Transverse Momentum Dependent parton distributions (TMDs)

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Lecture 1

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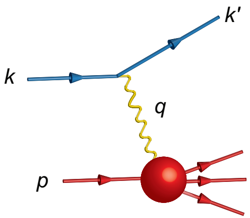
Outline

- Why TMDs?
- TMD Factorization
- Quark TMDs
  - Gauge invariance and process dependence
  - Unpolarized distributions
  - Sivers function
- Summary

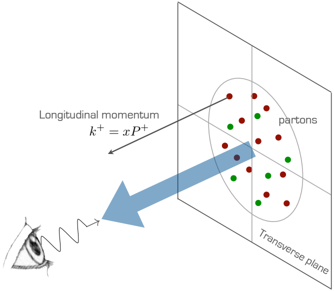
Motivation



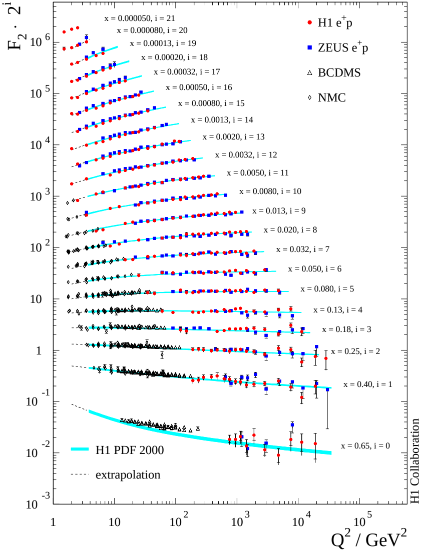
x: fraction of proton's momentum carried by the struck quark

$Q^2 \equiv -q^2$: defines the resolution of the measurement

The probe defines a longitudinal direction

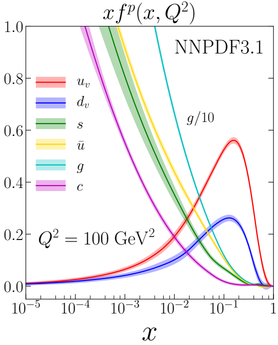


Structure Function F_2



$$F_2(x, Q^2) = x \sum_q e_q^2 f_1^q(x, Q^2)$$

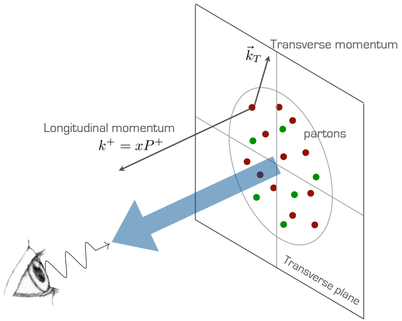
1D collinear parton distributions



Scaling violations: great success of QCD evolution!

Transverse momentum dependent distributions (TMDs)

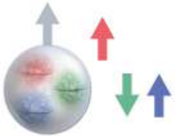
Three-dimensional distributions: provide information on the partonic longitudinal momentum and the two-dimensional transverse momentum



Renormalization scale μ and the rapidity scale ζ not shown explicitly

More detailed information on the proton's structure as compared to PDFs:
1D description is not always satisfactory, see i.e. spin effects

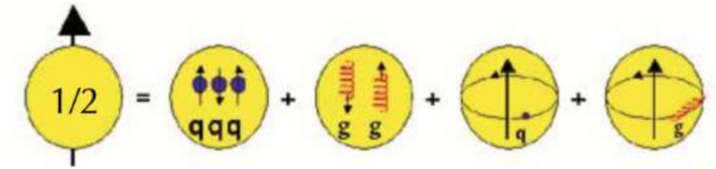
The proton has spin 1/2, the three *valence* quarks have also spin 1/2, we expect:



However: only 30% of the spin of the proton comes from the spin of the quarks

First measurement by the European Muon Collaboration (EMC, CERN 1987)

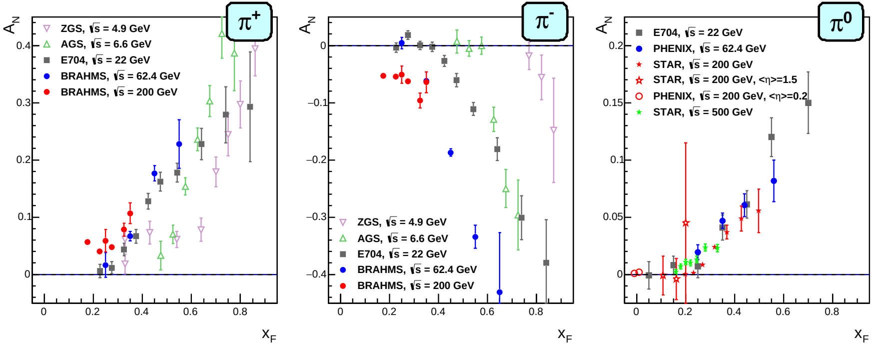
$$\vec{\mu}\vec{p} \to \mu X \qquad\qquad A = \frac{(\vec{\mu}\vec{p}) - (\vec{\mu}\,\overleftarrow{p})}{(\vec{\mu}\,\vec{p}) + (\vec{\mu}\,\overleftarrow{p})}$$



