

Transverse Momentum Dependent parton distributions (TMDs)

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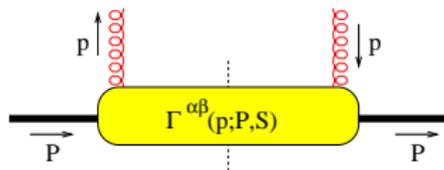
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Lecture 2

Gluon TMDs

- ▶ Definition and properties
- ▶ Linearly polarized gluons
- ▶ Phenomenology
 - ▶ Quarkonium production at the LHC
 - ▶ Open and hidden heavy-quark pair production at the EIC
- ▶ Summary

Gluon TMDs: definition and properties



Gauge invariant definition of $\Gamma^{\mu\nu}$

$$\Gamma^{[\mathcal{U},\mathcal{U}']\mu\nu} \propto \langle P, S | \text{Tr}_c [F^{+\nu}(0) \mathcal{U}_{[0,\xi]}^C F^{+\mu}(\xi) \mathcal{U}_{[\xi,0]}^{C'}] | P, S \rangle$$

Mulders, Rodrigues, PRD 63 (2001)

Buffing, Mukherjee, Mulders, PRD 88 (2013)

Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The gluon correlator depends on two path-dependent gauge links

$ep \rightarrow e' Q \bar{Q} X$, $ep \rightarrow e' \text{ jet jet } X$ probe gluon TMDs with $[++]$ gauge links

$pp \rightarrow \gamma\gamma X$ (and/or other CS final state) probes gluon TMDs with $[--]$ gauge links

$pp \rightarrow \gamma \text{ jet } X$ probes an entirely independent gluon TMD: $[+-]$ links (dipole)

GLUONS	<i>unpolarized</i>	<i>circular</i>	<i>linear</i>
U	f_1^g		$h_1^{\perp g}$
L		g_{1L}^g	$h_{1L}^{\perp g}$
T	$f_{1T}^{\perp g}$	g_{1T}^g	$h_{1T}^g, h_{1T}^{\perp g}$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)

Mulders, Rodrigues, PRD 63 (2001)

Meissner, Metz, Goeke, PRD 76 (2007)

- ▶ $h_1^{\perp g}$: T -even distribution of linearly polarized gluons inside an unp. hadron
- ▶ $f_{1T}^{\perp g}$: T -odd gluon Sivers function

Even unpolarized gluon TMDs are process dependent: *two relevant types*

This was first realized in the small- x framework:

Dominguez, Marquet, Xiao, Yuan, PRD (2011)

- ▶ Weizsäcker-Williams distribution (WW)
- ▶ Dipole distribution (DP)

Unpolarized (and in general T -even) gluon TMDs

$$\begin{aligned} [++] &= [--] \quad (\text{WW}) \\ [+ -] &= [- +] \quad (\text{DP}) \end{aligned}$$

In general they can differ in magnitude and width. Only constraint:

$$\int d^2 \mathbf{k}_T f_1^{[++]g}(x, \mathbf{k}_T^2) = \int d^2 \mathbf{k}_T f_1^{[+-]g}(x, \mathbf{k}_T^2)$$

Different processes can probe either types or a mixture of them

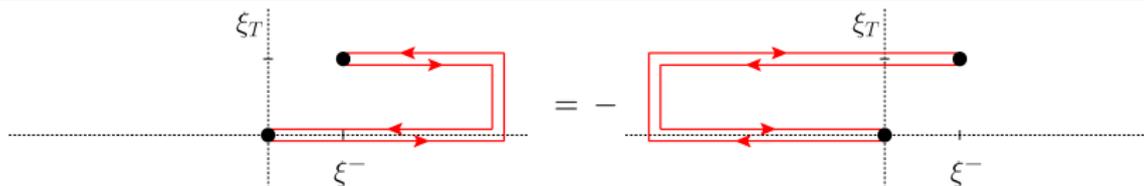
Related Processes

$ep^\uparrow \rightarrow e' Q\bar{Q}X$, $ep^\uparrow \rightarrow e' \text{jet jet } X$ probe GSF with $[++]$ gauge links (WW)

$p^\uparrow p \rightarrow \gamma\gamma X$ (and/or other CS final state) probe GSF with $[--]$ gauge links

Analogue of the sign change of $f_{1T}^{\perp g}$ between SIDIS and DY (true also for h_1^g and $h_{1T}^{\perp g}$)

$$f_{1T}^{\perp g}[e p^\uparrow \rightarrow e' Q\bar{Q} X] = -f_{1T}^{\perp g}[p^\uparrow p \rightarrow \gamma\gamma X]$$



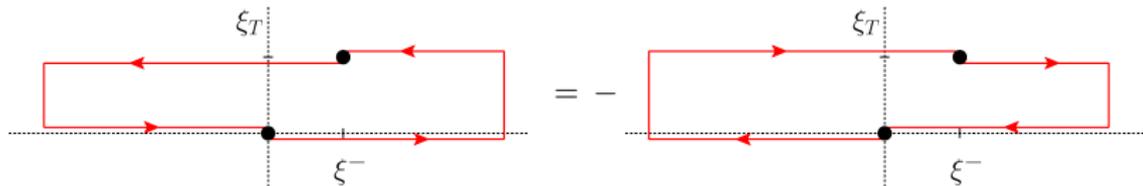
Boer, Mulders, CP, Zhou (2016)

