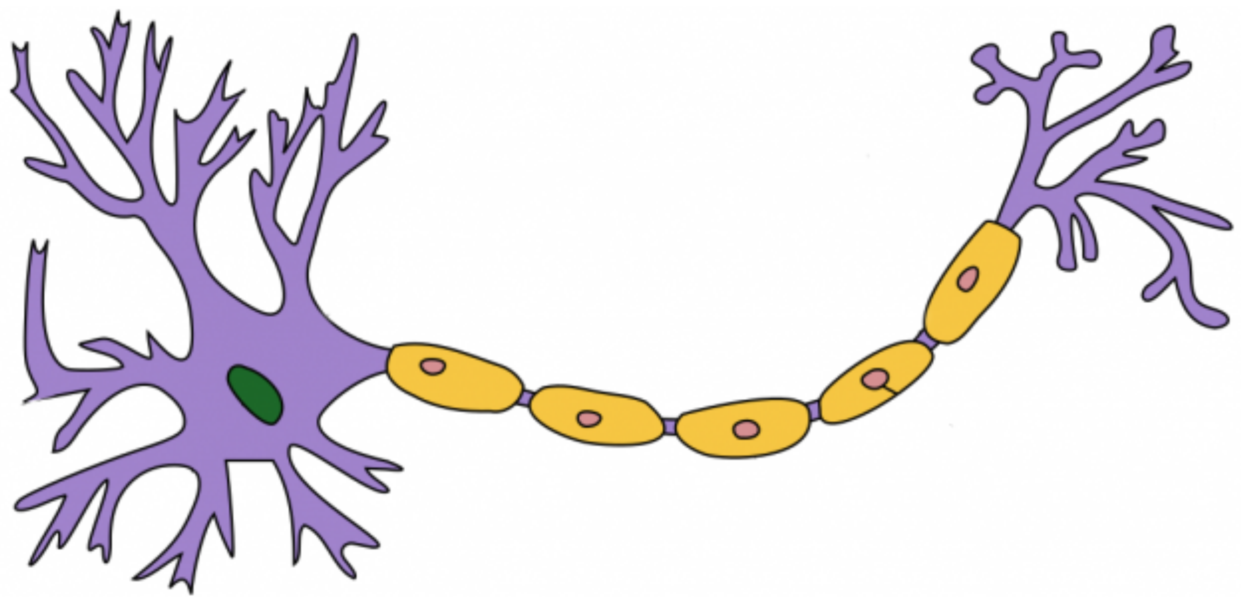
Lattice Determination of Parton Distributions Through Neural Network

Min-Huan Chu

Adam Mickiewicz University

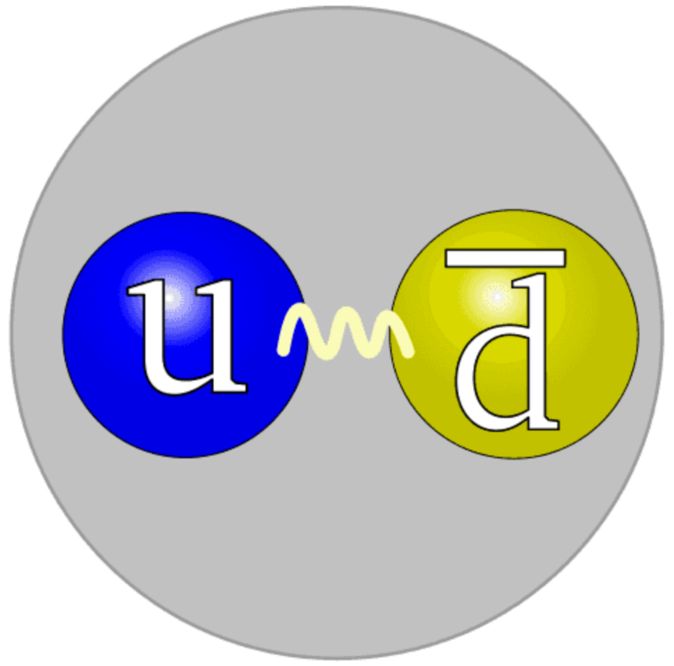
10/10/2025

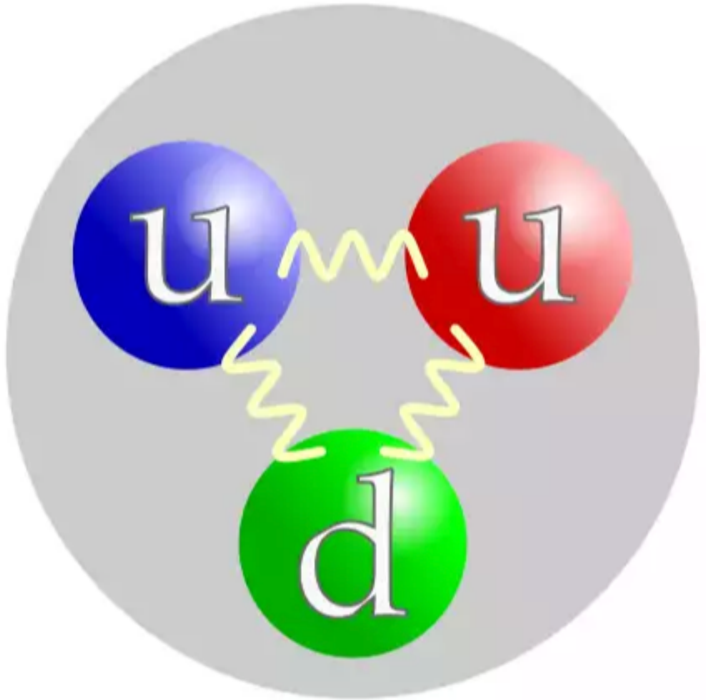


Collaborated with K. Cichy, M. Constantinou, P. Sznajder and J. Wagner

Outline

- Motivation
- neuron network reconstruction
- numerical results of PDFs
- numerical results of GPDs at $\xi = 0$
- summary and outlook





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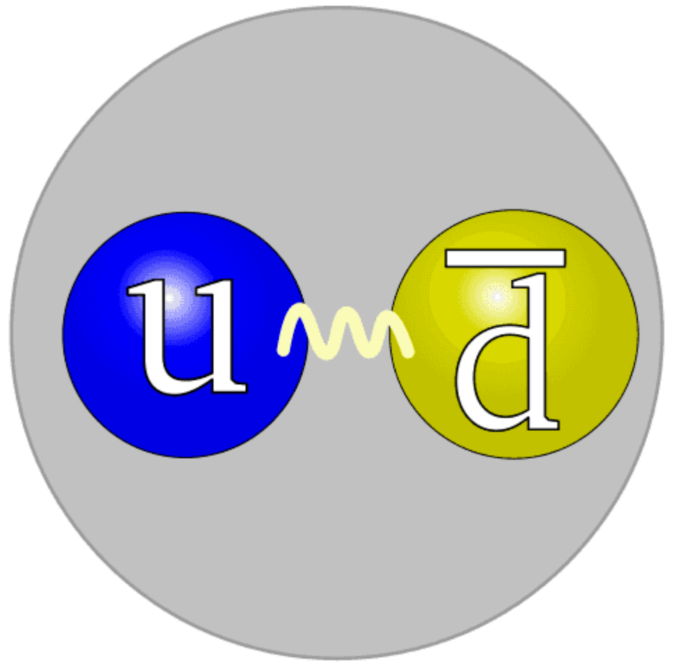
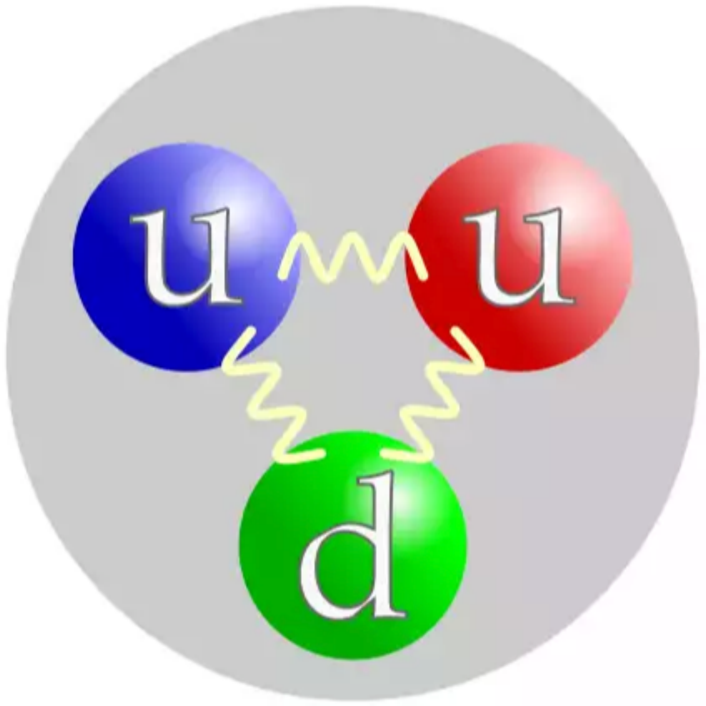
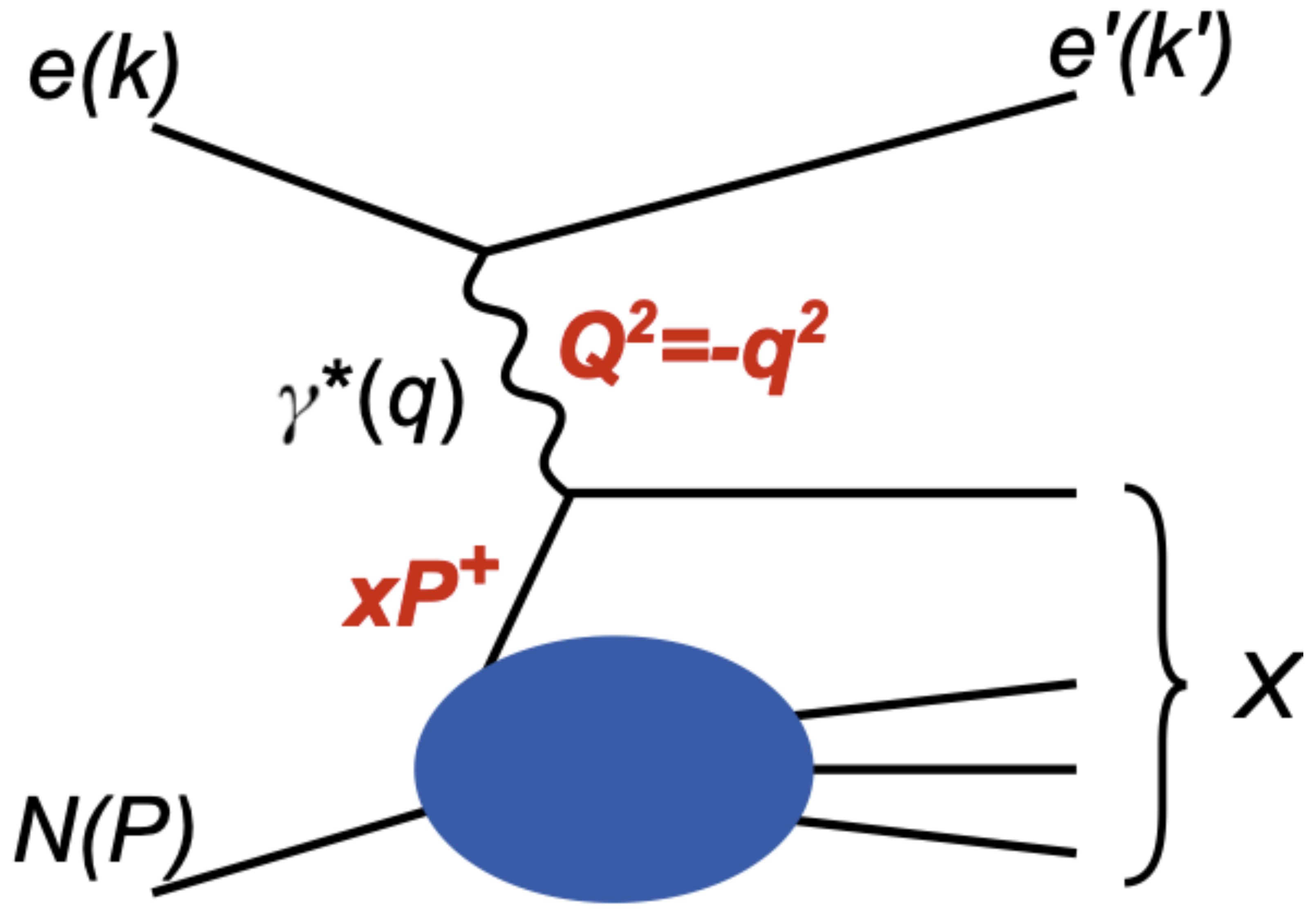




Motivation

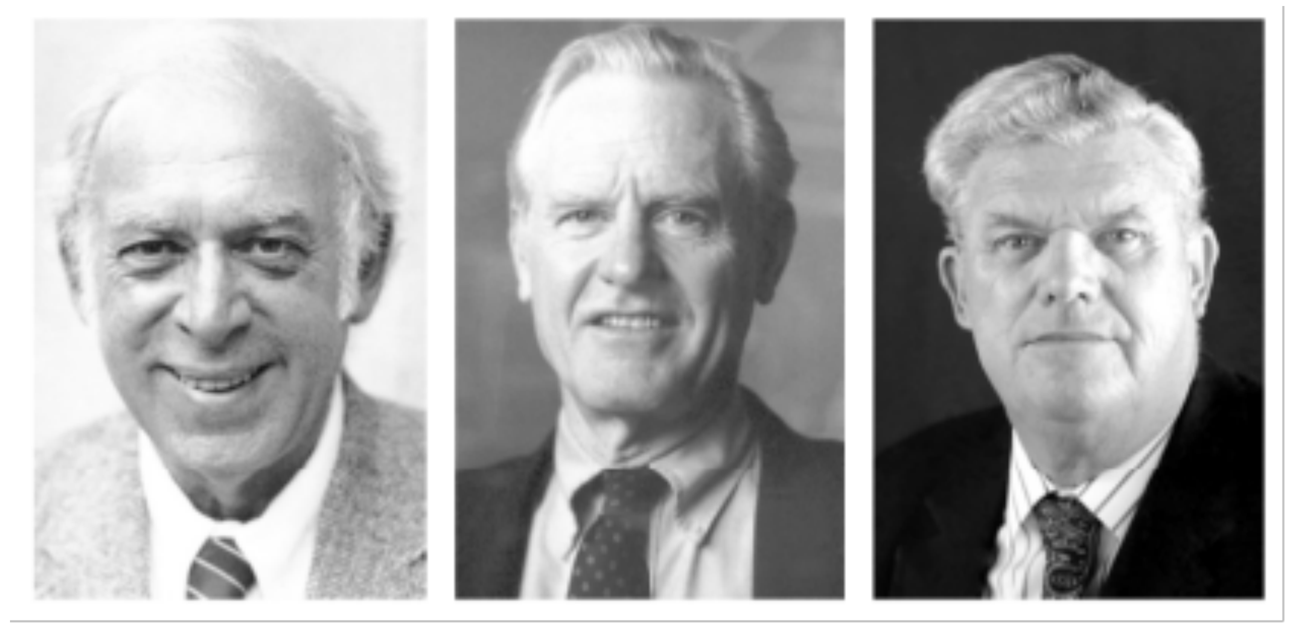
Deep Inelastic Scattering Process



hadronic part of cross section

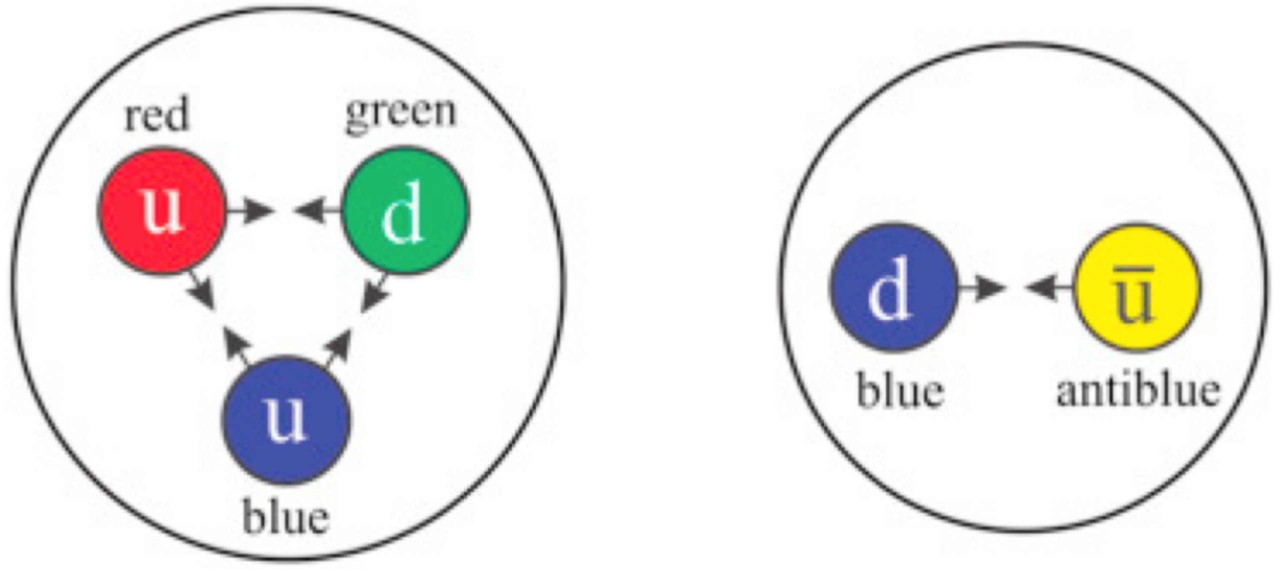
$$\frac{d\sigma}{d\Omega} \propto q(x)$$

Quarks in hadrons



J. Friedman
H. Kendall
R. Taylor
Nobel prize in 1990

pictures

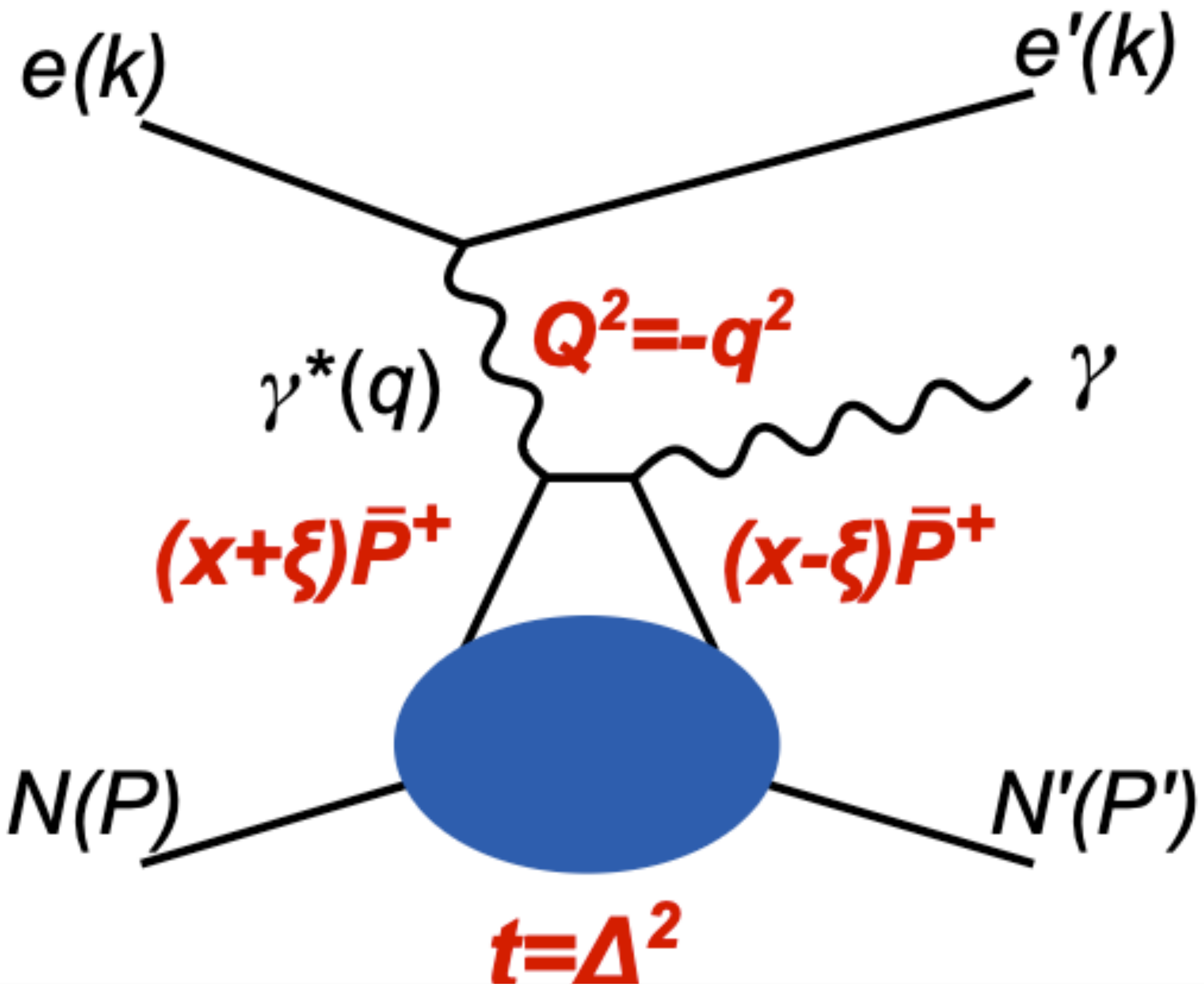


Baryon (proton, p^+)

Meson (negative pion, π^-)

Motivation

Deep Virtual Compton Scattering



nucleon tomographic figures ← → Ji's sum rule

hadronic part of cross section related to $q(x, \xi, t)$

$$q(x, \xi, t) = \bar{u}(P') \left[\gamma^+ H^q(x, \xi, t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E^q(x, \xi, t) \right] u(P)$$

$H(x, \xi, t)$ $E(x, \xi, t)$

forward limit $\xi \to 0,\ t \to 0$

$q(x)$

GPDs for nucleon 3D structure!

