Tomography of the Nucleon-1

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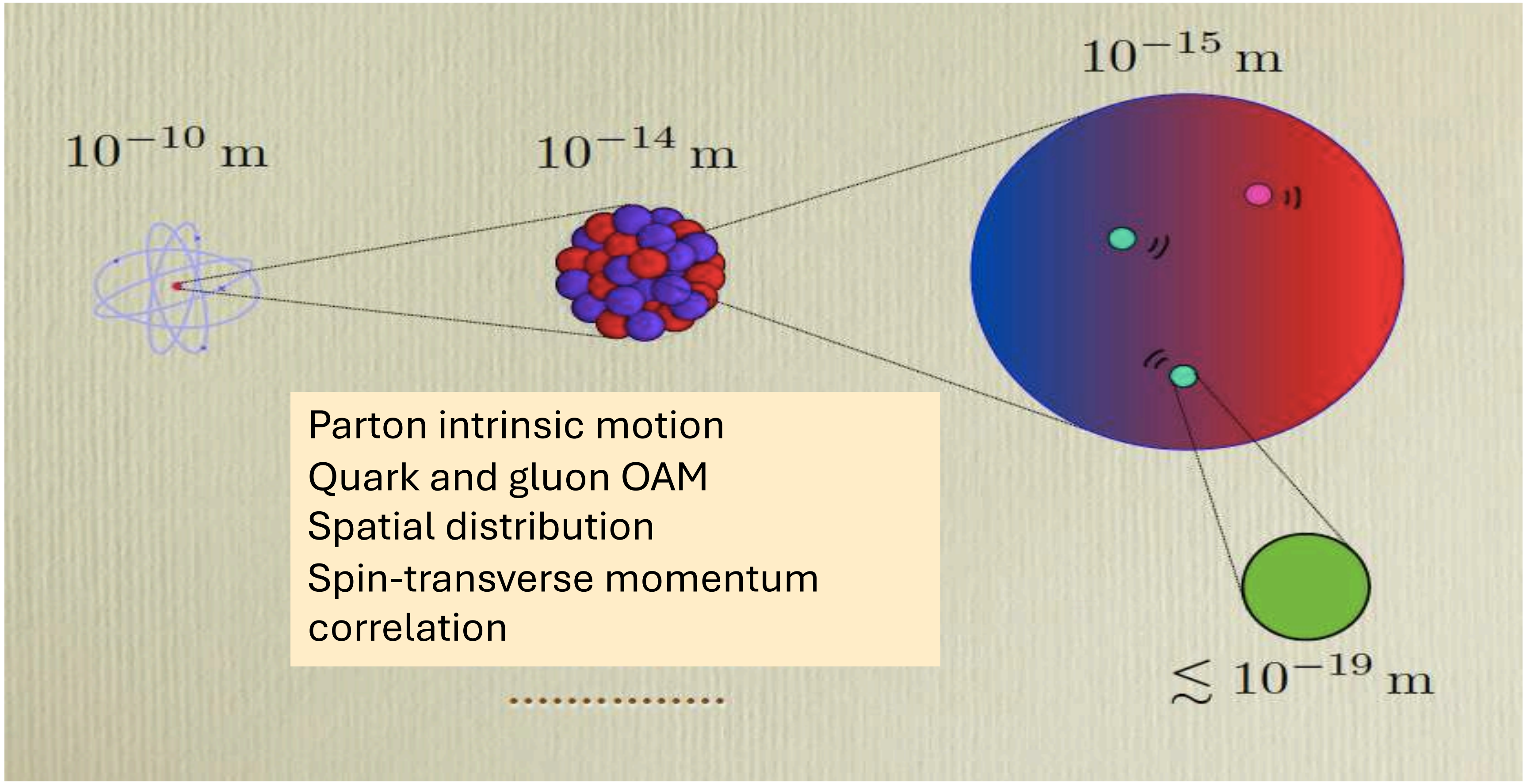
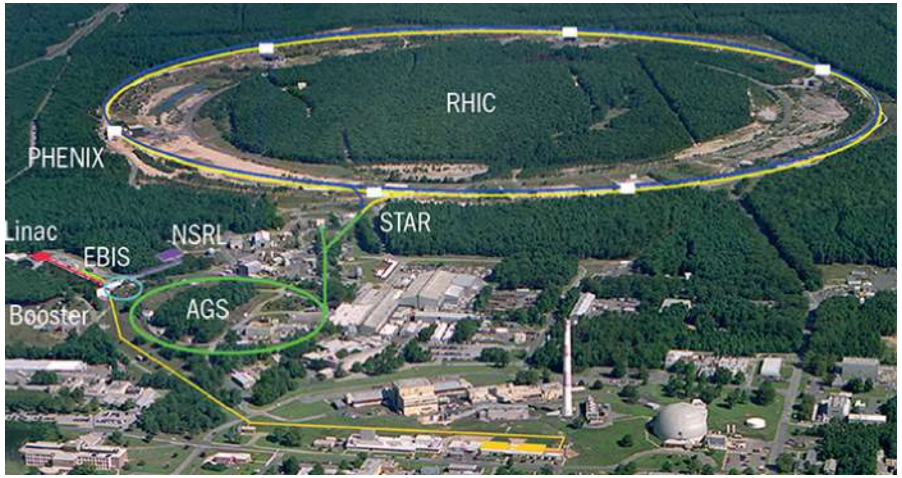
Structure of the nucleons in terms of quarks and gluons : still not understood completely



Pic: M. Anselmino

Experiments Around the World



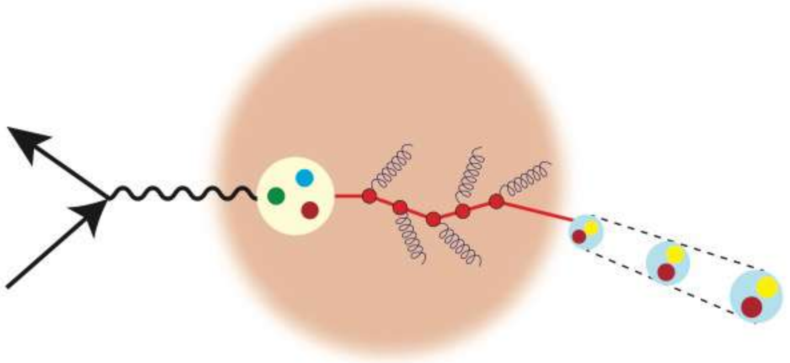
Upcoming Electron-Ion Collider (EIC)

The EIC to be built at Brookhaven National Lab, USA will collide highly energetic electron beam with proton/heavy ion to take 'snapshots' at high accuracy --tomography of the nucleon

How the quarks and gluons are distributed in space inside the nucleon

How do quarks and gluons bind together and for the nucleon ? What is the Origin of the mass of the nucleon ?

