Tomography of the Nucleon-2

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Overlap representation of GPDS in terms of light-front wave functions

Hadron state

$$|\psi_p(P^+, \vec{P}_\perp)\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i \mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}16\pi^3} 16\pi^3 \delta\left(1-\sum_{i=1}^{n} x_i\right)\delta^{(2)}\left(\sum_{i=1}^{n}\vec{k}_{\perp i}\right) \times \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)\left|n; x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i\right\rangle.$$

$$x_i = k_i^+/P^+ \qquad \vec{p}_{\perp i} = x_i \vec{P}_\perp + \vec{k}_{\perp i}.$$

In the region $\zeta < x < 1$ Diagonal overlap

$$\frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}} H_{(n\to n)}(x,\zeta,t) - \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(n\to n)}(x,\zeta,t)$$
$$= \left(\sqrt{1-\zeta}\right)^{2-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1-\sum_{j=1}^{n} x_j\right)\delta^{(2)}\left(\sum_{j=1}^{n}\vec{k}_{\perp j}\right) \times \delta(x-x_1)\psi^{\uparrow *}_{(n)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\psi^{\uparrow}_{(n)}(x_i, \vec{k}_{\perp i}, \lambda_i),$$

In the region $-\zeta < x < \zeta$ non diagonal overlap

$$\frac{\sqrt{1-\zeta}}{1-\frac{\zeta}{2}} H_{(n+1\to n-1)}(x,\zeta,t) - \frac{\zeta^2}{4(1-\frac{\zeta}{2})\sqrt{1-\zeta}} E_{(n+1\to n-1)}(x,\zeta,t)$$
$$= \left(\sqrt{1-\zeta}\right)^{3-n} \sum_{n,\lambda_i} \int \prod_{i=1}^{n+1} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{16\pi^3} 16\pi^3 \delta\left(1-\sum_{j=1}^{n+1} x_j\right)\delta^{(2)}\left(\sum_{j=1}^{n+1}\vec{k}_{\perp j}\right) \times 16\pi^3 \delta(x_{n+1}+x_1-\zeta)\delta^{(2)}(\vec{k}_{\perp n+1}+\vec{k}_{\perp 1}-\vec{\Delta}_\perp) \times \delta(x-x_1)\psi^{\uparrow *}_{(n-1)}(x'_i, \vec{k}'_{\perp i}, \lambda_i)\psi^{\uparrow}_{(n+1)}(x_i, \vec{k}_{\perp i}, \lambda_i)\delta_{\lambda_1 -\lambda_{n+1}},$$

$$P = \left(P^+, \vec{0}_\perp, \frac{M^2}{P^+}\right),$$
$$P' = \left((1-\zeta)P^+, -\vec{\Delta}_\perp, \frac{M^2+\vec{\Delta}_\perp^2}{(1-\zeta)P^+}\right),$$
$$\Delta = P - P' = \left(\zeta P^+, \vec{\Delta}_\perp, \frac{t+\vec{\Delta}_\perp^2}{\zeta P^+}\right),$$

Brodsky, Diehl, Hwang, Nucl. Phy. B (2001)

Gravitational Form Factors for the Nucleon

$$\langle P', S'|T_i^{\mu\nu}(0)|P, S\rangle = \bar{U}(P', S')\Big[-B_i(q^2)\frac{\bar{P}^\mu \bar{P}^\nu}{M} + (A_i(q^2) + B_i(q^2))\frac{1}{2}(\gamma^\mu \bar{P}^\nu + \gamma^\nu \bar{P}^\mu)$$
$$+ C_i(q^2)\frac{q^\mu q^\nu - q^2 g^{\mu\nu}}{M} + \bar{C}_i(q^2) M g^{\mu\nu}\Big] U(P, S),$$

$$\bar{P}^\mu = \tfrac{1}{2}(P' + P)^\mu,\ q^\mu = (P' - P)^\mu$$

$$P = (P^+, P_\perp, P^-) = \left(P^+, 0, \frac{M^2}{P^+}\right),$$
$$P' = (P'^+, P'_\perp, P'^-) = \left(P^+, q_\perp, \frac{q_\perp^2 + M^2}{P^+}\right)$$
$$q = P' - P = \left(0, q_\perp, \frac{q_\perp^2}{P^+}\right),$$

We choose Drell-Yan frame $Q^2 = -q^2 = \vec{q}_\perp^{\,2}$

GFFs give how matter couples to gravity

$A(Q^2)$ and $B(Q^2)$ are related to the mass and angular momentum of the proton

Gravitational Form Factors

The GPDs H_q and E_q are related to the GFFs A and B, and to the angular momentum carried by the quarks

$$\int dx x[H_q(x,0,0) + E_q(x,0,0)] = A_q(0) + B_q(0) = 2J_q$$

X. Ji, PRD, 1997

Similarly, for the gluon counterpart.

$$\sum_{a=q,G} A_a(0) = 1, \quad \sum_{a=q,G} B_a(0) = 0, \quad \sum_{a=q,G} \bar{C}_a(t) = 0,$$

Follow from Poincare invariance

$\bar{C}(Q^2)$ arises due to non-conservation of EM tensor separately for quarks and gluons, and must vanish when summed over both

Lorce, Moutarde, Trawinski, EPJC (2019)

However, $C(Q^2)$, also called the D-term, is not related to any Poincare generator and is unconstrained

Energy and pressure distributions

GFF C is related to the pressure and shear force distributions inside the nucleon

Polyakov and Schweitzer, IJMPA (2018)

Breit frame (nucleon rest frame **P**=0) is a popular choice for analysis of form factors

3D Fourier transform of the FF can be given as $$\mathcal{F}_a(r) = \int \frac{d^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\, F_a(t) \qquad t = -\boldsymbol{\Delta}^2.$$

Energy density

Isotropic pressure

EMT can be written as

$$\Theta^{\mu\nu}(\boldsymbol{r}) = [\varepsilon(r) + p(r)]\,u^\mu u^\nu - p(r)\eta^{\mu\nu} + s(r)\left(\chi^\mu\chi^\nu - \frac{1}{3}h^{\mu\nu}\right)$$

Pressure anisotropy

$$x^\mu = (0, r)$$

$$u^\mu \qquad \chi^\mu = x^\mu / r$$

Unit timelike and spacelike 4 vectors orthogonal to each other

$$h^{\mu\nu} = u^\mu u^\nu - \eta^{\mu\nu}$$

Pressure distributions

Isotropic pressure and pressure anisotropy are related to radial and tangential pressure

$$p(r) = \frac{p_r(r) + 2\, p_t(r)}{3}, \quad s(r) = p_r(r) - p_t(r).$$

These can be expressed in terms of the GFFs

$$\varepsilon_a(r) = M\left\{\mathcal{A}_a(r) + \bar{\mathcal{C}}_a(r) + \frac{1}{4M^2}\frac{1}{r^2} \times \frac{\mathrm{d}}{\mathrm{d}r}\left(r^2 \frac{\mathrm{d}}{\mathrm{d}r}[\mathcal{B}_a(r) - 4\mathcal{C}_a(r)]\right)\right\},$$

$$p_{r,a}(r) = M\left\{-\bar{\mathcal{C}}_a(r) + \frac{1}{M^2}\frac{2}{r}\frac{\mathrm{d}\mathcal{C}_a(r)}{\mathrm{d}r}\right\},$$

$$p_{t,a}(r) = M\left\{-\bar{\mathcal{C}}_a(r) + \frac{1}{M^2}\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\mathcal{C}_a(r)}{\mathrm{d}r}\right)\right\},$$

$$p_a(r) = M\left\{-\bar{\mathcal{C}}_a(r) + \frac{2}{3}\frac{1}{M^2}\frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}\mathcal{C}_a(r)}{\mathrm{d}r}\right)\right\},$$

