

Transverse momentum distributions of the proton and pion from Drell Yan

Leonard Gamberg

w/ Patrick Barry, Eric Moffat, Alexei Prokudin
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“JAM 3-D”

New Horizons with Fixed-Target Proton-Nucleus Experiments
CFNS Stony Brook 07/10/2025

Outline/Discussion

- Discuss our recent work on momentum imaging of protons & pions from Drell Yan Phys. Rev. D 108 (2023)
- Accessing TMDs (valence) thru π -nucleon (nuclear) Drell Yan JAM collaboration:
- First simultaneous fit of pion & proton TMD pdfs. w/ pion collinear pdfs — from collinear DY & LN data within JAM QCD analysis
- Analysis framework: Pheno Results & and status of proton-nuclear work (*prelim*)
- Comments: explore opportunities to perform momentum imaging @ fixed target nucleon nuclear DY @ AGS... ? Pion as secondary beam?

Opportunity for Advertisement



Preprints: JLAB-THY-23-3780, LA-UR-21-20798, MIT-CTP/5386

TMD Handbook

Renaud Boussarie¹, Matthias Burkardt², Martha Constantinou³, William Detmold⁴, Markus Ebert^{4,5},
Michael Engelhardt², Sean Fleming⁶, Leonard Gamberg⁷, Xiangdong Ji⁸, Zhong-Bo Kang⁹,
Christopher Lee¹⁰, Keh-Fei Liu¹¹, Simonetta Liuti¹², Thomas Mehen¹³, Andreas Metz³, John Negele⁴,
Daniel Pitonyak¹⁴, Alexei Prokudin^{7,16}, Jian-Wei Qiu^{16,17}, Abha Rajan^{12,18}, Marc Schlegel^{2,19},
Phiala Shanahan⁴, Peter Schweitzer²⁰, Iain W. Stewart⁴, Andrey Tarasov^{21,22}, Raju Venugopalan¹⁸,
Ivan Vitev¹⁰, Feng Yuan²³, Yong Zhao^{24,4,18}

TMD Handbook arXiv <https://arxiv.org/abs/2304.03302>
Aybat Rogers 2011 PRD
Collins 2011 red book
Collins Rogers 2015 PRD

**Quark
Polarization**

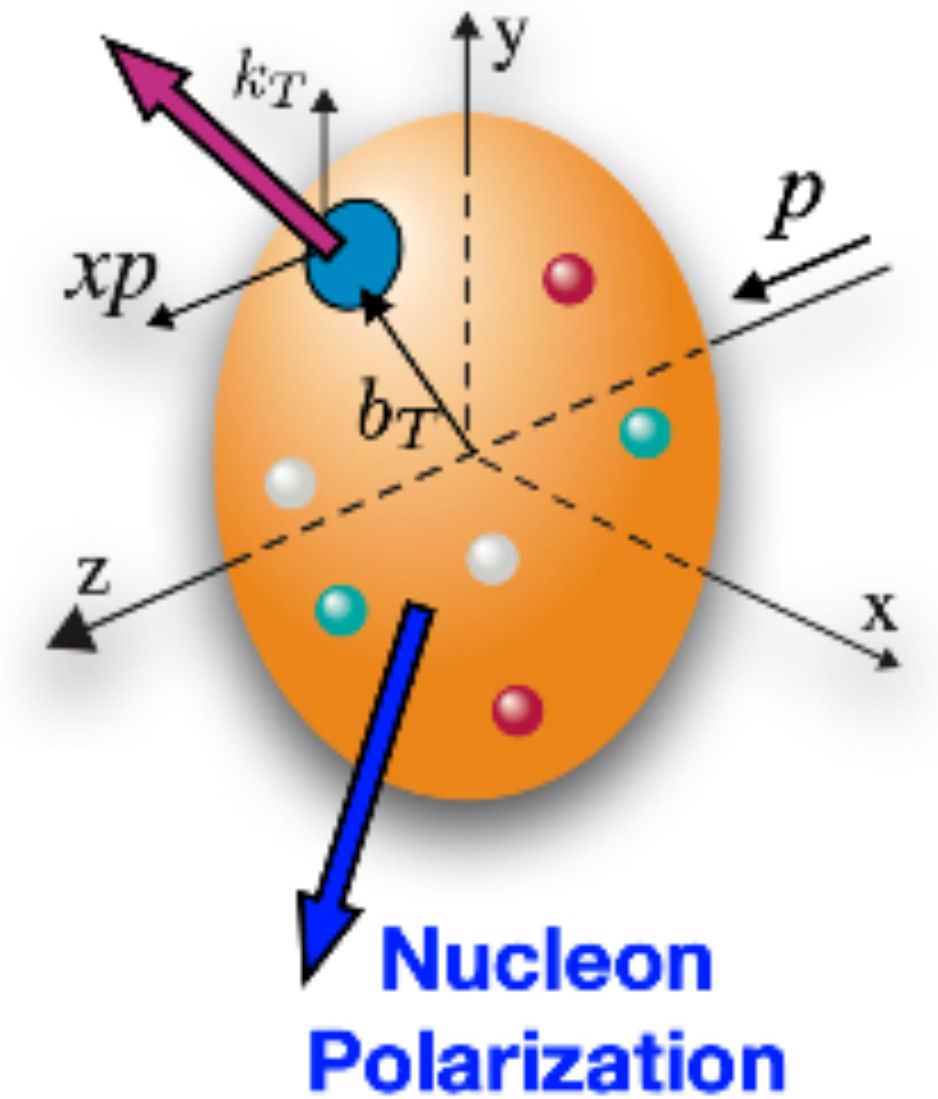


Figure 1.1: Illustration of the momentum and spin variables probed by TMD parton distributions.

What we did & how does it relates to the topic of workshop

Phys. Rev. D 108 (2023) <https://arxiv.org/abs/2302.01192>

- Carried out a simultaneous fit of pion & proton TMD pdfs w/ pion collinear pdfs — from collinear DY & LN data within JAM QCD analysis
- i.e. explored the **impact on JAM 21** pion pdfs extracted from a simultaneous fit of low energy fixed target p_T dependent DY & collinear π -nuclear cross section data

Motivation/Justification

- While the TMD is technically the object to be inferred from data, its small- b_T behavior can be written in terms of collinear PDFs (**TMD factorization/imaging**)

$$\tilde{f}(x, b_T; \mu, \zeta) = [C \otimes \mathbf{f}](x, b_T; \mu_0, \zeta_0) \times e^{S_{\text{evo}}(b_T; \mu, \mu_0, \zeta, \zeta_0)} f_{\text{NP}}(x, b_T)$$

From TMD factorization ... will fill in some gaps

Collins, Soper, Sterman Nucl. Phys. B 250 (1985)
Collins, Cambridge University Press, (2011)

What we did & how does it relates to the topic of workshop

Phys. Rev. D 108 (2023) <https://arxiv.org/abs/2302.01192>

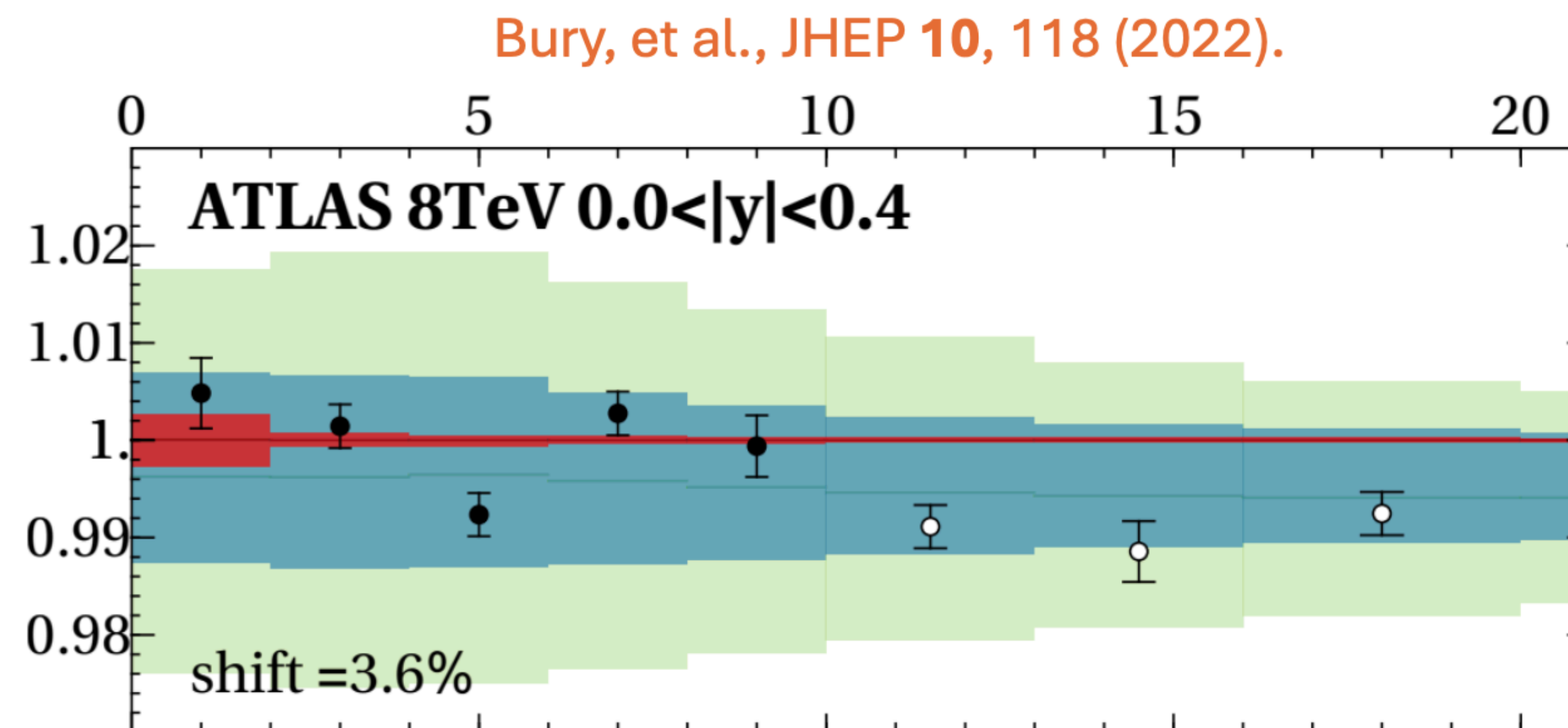
Motivation/Justification . . .

- **Most TMD** pheno extractions make use of this connection however fixing the collinear PDFs and focusing on the analysis of the nonperturbative large- b_T region
- However, such extractions are subject to the choices of the **input collinear PDFs** as discussed by Bury et al. JHEP 2022
- **Stuatus** now carrying out for **proton nuclear Drell Yan fixed target + collider data** [arXiv-xyz.2025](#)

Further motivation/Justification . . .

How sensitive are TMD observables to PDFs?

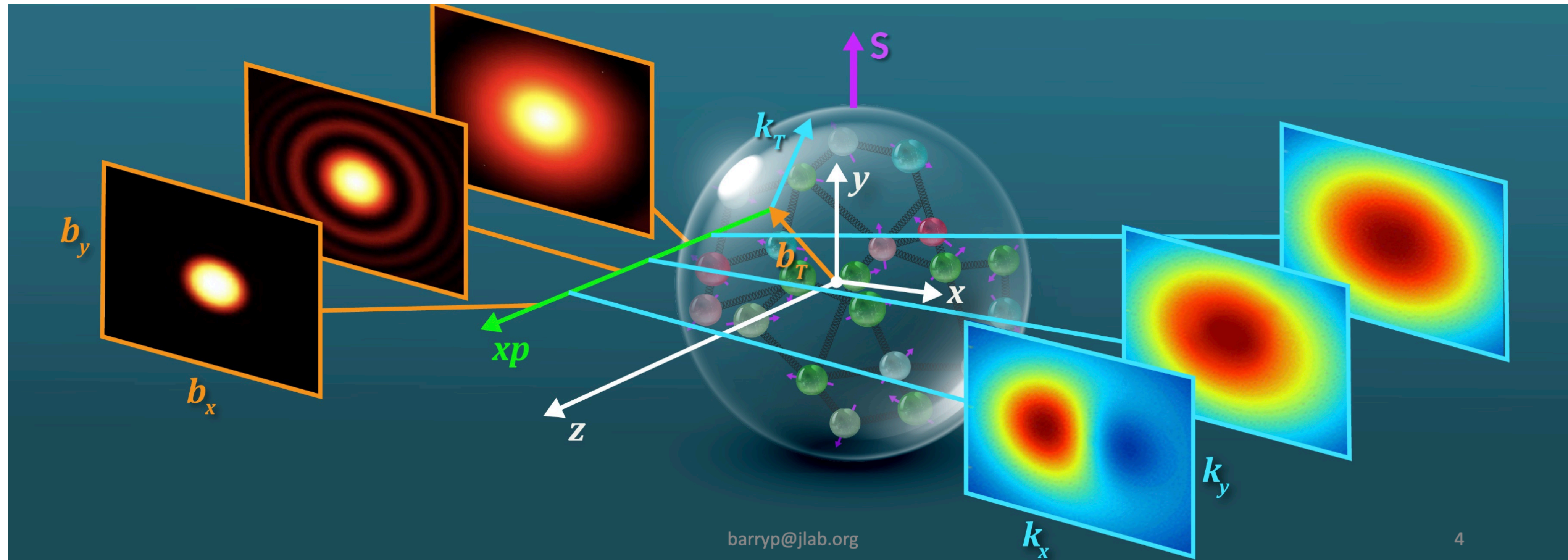
- **Red**: Bootstrapped fit with central PDFs
- **Green**: Unbootstrapped fit, varying the PDF replicas
- **Blue**: Weighted average
- **One needs to take a holistic approach and analyze both PDFs and TMDs simultaneously**



$$\tilde{f}(x, b_T; \mu, \zeta) = [C \otimes \textcolor{red}{f}](x, b_T; \mu_0, \zeta_0) \times e^{S_{\text{evo}}(b_T; \mu, \mu_0, \zeta, \zeta_0)} f_{\text{NP}}(x, b_T)$$