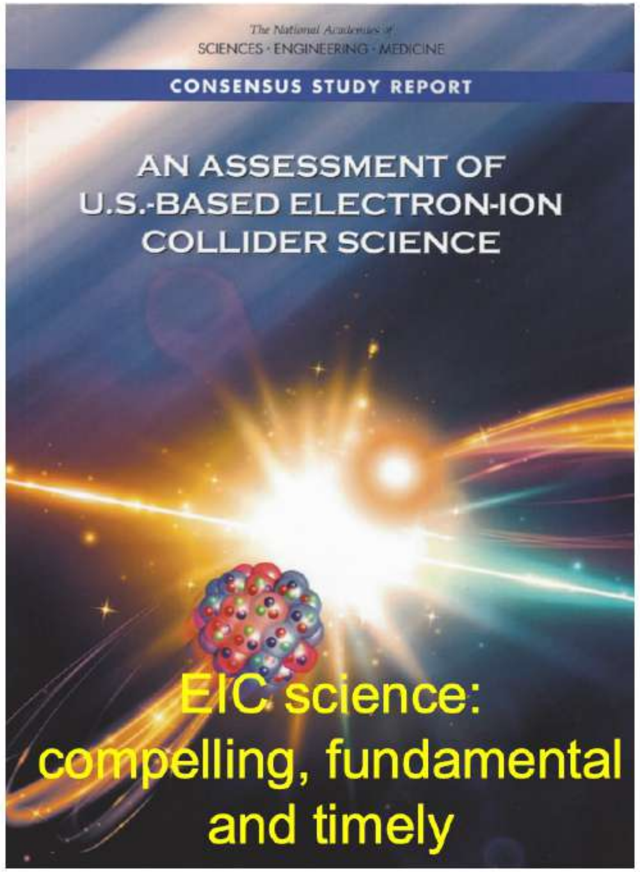
How the spin (1/2) of the proton is made from the spin and orbital angular momentum of the quarks and gluons



Will explore the correlations between spin/OAM and intrinsic transverse momentum

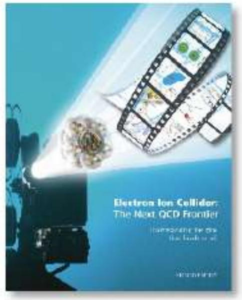
How does a dense nuclear environment affect quarks and gluons and their interactions ?

National Academy's Assessment



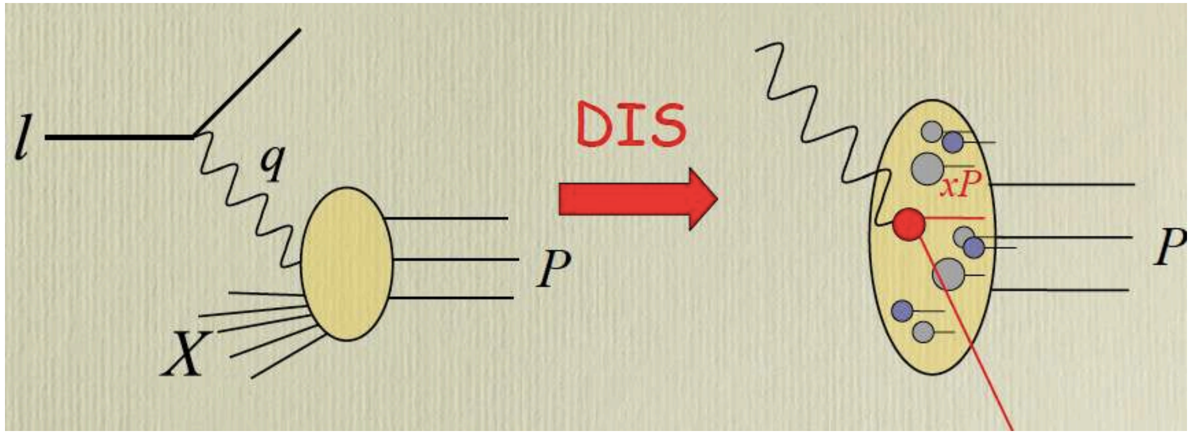
Machine Design Parameters:

- High luminosity: up to 10^{33}-10^{34} $cm^{-2}sec^{-1}$
  - a factor ~100-1000 times HERA
- Broad range in center-of-mass energy: ~20-100 GeV upgradable to 140 GeV
- Polarized beams e-, p, and light ion beams with flexible spin patterns/orientation
- Broad range in hadron species: protons…. Uranium
- Up to two detectors well-integrated detector(s) into the machine lattice



Slide : Abhay Deshpande

Nucleon structure : probed through electron-proton deep inelastic scattering



$$Q^2 = -q^2 \to \infty$$

$$x = \frac{Q^2}{2P \cdot q} \quad \text{fixed}$$

Virtual photon ‘sees ‘ the partons (quarks) inside the proton

Proton is Lorentz contracted, like a pancake in transverse plane

Target is a collection of partons moving with fraction x of proton momentum, and collinearly with the proton

In the deep inelastic limit, the electron passes target at almost zero time, sees partons frozen in transverse plane.

Electron can interact with the partons only if the impact parameter is less than 1/Q. Electron-parton scattering happens at a much shorter time scale than the hadronization scale of proton remnants

Factorized form of the cross section

$$\frac{d\sigma}{dx\, dQ^2} = \sum_f \left(\frac{d\hat{\sigma}}{dQ^2}\right)_f e_f^2\, \phi_f(x)$$

Probability density of finding a parton of momentum fraction x inside the proton

Differential scattering cross section

Elastic electron-parton scattering

Incoherent sum over all partons

Parton model : Partons are non-interacting

Factorization of the hard part, that is interaction of electron with the parton, and the soft part, that is the parton distributions in the cross section

Hard part can be calculated perturbatively but the parton distributions are non-perturbative. They are also not dependent on the process

In parton model, parton distributions show scaling : they are functions of x only

Bjorken & Paschos, Phys. Rev D185, 1975, (1969).

Parton model to QCD

$$\frac{d\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4} A(y)\ F_2(x)$$

$$F_2(x) = \sum_f e_f^2\, x\, \phi_f(x)$$

y = electron inelasticity $\sim \frac{\nu}{E}$

$A(y) = 1 + (1-y)^2$

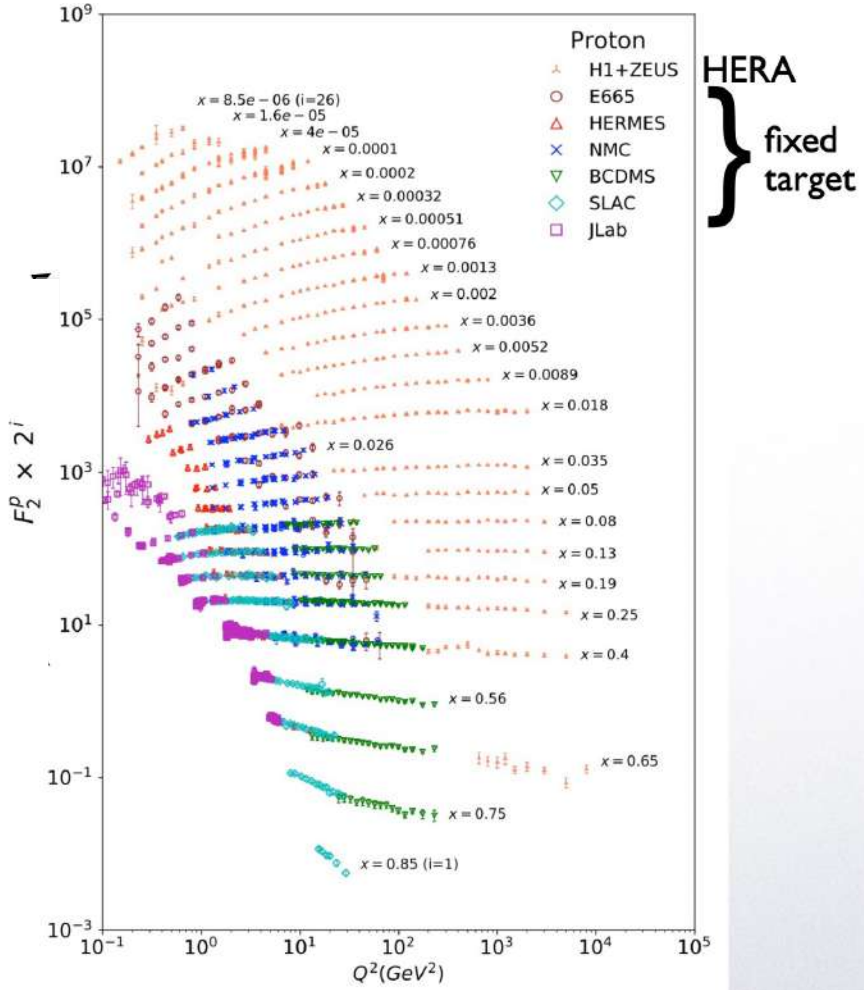
F_2 varies also with Q^2 : scale evolution

Scale evolution can be calculated using evolution equations

partons, or quarks are not free : they interact through gluons !