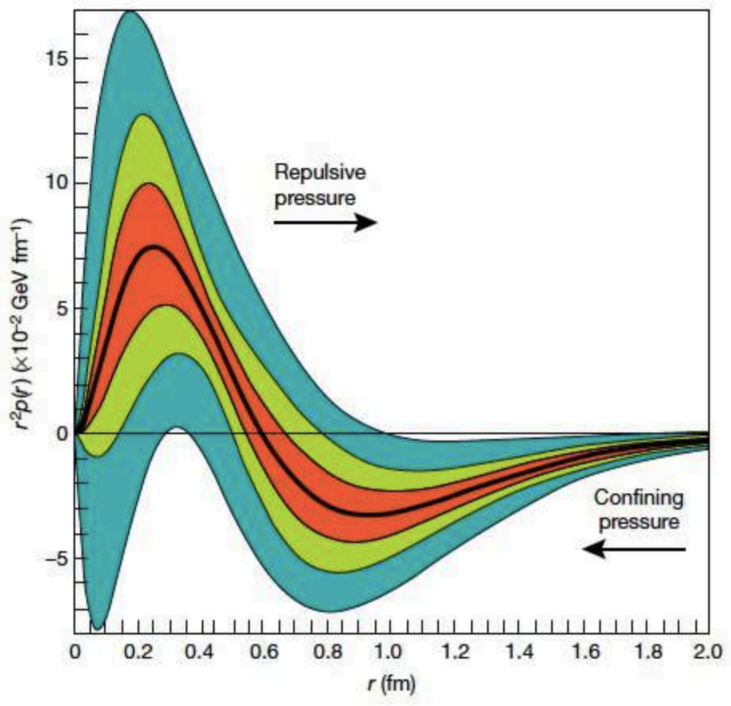
$$s_a(r) = M\left\{-\frac{1}{M^2}\, r\, \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{1}{r}\frac{\mathrm{d}\mathcal{C}_a(r)}{\mathrm{d}r}\right)\right\},$$

Where $$\mathcal{F}_a(r) = \int \frac{\mathrm{d}^3\boldsymbol{\Delta}}{(2\pi)^3}\, e^{-i\boldsymbol{\Delta}\cdot\boldsymbol{r}}\, F_a(t)$$

Average squared mass radius is defined as

$$R_M^2 = \frac{1}{M}\int \mathrm{d}^3\boldsymbol{r}\, r^2\, \varepsilon(r) = 6\left[\left.\frac{\mathrm{d}A(t)}{\mathrm{d}t}\right|_{t=0} - \frac{1}{M^2}C(0)\right].$$

Pressure distribution inside the nucleon



Pressure distribution obtained from fits to Jlab data to extract the GPDs, in particular the D-term

Pressure distribution is repulsive at the center of the nucleon and confining in the outer region

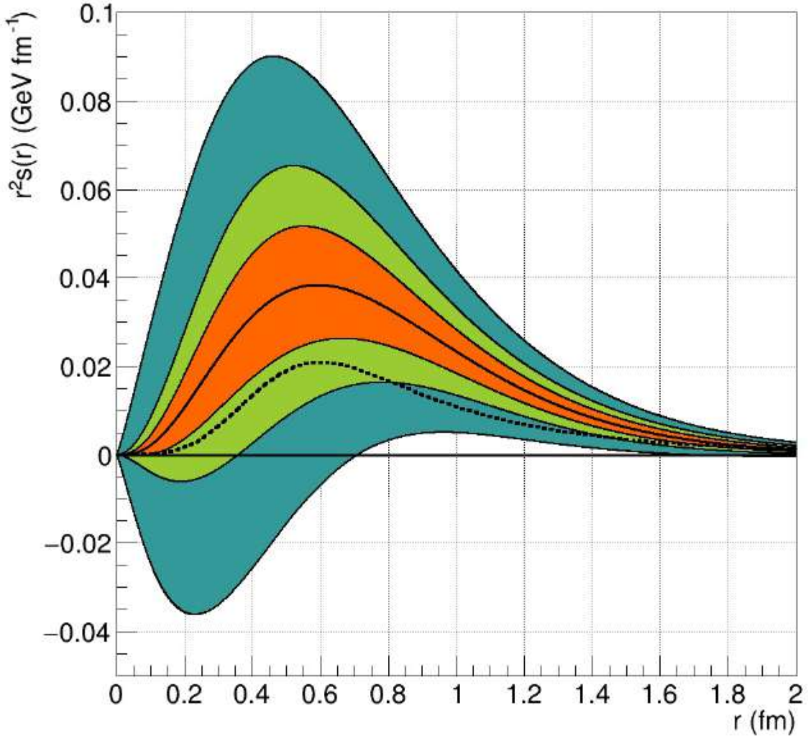
At the core it exceeds the pressure density of the most dense object that is neutron star , average peak pressure 10^{35} Pascals

This also connects a set of collider observables (GPDs) to the investigation of the equation of state (EoS) of neutron stars

Burkert, Elouadrhiri, Girod, Nature(2018)

Rajan, Gorda, Liuti, Yagi (2018)

Shear Distribution Inside the Nucleon



Shear (tangential) force inside the nucleon from DVCS data at JLab

Maximum shear force at 0.6 fm from the center of the nucleon : confinement may be dominant

Shear forces change direction at r=0.45 fm from the center

Burkert, Elouadrhiri, Girod, 2104.02031[nucl-ex]

GFFs and Pressure Distribution

Jlab result triggered a lot of interest : theoretical model calculations of the pressure distributions

Polyakov and Schweitzer, IJMPA (2018)

Most calculations are done in the Breit frame and are subject to relativistic corrections

2-D distributions in the infinite momentum frame or light-front formalism introduced in

Lorce, Moutarde, Trawinski, EPJC (2019), Freese and Miller, PRD(2021)

Because of transverse Galilean symmetry on the light-front these are free from relativistic corrections

Connection between 2D and 3D pressure distributions can be established through Abel transformation

Panteleeva and Polyakov, 2021

Problem with 3 D densities

R. L. Jaffe, PRD 103, 016017 (2021); A Freese and G Miller, PRD 103, 094023 (2021)

Coordinate r in the Fourier transform has to be defined wrt the location of the system

It is thus necessary to construct a localized wave packet whose center is the refrence point wrt which r is measured

The more precisely one tries to localize the system, the higher the momentum components one has to introduce in the wave packet, making the relativistic corrections larger

Thus this picture is not very accurate for a system having a size of the same order as Compton wavelength, for example the nucleon. The relativistic corrections become model dependent of the wave functions as Lorentz Boosts depend on the dynamics of the system.

One can define 2 D light front distributions instead, where the FT are taken wrt the transverse momentum transfer. Such distributions are free from relativistic corrections, as transverse boosts are Galilean or free from dynamics in light-front framework.