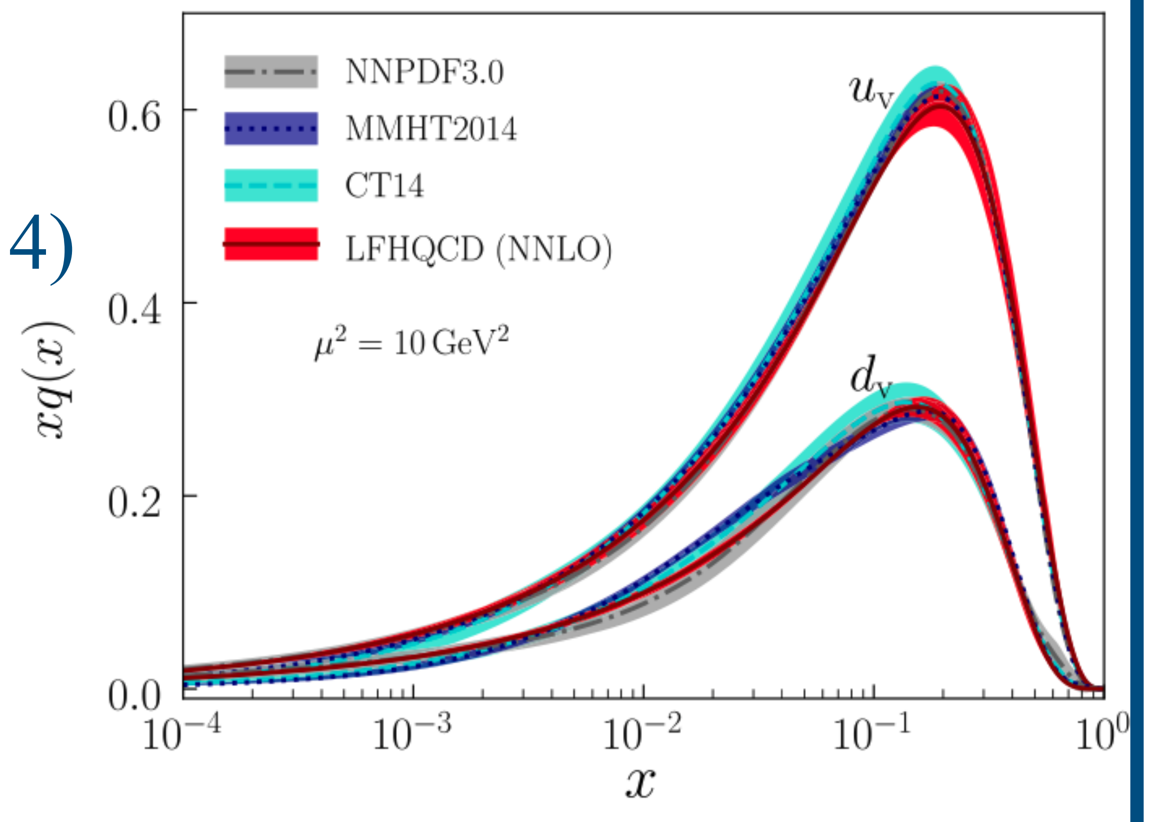
Motivation

experiment results

global fitting

- **PDFs**

Eur.Phys.J.C 75 (2015) 5, 204 (MMTH2014)
JHEP 04 (2015) 040 (NNPDF3.0)
Phys.Rev.D 93 (2016) 3, 033006 (CT14)

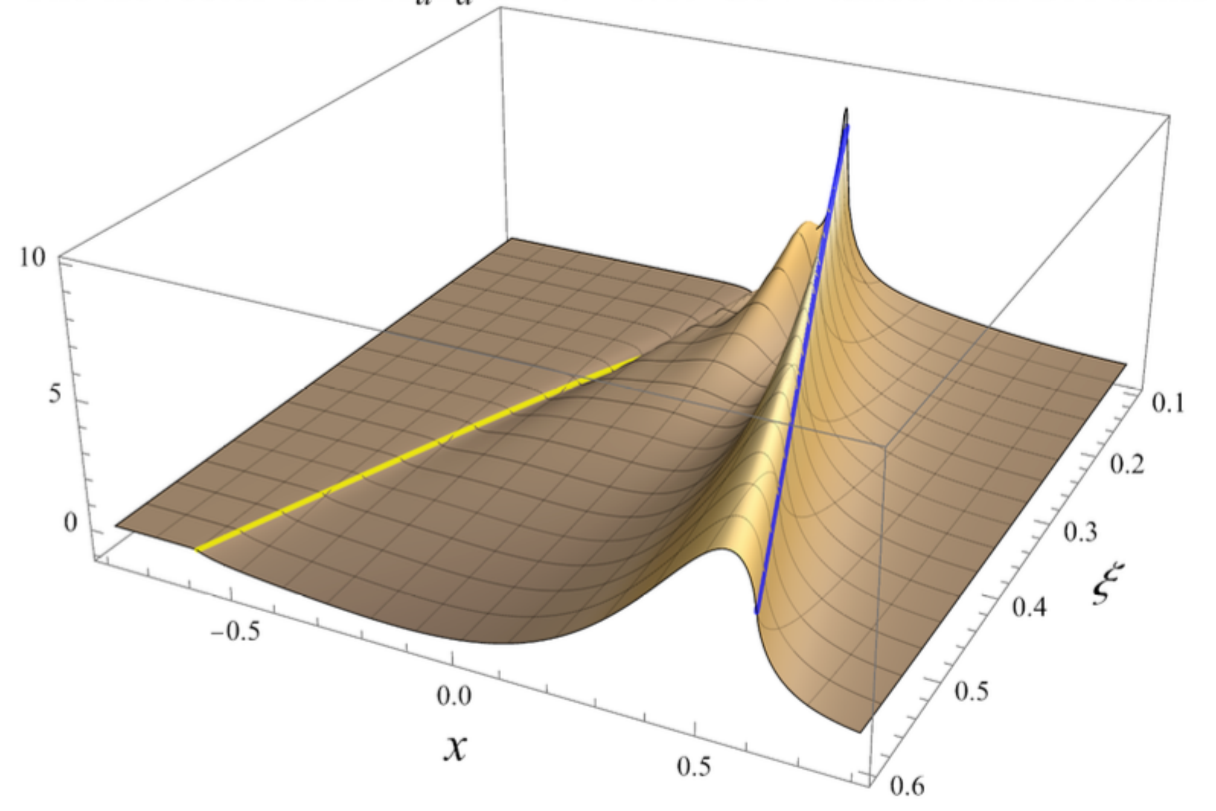


Phys.Rev.Lett. 120 (2018) 18, 182001

- **GPDs**

EPJ Web Conf. 112 (2016) 01012
JHEP 05 (2023) 150
Nature 557 (2018) 7705, 396-399

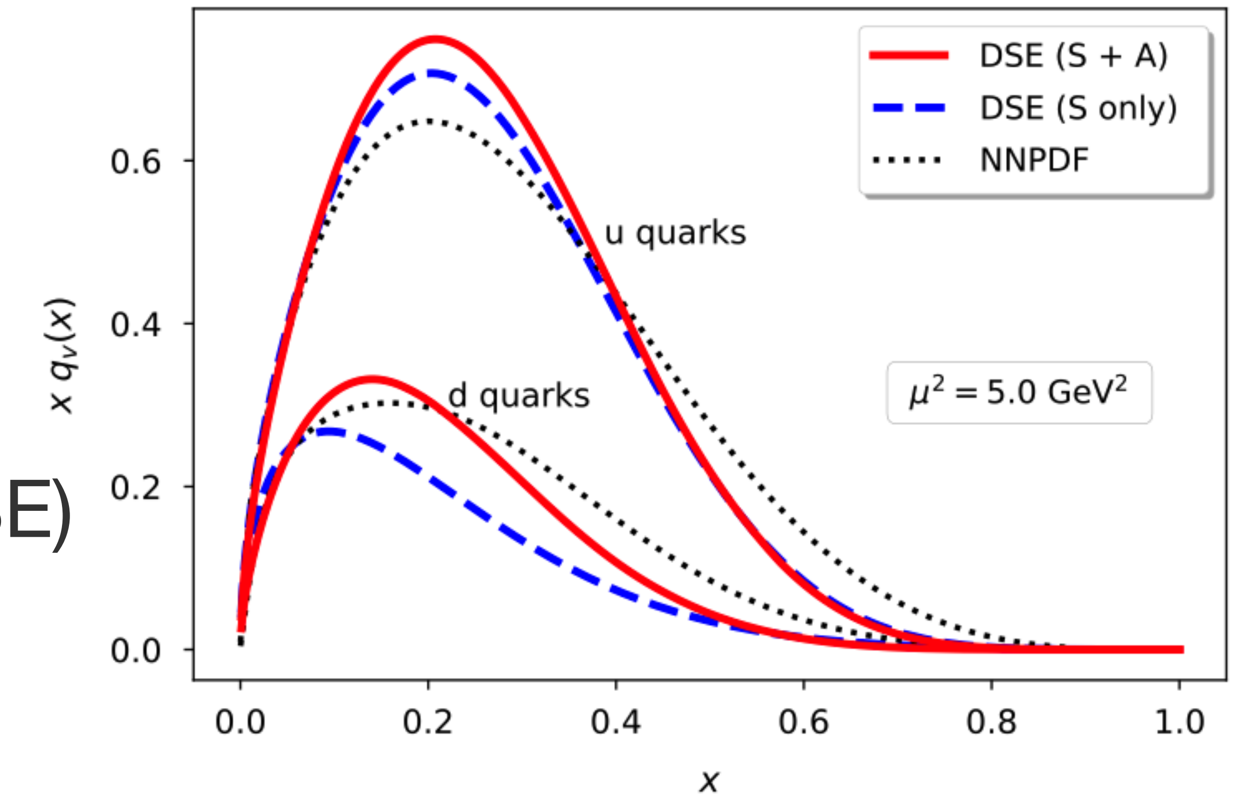
The isovector GPD H_{u-d} at $-t = 0.69\,\mathrm{GeV}^2$ tuned with DA terms



phenomenological results

- **PDFs**

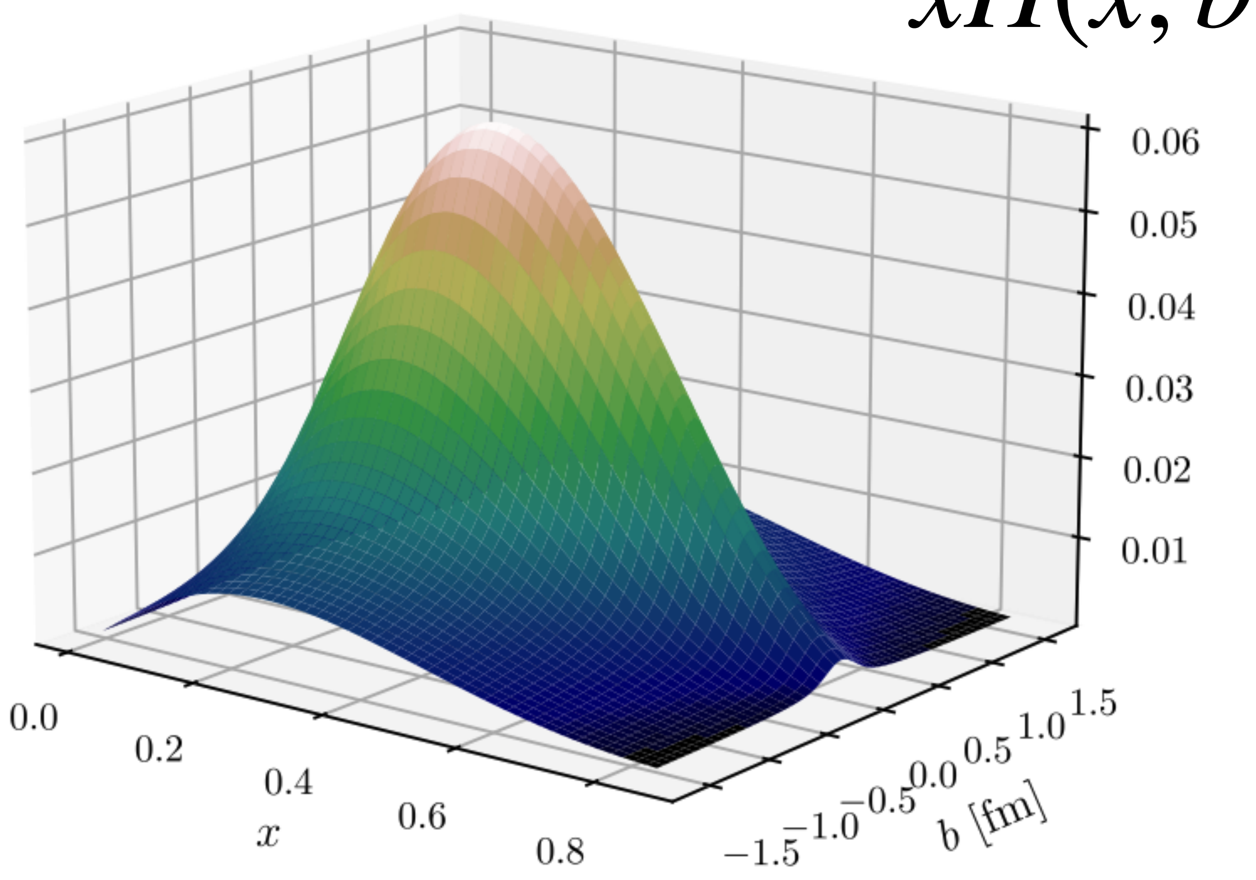
Phys.Lett.B 399 (1997) 287-296
Eur.Phys.J.C 23 (2002) 487-501
Phys.Lett.B 782 (2018) 675-681(DSE)



- **GPDs**

Eur.Phys.J.C 78 (2018) 11, 890
Phys.Rev.D 95 (2017) 1, 011501
Phys.Rev.D 110 (2024) 1, 014509

$xH(x, b)$

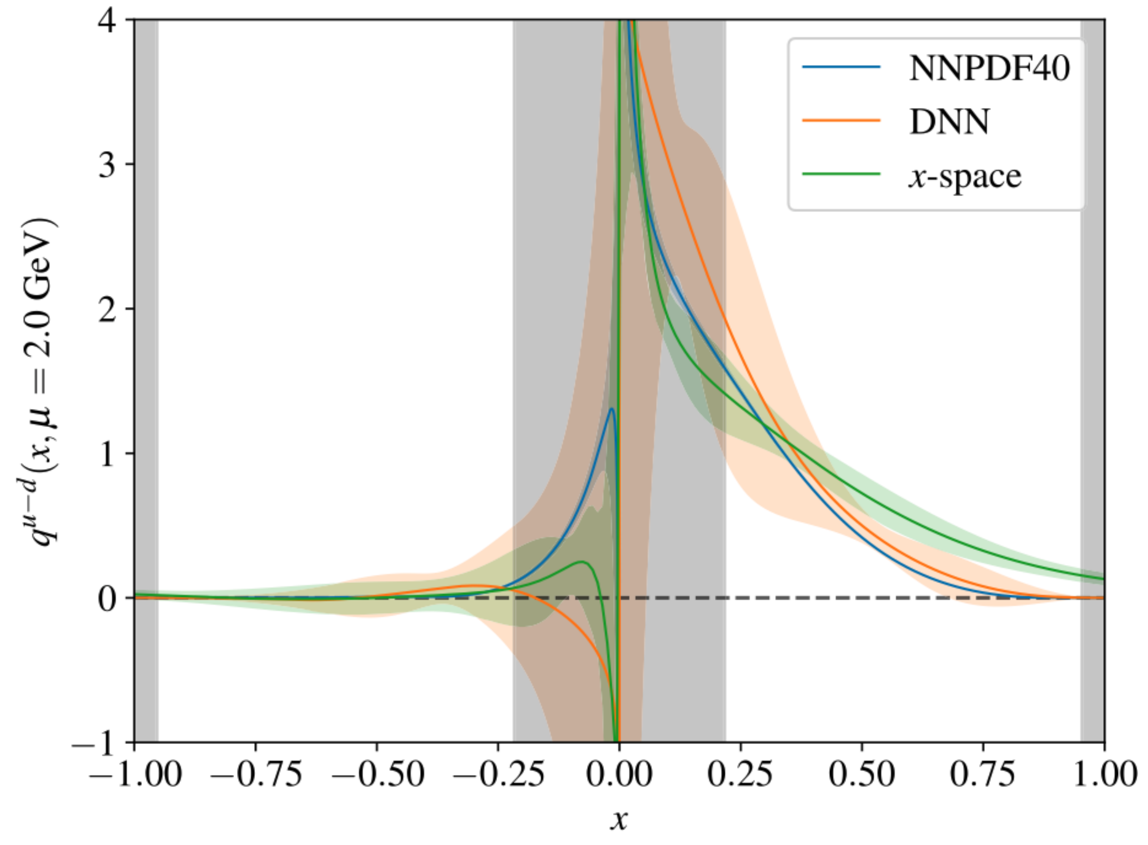


Motivation

Lattice results of PDFs LaMET

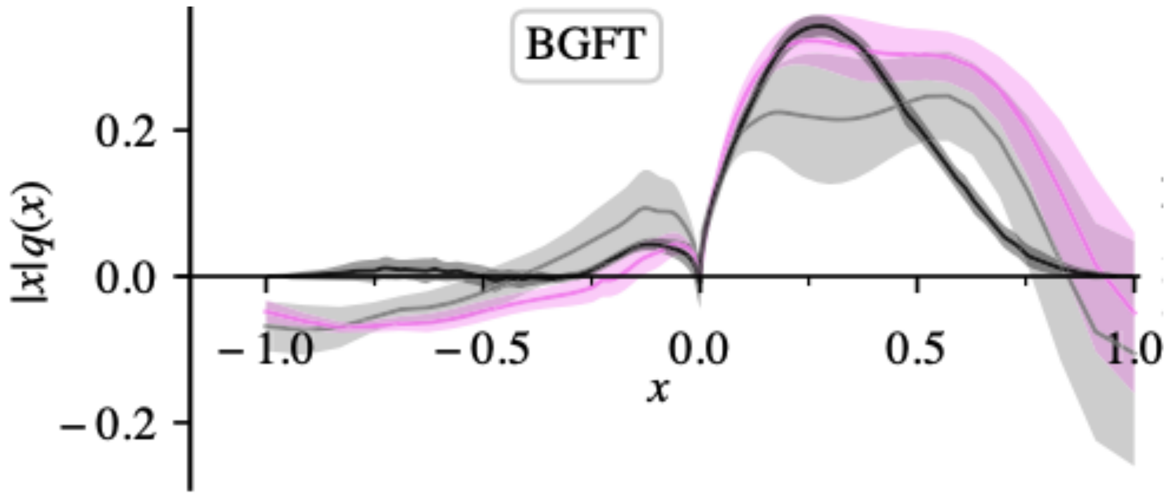
- **unpolarized**

ETMC, Phys.Rev.Lett. 121 (2018) 11, 112001
LPC, Phys.Rev.D 101 (2020) 3, 034020
BNL/ANL, Phys.Rev.D 107 (2023) 7, 074509