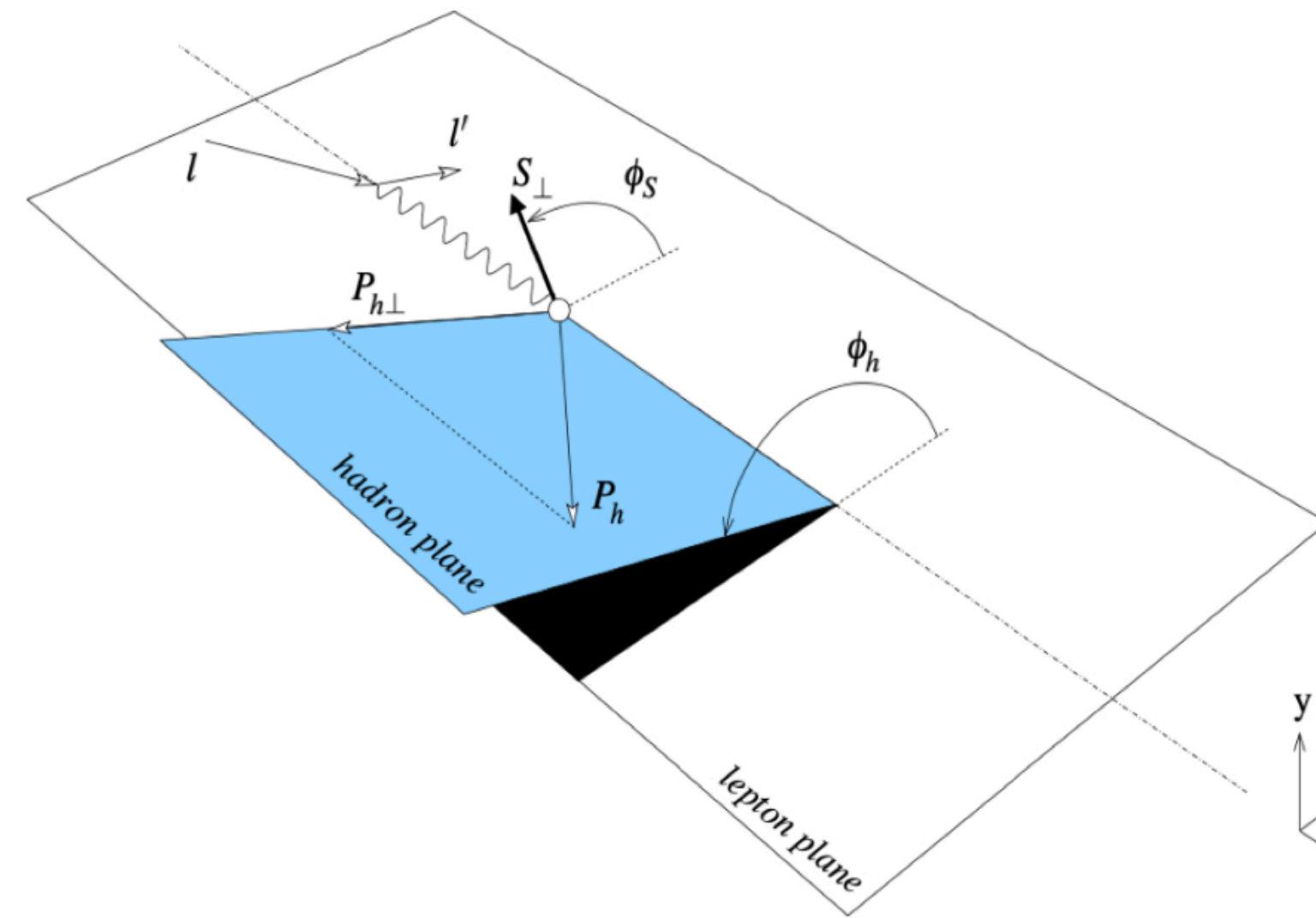
Some context Review of TMDs & Factorization

3D structures of hadrons



Reminder (DY & SIDIS): 2 theorems: TMD Factorization & Collinear P_\perp Factorization

- TMD: applicable $\Lambda_{QCD} \sim P_{h\perp} \ll Q$ Collinear: applicable $P_{h\perp} \sim Q \gg \Lambda_{QCD}$
- $P_{h\perp} \sim \mathbf{k}_T$ or \mathbf{p}_T intrinsic transverse momentum partons CS described via TMDs
- $P_{h\perp} \gg \mathbf{k}_T$ or \mathbf{p}_T generated transverse momentum in the final state as perturbative radiation & non-perturbative structure given by collinear pdfs & FFs



$$E'E_h \frac{d\sigma_{ep \rightarrow e'hX}}{d^3l' d^3P_h} \approx \hat{\sigma}_{eq \rightarrow e'q'} \otimes f_1 \tilde{\otimes} D_{h/q'}$$

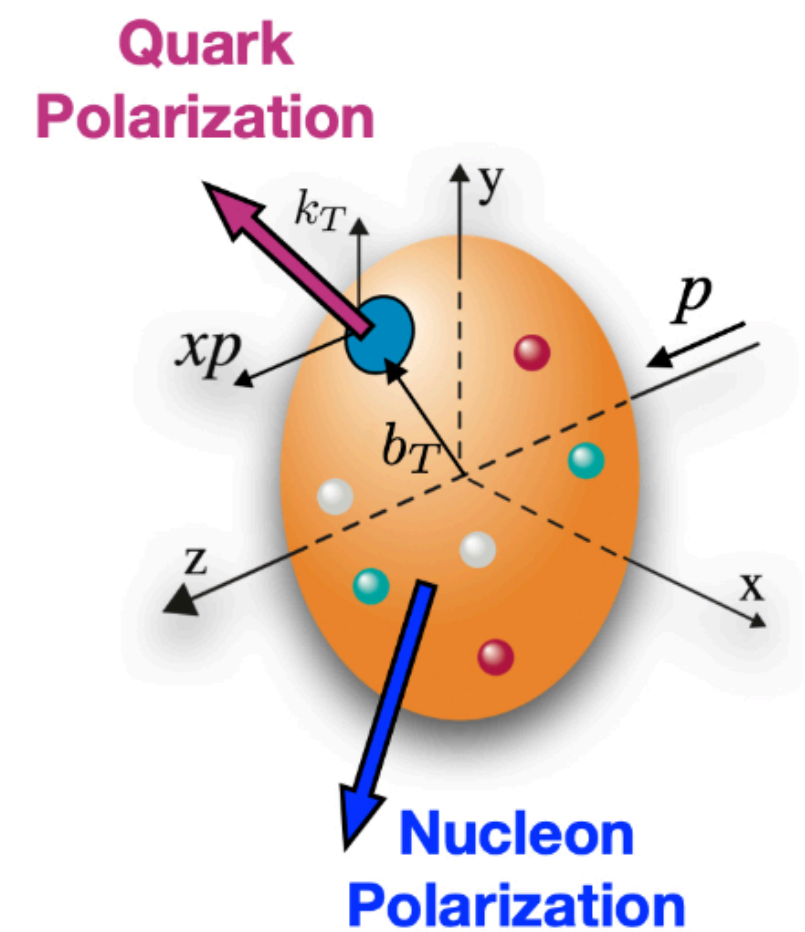
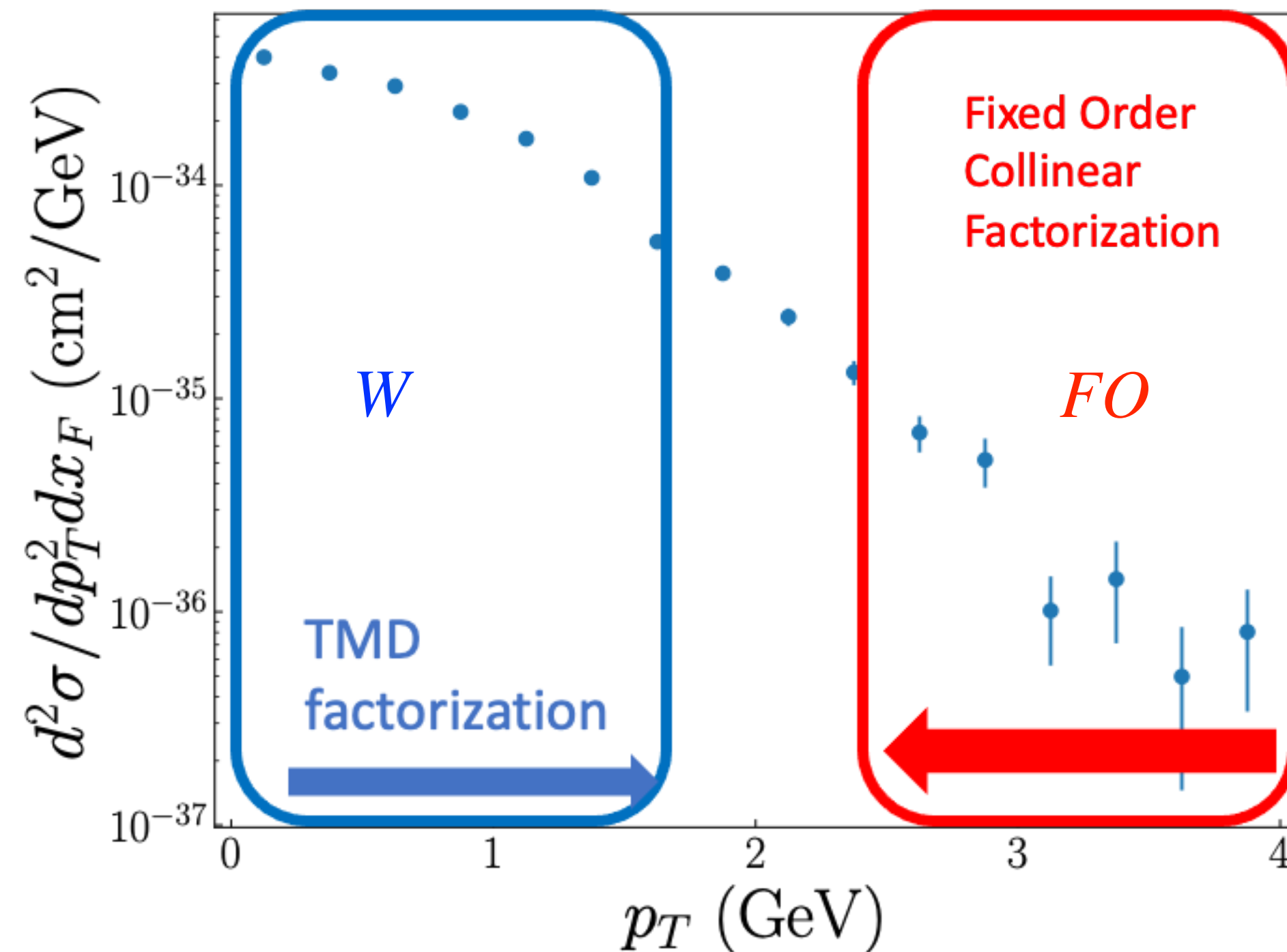


Figure 1.1: Illustration of the momentum and spin variables probed by TMD parton distributions.

$$\sigma_{\text{SIDIS}} \propto \left| \begin{array}{c} l \\ l' \\ q \\ P \\ X \end{array} \right|^2 \approx \left| \begin{array}{c} \xi P, k_T \\ P \end{array} \right|^2 \otimes \left| \begin{array}{c} l \\ l' \\ q \\ k' \\ \xi P, k_T \end{array} \right|^2 \otimes \left| \begin{array}{c} P_h \\ \frac{P_h}{\zeta}, k'_T \end{array} \right|^2$$

Matching of TMD & large $q_T \approx P_{hT}/z$ @ Leading power

- Factorization & Matching **unpolarized** Collins Soper Sterman NPB 1985, Sun, Isaacson, C.-P. Yuan, F. Yuan IJMPA(2018)
Collins Collins, Gamberg, Prokudin, Rogers, Sato, Wang PRD 2016
- **Polarization** Bacchetta, Boer, Diehl, Mulders JHEP (2008)
(N.B. Transverse polarization Ji, Qiu, Vogelsang Yuan PRL (2006); PRD (2006))



- **Cross section in terms of different “regions”**
- W valid for $q_T \sim k_T \ll Q$ TMD factorization
- FO valid for $k_T \ll p_T \sim Q$ Collinear factorization
- AY subtracts d.c. & in principle,
 $AY \rightarrow W, p_T \rightarrow \infty$ and $AY \rightarrow FO, p_T \rightarrow 0$
- $Y \equiv \rightarrow FO - AY$

$$\frac{d\sigma(m \lesssim p_T \lesssim Q, Q)}{dy dq^2 d^2 p_T} = W(p_T, Q) - AY(p_T, Q) + FO(p_T, Q) + \mathcal{O}\left(\frac{M}{Q}\right)^c$$

Recent progress collinear DY the Pion

@ high energies pion's partonic structure unfolded/revealed from DY process as predicted from Collinear Factorization \longrightarrow momentum distributions, $f_{i/\pi}(x, \mu)$

$$\frac{d\sigma^{\text{DY}}}{dQ^2 dy} = \sum_{a,b} \int dx_a dx_b H_{a,b}^{\text{DY}}(x_a, x_b, y, Q^2, \mu^2) f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2)$$

$$\frac{d^3\sigma^{\text{LN}}}{dx_B dQ^2 dx_L} = \frac{4\pi\alpha^2}{x_B Q^4} \left(1 - y_e + \frac{y_e^2}{2}\right) F_2^{\text{LN}}(x_B, Q^2, x_L)$$

Jefferson Lab Angular Momentum (JAM)

