Analytic Scaling of the Lensing Tail of the Cosmic Microwave Background Power Spectrum

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The CMB Power Spectrum

The Cosmic Microwave Background (CMB) is the photons from the surface of last scattering. In this signal, we see anisotropies that correspond to over- and under-densities from the primordial Universe. This can be represented by the temperature-temperature correlation function C_ℓ^{TT}, which quantifies the similarity of the temperatures of the CMB at two points in the sky separated by a given scale.

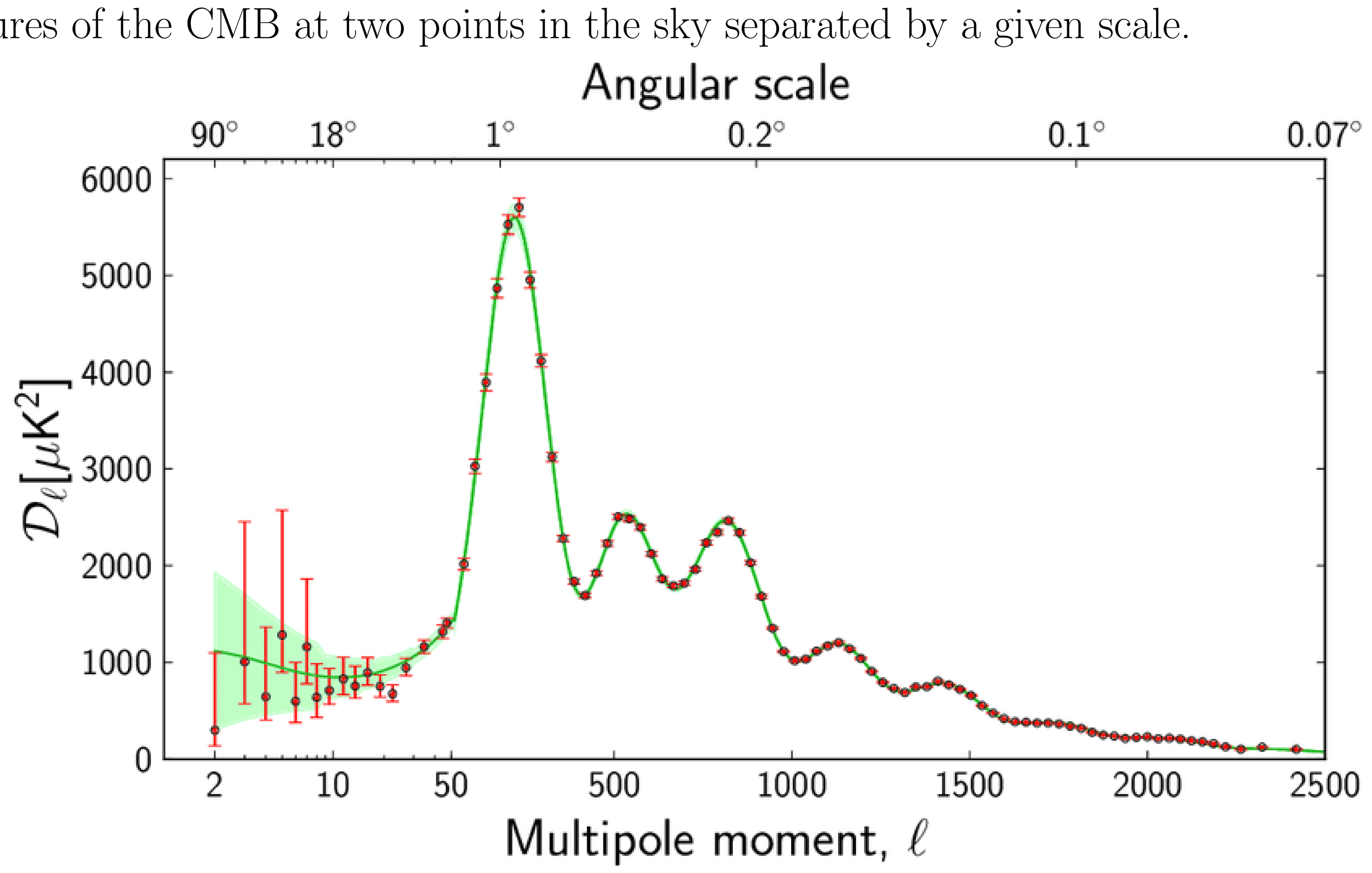


Image from [3]. The scaled CMB power spectrum $D_\ell = \ell(\ell+1)C_\ell/2\pi$

Gravitational Lensing

The force of gravity, such as from a cluster of galaxies, bends spacetime, thus changing the path light travels. We see light coming from a different place than it really is, causing for example a smearing in the CMB signal. This effect dominates at large ℓ, or small scales.

