQ^2\,dx_B\,d\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\boxed{\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\boldsymbol{\mathcal{F}_{1,2}}+\mathcal{G}_{1,2}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]}$$

$$+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\bigg\}$$

Surprise!

- Probe quark Sivers function through an unpolarized target

$$\boxed{\mathrm{Im}\left[\boldsymbol{F_{1,2}}\right]\Big|_{\Delta=0} = -\boldsymbol{f_{1T}^{\perp}}}$$

(Similar to the gluon GTMD $F_{1,2}$, as discussed in Boussarie, Hatta, Szymanowski, Wallon, 2019)

Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dtdQ^2dx_Bd\phi}=\frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2f_\pi^2\xi^3\Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\boxed{\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\Delta_\perp^2}|\boldsymbol{\mathcal{F}_{1,2}}+\boldsymbol{\mathcal{G}_{1,2}}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]\right.}$$

$$\left.+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\right\}$$

Surprise!

- Probe quark Sivers function through an unpolarized target $\boxed{\mathrm{Im}\left[\boldsymbol{F_{1,2}}\right]\big|_{\Delta=0}=-\boldsymbol{f_{1T}^{\perp}}}$

- Probe quark worm gear function through an unpolarized target $\boxed{\mathrm{Re}\left[\boldsymbol{G_{1,2}}\right]\big|_{\Delta=0}=\boldsymbol{g_{1T}}}$

Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dt\,dQ^2\,dx_B\,d\phi} = \frac{(N_c^2-1)^2\alpha_{em}^2\alpha_s^2 f_\pi^2 \xi^3 \boldsymbol{\Delta_\perp^2}}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)}\left[1+(1-y)^2\right]$$

$$\boxed{\times\left\{\left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2+2\frac{M^2}{\boldsymbol{\Delta_\perp^2}}|\boldsymbol{\mathcal{F}_{1,2}}+\boldsymbol{\mathcal{G}_{1,2}}|^2\right]+\cos(2\phi)a\left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2+|\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2\right]}$$

$$+\lambda\sin(2\phi)\,2a\,\mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4})\left(\mathcal{F}_{1,1}^*+\mathcal{G}_{1,1}^*\right)\right]\Big\}$$

Helicity flip terms persist even when $\Delta_\perp \to 0$

Probing quark OAM through π^0 production in ep collisions

Cross section

$$\frac{d\sigma}{dt dQ^2 dx_B d\phi} = \frac{(N_c^2-1)^2 \alpha_{em}^2 \alpha_s^2 f_\pi^2 \xi^3 \Delta_\perp^2}{2N_c^4(1-\xi^2)Q^{10}(1+\xi)} \left[1+(1-y)^2\right]$$

$$\times \left\{ \left[|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 + 2\frac{M^2}{\Delta_\perp^2}|\mathcal{F}_{1,2}+\mathcal{G}_{1,2}|^2 \right] + \cos(2\phi) a \left[-|\mathcal{F}_{1,1}+\mathcal{G}_{1,1}|^2 + |\mathcal{F}_{1,4}+\mathcal{G}_{1,4}|^2 \right] \right.$$

$$\left. + \lambda \sin(2\phi)\, 2a\, \mathrm{Re}\left[(i\mathcal{F}_{1,4}+i\mathcal{G}_{1,4}) \left(\mathcal{F}^*_{1,1} + \mathcal{G}^*_{1,1} \right) \right] \right\}$$

Since both unpolarized and polarized cross sections contribute at twist-3, the magnitudes of the asymmetries are not power-suppressed

Probing quark OAM through π^0 production in ep collisions

Theoretical complications

Probing quark OAM through π^0 production in ep collisions

Theoretical complications

End-point singularity & discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

Model-dependent method:

$$\boxed{\int_{\langle p_\perp^2\rangle/Q^2}^{1-\langle p_\perp^2\rangle/Q^2} dz}$$

$\langle p_\perp^2\rangle = 0.04\,\mathrm{GeV}^2$ determined based on a fit to CLAS data

S. V. Goloskokov and P. Kroll, 2005

Probing quark OAM through π^0 production in ep collisions

Theoretical complications

End-point singularity & discontinuity:

$$\mathcal{F}_{1,4} = \int_{-1}^{1} dx \frac{x\xi \int d^2k_\perp k_\perp^2 F_{1,4}^{u+d}(x,\xi,\Delta_\perp,k_\perp)}{M^2 \boldsymbol{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}} \times \int_0^1 dz \frac{\phi_\pi(z)(1+z^2-z)}{z^2(1-z)^2}$$

Model-dependent method:

$$\int_{\langle \boldsymbol{p}_\perp^2\rangle/\boldsymbol{Q}^2}^{1-\langle \boldsymbol{p}_\perp^2\rangle/\boldsymbol{Q}^2} dz$$

S. V. Goloskokov and P. Kroll, 2005

$$\boxed{\frac{1}{(x-\xi+i\epsilon)^2} \to \frac{1}{(x-\xi-\langle \boldsymbol{p}_\perp^2\rangle/\boldsymbol{Q}^2+i\epsilon)^2}}$$

I. V. Anikin, O. V. Teryaev, 2003

Probing quark OAM through π^0 production in ep collisions

Numerical results

Kinematics:

	$Q^2(\mathrm{GeV}^2)$	$\sqrt{s_{ep}}(\mathrm{GeV})$
EIC	10	100
EicC	3	16

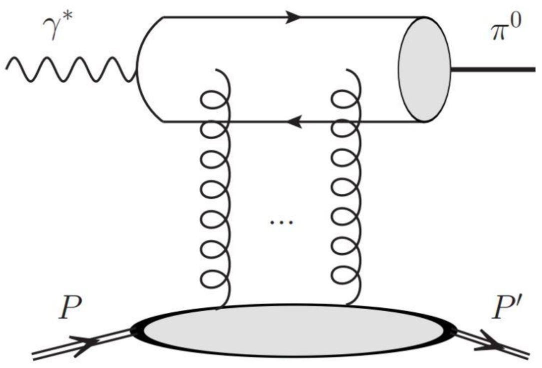
Probing quark OAM through π^0 production in ep collisions

Numerical results

Kinematics:

	$Q^2(\mathrm{GeV}^2)$	$\sqrt{s_{ep}}(\mathrm{GeV})$
EIC	10	100
EicC	3	16

- We focus on large skewness (ξ) region to suppress gluon contribution



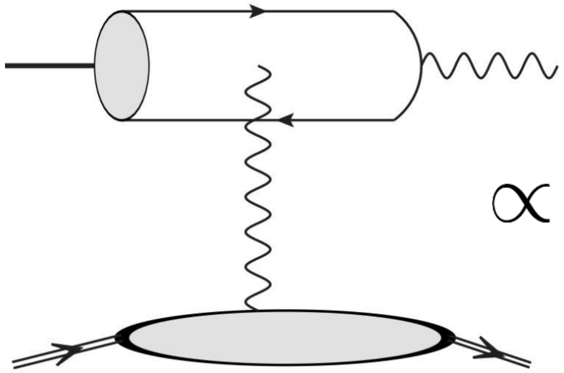
Probing quark OAM through π^0 production in ep collisions

Numerical results

Kinematics:

	$Q^2(\mathrm{GeV}^2)$	$\sqrt{s_{ep}}(\mathrm{GeV})$
EIC	10	100
EicC	3	16

- We focus on large skewness (ξ) region to suppress gluon contribution
- We focus on large momentum transfer (t) region to suppress contribution from Primakoff process