Barry, Sato, Melnitchouk, C.-R. Ji PRL 2018

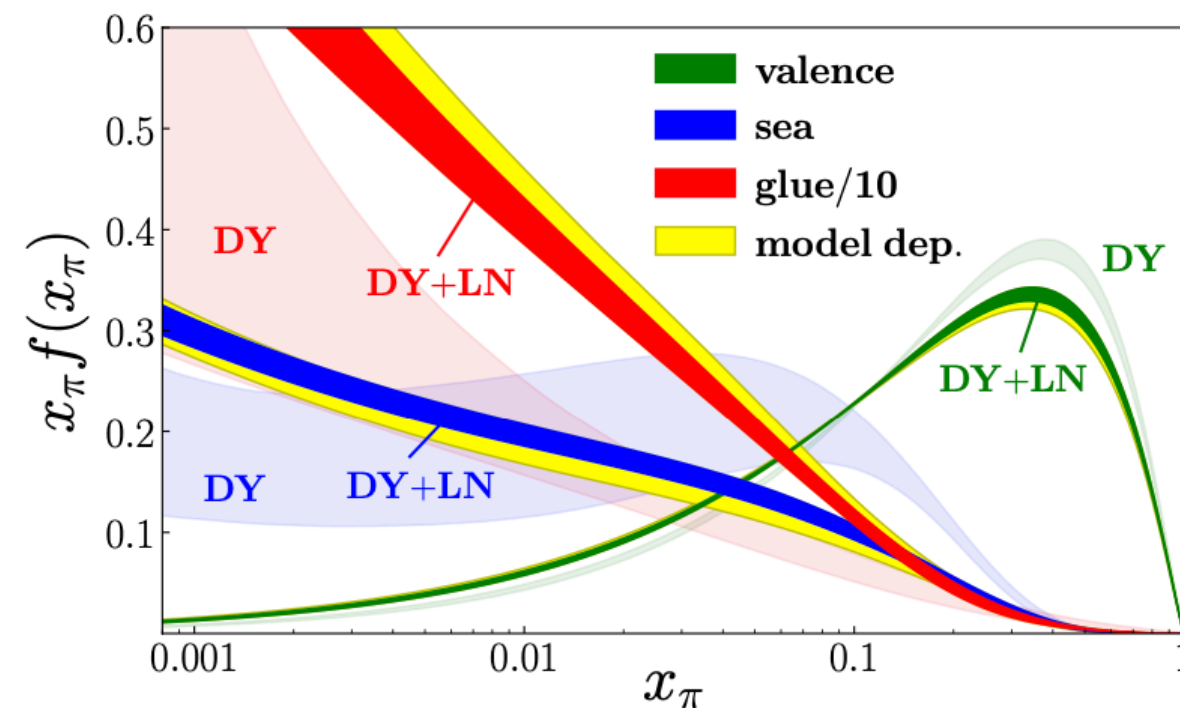
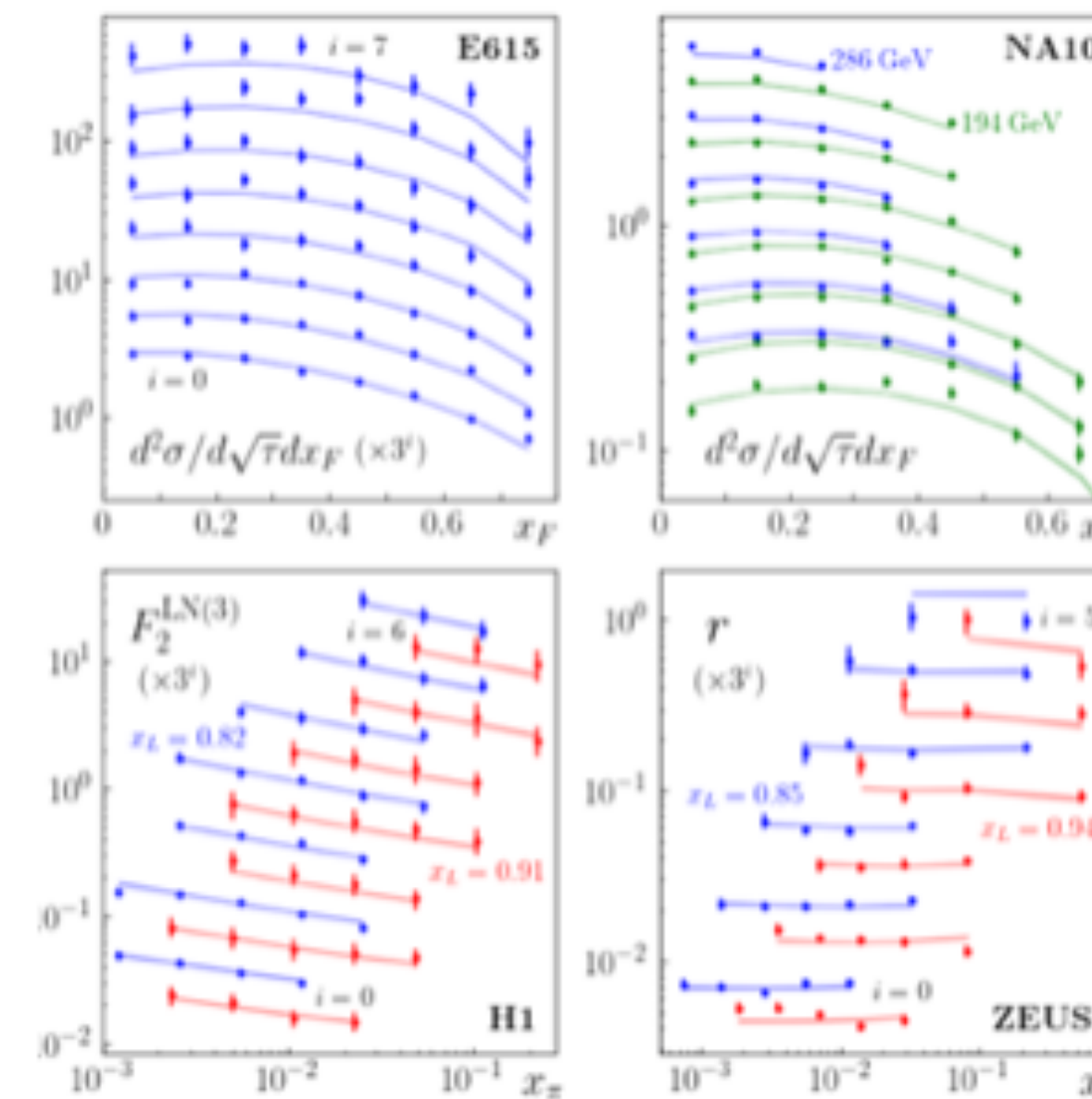
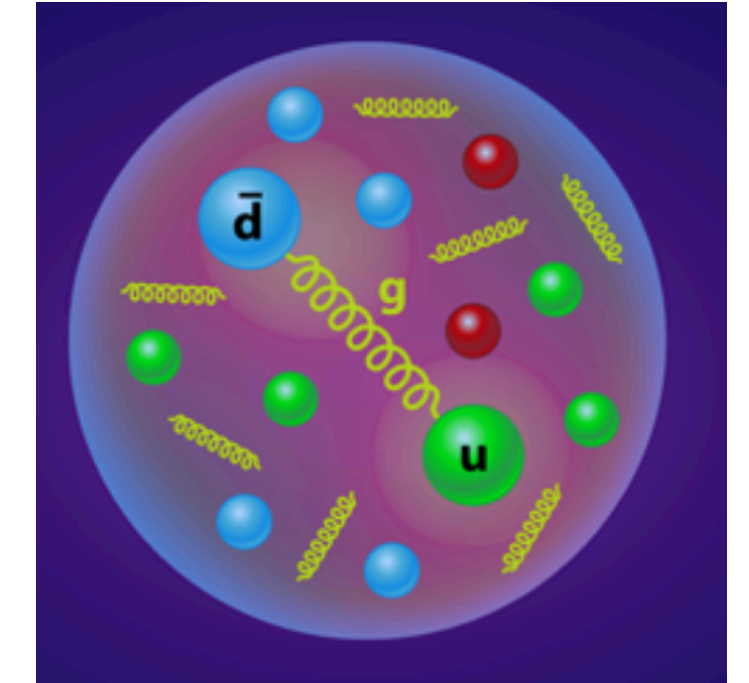
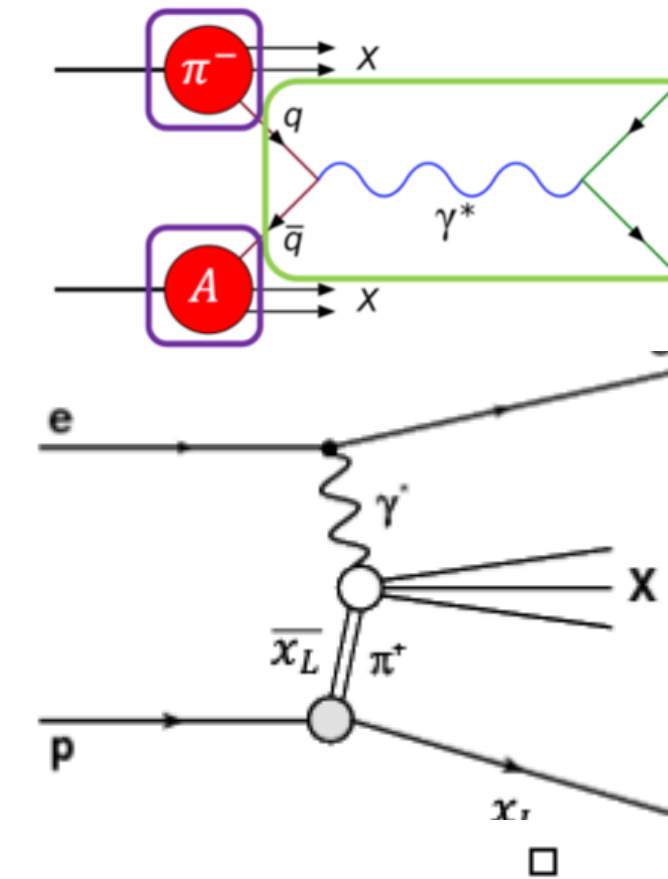


FIG. 2. Pion valence (green), sea quark (blue) and gluon (red, scaled by 1/10) PDFs versus x_π at $Q^2 = 10 \text{ GeV}^2$, for the full DY + LN (dark bands) and DY only (light bands) fits. The bands represent 1σ uncertainties, as defined in the standard Monte Carlo determination of the uncertainties [42] from the experimental errors. The model dependence of the fit is represented by the outer yellow bands.

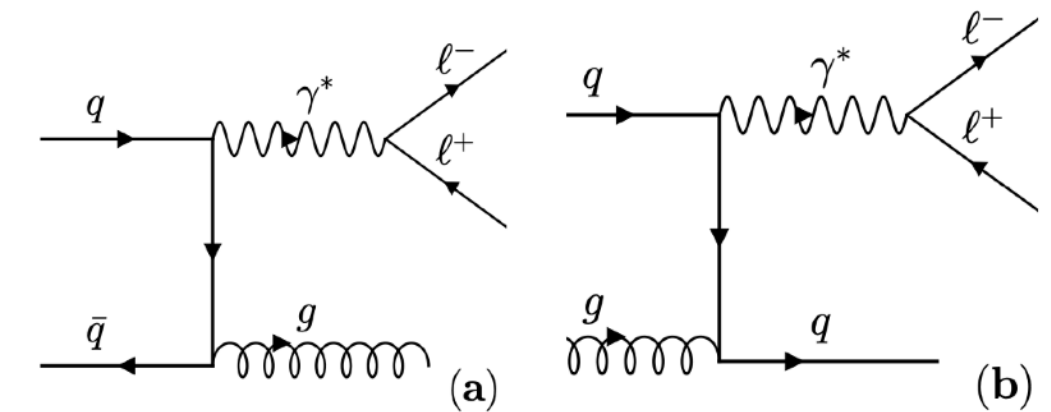


- Top row – Drell-Yan
- Bottom row – Leading neutron
- Good agreement with data
- $\chi^2_{\text{npts}} = 0.979$

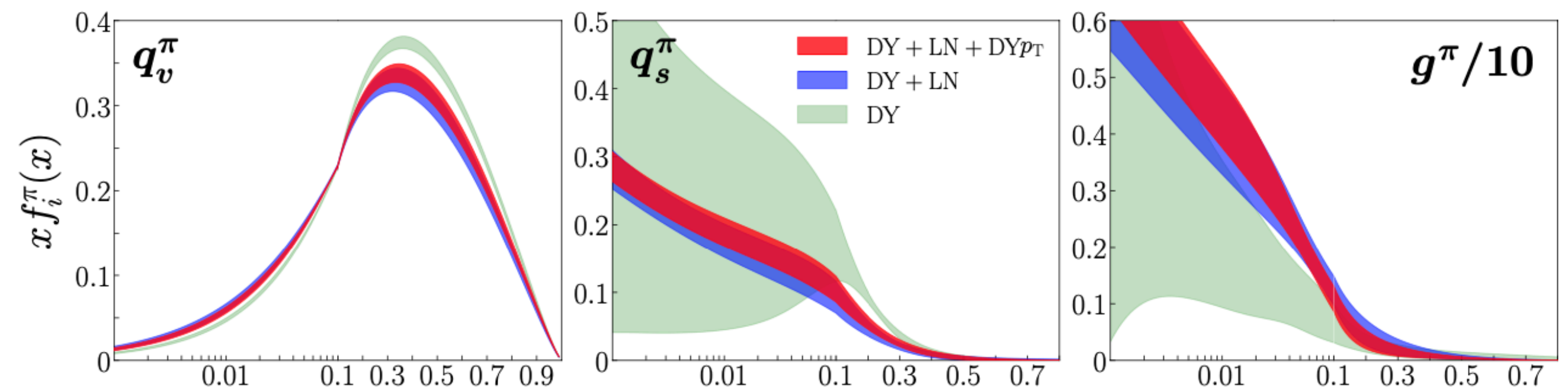
Recent progress large p_T FO -collinear

Cao et al *Phys.Rev.D* 103 (2021), added transverse momentum dependent DY data in a globalQCD analysis of large transverse momentum $p_T \sim Q$ dominated hard QCD radiation

$$\frac{d\sigma^{\text{DY}}}{dQ^2 dy dp_T} = \sum_{a,b} \int dx_a dx_b H_{a,b}^{\text{FO}}(x_a, x_b, y, p_T, Q^2, \mu^2) f_{a/A}(x_a, \mu^2) f_{b/B}(x_b, \mu^2)$$



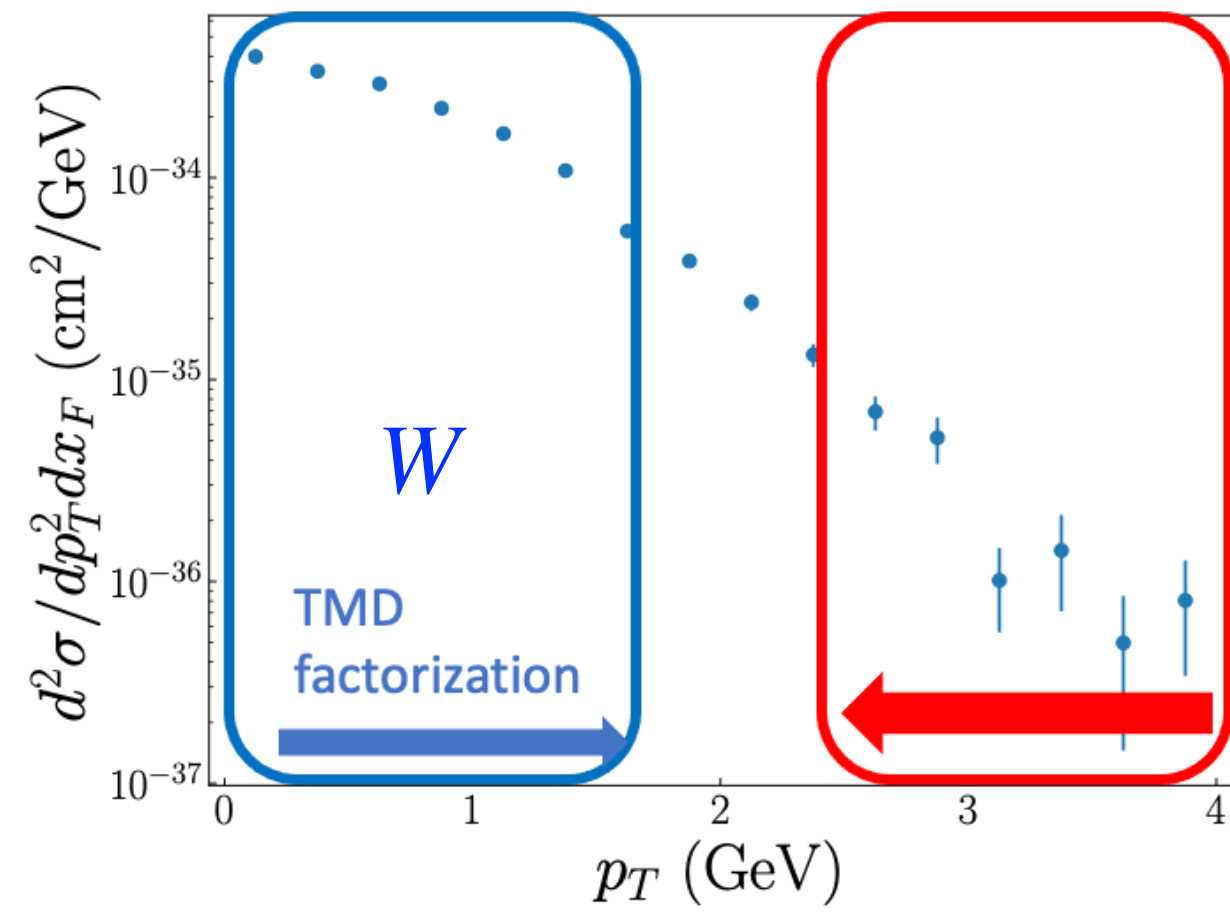
Inclusion of p_T -dependent data only slightly reduce uncertainties of the gluon distribution at large x & impacts on other distributions negligible