Interaction of quarks and gluons are called strong interaction or QCD

$$\frac{d\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[A(y)\, F_2(x,Q^2) - y^2\, F_L(x,Q^2)\right]$$



Fundamental Interactions

Interaction	Range	Typical Lifetime (sec)	Typical Cross Section (mb)	Typical Coupling α_i
Strong	$1\ \text{F} \simeq \frac{1}{m_\pi}$	10^{-23}	10	1
	Color confinement range[a]	e.g., $\Delta \to p\pi$	e.g., $\pi p \to \pi p$	
Electromagnetic	∞	$10^{-20} \sim 10^{-16}$ e.g., $\pi^0 \to \gamma\gamma$ $\Sigma \to \Lambda\gamma$	10^{-3} e.g., $\gamma p \to p\pi^0$	10^{-2}
Weak	$\frac{1}{M_W}$ with	10^{-12} or longer	10^{-11}	10^{-6}
	$M_W \simeq 100 m_p$	e.g., $\Sigma^- \to n\pi^-$ $\pi^- \to \mu^-\bar{\nu}$	e.g., $\nu p \to \nu p$ $\nu p \to \mu^- p \pi^+$	

Quarks and Gluons

The quarks interact with each other through exchange of gluons, the coupling is denoted by α_s

Quark-gluon interaction is called color gauge interaction or QCD (quantum chromodynamics) : it is strong interaction

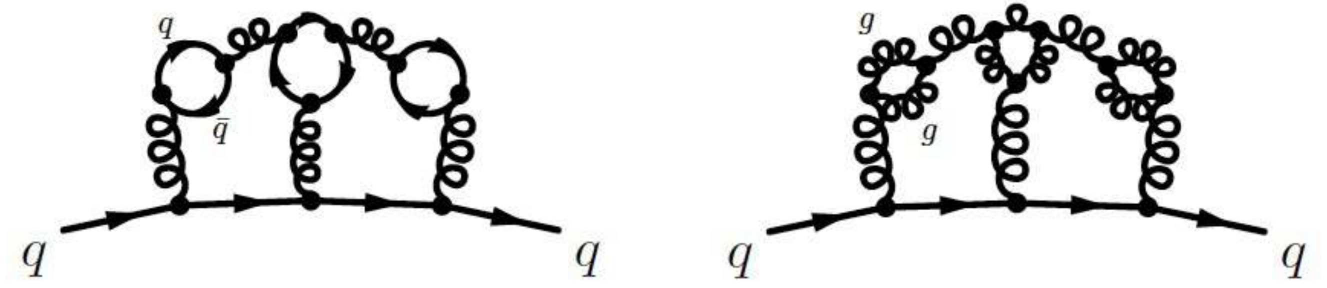
Lifetime (strong interaction): 10^{-23} sec

Lifetime (electromagnetic interaction) : 10^{-20}-10^{-16} sec

Coupling α_s is not constant (like QED) ; it changes with energy : running of the coupling

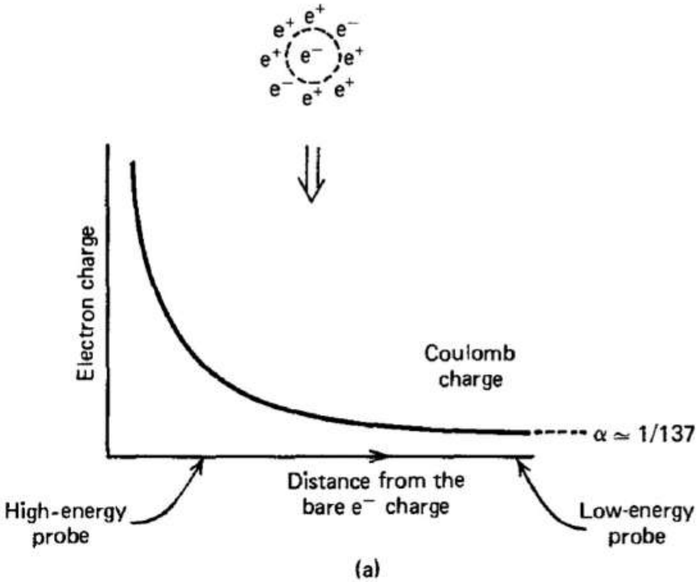
But this running in QCD is different compared to QED

Gluons interact with each other unlike photons : due to this the color charge is screened by both virtual quarks and gluons