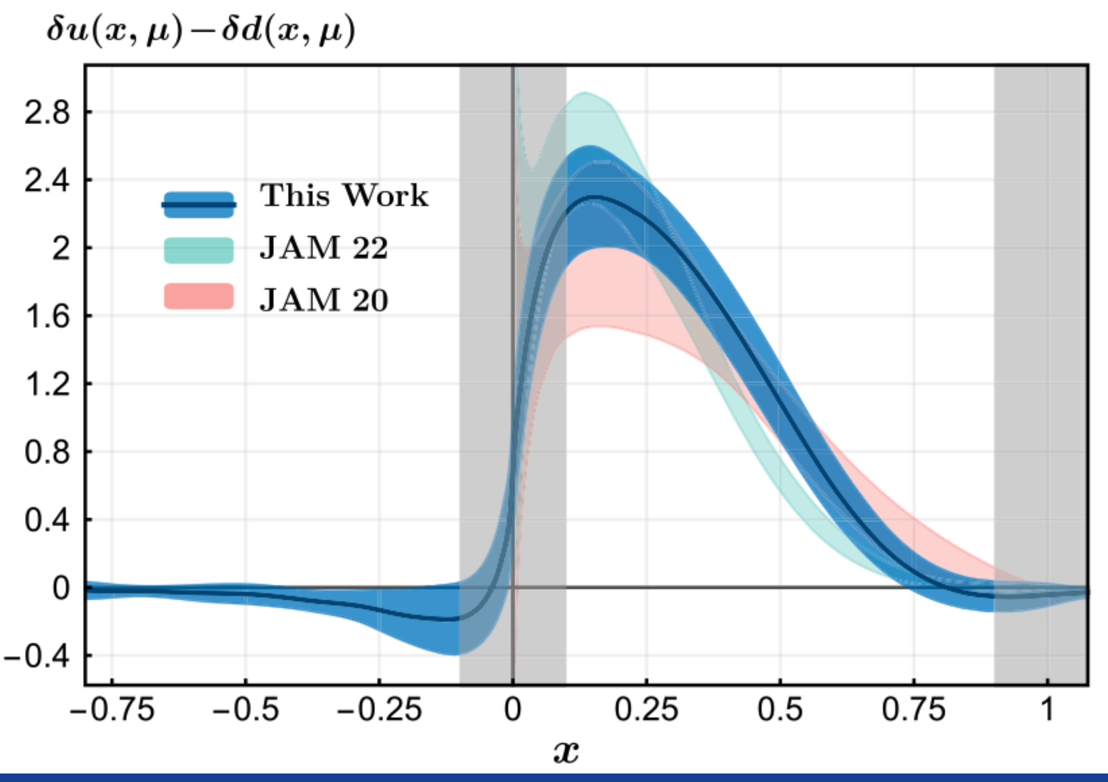
- **helicity**

ETMC, Phys.Rev.Lett. 121 (2018) 11, 112001
ETMC, Phys.Rev.D 103 (2021) 094512
MSU, Phys.Lett.B 854 (2024) 138731



- **transversity**

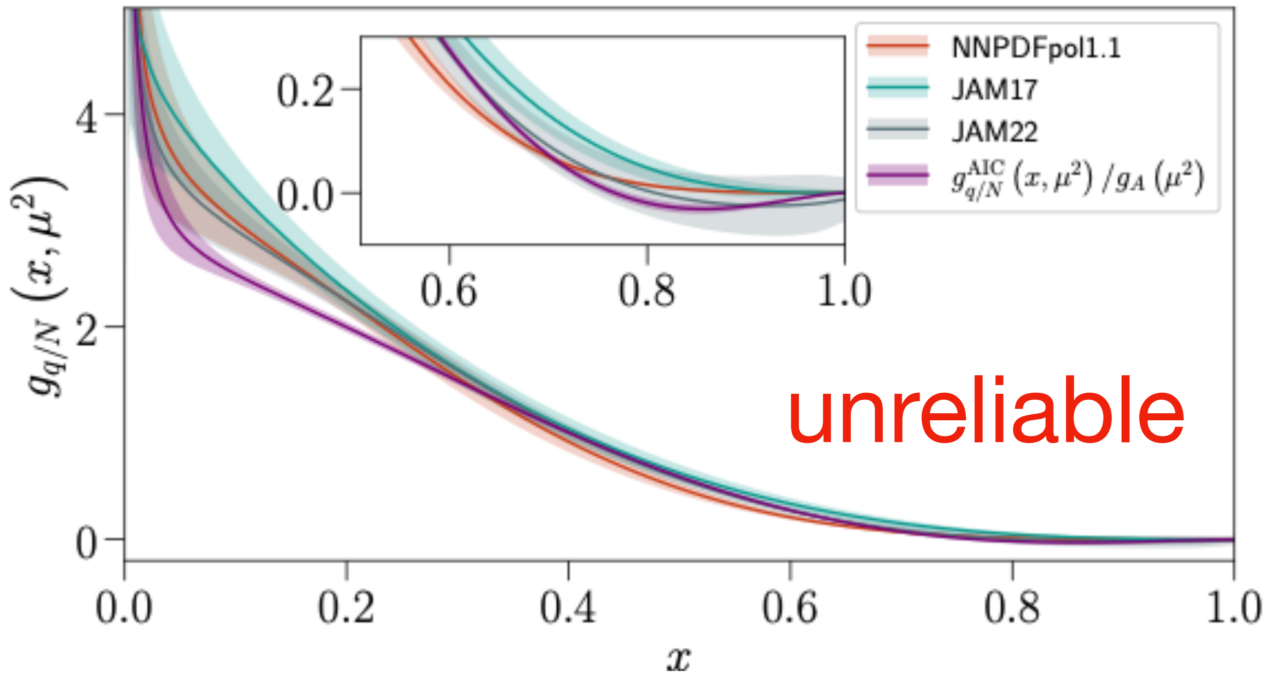
ETMC, Phys.Rev.D 98 (2018) 9, 091503
LPC, Phys.Rev.Lett. 131 (2023) 26, 261901
BNL/ANL, Phys.Rev.D 109 (2024) 5, 054506



Lattice results of PDFs SDE

- **short-distance expansion**

HadStruc, Phys.Rev.D 96 (2017) 9, 094503
HadStruc, JHEP 11 (2021) 148
HadStruc, JHEP 03 (2023) 086



- **power corrections**

SDE: $\mathcal{O}(z^2)$ not related to x

LaMET: $\mathcal{O}\left(1/x^2P_z^2\right)$,$\mathcal{O}\left(1/(1-x)^2P_z^2\right)$ dominate in end point region

idea from Ji's paper: Research 8 (2025) 0695

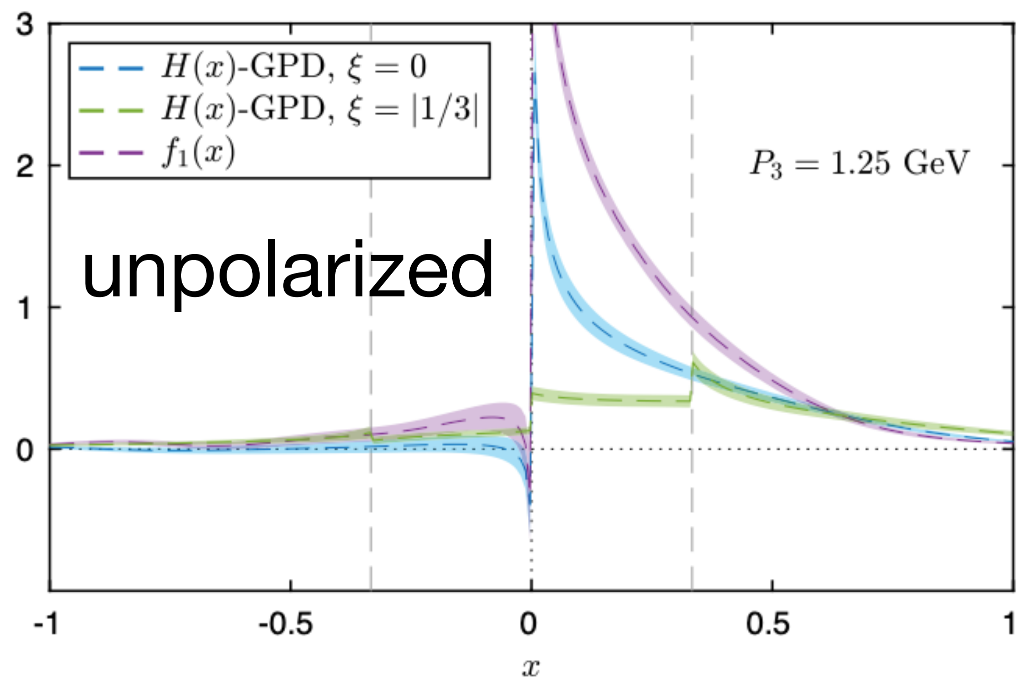
Motivation

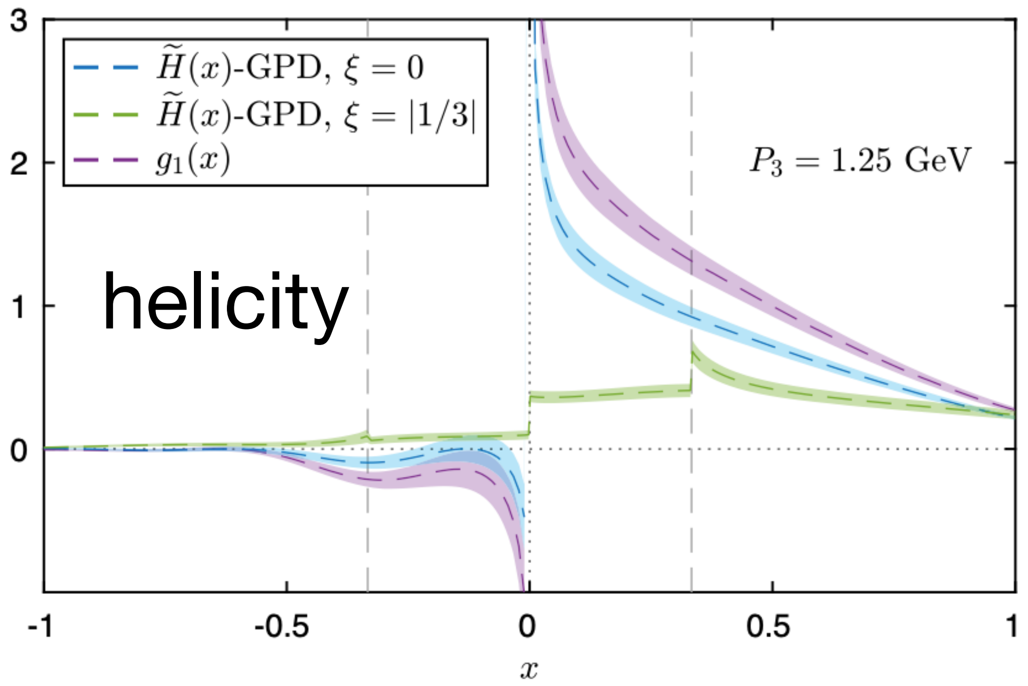
Lattice results of GPDs LaMET

- **symmetric frame**

ETMC, Phys.Rev.Lett. 125 (2020) 26, 262001 (unpolarized+helicity+one ξ)

ETMC, Phys.Rev.D 105 (2022) 3, 034501(transversity)



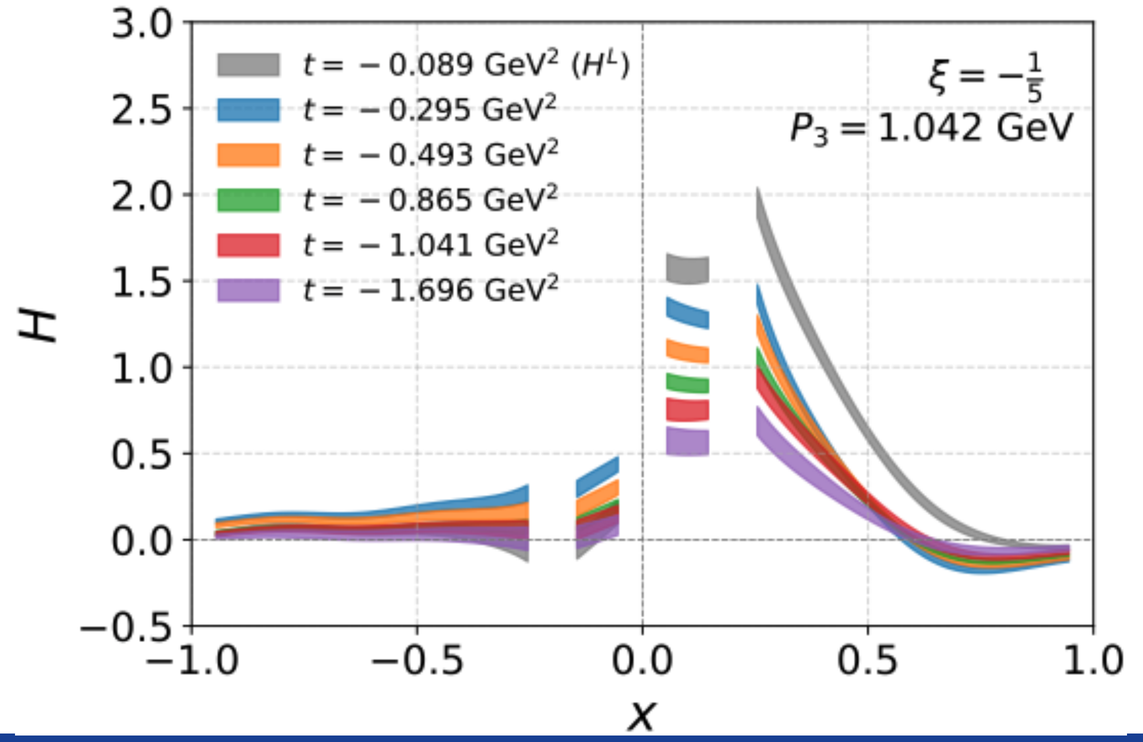


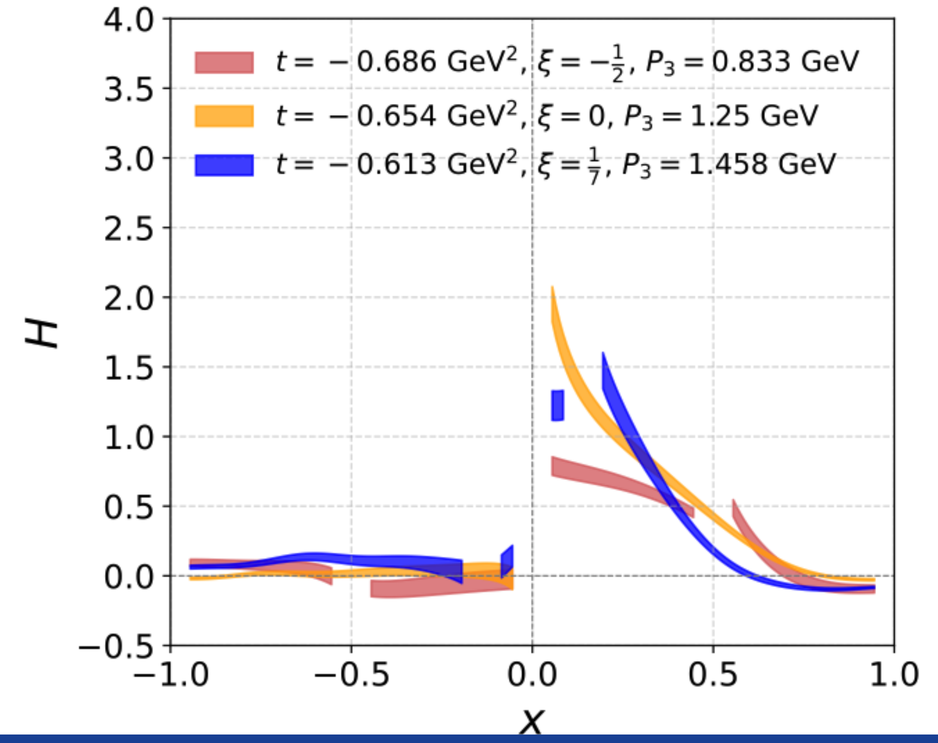
- **asymmetric frame**

ETMC, Phys.Rev.D 106 (2022) 11, 114512 (unpolarized)

ETMC, Phys.Rev.D 109 (2024) 3, 034508 (axial-vector)

ETMC, arxiv: 2508.17998 (unpolarized + ξ dependent)





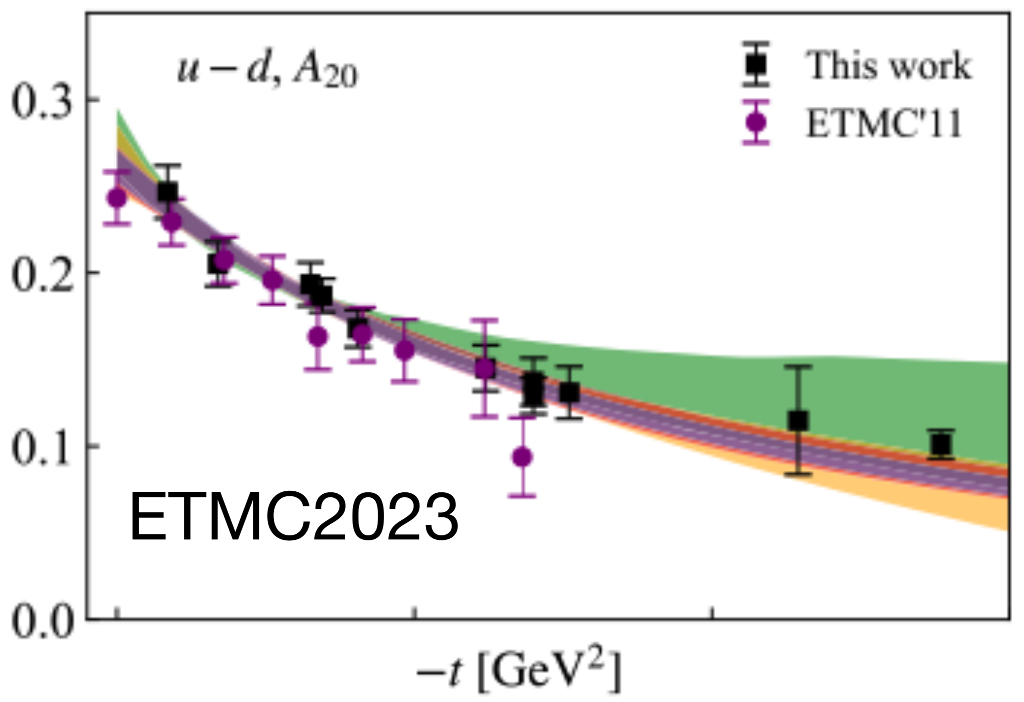
Lattice results of GPDs SDE

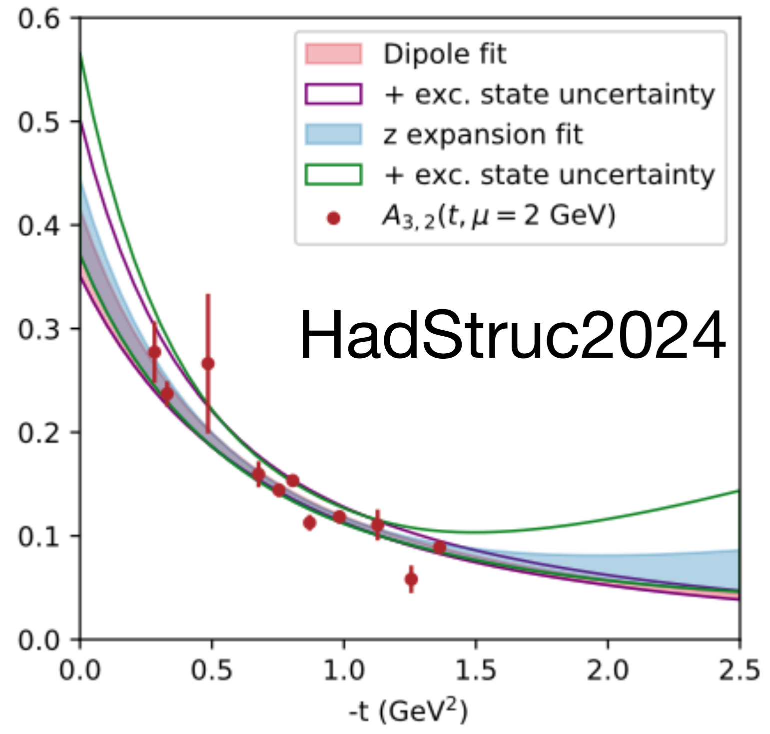
- **short-distance expansion (moments)**

ETMC, Phys.Rev.D 108 (2023) 1, 014507 (unpolarized + moments)

ETMC, Phys.Rev.D 110 (2024) 5, 054502 (unpolarized + reconstruction)

HadStruc, JHEP 08 (2024) 162 (unpolarized + moments + ξ dependent)





- **GPD features**

unpolarized → polarized (helicity, transversity…)

$\xi = 0 \rightarrow \xi \neq 0$

ANN reconstruction could be applied for all cases !