Understanding how these contrasting manifestations of the same $\bar{q}q$ bound state arise dynamically at different energy scales from first principles remains a major challenge in QCD

Extend fit to include “low” p_T Drell Yan data

- We consider impact on collinear pion pdfs from $p_T (\equiv q_T) \sim k_T \ll Q$ “TMD” region (*n.b.* smaller statistical uncertainties on the data)
- Pion induced DY scattering processes provide possibility to extract TMDs of the pion and nucleon when the cross section is kept differential in the transverse momentum of the produced lepton pair
- Factorized according to the framework of Collins-Soper-Sterman (CSS)



W TMD Term for DY & TMD correlator $\Phi_{q/N}$

$$\frac{d\sigma}{d^4q d\Omega} = \frac{\alpha_{\text{em}}^2}{4sQ^4} L_{\mu\nu} W^{\mu\nu},$$

$$d^4q = dQ^2 dy d^2\mathbf{q}_\perp$$

$$W_{\mu\nu}^{(2)} = \frac{1}{N_c} \sum_q e_q^2 \int d^2\mathbf{k}_{1\perp} d^2\mathbf{k}_{2\perp} \delta^{(2)}(\mathbf{q}_\perp - \mathbf{k}_{1\perp} - \mathbf{k}_{2\perp}) \\ \times \text{Tr} [\Phi_{q/P_1}(x_1, \mathbf{k}_{1\perp}, \mathbf{S}_1) \gamma^\mu \Phi_{\bar{q}/P_2}(x_2, \mathbf{k}_{2\perp}, \mathbf{S}_2) \gamma^\nu]$$

Well known correlator for the LP (& NLP correlation functions) in DY Cross section

$$\Phi(x, \mathbf{k}_T)$$

$$\begin{aligned} \Phi_{q/P}^{(2)}(x, \mathbf{k}_\perp, \mathbf{S}) = & \left(f_1 - \frac{\epsilon_{\perp}^{ij} k_{\perp i} S_{\perp j}}{M} f_{1T}^\perp \right) \frac{\not{\mathbf{k}}}{4} + \left(\lambda g_{1L} - \frac{\mathbf{k}_\perp \cdot \mathbf{S}_\perp}{M} g_{1T} \right) \frac{\gamma^5 \not{\mathbf{k}}}{4} \\ & + \left(S_\perp^i h_1 + \frac{\lambda k_\perp^i}{M} h_{1L}^\perp - \frac{\epsilon_{\perp}^{ij} k_{\perp j}}{M} h_1^\perp - \frac{k_\perp^i k_\perp^j - \frac{1}{2} k_\perp^2 g_{\perp}^{ij}}{M^2} S_{\perp j} h_{1T}^\perp \right) \frac{i \gamma^5 \sigma_{-i}}{4} \end{aligned}$$

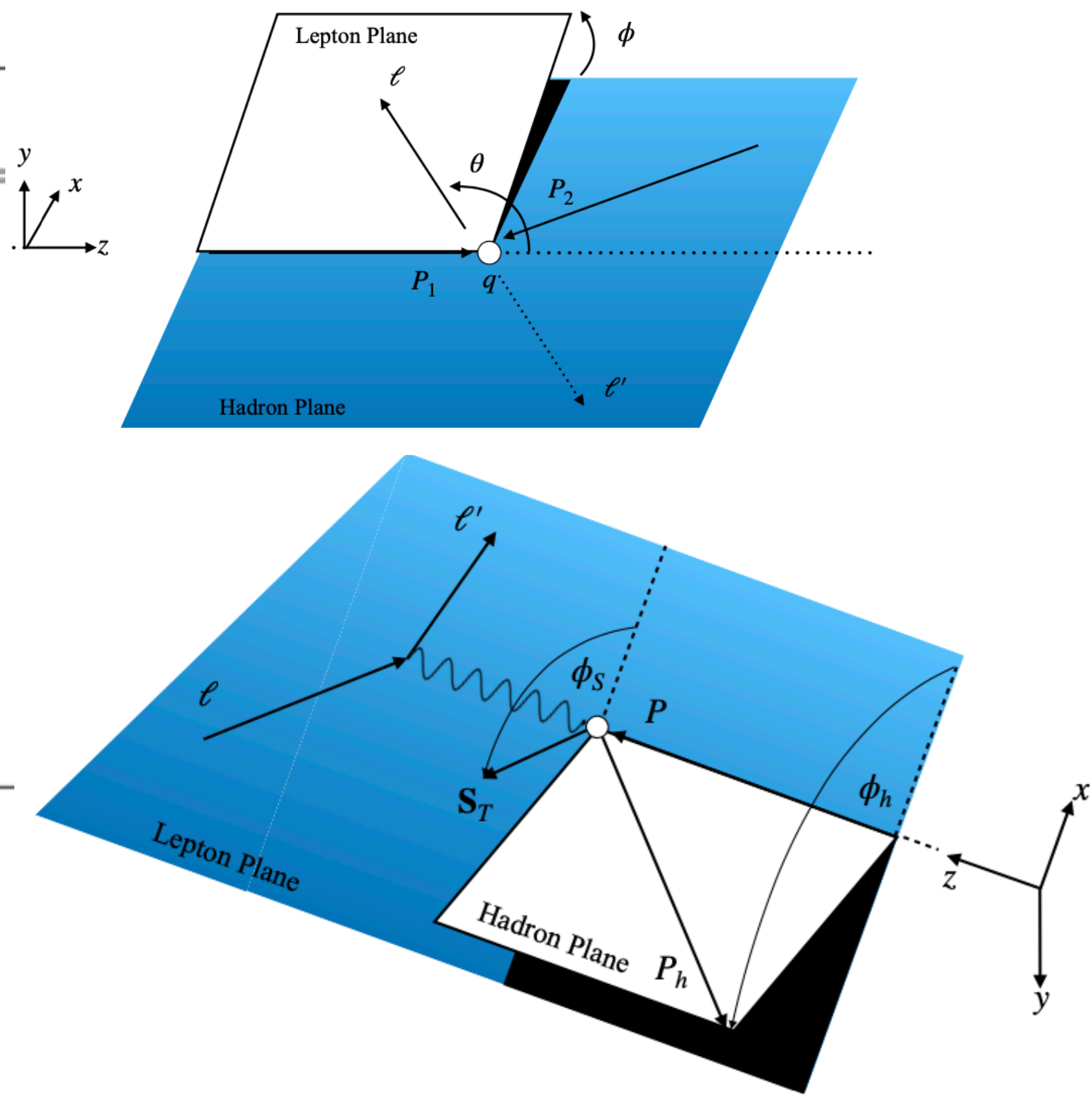
Leading Quark TMDPDFs

Nucleon Spin

Quark Spin

		Quark Polarization		
		Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
Nucleon Polarization	U	$f_1 = \text{Unpolarized}$		$h_1^\perp = \text{Boer-Mulders}$
	L		$g_1 = \text{Helicity}$	$h_{1L}^\perp = \text{Worm-gear}$
	T	$f_{1T}^\perp = \text{Sivers}$	$g_{1T}^\perp = \text{Worm-gear}$	$h_1 = \text{Transversity}$ $h_{1T}^\perp = \text{Pretzelosity}$

Twist 2	Twist 3
$\frac{1}{2} \not{\mathbf{k}}, \frac{1}{4} \not{\mathbf{k}}$	$\frac{1}{2}, \frac{1}{2}$
$\frac{1}{2} \not{\mathbf{k}} \gamma^5, \frac{1}{4} \gamma^5 \not{\mathbf{k}}$	$\frac{1}{2} \gamma^5, \frac{1}{2} \gamma^5$
$\frac{i}{2} \sigma^{k+} \gamma^5, \frac{i}{4} \gamma^5 \sigma_{-k}$	$\frac{1}{2} \gamma^k, \frac{1}{2} \gamma_k$
	$\frac{1}{2} \gamma^k \gamma^5, \frac{1}{2} \gamma^5 \gamma_k$
	$\frac{i}{2} \sigma^{kl} \gamma^5, \frac{i}{4} \gamma^5 \sigma_{lk}$
	$\frac{i}{4} \sigma^{+-} \gamma^5, \frac{i}{4} \gamma^5 \sigma_{+-}$



- ◆ Mulders Tangerman NPB 1995
- ◆ Goeke Metz Schlegel PLB 2005
- ◆ Bacchetta et al 2007 JHEP

Factorization for low $-q_T$ Drell-Yan $q_T \sim k_T \ll Q$

- W -term (CSS) depends on $\tau = Q^2/S$, rapidity Y , trans momentum q_T
- Cross section factorized in terms of hard part and two TMDs for beam and target
- *N.B.* W -term optimized to describe at low q_T

$$\frac{d^3\sigma}{d\tau dY dq_T^2} = \frac{4\pi^2\alpha^2}{9\tau S^2} \sum_q H_{q\bar{q}}(Q^2, \mu) \int d^2b_T e^{ib_T \cdot q_T} \times \tilde{f}_{q/\pi}(x_\pi, b_T, \mu, Q^2) \tilde{f}_{\bar{q}/A}(x_A, b_T, \mu, Q^2)$$

- Factorization commonly carried out in “Fourier” $b = (b^-, b^+, \mathbf{b}_T)$ space
- \mathbf{b}_T is the Fourier conjugate to the intrinsic transverse momentum of quarks \mathbf{k}_T

$$\tilde{f}_{q/N}(x, b_T) = \int d^2k_T e^{i\mathbf{b}_T \cdot \mathbf{k}_T} f_{q/N}(x, \mathbf{k}_T)$$

$$\tilde{f}_{q/\mathcal{N}}(x, b_T) = \int \frac{db^-}{4\pi} e^{-ixP^+b^-} \text{Tr}[\langle \mathcal{N} | \bar{\psi}_q(b) \gamma^+ \mathcal{W}(b, 0) \psi_q(0) | \mathcal{N} \rangle]$$

From TMD factorization

$$\tilde{f}(x, b_T; \mu, \zeta) = [C \otimes \textcolor{red}{f}](x, b_T; \mu_0, \zeta_0) \times e^{S_{\text{evo}}(b_T; \mu, \mu_0, \zeta, \zeta_0)} f_{\text{NP}}(x, b_T)$$

Transverse Momentum Dependent distribution (TMD)

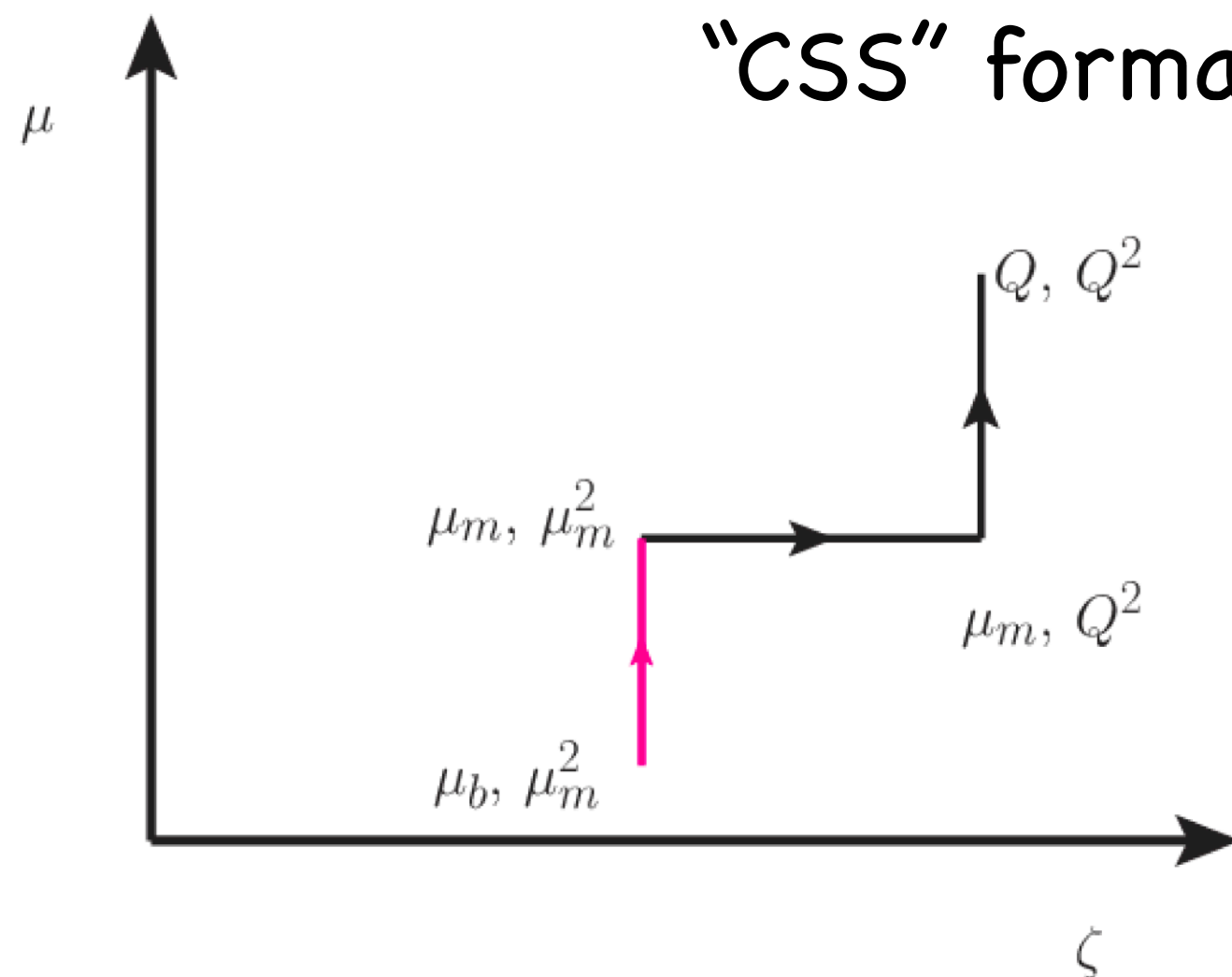
- Encode both the **collinear** and **transverse momentum** carried by partons
- TMDs are related to collinear PDFs via Operator Product Expansion
- Both TMDs and PDFs can be extracted from variety of experimentally measured processes where factorization is applicable, such as Drell-Yan (DY)