Motivation to study gluon Sivers effects at both RHIC and the EIC

Complementary Processes

$ep^\uparrow \rightarrow e' Q \bar{Q} X$ probes a GSF with $[++]$ gauge links (WW)

$p^\uparrow p \rightarrow \gamma \text{jet} X$ ($gq \rightarrow \gamma q$) probes a gluon TMD with $[+-]$ links (DP)



At small- x the WW Sivvers function appears to be suppressed by a factor of x compared to the unpolarized gluon function, unlike the dipole one

The DP gluon Sivvers function at small- x is the **spin dependent odderon** (single spin asymmetries from a single Wilson loop matrix element)

Boer, Echevarria, Mulders, Zhou, PRL 116 (2016)
Boer, Cotogno, Van Daal, Mulders, Signori, Zhou, JHEP 1610 (2016)

The first transverse moments of the WW and DP gluon Siverson functions

$$f_{1T}^{\perp(1)g(f/d)}(x) = \int d^2 \mathbf{k}_T \frac{k_T^2}{2M_p^2} f_{1T}^{\perp g(f/d)}(x, \mathbf{k}_T^2)$$

related to two different trigluon Qiu-Sterman functions $T_G^{(f/d)}$, involving the antisymmetric f_{abc} and symmetric d_{abc} color structures, respectively

Bomhof, Mulders, JHEP 0702 (2007)
Buffing, Mukherjee, Mulders, PRD 88 (2013)

The two distributions have a different behavior under charge conjugation

The Burkardt sum rule constraints only the f -type gluon Siverson function

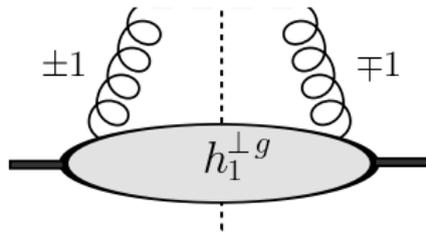
$$\sum_{a=q, \bar{q}, g} \int dx f_{1T}^{\perp(1)a}(x) = 0$$

Boer, Lorcé, CP, Zhou, AHEP 2015 (2015)

The distribution of linearly polarized gluons
inside an unpolarized proton: $h_1^{\perp g}$

Gluons inside an unpolarized hadron can be linearly polarized

It requires nonzero transverse momentum



Interference between ± 1 gluon helicity states

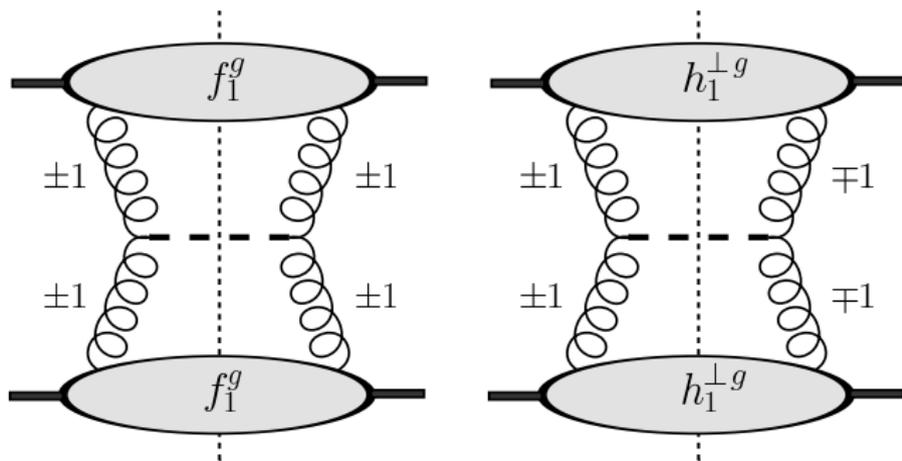
Like the unpolarized gluon TMD, it is T -even and exists in different versions:

- ▶ $[++] = [--]$ (WW)
- ▶ $[+-] = [-+]$ (DP)

Higgs boson production happens mainly via $gg \rightarrow H$

Pol. gluons affect the Higgs transverse spectrum at NNLO pQCD

Catani, Grazzini, NPB 845 (2011)



The nonperturbative distribution can be present at tree level and would contribute to Higgs production at low q_T

Boer, den Dunnen, CP, Schlegel, Vogelsang, PRL 108 (2012)