M. Burkardt, Int.J.Mod.Phys.A 18 (2003) 173

Lorce, Moutarde, Trawinski, EPJC 79:89 (2019)

Light-front distributions

Drell-Yan frame $\Delta^+ = 0$

2D Fourier transform of the GFFs

$$\tilde{F}(b_\perp) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot b_\perp} F(-\Delta_\perp^2)$$

Momentum transfer is in the transverse direction

Lorce, Moutarde, Trawinski, EPJC 79:89 (2019)

2D Galilean energy density

$$\mu_a(b) = M\left\{\frac{\mathsf{A}_a(b)}{2} + \bar{\mathsf{C}}_a(b) + \frac{1}{4M^2}\frac{1}{b}\frac{\mathrm{d}}{\mathrm{d}b}\left(b\frac{\mathrm{d}}{\mathrm{d}b}\left[\frac{\mathsf{B}_a(b)+\mathsf{D}_a(b)}{2} - 4\mathsf{C}_a(b)\right]\right)\right\},$$

2D pressure distributions

$$\sigma_{r,a}(b) = M\left\{-\bar{\mathsf{C}}_a(b) + \frac{1}{M^2}\frac{1}{b}\frac{\mathrm{d}\mathsf{C}_a(b)}{\mathrm{d}b}\right\},$$

$$\sigma_{t,a}(b) = M\left\{-\bar{\mathsf{C}}_a(b) + \frac{1}{M^2}\frac{\mathrm{d}^2\mathsf{C}_a(b)}{\mathrm{d}b^2}\right\},$$

$$\sigma_a(b) = M\left\{-\bar{\mathsf{C}}_a(b) + \frac{1}{2}\frac{1}{M^2}\frac{1}{b}\frac{\mathrm{d}}{\mathrm{d}b}\left(b\frac{\mathrm{d}\mathsf{C}_a(b)}{\mathrm{d}b}\right)\right\},$$

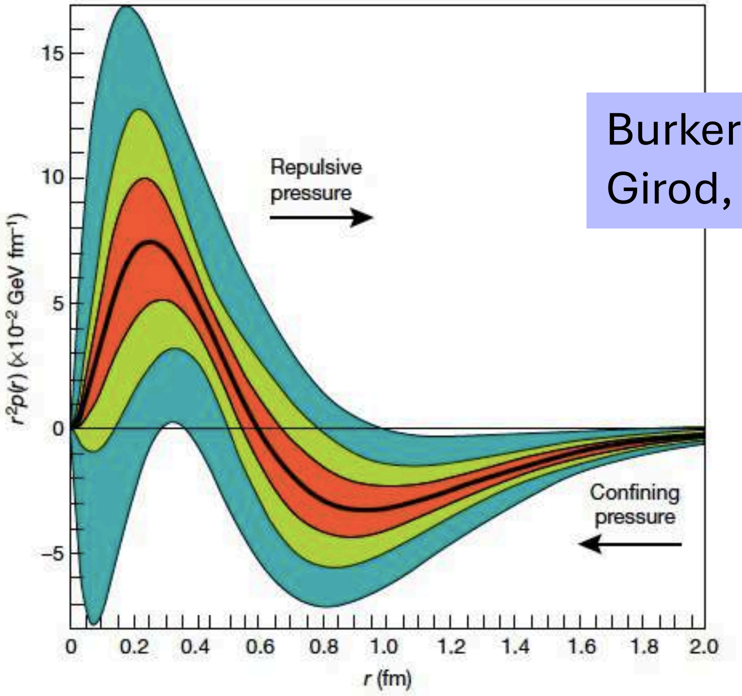
$$\Pi_a(b) = M\left\{-\frac{1}{M^2}b\frac{\mathrm{d}}{\mathrm{d}b}\left(\frac{1}{b}\frac{\mathrm{d}\mathsf{C}_a(b)}{\mathrm{d}b}\right)\right\}.$$

Pressure anisotropy

Pressure distributions

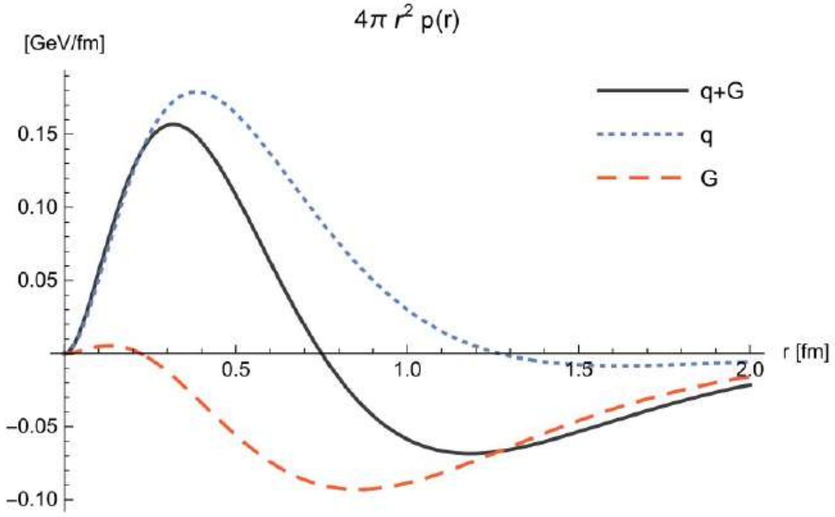
$$\int_0^\infty \mathrm{d}r\, r^2\, p(r) = 0,$$ Von Laue Condition   states that isotropic pressure has to change sign

$$\int_0^\infty \mathrm{d}b\, b\, \sigma(b) = 0,$$ 2 D version of Von Laue condition



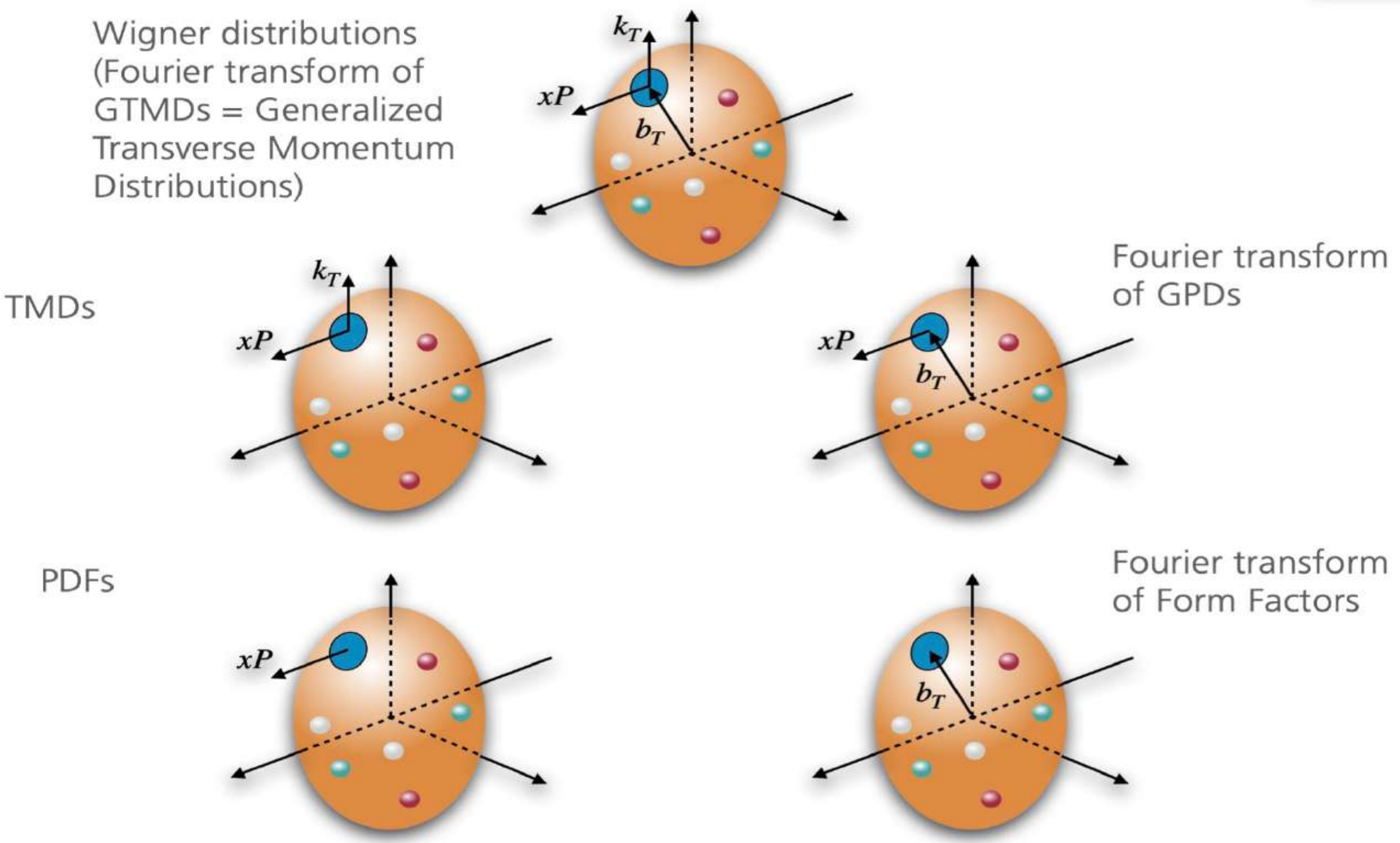
Burkert, Elouadrhiri, Girod, Nature(2018)