The partonic orbital angular momentum needs to be considered

A_N in $p^\uparrow p \to \pi X$ is a long standing puzzle, only a few % in twist-2 collinear QCD

$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow} \qquad x_F = \frac{2p_L}{\sqrt{2}}$$



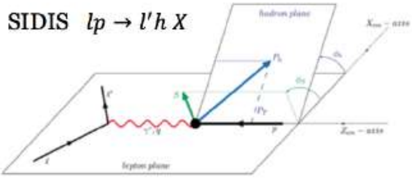
Aschenauer, D'Alesio, Murgia, EPJA52 (2016)

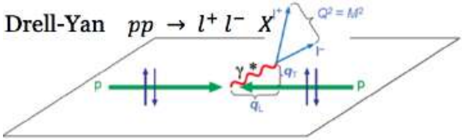
Almost energy independent

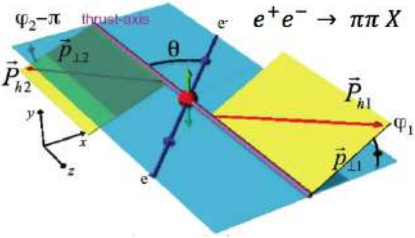
TMD factorization does not hold, collinear twist-3 approach more solid

Quark TMDs

Two scale processes $Q^2 \gg q_T^2$







Factorization proven

All orders in α_s
Leading order in powers of $1/Q$ (twist)

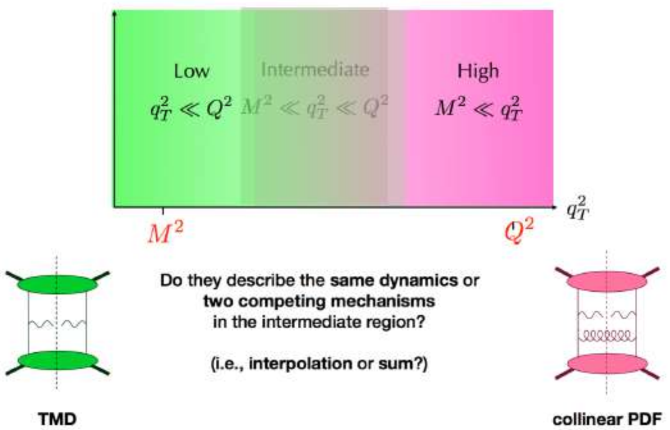
Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

Attempts to establish factorization at one-loop and next-to-leading power

Rodini, Vladimirov, PRD 110 (2024)
Gamberg, Kang, Shao, Terry, Zhao, 2211.13209

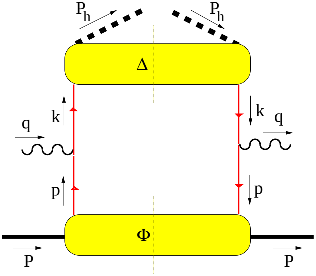
Three physical scales, two theoretical tools

Bacchetta, Boer, Diehl, Mulders, JHEP 08 (2008)



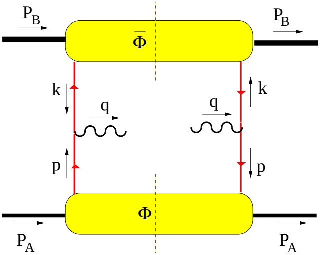
Hard partonic interactions can be separated from nonperturbative correlators