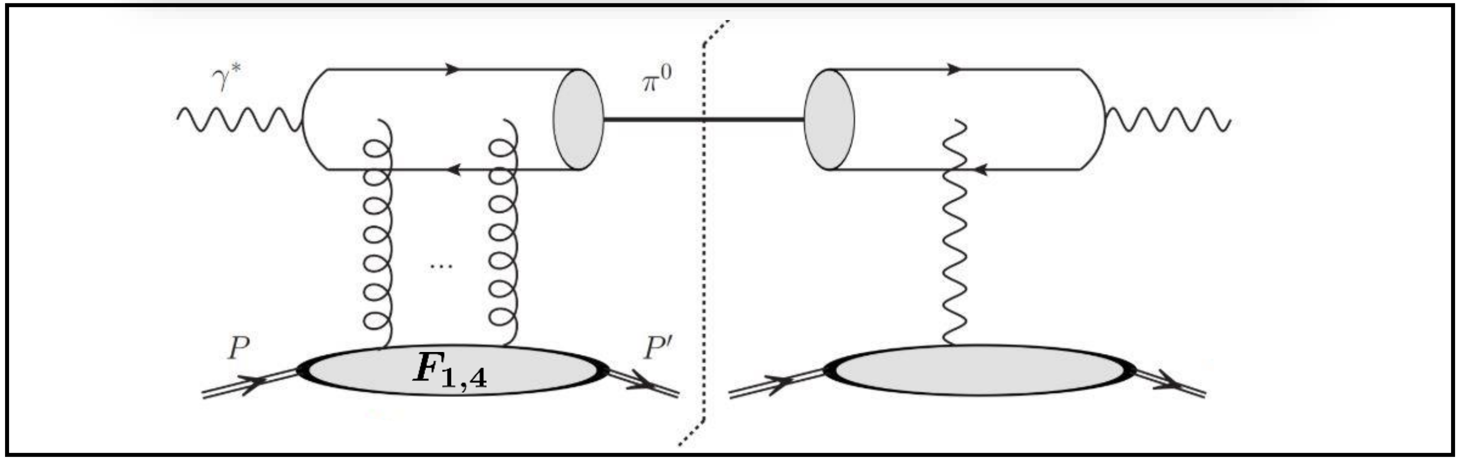
$$\propto \frac{1}{t}$$

Probing quark OAM through π^0 production in ep collisions

Remark:

Accessing the gluon GTMD $F_{1,4}$ in exclusive π^0 production in ep collisions

Shohini Bhattacharya,[1] Duxin Zheng,[2] and Jian Zhou[3]



$$\frac{d\Delta\sigma}{dt\,dQ^2\,dx_B\,d\phi} = -\sin(2\phi)\frac{\alpha_{em}^3\alpha_s f_\pi^2(1-y)\xi x_B\mathcal{F}(t)}{3Q^8N_c}\left[\int_0^1 dz\frac{\phi_\pi(z)}{z(1-z)}\right]^2 \mathrm{Im}\left[\int_{-1}^1 dx\frac{F_{1,4}^{(1)}(x,\xi,\Delta_\perp)/M^2}{(x+\xi-i\epsilon)^2(x-\xi+i\epsilon)^2}\right]$$

The same azimuthal asymmetry, precisely mirroring what we observe in this study, also emerges from the interference between the Primakoff process and the contribution from the gluon GTMD

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\ \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

Example:

$$\boxed{H^q(x, \boldsymbol{\xi}, \boldsymbol{t}) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\, \delta(\beta + \boldsymbol{\xi}\alpha - x) \times \frac{3}{4}|\beta|^{\boldsymbol{-1.3\, t}} \frac{[(1-|\beta|)^2 - \alpha^2]}{(1-|\beta|)^3} q(|\beta|)}$$

JAM 2022 PDFs

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\ \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

Example:

$$\boxed{H^q(x,\xi,t) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha\,\delta(\beta+\xi\alpha-x) \times \frac{3}{4}|\beta|^{-1.3\,t}\frac{[(1-|\beta|)^2-\alpha^2]}{(1-|\beta|)^3}q(|\beta|)}$$

The t-dependence is determined based on a fit to CLAS data

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

- Model for OAM:

  1. "OAM density": (Hatta, Yoshida, 1207.5332)

$$L^q_{can}(\boldsymbol{x}) \;=\; x\int_x^1 \frac{dx'}{x'}\,q(x') \;-\; x\int_x^1 \frac{dx'}{x'^2}\,\Delta q(x') + \text{ genuine twist-three}$$

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\ \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

- Model for OAM:

1. “OAM density”: (Hatta, Yoshida, 1207.5332)

$$L^q_{can}(\boldsymbol{x}) \overset{\text{WW approx}}{=} x\int_x^1 \frac{dx'}{x'}\,q(x') - x\int_x^1 \frac{dx'}{x'^2}\,\Delta q(x') + \cancel{\text{genuine twist-three}}$$

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

- Model $(H^q,\, \tilde{H}^q)$ according to the Double distribution approach (see Radyushkin, 9805342)

- Model for OAM:

1. "OAM density": (Hatta, Yoshida, 1207.5332)

$$L^q_{can}(\boldsymbol{x}) \overset{\text{WW approx}}{=} x\int_x^1 \frac{dx'}{x'}q(x') - x\int_x^1 \frac{dx'}{x'^2}\Delta q(x') + \cancel{\text{genuine twist-three}}$$

2. Use the Double distribution approach to construct $xL^q(x,\boldsymbol{\xi})e^{\boldsymbol{t}/\Lambda}$ from $xL^q(x)$

Probing quark OAM through π^0 production in ep collisions

Model input for numerical estimations

Ingredients for non-perturbative functions:

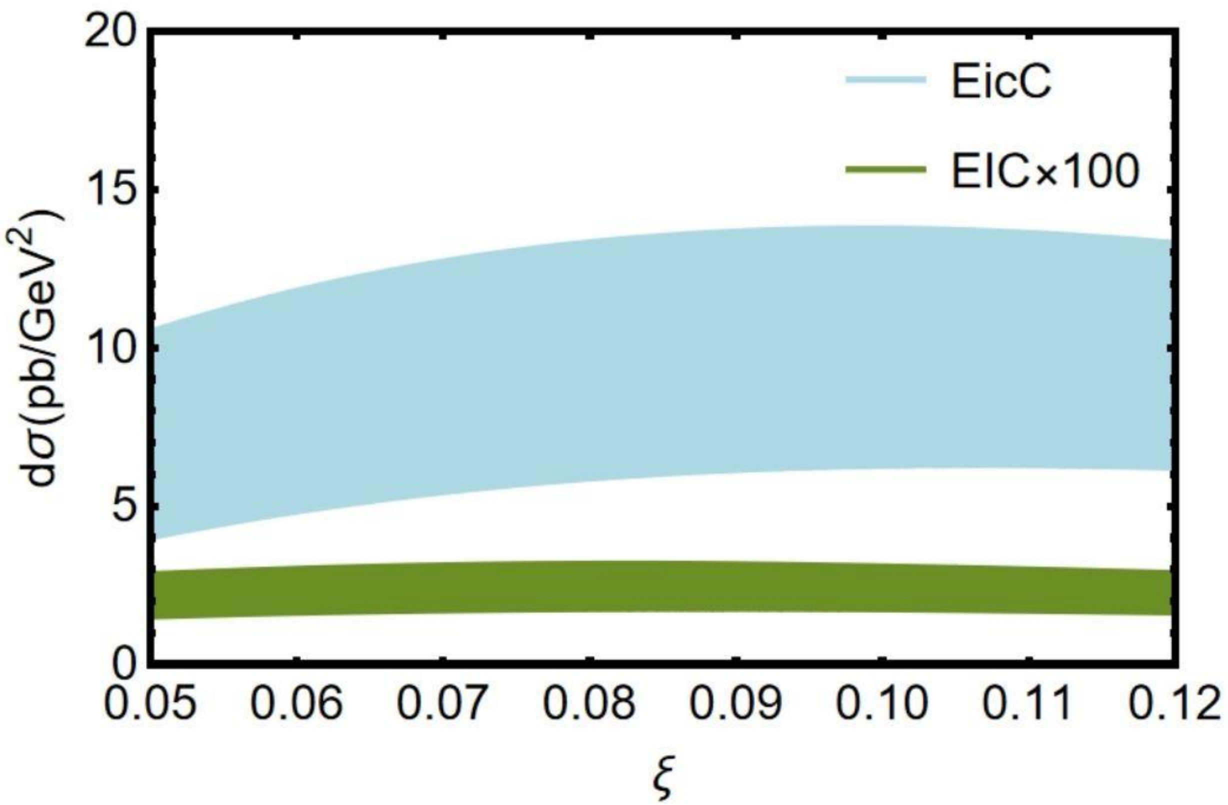
- Pion distribution amplitude:

Asymptotic form $\boxed{\phi_\pi(z) = 6z(1-z)}$

Probing quark OAM through π^0 production in ep collisions

Numerical results

Unpolarized cross section



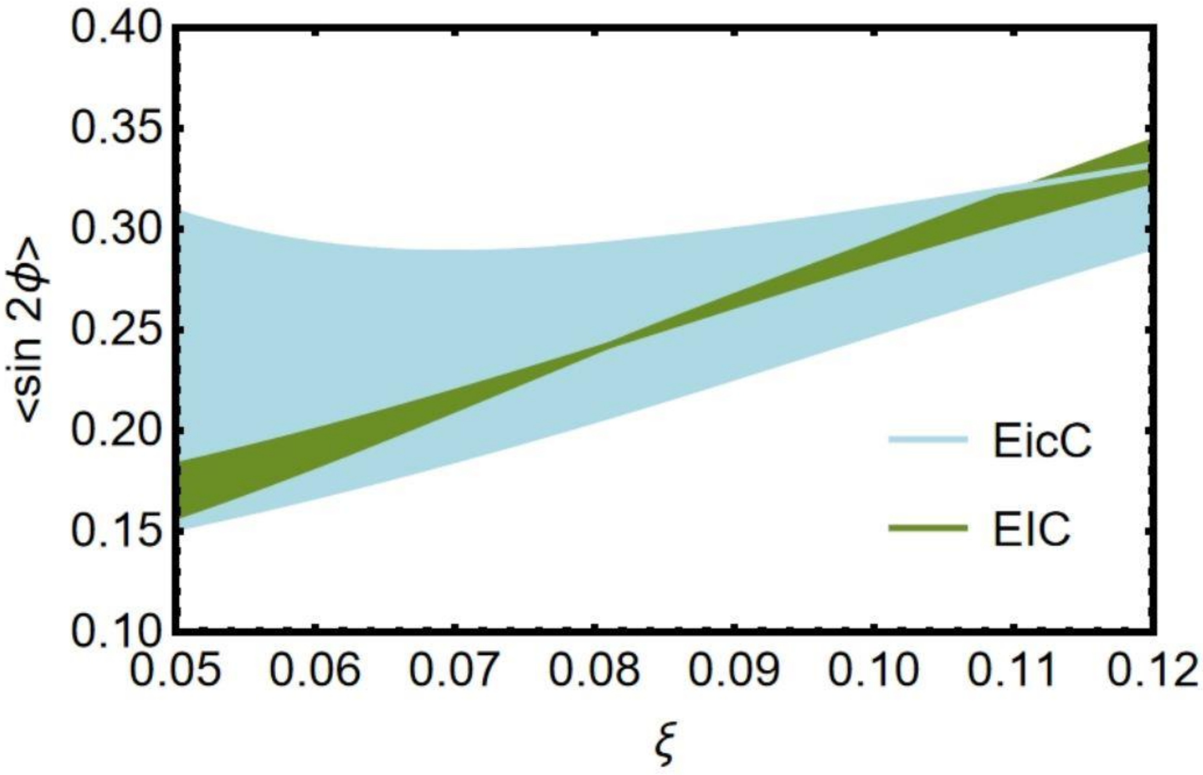
Findings:

- The unpolarized cross section exhibits a notable magnitude at EicC energy
- Relatively small at EIC energy

Probing quark OAM through π^0 production in ep collisions

Numerical results

Asymmetry



$$\langle \sin(2\phi) \rangle = \frac{\int \frac{d\Delta\sigma}{d\mathcal{P}.\mathcal{S}.} \sin(2\phi)\, d\mathcal{P}.\mathcal{S}.}{\int \frac{d\sigma}{d\mathcal{P}.\mathcal{S}.} d\mathcal{P}.\mathcal{S}.}$$

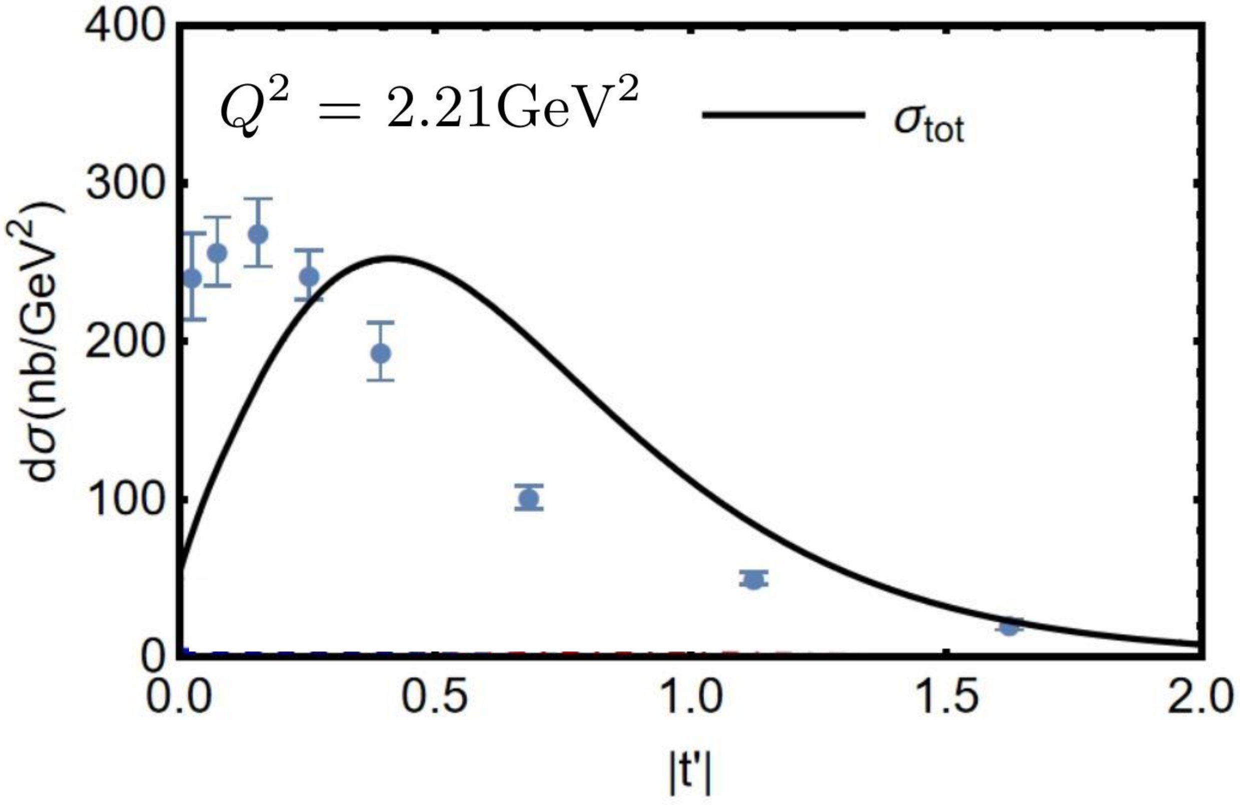
Findings:

The asymmetries are substantial for both EIC & EicC kinematics

Probing quark OAM through π^0 production in ep collisions

Numerical results

Comparison with CLAS data



Unpolarized cross section:

$$\boxed{\frac{d\sigma_T}{dt} + a\frac{d\sigma_L}{dt}}$$

Probing quark OAM through π^0 production in ep collisions

Numerical results

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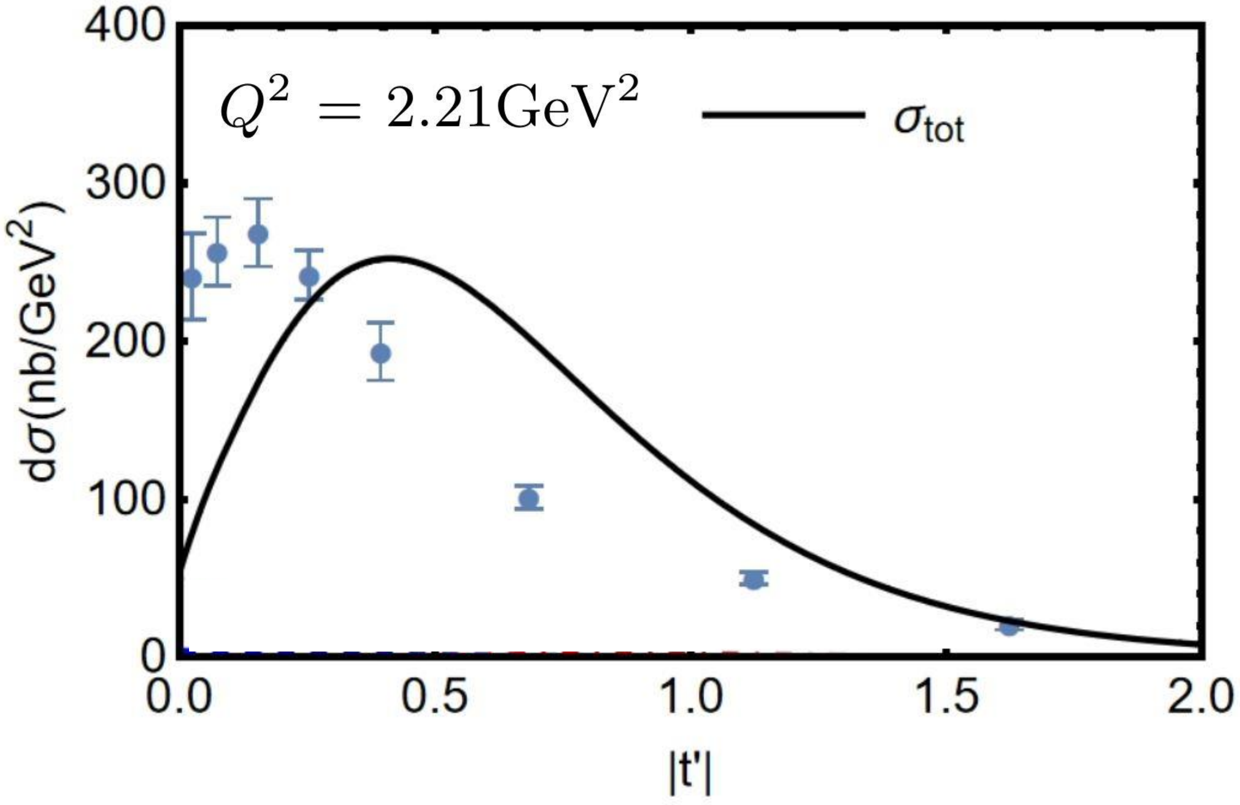


Findings:

- Our theoretical model is in reasonable agreement with experimental data

Developments

arXiv: 1612.02438 (2016)

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]

arXiv: 1702.04387 (2017)

Generalized TMDs and the exclusive double Drell-Yan process

Shohini Bhattacharya,[1] Andreas Metz,[1] and Jian Zhou[2]

arXiv: 1802.10550 (2018)

Exclusive double quarkonium production and generalized TM

Shohini Bhattacharya,[1] Andreas Metz,[1] Vikash Kumar Ojha,[2] Jeng-Yuan Tsai,[1]

arXiv: 1807.08697 (2018)

Probing the Weizsäcker-Williams gluon Wigner distribution in pp collisions

Renaud Boussarie,[1] Yoshitaka Hatta,[2] Bo-Wen Xiao,[3,4] and Feng Yuan[5]

arXiv: 1912.08182 (2019)

Probing the gluon Sivers function with an unpolar GTMD distributions and the Oderons

Renaud Boussarie,[1] Yoshitaka Hatta,[1] Lech Szymanowski,[2] and S

arXiv: 2106.13466 (2021)

Probing the gluon tomography in photoproduction of di-pions

Yoshikazu Hagiwara, Cheng Zhang, Jian Zhou, and Ya-jin Zhou

arXiv: 2201.08709 (2022/2024)

Signature of the gluon orbital angular momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]

arXiv: 2205.00045 (2022)

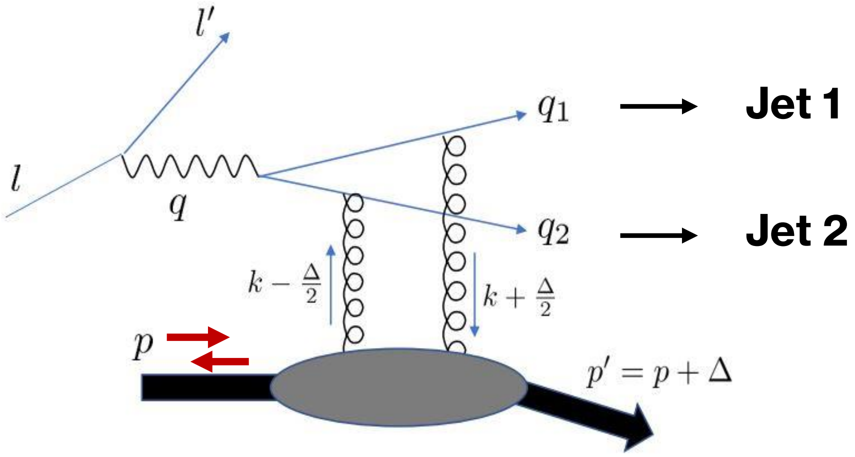
Angular correlations in exclusive dijet photoproduction in ultra-peripheral PbPb collisions at $\sqrt{s_{NN}} = 5.02$ TeV

Selected works on gluon GTMDs

Probing gluon OAM through exclusive di-jet production

Hunting the Gluon Orbital Angular Momentum at the Electron-Ion Collider

Xiangdong Ji,[1,2] Feng Yuan,[3] and Yong Zhao[1,3]



Main result: Single spin asymmetry as a probe of gluon OAM

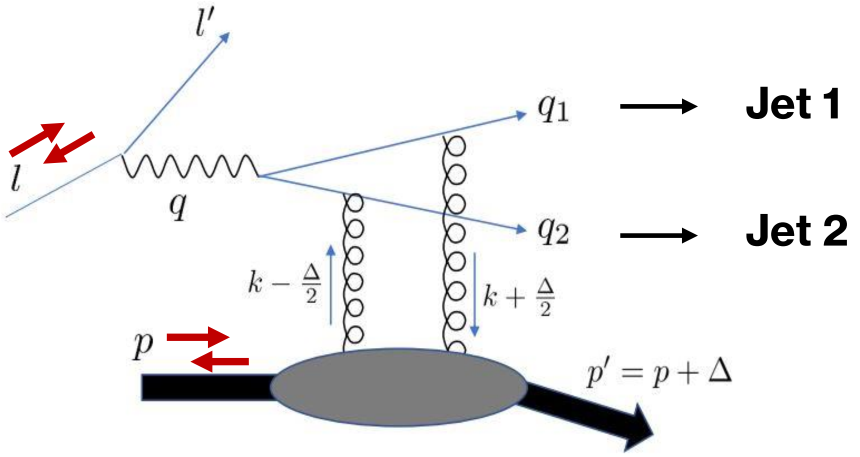
Selected works on gluon GTMDs

Probing gluon OAM through exclusive di-jet production

PHYSICAL REVIEW LETTERS **128,** 182002 (2022)