Multipole model



Lorce, Moutarde, Trawinski, EPJC 79:89 (2019)

Hadron Structure in three dimensions



TMDs : Lectures by C Pisano

Lorce, Pasquini, Vanderhaeghen, JHEP 1105 (11)

Generalized parton correlation functions

GPCFs are the most general correlation functions in the hadron-often called the ‘mother distributions’

Both GPDs an TMDs can be obtained from the GPCFs-rich in information about the internal structure of the hadron

Integrating the GPCFs over the minus component of momentum, one gets the correlator that is parametrized in terms of GTMDs (generalized transverse momentum dependent parton distributions)

$$
\begin{aligned}
W^{[\Gamma]}_{\lambda\lambda'}(P,x,\vec{k}_T,\Delta,N;\eta) &= \int dk^-\, W^{[\Gamma]}_{\lambda\lambda'}(P,k,\Delta,N;\eta) \\
&= \frac{1}{2}\int \frac{dz^-\, d^2\vec{z}_T}{(2\pi)^3}\, e^{ik\cdot z}\, \langle p',\lambda'|\, \bar{\psi}\Big(-\frac{1}{2}z\Big)\, \Gamma\, \mathcal{W}\Big(-\frac{1}{2}z, \frac{1}{2}z\,|\,n\Big)\, \psi\Big(\frac{1}{2}z\Big)\, |p,\lambda\rangle \Big|_{z^+=0} .
\end{aligned}
$$

Gauge link

Required for color gauge invariance

Meissner, Metz, Schlegel, JHEP 08, (2009) 056

Twist Two GTMDs

$$W_{\lambda\lambda'}^{[\gamma^+]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[F_{1,1} + \frac{i\sigma^{i+}k_T^i}{P^+}F_{1,2} + \frac{i\sigma^{i+}\Delta_T^i}{P^+}F_{1,3} + \frac{i\sigma^{ij}k_T^i\Delta_T^j}{M^2}F_{1,4}\right]u(p,\lambda)\,,$$

$$W_{\lambda\lambda'}^{[\gamma^+\gamma_5]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon_T^{ij}k_T^i\Delta_T^j}{M^2}G_{1,1} + \frac{i\sigma^{i+}\gamma_5 k_T^i}{P^+}G_{1,2} + \frac{i\sigma^{i+}\gamma_5\Delta_T^i}{P^+}G_{1,3} + i\sigma^{+-}\gamma_5\,G_{1,4}\right]u(p,\lambda)\,,$$

$$W_{\lambda\lambda'}^{[i\sigma^{j+}\gamma_5]} = \frac{1}{2M}\,\bar{u}(p',\lambda')\left[-\frac{i\varepsilon_T^{ij}k_T^i}{M}H_{1,1} - \frac{i\varepsilon_T^{ij}\Delta_T^i}{M}H_{1,2} + \frac{M\,i\sigma^{j+}\gamma_5}{P^+}H_{1,3} + \frac{k_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}H_{1,4} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5 k_T^k}{M\,P^+}H_{1,5} + \frac{\Delta_T^j\,i\sigma^{k+}\gamma_5\Delta_T^k}{M\,P^+}H_{1,6} + \frac{k_T^j\,i\sigma^{+-}\gamma_5}{M}H_{1,7} + \frac{\Delta_T^j\,i\sigma^{+-}\gamma_5}{M}H_{1,8}\right]u(p,\lambda)\,.$$

All GTMDs are functions of

$$(x, \xi, \vec{k}_T^2, \vec{k}_T\cdot\vec{\Delta}_T, \vec{\Delta}_T^2; \eta).$$

↑ gives the direction of the gauge link

After integrating over $k^-,\ k_T$

dependence on η goes out

In the limit of vanishing momentum transfer **Δ** =0 one gets TMDs and integration over k_T gives GPDs.

Wigner Distributions

A one dimensional quantum system with wave function $\psi(x)$

Wigner distribution is defined as

$$W(x,p)=\int d\eta e^{ip\eta}\psi^*(x-\eta/2)\psi(x+\eta/2),$$

If we integrate over p, we get a positive definite coordinate space density $|\psi(x)|^2$

If we integrate over x, we get a positive definite momentum space density $|\psi(p)|^2$

For arbitrary p and x Wigner distribution is not positive definite and does not have probabilistic interpretation

Can be used to calculate average of observables

$$\langle\hat{O}(x,p)\rangle=\int dxdpW(x,p)O(x,p),$$

Quantum mechanical 'phase space' distribution

E.P. Wigner, Phys. Rev. **40**, 749. 1932.

Wigner distribution for quarks

Introduce the Wigner operator

$$\hat{\mathcal{W}}_\Gamma(\vec{r},k) = \int d^4\eta e^{ik\cdot\eta}\bar{\Psi}(\vec{r}-\eta/2)\Gamma\Psi(\vec{r}+\eta/2),$$

Quark phase space position

Phase space momentum conjugate to η

Bilocal operator, need gauge link for gauge invariance

$$\Psi(\eta) = \exp\left(-ig\int_0^\infty d\lambda n\cdot A(\lambda n+\eta)\right)\psi(\eta),$$

Define reduced Wigner operator

$$W_\Gamma(\vec{r},\vec{k}) = \int \frac{dk^-}{(2\pi)^2} W_\Gamma(\vec{r},k),$$

Cannot be measured experimentally

5 D Wigner operator is defined as

$$\hat{W}^{[\Gamma]}(\vec{b}_\perp,\vec{k}_\perp,x) \equiv \frac{1}{2}\int\frac{dz^- d^2z_\perp}{(2\pi)^3} e^{i(xp^+z^- - \vec{k}_\perp\cdot\vec{z}_\perp)} \times \bar{\psi}\left(y-\frac{z}{2}\right)\Gamma\mathcal{W}\psi\left(y+\frac{z}{2}\right)\Big|_{z^+=0}$$

Belitsky, Ji, Yuan, PRD 69,074014 (2004)

Wigner distribution for quarks

$$\hat{W}^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x) \equiv \frac{1}{2}\int \frac{dz^- d^2 z_\perp}{(2\pi)^3} e^{i(xp^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)} \times \bar{\psi}\left(y - \frac{z}{2}\right)\Gamma \mathcal{W} \psi\left(y + \frac{z}{2}\right)\Big|_{z^+=0}$$

$$y^\mu = [0, 0, \vec{b}_\perp],$$

x is the average fraction of the nucleon momentum carried by active quark

$b_\perp, k_\perp$ Are not Fourier conjugate variables But do not commute, so subject to uncertainty principle

Conjugate to $\Delta_\perp = k_{f\perp} - k_{i\perp}$

Average quark momentum

Wigner distribution is the 2D Fourier transformation of the GTMD correlator introduced earlier

GTMDs are in general complex values functions, however, their 2D FT are real valued

Wigner distributions do not have probabilistic interpretation due to Heisenberg uncertainty principle, and they are not positive definite

One can obtain three dimensional probability densities from Wigner distributions.