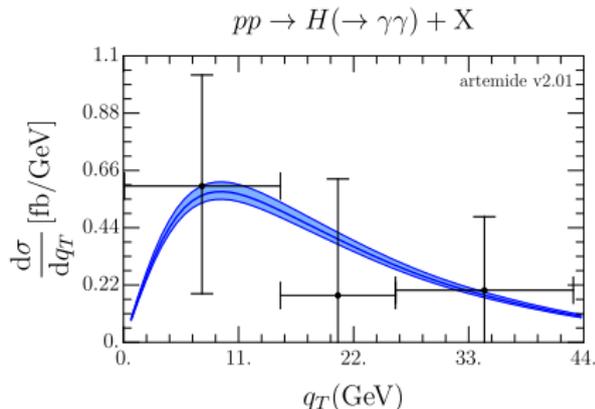
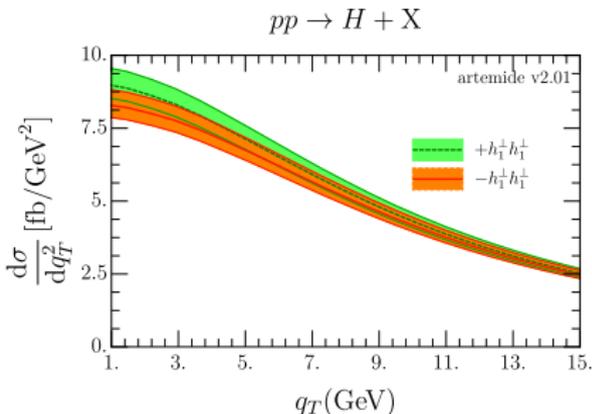
Boer, den Dunnen, CP, Schlegel, PRL 111 (2013)

tinyEchevarria, Kasemets, Mulders, CP, JHEP 1507 (2015)

q_T -distribution of the Higgs boson

$$\frac{1}{\sigma} \frac{d\sigma}{dq_T^2} \propto 1 + R(q_T^2) \quad R = \frac{h_1^{\perp g} \otimes h_1^{\perp g}}{f_1^g \otimes f_1^g} \quad |h_1^{\perp g}(x, \mathbf{p}_T^2)| \leq \frac{2M_p^2}{\mathbf{p}_T^2} f_1^g(x, \mathbf{p}_T^2)$$

The perturbative tails of f_1^g and $h_1^{\perp g}$ (matching coefficients to collinear PDFs) are known up to $\mathcal{O}(\alpha_s^2)$ (NNLO); g_{1L} up to $\mathcal{O}(\alpha_s)$ (NLO)



Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov, JHEP 11 (2019) 121

The matching coefficients for the other gluon TMDs are still unknown

Gluon TMDs and quarkonium production

Color Octet (CO) production in pp collisions involves a complicated link structure

Color Singlet (CS) production of C -even quarkonia from two gluons is possible

This is not allowed for J/ψ or Υ because of the Landau-Yang theorem

$$pp \rightarrow [Q\bar{Q}]X \quad (gg \rightarrow [Q\bar{Q}])$$

Hard scale can only be the particle mass: $Q = M$

TMD Factorization requires the resulting particle Q to have small q_T ($q_T \ll M$)

C = +1 quarkonium production

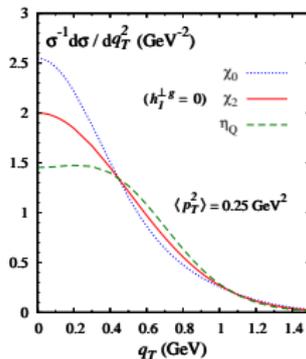
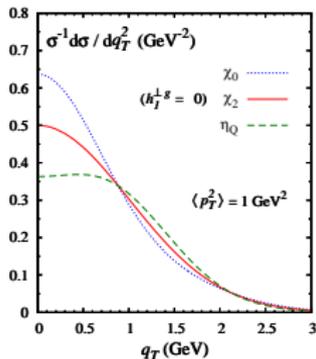
q_T -distribution of η_Q and χ_{QJ} ($Q = c, b$) in the kinematic region $q_T \ll 2M_Q$

$$\frac{1}{\sigma(\eta_Q)} \frac{d\sigma(\eta_Q)}{dq_T^2} \propto f_1^g \otimes f_1^g [1 - R(q_T^2)] \quad [\text{pseudoscalar}] \quad R(q_T^2) = \frac{h_1^{\perp g} \otimes h_1^{\perp g}}{f_1^g \otimes f_1^g}$$

$$\frac{1}{\sigma(\chi_{Q0})} \frac{d\sigma(\chi_{Q0})}{dq_T^2} \propto f_1^g \otimes f_1^g [1 + R(q_T^2)] \quad [\text{scalar}]$$

$$\frac{1}{\sigma(\chi_{Q2})} \frac{d\sigma(\chi_{Q2})}{dq_T^2} \propto f_1^g \otimes f_1^g$$

Boer, CP, PRD 86 (2012)



Proof of factorization at NLO for $pp \rightarrow \eta_Q X$ in the Color Singlet Model (CSM)

Ma, Wang, Zhao, PRD 88 (2013); PLB 737 (2014)
Echevarria, JHEP 1910 (2019)