Discussion from TMD factorization Evolution

$$\tilde{f}_{q/N}(x, b_T; \mu_0, \zeta_0) = f_{q/N}^{\text{NP}}(x, b_T) \sum_j \int_x^1 \frac{d\xi}{\xi} \tilde{C}_{q/j}(x/\xi, b_T; \mu_0, \zeta_0) f_{j/N}(\xi; \mu_0)$$

Transverse Momentum Dependent distributions

- f^{NP} describes the non-perturbative structure of the TMD at large- b_T
- Convolution is the OPE, which describes small- b_T behavior
- Explicit dependence on the collinear PDF $f_{i/N}$
- \tilde{C} coefficient function is perturbatively calculable
- **TMD evolution** in μ & ζ needs to be incorporated to match w/ data
- “CSS” formalism see Ch. 4 TMD handbook



TMD evolution depends on UV and rapidity renormalization scales

The TMDs: factorization, renormalization and evolution

“Bare” TMD Factorization Parton Model

♦Mulders Tangerman NPB1995, Boer Mulders PRD 1997

$$q_T \sim k_T \ll Q$$

Leading Quark TMDPDFs

Quark Spin

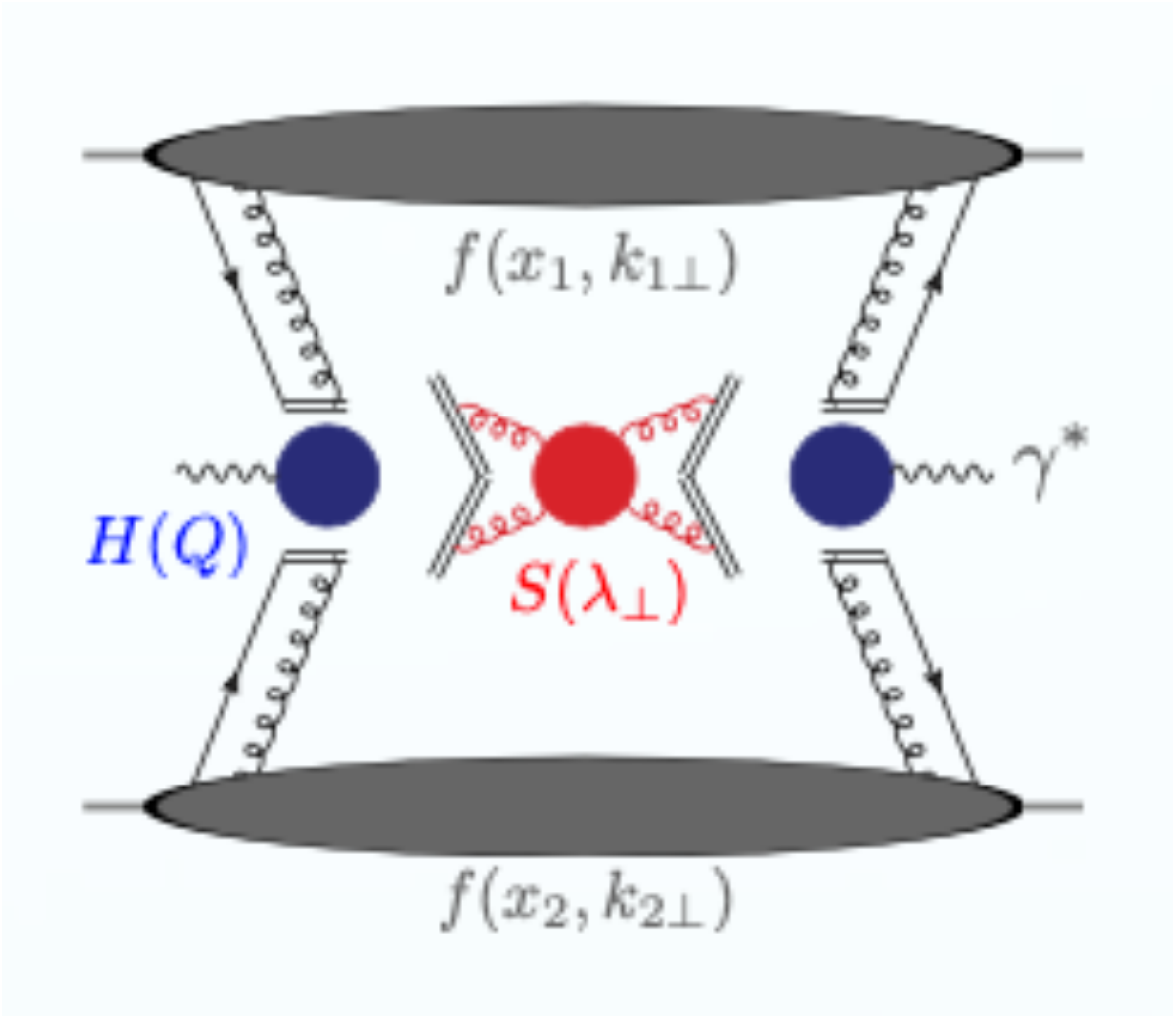
	Quark Polarization		
	Un-Polarized (U)	Longitudinally Polarized (L)	Transversely Polarized (T)
U	$f_1 = $ <div> <div></div> <div>Unpolarized</div> </div>		$h_1^\perp = $ <div> <div></div> <div>Boer-Mulders</div> </div>

$$f(x,k_T,s_T)=\frac{1}{2}\left[f_1^\pi(x,k_T^2)+\frac{s_T^i\epsilon^{ij}k_T^j}{m_\pi}h_1^{\pi\perp}(x,k_T^2)\right]$$

- Factorization carried out Fourier b_T space FT TMDs $\tilde{f}(x,b_T)$
- Real QCD need QFT definitions of TMDs LC & UV divergences reflected in the CS & RG Eqs. $\tilde{f}(x,b_T;\mu,\zeta)$
- TMD Evolution depends on rapidity ζ and RGE scales μ

$$\tilde{f}_{q/\mathcal{N}}(x,b_T)=\int\frac{db^-}{4\pi}e^{-ixP^+b^-}\text{Tr}[\langle\mathcal{N}|\bar{\psi}_q(b)\gamma^+\mathcal{W}(b,0)\psi_q(0)|\mathcal{N}\rangle]$$

$$\tilde{f}_{i/p}(x,\mathbf{b}_T,\mu,\zeta)=\lim_{\epsilon\rightarrow 0}Z_{\text{uv}}(\mu,\zeta,\epsilon)\lim_{y_B\rightarrow-\infty}\frac{\tilde{f}_{i/p}^{0(u)}(x,\mathbf{b}_T,\epsilon,y_B,xP^+)}{\sqrt{\tilde{S}_{n_A(2y_n)n_B(2y_B)}^0(b_T,\epsilon,2y_n-2y_B)}}.$$



TMD Factorization

- ♦Collins Soper Sterman NPB 1985
- ♦Ji Ma Yuan PRD PLB ...2004, 2005
- ♦Aybat Rogers PRD 2011
- ♦Collins 2011 Cambridge Press
- ♦Echevarria, Idilbi, Scimemi JHEP 2012, ...
- ♦SCET Becher & Neubert, 2011 EJPC

Aybat Rogers 2011 PRD
 Collins 2011 red book
 Collins Rogers 2015 PRD
 TMD Handbook <https://arxiv.org/abs/2304.03302>

Pheno analysis

Phys. Rev. D 108 (2023)

<https://arxiv.org/abs/2302.01192>

Analysis:

- **First** we carry out a fit of the non-perturbative parameters of the π TMD from the available data (scant and old E615—COMPASS has new data ...)
- As a **second step** we open up the fit of both collinear pion pdf parameters along with non-perturbative parameters
- As a **final step**, we perform a fit of the p_T integrated (collinear) and p_T dependent data to carry out a simultaneous fit of the pion collinear pdfs and pion TMDs
This constitutes a first such study
- We also compare the impact of various scenarios for describing non-perturbative content of the TMD contribution.

Further details of analysis framework

Nuclear TMD PDFs

- The TMD factorization allows for the description of a quark inside a nucleus to be $\tilde{f}_{q/A}$
- However, the intrinsic non-perturbative structure will in-principle change from nucleus-to-nucleus
- Want to model these in terms of protons and neutrons as we don't have enough observables to separately parametrize different nuclei

Nuclear TMD PDFs – working hypothesis

- We must model the nuclear TMD PDF from proton

$$\tilde{f}_{q/A}(x, b_T, \mu, \zeta) = \frac{Z}{A} \tilde{f}_{q/p/A}(x, b_T, \mu, \zeta) + \frac{A-Z}{A} \tilde{f}_{q/n/A}(x, b_T, \mu, \zeta)$$

- Each object on the right side independently obeys the CSS equation
 - **Assumption** that the bound proton and bound neutron follow TMD factorization
- Make use of isospin symmetry in that $u/p/A \leftrightarrow d/n/A$, etc.

Building of the nuclear TMD PDF

- Then taking into account the intrinsic non-perturbative, we model the flavor-dependent pieces of the TMD PDF as

$$(C \otimes f)_{u/A}(x) e^{-g_{u/A}(x, b_T)} \rightarrow \frac{Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x, b_T)} \\ + \frac{A-Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x, b_T)}$$

and

$$(C \otimes f)_{d/A}(x) e^{-g_{d/A}(x, b_T)} \rightarrow \frac{Z}{A} (C \otimes f)_{d/p/A}(x) e^{-g_{d/p/A}(x, b_T)} \\ + \frac{A-Z}{A} (C \otimes f)_{u/p/A}(x) e^{-g_{u/p/A}(x, b_T)}.$$

Perform the Monte Carlo

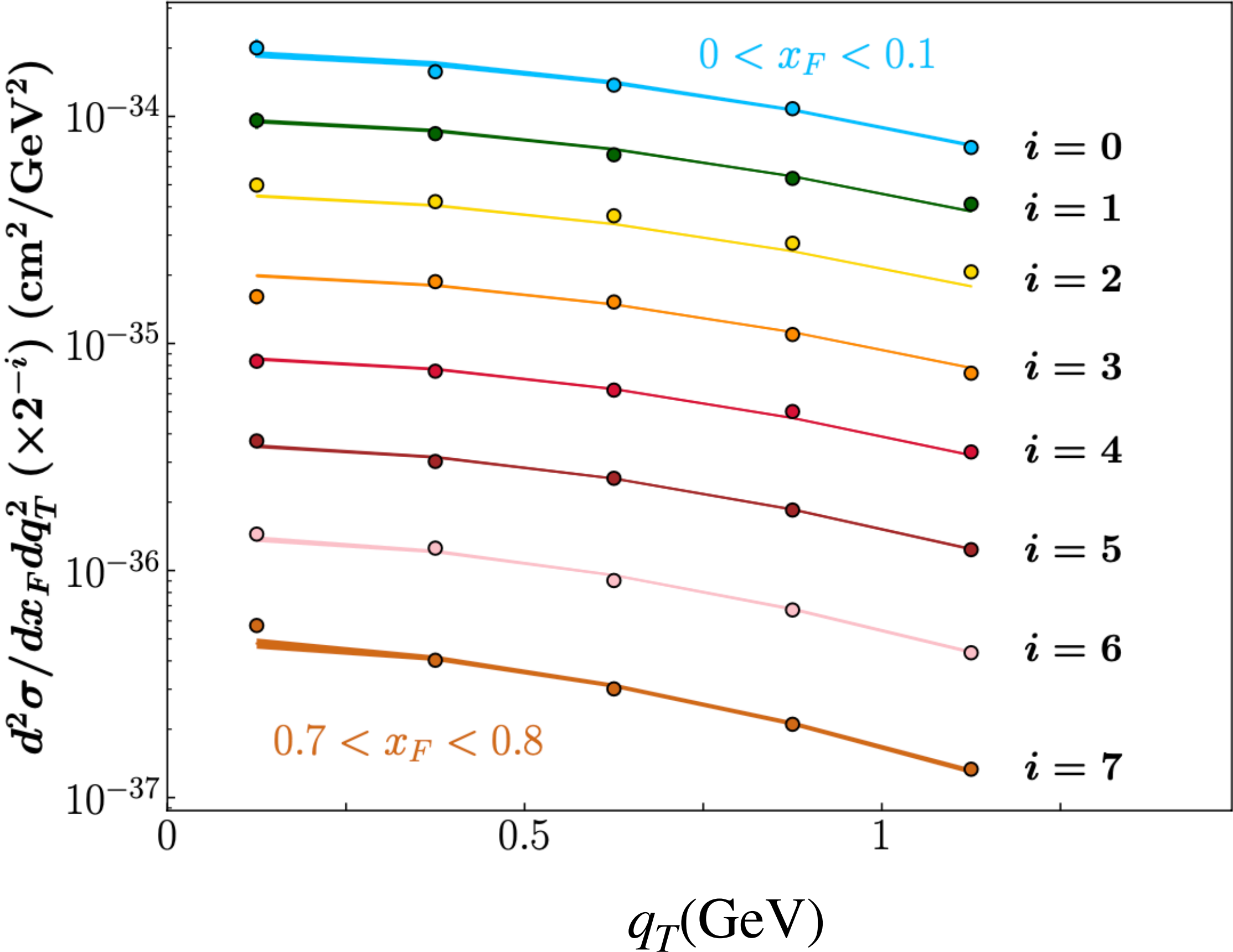
- We use the MAP parametrization [JHEP10\(2022\)127 Bacchetta et al.](#)
- Now, we can include the **pion collinear PDF** and its collinear datasets
- Include an additional 225 collinear data points
- Simultaneously extract
 1. Pion TMD PDFs
 2. Pion collinear PDFs
 3. Proton TMD PDFs
 4. Nuclear dependence
 5. Non-perturbative CS kernel

Pheno-Aspects of the fit

In this analysis both q_T dependent and collinear data we are able to first time simultaneously extract the pion's TMD and collinear PDFs

Fit both pA and πA DY data and achive good agreement in both

Process	Experiment	\sqrt{s} (GeV)	χ^2/N	Z-score
TMD				
q_T -dep. pA DY $pA \rightarrow \mu^+ \mu^- X$	E288 [90]	19.4	1.07	0.34
	E288 [90]	23.8	0.99	0.05
	E288 [90]	24.7	0.82	0.99
	E605 [91]	38.8	1.22	1.03
	E772 [92]	38.8	2.54	5.64
	(Fe/Be) E866 [93]	38.8	1.10	0.36
(W/Be)	E866 [93]	38.8	0.96	0.15
q_T -dep. πA DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [94]	21.8	1.45	1.85
	E537 [95]	15.3	0.97	0.03
collinear				
q_T -integr. DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [94]	21.8	0.90	0.48
	NA10 [96]	19.1	0.59	1.98
	NA10 [96]	23.2	0.92	0.16
leading neutron $ep \rightarrow enX$	H1 [97]	318.7	0.36	4.59
	ZEUS [98]	300.3	1.48	2.15
Total			1.12	1.86