Lorce and Pasquini, PRD 84, 014015 (2011)

Wigner distribution for quarks

$$\rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv \int \frac{d^2\Delta_\perp}{(2\pi)^2} \left\langle p^+, \frac{\vec{\Delta}_\perp}{2}, \vec{S} \middle| \hat{W}^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x) \middle| p^+, -\frac{\vec{\Delta}_\perp}{2}, \vec{S} \right\rangle.$$

Wigner operator sandwiched between nucleon states.

Integration over $b_\perp$ effectively sets $\Delta_\perp = 0$ and gives the TMD correlator

$$\int d^2 b_\perp \rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) = W^{[\Gamma]}(\vec{0}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv \Phi^{[\Gamma]}(\vec{k}_\perp, x, \vec{S}),$$

Integration over $k_\perp$ sets $z_\perp = 0$ and one gets 2 D Fourier transform of GPDs

$$\int d^2 k_\perp \rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} F^{[\Gamma]}(\vec{\Delta}_\perp, x, \vec{S})$$

These are the impact parameter dependent pdfs.

Wigner distribution for quarks

Integrating over b_y and k_x effectively sets $\Delta_y = z_x = 0$

Lorce and Pasquini, PRD 84, 014015 (2011)

One gets

$$\int db_y dk_x \rho^{[\Gamma]}(\vec{b}_\perp, \vec{k}_\perp, x, \vec{S}) \equiv \tilde{\rho}^{[\Gamma]}(b_x, k_y, x, \vec{S}).$$

Wigner distribution for quark with longitudinal polarization λ in a nucleon with longitudinal polarization Λ is given as

$$\rho_{\Lambda\lambda}(\vec{b}_\perp, \vec{k}_\perp, x) \equiv \frac{1}{2}[\rho^{[\gamma^+]}(\vec{b}_\perp, \vec{k}_\perp, x, \Lambda\vec{e}_z) + \lambda\rho^{[\gamma^+\gamma_5]}(\vec{b}_\perp, \vec{k}_\perp, x, \Lambda\vec{e}_z)].$$

Unpolarized quark in unpolarized nucleon

Longitudinally polarized quark in unpolarized nucleon

This can be decomposed as

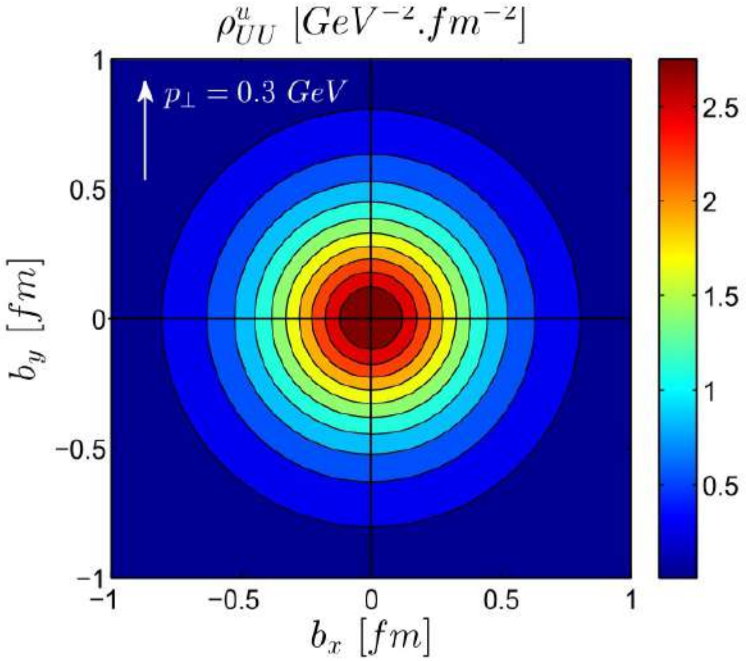
$$\rho_{\Lambda\lambda}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{2}[\rho_{UU}(\vec{b}_\perp, \vec{k}_\perp, x) + \Lambda\rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) + \lambda\rho_{UL}(\vec{b}_\perp, \vec{k}_\perp, x) + \Lambda\lambda\rho_{LL}(\vec{b}_\perp, \vec{k}_\perp, x)],$$

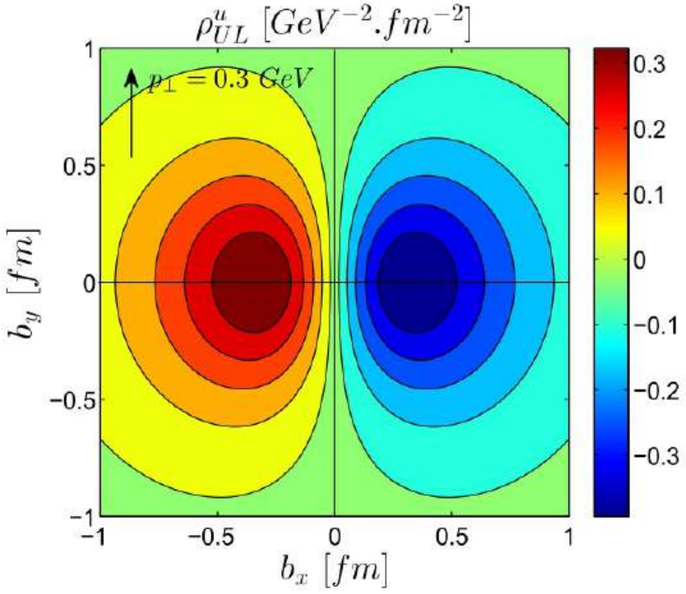
Unpol quark in long pol nucleon

Long pol quark in long pol nucleon

Model calculations of transverse Wigner distributions

(Wigner distributions integrated over x)





Wigner distributions in a spectator model (ADS-QCD)

Chakrabarti, Maji, Mondal and Mukherjee, PRD 95, 074028 (2017)

Wigner distributions and GTMDs

In terms of GTMDs we have

$$\rho_{UU}(\vec{b}_\perp, \vec{k}_\perp, x) = \mathcal{F}_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2),$$

$$\rho_{LU}(\vec{b}_\perp, \vec{k}_\perp, x) = -\frac{1}{M^2}\epsilon_\perp^{ij} k_\perp^i \frac{\partial}{\partial b_\perp^j} \times \mathcal{F}_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2),$$

$$\rho_{UL}(\vec{b}_\perp, \vec{k}_\perp, x) = \frac{1}{M^2}\epsilon_\perp^{ij} k_\perp^i \frac{\partial}{\partial b_\perp^j} \mathcal{G}_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot b_\perp, b_\perp^2),$$

$$\rho_{LL}(\vec{b}_\perp, \vec{k}_\perp, x) = \mathcal{G}_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2),$$