Future fixed target experiments at LHC

C = +1 quarkonium production

$\eta_Q, \chi_{Q0}, \chi_{Q2}$

Structure of the cross section for the doubly polarized process $p(S_A) + p(S_B) \rightarrow Q X$

$$\begin{aligned} \frac{d\sigma[Q]}{dy d^2\mathbf{q}_T} = & F_{UU}^Q + F_{UL}^Q S_{BL} + F_{LU}^Q S_{AL} + F_{UT}^{Q,\sin\phi_{S_B}} |\mathbf{S}_{BT}| \sin\phi_{S_B} + F_{TU}^{Q,\sin\phi_{S_A}} |\mathbf{S}_{AT}| \sin\phi_{S_A} \\ & + F_{LL}^Q S_{AL} S_{BL} + F_{LT}^{Q,\cos\phi_{S_B}} S_{AL} |\mathbf{S}_{BT}| \cos\phi_{S_B} + F_{TL}^{Q,\cos\phi_{S_A}} |\mathbf{S}_{AT}| S_{BL} \cos\phi_{S_A} \\ & + |\mathbf{S}_{AT}| |\mathbf{S}_{BT}| \left[F_{TT}^{Q,\cos(\phi_{S_A}-\phi_{S_B})} \cos(\phi_{S_A} - \phi_{S_B}) + F_{TT}^{Q,\cos(\phi_{S_A}+\phi_{S_B})} \cos(\phi_{S_A} + \phi_{S_B}) \right] \end{aligned}$$

Kato, Maxia, CP, PRD 110 (2024)

Single spin asymmetries for different quarkonia are sensitive to different TMDs

$$\begin{aligned} F_{UT}^{\eta_Q,\sin\phi_{S_B}} & \propto -f_1^g \otimes f_{1T}^{\perp g} + h_1^{\perp g} \otimes h_1^g - h_1^{\perp g} \otimes h_{1T}^{\perp g} \\ F_{UT}^{\chi_{Q0},\sin\phi_{S_B}} & \propto -f_1^g \otimes f_{1T}^{\perp g} - h_1^{\perp g} \otimes h_1^g + h_1^{\perp g} \otimes h_{1T}^{\perp g} \\ F_{UT}^{\chi_{Q2},\sin\phi_{S_B}} & \propto -f_1^g \otimes f_{1T}^{\perp g} \end{aligned}$$

Such observables are in principle measurable at the planned LHCspin experiment

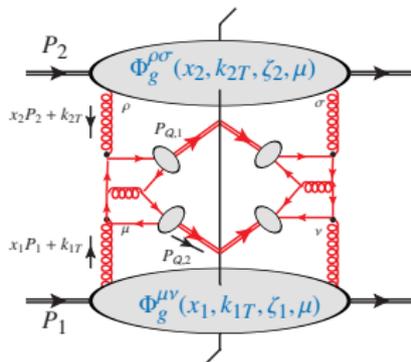
J/ψ -pair production at the LHC

J/ψ 's are relatively easy to detect. Accessible at the LHC: already studied by LHCb, CMS & ATLAS

LHCb PLB 707 (2012)
 CMS JHEP 1409 (2014)
 ATLAS EPJC 77 (2017)

gg fusion dominant, negligible $q\bar{q}$ contributions even at fixed target energies

Lansberg, Shao, NPB 900 (2015)

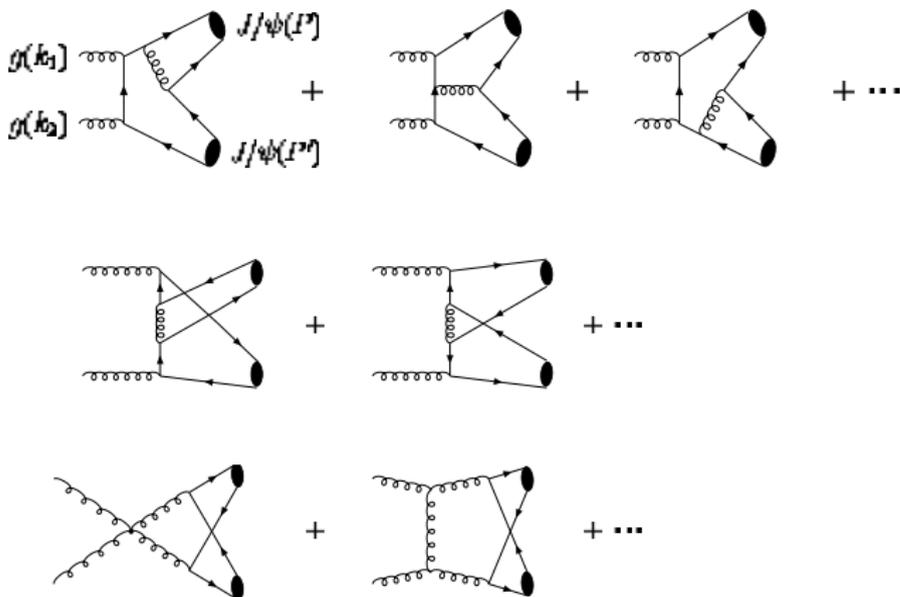


No final state gluon needed for the Born contribution in the Color Singlet Model. Pure colorless final state, hence simple color structure because one has only ISI

Lansberg, Shao, PRL 111 (2013)

Negligible Color Octet contributions, in particular at low $P_T^{\psi\psi}$

At LO pQCD in the Color Singlet Model, one needs to consider 36 diagrams



$$\frac{d\sigma}{dQdYd^2q_Td\Omega} \approx A f_1^g \otimes f_1^g + B f_1^g \otimes h_1^{\perp g} \cos(2\phi_{CS}) + C h_1^{\perp g} \otimes h_1^{\perp g} \cos(4\phi_{CS})$$

Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)

- ▶ valid up to corrections $\mathcal{O}(q_T/Q)$
- ▶ Y : rapidity of the J/ψ -pair, along the beam in the hadronic c.m. frame
- ▶ $d\Omega = d\cos\theta_{CS} d\phi_{CS}$: solid angle for J/ψ -pair in the Collins-Soper frame

Analysis similar to the one for $pp \rightarrow \gamma\gamma X$, $pp \rightarrow J\psi \gamma^{(*)} X$, $pp \rightarrow H \text{jet} X$

Qiu, Schlegel, Vogelsang, PRL 107 (2011)

den Dunnen, Lansberg, CP, Schlegel, PRL 112 (2014)

Lansberg, CP, Schlegel, NPB 920 (2017)

Boer, CP, PRD 91 (2015)

The three contributions can be disentangled by defining the transverse moments

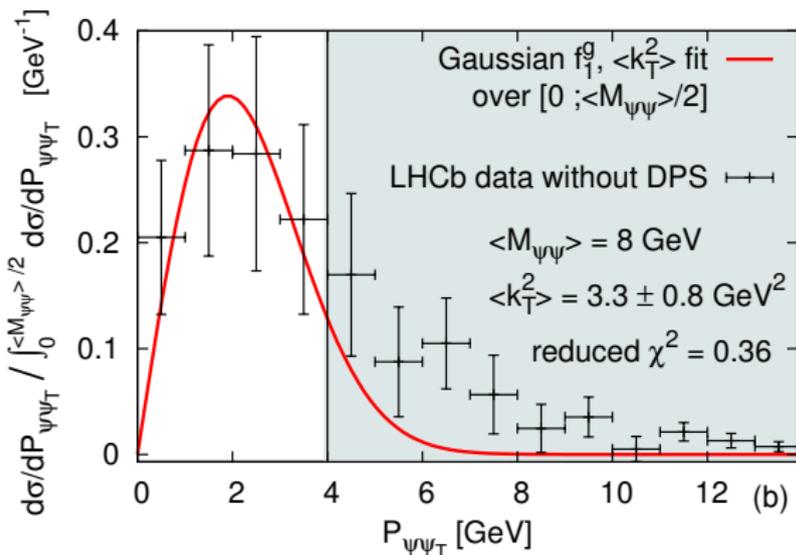
$$\langle \cos n\phi_{CS} \rangle \equiv \frac{\int_0^{2\pi} d\phi_{CS} \cos(n\phi_{CS}) \frac{d\sigma}{dQdYd^2q_Td\Omega}}{\int_0^{2\pi} d\phi_{CS} \frac{d\sigma}{dQdYd^2q_Td\Omega}} \quad (n = 2, 4)$$

$$\int d\phi_{CS} d\sigma \implies f_1^g \otimes f_1^g$$

$$\langle \cos 2\phi_{CS} \rangle \implies f_1^g \otimes h_1^{\perp g}$$

$$\langle \cos 4\phi_{CS} \rangle \implies h_1^{\perp g} \otimes h_1^{\perp g}$$

We consider $q_T = P_T^{\psi\psi} \leq M_{\psi\psi}/2$ in order to have two different scales

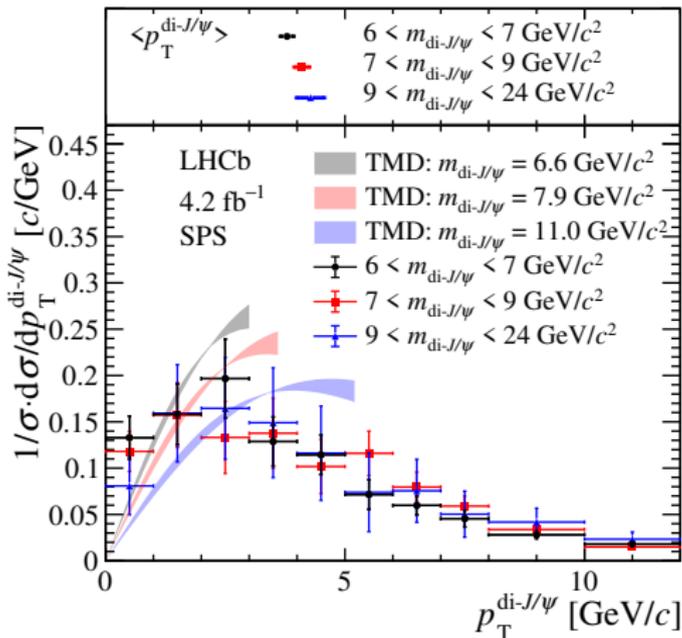


Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., JHEP 06 (2017)

Gaussian model:

$$f_1^g(x, k_T^2) = \frac{f_1^g(x)}{\pi \langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right)$$

