Twist-2 and twist-3 tensor GPDs for the proton from lattice QCD

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LaMET 2025

Stony Brook University

08/10/2024

Outline

- ❖ Work extends the approach for the unpolarized and axial cases
- ❖ Theoretical Formulation
- ❖ Lattice Formulation (focus of this talk)
  - ❖ Background
  - ❖ Lattice Methodology
  - ❖ Results: Twist-2
    - Matrix Elements
    - Lorentz invariant amplitudes
    - Quasi-GPDs
    - Light-Cone GPDs
  - ❖ Results: Twist-3

PHYSICAL REVIEW D **106,** 114512 (2022)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[3,†] Jack Dodson,[3] Xiang Gao,[4] Andreas Metz,[3] Swagato Mukherjee,[1] Aurora Scapellato,[3] Fernanda Steffens,[5] and Yong Zhao[4]

PHYSICAL REVIEW D **109,** 034508 (2024)

Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Axial-vector case

Shohini Bhattacharya,[1,*] Krzysztof Cichy,[2] Martha Constantinou,[2,†] Jack Dodson,[2] Xiang Gao,[3] Andreas Metz,[2] Joshua Miller,[2,‡] Swagato Mukherjee,[4] Peter Petreczky,[4] Fernanda Steffens,[5] and Yong Zhao[3]

Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Tensor case

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Generalized Parton Distributions

❖ **GPDs are rich in information:**

- Reflect spatial distribution of partons in transverse plane
- Hadron mechanical properties are stored in GPDs
- Information on spin

❖ **… but not well studied:**

- extracted from off-forward kinematic (unlike PDFs)
- Multi-variable quantities; dependence upon x, t and ξ (unlike PDFs)
- Inferred from Compton form factors from experimental data (e.g., DVCS)
- Other processes proposed (**SDHEP [J. Qiu et al, arXiv:2205.07846]**) still require theoretical developments

❖ **Transversity proton GPDs:**

- Four GPDs: H_T, E_T, $\widetilde{H}_T$, $\widetilde{E}_T$

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

Methodology on the Lattice

❖ Choice of frame:

- Symmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p}_f = P_3\hat{z} + \vec{\Delta}/2$
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \ \vec{p}_f = P_3\hat{z}$

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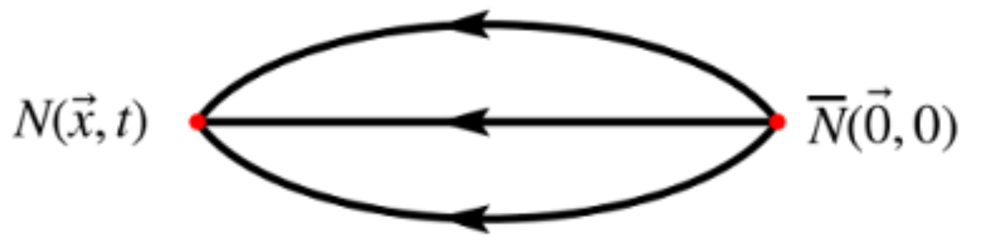
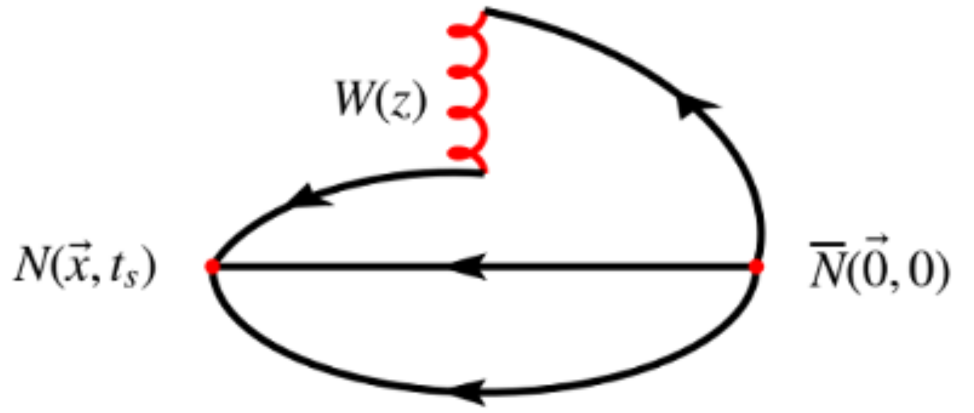
- Asymmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}, \vec{p}_f = P_3\hat{z}$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen



$\langle N(P) | N(P) \rangle$



$\langle N(P_f) | \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) | N(P_i) \rangle$

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- Symmetric: $\vec{p}_i = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p}_f = P_3\hat{z} + \vec{\Delta}/2$
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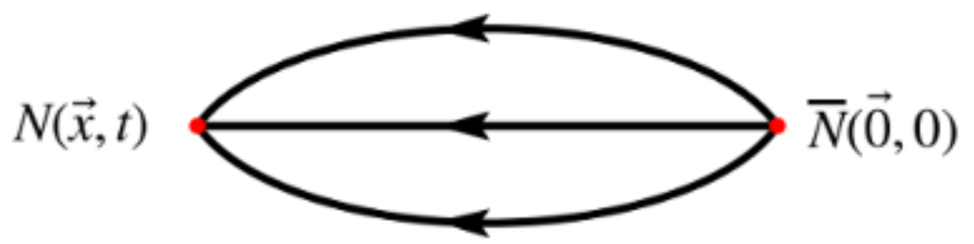

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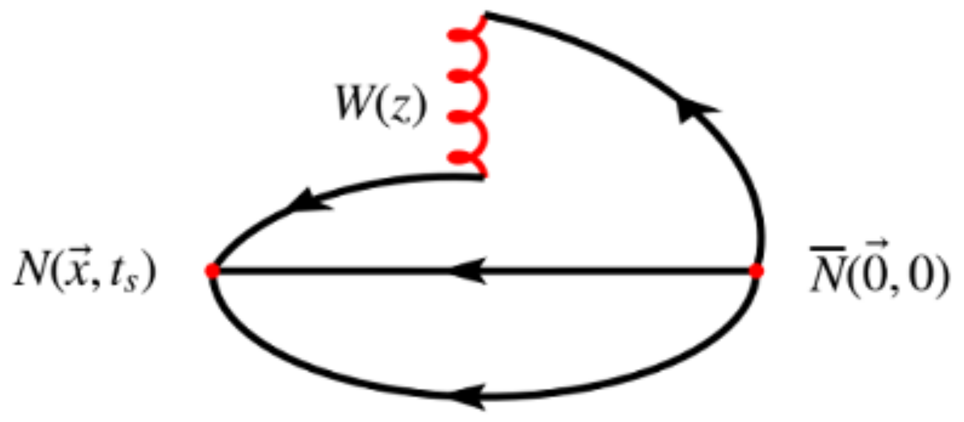
❖ Calculate the 2-, 3-point correlation functions in the frame chosen



$\langle N(P_f) | \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) | N(P_i) \rangle$

❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C_\mu^{3\text{pt}}(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

Methodology on the Lattice

❖ Choice of frame:

- Symmetric: $\vec{p_i} = P_3\hat{z} - \vec{\Delta}/2, \quad \vec{p_f} = P_3\hat{z} + \vec{\Delta}/2$

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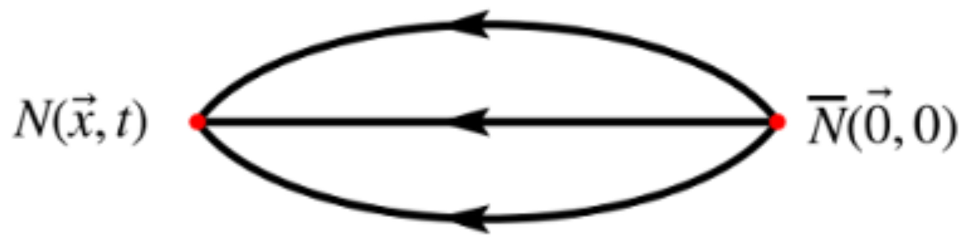
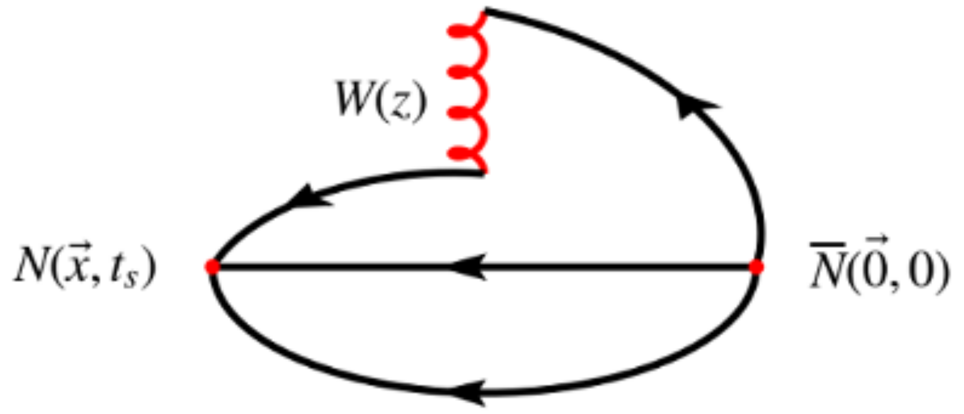


$\langle N(P) \,|\, N(P) \rangle$



$\langle N(P_f) \,|\, \bar{\Psi}(z) i\sigma^{\mu\nu}\gamma_5 \mathscr{W}(z,0)\Psi(0) \,|\, N(P_i) \rangle$

❖ Calculate the 2-, 3-point correlation functions in the frame chosen

❖ Take an appropriate ratio of the 2- and 3-point correlation functions and isolate the ground state (plateau fit)

$$R_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau) = \frac{C^{3\text{pt}}_\mu(\Gamma_\kappa, z, p_f, p_i; t_s, \tau)}{C^{2\text{pt}}(\Gamma_0, p_f; t_s)} \sqrt{\frac{C^{2\text{pt}}(\Gamma_0, p_i, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_f, \tau) C^{2\text{pt}}(\Gamma_0, p_f, t_s)}{C^{2\text{pt}}(\Gamma_0, p_f, t_s - \tau) C^{2\text{pt}}(\Gamma_0, p_i, \tau) C^{2\text{pt}}(\Gamma_0, p_i, t_s)}} \xrightarrow[\tau \gg a]{t_s - \tau \gg a} \Pi_\mu(\Gamma_\kappa, z, p_f, p_i; t_s)$$

❖ Parameterize the matrix elements in terms of (Lorentz invariant) amplitudes

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

$$+ i\epsilon^{\mu\nu P z}A_{11} + i\epsilon^{\mu\nu z\Delta}A_{12}$$

→ 12 linearly independent Lorentz invariant amplitudes

Methodology on the Lattice

SF: Symmetric Frame

AF: Asymmetric Frame

❖ Equate and relate the amplitude and quasi-GPD decomposition

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

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GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

$$\longrightarrow \quad \widetilde{E}_T = -2A_6 - \frac{2E^2}{P}zA_8$$

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GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8 + A_{10}$$

$$\longrightarrow \quad E_T = 2A_2 - A_4 + \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8$$

$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

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GPDs defined in SF with A_i in SF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{E^2 - P_3^2}{m^2}\right) + A_4 + A_{10}$$

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$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8 + A_{10}$$

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❖ Renormalization functions: RI-MOM

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GPDs defined in SF with A_i in SF

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GPDs defined in SF with A_i in AF

$$\longrightarrow \quad H_T = -2A_2\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_4 - \frac{(E_f + E_i)(E_f - E_i)}{2P_3}zA_8 + A_{10}$$

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❖ Renormalization functions: RI-MOM

❖ Transform from position to momentum space (Backus-Gilbert) [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

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$$\longrightarrow \quad \widetilde{H}_T = -A_2 - \frac{m^2}{P_3}zA_{12}$$

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❖ Renormalization functions: RI-MOM

❖ Transform from position to momentum space (Backus-Gilbert) [Backus & Gilbert, Geophysical Journal International 16, 169 (1968)]

❖ Extract light cone-GPDs using one-loop matching formalism [Liu, et al., Phys. Rev. D 100, 034006 (2019)]

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^s_{03}(\Gamma_0) = iK\left(\frac{P_3(\Delta_1^2+\Delta_2^2)}{4m^3}A_{T6} - \frac{E(E(E+m)-P_3^2)}{2m^2}zA_{T11}\right)$$

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

$$\Pi^a_{03}(\Gamma_0) = iK\left(\frac{(E_f-E_i)(E_f+E_i)P_3}{8m^3}A_{T4} + \frac{(E_i^2-E_f^2)P_3}{4m^3}A_{T6} + \frac{(E_i-E_f)P_3}{4m^2}A_{T10} + \frac{(E_f+E_i)(E_f-E_i-2m)(E_f+m)}{8m^2}zA_{T11} - \frac{(E_f-E_i)(E_f-E_i-2m)(E_f+m)}{4m^2}zA_{T12}\right)$$

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$$\Pi^s_i \neq \Pi^a_i$$

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Coefficients become more complicated

Amplitudes frame independent

Asymmetric frame ($\xi = 0$)

$$\Pi^a_{01}(\Gamma_0) = iK\left(\frac{(m-E_f)(m+E_f)\Delta_1}{4m^3}A_{T4} + \frac{P_3\Delta_1}{4m}zA_{T5} + \frac{(E_f+m)\Delta_1}{4m^2}A_{T10}\right)$$

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$$\Pi^s_i \neq \Pi^a_i \qquad A^s_{Ti} = A^a_{Ti}$$

Decomposition

Symmetric frame ($\xi = 0$)

$$\Pi^s_{01}(\Gamma_0) = iK\left(-\frac{\Delta_1 P_3^2}{4m^3}A_{T4} + \frac{\Delta_1 P_3}{4m}zA_{T5} + \frac{(E+m)\Delta_1}{4m^2}A_{T10}\right)$$

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$$\Pi^s_i \neq \Pi^a_i \qquad A^s_{Ti} = A^a_{Ti}$$

★ **Frame dependence of matrix elements due to kinematic coefficients of A_{Ti}** ★