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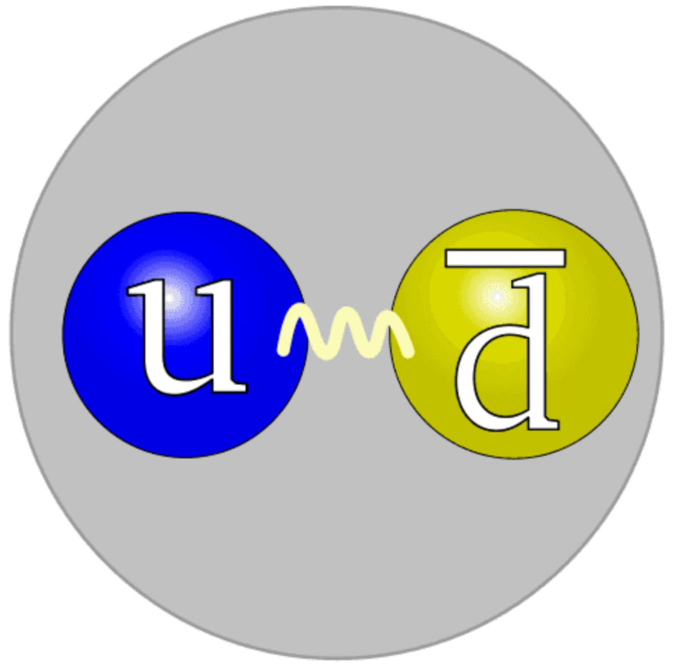
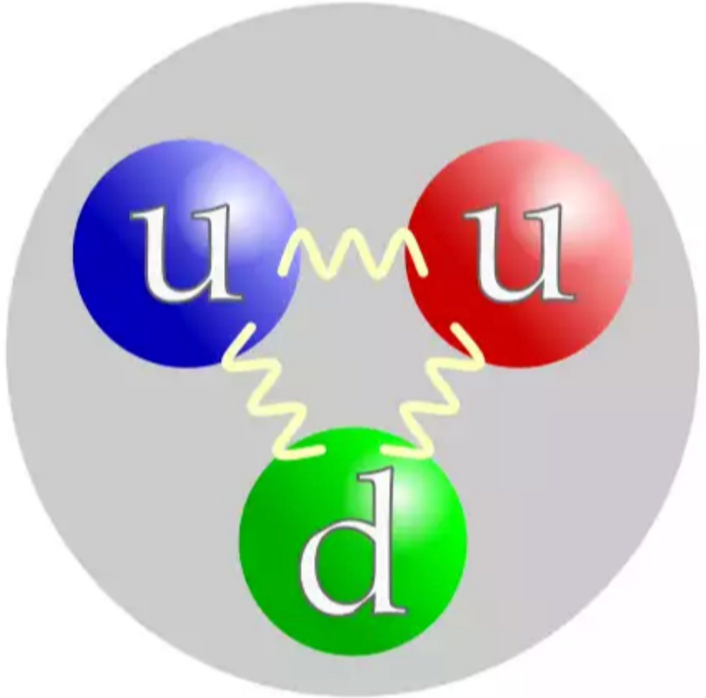




neuron network reconstruction

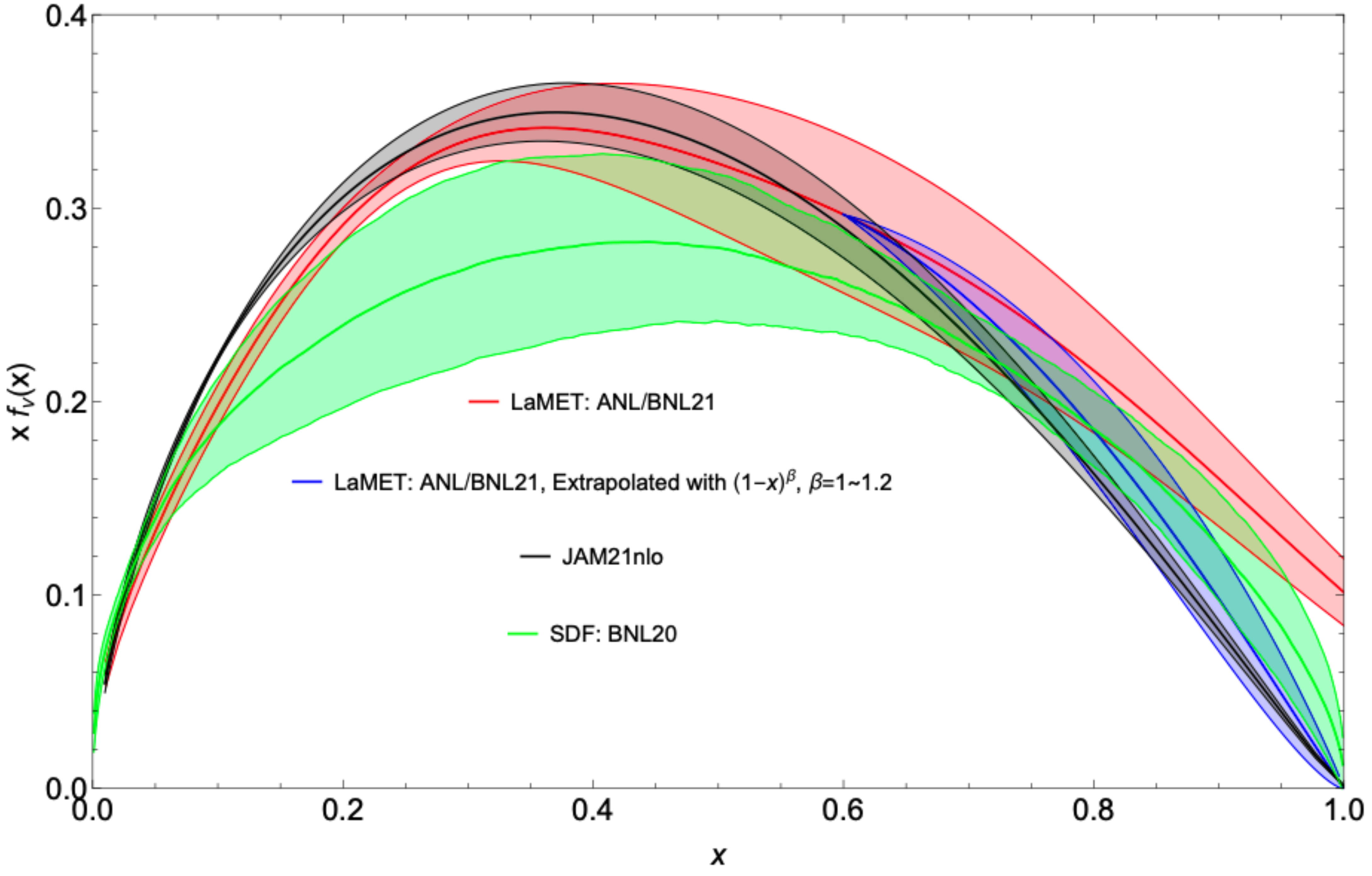
light-cone PDF

X. Ji, et,al., Research 8 (2025) 0695

0.2 — 0.8

SDE | LaMET | SDE

end point region: $$f(x)=\begin{cases} Ax^{\alpha}\left(1+a\sqrt{x}+\ldots\right), & x\in[0,x_{\min}] \\ B(1-x)^{\beta}(1+b(1-x)+\ldots), & x\in[x_{\max},1] \end{cases}$$



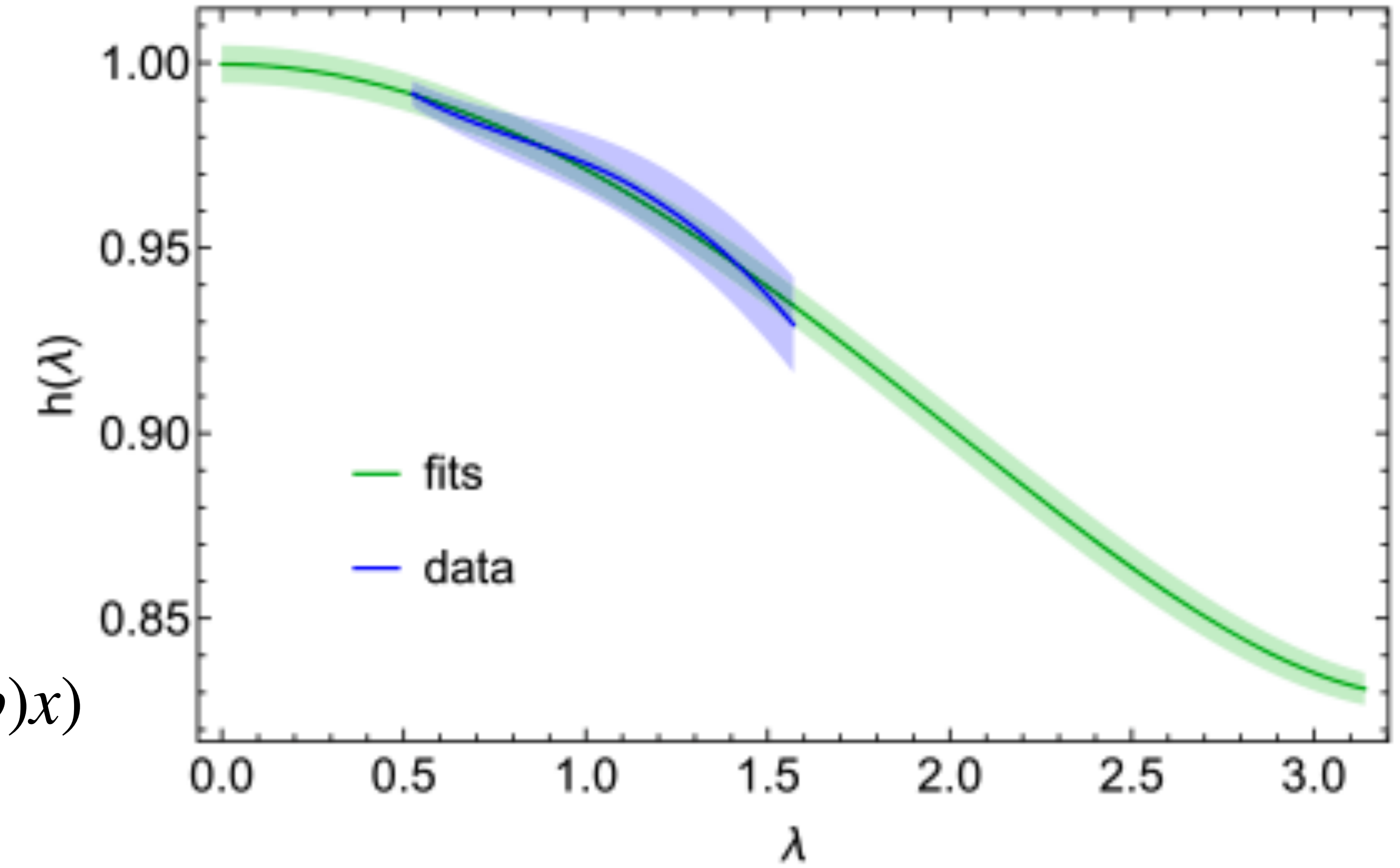
light-cone DA

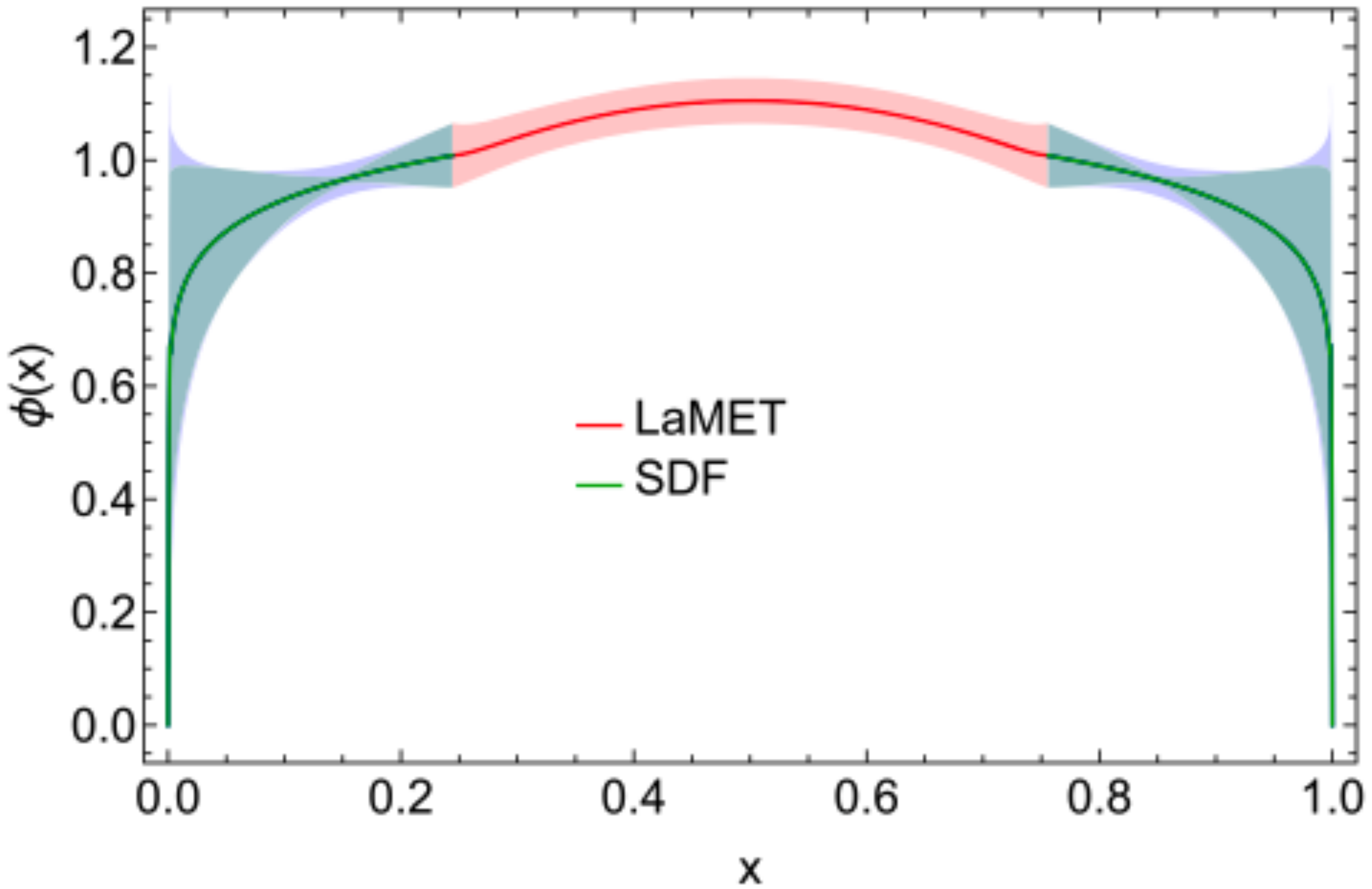
J. Holligan, et,al., Nucl.Phys.B 993 (2023) 116282