No obvious broadening can be seen due to the large uncertainties



Lansberg, CP, Scarpa, Schlegel, PLB 784 (2018)
LHCb Coll., 2311.14085

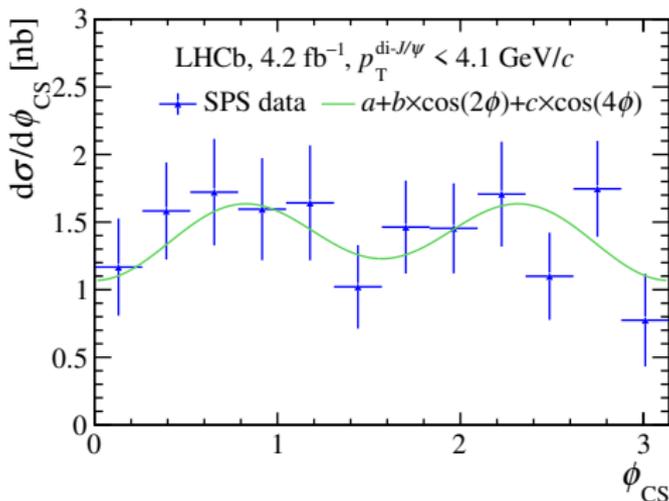
The average values of the p_T distributions slightly increase with mass

$$\langle \cos 2\phi \rangle = -0.029 \pm 0.050 \text{ (stat)} \pm 0.009 \text{ (syst)}$$

$$\langle \cos 4\phi \rangle = -0.087 \pm 0.052 \text{ (stat)} \pm 0.013 \text{ (syst)}$$

Theoretical predictions consistent with measurements

Scarpa, Boer, Echevarria, Lansberg, CP, Schlegel EPJC 80 (2020)



LHCb Coll., 2311.14085

The results are consistent with zero, but the presence of an azimuthal asymmetry at a few percent level is allowed

Heavy quark pair production at the EIC

$e p \rightarrow e J/\psi X$ (with the inclusion of TMD shape functions)

Mukherjee, Rajesh, EPJ.C 77 (2017)

Kishore, Mukherjee, PRD 99 (2019)

Bacchetta, Boer, CP, Taels, EPJ.C 80 (2020)

Boer, Bor, Maxia, CP, Yuan, JHEP 08 (2023)

$e p \rightarrow e J/\psi \text{jet} X$

D'Alesio, Murgia, CP, Taels, PRD 100 (2019)

Kishore, Mukherjee, Pawar, Siddiqah, PRD 106 (2022)

Maxia, Yuan, PRD 110 (2024)

R. Kishore, A. Mukherjee, A. Pawar, S. Rajesh, M. Siddiqah, PRD 111 (2025)

$e p \rightarrow e J/\psi \gamma X$

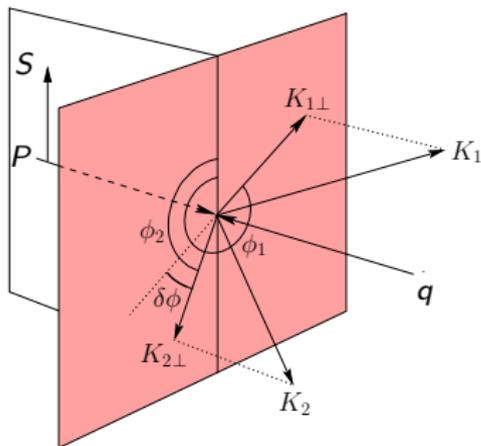
Chakrabarti, Kishore, Mukherjee, Rajesh, PRD 107 (2023)

$e p \rightarrow e D \text{jet} X$

Banu, Mukherjee, Pawar, Rajesh, PRD 108 (2023)

Gluon TMDs probed directly in $e(\ell) + p(P, S) \rightarrow e(\ell') + Q(K_1) + \bar{Q}(K_2) + X$
Boer, Mulders, CP, Zhou, JHEP 1608 (2016)

- ▶ the $Q\bar{Q}$ pair is almost back to back in the plane \perp to q and P
- ▶ $q \equiv \ell - \ell'$: four-momentum of the exchanged virtual photon γ^*



$$q_T \equiv K_{1\perp} + K_{2\perp}$$

$$K_{\perp} \equiv (K_{1\perp} - K_{2\perp})/2$$

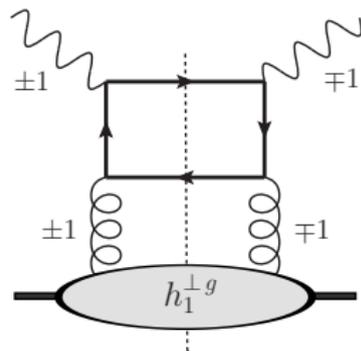
\Rightarrow Correlation limit: $|q_T| \ll |K_{\perp}|$, $|K_{\perp}| \approx |K_{1\perp}| \approx |K_{2\perp}|$

Heavy quark pair production in DIS

Angular structure of the cross section

$\phi_T, \phi_\perp, \phi_S$ azimuthal angles of q_T, K_\perp, S_T

At LO in pQCD: only $\gamma^* g \rightarrow Q\bar{Q}$ contributes



$$d\sigma(\phi_S, \phi_T, \phi_\perp) = d\sigma^U(\phi_T, \phi_\perp) + d\sigma^T(\phi_S, \phi_T, \phi_\perp)$$