FT of GTMDs

$$\mathcal{X}(x, \xi, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\vec{\Delta}_\perp \cdot \vec{b}_\perp} X(x, \xi, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2).$$

Connection with TMD and GPDs

$$f_1(x, \vec{k}_\perp^2) = \int d^2 b_\perp \mathcal{F}_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = F_{1,1}(x, 0, \vec{k}_\perp^2, 0, 0),$$

$$H(x, 0, \vec{\Delta}_\perp^2) = \int d^2 k_\perp F_{1,1}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2),$$

$$g_{1L}(x, \vec{k}_\perp^2) = \int d^2 b_\perp \mathcal{G}_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{b}_\perp, \vec{b}_\perp^2) = G_{1,4}(x, 0, \vec{k}_\perp^2, 0, 0),$$

$$\tilde{H}(x, 0, \vec{\Delta}_\perp^2) = \int d^2 k_\perp G_{1,4}(x, 0, \vec{k}_\perp^2, \vec{k}_\perp \cdot \vec{\Delta}_\perp, \vec{\Delta}_\perp^2).$$

Lorce and Pasquini, PRD 84, 014015 (2011)

Quark orbital angular momentum

Ji's sum rule $$J_z^q = \frac{1}{2}\int dx x[H^q(x,0,0) + E^q(x,0,0)].$$

So the OAM is given by $$L_z^q = \frac{1}{2}\int dx\{x[H^q(x,0,0) + E^q(x,0,0)] - \tilde{H}^q(x,0,0)\}.$$

This is called kinetic quark OAM.

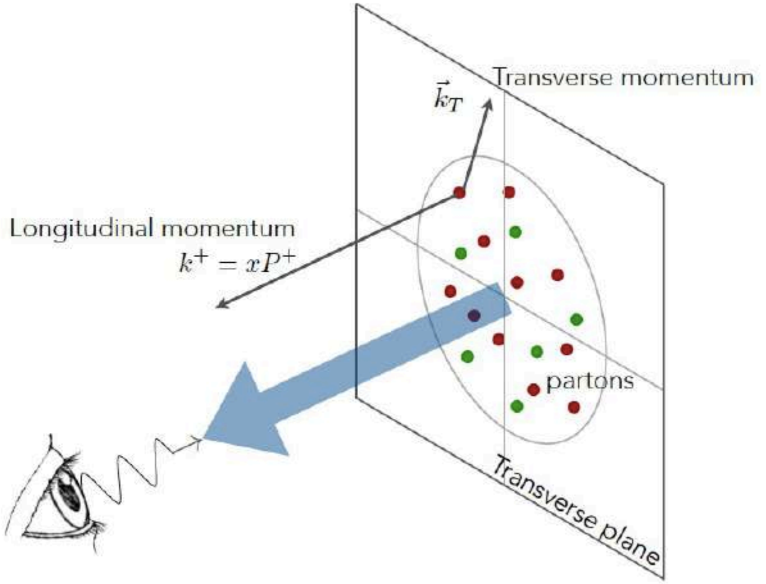
On the other hand, canonical quark OAM can be given by $$\ell_z^q = -\int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} F_{1,4}^q(x,0,\vec{k}_\perp^2,0,0).$$

OAM obtained using these two definitions agree in models with no gluonic degree of freedom

Correlation between quark spin and OAM is given by
$$C_z^q \equiv \int dx d^2k_\perp d^2b_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z \rho_{UL}^q(\vec{b}_\perp, \vec{k}_\perp, x)$$
$$= \int dx d^2k_\perp \frac{\vec{k}_\perp^2}{M^2} G_{1,1}^q(x,0,\vec{k}_\perp^2,0,0),$$

If it is greater than zero, quark spin and OAM are aligned

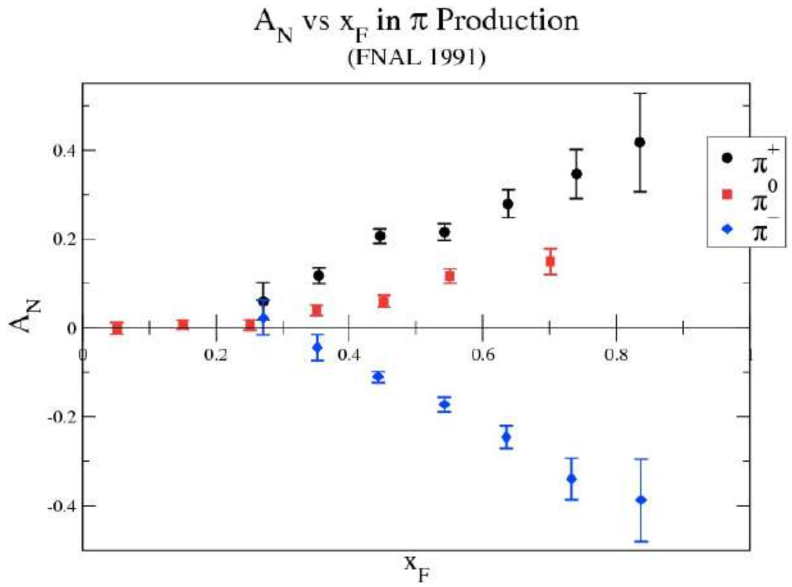
TRANSVERSE MOMENTUM DEPENDENT PARTON DISTRIBUTIONS (TMDS)



Large (30-40%) Single transverse spin asymmetries were seen at FermiLab and RHIC experiments

Such large asymmetries cannot be explained in terms of collinear leading twist pdfs : need TMDs, or twist three pdfs

$$A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}$$

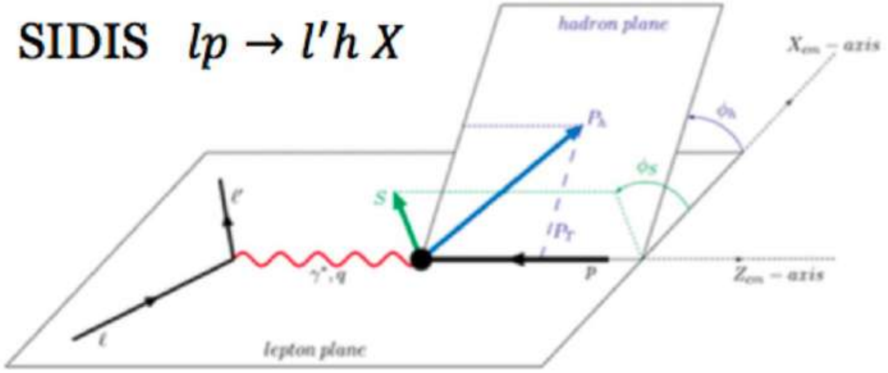


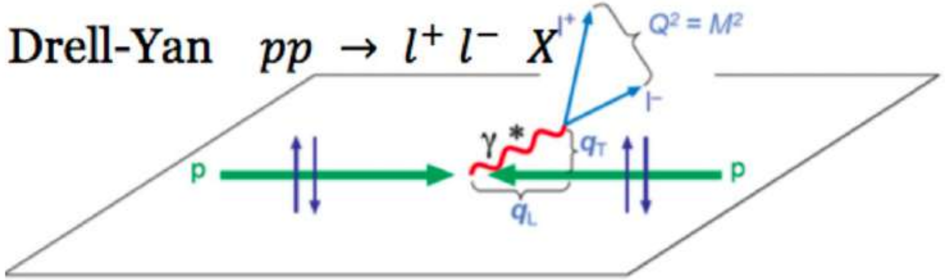
TMDs : functions of x and intrinsic transverse momentum : Gives a 3 D picture of the nucleon in momentum space