Lattice Setup

❖ $N_f = 2 + 1 + 1$ Twisted mass fermions with a clover term

Parameters							
Ensemble	β	a [fm]	volume $L^3 \times T$	N_f	m_π [MeV]	Lm_π	L [fm]
cA211.32	1.726	0.093	$32^3 \times 64$	u, d, s, c	260	4	3.0

Lattice Setup

❖ $N_f = 2+1+1$ Twisted mass fermions with a clover term

Parameters							
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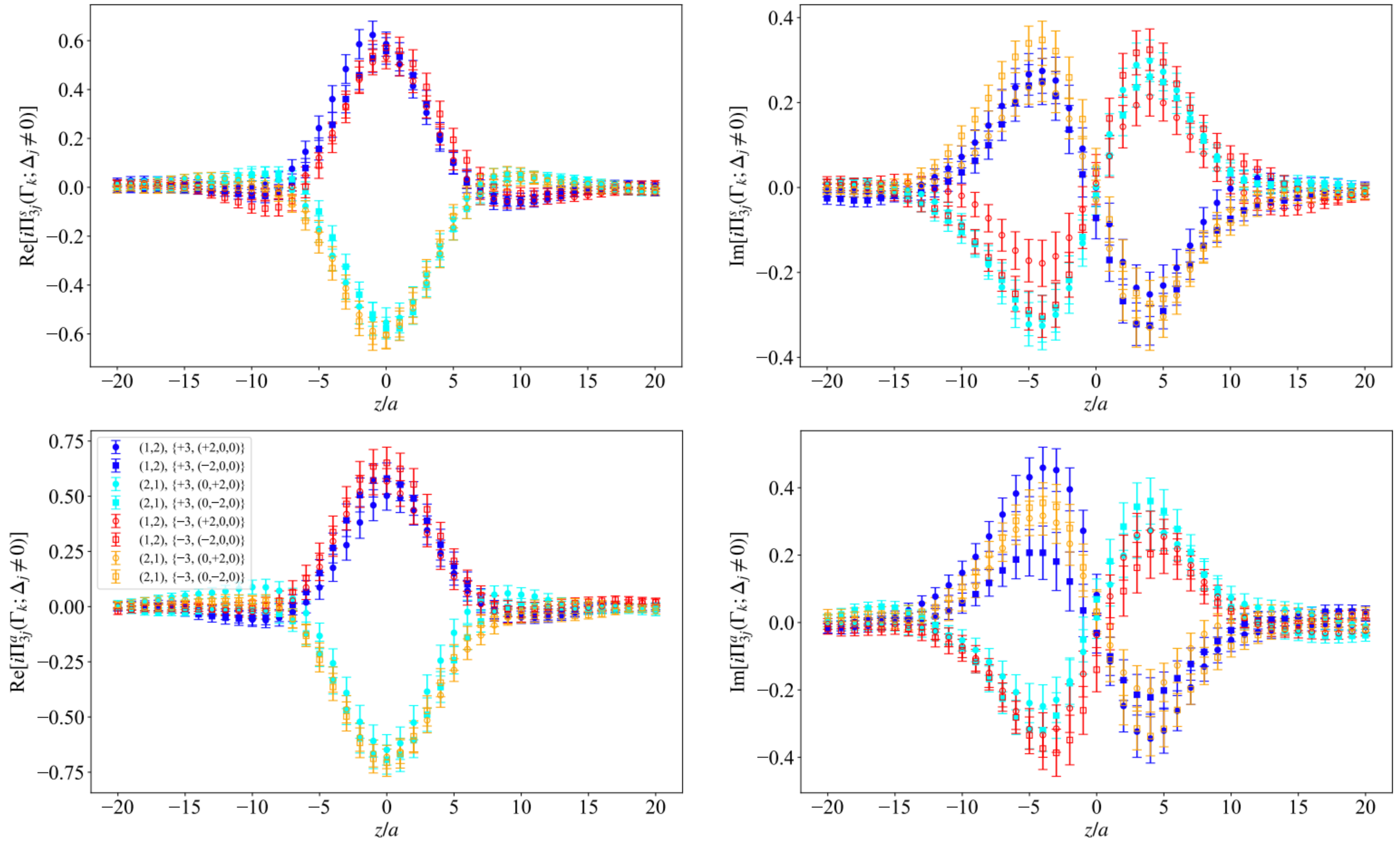
❖ Exploitation of $\tilde{A}_i$ symmetry properties with respect to $(\pm P_3, \pm\vec{\Delta}, \pm z)$

Matrix Elements: $\Pi^{s/a}_{3j}(\Gamma_k, \Delta_j \neq 0)$

$|P_3| = 1.25\,\text{GeV} \quad -t = 0.69\ \text{GeV}$



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Matrix Elements: $\Pi^{s/a}_{3j}(\Gamma_k, \Delta_j = 0)$

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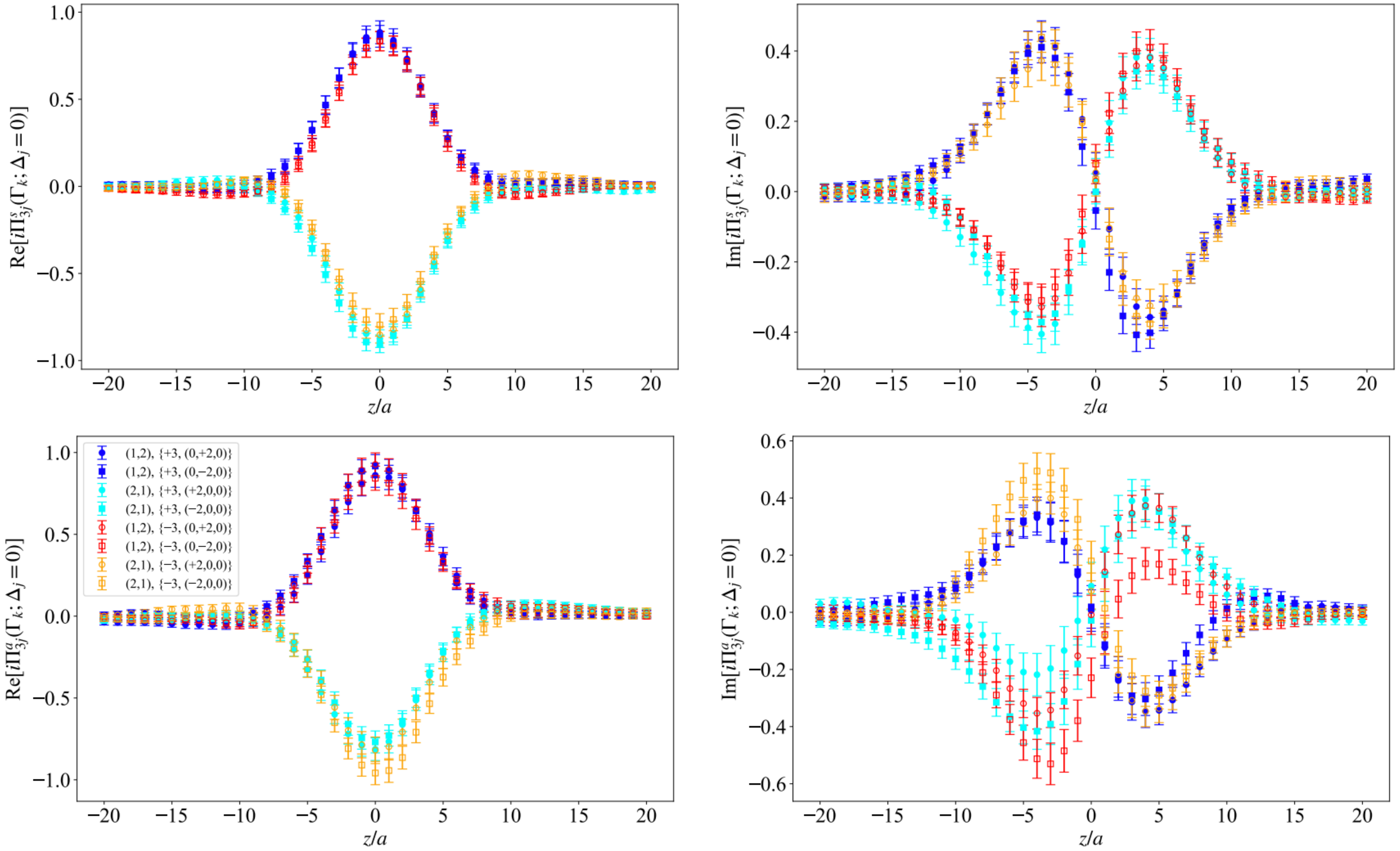

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- Disentangling amplitudes gets more difficult in (any) non-symmetric frame compared to the symmetric one
- Amplitudes are frame independent by construction
- Coefficients are frame dependent

Transversity Amplitudes

Ratio

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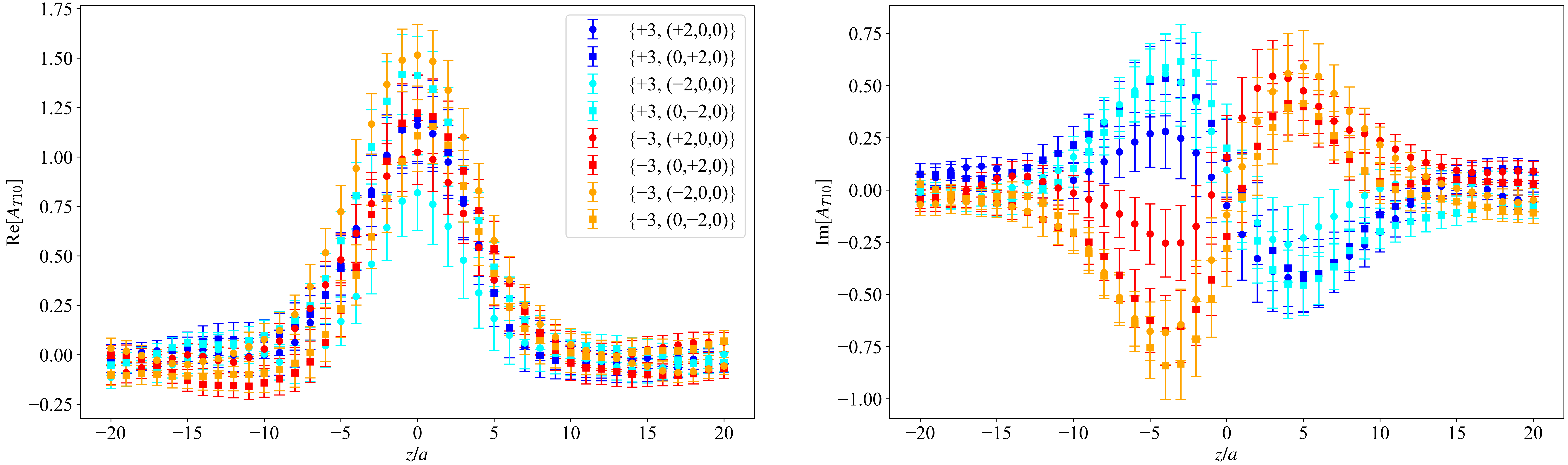
Ratio → *Plateau*

Transversity Amplitudes

Ratio → *Plateau* → *Amplitudes*

Example: A_{T10}



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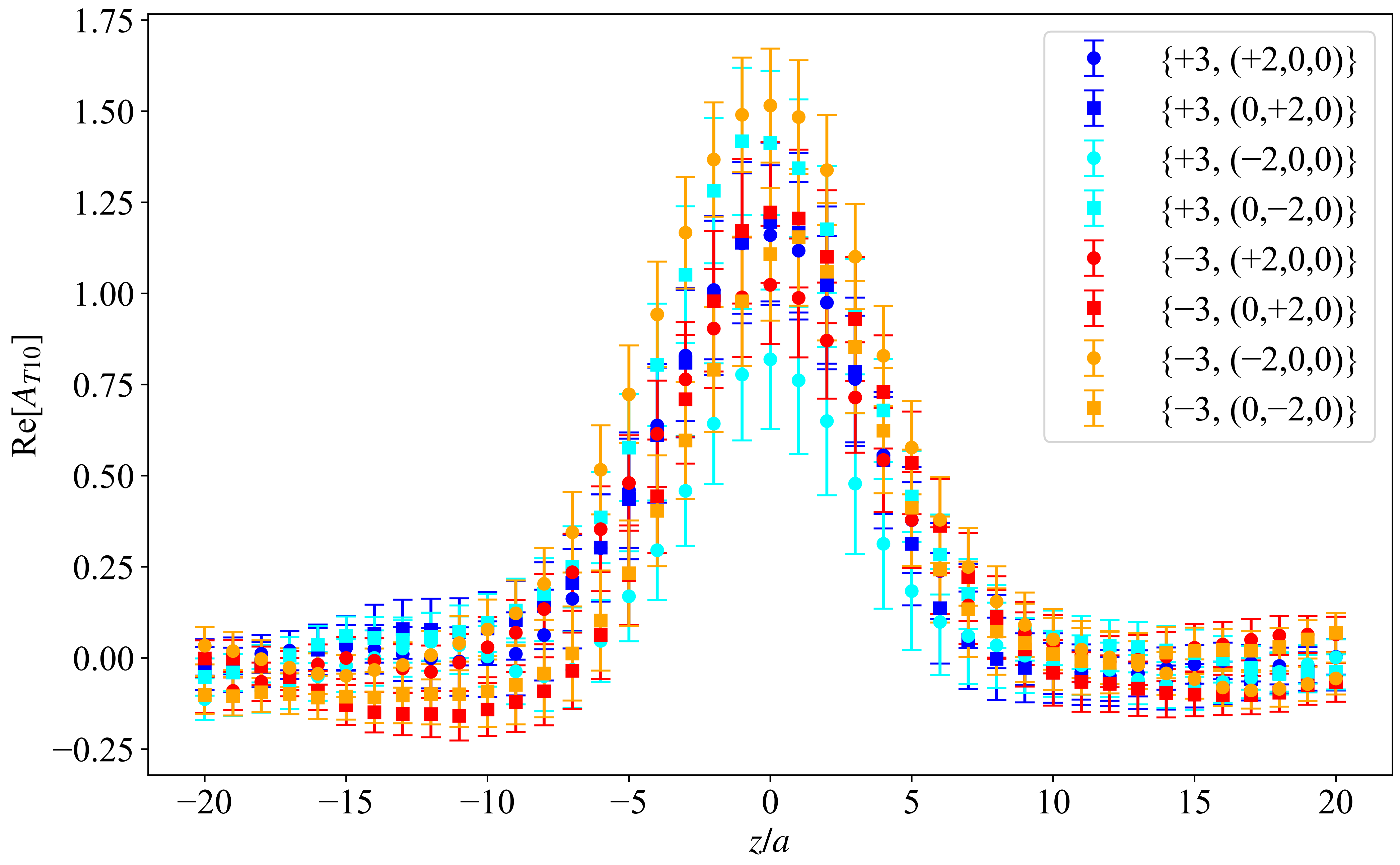