Pic : Tina Potter

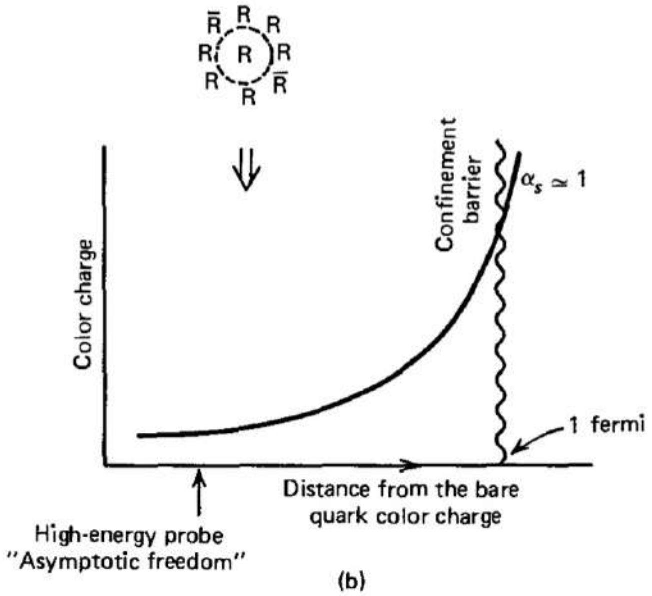
Electromagnetic vs strong interaction

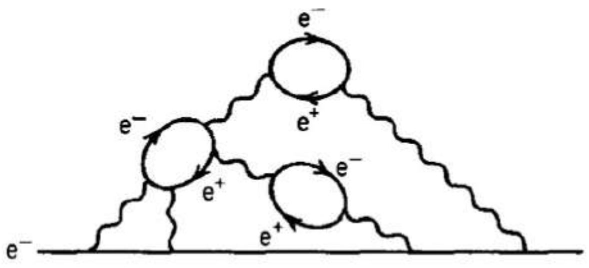


Photons do not carry electric charge : do not interact with photons

Gluons carry color and interact with themselves

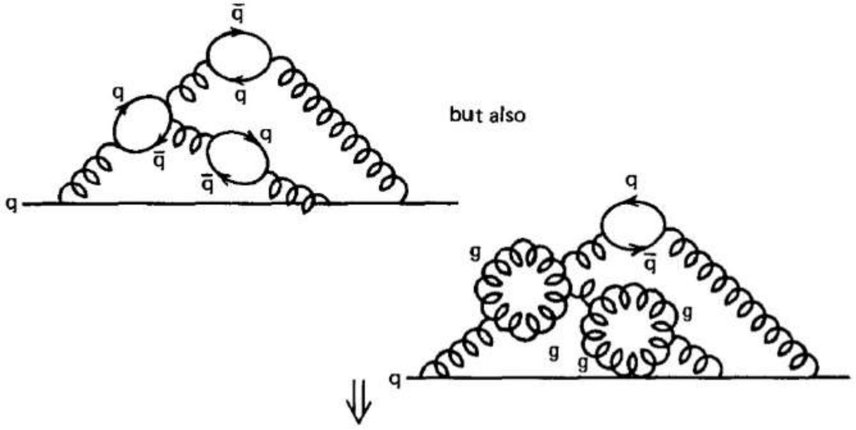
Asymptotic freedom



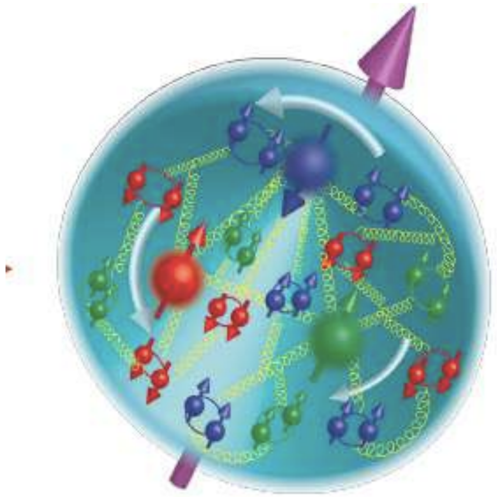


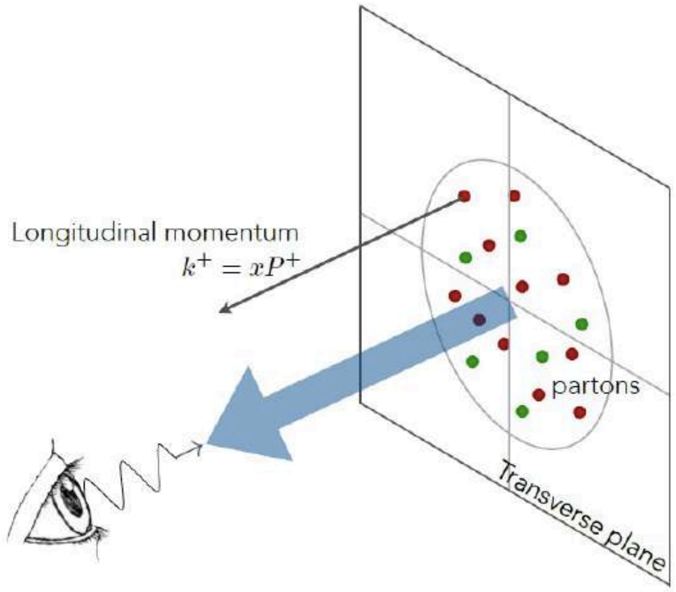
Halzen & Martin; Quarks and Leptons

Published by John Wiley & sons.



Collinear pdfs : nucleon structure In 1-D





Motion of quarks in the transverse plane ignored

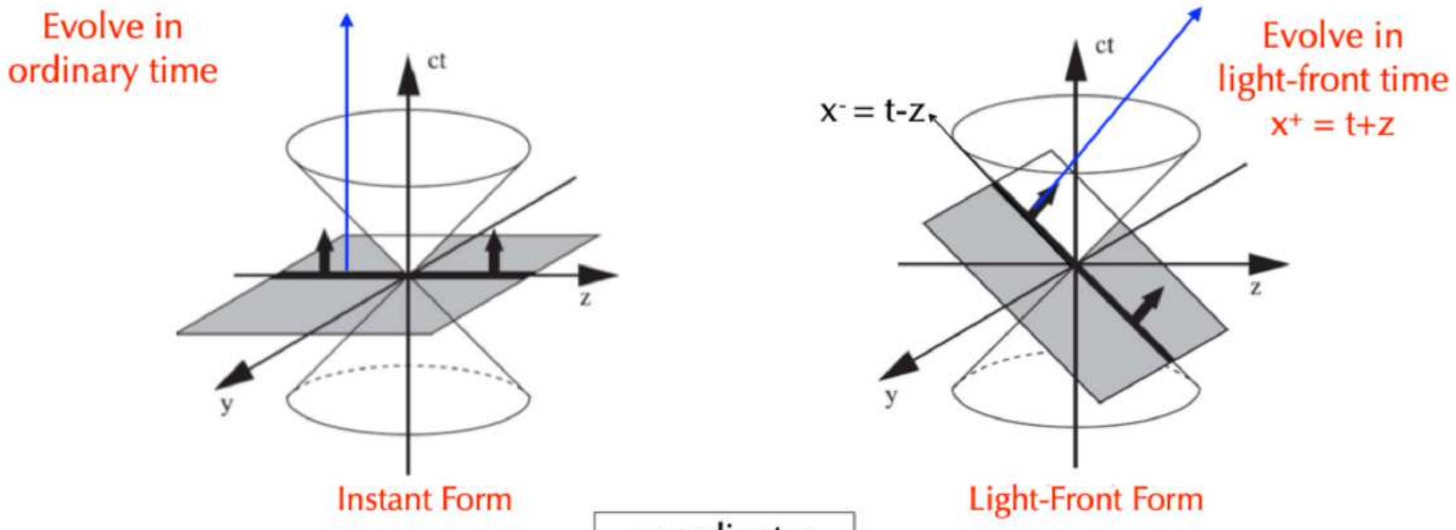
Non-perturbative : Is extracted by fitting experimental data

Scale evolution of pdfs can be calculated using **Dokshitzer–Gribov–Lipatov–Altarelli–Parisi** (DGLAP) evolution equations

Independent of process : once extracted can be used to predict cross section of another process as the scale evolution is known

One can also perform a polarized scattering experiment : probes polarized structure functions

Introduction to light-front coordinates



Instant Form		Light-Front Form	
coordinates			
x^0	time	$\frac{x^0 + x^3}{\sqrt{2}}$	time
x^1, x^2, x^3	space	$\frac{x^0 - x^3}{\sqrt{2}}, \vec{x}_\perp = (x^1, x^2)$	space
Hamiltonian			
$H = \sqrt{P^2 + M_0^2} + V$		$P^- = \frac{\vec{P}_\perp^2 + M_0^2}{P^+} + V$	

Also see A. Harindranath, *An Introduction to light-front dynamics for pedestrians*; hep-ph/9612244

$x^+=0$ plane becomes the plane of quantization in quantum theory

P. A. M Dirac, Rev. Mod Phys 21, 392 (1949)

Light front dynamics

- LF coordinates

$\tau = x^+ = x^0 + x^3$	light-front time	$P^+ = P^0 + P^3$	longitudinal momentum
$x^- = x^0 - x^3$	longitudinal space variable	$P^- = P^0 - P^3$	light-front Hamiltonian
$\mathbf{x}_\perp = (x^1, x^2)$	transverse space variable	$\mathbf{P}_\perp = (P^1, P^2)$	transverse momentum