SIDIS



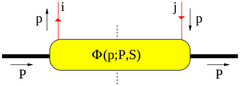
$\gamma^* p \to h X$

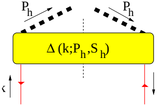
Drell-Yan



$p\, p \to \gamma^* X$

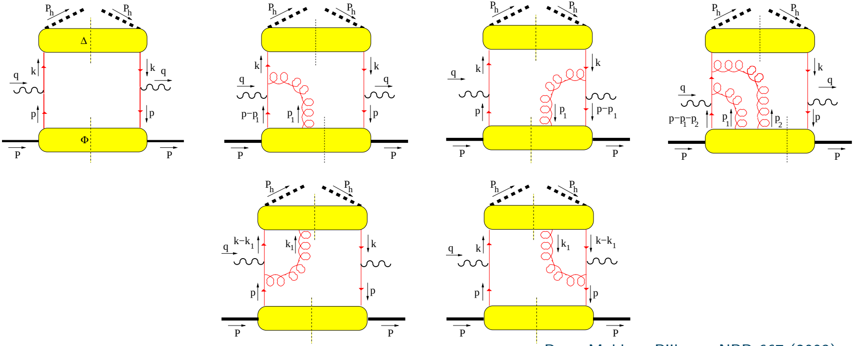
Parton correlators Φ and Δ describe the soft hadron ↔ parton transitions





Parametrized in terms of distribution and fragmentation functions

Resummation of all gluon exchanges leads to *gauge links* in the correlators Φ, Δ



Boer, Mulders, Pijlman, NPB 667 (2003)

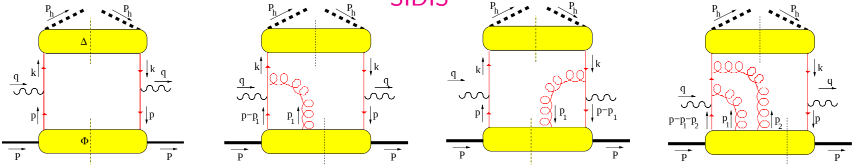
$$\mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} \mathrm{d}s_\mu A^\mu(s)\right)$$

The path $\mathcal{C}$ depends on the color interactions, *i.e.* on the specific process

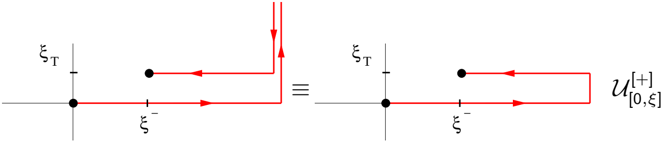
Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P,S \left| \overline{\psi}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,\psi(\xi) \right| P,S \right\rangle$$

SIDIS

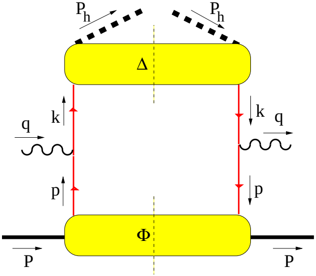


Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)

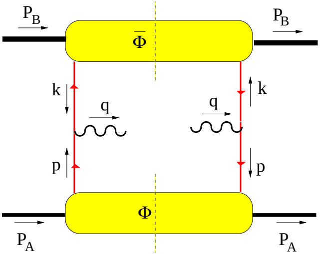


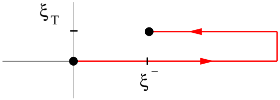
Possible effects in transverse momentum observables (ξ_T is conjugate to k_T)

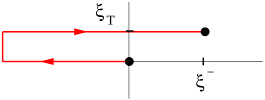
SIDIS



Drell-Yan



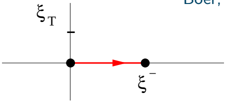




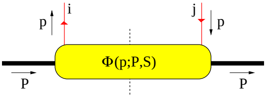
$\mathcal{U}^{[+]}_{[0,\xi]}$ $\mathcal{U}^{[-]}_{[0,\xi]}$

Belitsky, Ji, Yuan, NPB 656 (2003)
Boer, Mulders, Pijlman, NPB 667 (2003)
Boer, talk at RBRC Synergies workshop (2017)

$\int \mathrm{d}\boldsymbol{p}_T \longrightarrow \xi_T = 0 \longrightarrow$



the same in both cases



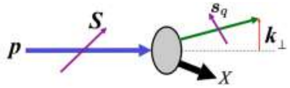
The quark correlator can be decomposed into eight leading-power TMDs

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Tangerman, NPB 461 (1996)
Boer, Mulders, PRD 57 (1998)

They are all known and can all be accessed in SIDIS (mostly COMPASS data)

Beyond the unpolarized f_1, helicity g_{1L} and transversity h_1 surving the collinear limit, we have five more. In particular the Sivers ($f_{1T}^{\perp}$) and Boer-Mulders ($h_1^{\perp}$):



$\boldsymbol{S} \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$
Sivers effect

$\boldsymbol{s}_q \cdot (\boldsymbol{p} \times \boldsymbol{k}_\perp)$
Boer-Mulders effect