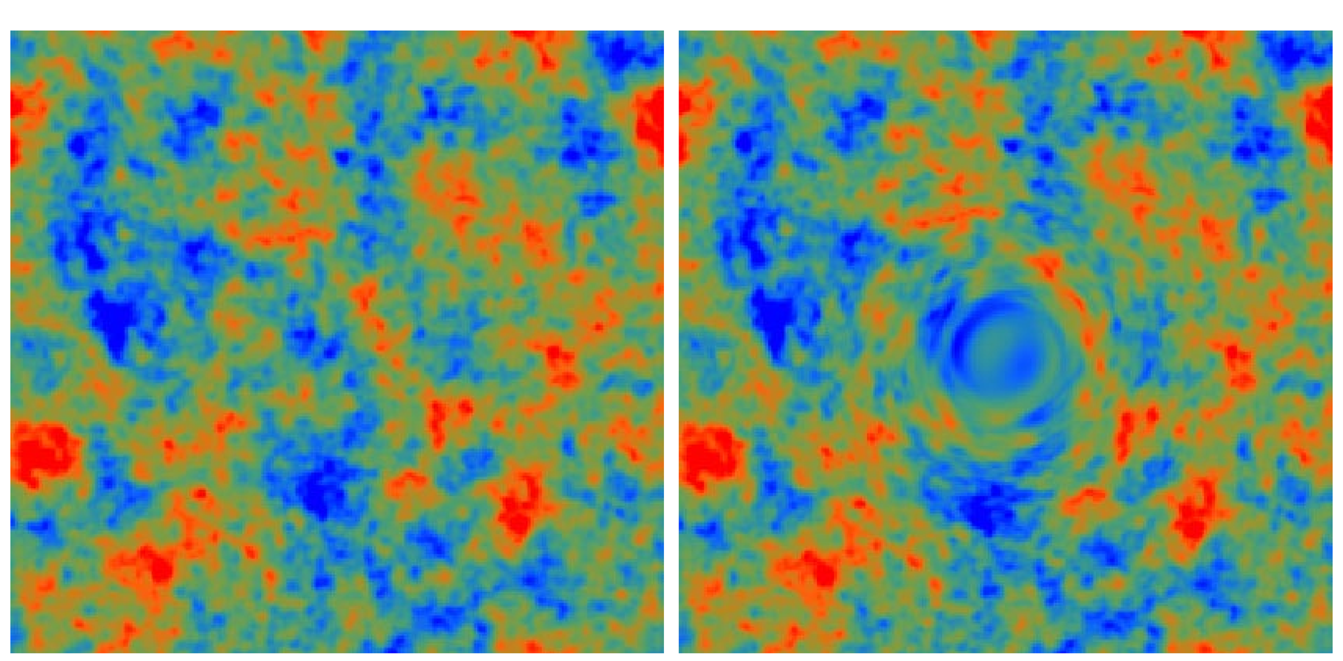


Image from [4]. Left is the unlensed temperature field of the CMB, and right is with an exaggerated lensing effect.

Motivation

Markov Chain Monte Carlo (MCMC) chains are used to infer cosmological parameters from a dataset, but as the precision of future measurements demands more extensive analysis, they are becoming increasingly computationally expensive. Using machine learning to emulate CMB power spectra from CAMB data can reduce computation time for MCMC chains.

In this project, we analytically scale the CMB lensing tail. By pre-processing in this way, we can reduce the span of the data vectors, hopefully decreasing training time.