- On shell relation $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = \mathcal{M}^2$ leads to dispersion relation for LF Hamilnotian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \quad P^+ > 0$$

- Hamiltonian equation for the relativistic bound state

$$i\frac{\partial}{\partial x^+}|\psi(P)\rangle = P^-|\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+}|\psi(P)\rangle$$

where P^- is derived from the QCD Lagrangian: kinetic energy of partons plus confining interaction

Some features of light-front framework

Dot product of two 4-vectors

$$a \cdot b = a^\mu b_\mu = \frac{1}{2}a^+b^- + \frac{1}{2}a^-b^+ - a^\perp \cdot b^\perp.$$

Dispersion relation for an on-mass-shell particle

$$k^- = \frac{(k^\perp)^2 + m^2}{k^+}$$

This is remarkable because of the following :

(i) Even though this is a relativistic equation there is no square root factor. This is helpful if one tries to solve the eigenvalue equation

(ii) The dependence of the energy k^- on the transverse momentum is similar to non-relativistic dispersion relation

(iii) From the dispersion relation it can be seen that for an on-mass-shell particle positive(negative) k^+ gives positive(negative) k^- . In fact particles with negative energy and negative momentum can be mapped into positive energy and positive momentum. As a result, we always have k+ greater than or equal to zero. This has interesting consequences, in particular in the vacuum structure of light-front field theories.

Poincare generators

Poincare transformation is generated by ten generators satisfying the algebra

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = i(-g^{\mu\rho} P^\nu + g^{\nu\rho} P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = i(-g^{\mu\rho} M^{\nu\sigma} + g^{\nu\rho} M^{\mu\sigma} - g^{\mu\sigma} M^{\rho\nu} + g^{\nu\sigma} M^{\rho\mu}).$$

A. Harindranath, *An Introduction to light-front dynamics for pedestrians*; hep-ph/9612244

These are constructed from the symmetric energy-momentum tensor in field theory

$$P^\mu = \int d^3x \theta^{0\mu},$$

$$M^{\mu\nu} = \int d^3x [x^\mu \theta^{0\nu} - x^\nu \theta^{0\mu}].$$

Rotation generators $M_{ij} = \epsilon_{ijk} J^k$

Boost generators $M^{0i} = K^i$

Hamiltonian and the three Boost generators are dynamical (contain dynamics) and the rotation generators are kinematical (do not contain dynamics)

Poincare generators in light front field theory

$$P^{\mu} = \frac{1}{2}\int dx^{-} d^{2}x^{\perp} \theta^{+\mu},$$

$$M^{\mu\nu} = \frac{1}{2}\int dx^{-} d^{2}x^{\perp} [x^{\mu}\theta^{+\nu} - x^{\nu}\theta^{+\mu}].$$

x^+ is the light-front time and P^- is the LF Hamiltonian that generates x^+ evolution, P^+ is the longitudinal momentum

$M^{+-} = 2K^3$ $\quad$ $M^{+i} = E^i$ $\quad$ Light-front Boosts

$M^{12} = J^3$ $\quad$ $M^{-i} = F^i$ $\quad$ Light-front rotation

Longitudinal Boosts produce a scale transformation and transverse Boosts are kinematical , that is they keep the $x^+=0$ plane invariant.

Transverse rotation generators are dynamical

$$E^1 = -K_1 + J_2, \quad E^2 = -K_2 - J_1$$
$$F^1 = -K_1 - J_2, \quad F^2 = -K_2 + J_1$$

Linear combination of equal-time rotation and Boosts

Light front wave functions

A hadronic bound state of momentum P and helicity λ can be expanded in Fock space as

$$| P^+, P^\perp, \lambda\rangle = \sum_{n,\lambda_i} \int' dx^i d^2\kappa_i^\perp \, | n, x_i P^+, x_i P^\perp + \kappa_i^\perp, \lambda_i\rangle \Phi_n^\lambda(x_i, \kappa_i^\perp, \lambda_i).$$

Multiparticle Light front wave function

Brodsky, Pauli, Pinsky, Phy. Rep 301 (1998) ,299

$$\sum_i x_i = 1, \qquad \sum_i \kappa_i^\perp = 0,$$

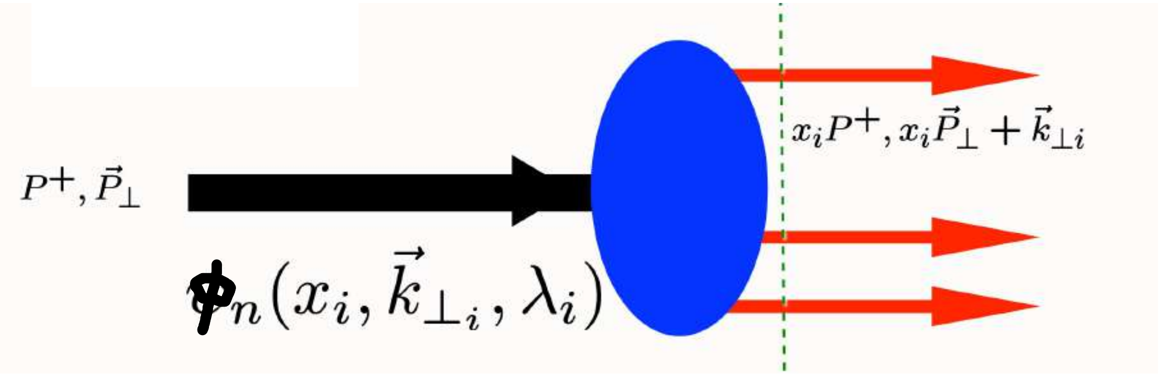
$$x_i = \frac{p_i^+}{P^+}, \qquad \kappa_i^\perp = p_i^\perp - x_i P^\perp.$$

Normalization

$$\sum_{n,\lambda_i} \int' dx_i d^2\kappa_i^\perp | \Phi_n^\lambda(x_i, \kappa_i^\perp, \lambda_i) |^2 = 1.$$

These states obey the LF eigenvalue equation

$$P^- | P, \lambda\rangle = \frac{(P^\perp)^2 + M^2}{P^+} | P, \lambda\rangle.$$

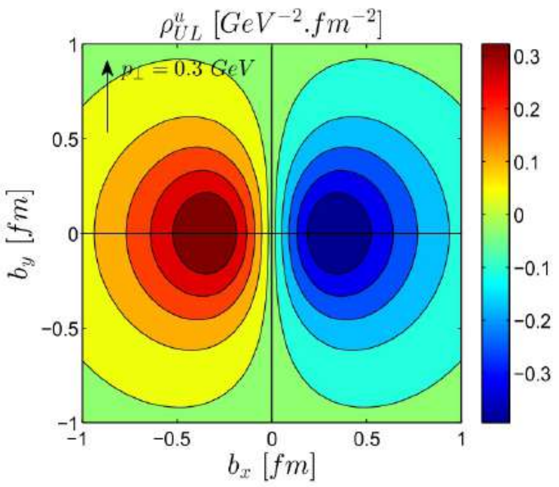


Light-front wave functions

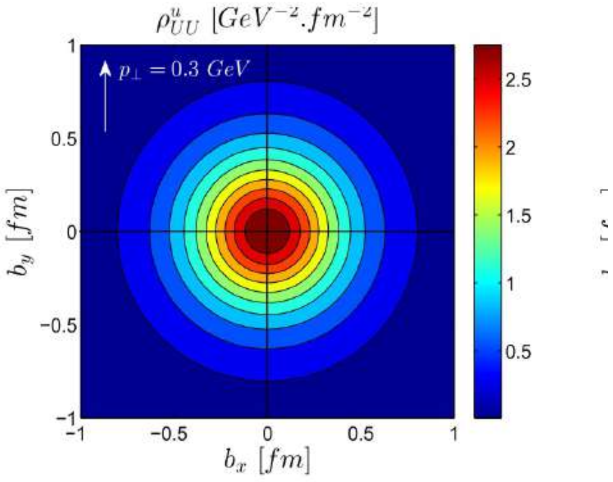
Light font wave functions are Boost invariant ; one can truncate the Fock space expansion to a few particle sector.