Signature of the Gluon Orbital Angular Momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]



Main result (double spin asymmetry):

Signature of gluon OAM is cosine angular modulation

$$d\sigma^{\mathbf{asym}} \sim -\,\mathfrak{Re}\left[\left\{\mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{H}_g^{(2)*}(\xi)\right\}\mathcal{L}_g(\xi)\right]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

$$+\,\mathfrak{Re}\left[\mathcal{H}_g^{(1)*}(\xi)\,\tilde{\mathcal{H}}_g^{(2)}(\xi)\right]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

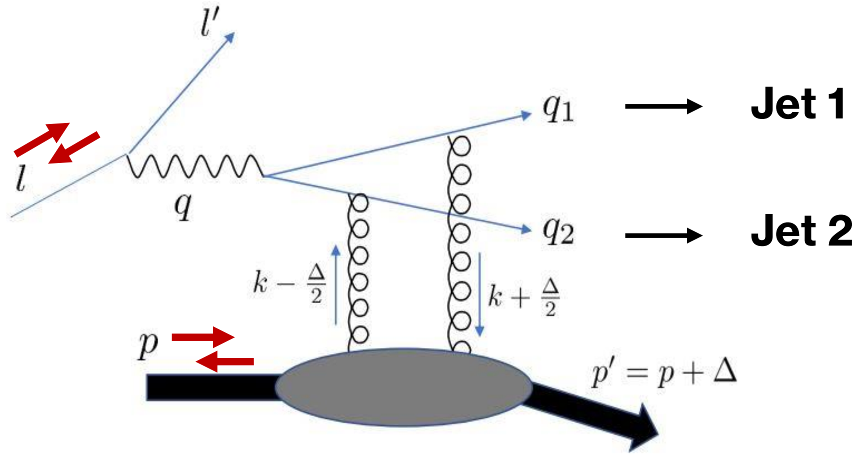
Selected works on gluon GTMDs

Probing gluon OAM through exclusive di-jet production

PHYSICAL REVIEW LETTERS **128,** 182002 (2022)

Signature of the Gluon Orbital Angular Momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]



Gluon helicity contributes to the same angular modulation as that of OAM

$$d\sigma^{\mathbf{asym}} \sim -\Re e\left[\left\{\mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{H}_g^{(2)*}(\xi)\right\}\mathcal{L}_g(\xi)\right]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

Helicity GPD
(intrinsic spin)

$$+\Re e\left[\mathcal{H}_g^{(1)*}(\xi)\,\tilde{\mathcal{H}}_g^{(2)}(\xi)\right]\cos(\phi_{l_\perp} - \phi_{\Delta_\perp})$$

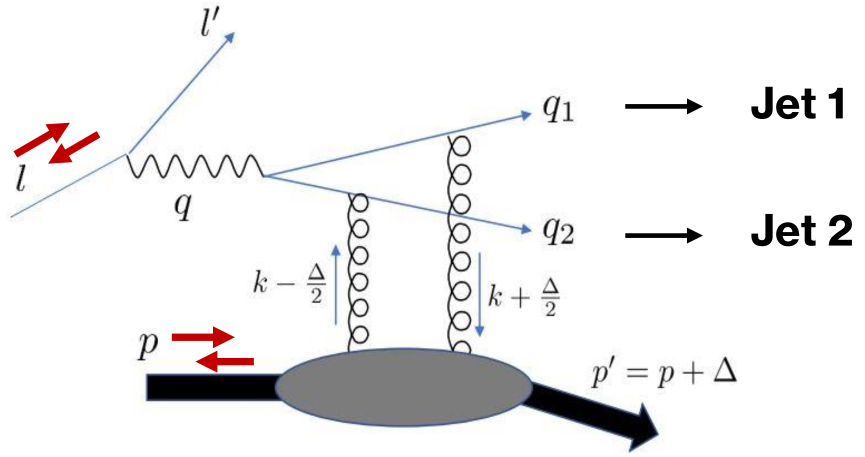
Selected works on gluon GTMDs

Probing gluon OAM through exclusive di-jet production

PHYSICAL REVIEW LETTERS 128, 182002 (2022)

Signature of the Gluon Orbital Angular Momentum

Shohini Bhattacharya,[1,*] Renaud Boussarie,[2,†] and Yoshitaka Hatta[1,3,‡]



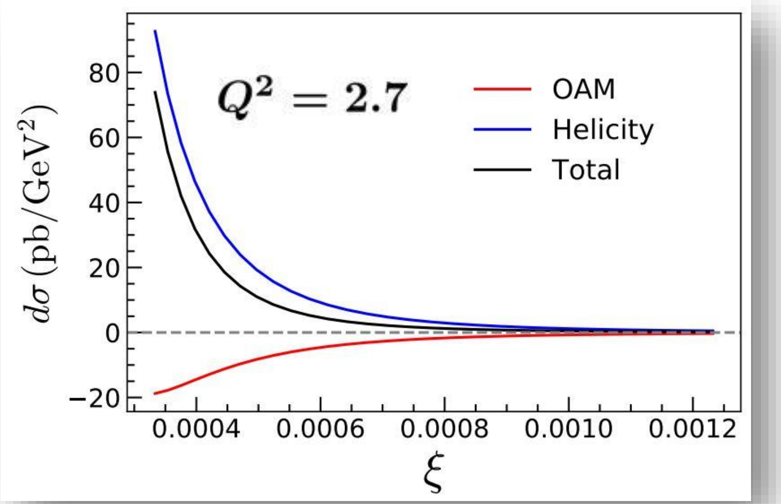
Our observable is a simultaneous probe of gluon OAM & it's helicity

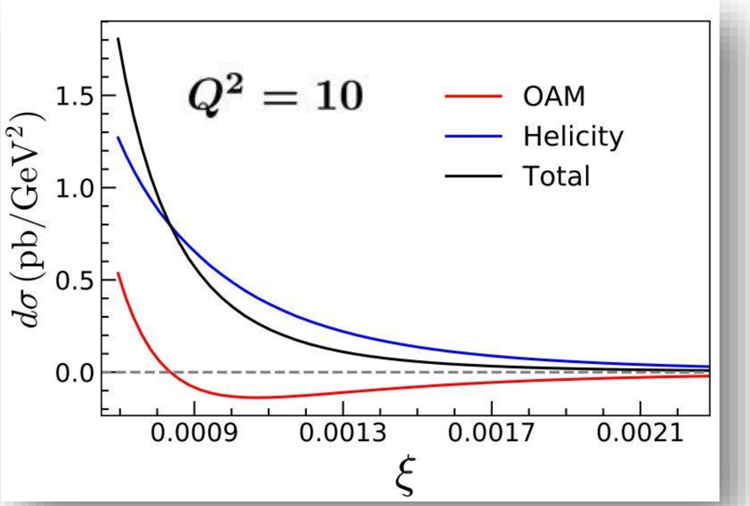
$$d\sigma^{\mathbf{asym}} \sim -\Re\left[\left\{\mathcal{H}_g^{(1)*}(\xi) + \frac{4q_\perp^2}{q_\perp^2+\mu^2}\mathcal{H}_g^{(2)*}(\xi)\right\}\mathcal{L}_g(\xi)\right]\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})$$

$$+\Re\left[\mathcal{H}_g^{(1)*}(\xi)\tilde{\mathcal{H}}_g^{(2)}(\xi)\right]\cos(\phi_{l_\perp}-\phi_{\Delta_\perp})$$