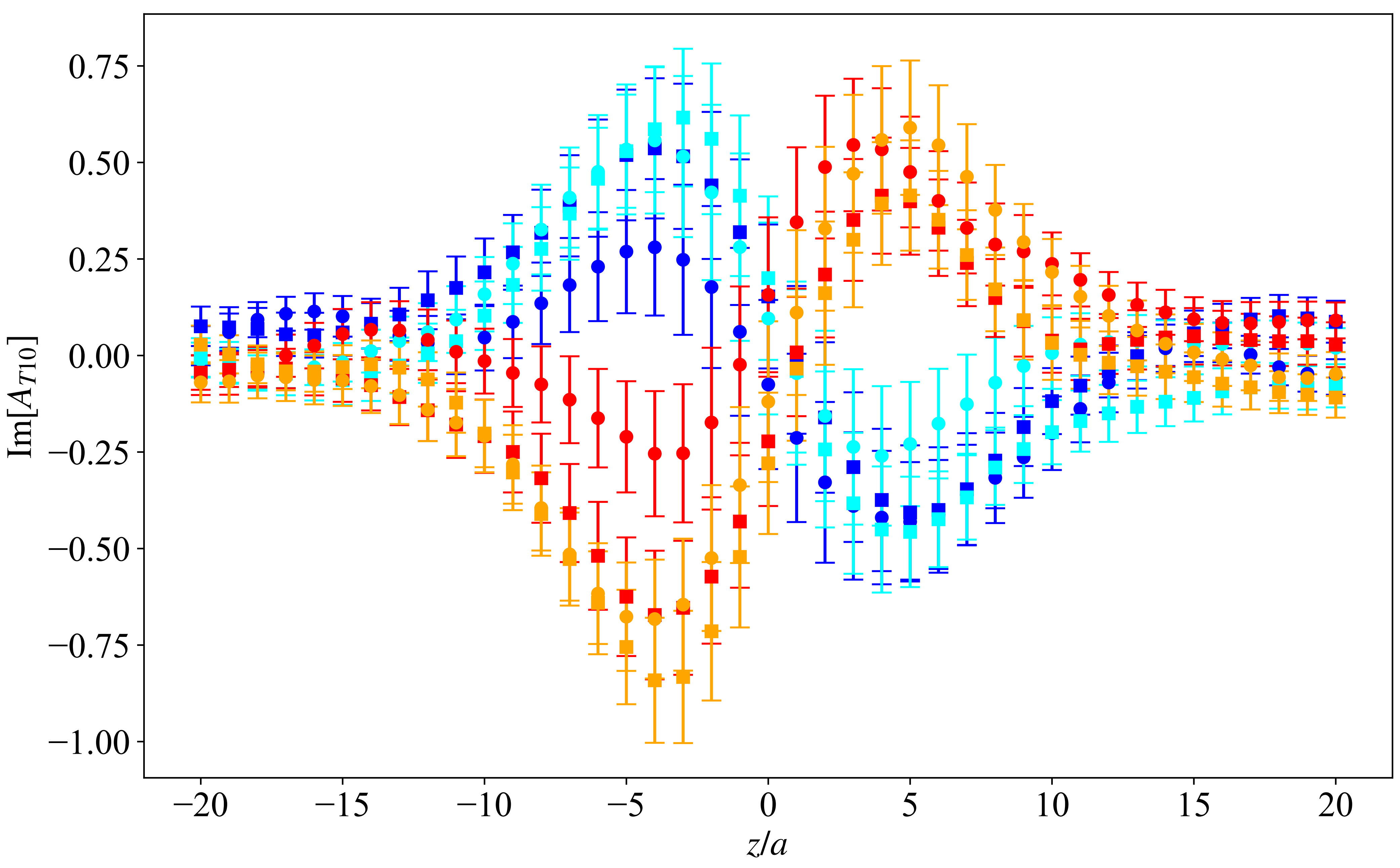

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→ *Average*

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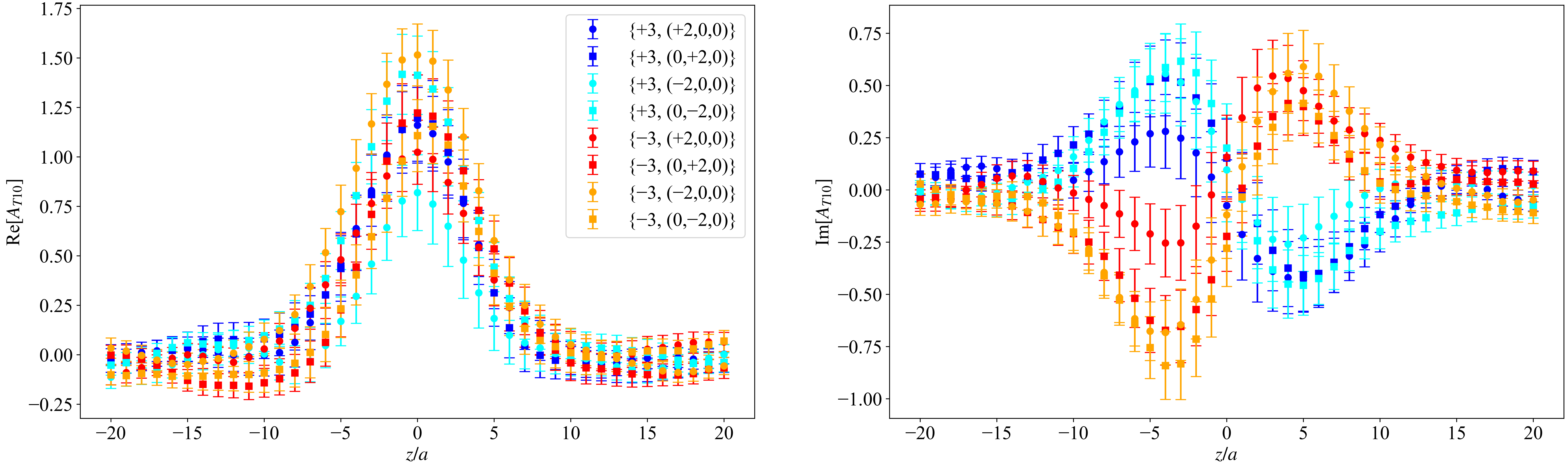

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→ *Average* → *Quasi-GPDs*

Transversity Amplitudes

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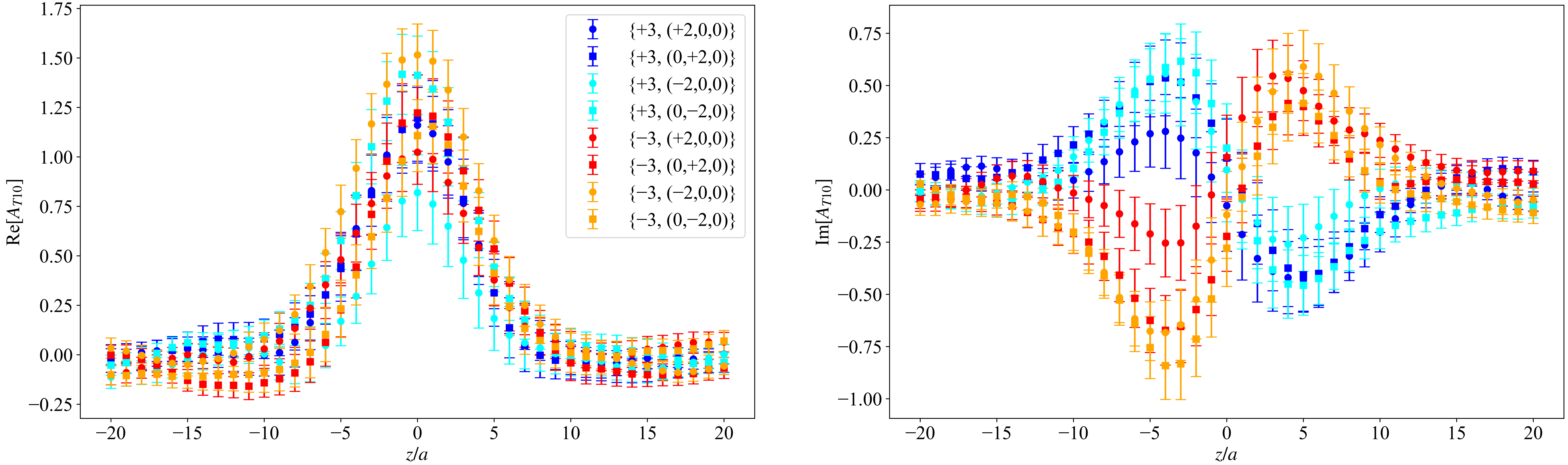

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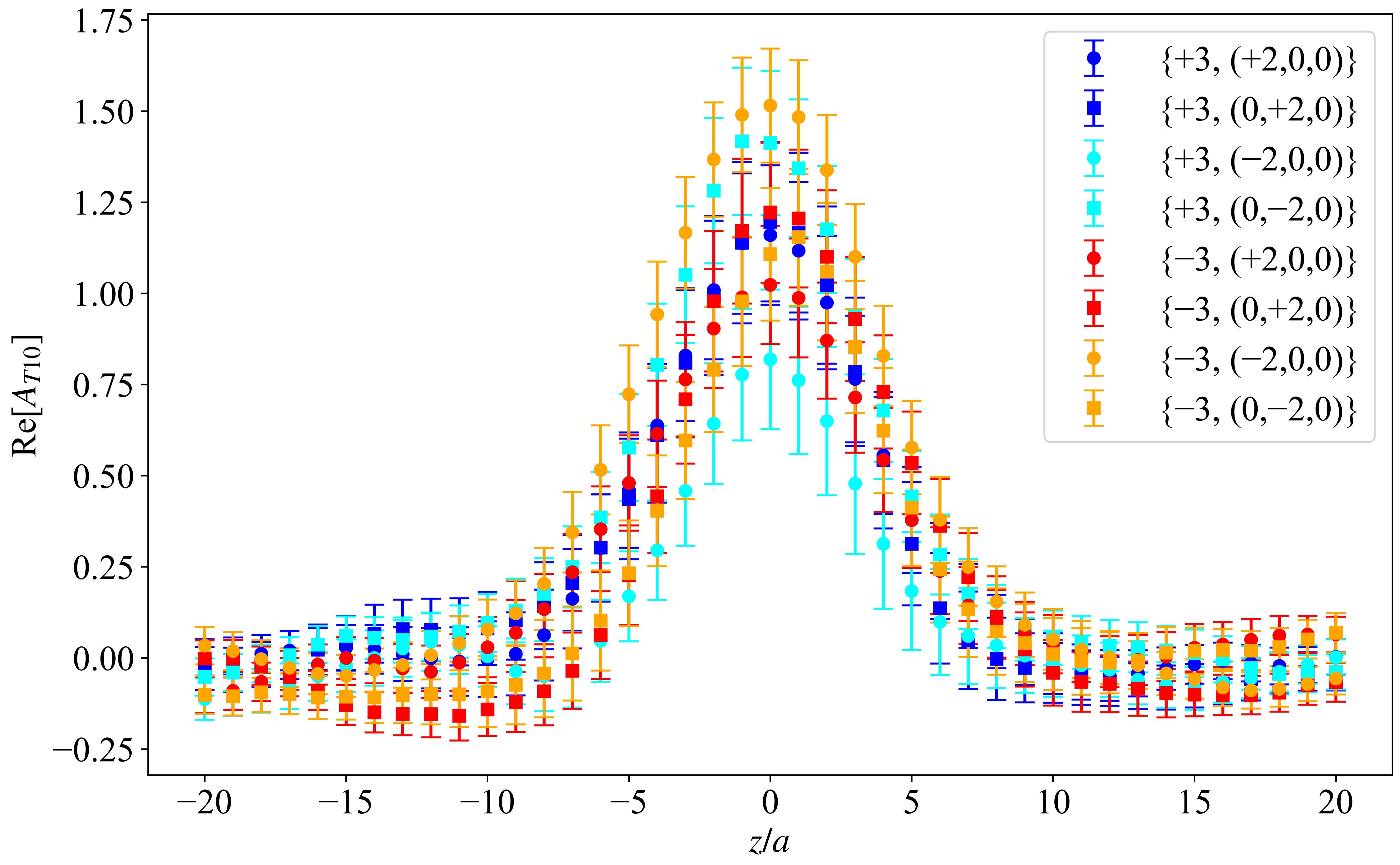