Correlations between (proton or quark) spin and quark transverse momentum

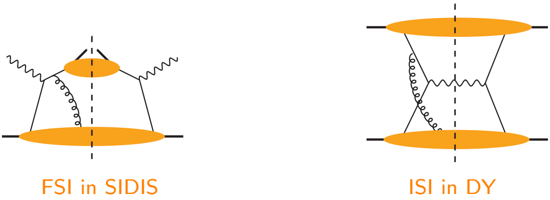
The Sivers effect is expected to give rise to transverse single spin asymmetries

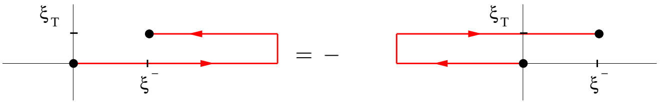
Sivers, PRD 41 (1990)

Fundamental test of TMD theory

$$f_{1T}^{\perp\,[DY]}(x,\boldsymbol{k}_\perp^2) = -f_{1T}^{\perp\,[SIDIS]}(x,\boldsymbol{k}_\perp^2) \qquad h_1^{\perp\,[DY]}(x,\boldsymbol{k}_\perp^2) = -h_1^{\perp\,[SIDIS]}(x,\boldsymbol{k}_\perp^2)$$

Collins, PLB 536 (2002)





ISI/FSI lead to process dependence of TMDs, could even break factorization

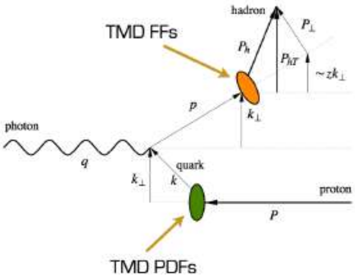
Collins, Qiu, PRD 75 (2007)
Collins, PRD 77 (2007)
Rogers, Mulders, PRD 81 (2010)

TMDs are, or will be, under experimental investigation all over the world

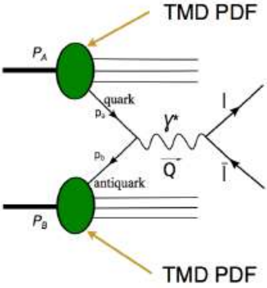
Bacchetta, Contalbrigo (2012) [updated by F. Murgia]

Unpolarized quark TMDs

$$l(\ell) + N(\mathcal{P}) \rightarrow l(\ell') + h(\mathcal{P}_h) + X$$

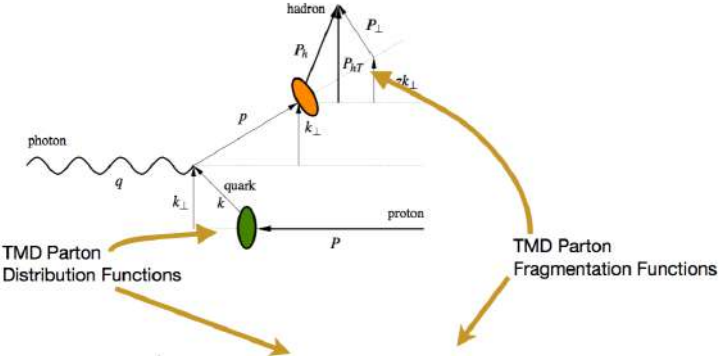


$$A + B \rightarrow \gamma^* \rightarrow l^+ l^-$$
$$A + B \rightarrow Z \rightarrow l^+ l^-$$



Highly complementary processes:

- DY: insight into quark-antiquark TMD PDFs
- SIDIS: particularly sensitive to flavor differences through TMD FFs



$$F_{UU,T}(x,z,P_{hT}^2,Q^2) = \sum_a \mathcal{H}^a_{UU,T}(Q^2;\mu^2) \int d^2\boldsymbol{k}_\perp \, d^2\boldsymbol{P}_\perp \, f_1^a(x,\boldsymbol{k}_\perp^2;\mu^2) \, D_1^{h/a}(z,\boldsymbol{P}_\perp^2;\mu^2) \times \delta^2(z\boldsymbol{k}_\perp - \boldsymbol{P}_{hT} + \boldsymbol{P}_\perp) + Y_{UU,T}(Q^2,\boldsymbol{P}_{hT}^2) + \mathcal{O}(M/Q)$$