short-distance correlations

$$\phi(x<x_0) = Ax^{m}(1+\sin(b)x)$$

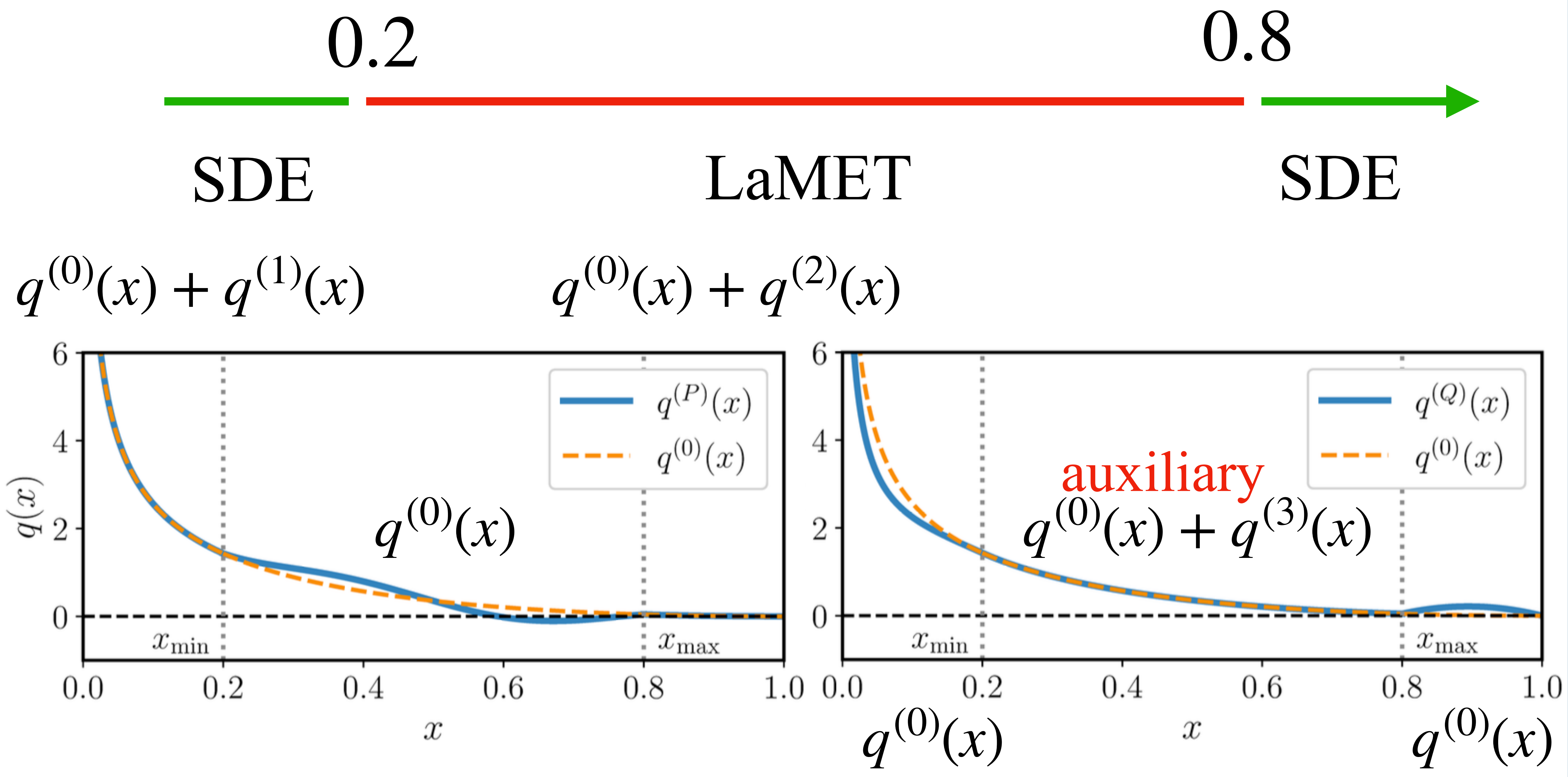
applied in pion DA





light-cone PDF

X. Ji, et,al., Research 8 (2025) 0695



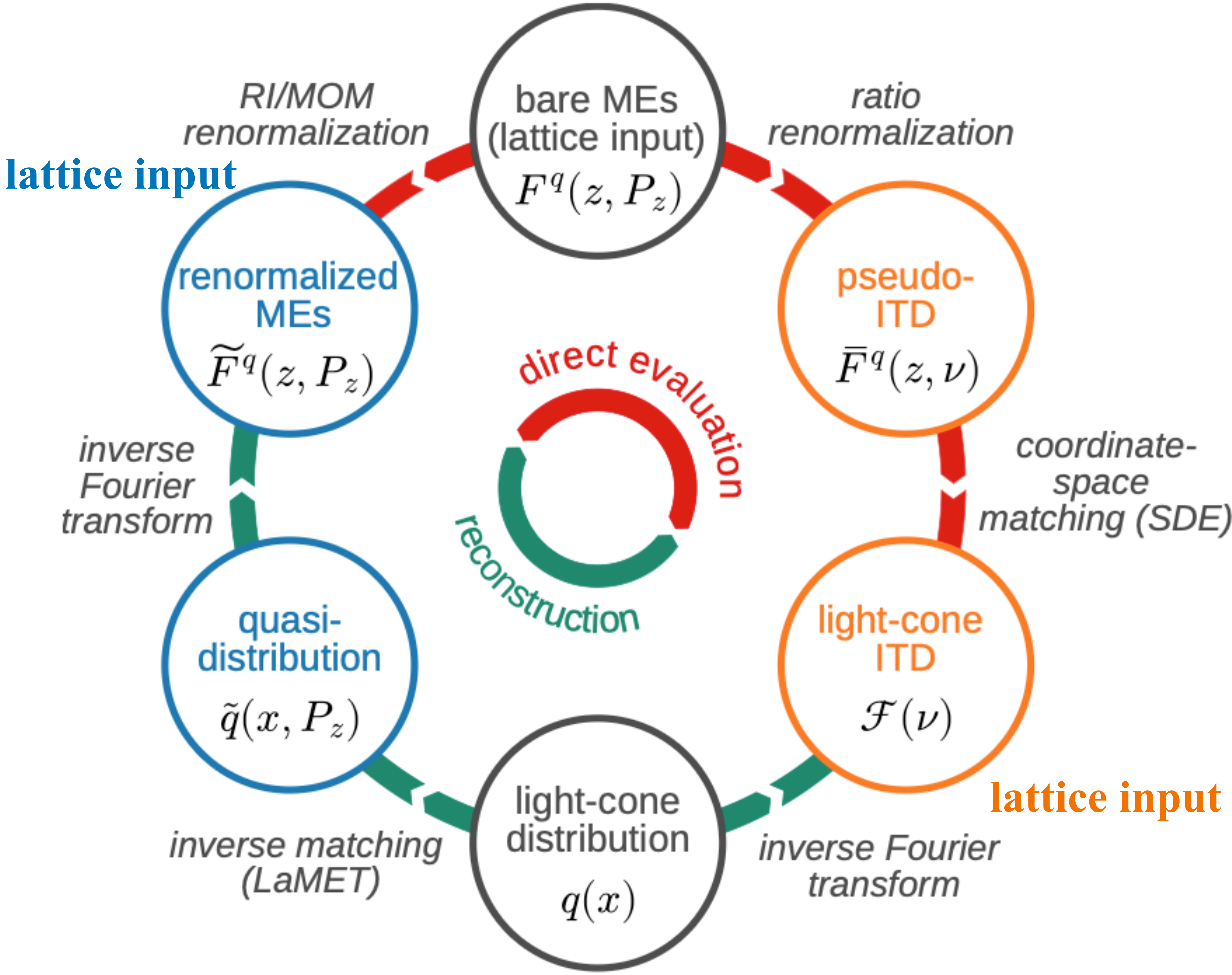
M. Chu, et,al., arxiv: 2509.15931

$$q^{(Q)}(x) = \begin{cases} q^{(0)}(x) + q^{(1)}(x)\,, & 0 < x < x_{\min}\,, \\ q^{(0)}(x) + q^{(2)}(x)\,, & x_{\max} < x < 1\,, \\ q^{(0)}(x)\,, & \text{otherwise}\,, \end{cases}$$

$$q^{(P)}(x) = \begin{cases} q^{(0)}(x) + q^{(3)}(x)\,, & x_{\min} < x < x_{\max}\,, \\ q^{(0)}(x)\,, & \text{otherwise}\,, \end{cases}$$

unified ANN reconstruction

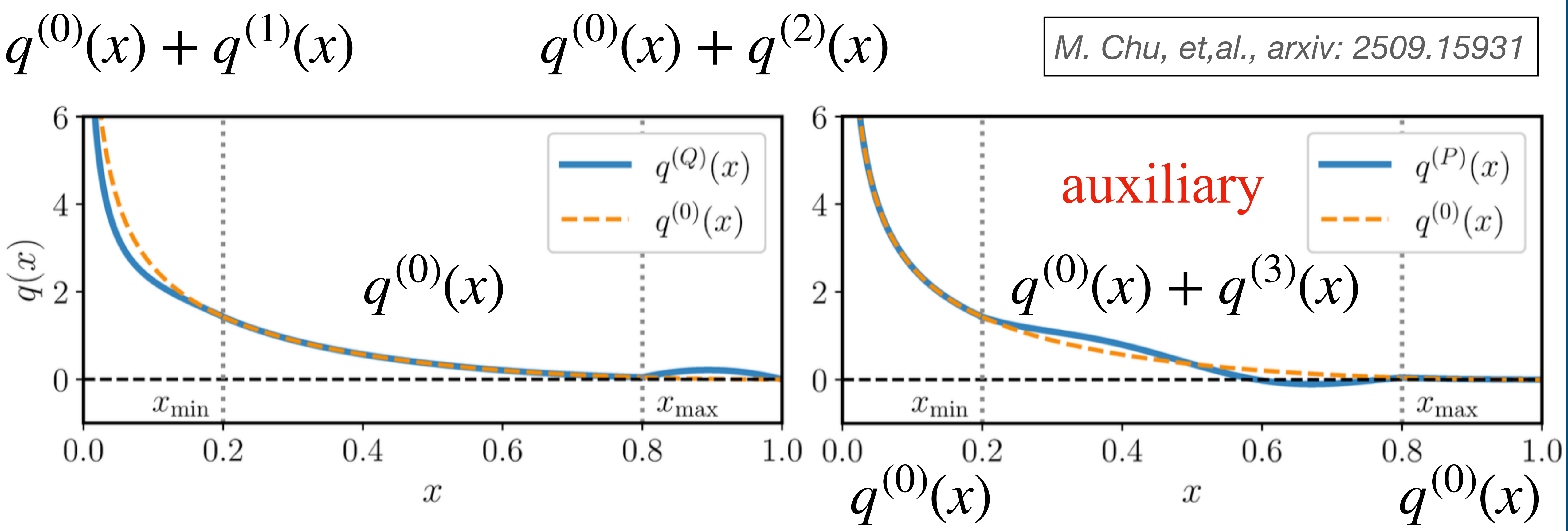
M. Chu, et,al., arxiv: 2509.15931



neuron network reconstruction

network structure

M. Chu, et,al., arxiv: 2509.15931



parametrization of light-cone PDF

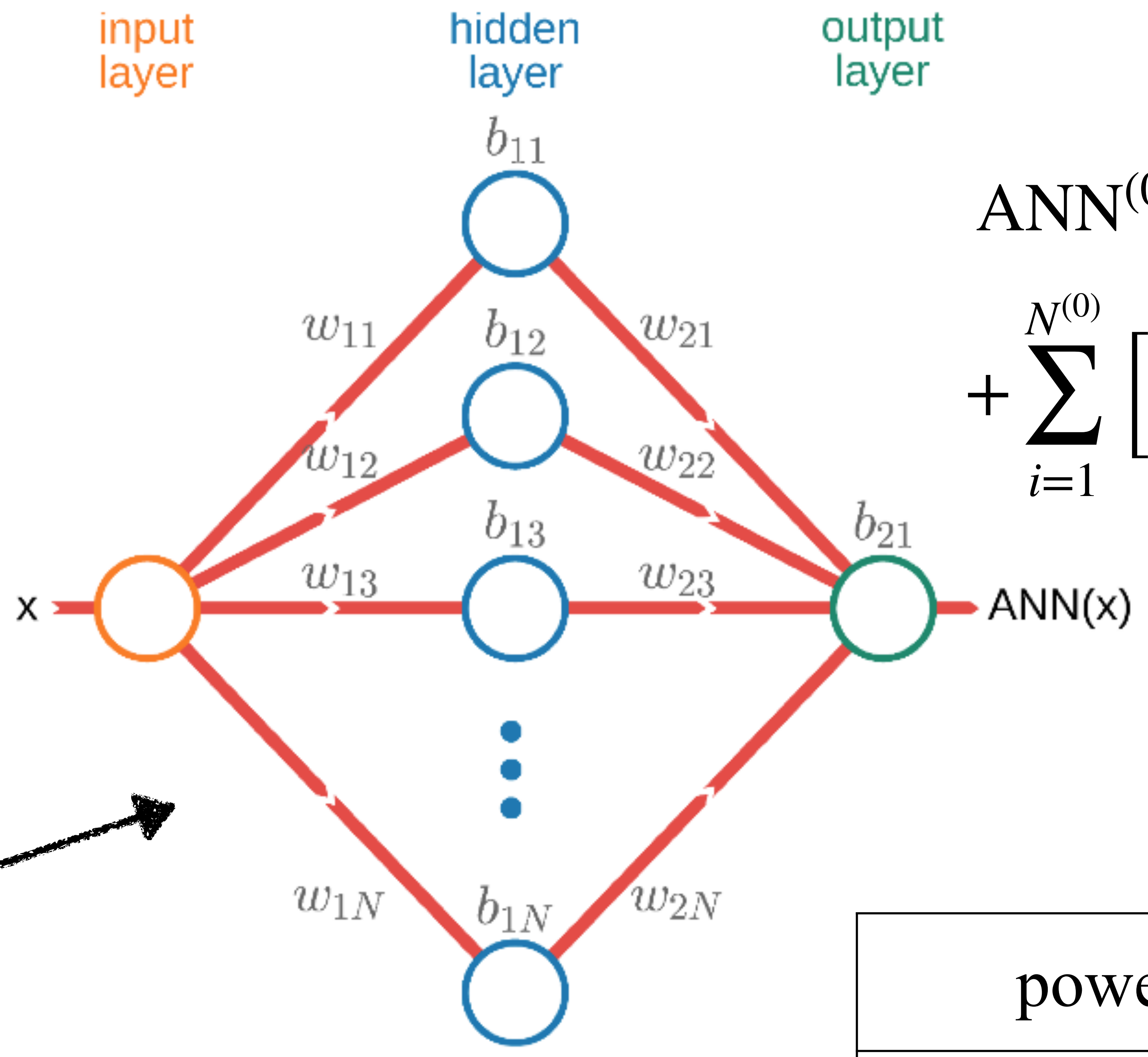
$$q^{(0)}(x) = x^{\delta^{(0)}}(1-x)^{\rho^{(0)}}\,\mathrm{ANN}^{(0)}(x)$$

parametrization of auxiliary PDFs

$$q^{(1)}(x) = x^{\delta^{(1)}}(x_{\min}-x)^{\rho^{(1)}}\,\mathrm{ANN}^{(1)}(x)$$
$$q^{(2)}(x) = (x-x_{\max})^{\delta^{(2)}}(1-x)^{\rho^{(2)}}\,\mathrm{ANN}^{(2)}(x)$$
$$q^{(3)}(x) = (x-x_{\min})^{\delta^{(3)}}(x_{\max}-x)^{\rho^{(3)}}\,\mathrm{ANN}^{(3)}(x)$$

network structure

M. Chu, et,al., arxiv: 2509.15931



$$\mathrm{ANN}^{(0)}(x) = b_{21}^{(0)} + \sum_{i=1}^{N^{(0)}}\left[w_{2i}^{(0)}\ln\left(1+\exp(b_{1i}^{(0)}+w_{1i}^{(0)}x)\right)\right]$$

activation: soft plus

parameters in fits:

power	$\delta^{(0,1,2,3)}, \rho^{(0,1,2,3)}$
weights	$w_{jk}^{(0,1,2,3)}$
bias	$b_{jk}^{(0,1,2,3)}$

parameters in system:

truncation in quasi	$z_{\mathrm{cut}} = 9a$
truncation in SDE	$z_{\max} = 4a$
boarder of regions	$x_{\min}, x_{\max} = 0.2, 0.8$

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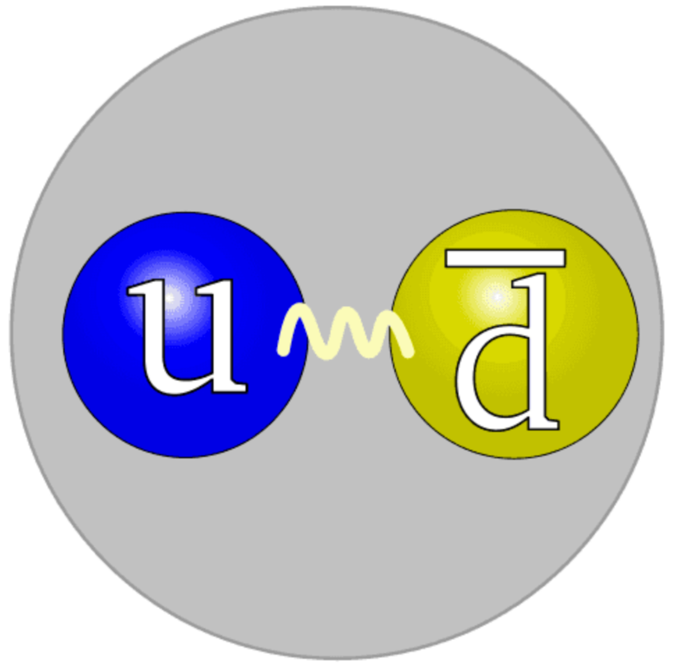
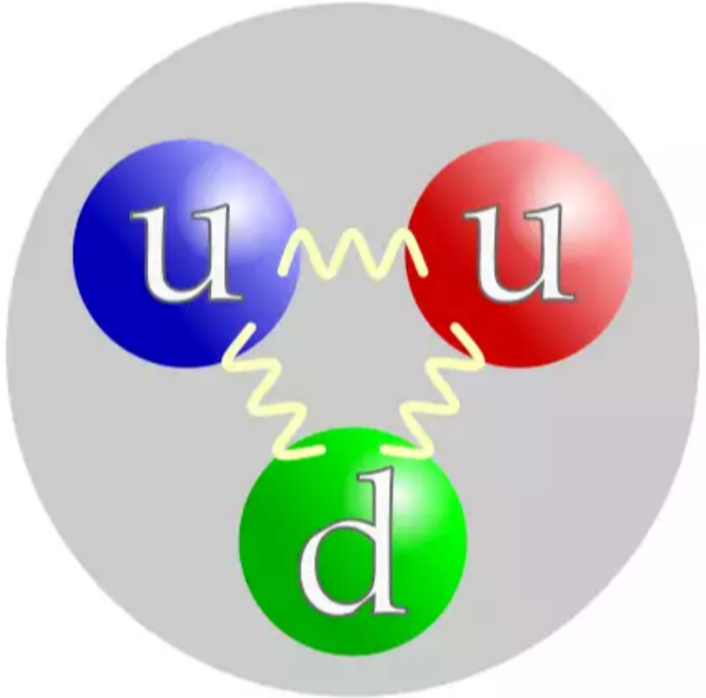
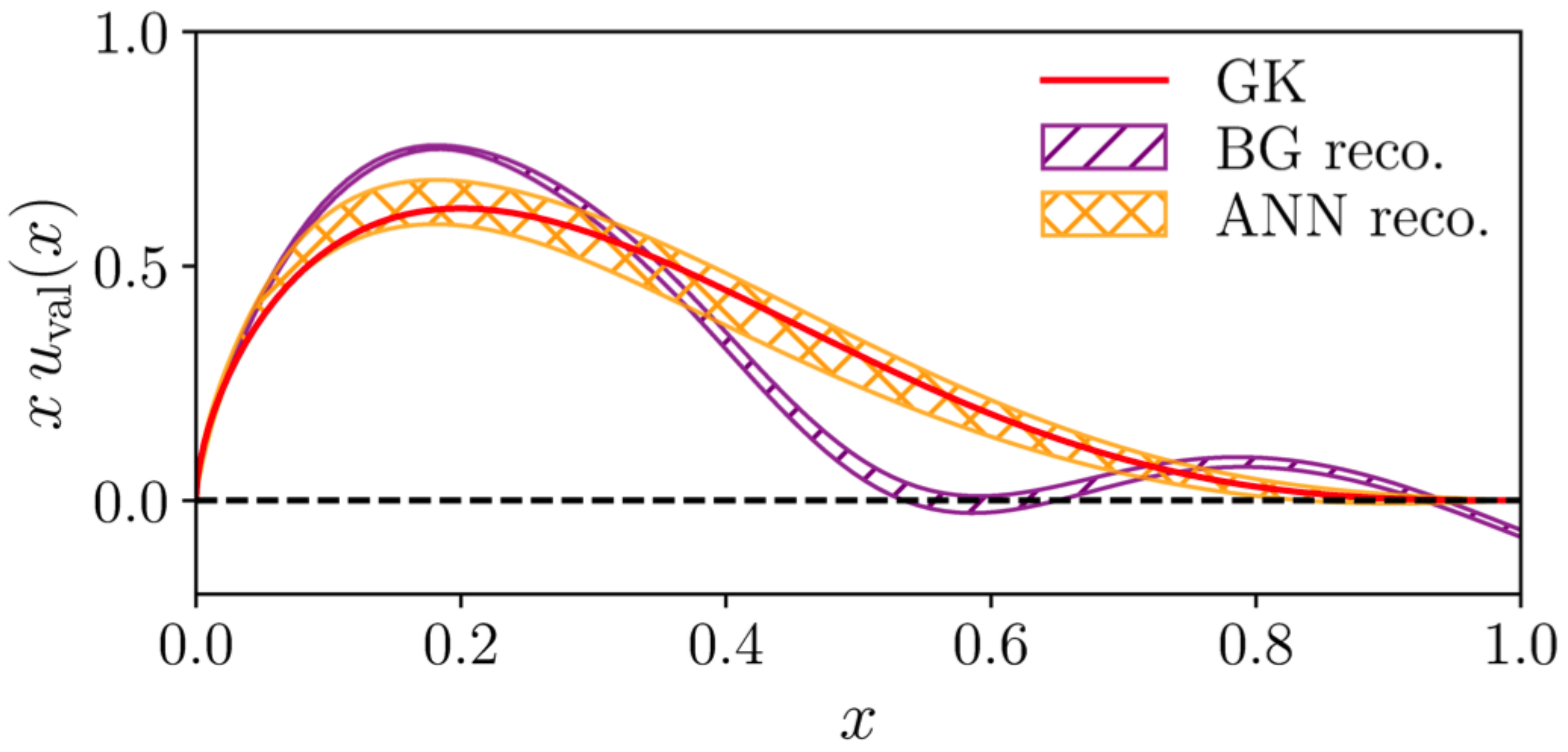