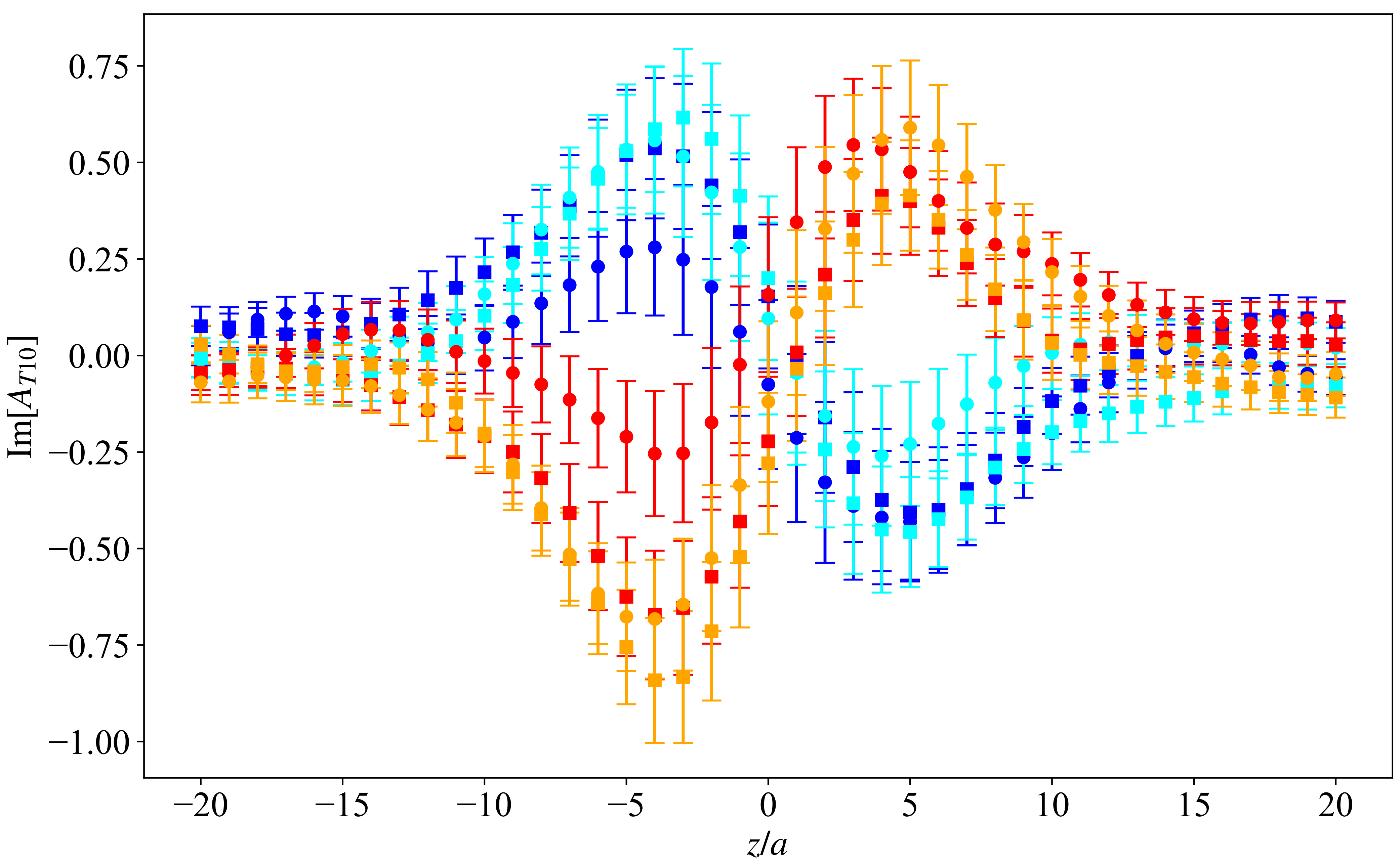

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→ *Average* → *Quasi-GPDs* → *Backus-Gilbert* → *Match*

Amplitude Symmetry

❖ Symmetry properties of amplitudes

$$A^*_{Ti}(-z\cdot P, z\cdot\Delta, \Delta^2, z^2) = A_{Ti}(z\cdot P, z\cdot\Delta, \Delta^2, z^2) \qquad i = 1,\ 2,\ 4,\ 7,\ 8,\ 10,\ 11$$

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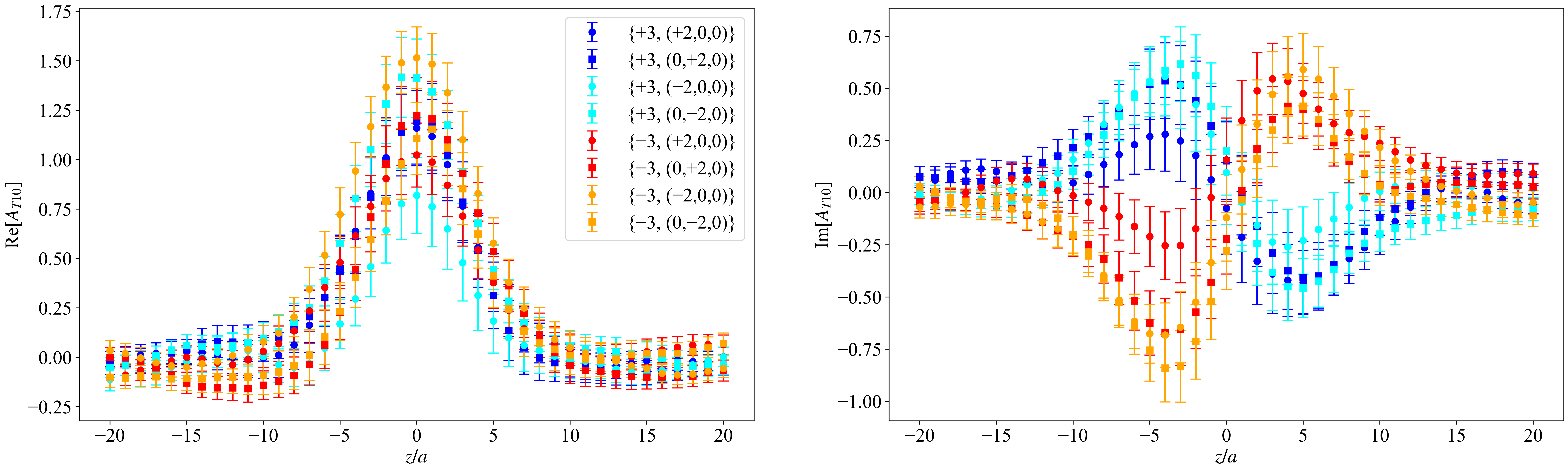

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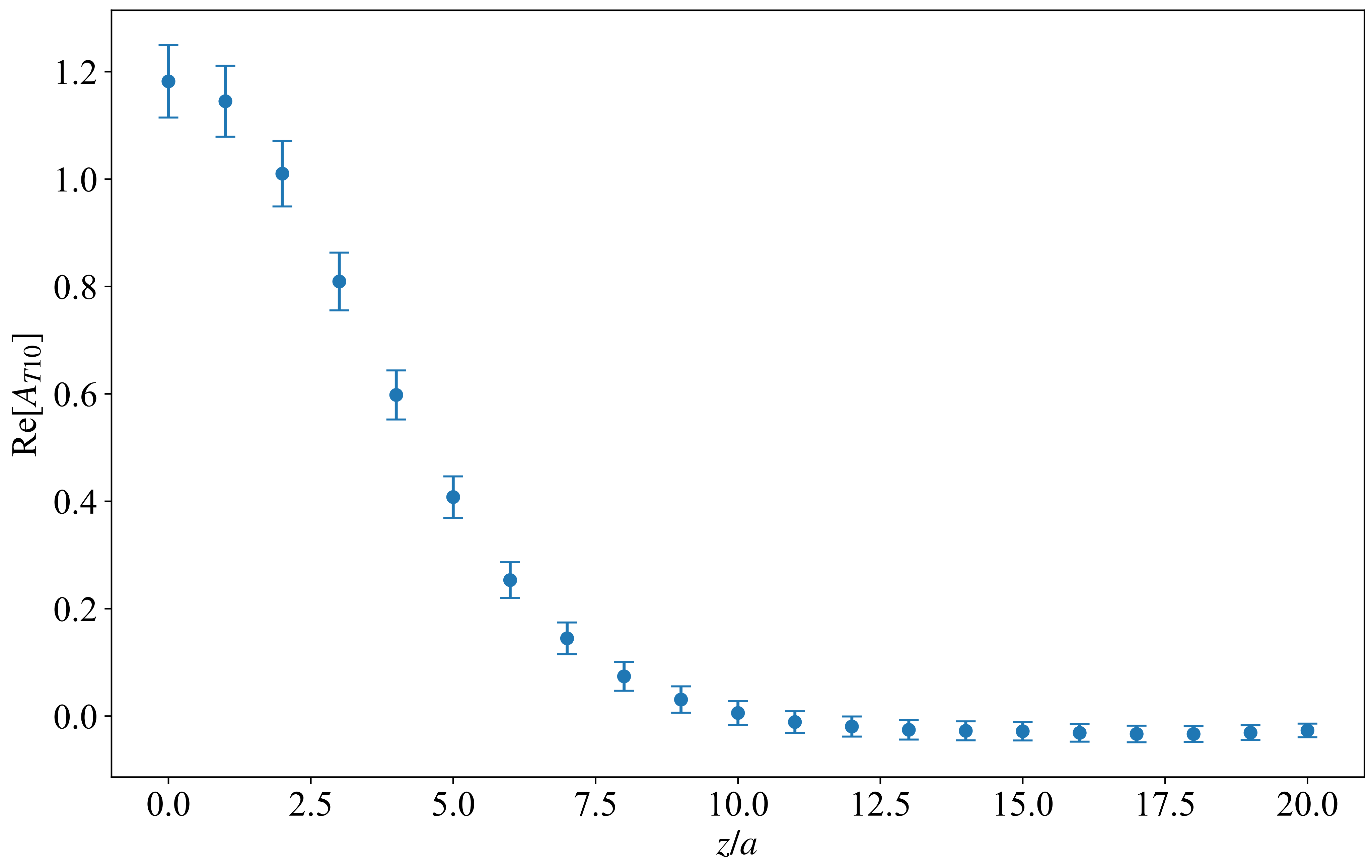
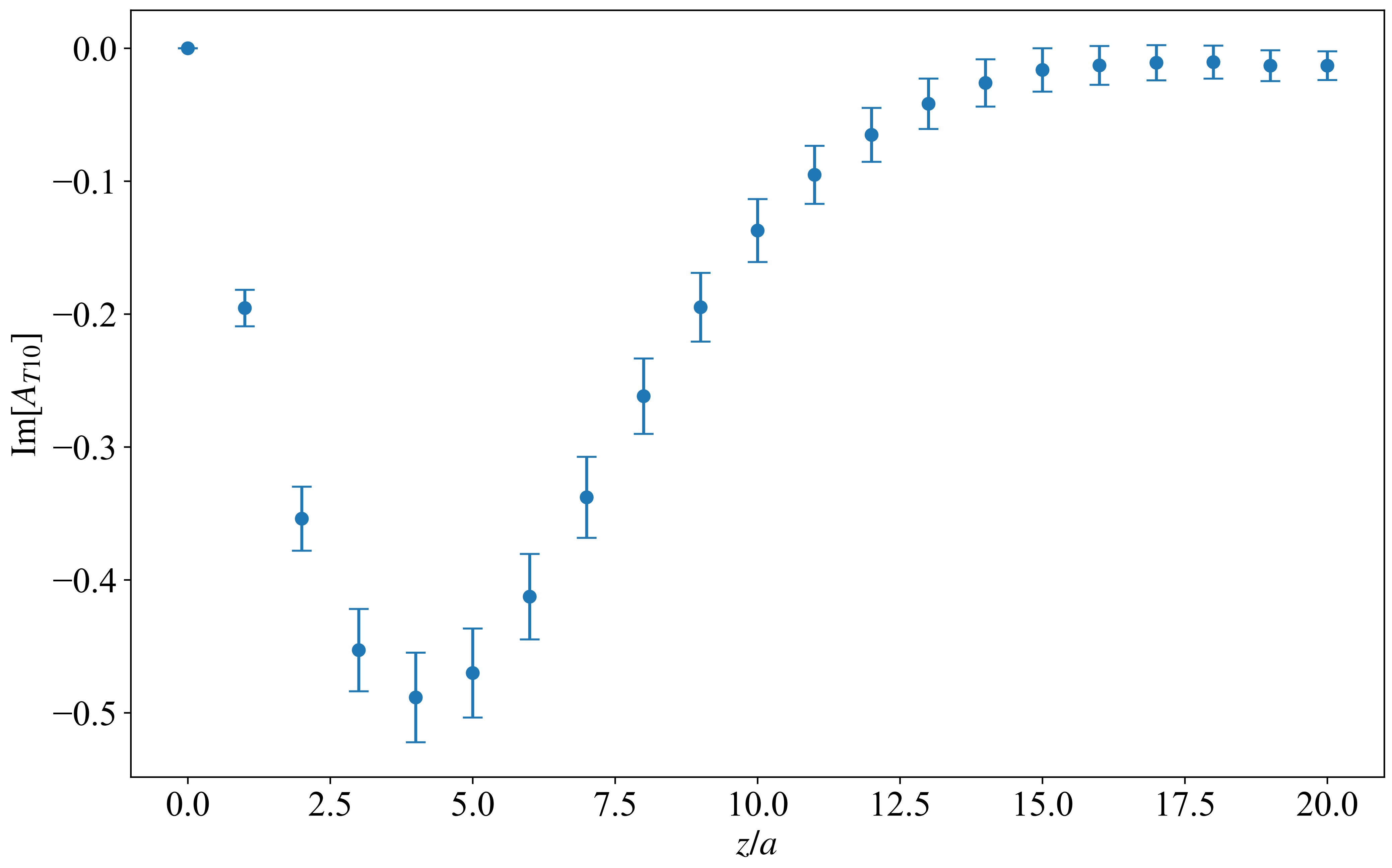
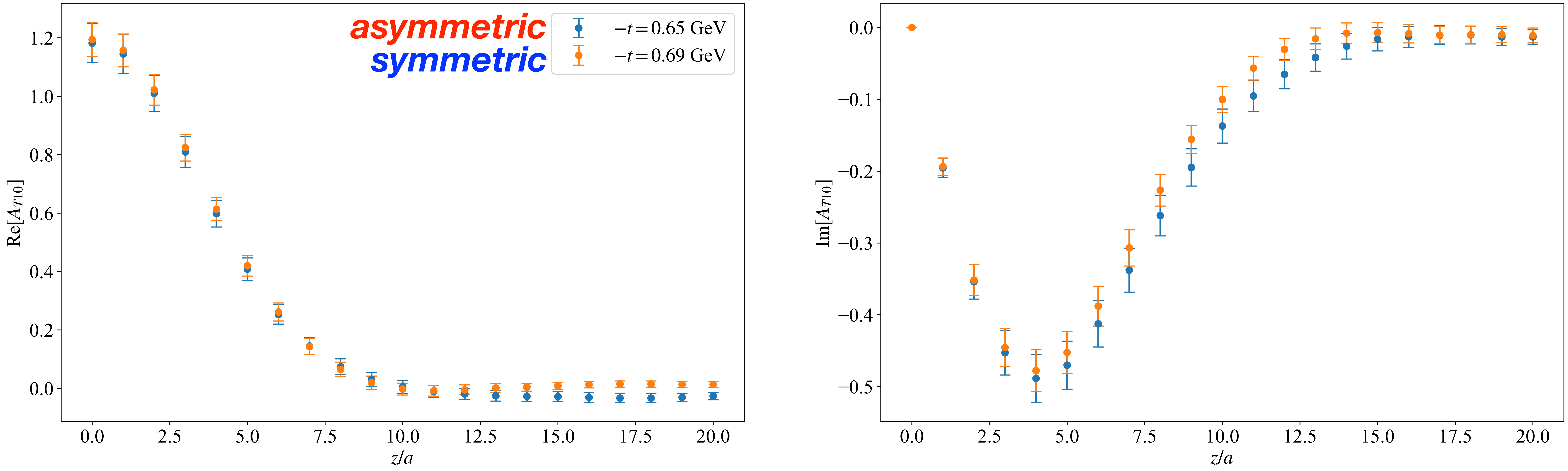