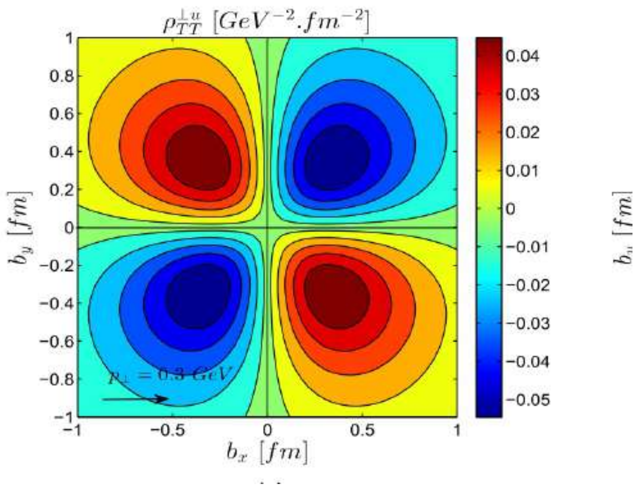
LFWFs are useful descriptions of relativistic multiparticle system like a hadron

Useful for model calculations for hadrons like a proton or a meson



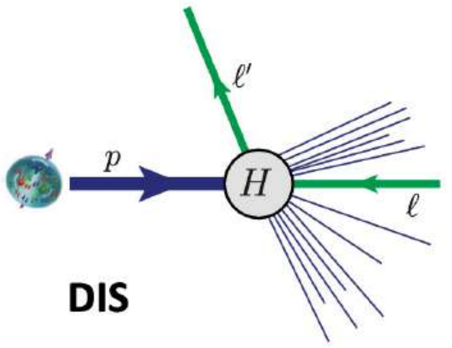
Distribution of longitudinally polarized quark in an unpolarized hadron

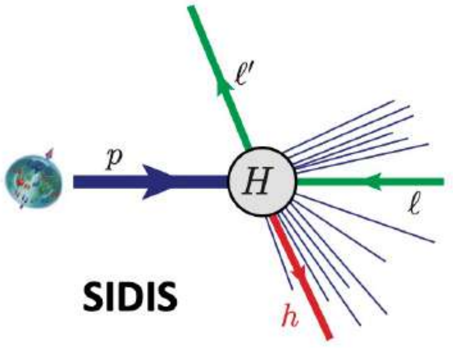


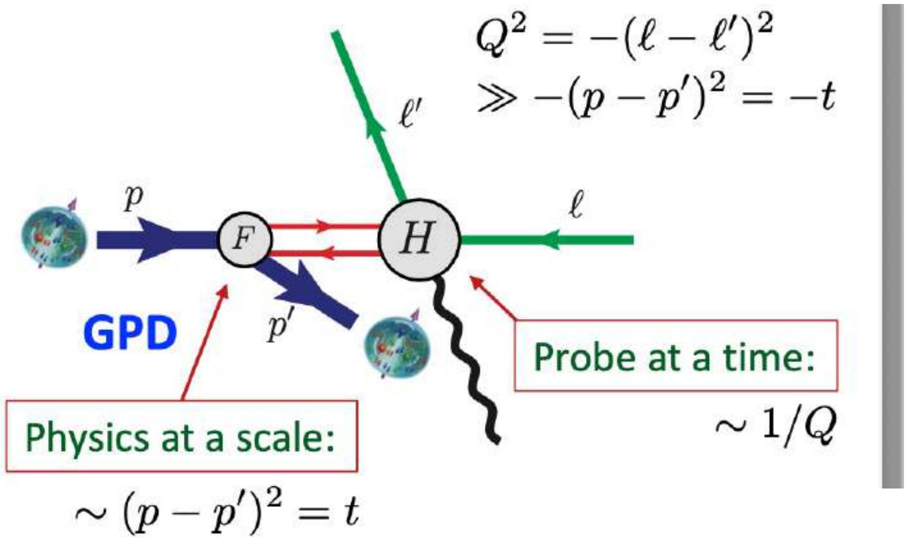


Distribution of transversely polarized quarks in a transversely polarized hadron

Exploring partonic structure of the hadron while breaking/not breaking the proton







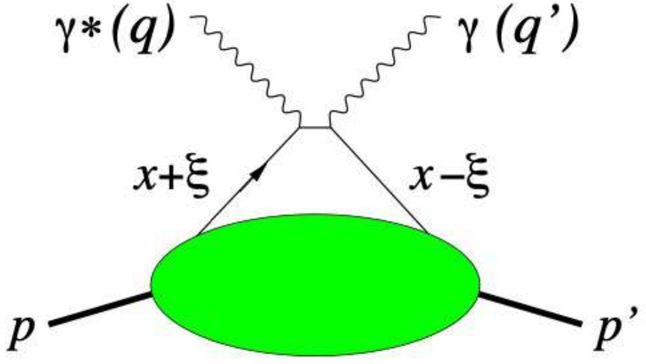
Deep inelastic scattering (DIS) and semi-inclusive deep inelastic scattering (SIDIS)

Exclusive process; Deeply virtual Compton scattering (DVCS)

SIDIS probes the transverse momentum dependent parton distributions (TMDs)

The proton remains intact, there is a momentum transfer and a real photon is observed in the final state

Deeply virtual Compton Scattering and GPDs



$$e + P \to e + \gamma + P'$$

A real photon detected in the final state in addition to the electron and proton remains intact, but there is a momentum transfer

At leading order, the amplitude can be in a factorized form by the handbag diagram

Upper part : perturbative ; lower part non-perturbative: parametrized in terms of GPDs

$$t = (p - p')^2; \qquad Q^2 = -q^2$$

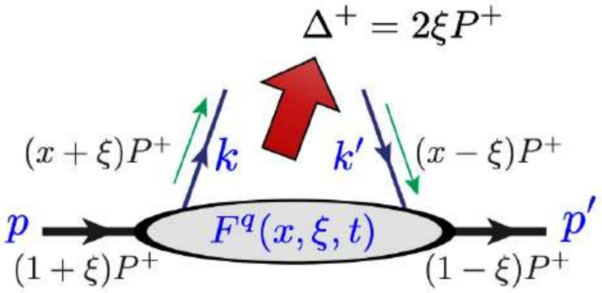
t : momentum transfer of the proton, is much less compared to Q^2

$$x_b = \frac{Q^2}{2p \cdot q}$$

$$\xi \approx \frac{x_B}{2 - x_B}$$

Skewness variable

Generalized parton distributions (GPDs)



Factorization of DVCS amplitude

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$

- $\frac{x+\xi}{1+\xi}$ and $\frac{x-\xi}{1-\xi}$ are initial and final **light cone momentum fractions** of struck quark.
- t is the invariant momentum transfer **to the target**.
- Q^2 is the invariant momentum transfer **from the electron**.
- $t \neq -Q^2$, in contrast to elastic scattering.
- Q^2 acts as a **resolution scale**, like in deeply inelastic scattering (DIS).
- t tells us about structure seen from redistribution of momentum kick.
- t is the "form factor variable" rather than Q^2.