numerical results for PDFs

mock data test

LaMET alone: BG v.s. ANN



mock data: generated by GK model with same correlation of lattice data

S.V. Goloskokov, et,al., Eur.Phys.J.C 50 (2007) 829-842

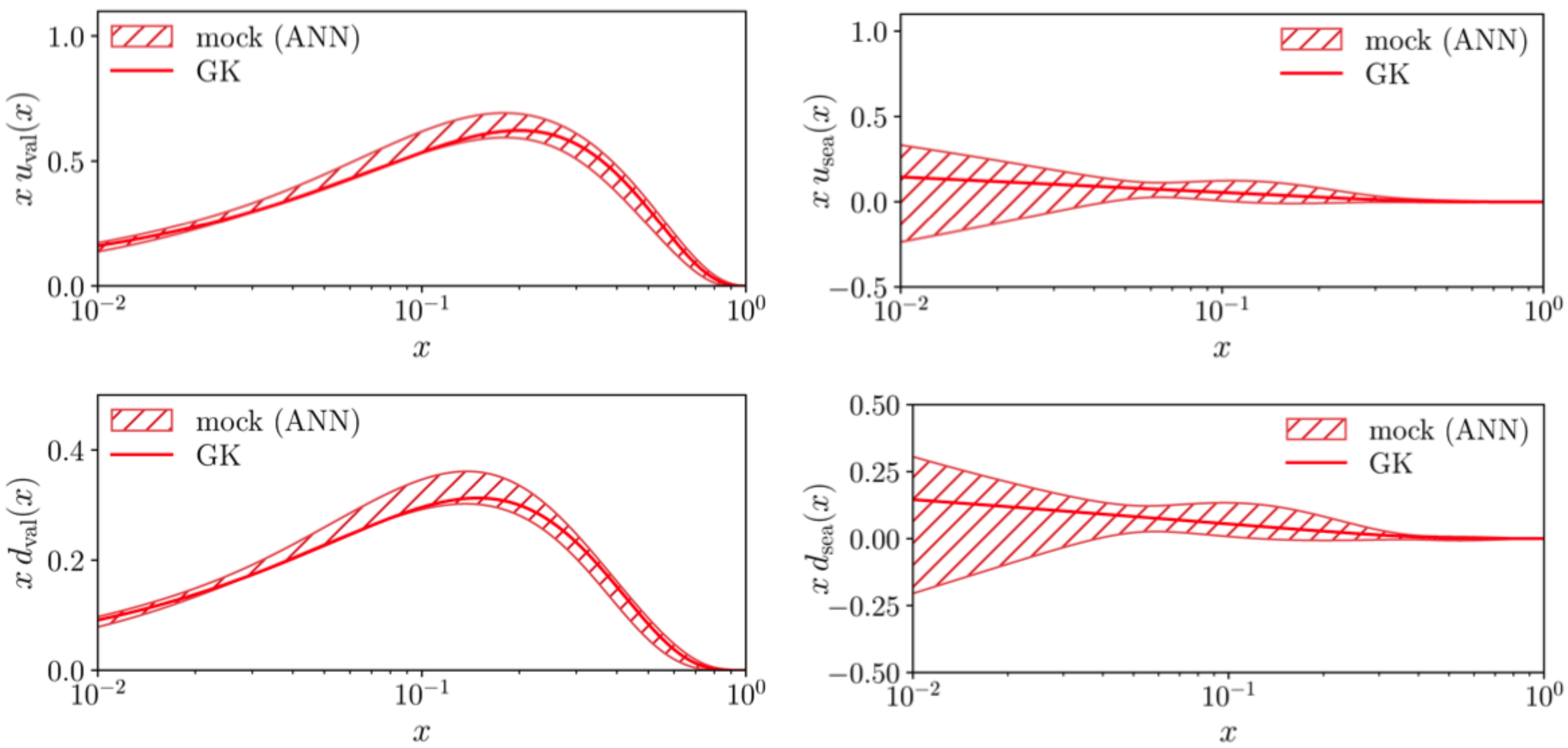
BG + matching

1. fails in recovering;
2. tiny error;
3. oscillation in large x;

ANN

1. success in recovering;
2. reasonable error;
3. no oscillations;

mock data test



valence (positivity constrain): success in recovering.

sea (no positivity constrain): success but large uncertainties.

Tiny difference between LaMET and LaMET + SDE

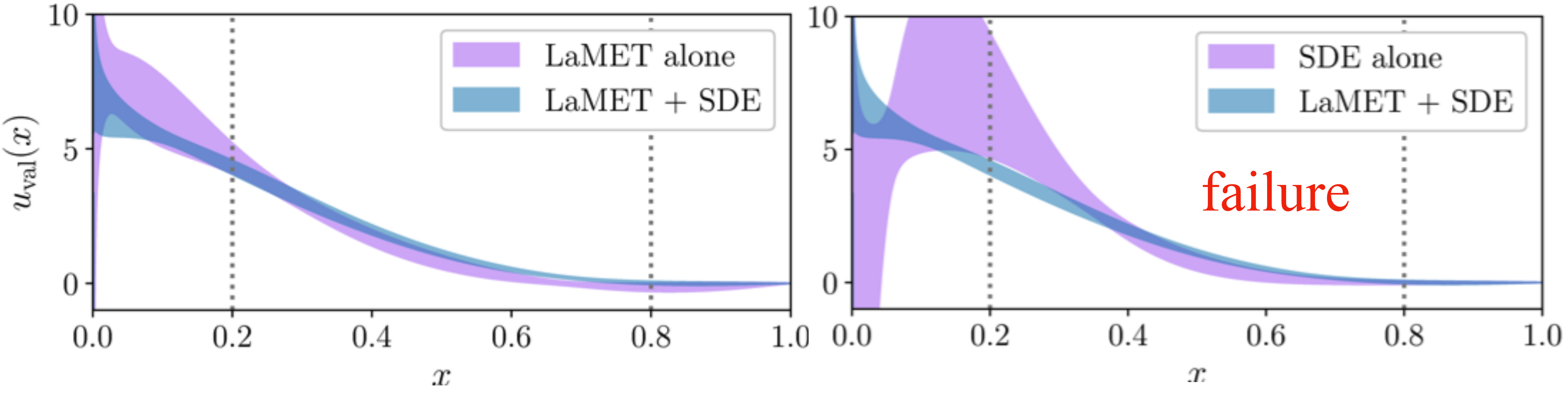
numerical results for PDFs

application to lattice data

S. Bhattacharya, et,al., Phys.Rev.D 106 (2022) 11, 114512

lattice inputs: ETMC GPD/PDF data

LaMET alone v.s. SDE alone v.s. LaMET + SDE

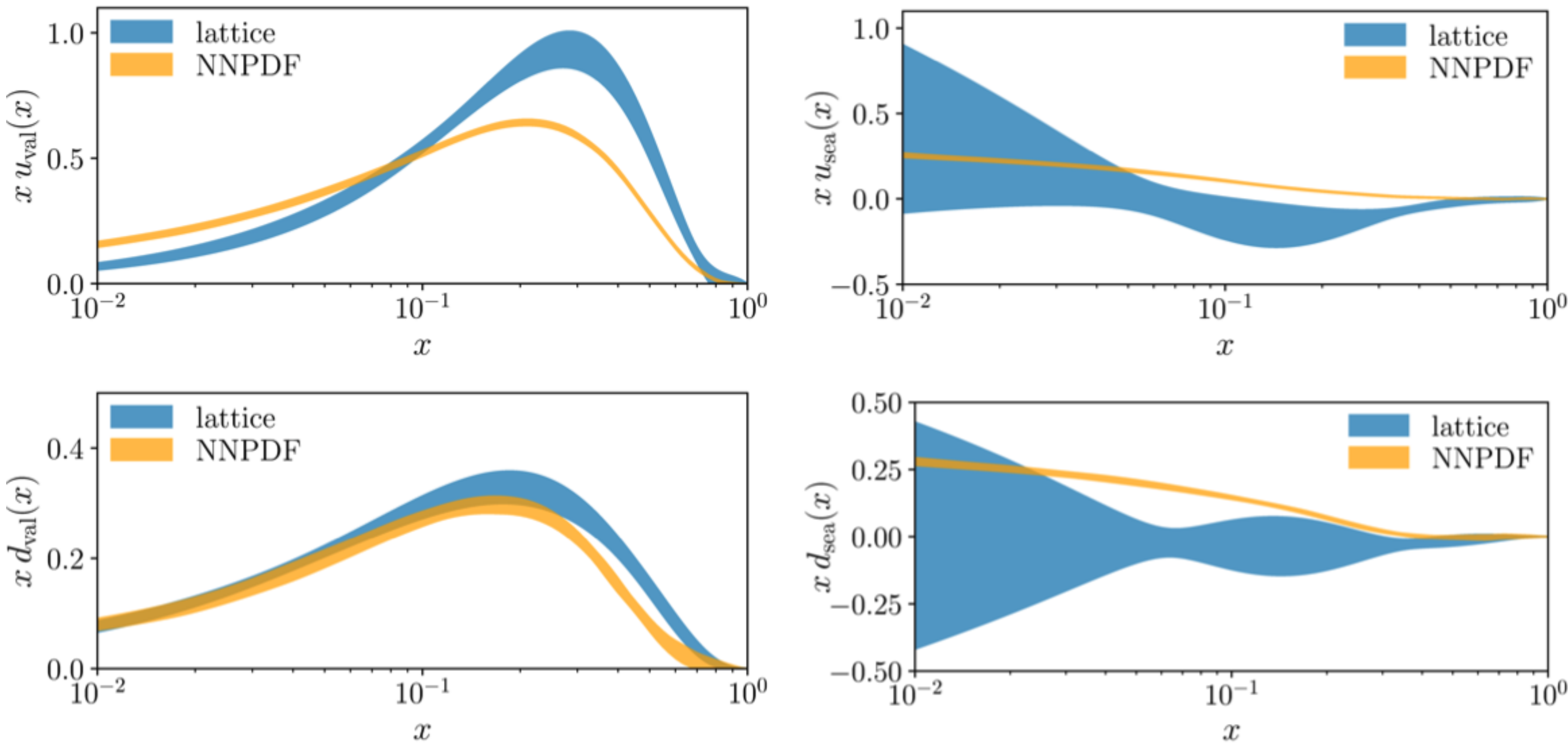


important indications

1. SDE alone fails because limited data points are used;
2. LaMET + SDE have smaller error than LaMET alone;

application to lattice data

M. Chu, et,al., arxiv: 2509.15931



- not compatible with NNPDF (JHEP 04 (2015) 040);
- sea distributions have large uncertainties;
- unaddressed systematic uncertainties in lattice inputs;