Cosmological Parameters

Cosmological parameters describe the evolution and current state of the Universe. In this project, we examined the effect of gravitational lensing on the CMB considering five parameters: the Hubble constant H_0, the baryon density $\Omega_b h^2$, the cold dark matter density $\Omega_c h^2$, the power of the primordial curve perturbations A_s, and the scalar spectrum power-law index n_s. We explored the following parameter space:

Ranges for Cosmological Parameters					
	H_0 (km/s/Mpc)	$\Omega_b h^2$	$\Omega_c h^2$	$a = \ln(A_s \times 10^{10})$	n_s
Minimum	50	0.007	0.04	1.61	0.8
Fiducial	80	0.0224	0.12	3.043	0.965
Maximum	67.4	0.035	0.23	3.6	1.2

Fitting Model & Optimization

The lensing tail is defined as the lensed power spectrum divided by the unlensed power spectrum: $L(\ell) = \frac{C_\ell^{\text{lensed}}}{C_\ell^{\text{unlensed}}}$. We model the lensing tail as:

$$L(\ell) = 1 + w(\ell)\Big[\beta_1 \cdot \Big(\frac{\ell}{\beta_2}\Big)^\alpha - 1\Big]$$

$$w(\ell) = \frac{1}{1 + e^{-(\ell-\beta_3)/\beta_4}}$$

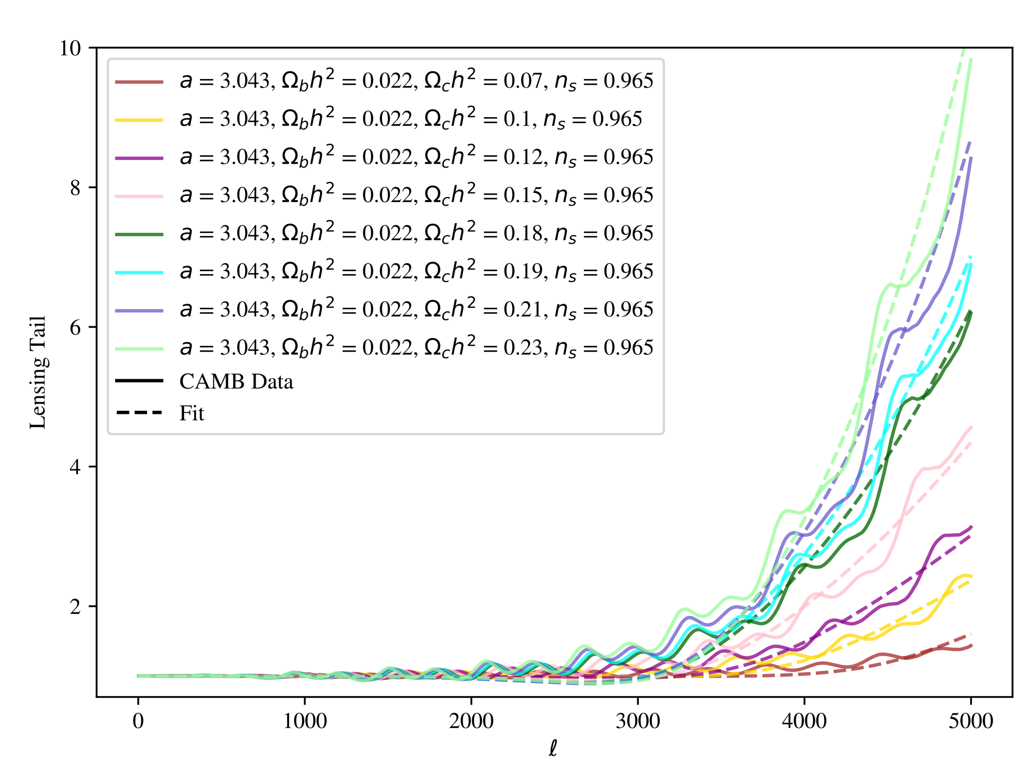
where $w(\ell)$ is a weighting function. Here each of the parameters, β_1, β_2, β_3, β_4, and α, is allowed to be a function of the cosmological parameters centered around the fiducial values (except H_0).

Comparison with CAMB Lensing Tail

To create a fit, we employed linear and quadratic regression, using the Levenberg-Marquardt algorithm. For example, for the behavior of β_1 for changes in n_s is:

$$\beta_1(n_s) = b_1\Big(\frac{n_s}{0.965} - 1\Big) + b_2\Big(\frac{n_s}{0.965} - 1\Big)^2$$

We developed a fit using 600 random parameters in the selected range, with the lensing tail behavior found using CAMB. The fit was trained to each parameter in isolation, then in pairs, then altogether.