Selected works on gluon GTMDs

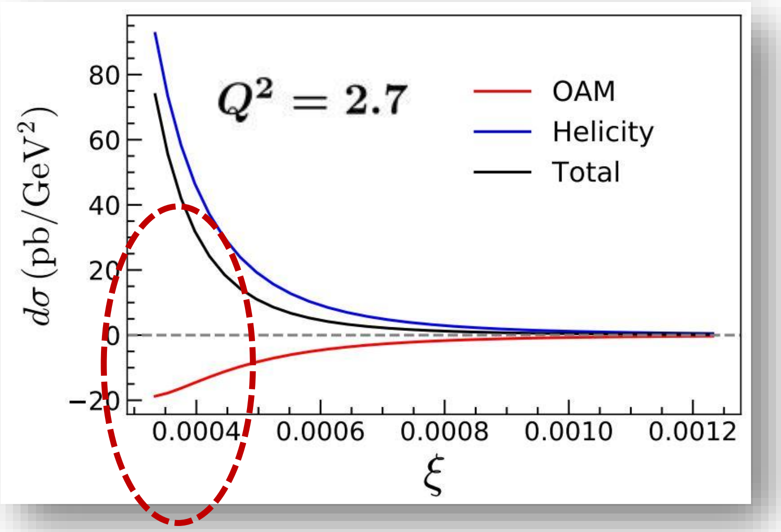
Interplay between OAM and helicity at small x

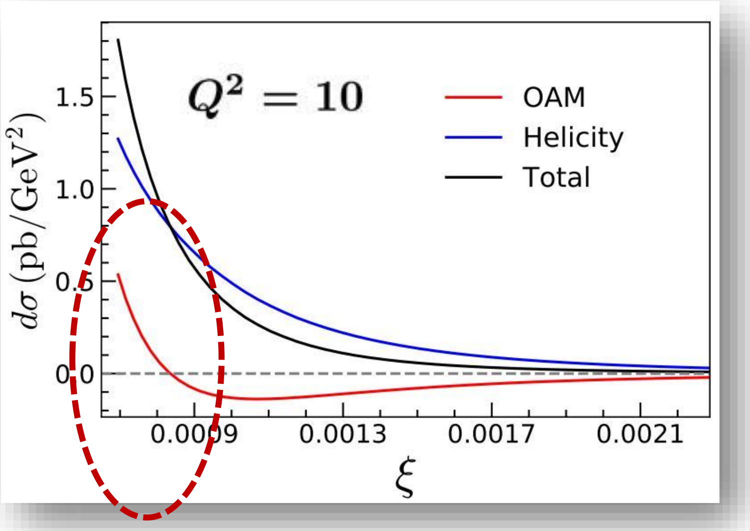




Selected works on gluon GTMDs

Interplay between OAM and helicity at small x





Schematic structure of our observable:

$$d\sigma^{\mathrm{asym}} \sim \mathcal{H}_g^{(1)*}(\xi)\left(\tilde{\mathcal{H}}_g^{(2)}(\xi) + \frac{q_\perp^2 - Q^2/4}{q_\perp^2 + Q^2/4}\mathcal{L}_g(\xi)\right)$$

$$\tilde{\mathcal{H}}_g^{(2)}(\xi) \downarrow \Delta G(x) \qquad \mathcal{L}_g(\xi) \downarrow L_g(x)$$

Cancellation expected between helicity & OAM at small x

$$\boxed{\Delta G(x) \approx -\frac{2}{1+c} L_g(x)}$$

Boussarie, Hatta, Yuan (2019)
Kovchegov, Manley (2023, 2024)

Selected works on gluon GTMDs

Contribution from spin-orbit correlation at small x?

Another non-negligible contribution to the process:

$$\boxed{d\sigma^{\mathbf{asym}} \sim \frac{4q_\perp^2}{q_\perp^2 + \mu^2} C_g^{(2)}(\xi)\tilde{\mathcal{H}}_g^{(2)}(\xi)}$$

Spin-orbit correlation:

$$C^g(x) = \int d^2\vec{k}_\perp \frac{\vec{k}_\perp^2}{M^2} \boldsymbol{G^g_{1,1}(x, \vec{k}_\perp^2)}$$

Selected works on gluon GTMDs

Contribution from spin-orbit correlation at small x?

Another non-negligible contribution to the process:

$$\boxed{d\sigma^{\text{asym}} \sim \frac{4q_\perp^2}{q_\perp^2+\mu^2} C_g^{(2)}(\xi)\tilde{\mathcal{H}}_g^{(2)}(\xi)}$$

First insight into the small-x behavior of spin-orbit correlation:

$$\boxed{C^g(x) \approx -2x\int_x^1 \frac{dx'}{x'^2}G(x') + \ldots \propto -G(x)}$$

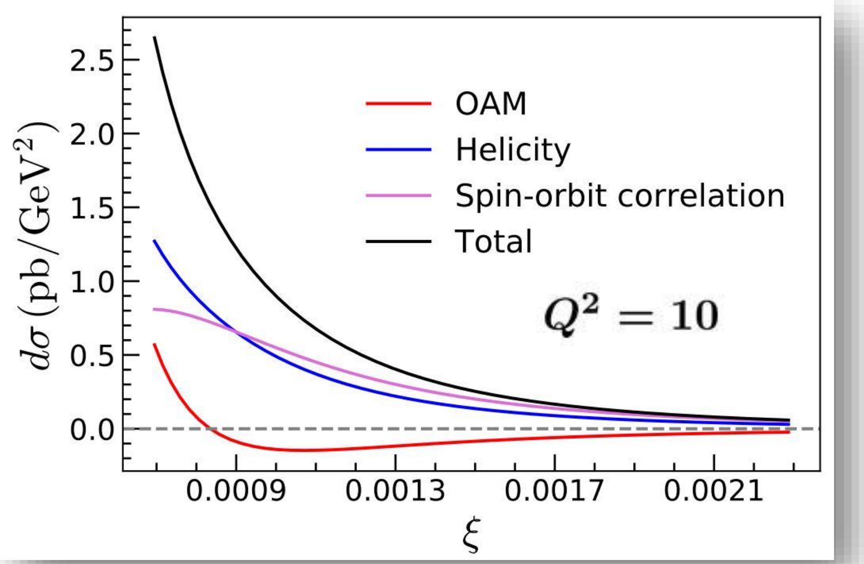
(SB, Boussarie, Hatta, 2404.04208, 2404.04209)

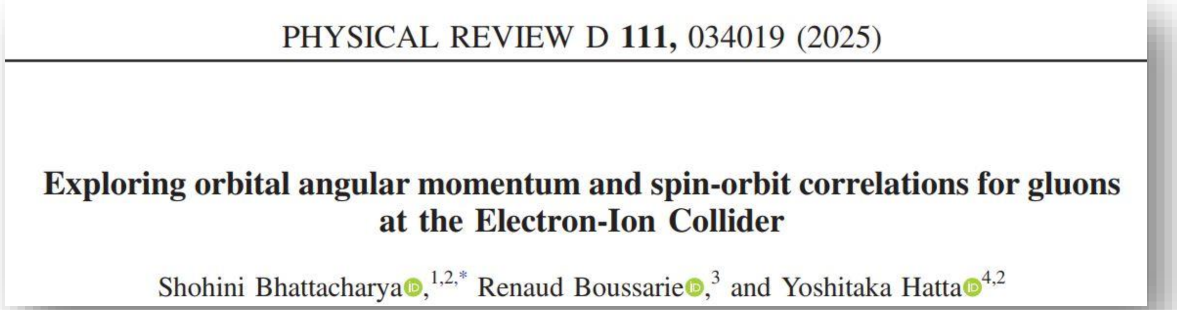
For a complete twist structure of spin-orbit correlation, see Hatta, Schoenleber, 2404.18872

Selected works on gluon GTMDs

Probing gluon OAM & spin-orbit correlation at small x

Updated numerical results (SB, Boussarie, Hatta, 2404.04209)**:**




PHYSICAL REVIEW D **111,** 034019 (2025)