hybrid renormalization

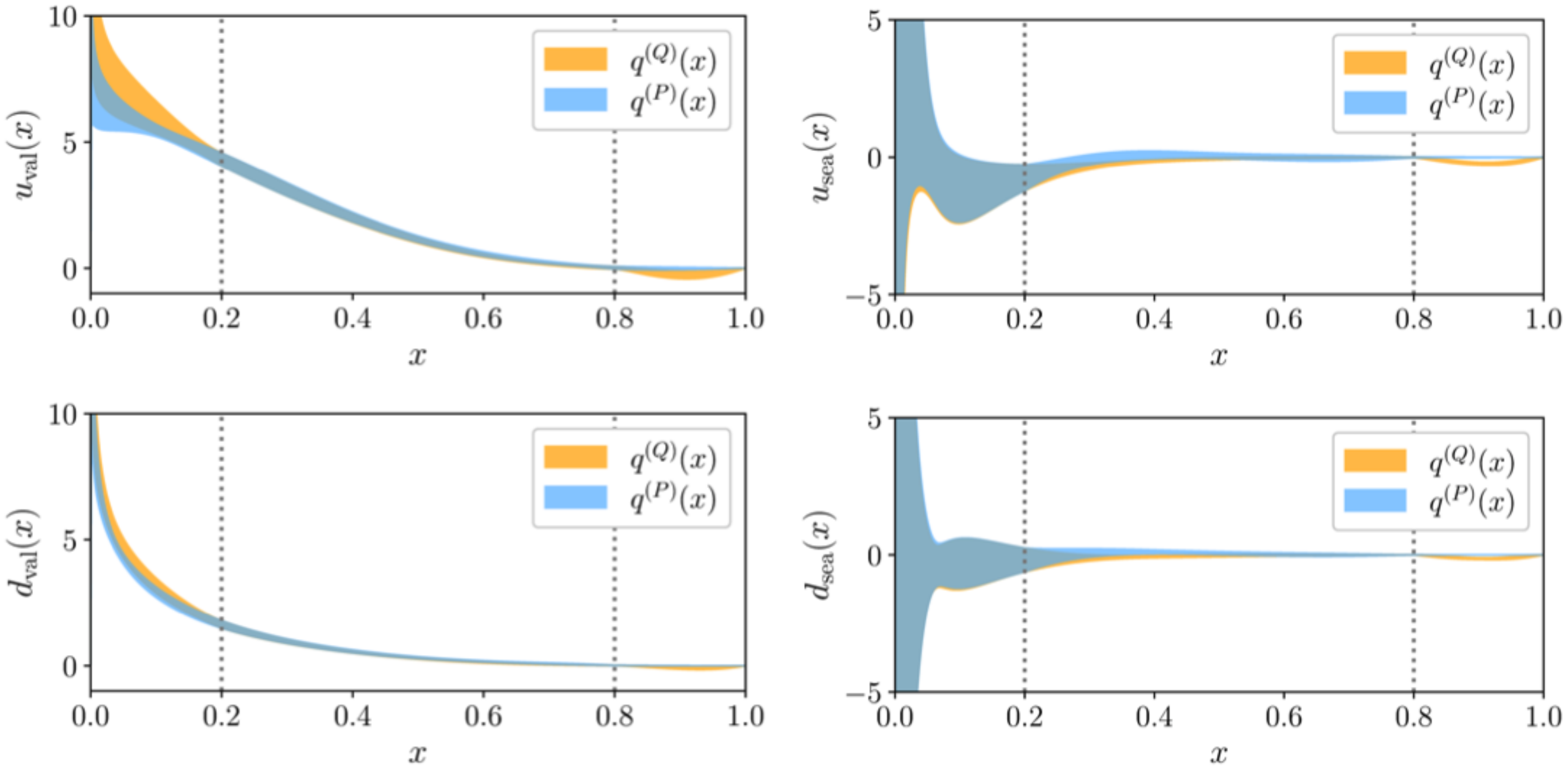
large P_z extrapolation

continuum limit…

numerical results for PDFs

fit qualities

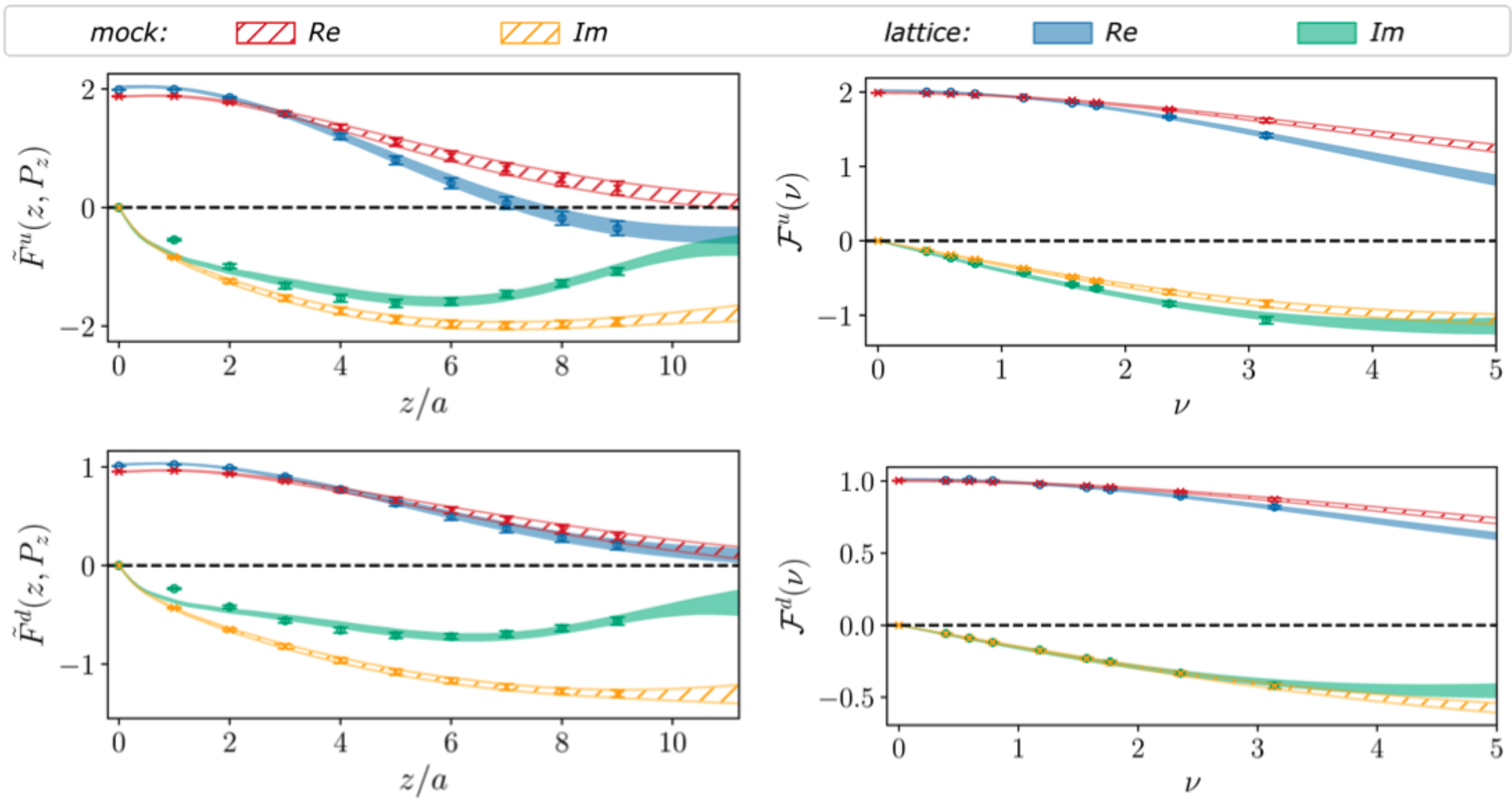
results of $q^{(Q)}$ and $q^{(P)}$



$q^{(0)} = q^{(P)}$ $\quad$ $q^{(0)} = q^{(Q)}$ $\quad$ $q^{(0)} = q^{(P)}$

fit qualities

compared with mock/lattice inputs

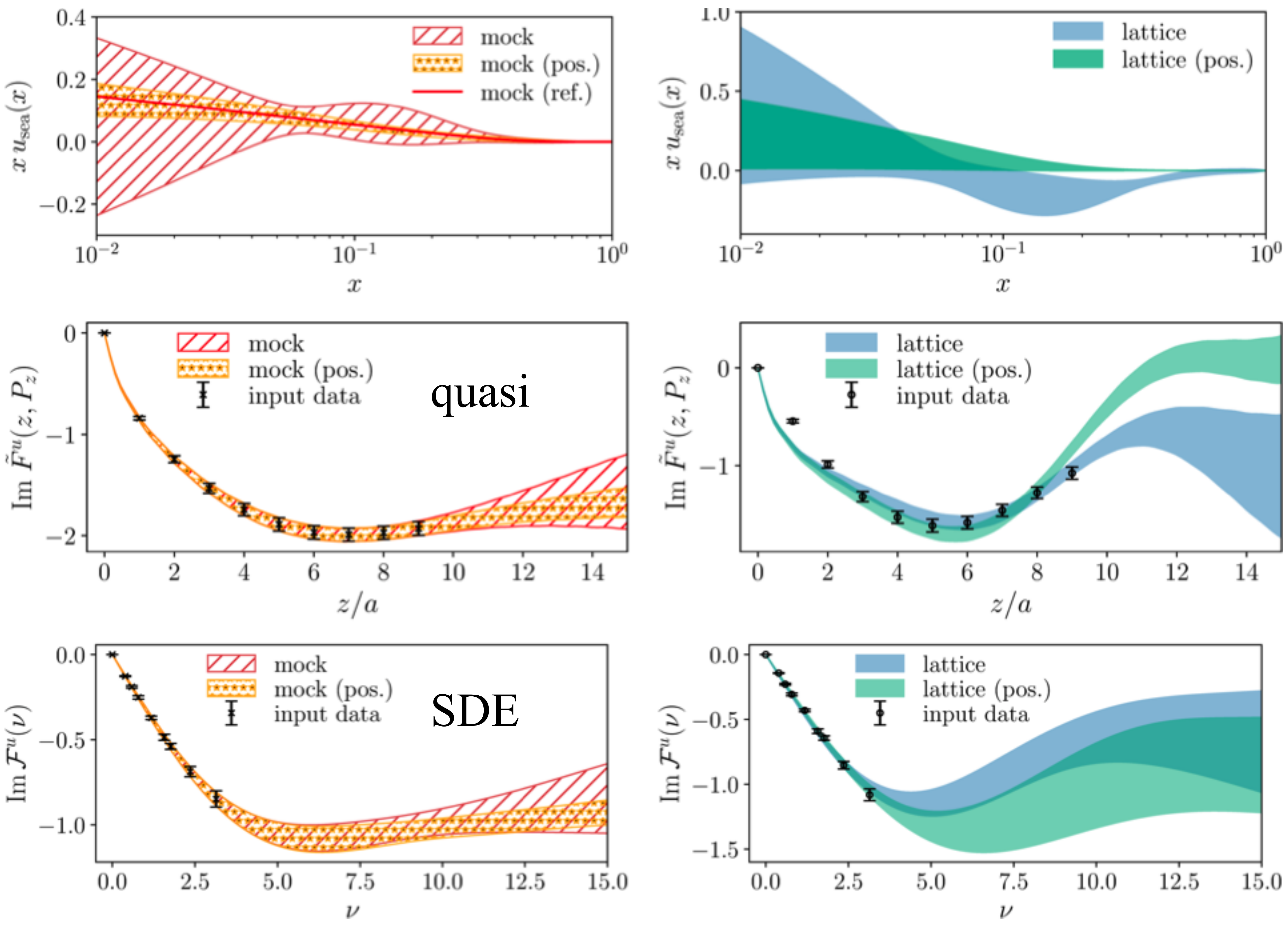


Within the range of the input data, ANN reconstruction accurately reproduces the original data.

numerical results for PDFs

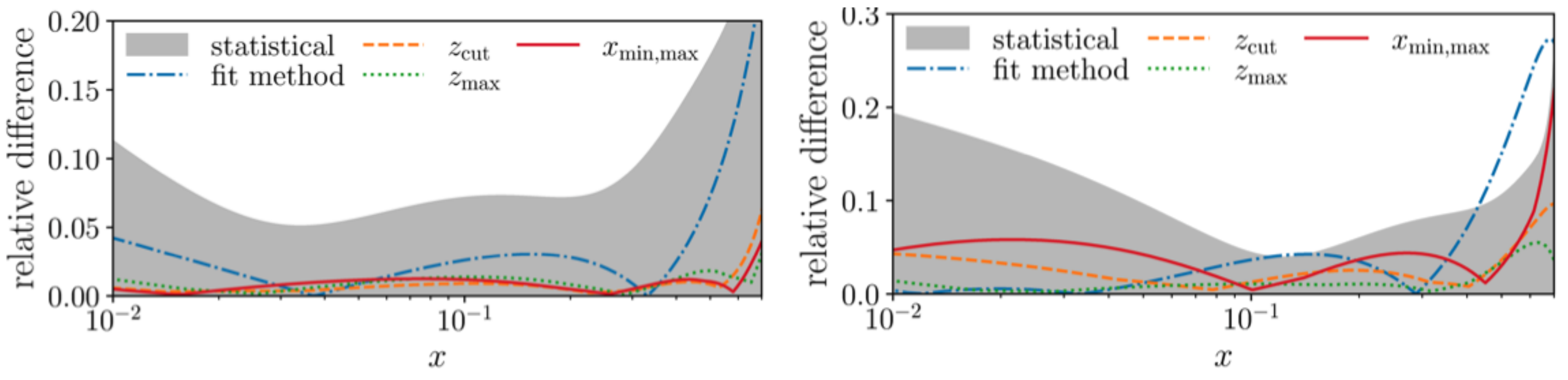
positivity test

mock data | lattice data



positivity constrain of $q^{(0)}(x)$ → better control of uncertainties

systematic study



$$\delta_{sys}(x) = \frac{q^{(0)}_{\rm ref}(x) - q^{(0)}_{sys}(x)}{q^{(0)}_{\rm ref}(x)}$$

- fit method: separate or joint fit of real and imaginary parts;
- $z_{\rm cut}$: truncation of quasi data. choices $z_{\rm cut} = 8a, 9a, 10a$;
- $z_{\rm max}$: maximum z used for SDE. choices: $z_{\rm cut} = 3a, 4a, 5a$;
- $x_{\rm min,max}$: boarders of regions. choices: {0.15,0.85}, {0.2,0.8}, {0.25,0.75};

conclusion

1. only fit method in large x region is compatible with statistical.
2. separate fit for lattice data is better to exclude imaginary contaminations of valence.

$q_{\rm val}(x) \to \mathrm{Re}[F(z)]$
$q_{\rm val}(x) + 2q_{\rm sea}(x) \to \mathrm{Im}[F(z)]$

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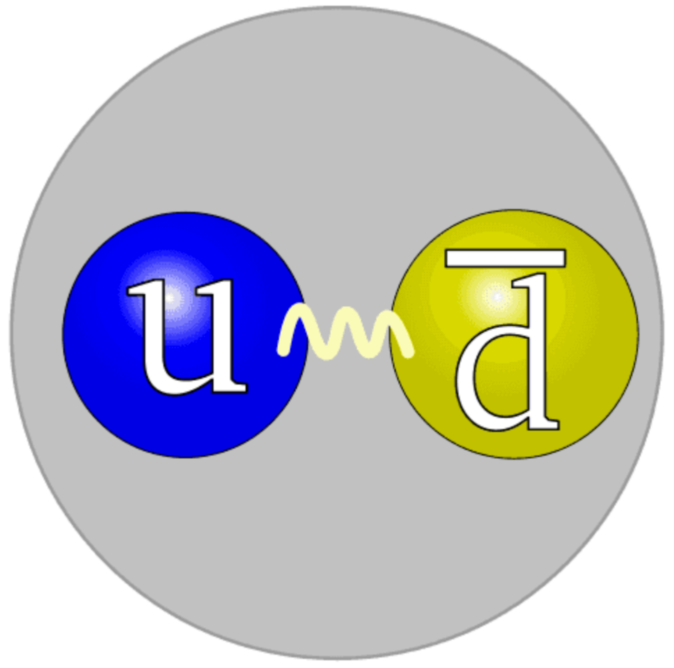
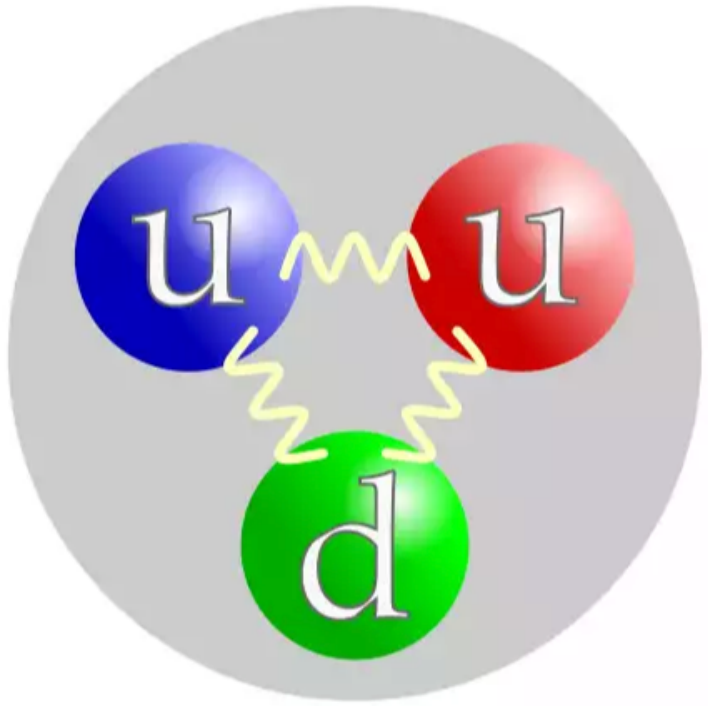




numerical results for GPDs

networks for GPDs

parametrization of light-cone GPDs

$$q^{(0)}(x) = x^{\delta^{(0)}}(1-x)^{\rho^{(0)}}\,\underline{\mathrm{ANN}^{(0)}(x,t)}$$

$$\mathrm{ANN}^{(0)}(x,t) = b_{21}^{(0)} + \sum_{i=1}^{N^{(0)}}\left[w_{2i}^{(0)}\ln\left(1+\exp(b_{1i}^{(0)}+w_{1i}^{(0)}x)\right)\times\exp\left(s_i^{(0)}(1-x)^2 t\right)\right]$$

t dependence (semi activation function)

normalization of H GPDs: $\int_0^1 dx\, H^u_{\mathrm{val}}(x,0) = 2,\ \int_0^1 dx\, H^d_{\mathrm{val}}(x,0) = 1$

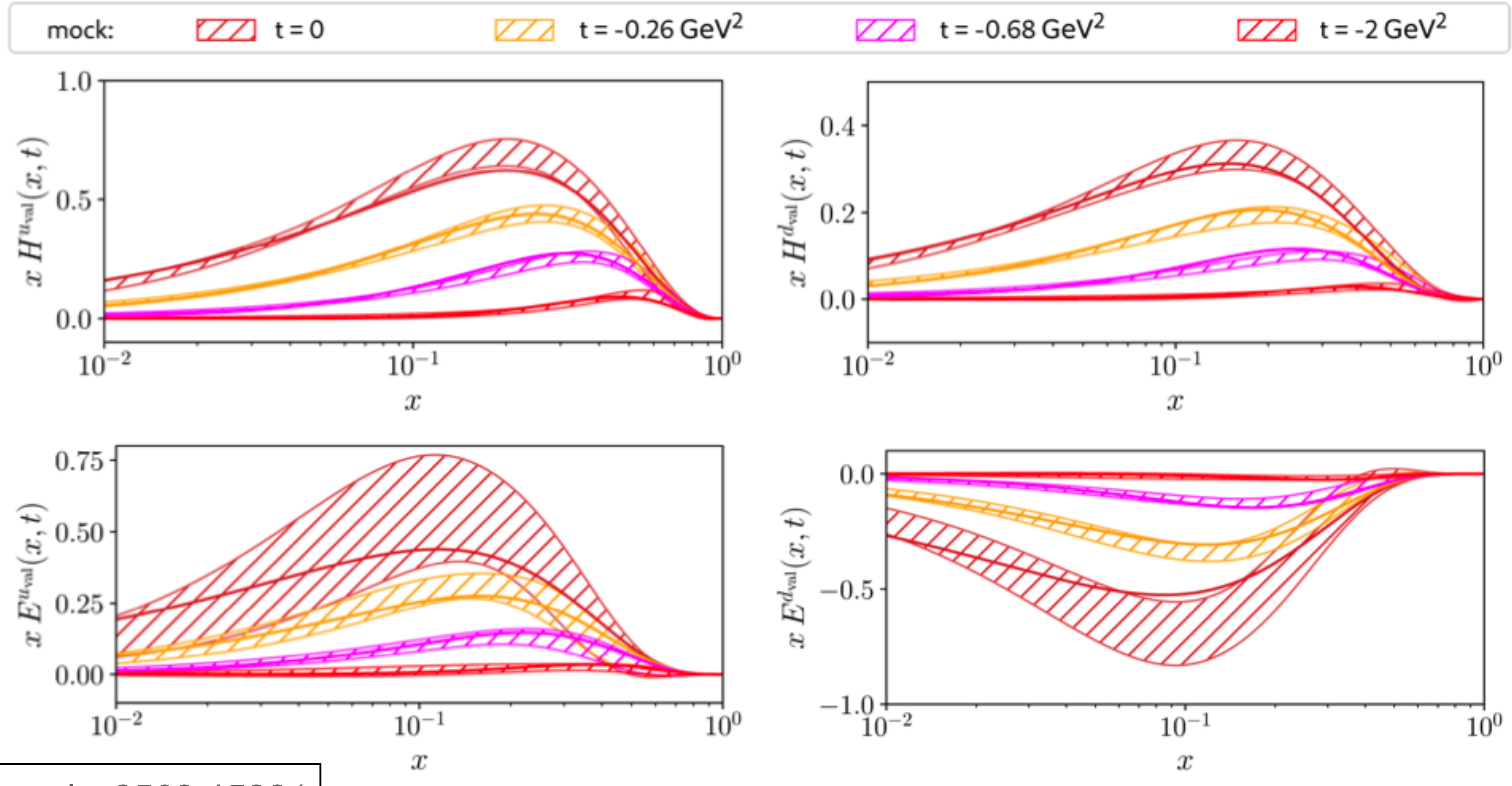
constrain in reconstruction

normalization of E GPDs: $\int_0^1 dx\, E^u_{\mathrm{val}}(x,0) = \kappa_u,\ \int_0^1 dx\, E^d_{\mathrm{val}}(x,0) = \kappa_d$

not constrained

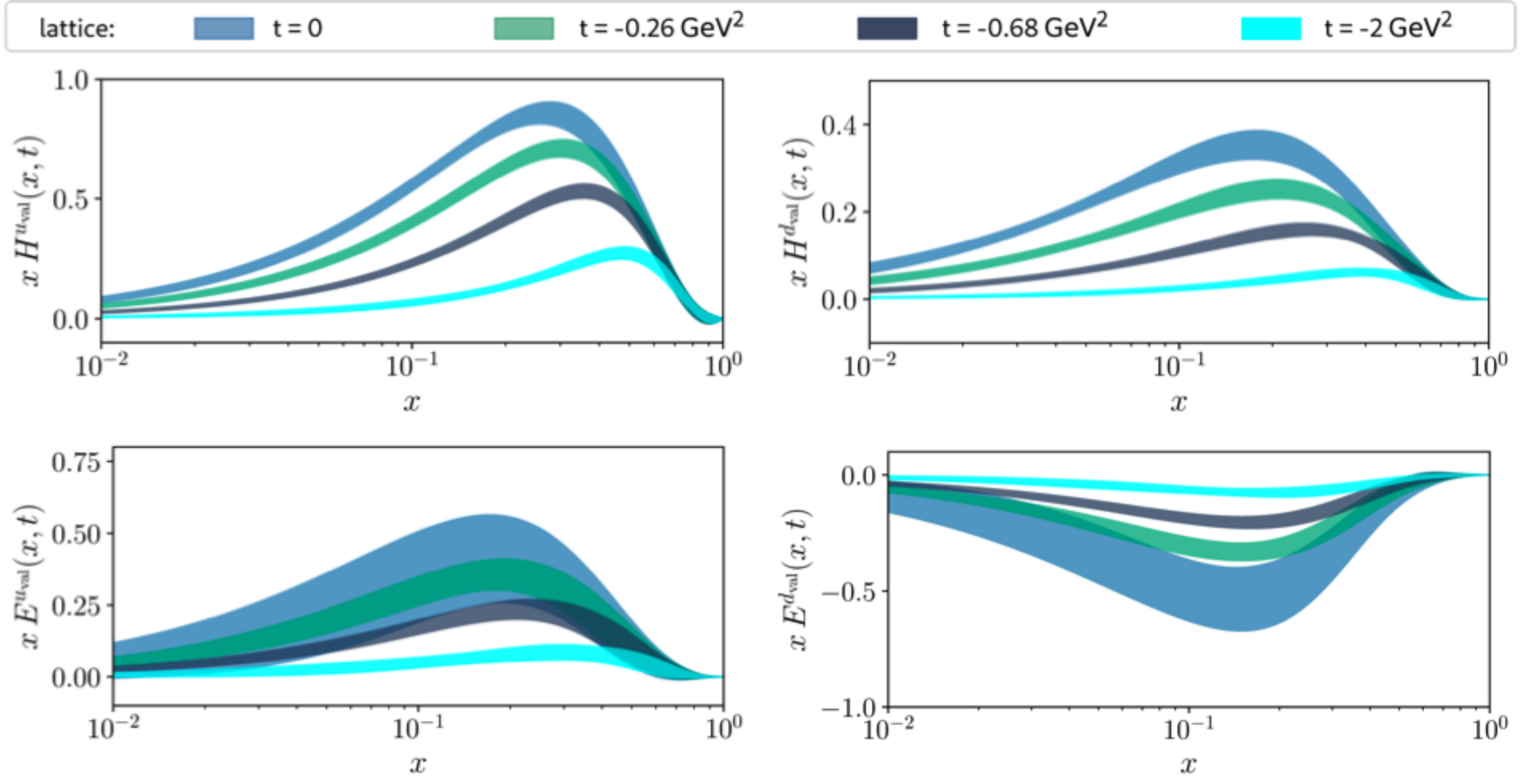
$\kappa_{u,d}$ contribute to anomalous magnetic moment of the proton