These are parametrized in terms of GPDs

$$F^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\,\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right],$$
$$\widetilde{F}^q(x,\xi,t) = \int \frac{\mathrm{d}z^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+\gamma_5 q(-z^-/2)|p\rangle$$
$$= \frac{1}{2P^+}\left[\widetilde{H}^q(x,\xi,t)\,\bar{u}(p')\,\gamma^+\gamma_5 u(p) - \widetilde{E}^q(x,\xi,t)\,\bar{u}(p')\,\frac{\gamma_5\Delta^+}{2m}u(p)\right].$$

Off-forward matrix elements

Properties of GPDs

As GPDs appear in the amplitude and are expressed as off-forward matrix elements, they do not have a probabilistic interpretation like parton distributions

A Fourier transform wrt momentum transfer in the transverse direction, $\Delta^{\perp}$ gives parton distribution in impact parameter space, which have probabilistic interpretation (more later)

Forward limit of GPDs give parton distributions $H_q(x,0,0) = q(x)$ $\quad \tilde{H}^q(x,0,0) = \Delta q(x)$

$$\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t), \qquad \int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t),$$

$$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = g_A^q(t), \qquad \int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = g_P^q(t),$$

Moment of GPDs give Form factors

$$\langle p'|\, \bar{q}(0)\gamma^\mu q(0)\, |p\rangle = \bar{u}(p')\left[F_1^q(t)\,\gamma^\mu + F_2^q(t)\,\frac{i\sigma^{\mu\alpha}\Delta_\alpha}{2m}\right] u(p),$$

Dirac and Pauli Form factors

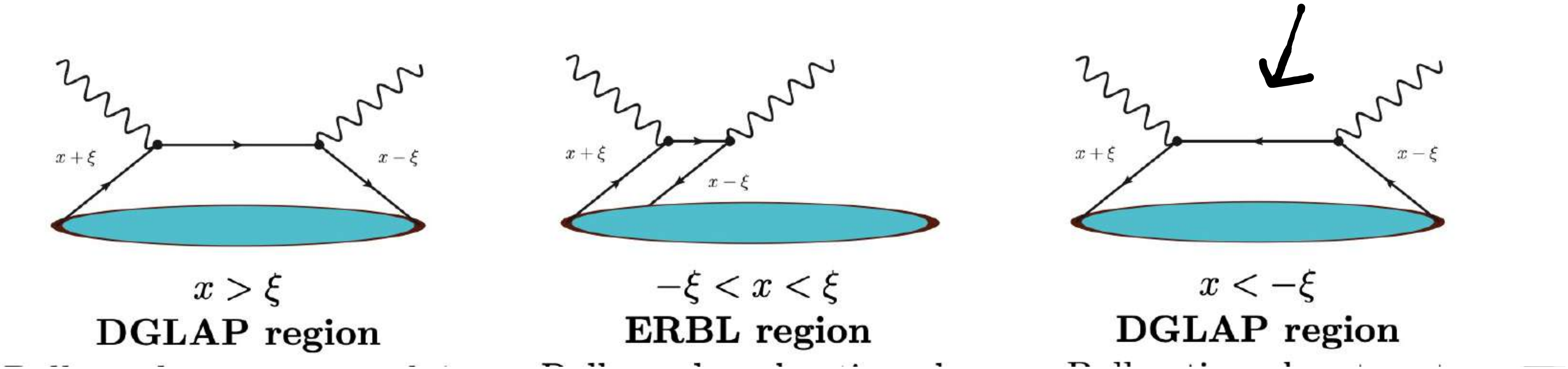
$$\langle p'|\, \bar{q}(0)\gamma^\mu\gamma_5 q(0)\, |p\rangle = \bar{u}(p')\left[g_A^q(t)\,\gamma^\mu\gamma_5 + g_P^q(t)\,\frac{\gamma_5\Delta^\mu}{2m}\right] u(p),$$

Axial and pseudoscalar form factors

M. Diehl; Phys.Rept.388:41-277,2003

Evolution of generalized parton distributions

Antiquark



(Dokshitzer–Gribov–Lipatov–Altarelli–Parisi)

(Efremov-Radyushkin-Brodsky-Lepage)

ξ is called skewness, it gives momentum transfer in the light-cone plus or longitudinal direction

GPDs are functions of x, **ξ** and t as well as the scale Q^2.

GPDs evolve with the scale, evolution depending on whether skewness is greater or less than x

Nucleon Spin Puzzle

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Proton spin — Quark spin — Quark OAM — Gluon spin — Gluon OAM

EMC (European Muon Collaboration) at CERN in 1989 measured spin asymmetry in polarized muon-proton scattering experiment, and found that the contribution coming from the intrinsic spin of quarks is very small

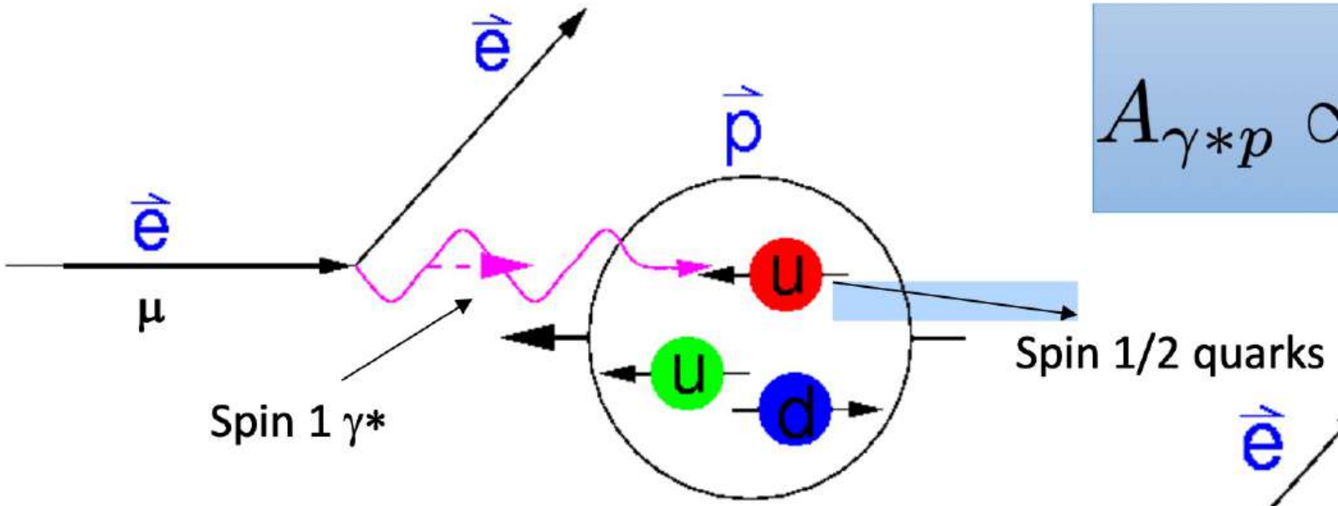
$$\Delta\Sigma /2 = (0.12) +/- (0.17) \text{ (EMC, 1989)}$$

Significant contribution comes from gluons as well as the orbital angular momentum of quarks and gluons

How to measure the orbital angular momentum ? Observables ? Can one separate the gluon part intro intrinsic and orbital in a gauge invariant way ?