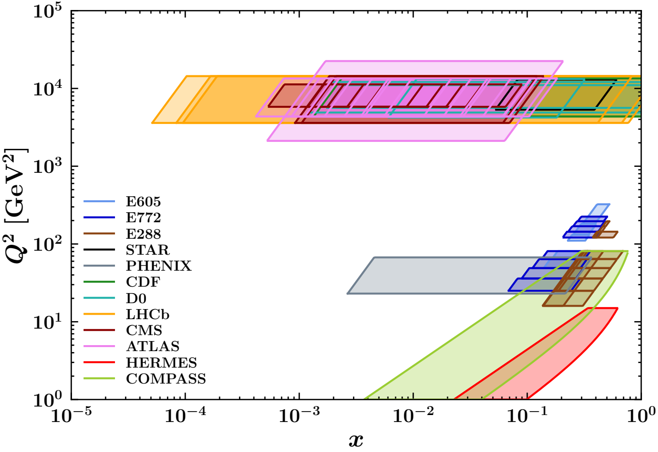
In all TMD studies so far $Y_{UU,T}\left(Q^2,\boldsymbol{P}_{hT}^2\right) \approx 0$

Multiplicities: $m_N^h(x,z,\boldsymbol{P}_{hT}^2,Q^2) = \dfrac{\mathrm{d}\sigma_N^h/\mathrm{d}x\,\mathrm{d}z\,\mathrm{d}\boldsymbol{P}_{hT}^2\,\mathrm{d}Q^2}{\mathrm{d}\sigma_{\mathrm{DIS}}/\mathrm{d}x\,\mathrm{d}Q^2} \approx \dfrac{2\pi|\boldsymbol{P}_{hT}|F_{UU,T}(x,z,\boldsymbol{P}_{hT}^2,Q^2)}{F_T(x,Q^2)}$

	Accuracy	SIDIS HERMES	SIDIS COMPASS	DY fixed target	DY collider	N of points	χ^2/N_{points}
Pavia 2017 arXiv:1703.10157	NLL	✓	✓	✓	✓	8059	1.55
SV 2019 arXiv:1912.06532	N^3LL^-	✓	✓	✓	✓	1039	1.06
MAP22 arXiv:2206.07598	N^3LL^-	✓	✓	✓	✓	2031	1.06
MAP24 arXiv:2405.13833	N^3LL	✓	✓	✓	✓	2031	1.08
ART25 arXiv:2503.11201	N^3LL	✓	✓	✓	✓	1209	1.05

Unpolarized quark TMDs

$x - Q^2$ coverage



Bacchetta et al. (MAP24), JHEP 08 (2024)

HERMES and COMPASS are SIDIS experiments; other acronyms refer to DY data

$\langle Q \rangle > 1.4$ GeV

$0.2 < z < 0.7$

Drell-Yan $q_T < 0.2\,Q$

$P_{hT} < \min\left[\min\left[0.2Q, 0.5Qz\right] + 0.3\text{GeV}, zQ\right]$

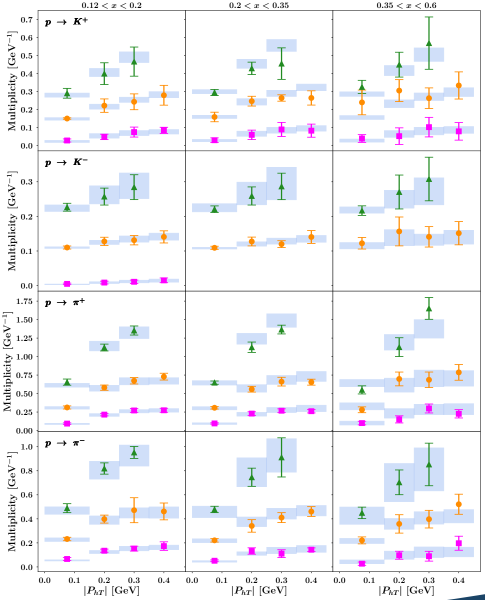
Fit of 100 replicas of the data

2031 data points (484 from DY and 1547 from SIDIS)

96 free parameters (flavor dependence of TMDs fully taken into account)