Averaging across 8 kinematic cases reduces the error by $1/\sqrt{8}$

Amplitude Comparison for different frames



- $-t = 0.65\ \mathrm{GeV}^2$ corresponds to $|P_3| = 3$ and $\vec{\Delta} = (2,0,0)$+permutations in ***asymmetric*** frame
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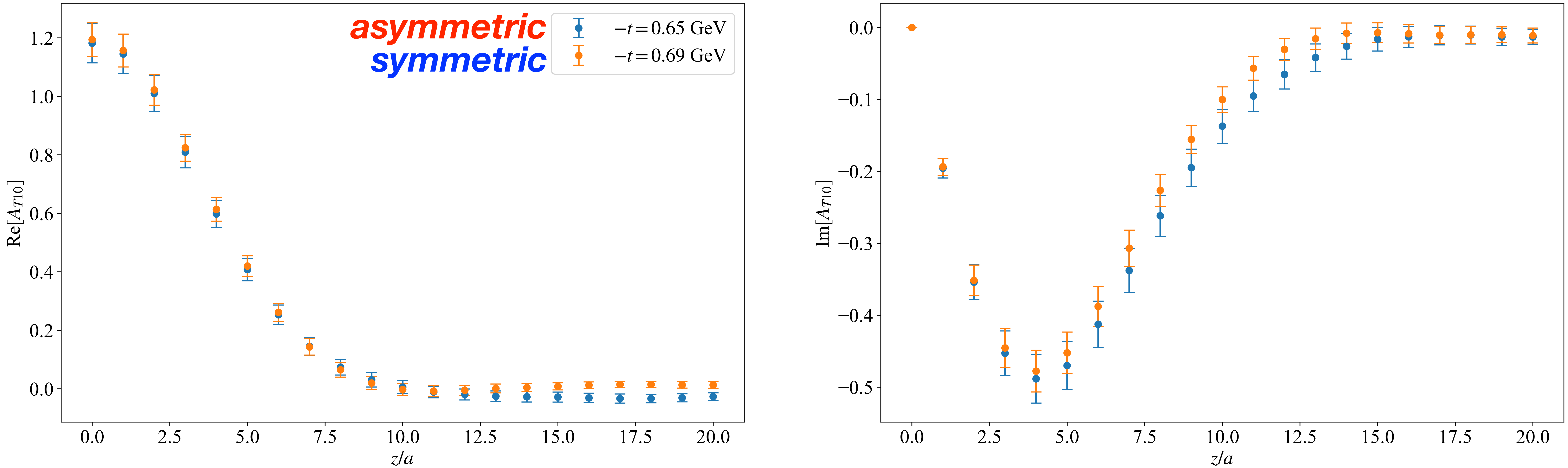

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Amplitudes are frame independent!

Quasi-GPDs

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

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$$\mathscr{H}_T = -2A_{T2}\left(1 + \frac{P^2}{M^2}\right) + A_{T4} + A_{T10}$$

$$\mathscr{E}_T = 2A_{T2} - A_{T4}$$

$$\tilde{\mathscr{H}}_T = -A_{T2}$$

$$\tilde{\mathscr{E}}_T = -2A_{T6} - 2P_3 z A_{T8}$$

- ❖ Focus only on LI definitions
  - ❖ Can substitute P^a, Δ^a directly
- ❖ Differences between LI and standard definitions
  - ❖ $\mathscr{H}^s_T$, $\mathscr{E}^s_T$ have a factor of A_{T8}
  - ❖ $\tilde{\mathscr{H}}^s_T$ has a factor of A_{T12}
  - ❖ $\tilde{\mathscr{E}}^s_T$ has a different coefficient for A_{T8}

Quasi-GPDs

$$F^{[i\sigma^{j+}\gamma_5]}_{\lambda,\lambda'}(z,\Delta,P) = -i\epsilon^{-+ij}\bar{u}(p',\lambda')\left[i\sigma^{+i}H_T + \frac{\gamma^+\Delta^i_\perp - \Delta^+\gamma^i_\perp}{2M}E_T + \frac{P^+\Delta^i_\perp - P^i_\perp\Delta^+}{M^2}\widetilde{H}_T + \frac{\gamma^+P^i_\perp - P^+\gamma^i_\perp}{M}\widetilde{E}_T\right]u(p,\lambda)$$

$$\Updownarrow$$

$$F^{[i\sigma^{\mu\nu}\gamma_5]}_{\lambda,\lambda'} = P^{[\mu}z^{\nu]}A_1 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}\gamma_5 A_2 + z^{[\mu}\Delta^{\nu]}\gamma_5 A_3 + \gamma^{[\mu}\left(\frac{P^{\nu]}}{M}A_4 + Mz^{\nu]}A_5 + \frac{\Delta^{\nu]}}{M}A_6\right)\gamma_5 + M\gamma_\alpha z^\alpha\left(P^{[\mu}z^{\nu]}A_7 + \frac{P^{[\mu}\Delta^{\nu]}}{M^2}A_8 + z^{[\mu}\Delta^{\nu]}A_9\right) + i\sigma^{\mu\nu}\gamma_5 A_{10}$$

$$\mathcal{H}_T = -2A_{T2}\left(1 + \frac{P^2}{M^2}\right) + A_{T4} + A_{T10}$$

$$\mathcal{E}_T = 2A_{T2} - A_{T4}$$

$$\tilde{\mathcal{H}}_T = -A_{T2}$$

$$\tilde{\mathcal{E}}_T = -2A_{T6} - 2P_3 z A_{T8}$$

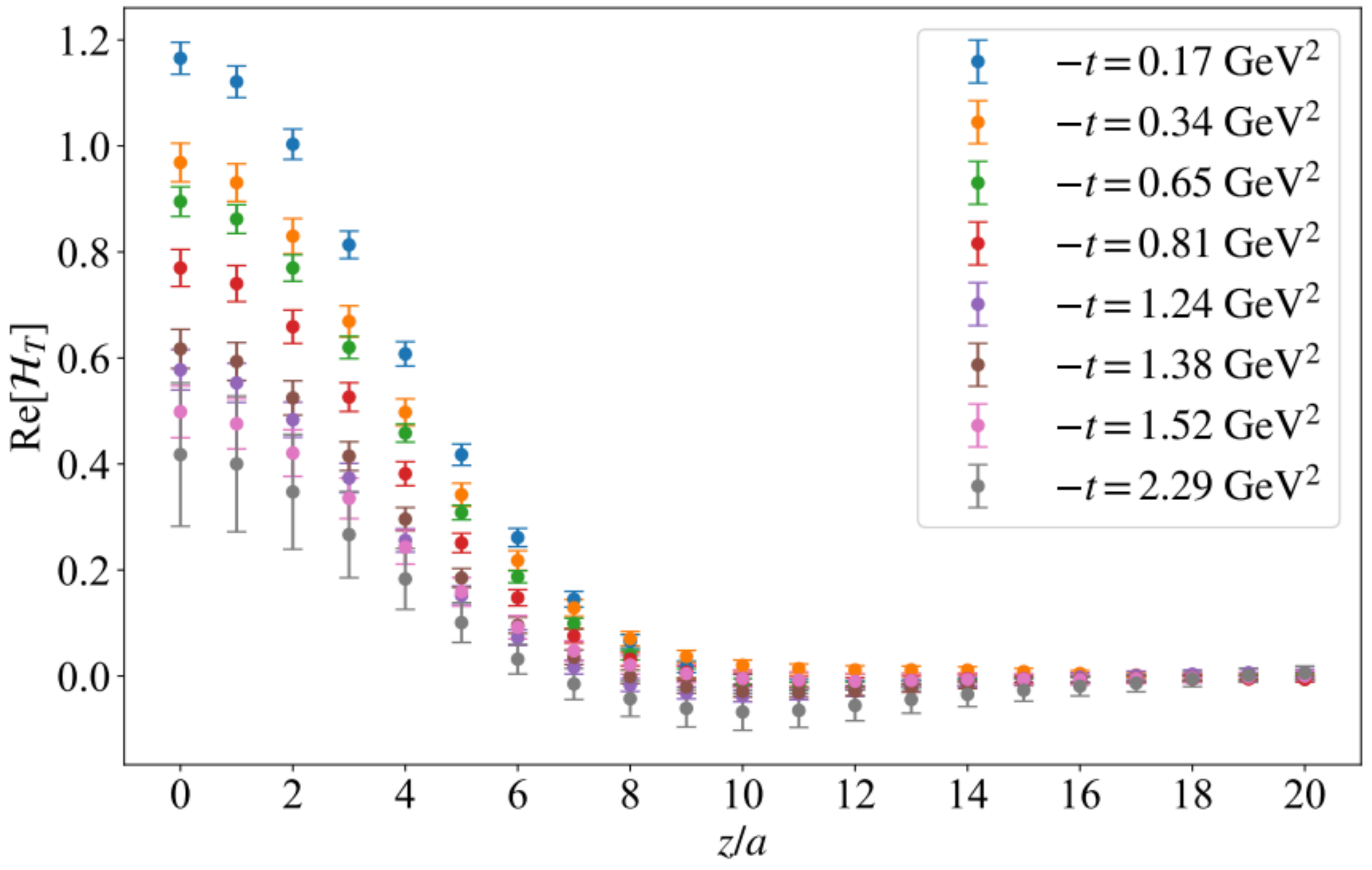
- ❖ Focus only on LI definitions
  - ❖ Can substitute P^a, Δ^a directly
- ❖ Differences between LI and standard definitions
  - ❖ $\mathcal{H}^s_T$, $\mathcal{E}^s_T$ have a factor of A_{T8}
  - ❖ $\tilde{\mathcal{H}}^s_T$ has a factor of A_{T12}
  - ❖ $\tilde{\mathcal{E}}^s_T$ has a different coefficient for A_{T8}

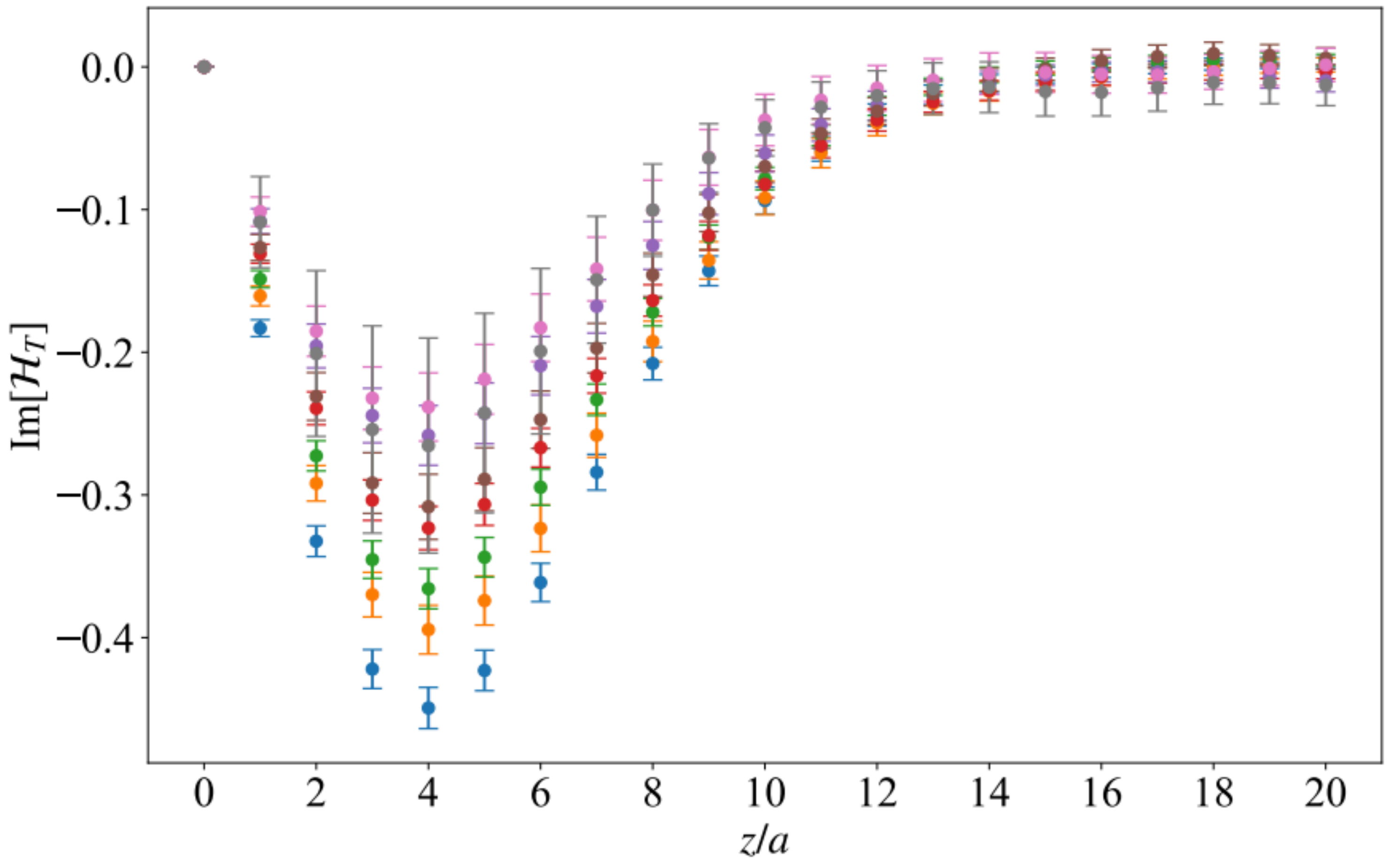
Amplitude decomposition matches results from previous results **[ETMC, PRD 105, 034501 (2022)]**