Angular structure of the unpolarized cross section for $ep \rightarrow e' Q\bar{Q}X$, $|q_T| \ll |K_\perp|$

$$\frac{d\sigma^U}{d^2q_T d^2K_\perp} \propto \left\{ A_0^U + A_1^U \cos \phi_\perp + A_2^U \cos 2\phi_\perp \right\} f_1^g(x, q_T^2) + \frac{q_T^2}{M_p^2} h_1^{\perp g}(x, q_T^2) \\ \times \left\{ B_0^U \cos 2\phi_T + B_1^U \cos(2\phi_T - \phi_\perp) + B_2^U \cos 2(\phi_T - \phi_\perp) + B_3^U \cos(2\phi_T - 3\phi_\perp) + B_4^U \cos 2(\phi_T - 2\phi_\perp) \right\}$$

The different contributions can be isolated by defining

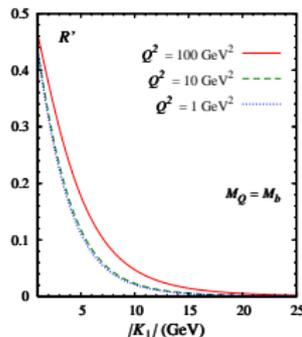
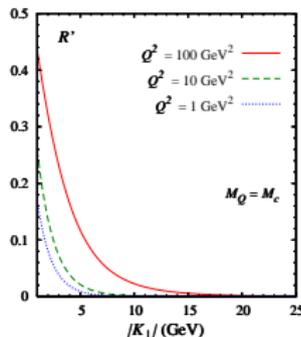
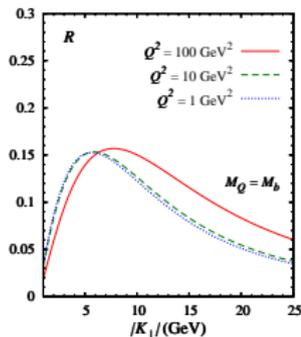
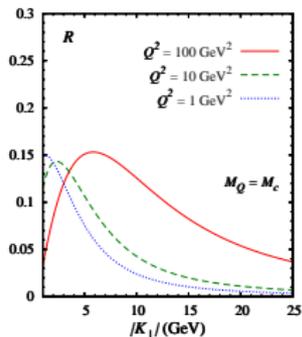
$$\langle W(\phi_\perp, \phi_T) \rangle = \frac{\int d\phi_\perp d\phi_T W(\phi_\perp, \phi_T) d\sigma}{\int d\phi_\perp d\phi_T d\sigma}, \quad W = \cos 2\phi_T, \cos 2(\phi_\perp - \phi_T), \dots$$

Positivity bound for $h_1^{\perp g}$: $|h_1^{\perp g}(x, \mathbf{p}_T^2)| \leq \frac{2M_p^2}{p_T^2} f_1^g(x, \mathbf{p}_T^2)$

It can be used to estimate maximal values of the asymmetries

Asymmetries usually larger when Q and \bar{Q} have same rapidities

Upper bounds on $R \equiv |\langle \cos 2(\phi_T - \phi_{\perp}) \rangle|$ and $R' \equiv |\langle \cos 2\phi_T \rangle|$ at $y = 0.01$



CP, Boer, Brodsky, Buffing, Mulders, JHEP 1310 (2013)
Boer, Brodsky, Mulders, CP, PRL 106 (2011)

Spin asymmetries in $ep^\uparrow \rightarrow e'Q\bar{Q}X$

Angular structure of the single polarized cross section for $ep^\uparrow \rightarrow e'Q\bar{Q}X$, $|q_T| \ll |K_\perp|$

$$\begin{aligned}
 d\sigma^T \propto & \sin(\phi_S - \phi_T) \left[A_0^T + A_1^T \cos \phi_\perp + A_2^T \cos 2\phi_\perp \right] f_{1T}^{\perp g} + \cos(\phi_S - \phi_T) \left[B_0^T \sin 2\phi_T \right. \\
 & + B_1^T \sin(2\phi_T - \phi_\perp) + B_2^T \sin 2(\phi_T - \phi_\perp) + B_3^T \sin(2\phi_T - 3\phi_\perp) + B_4^T \sin(2\phi_T - 4\phi_\perp) \left. \right] h_{1T}^{\perp g} \\
 & + \left[B_0'^T \sin(\phi_S + \phi_T) + B_1'^T \sin(\phi_S + \phi_T - \phi_\perp) + B_2'^T \sin(\phi_S + \phi_T - 2\phi_\perp) \right. \\
 & \left. + B_3'^T \sin(\phi_S + \phi_T - 3\phi_\perp) + B_4'^T \sin(\phi_S + \phi_T - 4\phi_\perp) \right] h_{1T}^g
 \end{aligned}$$