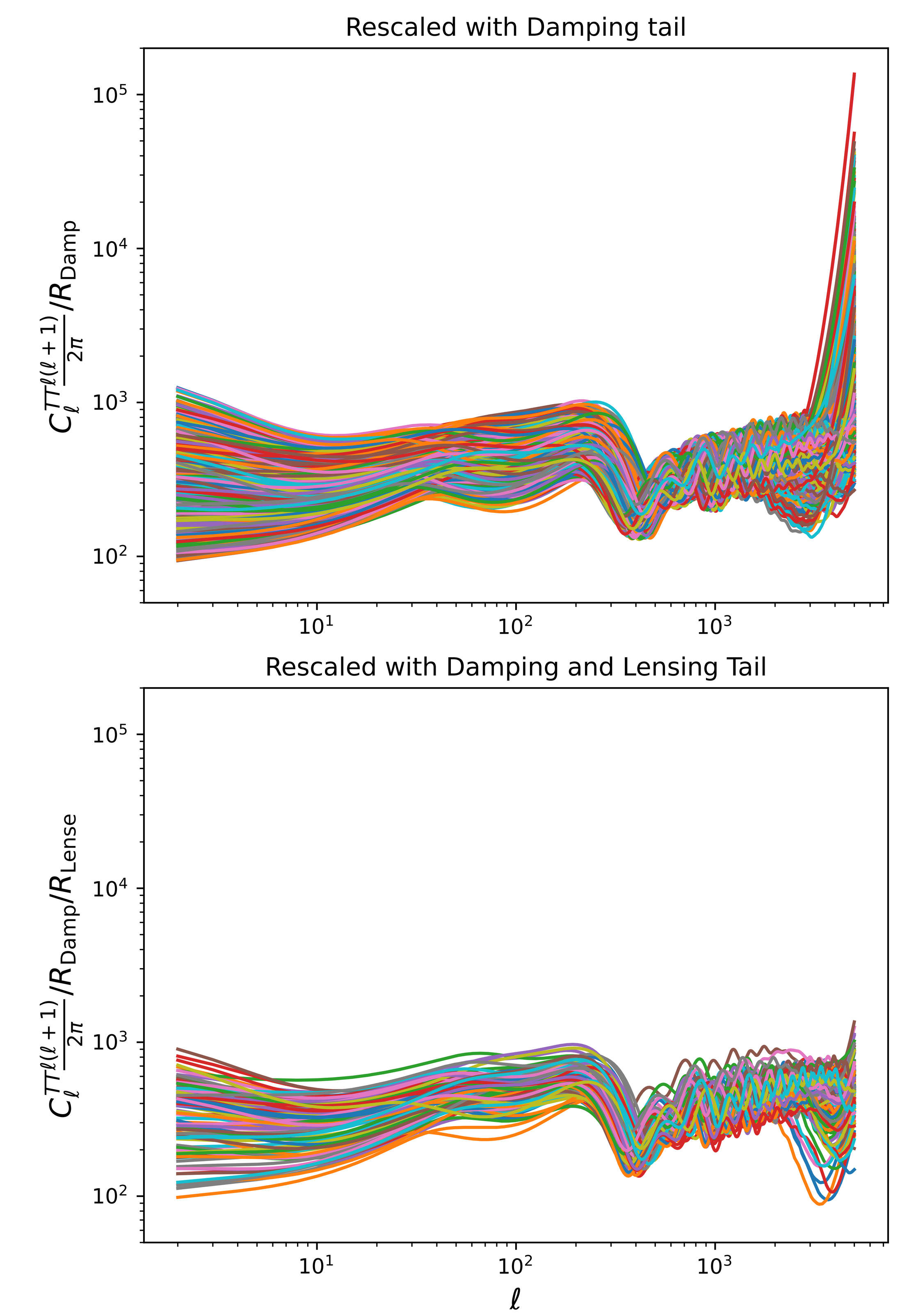


A plot of the fit for varied $\Omega_c h^2$. As shown, the fit captures the order of magnitude of the tail, not its oscillatory behavior.

The overall mean error compared to CAMB for the 600 cosmologies used for training was 12.3%. The error was low near the fiducial cosmology.

Effect of Rescaling

Scaling the lensing tail decreased the span of the vectors:



A plot of CAMB data before and after dividing by the equation found for the lensing tail.

Conclusion

We have found that we can model the scale of the CMB lensing tail for pre-processing in machine learning.

In the future, we will expand this study by using machine learning algorithms to create a more precise fit as well as expand the parameter space.

Bibliography & Acknowledgments

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References

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