Quasi-GPDs

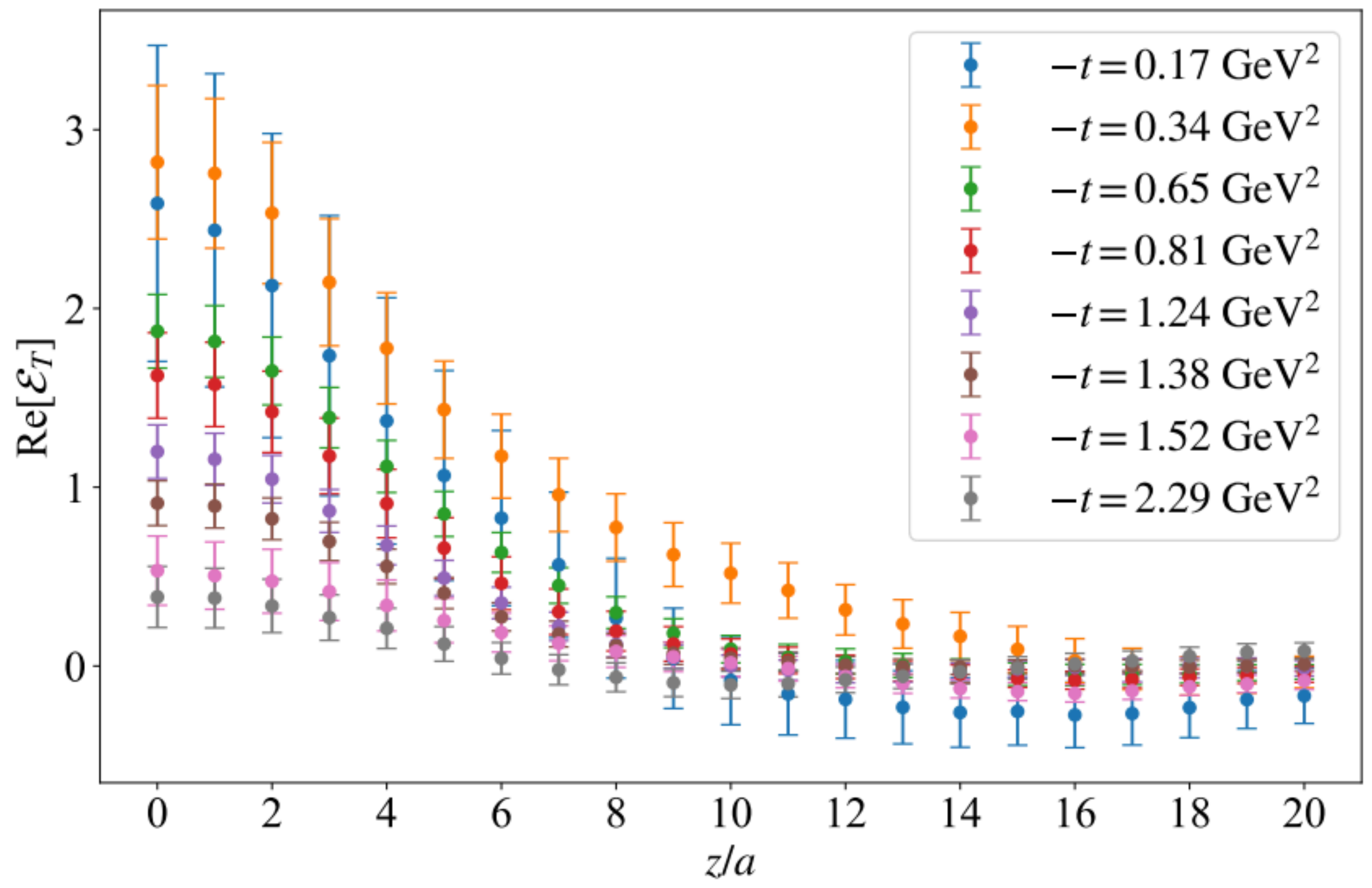
- ❖ Results in agreement between asymmetric and symmetric frames!
- ❖ Asymmetric frame is **computationally much more efficient** to produce data for a dense range of $-t$

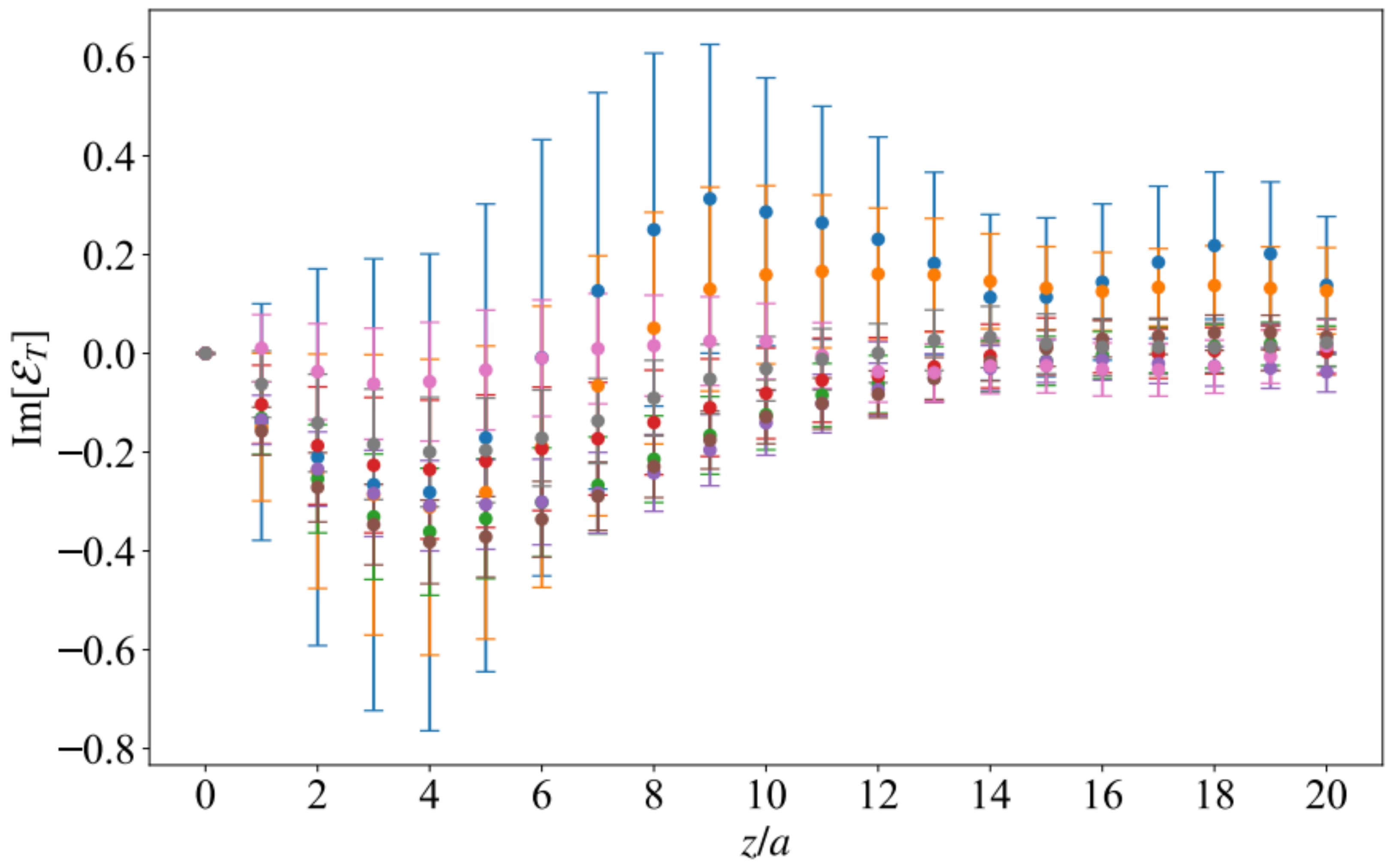
$$\mathscr{H}_T = -2A_{T2}\left(1 - \frac{(E_i + E_f)^2 - \vec{\Delta}^2 - 4P_3^2}{4m^2}\right) + A_{T4} + A_{T10}$$





- ❖ Large contribution from A_{T10}





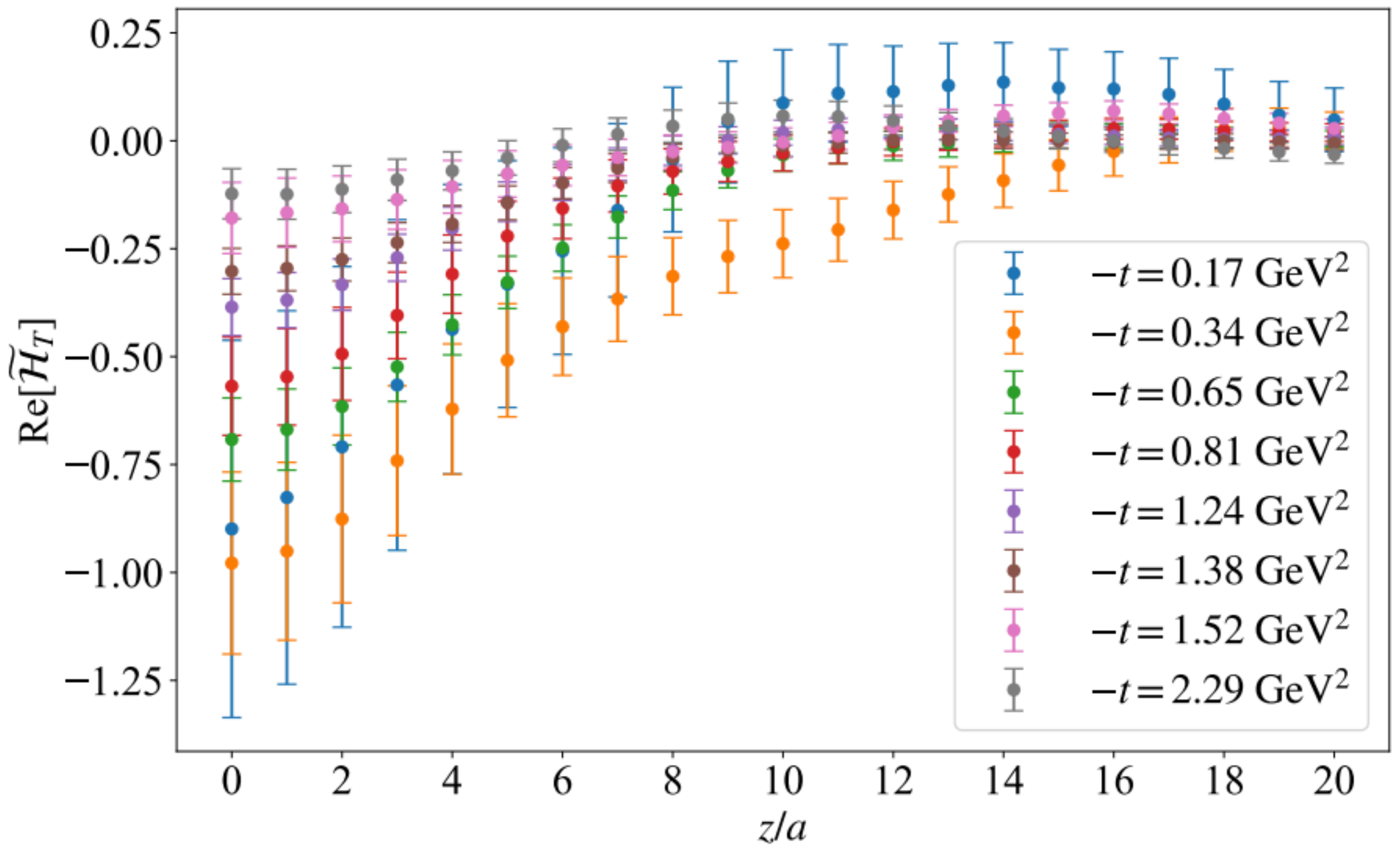
$$E_T = 2A_{T2} - A_{T4}$$

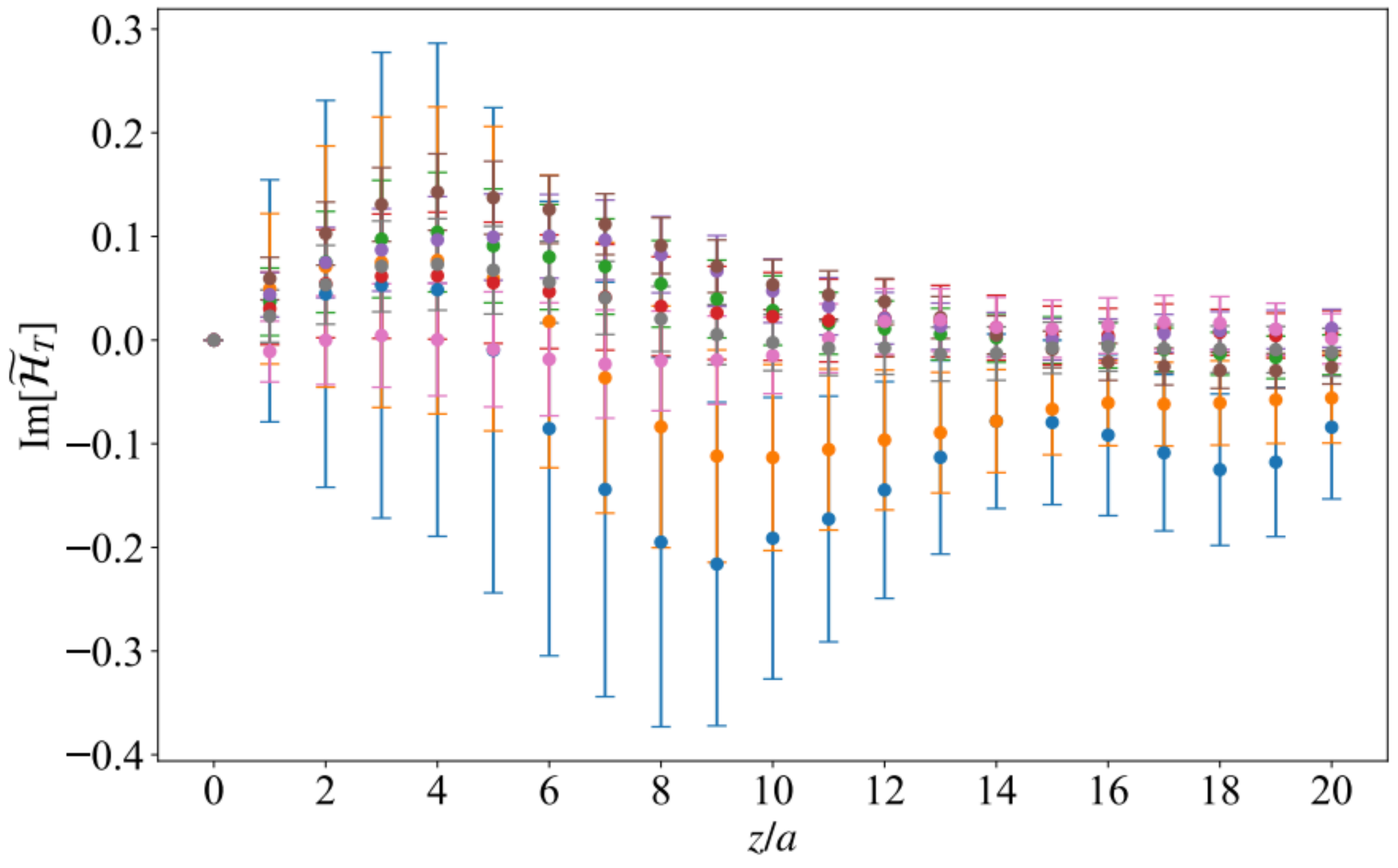
- ❖ Magnitude decreases as $-t$ increases