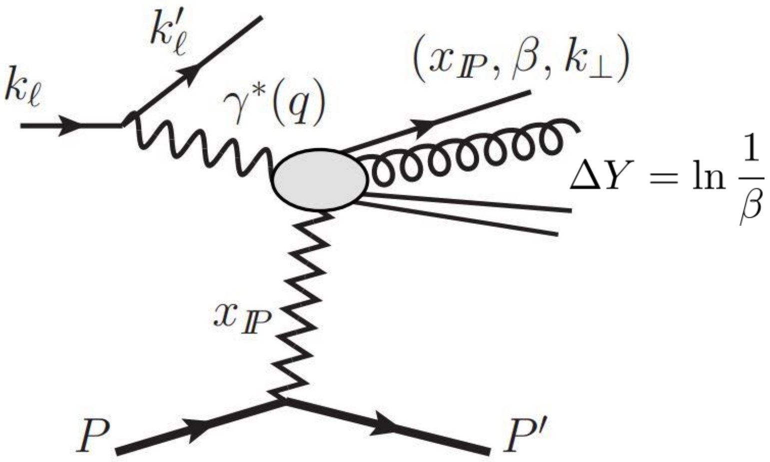
Exploring orbital angular momentum and spin-orbit correlations for gluons at the Electron-Ion Collider

Shohini Bhattacharya[1,2,*] Renaud Boussarie[3] and Yoshitaka Hatta[4,2]

Spin-orbit correlation is more accurately constrained than **OAM** because the latter necessitates the precise determination of <u>both</u> unpolarized and polarized gluon distributions

Selected works on gluon GTMDs

Probing gluon OAM through Semi Inclusive Diffractive Deep Inelastic Scattering



Hatta, Xiao, Yuan (2022)

- Measure invariant mass of diffractively produced system instead of reconstructing jets

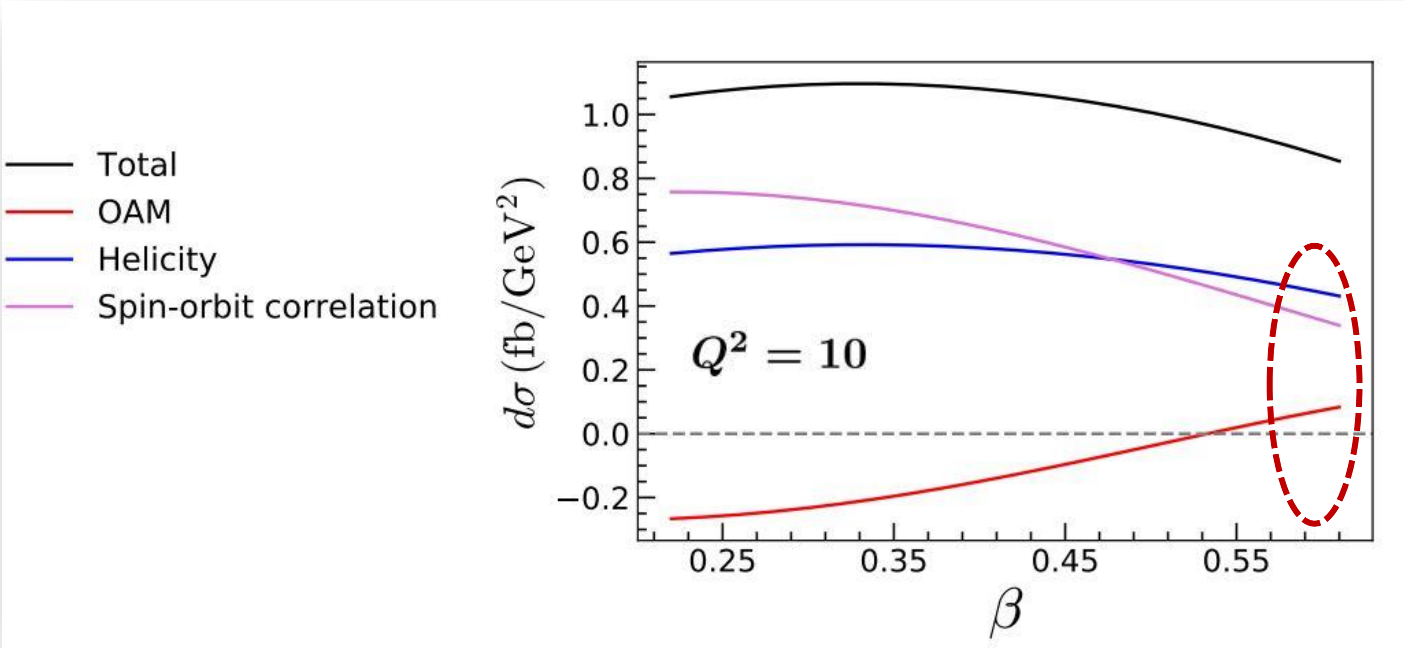
$$M_X^2 = \frac{q_\perp^2}{z\bar{z}} = \frac{1-\beta}{\beta} Q^2$$

- Tag hadron species out of the diffractively produced system

Selected works on gluon GTMDs

Probing gluon OAM through Semi Inclusive Diffractive Deep Inelastic Scattering

Numerical results (SB, Boussarie, Hatta, 2404.04209):



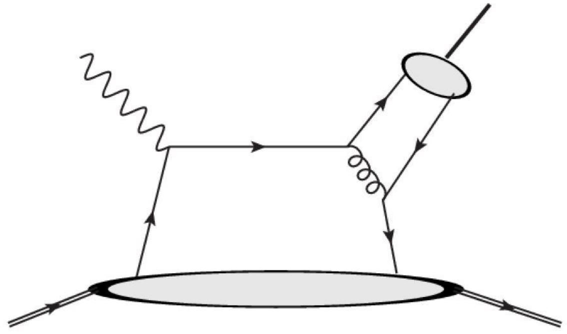
Challenging, yet there is <u>no</u> requirement to reconstruct jets & we still maintain sensitivity to OAM

Summary

- Generalized TMDs/Wigner functions are the holy grail of spin physics

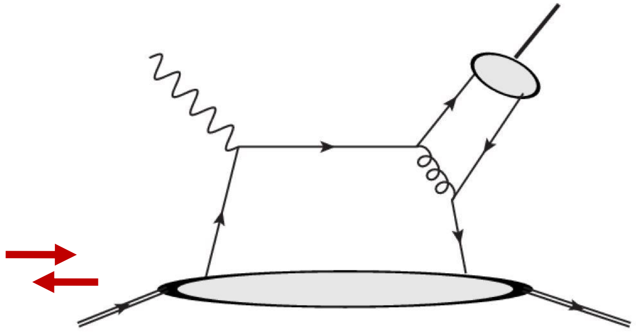
Summary

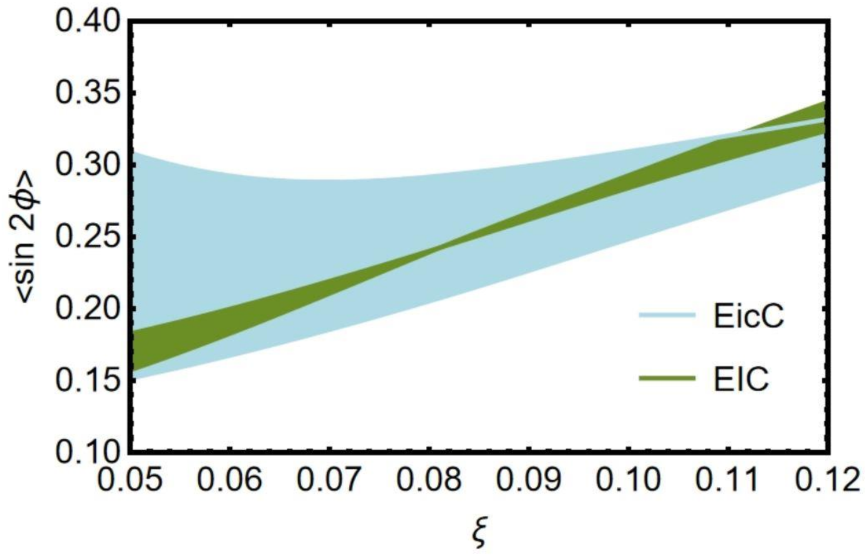
- Generalized TMDs/Wigner functions are the holy grail of spin physics
- Probe **quark OAM** via exclusive π^0 production in ep collisions
- Circumvent challenges associated with double Drell-Yan process