Pheno-Aspects of the fit

In this analysis both q_T dependent and collinear data we are able to first time simultaneously extract the pion's TMD and collinear PDFs

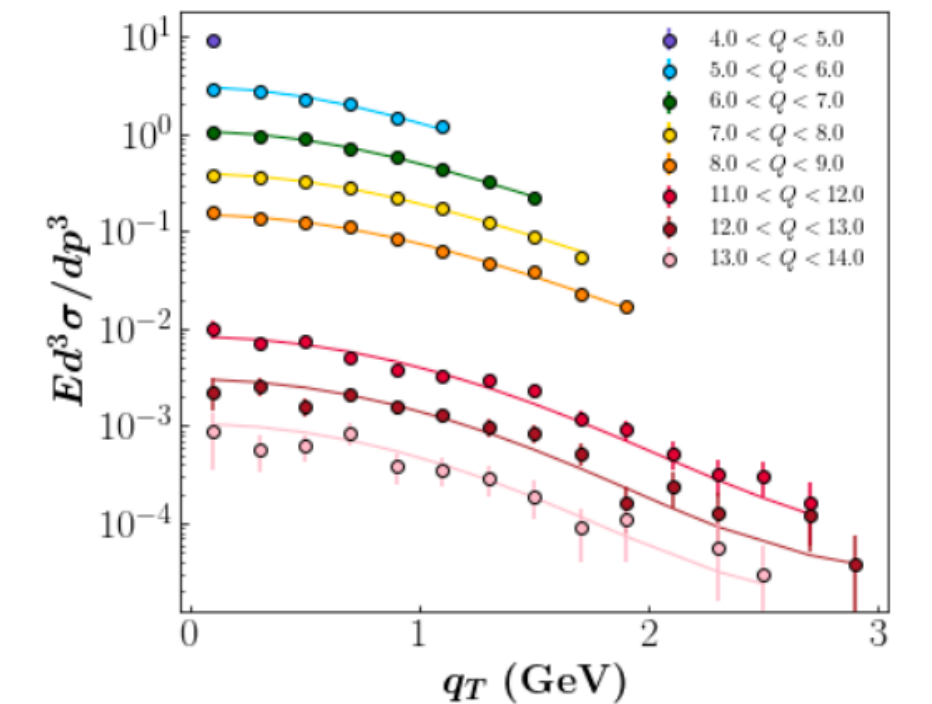
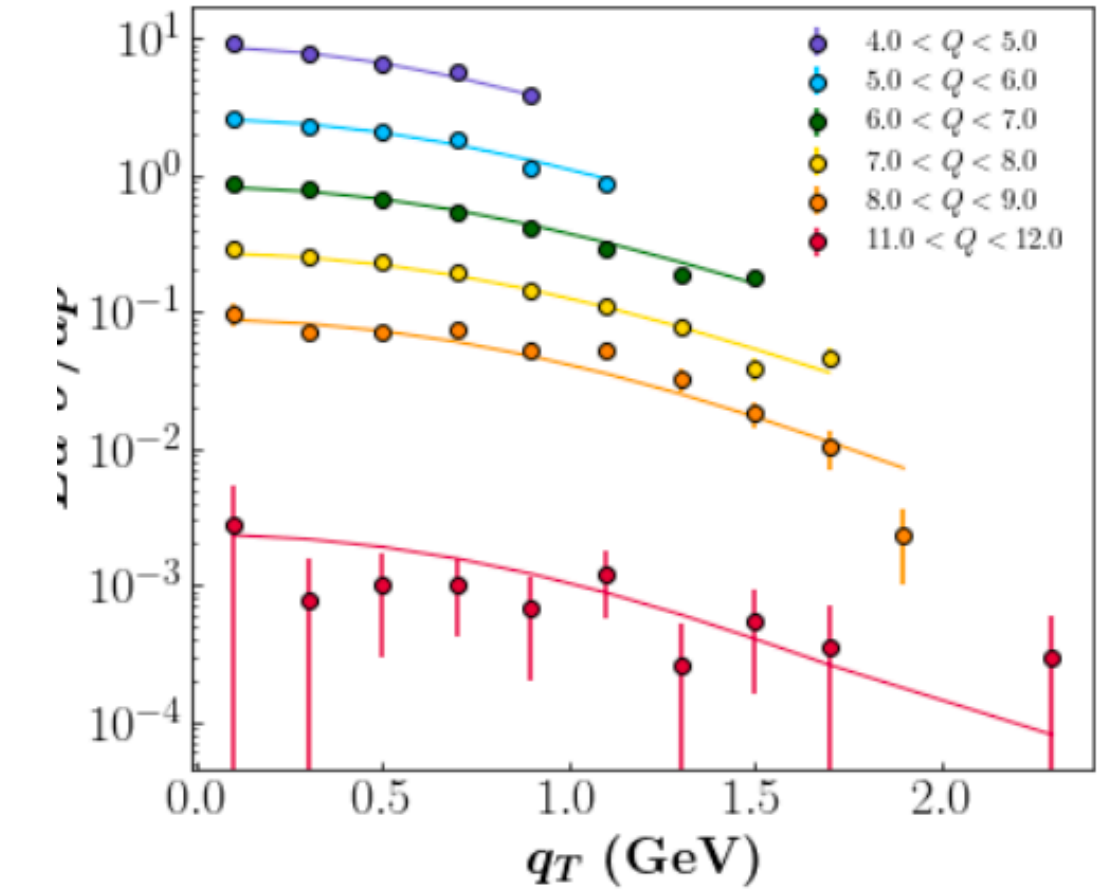
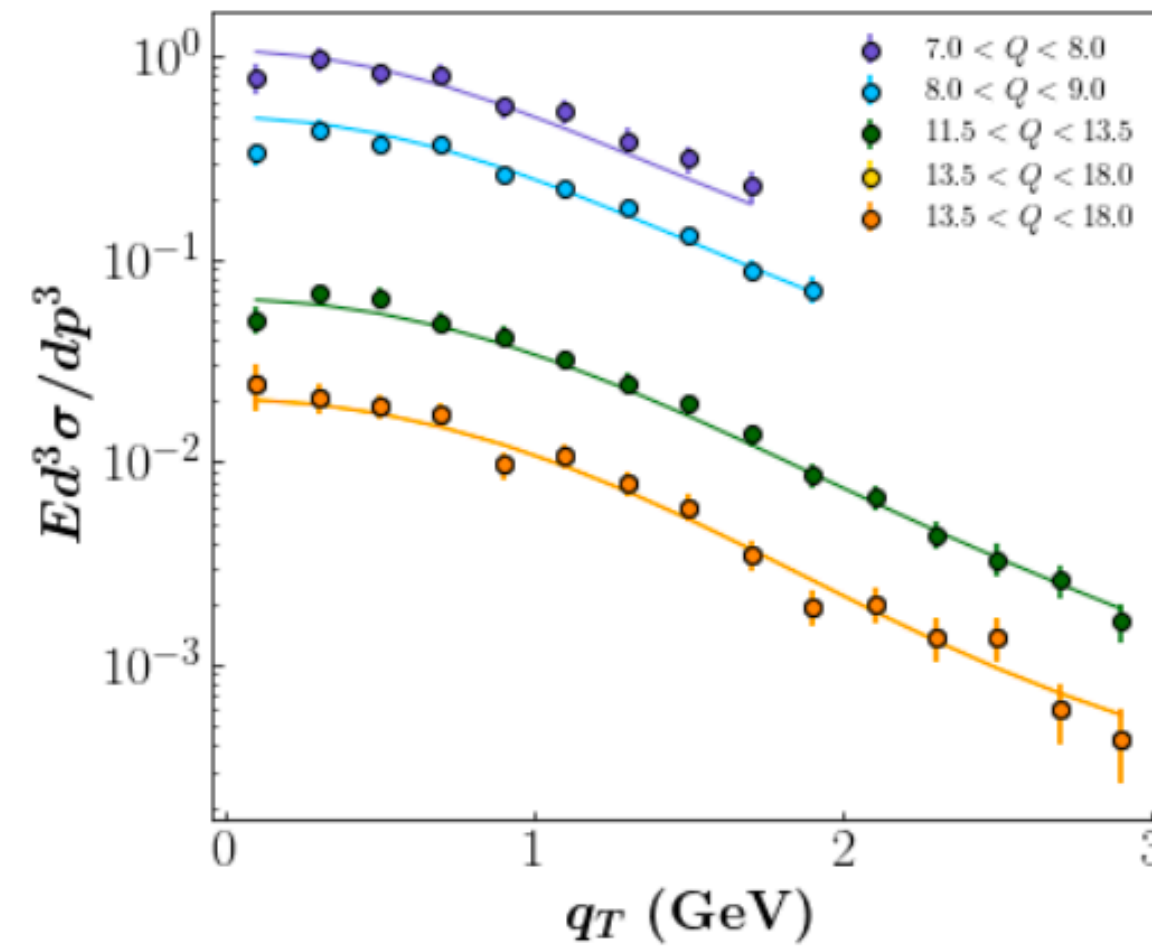
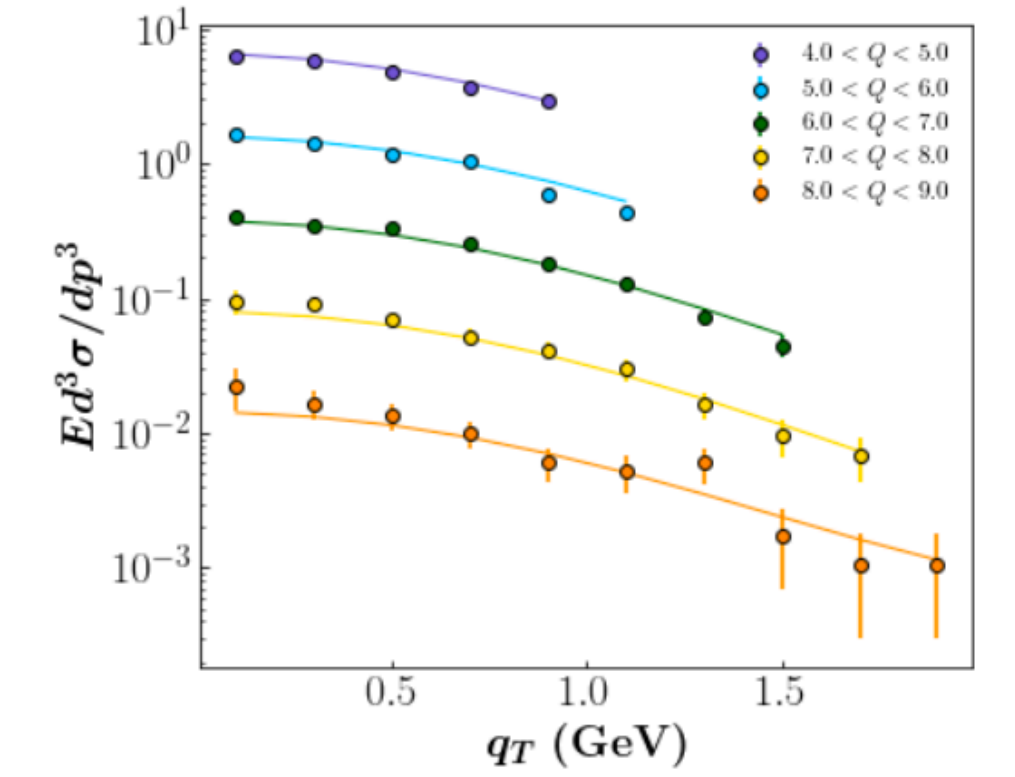


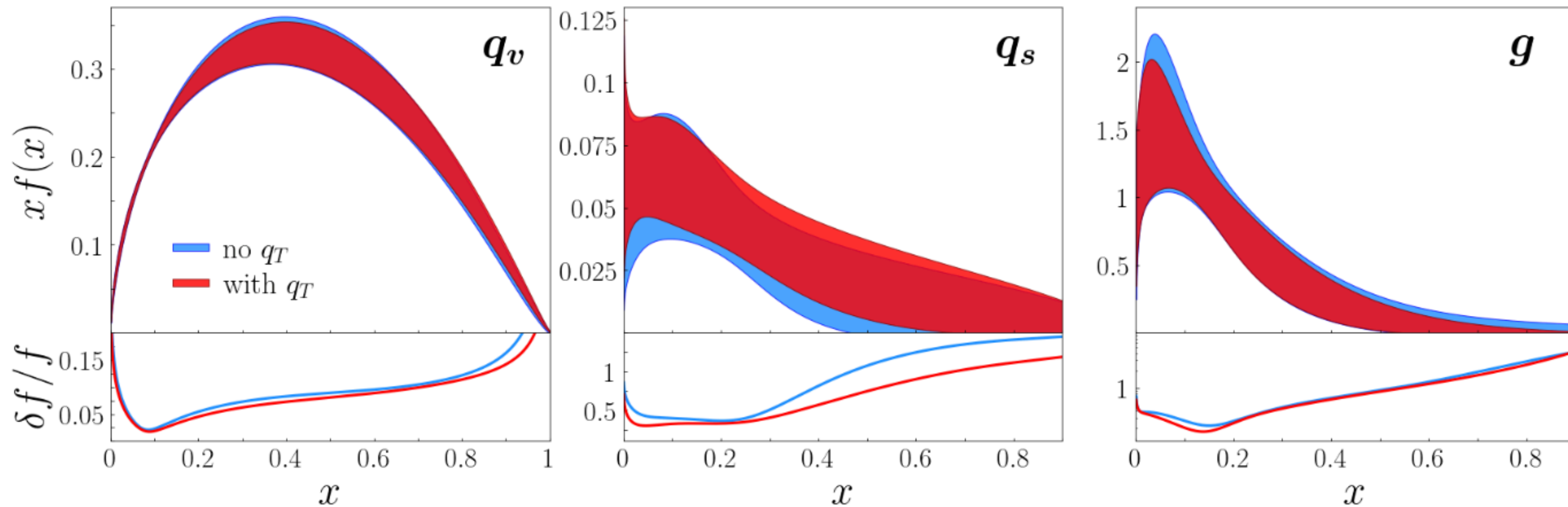
Figure 4: Comparison of the data and the results of the analysis for pCu Drell-Yan data from E605 experiment [42] as a function of q_T GeV for bins of Q .

Figure 5: Comparison of the data and the results of the analysis for pPt Drell-Yan data from E288 experiment [41] as a function of q_T GeV for bins of Q .