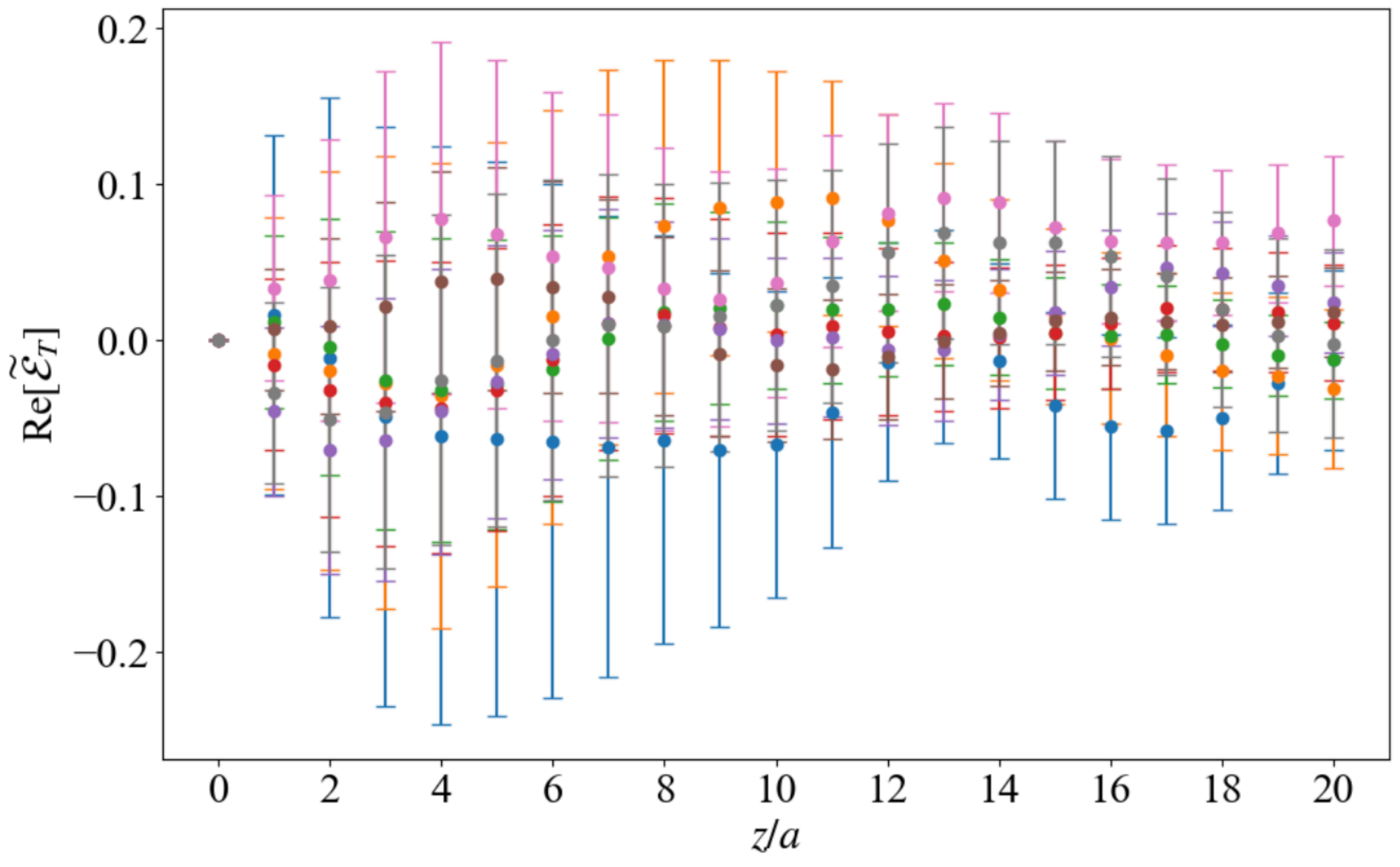
Quasi-GPDs

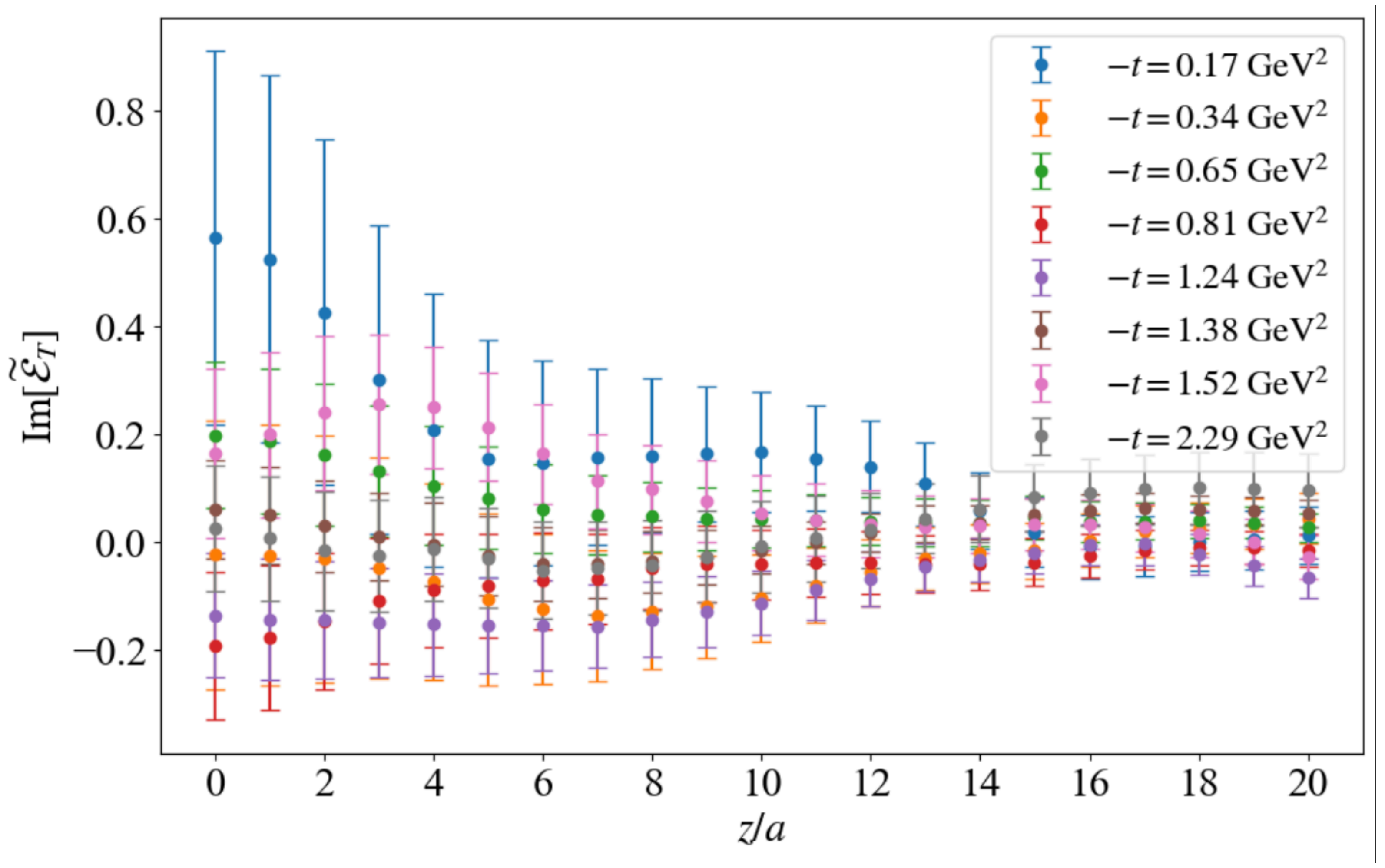
$\tilde{\mathscr{H}}_T = -A_{T2}$

- Magnitude decreases as $-t$ increases









$\tilde{\mathscr{E}}_T = -2A_{T6} - P_3 z A_{T8}$

- Odd in $\xi \to -\xi$
- Zero in $\xi = 0$

[Meissner, et al., JHEP 08, 056 (2009)]

Position space to Momentum space

- Multiple methods to consider
  - Standard Fourier Transform

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Position space to Momentum space

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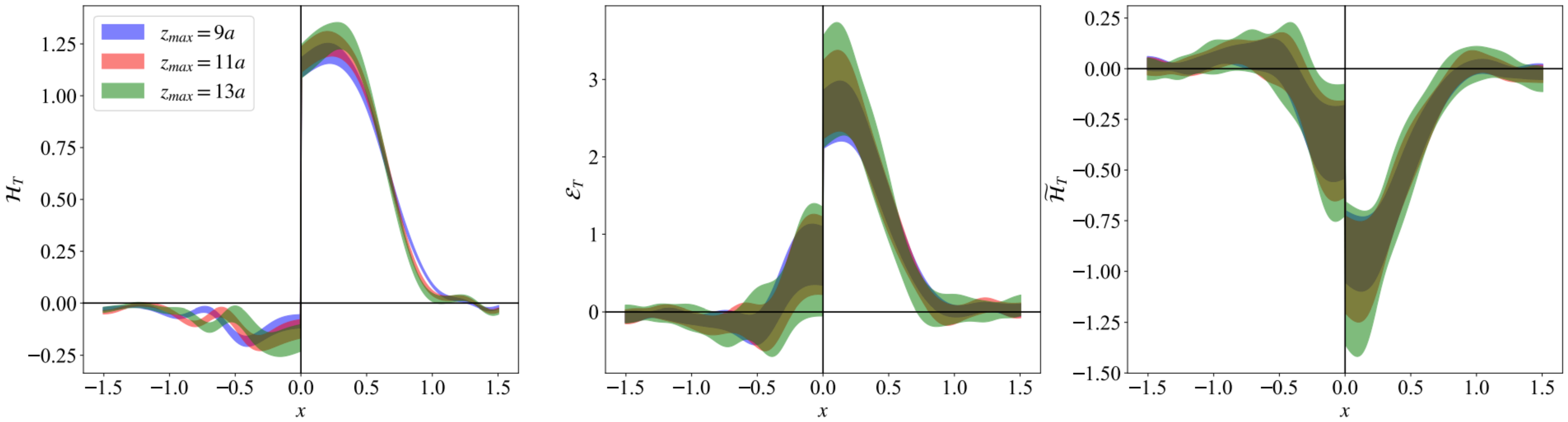
- Backus-Gilbert:
  - Model independent
  - Criterion: variance of solution with respect to statistical variation of input data is minimal
- Test the dependence on z_{max} in reconstruction



All calculations use $z_{max} = 11a$ to match to light-cone

Matching to the Light-Cone

[Liu, et al., Phys. Rev. D 100, 034006 (2019)]

$$GPD(x,\xi,t) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C_\Gamma\left(\frac{x}{y},\frac{\xi}{y},r,\frac{yP^z}{\mu},\frac{yP^z}{p_R^z}\right) qGPD(y,\xi,t)$$

$$C_{i\sigma^{z\perp}}\left(x,\xi=0,\frac{p^a}{\mu}\right) = \frac{\alpha_s C_F}{2\pi} f_1\left(x,\xi=0,\frac{p^z}{\mu}\right)_+ + 2\delta(1-x)\frac{\alpha_s C_F}{4\pi}\left[\frac{-1}{\epsilon_{UV}} + \ln\left(\frac{\mu^2}{\mu'^2}\right)\right] + \left[\left|\frac{p^z}{p_R^z}\right| \frac{\alpha_s C_F}{2\pi} f_2\left(\frac{p^z}{p_R^z}(x-1)+1, r\right)\right]_+$$

$x < 0:$

$$f_1(x,\xi=0) = \frac{-2x}{x-1}\ln\left(\frac{x-1}{x}\right) \qquad f_2(x,\xi=0) = \frac{-3}{2(1-x)} - \frac{r-2x}{(r-1)(r-4x+4x^2)} - \frac{-r+2x-rx}{(r-1)^{3/2}(x-1)}\tan^{-1}\left(\frac{\sqrt{r-1}}{2x-1}\right)$$

$0 < x < 1:$

$$f_1(x,\xi=0) = \frac{2x}{1-x}\left[\ln\left(\frac{4x(1-x)(p^z)^2}{\mu^2}\right) - 1\right] \qquad f_2(x,\xi=0) = \frac{1-3r+2x}{2(r-1)(1-x)} + \frac{r-2x+rx}{(r-1)^{3/2}(1-x)}\tan^{-1}\left(\sqrt{r-1}\right)$$