Summary

- Generalized TMDs/Wigner functions are the holy grail of spin physics
- Probe **quark OAM** via exclusive π^0 production in ep collisions
- Circumvent challenges associated with double Drell-Yan process

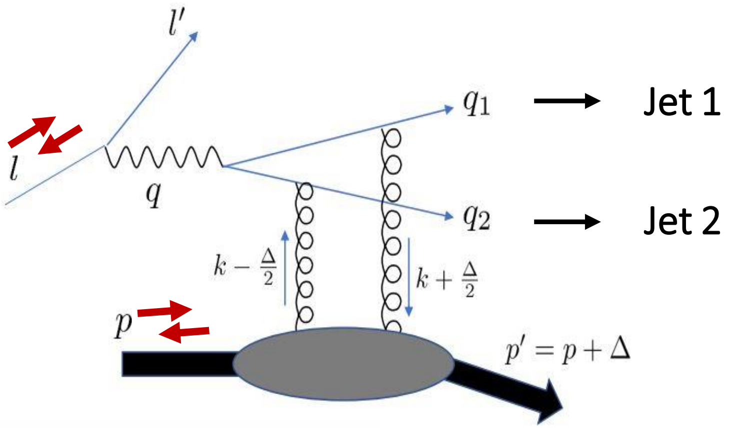


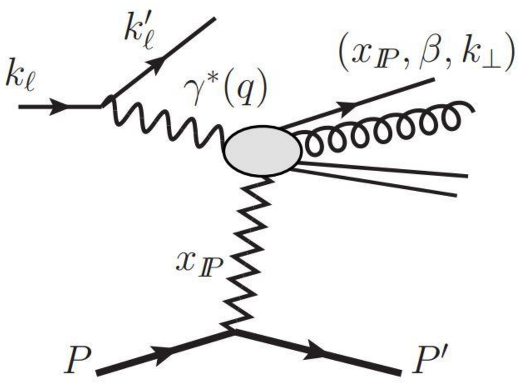


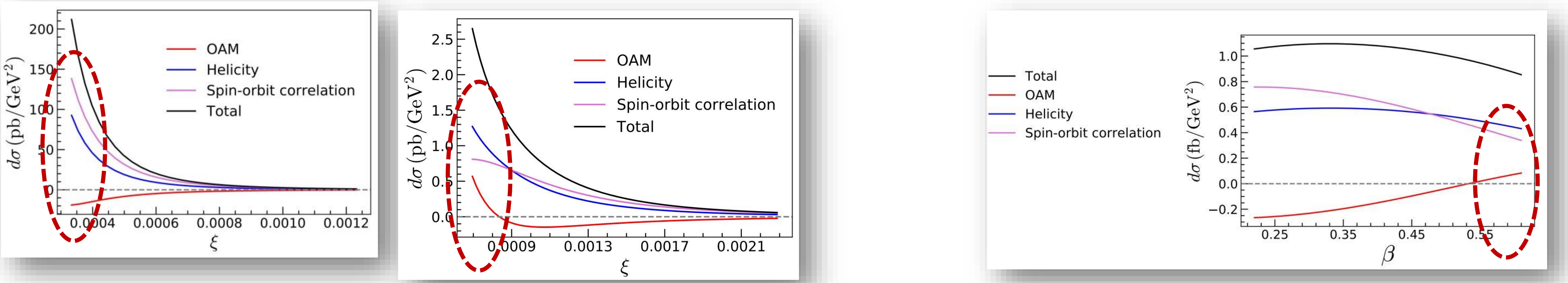
- Longitudinal single-target spin asymmetry is not power suppressed
- Asymmetry is substantial & thus exclusive π^0 production in ep collisions maybe a promising route to constrain quark OAM

Summary

- Probe **gluon OAM** via <u>double spin asymmetry</u> in exclusive di-jet production/ SID**D**IS in ep collisions







Backup slides

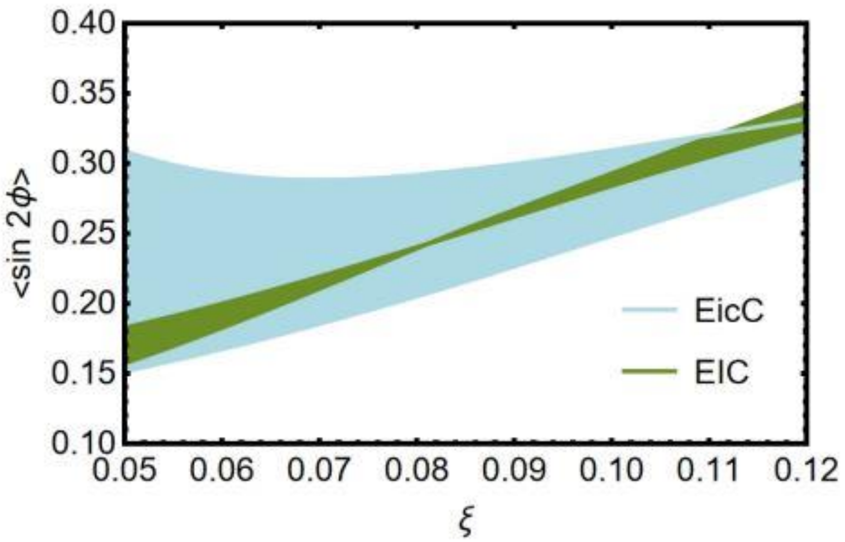


FIG. 3: The unpolarized cross section, as given by Eq. (15), is displayed in the top plot for EIC kinematics with $Q^2 = 10\,\mathrm{GeV}^2$ and $\sqrt{s_{ep}} = 100\,\mathrm{GeV}$, as well as for EicC kinematics with $Q^2 = 3\,\mathrm{GeV}^2$ and $\sqrt{s_{ep}} = 16\,\mathrm{GeV}$. The unpolarized cross section for the EIC case is re-scaled by a factor of 100. The bottom plot shows the average value of $\langle \sin(2\phi) \rangle$ given by Eq. (16). The variable t is integrated over the range $[-0.5\,\mathrm{GeV}^2, -\frac{4\xi^2 M^2}{1-\xi^2}]$.The error bands are obtained by varying the value of $\sqrt{\langle p_\perp^2 \rangle}$ from 150 MeV to 250 MeV and the value of α', which determines the t-dependence of the various distributions in the double distribution approach (see supplementary material), from 1.2 to 1.4.

We notice that other fitting for $g_{1T}^{(1)}(x)$ exist too [82]. Meanwhile, the $k_\perp$ moment of $F_{1,2}$ can be related to the Qiu-Sterman function,

$$\int d^2k_\perp \frac{k_\perp^2}{M} \mathrm{Im}[F_{1,2}(x, \xi = 0, \Delta_\perp = 0, k_\perp)]$$
$$= -\int d^2k_\perp \frac{k_\perp^2}{M} f_{1T}^\perp(x, k_\perp) = T_F(x, x) \tag{6}$$

where the Qiu-Sterman function is parametrized as [83],

$$T_F(x, x) = N_q \frac{(\alpha_q + \beta_q)^{(\alpha_q+\beta_q)}}{\alpha_q^{\alpha_q} \beta_q^{\beta_q}} x^{\alpha_q}(1-x)^{\beta_q} q(x) \tag{7}$$

with $\alpha_u = 1.051, \alpha_d = 1.552$, $\beta_u = \beta_d = 4.857$, and $N_u = 1.06, N_d = -0.163$. See also Refs. [84], [85], and [86] for the state-of-the-art extractions of the Sivers functions. Once the x-dependence of the $k_\perp$ moments of $F_{1,2}$ and $G_{1,2}$ is reconstructed as explained above, we reconstruct their (ξ, t)-dependence in accordance with the double distribution