Process	Experiment	\sqrt{s} (GeV)	χ^2/N	Z-score
TMD				
q_T -dep. pA DY $pA \rightarrow \mu^+ \mu^- X$	E288 [90]	19.4	1.07	0.34
	E288 [90]	23.8	0.99	0.05
	E288 [90]	24.7	0.82	0.99
	E605 [91]	38.8	1.22	1.03
	E772 [92]	38.8	2.54	5.64
	(Fe/Be) E866 [93]	38.8	1.10	0.36
	(W/Be) E866 [93]	38.8	0.96	0.15
q_T -dep. πA DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [94]	21.8	1.45	1.85
	E537 [95]	15.3	0.97	0.03
collinear				
q_T -integr. DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [94]	21.8	0.90	0.48
	NA10 [96]	19.1	0.59	1.98
	NA10 [96]	23.2	0.92	0.16
leading neutron $ep \rightarrow enX$	H1 [97]	318.7	0.36	4.59
	ZEUS [98]	300.3	1.48	2.15
Total			1.12	1.86

Pheno-Aspects of the fit

Extracted pion PDFs



- The small- q_T data do not constrain much the PDFs

TMD presentation

By def., the TMDPDF is a $2 - D$ number density dependent on x & b_T “joint” “probability distribution”

$$\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)$$

Here we study the probability distribution in b_T for a given x : this is a quantity in which describes the ratio of the $2 - D$ density to the integrated or b_T —independent density; that is dependent on “ b_T given x ”
→ “conditional density”

$$\equiv \tilde{f}_{q/\mathcal{N}}(b_T|x; \mu, \zeta)$$

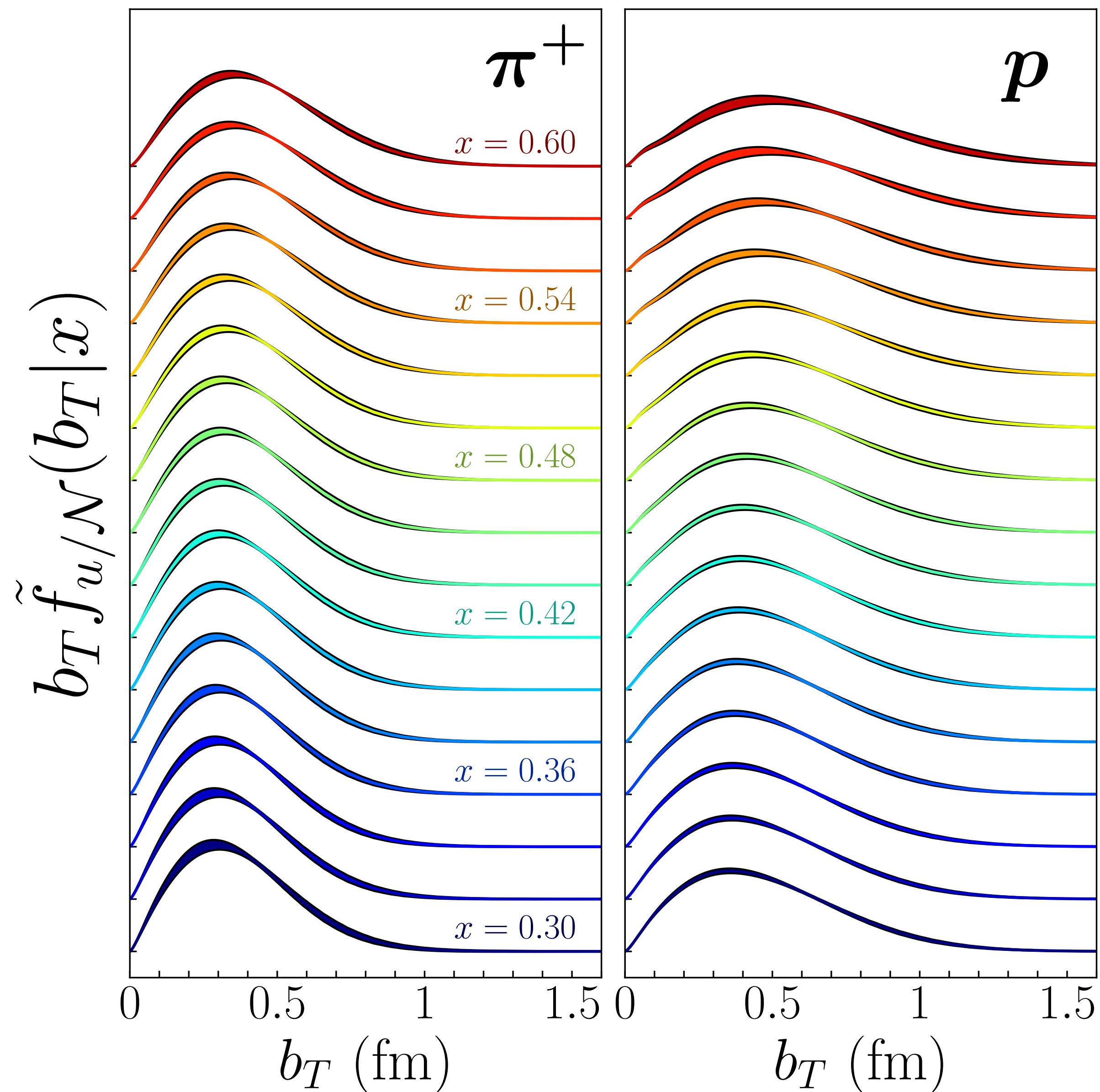
Naturally follow from Bayes’ theorem define a conditional density on b_T — given x

$$\tilde{f}(x, b_T; \mu, \zeta) = \tilde{f}(b_T|x; \mu, \zeta) f(x, \mu)$$

Operationally :

$$\tilde{f}_{q/\mathcal{N}}(b_T|x; \mu, \zeta) = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\int d^2\mathbf{b}_T \tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}$$

Resulting TMD PDFs of proton & pion



$$\tilde{f}_{q/\mathcal{N}}(b_T|x; \mu, \zeta) = \frac{\tilde{f}_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}{\int d^2\mathbf{b}_T f_{q/\mathcal{N}}(x, b_T; \mu, \zeta)}$$

- Shown in x range $x \in [0.3, 0.6]$ where π and p are both constrained: each TMDpdf show w/ 1σ uncertainty band from analysis
- u -quark in π is narrower than u -quark in p & “both” become wider w/increasing x

To make quantitative comparison btwn hadron distributions we consider the average b_T as a function of x

defined as

$$\langle b_T|x \rangle_{q/\mathcal{N}} = \int d^2\mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T|x; Q, Q^2)$$

“Average b_T ”

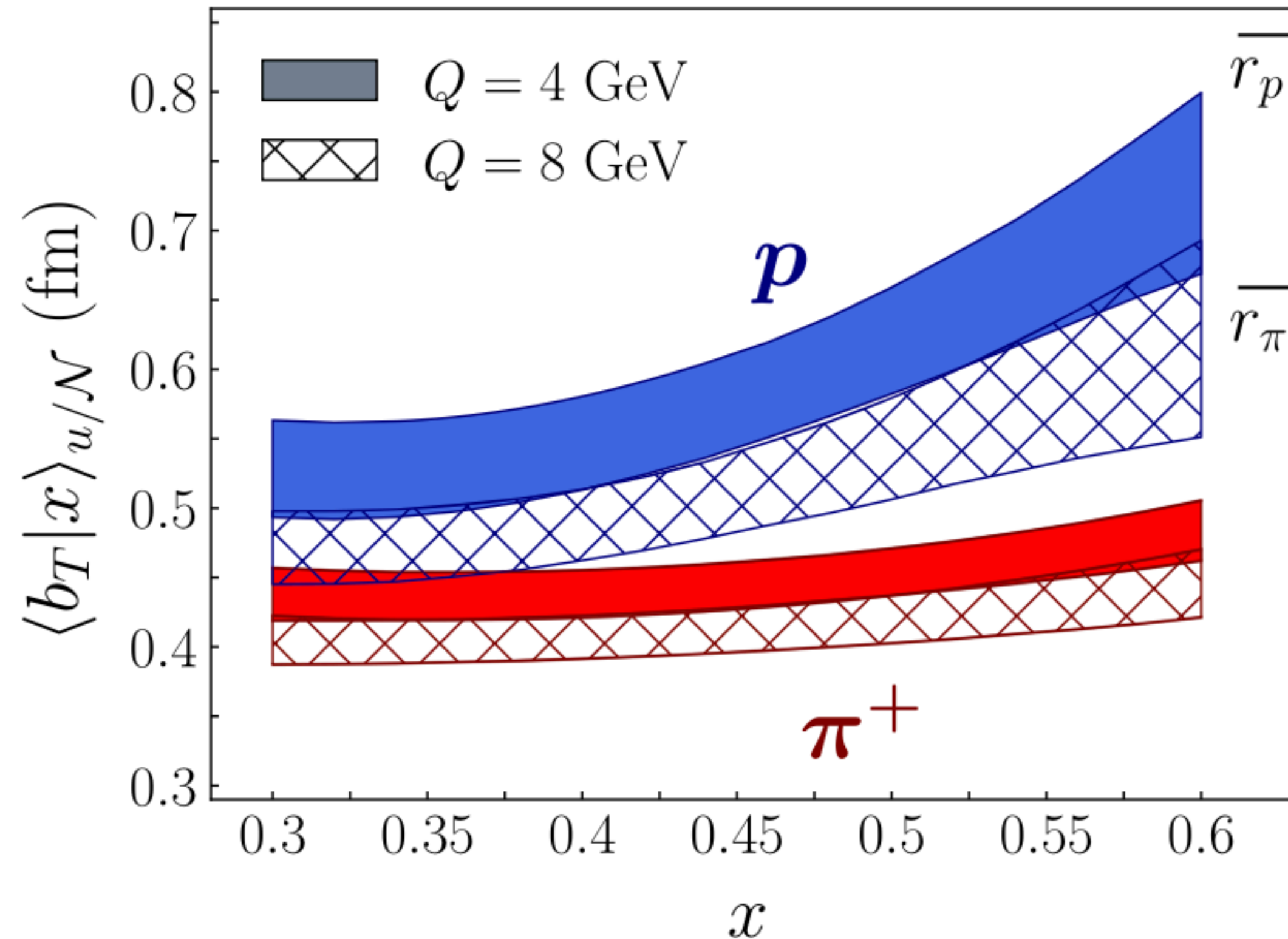
The conditional expectation value of b_T for a given x

$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2 \mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$

Gives a measure of the transverse correlation in Fourier space of the quark in a hadron for a given x

The conditional expectation value of b_T for a given x

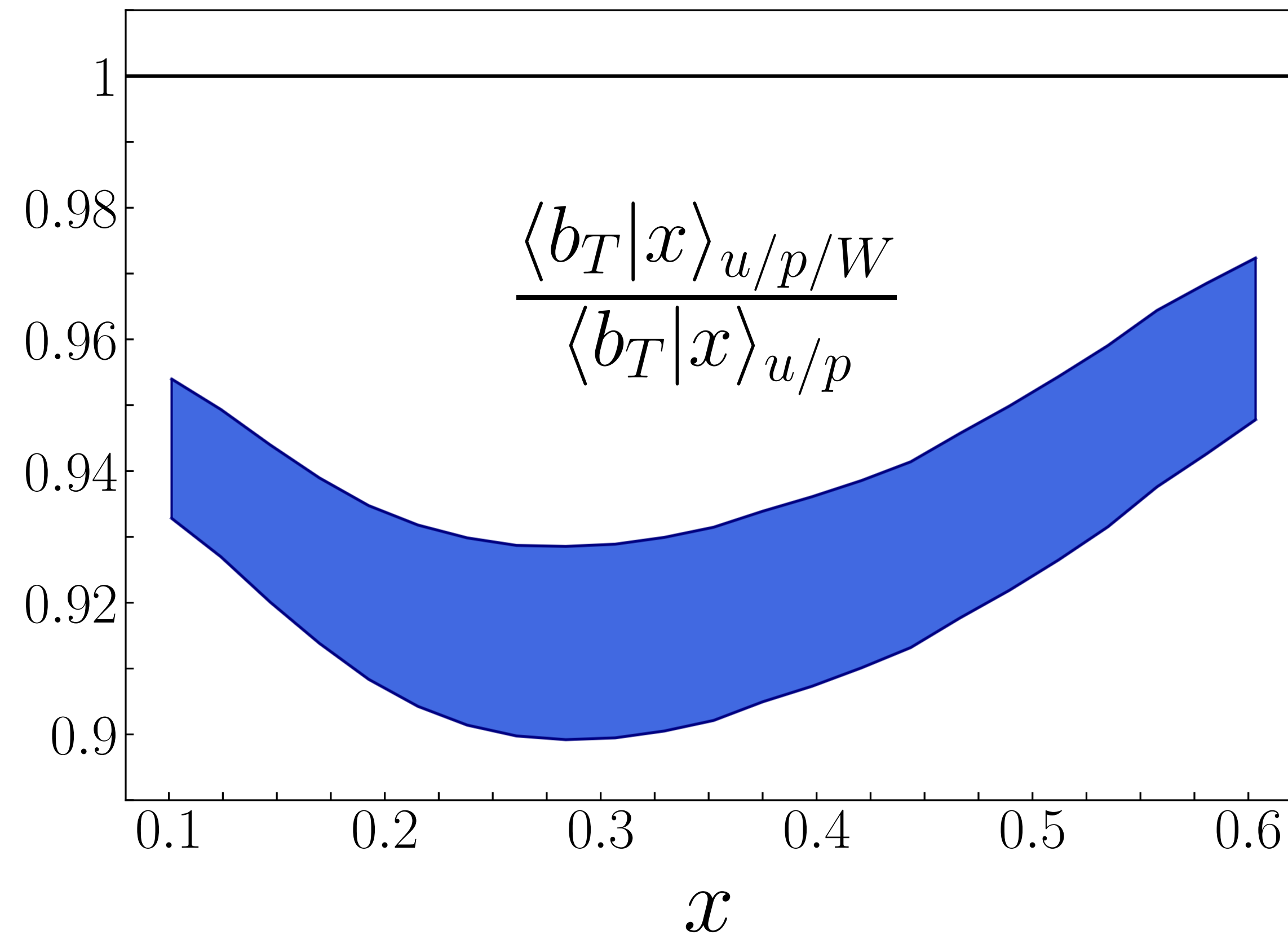
$$\langle b_T | x \rangle_{q/\mathcal{N}} = \int d^2 \mathbf{b}_T b_T \tilde{f}_{q/\mathcal{N}}(b_T | x; Q, Q^2)$$



- On av. $\sim 20\%$ reduction of u -quark transverse correlations in pion relative to proton within a $(4.0 - 5.2) \sigma$ confidence level
- Interestingly: charge radius of pion about 20% smaller than that of the proton, using the nominal PDG values, $r_p = 0.8409(4) \text{ fm}$, $r_\pi = 0.659(4) \text{ fm}$ [PDG]
- Also, within each hadron, the average spatial separation of quark fields in the transverse direction does not exceed its charge radius, as shown at right edge of fig.
- As $x \rightarrow 1$ phase space for the transverse motion k_T of partons becomes smaller, since most of the momentum is along the light-cone direction, thus one expects an increase in the transverse correlations in b_T space
- & as Q increases more glue is radiated, which makes TMDpdf wider in k_T & therefore narrow in b_T space

The effect of nuclear environment on quark correlations inside nucleon

EMC effect for conditional $\langle b_T | x \rangle$ av



- taking the ratio of $\langle b_T | x \rangle$ for a bound proton in a nucleus to that of a free proton
- Find analogous suppression at $x \sim 0.3$ similar to that found for collinear distributions Aubert et al. PLB 1983... “transverse EMC” effect:
- Have verified that effect is genuinely produced by the nonperturbative nuclear dependence in the TMD and not from the collinear dependence in the OPE by substituting nCTEQ15 for the EPPS16 nuclear PDFs, and seeing no difference in Fig.
- Results consistent with Alrasheed et al. PRL 129 (2022): extend by looking at x dependence of non-perturbative transverse structure within a simultaneous collinear & TMD QCD global analysis

Extension to proton-nuclear DY

Prospects of high-energy data for protons

- There are two major reasons to have hope for improvement of PDFs in the proton sector

1. LHC data are much more precise than their fixed-target low-energy counterparts

- Peaks of the cross-section in the Z -boson region gather high statistics

2. High-energy data shifts the peak of the b_T -spectrum into the

small b_T region, where the operator product expansion and

perturbative evolution dominates

- We are performing the simultaneous extraction of PDFs and TMDs from high-energy data to find out!