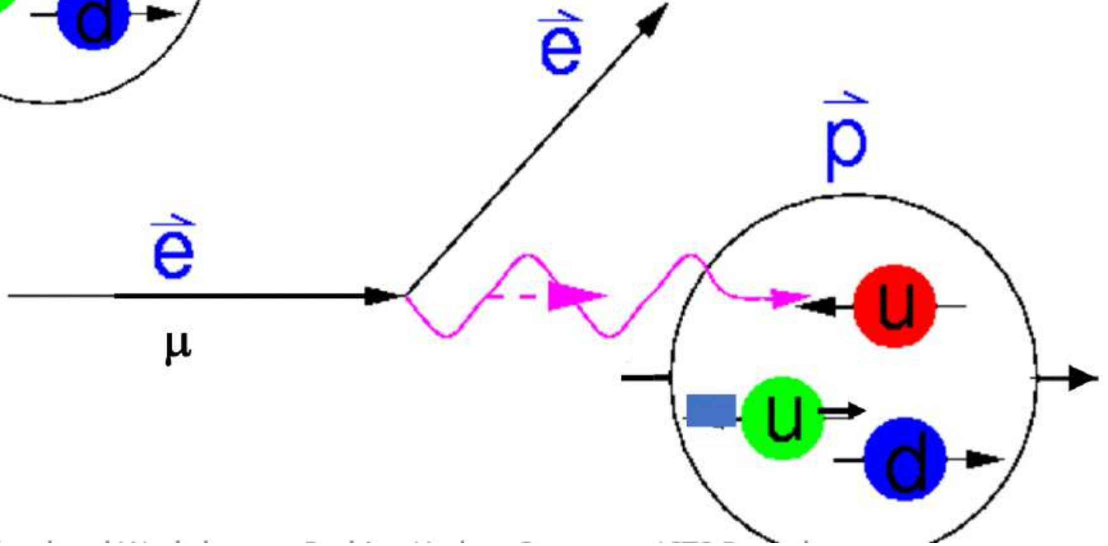
How to measure quark contribution to the spin ?

Polarized deep inelastic scattering experiment : electron and proton longitudinally polarized



$$A_{\gamma * p} \propto \frac{N^{\uparrow\downarrow} - N^{\uparrow\uparrow}}{N^{\uparrow\downarrow} + N^{\uparrow\uparrow}}$$

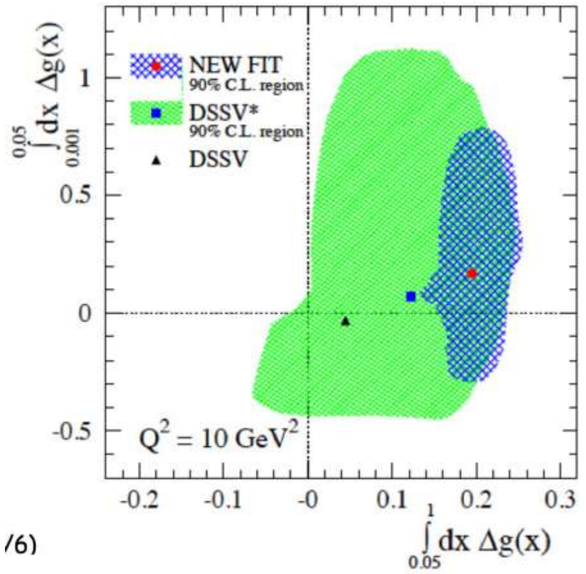
One can measure double longitudinal spin asymmetry : sensitive to the intrinsic spin carried by quarks



Pic : Abhay Deshpande

Nucleon Spin Puzzle

ΔG = 0.2 +/- 0.02 +/- 0.5



D. deFlorian et al., arXiv:1404.4293

RHIC data shows significant contribution from gluon spin.

Several lattice calculations of quark and gluon angular momentum contributions

Total quark angular momentum contribution about 54-57 %, total gluon angular momentum about 38-46 %, quark OAM about 13-18 %

How to measure OAM of quarks and gluon experimentally ? Intrinsic transverse momentum ?

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Status on lattice calculations of the proton spin decomposition

Check for updates

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Ji's sum rule : GPDs

$$\int dx x[H_q(x,0,0) + E_q(x,0,0)] = A_q(0) + B_q(0) = 2J_q$$

Angular momentum of quarks

$$\langle P', S'|T_i^{\mu\nu}(0)|P, S\rangle = \bar{U}(P', S')\Big[- B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2) + B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu) + C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big] U(P, S),$$

A and B are the form factors of the energy-momentum tensor, called Gravitational form factors

Similarly one can define gluon GPDs and a similar sum rule gives the total angular momentum of the gluons

So GPDs can help us to understand how proton spin is made from quarks and gluons.

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta g + L_g$$

Jaffe Manohar Sum rule

Extensive theoretical works to connect the two sum rules .

Impact parameter dependent pdfs

Consider momentum transfer only in the transverse direction that is zero skewness

$$q(x, \mathbf{b}_\perp) = |\mathcal{N}|^2 \int \frac{d^2\mathbf{p}_\perp}{(2\pi)^2} \int \frac{d^2\mathbf{p}'_\perp}{(2\pi)^2} H_q(x, -(\mathbf{p}_\perp - \mathbf{p}'_\perp)^2) e^{i\mathbf{b}_\perp\cdot(\mathbf{p}_\perp - \mathbf{p}'_\perp)}$$
$$= \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} H_q(x, -\mathbf{\Delta}_\perp^2) e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp},$$

M. Burkardt, Int.J.Mod.Phys.A 18 (2003) 173

Fourier transform wrt $\Delta^\perp$ gives parton distribution in transverse impact parameter space

Although GPDs do not have probabilistic interpretation, this can be interpreted as probability distribution

When the proton is transversely polarized, the ipdpdfs are distorted--- related to orbital motion of the quarks

$$q(x, \mathbf{b}_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \mathcal{E}_q(x, \mathbf{b}_\perp),$$

This distortion is related to the Fourier transform of the GPD E (nucleon polarization in x direction)

Impact parameter dependent pdfs

Reference point for the impact parameter is the transverse center-of-momentum

$$\mathbf{R}_\perp \equiv \frac{1}{p^+}\int d^2\mathbf{x}_\perp \int dx^- T^{++}\mathbf{x}_\perp = \sum_{i\in q,g} x_i \mathbf{r}_{\perp,\mathbf{i}},$$

Sum over all partons

Momentum fraction of the parton

Impact parameter dependent distribution is defined by introducing the light-cone correlator in transverse b space

$$q(x,\mathbf{b}_\perp) \equiv \langle p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda | \hat{O}_q(x,\mathbf{b}_\perp) | p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda \rangle,$$

Where

$$|p^+, \mathbf{R}_\perp = \mathbf{0}_\perp, \lambda\rangle \equiv \mathcal{N}\int d^2\mathbf{p}_\perp |p^+, \mathbf{p}_\perp, \lambda\rangle$$

$$\int d^2\mathbf{b}_\perp q(x,\mathbf{b}_\perp) = q(x)$$

$$q(x,\mathbf{b}_\perp) \geqslant 0 \quad (x>0)$$

$$q(x,\mathbf{b}_\perp) \leqslant 0 \quad (x<0)$$

GPDs at zero skewness can be used to construct tomographic images of the nucleon. One can study ‘slices’ of the nucleon in transverse impact parameter space, for different values of the light-cone momentum fraction x

M. Burkardt, Int.J.Mod.Phys.A 18 (2003) 173

To be continued ....