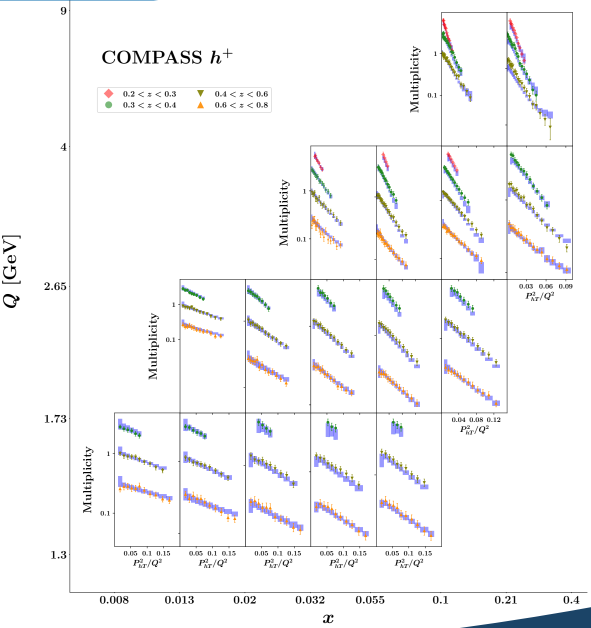
SIDIS proton target
HERMES data

$\chi^2/\text{dof} = 1.05$

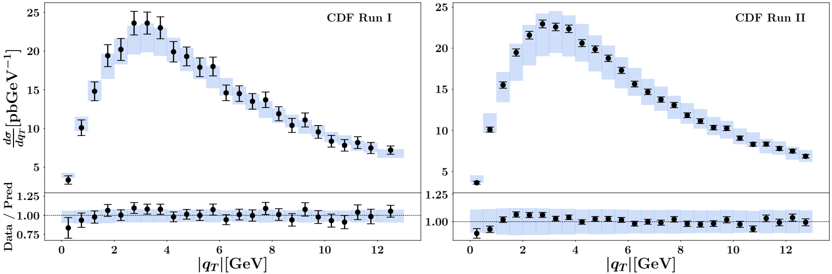


SIDIS h^+ production
COMPASS data

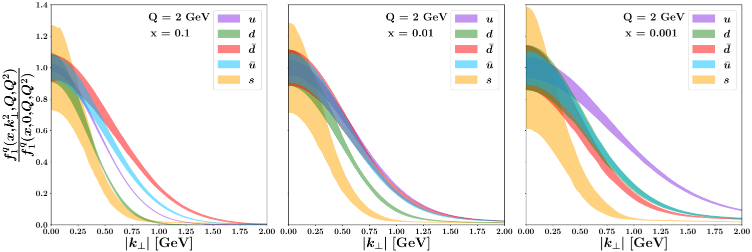
$\chi^2/\text{dof} = 0.94$



Z-boson production in $p\bar{p}$ collisions
CDF data



$$\frac{f_1(x_1, k_T; Q)}{f_1(x, 0; Q)}$$

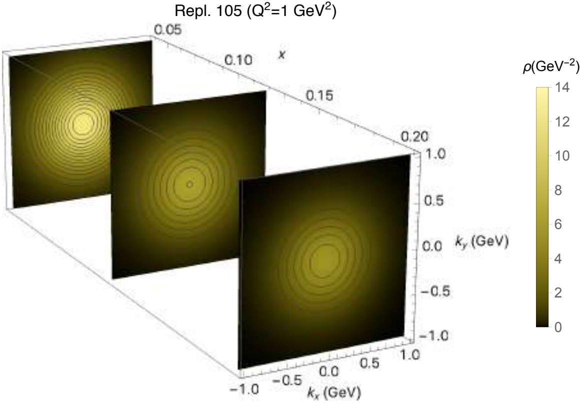


Different flavors have very different $k_\perp$-dependence

This behavior changes with x

The u-quark TMD is the most constrained and widest at small and intermediate x

Sivers effect



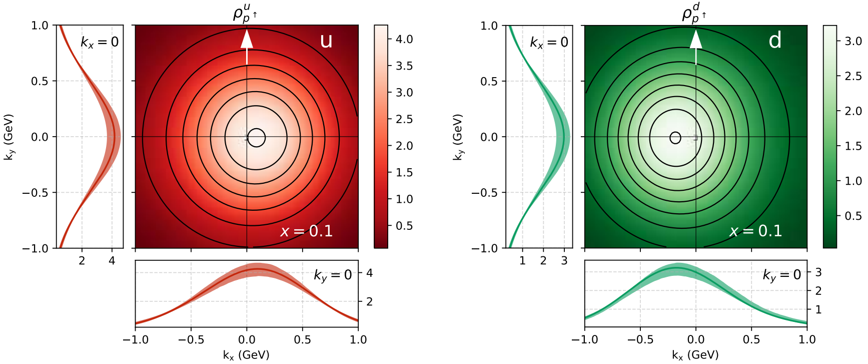
For unpolarized protons, the distribution of unp. quarks is cylindrically symmetric

What happens if the proton is transversely polarized?