TABLE I. Datasets included in this analysis, along with the resulting χ^2 per datum and Z -scores from the MC analysis.

Process	Experiment	\sqrt{s} (GeV)	χ^2/N	Z-score
TMD				
q_T -dep. pA DY $pA \rightarrow \mu^+ \mu^- X$	E288 [62]	19.4	1.07	0.34
	E288 [62]	23.8	0.99	0.05
	E288 [62]	24.7	0.82	0.99
	E605 [63]	38.8	1.22	1.03
	E772 [64]	38.8	2.54	5.64
$p\bar{p} \rightarrow \mu^+ \mu^- X$	CDF [76]	1800		
	CDF [77]	1960		
	CDF [78]	1800		
	D0 [79]	1800		
	D0 [80]	1960		
	D0 [81]	1960		
	D0 [82]	1800		
	ATLAS [83]	8000		
	CMS [84]	7000		
LHCb [85]	13000			
collinear				
q_T -integr. DY $\pi W \rightarrow \mu^+ \mu^- X$	E615 [86]	21.8	0.90	0.48
	NA10 [87]	19.1	0.59	1.98
	NA10 [87]	23.2	0.92	0.16
leading neutron $ep \rightarrow enX$	H1 [88]	318.7	0.36	4.59
	ZEUS [89]	300.3	1.48	2.15
Total			1.12	1.86

Fit results

- Using NLO+N2LL accuracy, we performed fits with a JAM replica (Anderson, Melnitchouk, and Sato, 2501.00665 [hep-ph]) by
 1. Fixing the PDF and fitting TMDs only
 2. Opening the PDF and the collinear datasets
- Flexibility of the collinear PDF allowed for an improved fit

TMD – Drell–Yan, Z-boson				
Process	Experiment	N_{pts}	χ^2/N_{pts}	
			(TMD-only)	(TMD+PDF)
Fixed target DY	E288, E605, E772	224	1.19	0.84
TeVatron	CDF, D0	80	0.79	0.88
RHIC	STAR, PHENIX	12	2.00	1.15
LHC	ATLAS 8 TeV	30	2.40	1.63
	CMS 13 TeV	64	1.82	0.83
	LHCb 7, 8, 13 TeV	26	0.68	0.65
Total		436	1.50	1.13

Fit results – collinear

- Use the datasets sensitive only to PDFs from the prior
- Good agreement with all collinear datasets

Collinear (TMD+PDF)			
Process	Experiment	N_{pts}	χ^2/N_{pts}
DIS	SLAC, BCDMS, NMC	1495	1.04
	HERA	1185	1.25
Drell-Yan	E866, E906	205	1.12
W -lepton asymmetry	CMS, LHCb, STAR	70	0.87
W charge asymmetry	CDF, D0	27	1.16
Z rapidity	CDF, D0	56	1.10
Inclusive jets	CDF, D0, STAR	198	1.03
W + charm	ATLAS, CMS	37	0.57
Total		3273	1.12

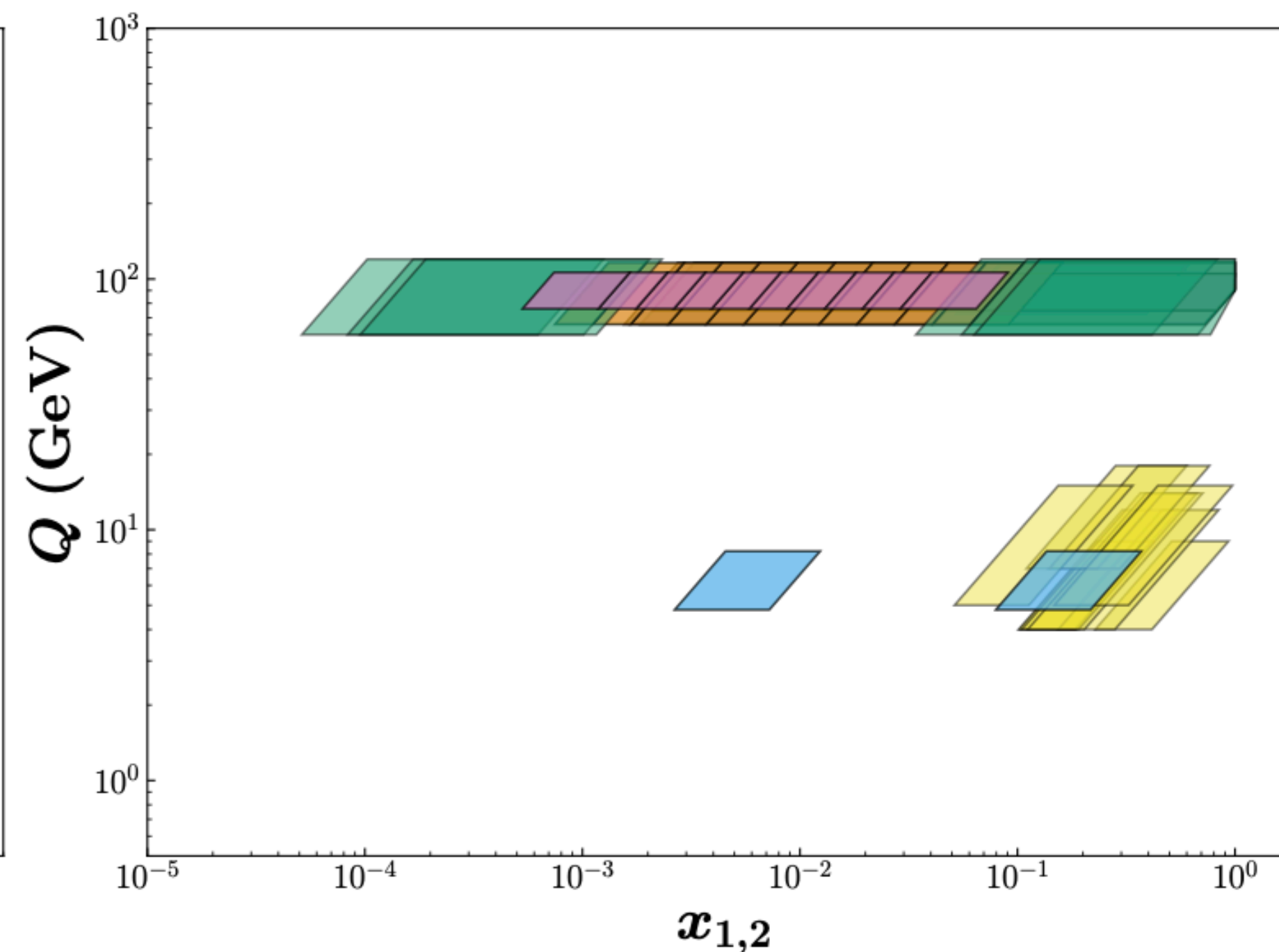
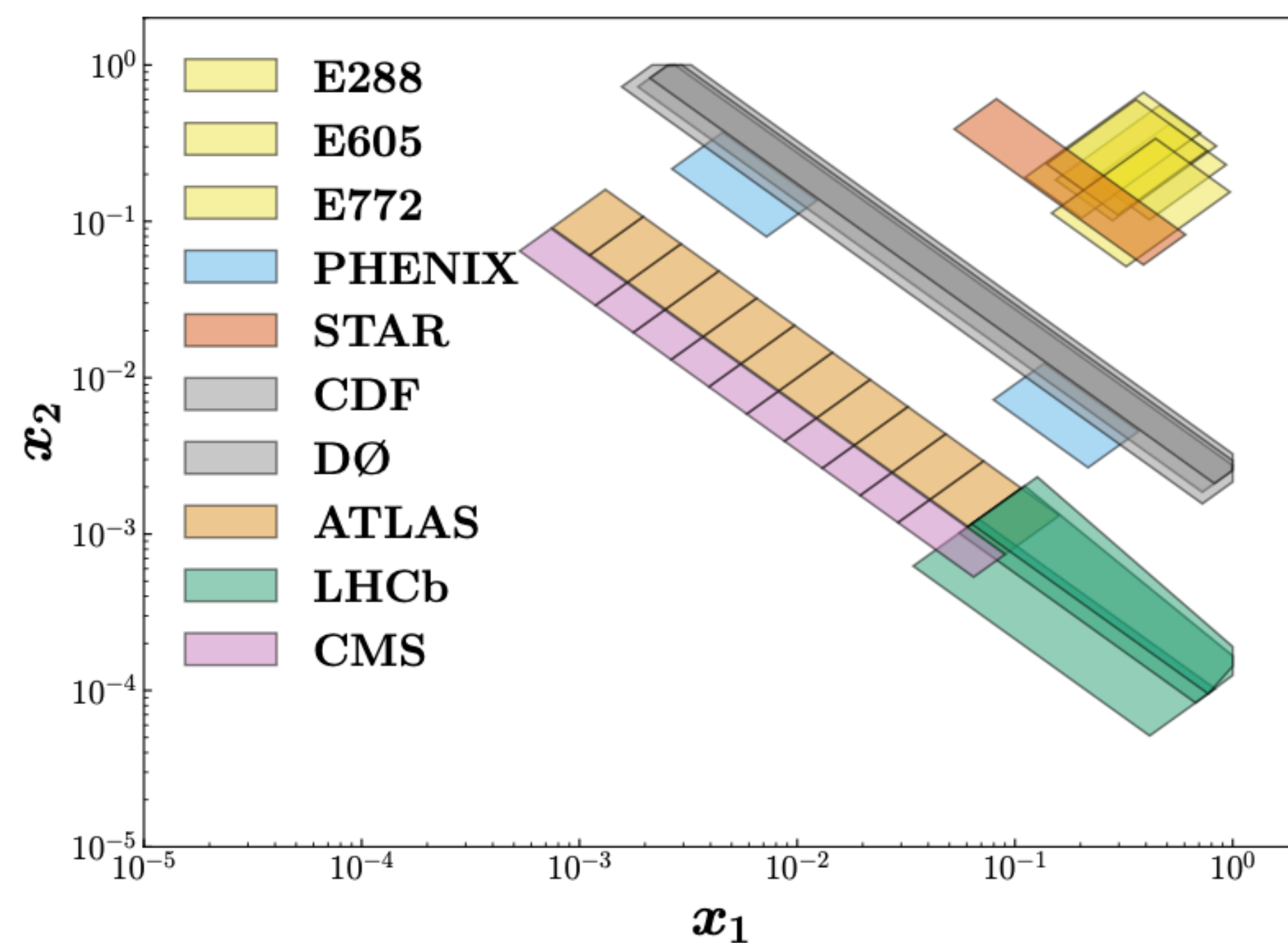
Summary

- Carried out a comprehensive analysis of π and P TMD PDFs at N2LL perturbative precision using fixed-target DY data.
- For the first time used both q_T -integrated and q_T - differential DY data, as well as LN measurements, to simultaneously extract π collinear and TMD PDFs and proton TMD PDFs.
- Combined analysis, include an exploration of the nuclear dependence of TMDs, allows us to perform a detailed comparison of π and P TMDs and to study the similarities and differences of their transverse spatial & momentum dependence
- We determined conclusively that the transverse correlations of quarks in a pion are $\approx 20\%$ smaller than those in a proton
- We found evidence for a transverse EMC effect, as discussed earlier by Alrashed et al.
- Extending global QCD analysis of collinear and transverse momentum dependent parton distribution functions (PDFs and TMD PDFs) in proton
- For the first time, we are able to study and quantify the impact of simultaneous inclusion of both data sets in the global fit
- We find that this combined analysis leads to improvements in the knowledge of both TMDs and collinear PDFs.

Outlook

Comment to proton-nuclear DY

- **Fixed-target** low-energy datasets: more sensitivity to non-perturbative TMD structures
- **Collider high-energy** datasets: more sensitive to perturbative information while complementing the non-perturbative evolution in Q
- Where would a fixed target **xyz** GeV beam sit?
Can add to unfolding momentum imaging of nucleon
- Is a secondary pion beam feasible to probe TMD valence distributions? Supplement COMPASS fixed target measurements?



Extras

CSS evolution F.T.-TMD B.C. OPE & b_* prescription

$$\{\zeta, \mu\} \rightarrow \zeta = Q^2, \quad \mu = \mu_Q \sim Q$$

TMD PDF within the b_* prescription

$$\mathbf{b}_*(\mathbf{b}_T) \equiv \frac{\mathbf{b}_T}{\sqrt{1 + b_T^2/b_{\text{max}}^2}}.$$

Low- b_T : perturbative
high- b_T : non-perturbative

$$\tilde{f}_{q/\mathcal{N}(A)}(x, b_T, \mu_Q, Q^2) = (C \otimes f)_{q/\mathcal{N}(A)}(x; b_*) \times \exp \left\{ -g_{q/\mathcal{N}(A)}(x, b_T) - g_K(b_T) \ln \frac{Q}{Q_0} - S(b_*, Q_0, Q, \mu_Q) \right\}$$

Relates the TMD at small- b_T to the **collinear** PDF
 \Rightarrow TMD is sensitive to collinear PDFs

$g_{q/\mathcal{N}(A)}$: intrinsic non-perturbative structure of the TMD
 g_K : universal non-perturbative Collins-Soper kernel

Controls the perturbative evolution of the TMD

Collins, Soper, Sterman, NPB **250**, 199 (1985).