mock



M. Chu, et,al., arxiv: 2509.15931

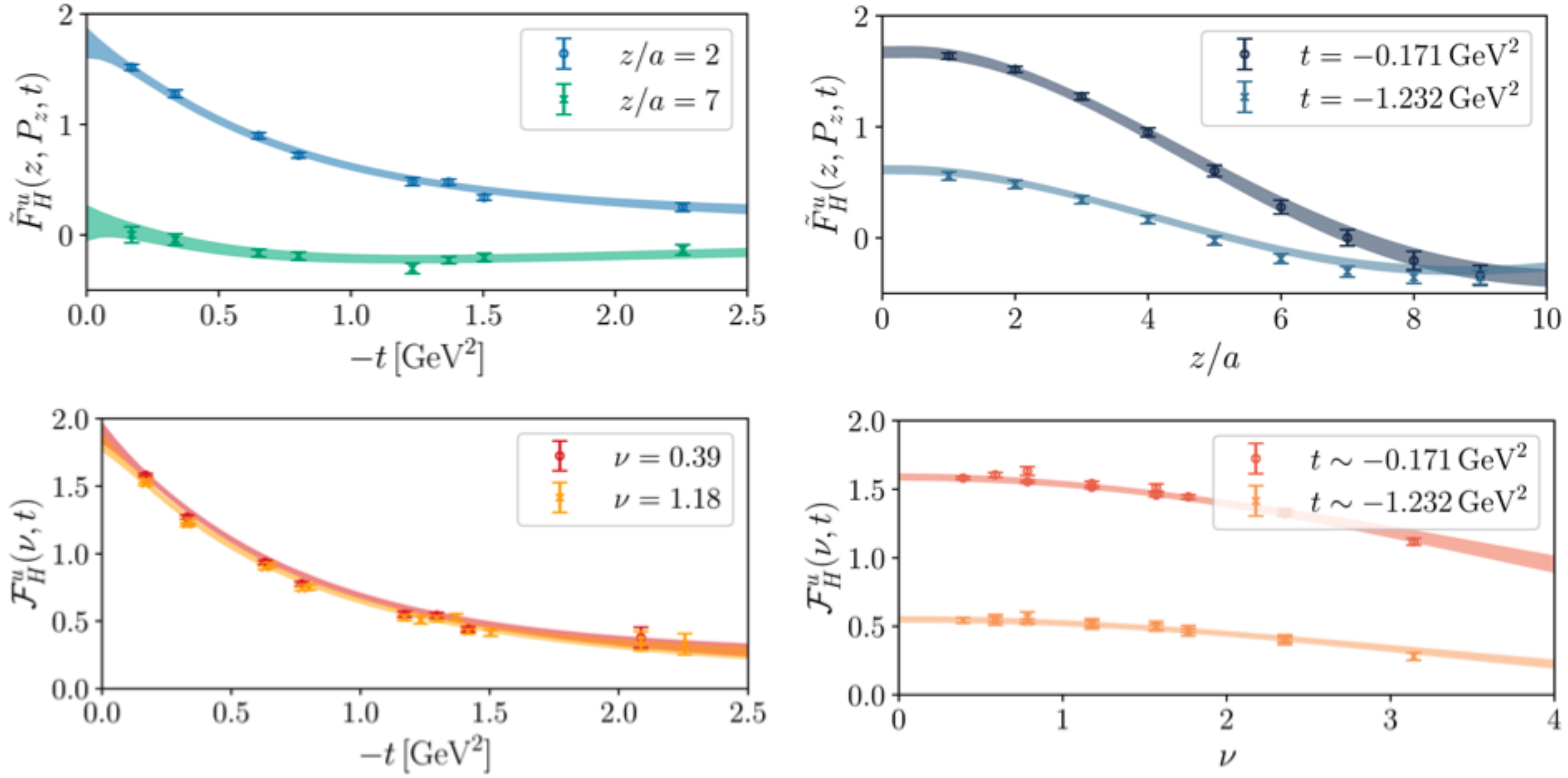
lattice



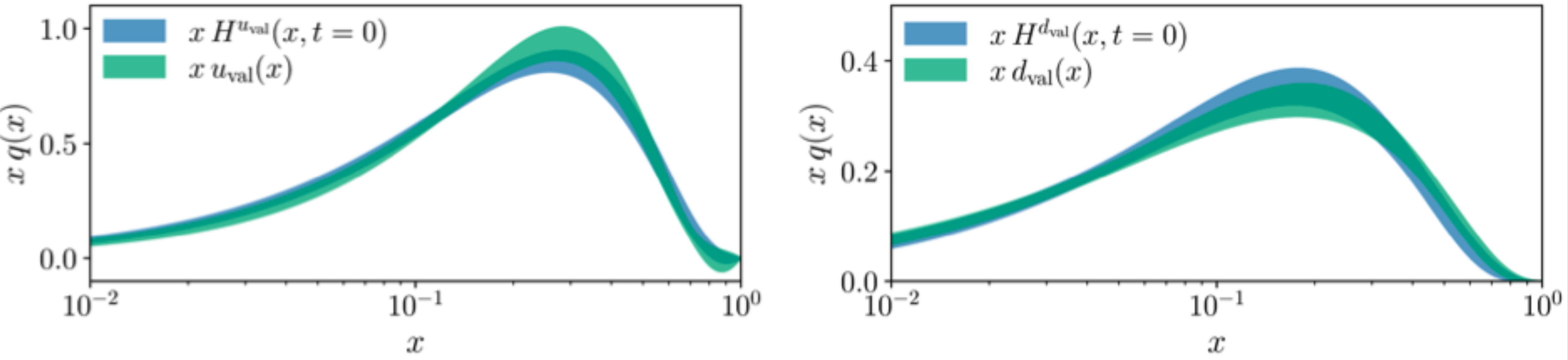
numerical results for GPDs

fit qualities

compared with mock/lattice inputs



forward limit of H GPDs and PDFs

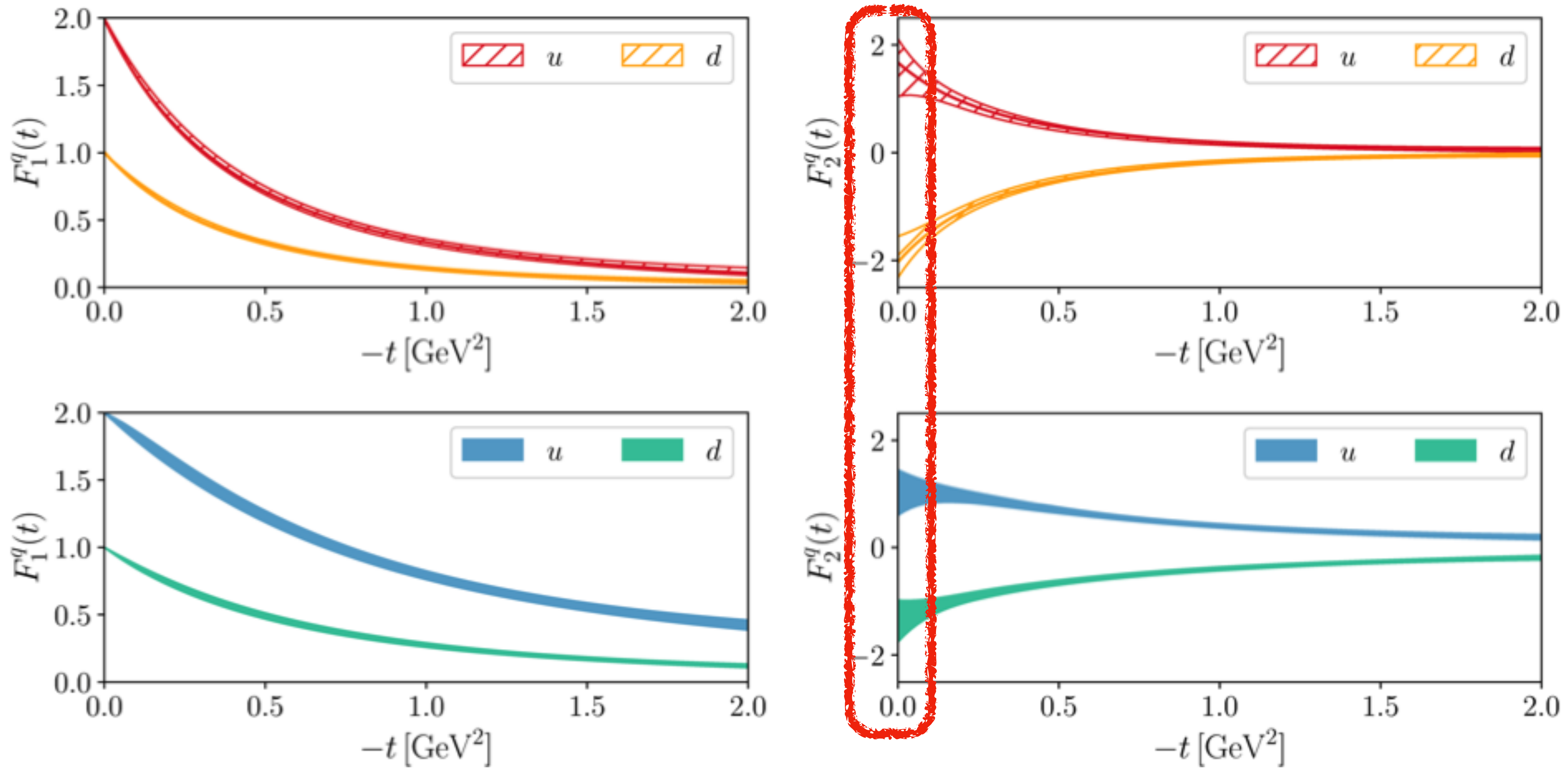


The agreement indicates the robustness of lattice inputs and our method!

elastic form factors

Dirac | Pauli

$$\int_{-1}^{1} dx\, H^q(x, \xi, t) \equiv F_1^q(t), \int_{-1}^{1} dx\, E^q(x, \xi, t) \equiv F_2^q(t)$$



anomalous magnetic moment: $\kappa_p = \frac{2}{3}\kappa_u - \frac{1}{3}\kappa_d = 1.14 \pm 0.26$

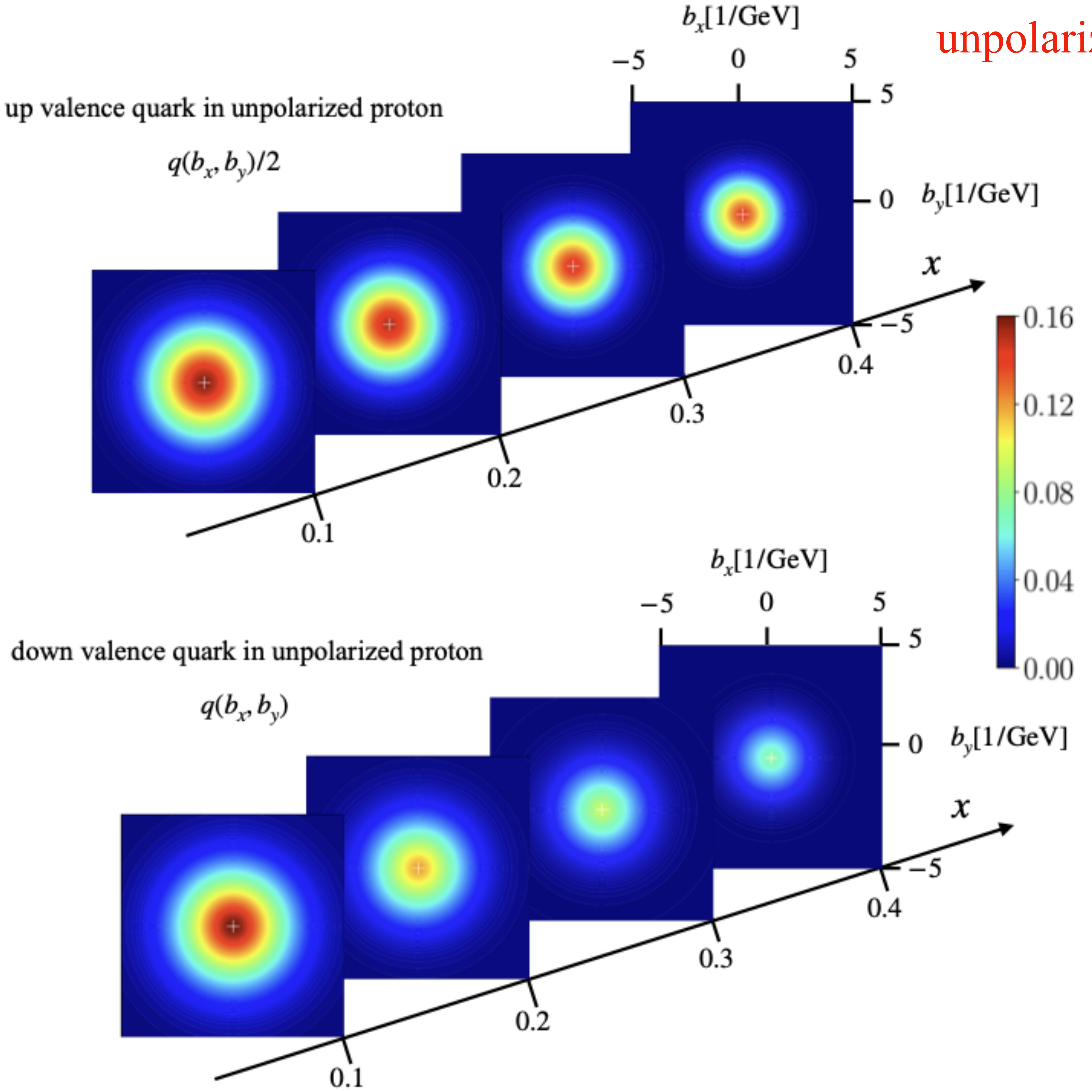
$\kappa_p^{\mathrm{exp}} = 1.792847350(9)$

A. Mooser, et,al., Nature 509 (2014) 596-599

numerical results for GPDs

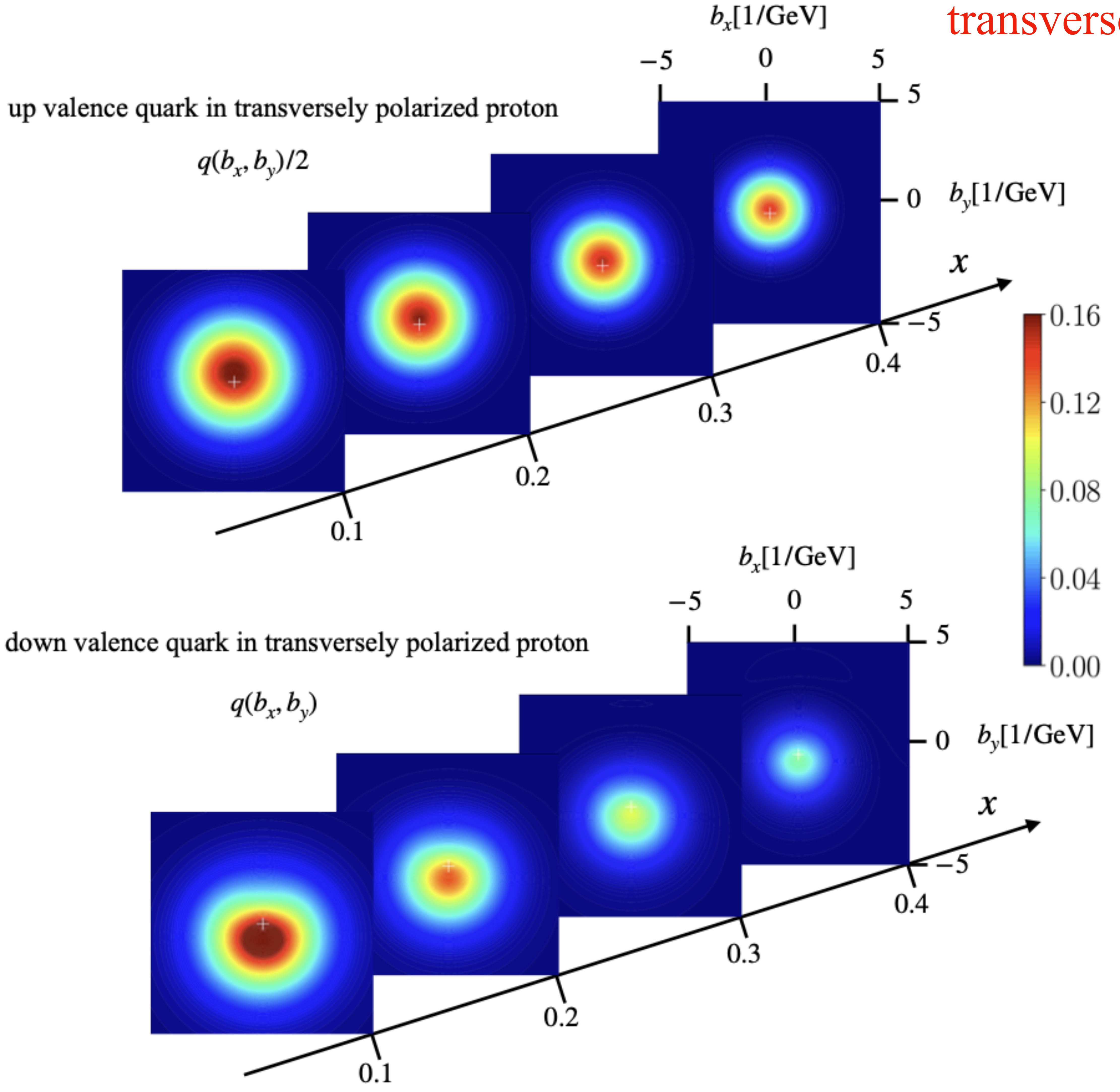
tomography: $q(x,\mathbf{b}_\perp) = \int \frac{d^2\mathbf{\Delta}_\perp}{(2\pi)^2} e^{-i\mathbf{b}_\perp\cdot\mathbf{\Delta}_\perp} H^q(x,0,-\mathbf{\Delta}_\perp^2)$

unpolarized



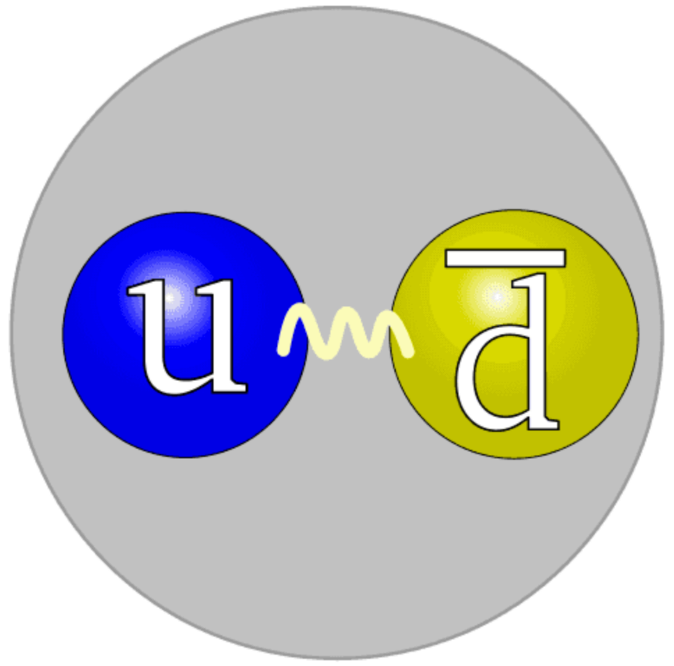
tomography: $q_X(x,\mathbf{b}_\perp) = q(x,\mathbf{b}_\perp) - \frac{1}{2M}\frac{\partial}{\partial b_y} e_q(x,\mathbf{b}_\perp)$

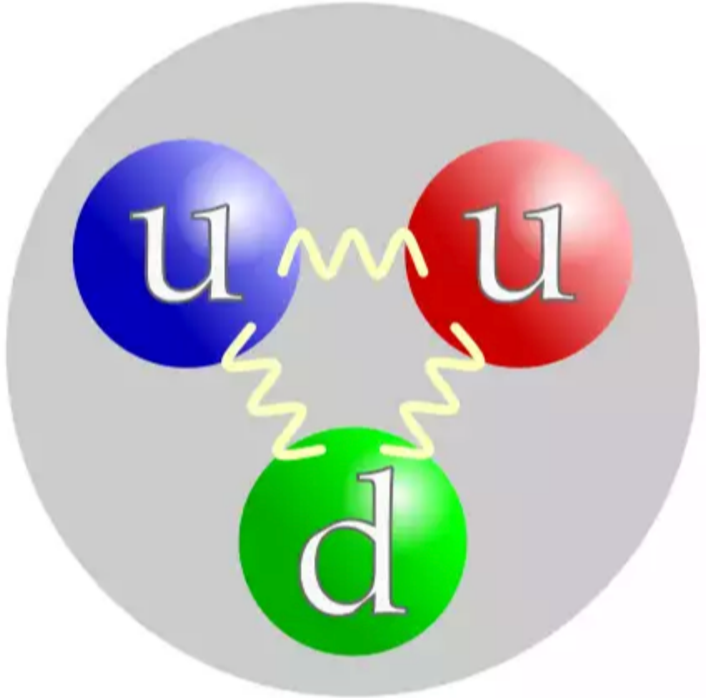
transversely



Outline

- Motivation
- neuron network reconstruction
- numerical results of PDFs
- numerical results of GPDs at $\xi = 0$
- summary and outlook





Summary and outlook

- This work introduces a unified neural-network framework that extracts light-cone PDFs and zero-skewness GPDs from lattice QCD by fitting of LaMET inputs with SDE ones.
- Using neuron networks with power-law factors and auxiliary components, the method outperforms traditional approaches, yielding stable results of PDFs.
- The framework is extended to zero-skewness GPDs, providing the reconstruction of H and E GPDs along with elastic form factors, nucleon tomography, and proton angular momentum estimates.
- Future improvements in lattice inputs will reduce uncertainties, enhance the reliability, and improve nucleon structure studies with experiments like the Electron-Ion Collider.

Thank you!