Correlations of spin, OAM and k_T : in terms of TMDs

TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

TMDs play a role in processes where two scales are present $Q^2 >> q_T^2$





For semi-inclusive DIS and Drell-Yan process, TMD factorization is proven to all orders in α_s and leading twist

Collins, Cambridge University Press (2011)
Boussarie et al, TMD handbook 2304.03302

TRANSVERSE MOMENTUM DEPENDENT PDFS (TMDS)

$$\underbrace{}_{}\mathrm{d}\sigma^{\ell p\to\ell hX} = \sum_q \underset{\text{TMD-PDFs}}{f_q(x,\boldsymbol{k}_\perp;Q^2)} \otimes \underset{\text{hard scattering}}{\mathrm{d}\hat{\sigma}^{\ell q\to\ell q}(y,\boldsymbol{k}_\perp;Q^2)} \otimes \underset{\text{TMD-FFs}}{D_q^h(z,\boldsymbol{p}_\perp;Q^2)}$$

Fragmentation function for final hadron

TMDs play an important role in single spin and azimuthal asymmetries

Process dependent due to the gauge link or Wilson line in the operator

Gauge invariant definition of Φ (not unique)

$$\Phi^{[\mathcal{U}]} \propto \left\langle P,S \left| \overline{\psi}(0)\,\mathcal{U}^{\mathcal{C}}_{[0,\xi]}\,\psi(\xi) \right| P,S \right\rangle \qquad \mathcal{U}^{\mathcal{C}}_{[0,\xi]} = \mathcal{P}\exp\left(-ig\int_{\mathcal{C}[0,\xi]} \mathrm{d}s_\mu A^\mu(s)\right)$$

Φ : quark correlator, parametrized in terms of TMDs

Gauge link : resummation of initial and/or final state gluon exchanges : process dependent

QUARK TMDS

Talk by C Pisano

QUARKS	*unpolarized*	*chiral*	*transverse*
U	f_1		$h_1^\perp$
L		g_{1L}	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	$h_{1T}, h_{1T}^\perp$

Angeles-Martinez *et al.*, Acta Phys, Pol. B46 (2015)
Mulders, Rodrigues, PRD 63 (2001)
Meissner, Metz, Goeke, PRD 76 (2007)

There are eight quark TMDs at leading twist

Only three of them survive after transverse momentum integration

Two TMDs, Sivers function and Boer-Mulders function are odd under time reversal

TMDs contribute in different azimuthal angle asymmetries

Quite a lot of advances in extracting the quark TMDs

Also gluon TMDS can be defined.

Pavia 2017, JHEP 06 (2017)
Scimemi, Vladimirov, JHEP 06 (2020)
MAP Collaboration, JHEP (2022)

Bury, Prokudin, Vladimirov, PRL 126 (2021)
Echevarria, Kang, Terry, JHEP 01 (2021)
Bacchetta, Delcarro, Pisano, Radici, CP, PLB 827 (2022)

Quark TMDs for the nucleon

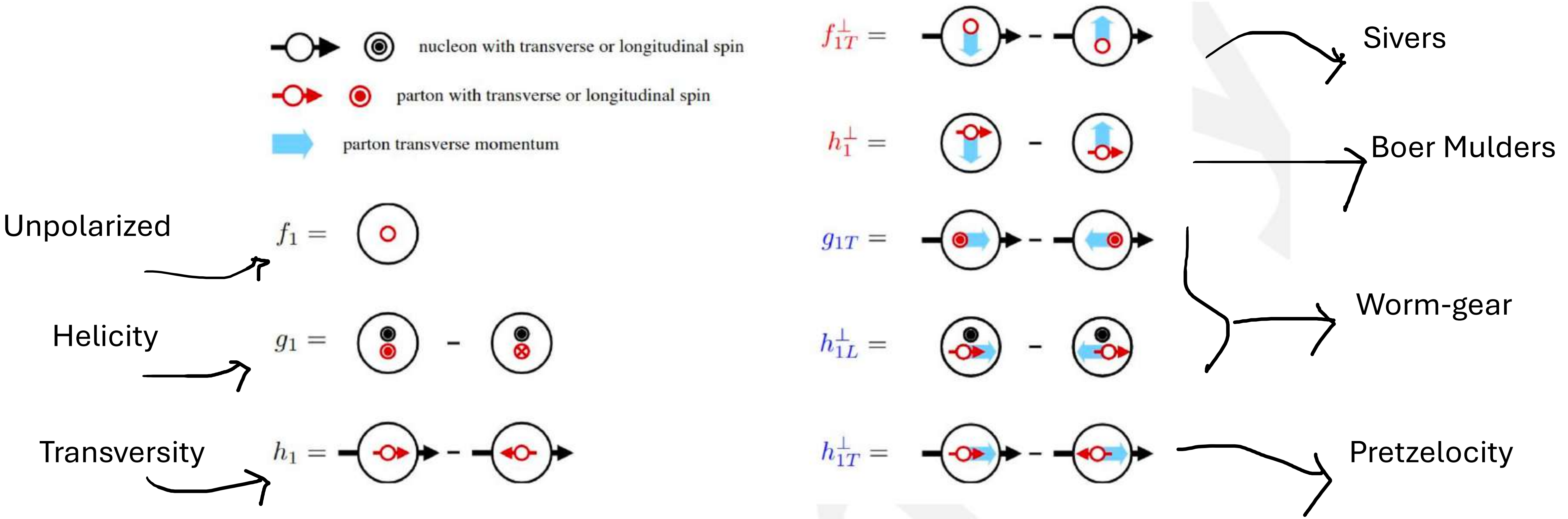


Figure 3.5: Probabilistic interpretation of twist-2 transverse-momentum-dependent distribution functions. To avoid ambiguities, it is necessary to indicate the directions of quark's transverse momemtum, target spin and quark spin, and specify that the proton is moving out of the page, or alternatively the photon is moving into the page.

Lecture notes, A. Bacchetta

SIMPLE EXAMPLE OF PROCESS DEPENDENCE OF TMDS : SIVERS EFFECT

Diff cross section for SIDIS with transversely polarized proton can be written as

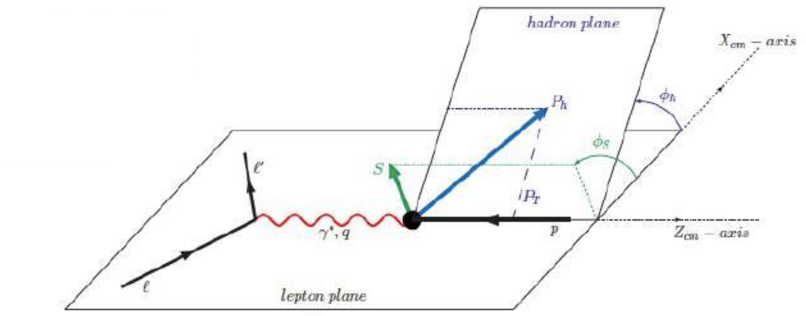
$$\frac{d\sigma^{\ell+p(S_T)\to\ell' hX}}{dx_B\, dQ^2\, dz_h\, d^2\boldsymbol{P}_T\, d\phi_S} = \frac{2\,\alpha^2}{Q^4} \times$$