Same formalism can be used to have a consistent picture (125 data points)

Distortion in the transverse plane of the TMD quark distribution in a $p^\uparrow$

$$\Phi^{[\gamma^+]}_{q/p^\uparrow}(x, k_x, k_y) = f_1^q(x, k_T^2) - \frac{k_x}{M} f_{1T}^{\perp q}(x, k_T^2) \qquad [Q^2 = 4\ \text{GeV}^2]$$

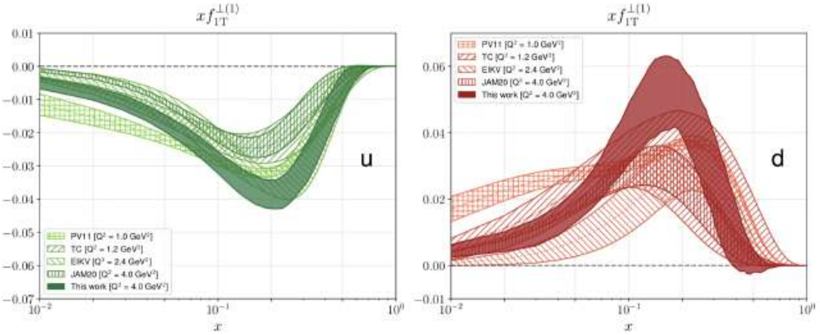


Bacchetta, Delcarro, CP, Radici, PLB 827 (2022)

Non zero Sivers effect related to parton orbital angular momentum

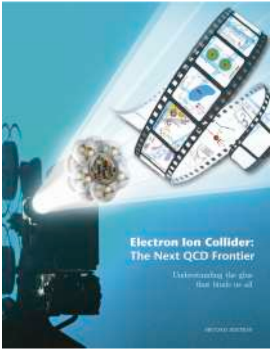
Sivers effect
First transverse moments

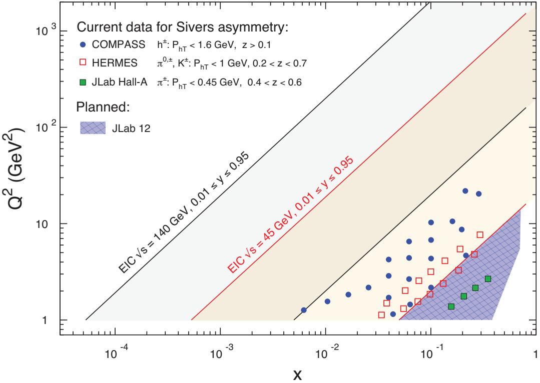
$$f_{1T}^{\perp(1)\,q}(x) = \int \mathrm{d}^2\boldsymbol{k}_T\, \frac{k_T^2}{2M_p^2}\, f_{1T}^{\perp\,q}(x, \boldsymbol{k}_T^2)$$



Bacchetta, Delcarro, CP, Radici, PLB 827 (2022)

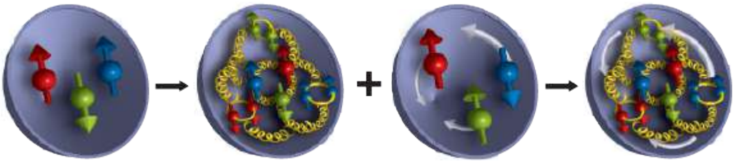
More data from CERN, JLab, EIC will help to reduce error bands and extend the ranges in x and Q^2





- Access to small-x domain
- Space, momentum and spin distributions of gluon and sea quarks
- Missing and complementary information on TMDs and GPDs

Summary



- Much progress in our understanding of the nucleon: evidence for going beyond 1D
- TMDs provide a 3D description of the partonic structure of the nucleon; their determination requires a continuous interplay between theory and experiment
- TMD distributions are affected by ISI/FSI, encoded in the gauge links, which render them gauge invariant but process dependent
- Unpolarized quark TMDs extracted from several hundred data points
- Evidence for nonzero Sivers function and its sign change

Back-up slides

Convolutions of TMDs are products in position space; the FT of a TMD is defined as:

$$\hat{f}_1^q(x, \boldsymbol{b}_T^2; \mu, \xi) = \int \mathrm{d}^2 k_\perp \, e^{i \boldsymbol{b}_T \cdot \boldsymbol{k}_\perp} \, f_1^q(x, \boldsymbol{k}_\perp^2; \mu, \xi)$$

Collins-Soper-Sterman (CSS) evolution scheme

$$\hat{f}_1^q(x, \boldsymbol{b}_T^2; \mu_f, \xi_f) = \mathrm{Evo}\left[(\mu_f, \xi_f) \leftarrow (\mu_i, \xi_i)\right] \times \hat{f}_1^q(x, \boldsymbol{b}_T^2; \mu_i, \xi_i)$$