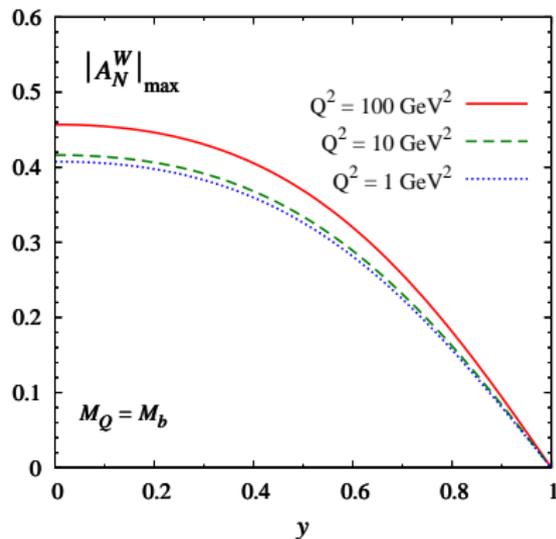
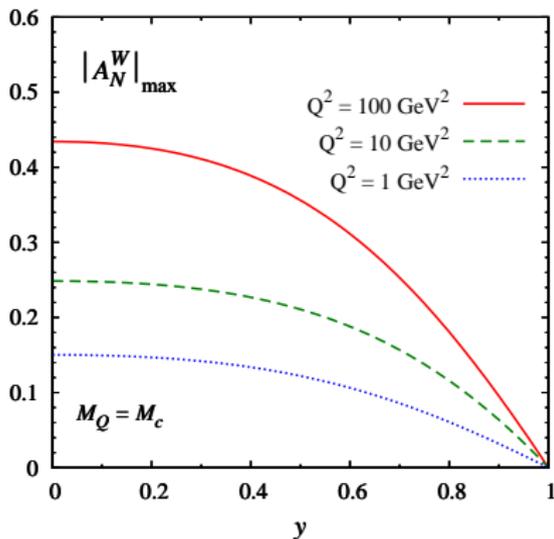
The ϕ_S dependent terms can be singled out by means of azimuthal moments A_N^W

$$A_N^{W(\phi_S, \phi_T)} \equiv 2 \frac{\int d\phi_T d\phi_\perp W(\phi_S, \phi_T) d\sigma_T(\phi_S, \phi_T, \phi_\perp)}{\int d\phi_T d\phi_\perp d\sigma_U(\phi_T, \phi_\perp)}$$

$$A_N^{\sin(\phi_S - \phi_T)} \propto \frac{f_{1T}^{\perp g}}{f_1^g} \quad A_N^{\sin(\phi_S + \phi_T)} \propto \frac{h_1^g}{f_1^g} \quad A_N^{\sin(\phi_S - 3\phi_T)} \propto \frac{h_{1T}^{\perp g}}{f_1^g}$$

Same modulations as in SIDIS for quark TMDs ($\phi_T \rightarrow \phi_h$)

Maximal values for $|A_N^W|$, $W = \sin(\phi_S + \phi_T)$, $\sin(\phi_S - 3\phi_T)$ ($|K_\perp| = 1 \text{ GeV}$)



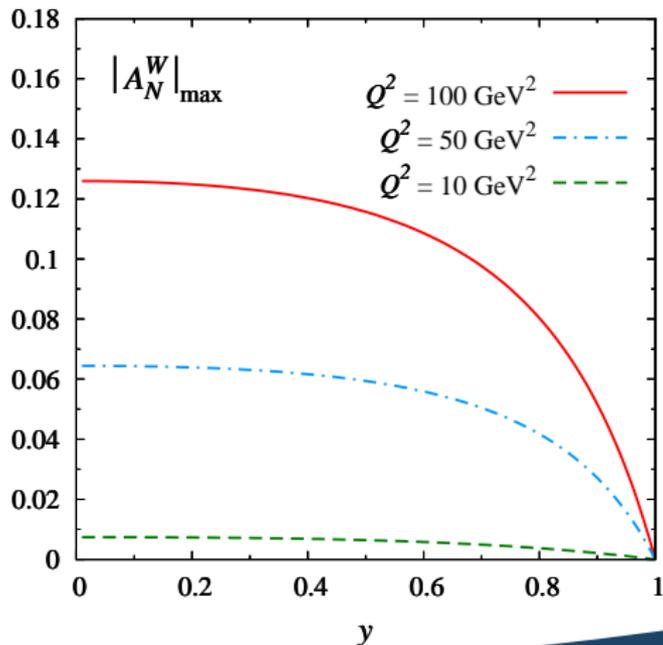
Asymmetries in $ep^\uparrow \rightarrow e'\text{jet jet } X$

Upper bounds

Contribution to the denominator also from $\gamma^* q \rightarrow gq$, negligible at small- x

Asymmetries much smaller than in $c\bar{c}$ case for $Q^2 \leq 10 \text{ GeV}^2$

Upper bounds for A_N^W for $K_\perp \geq 4 \text{ GeV}$



- ▶ TMD distributions are affected by ISI/FSI, encoded in the gauge links, which render them gauge invariant but process dependent
- ▶ Different behavior of unpolarized WW and dipole gluon TMDs, both accessible at RHIC, could be tested experimentally
- ▶ Same considerations apply to linearly polarized gluons, which affect transverse spectra of (pseudo) scalar particles, and generate azimuthal asymmetries in associated quarkonium production at the LHC
- ▶ Two distinct gluon Sivers functions can be measured in pp collisions (RHIC and AFTER@LHC); the WW-type allows for a sign-change test (EIC)
- ▶ Observables related to gluon TMDs could be part of both the *spin* and the *small-x* program at a future EIC