$$\Big\{ \frac{1+(1-y)^2}{2} F_{UU} + (2-y)\sqrt{1-y}\,\cos\phi_h\, F_{UU}^{\cos\phi_h} + (1-y)\,\cos 2\phi_h\, F_{UU}^{\cos 2\phi_h}$$

$$+ \Big[\frac{1+(1-y)^2}{2}\,\sin(\phi_h-\phi_S)\, F_{UT}^{\sin(\phi_h-\phi_S)} + (1-y)\,\sin(\phi_h+\phi_S)\, F_{UT}^{\sin(\phi_h+\phi_S)}$$

$$+ \quad (1-y)\,\sin(3\phi_h-\phi_S)\, F_{UT}^{\sin(3\phi_h-\phi_S)}$$

$$+ \quad (2-y)\sqrt{1-y}\,\Big(\sin\phi_S\, F_{UT}^{\sin\phi_S} + \sin(2\phi_h-\phi_S)\, F_{UT}^{\sin(2\phi_h-\phi_S)}\Big)\Big]\Big\}$$



$$F_{UT}^{\sin(\phi-\phi_S)} \sim \sum e_a^2\, f_{1T}^{\perp a} \otimes D_1^a$$

Sivers Function

F functions contain different TMDs : each come with a different azimuthal modulation

PROCESS DEPENDENCE OF TMDS

Gauge link is also present in collinear pdfs : but it is possible to choose a gauge (light-front gauge) where the gauge link becomes unity.

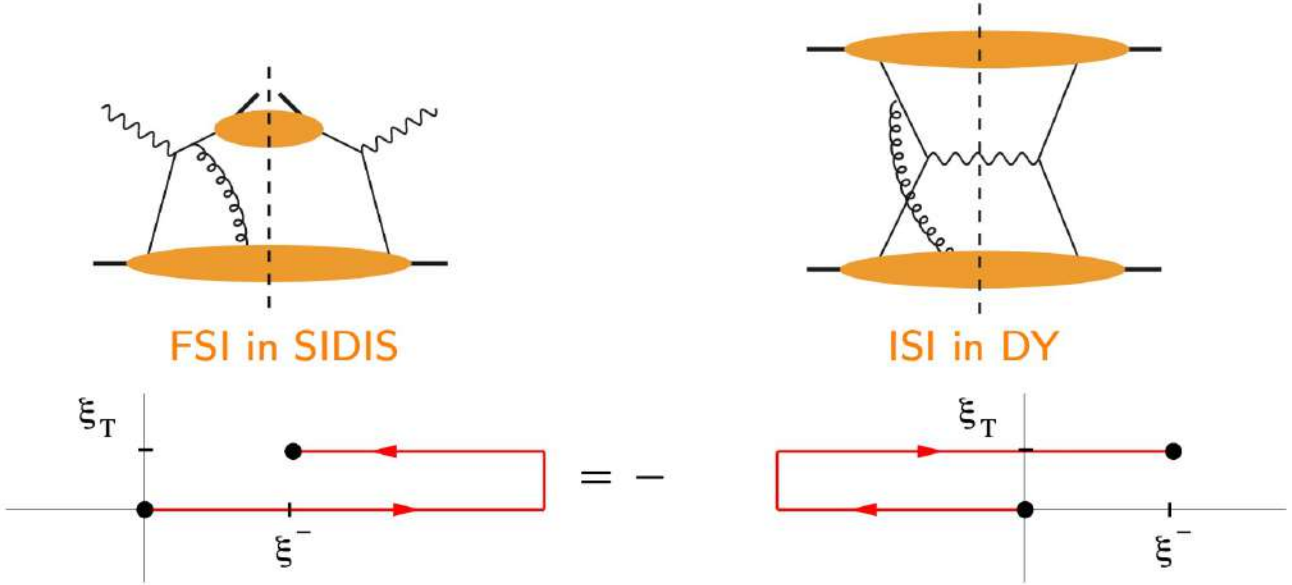
This is because the collinear pdf operator is bilocal only in the longitudinal direction but TMD operator is bilocal both in longitudinal and transverse direction

$\bar{\psi}(y^-)\Gamma\psi(0)$ $\bar{\psi}(y^-, y^\perp)\Gamma\psi(0)$ $y^- = y^0 - y^3$

For TMDs, even if one chooses the light cone gauge the effect of the gauge link remains and in fact plays a very important role for the T-odd TMDs like Sivers function.
Such TMDs would be zero if the gauge link is not taken into account

J. C. Collins, Phys. Lett. B 536 (2002) 43

SIVERS FUNCTION : PROCESS DEPENDENCE



Gauge link : depends on specific process . Example : SIDIS (final state interaction, future pointing gauge link) and Drell Yan (initial state interaction, past pointing gauge link)

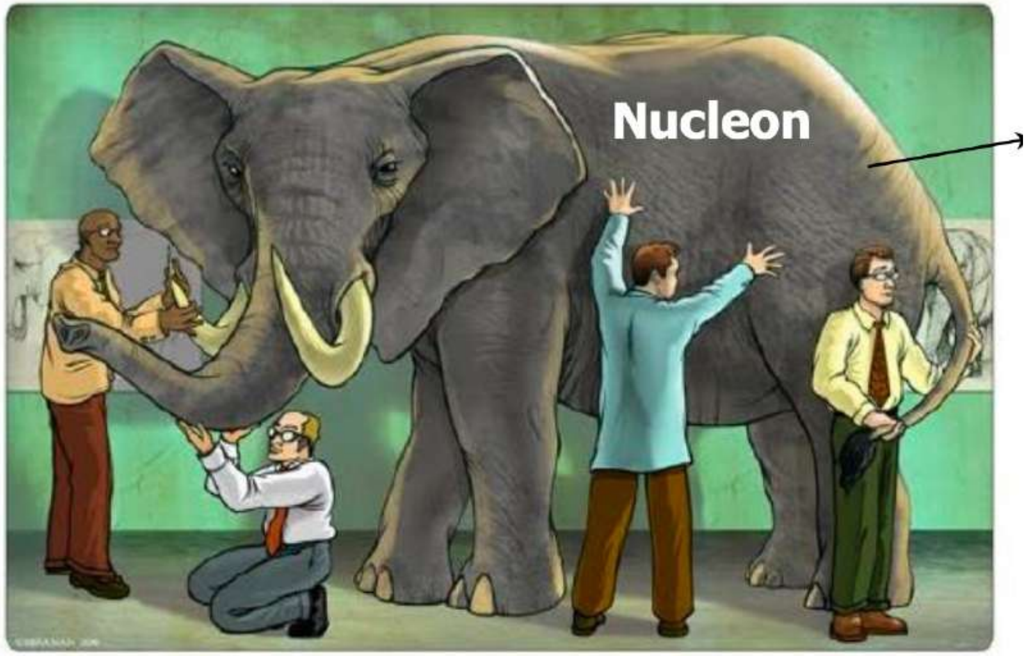
Sivers function in Drell-Yan process is same in magnitude but opposite in sign compared to the Sivers function probed in semi-inclusive DIS

$$f_{1T}^{\perp\,[DY]}(x, \boldsymbol{k}_\perp^2) = -f_{1T}^{\perp\,[SIDIS]}(x, \boldsymbol{k}_\perp^2)$$

Collins, PLB (2002); Boer, Mulders, Pijlman, Nucl. Phy. B (2003)

However, more complex processes have complex gauge link structure, and factorization is not always guaranteed

Summary and Outlook



GPDs **TMDs** **FFs** **PDFs**