At small b_T (large $k_\perp$), TMDs can be calculated in perturbative QCD:

$$\hat{f}_1^q(x, \boldsymbol{b}_T^2; \mu_i, \xi_i) = \sum_a C_{qa}(x, \boldsymbol{b}_T^2, \alpha_s(\mu_i), \mu_i, \xi_i) \otimes f_1^a(x, \mu_i)$$

The dependence on the scales is governed by the evolution equations

$$\frac{\partial \hat{f}_1}{\partial \ln \mu} = \gamma(\mu, \zeta) \qquad \frac{\partial \hat{f}_1}{\partial \ln \sqrt{\zeta}} = K(\mu) \qquad \frac{\partial K}{\partial \ln \mu} = \frac{\partial \gamma}{\partial \ln \sqrt{\zeta}} = -\gamma_K(\alpha_s(\mu))$$

γ_K : cusp anomalous dimension

After solving the evolution equations:

$$\hat{f}_1(x,\, \boldsymbol{b}_T^2;\, \mu_f, \zeta_f) = \left[C \otimes f_1\right](x,\, \boldsymbol{b}_T^2;\, \mu_i, \zeta_i)$$
$$\times \exp\left\{K(\mu_i) \ln \frac{\sqrt{\zeta_f}}{\sqrt{\zeta_i}} + \int_{\mu_i}^{\mu_f} \frac{d\mu}{\mu} \left[\gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta_f}}{\mu}\right]\right\}$$
$$\gamma_F(\alpha_s(\mu)) = \gamma(\mu, \mu^2)$$

In order to avoid large logarithms in the evolution kernel K and the coefficients C:

$$\mu_i = \sqrt{\zeta_i} \equiv \mu_b = 2e^{-\gamma_E}/|b_T|$$

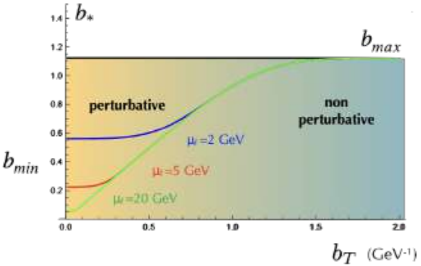
Log accuracies of the TMD evolution vs. $\mathcal{O}(\alpha_s^m)$ corrections in TMD ingredients

	$\mathcal{O}(\alpha_s^m)$ perturbative order				
Accuracy N^{n}LL	$\mathcal{H}$ and C	K and γ_F	γ_K	PDF and α_s evolution	FF evolution
NLL	0	1	2	LO	LO
N^{2}LL	1	2	3	NLO	NLO
N^{3}LL	2	3	4	NNLO	NNLO

To avoid hitting the Landau pole at large b_T and to smoothly match the fixed-order calculation at small b_T:

$$\mu_{b_\star} = \frac{2e^{-\gamma_E}}{b_\star} \qquad b_\star \equiv b_{\max}\left(\frac{1-e^{-b_T^4/b_{\max}^4}}{1-e^{-b_T^4/b_{\min}^4}}\right)^{\frac{1}{4}} \qquad 1\ \text{GeV} \le \mu_{b_\star} \le \mu_f$$

$$b_{\max} = 2e^{-\gamma_E} \approx 1.123\ \text{GeV}^{-1} \qquad b_{\min} = \frac{2e^{-\gamma_E}}{\mu_f}$$



These choices are arbitrary: they should be checked/challenged in the future

$$\hat{f}_1^q(x,\, \boldsymbol{b}_T^2;\, \mu_f, \zeta_f) \quad = \quad \sum_q \left[C_{qa} \otimes f_1\right](x,\, \boldsymbol{b}_T^2;\, \mu_{b_*}, \mu_{b_*}^2)$$
$$\times \exp\left\{K(b_*, \mu_{b_*}) \ln \frac{\sqrt{\zeta_f}}{\sqrt{\mu_{b_*}^2}} + \int_{\mu_{b_*}}^{\mu_f} \frac{d\mu}{\mu} \left[\gamma_F(\alpha_s(\mu)) - \gamma_K(\alpha_s(\mu)) \ln \frac{\sqrt{\zeta_f}}{\mu}\right]\right\}$$
$$\times f_{1NP}(x, \boldsymbol{b}_T^2;\, \zeta_f, Q_0)$$

$K(b_\star, \mu_{b_\star})$: perturbative part of the Collins-Soper kernel K

The nonperturbative part g_K is contained in f_{1NP}

$f_{1NP} \to 1$ for $b_T \to 0$: it is parametrized at Q_0 and fitted to the data