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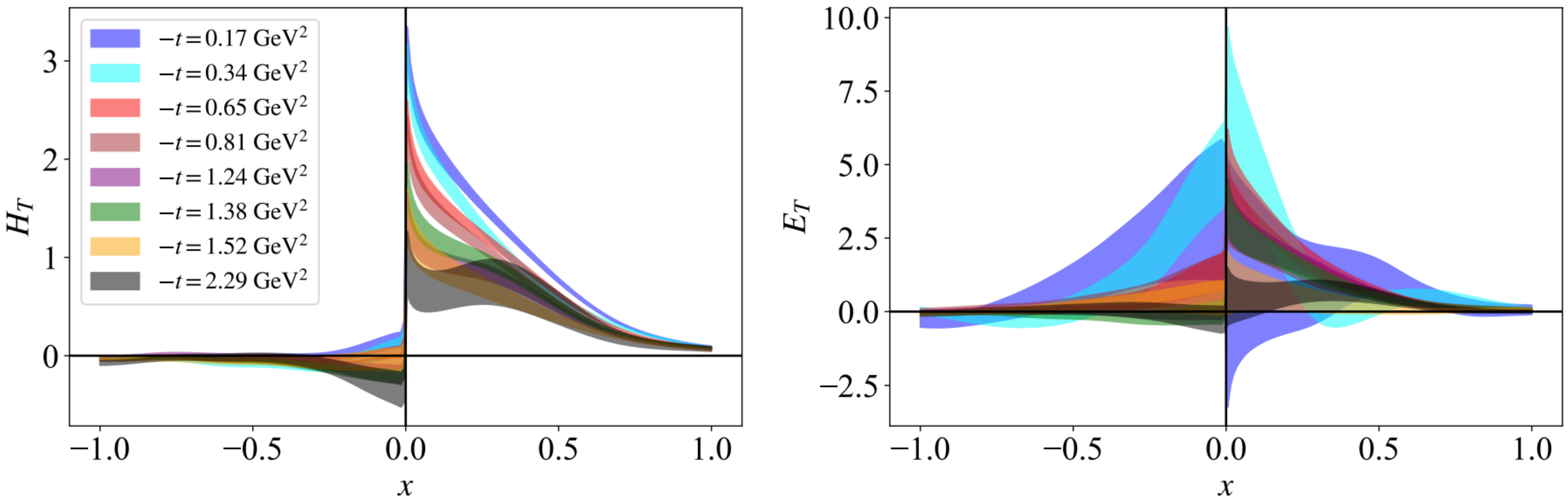
Light-Cone Results



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❖ Very good signal!

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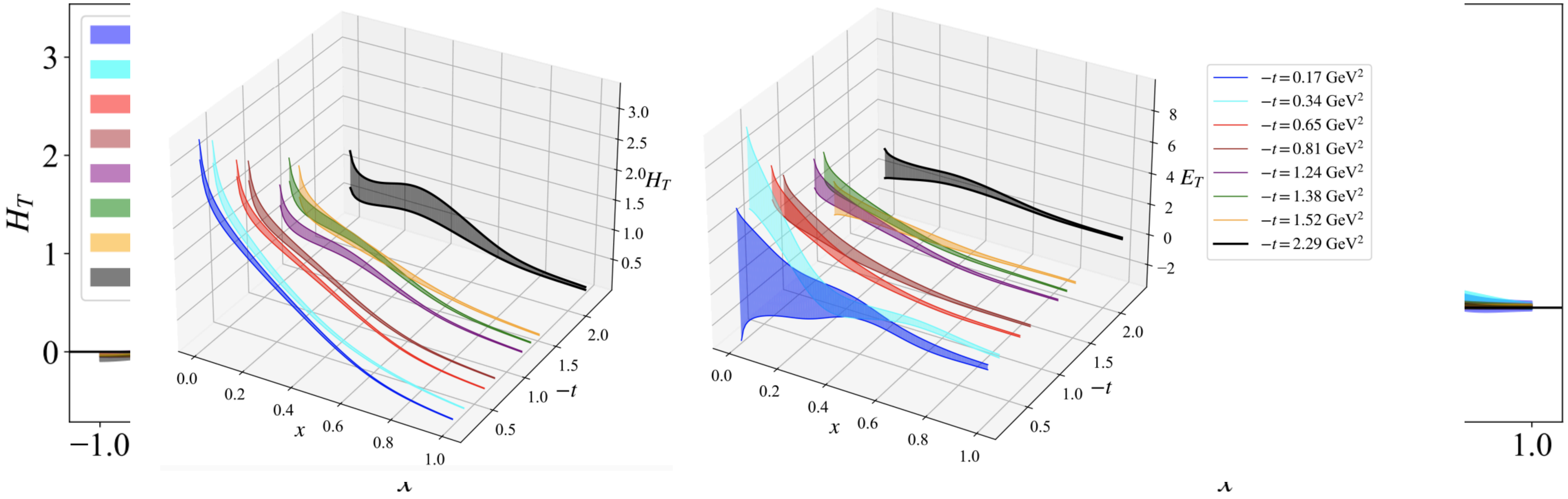

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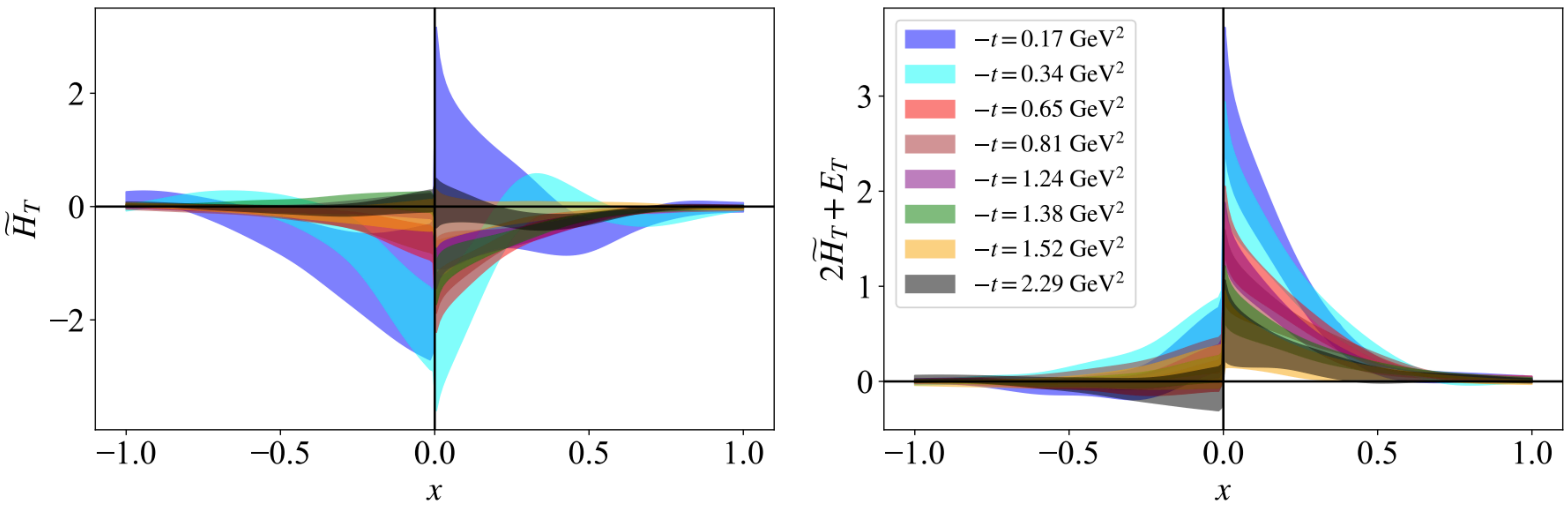

- Can combine $2\widetilde{H}_T + E_T$
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  - Interpretation is lateral deformation of transversely polarized quark in an unpolarized proton
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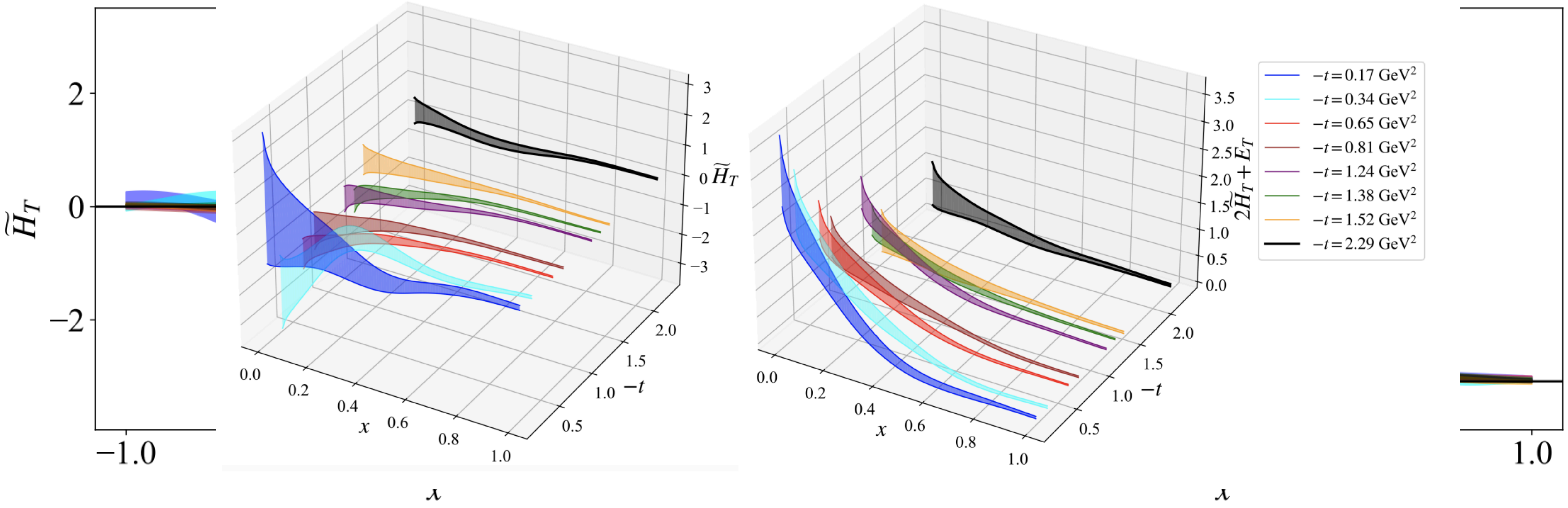

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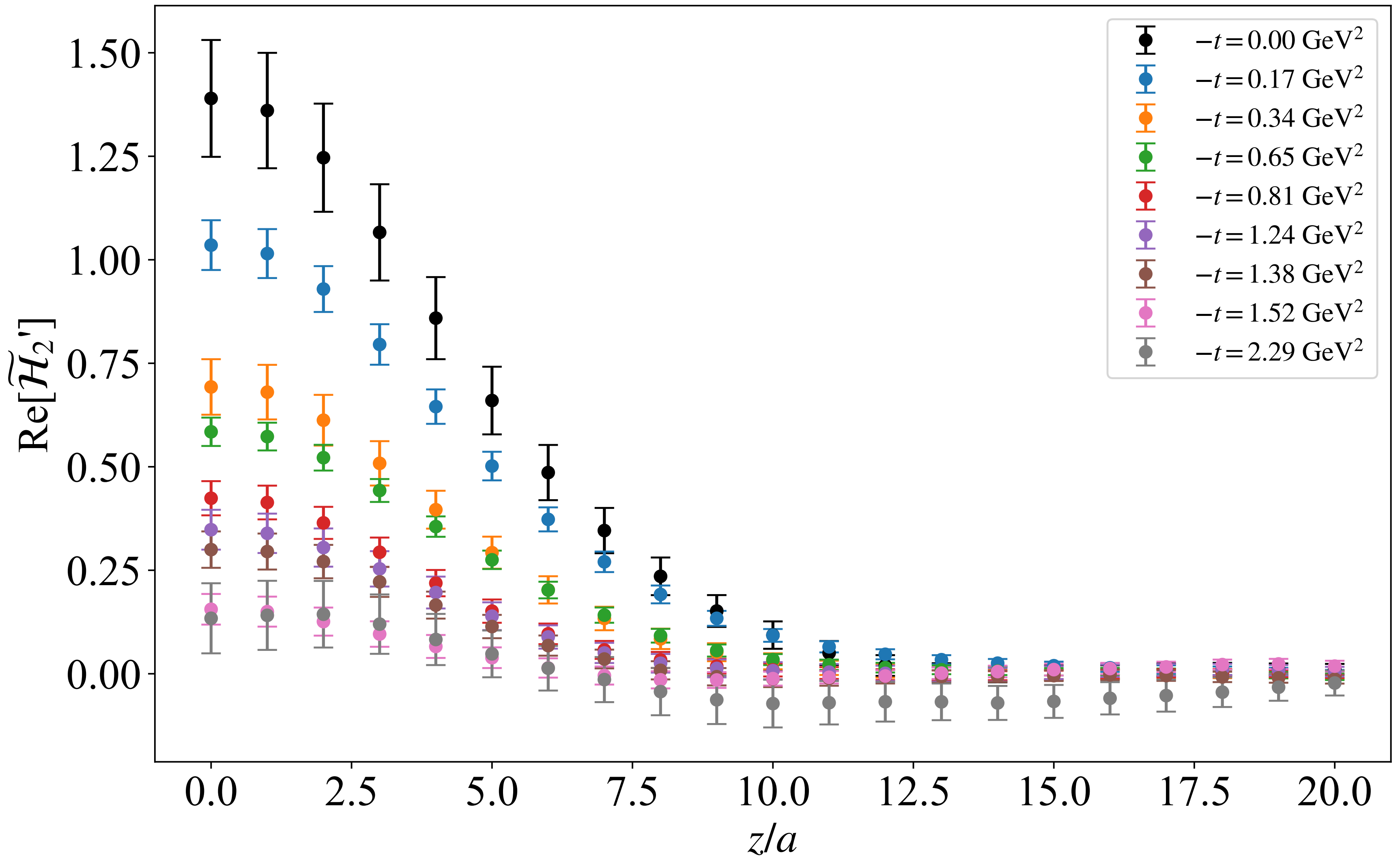
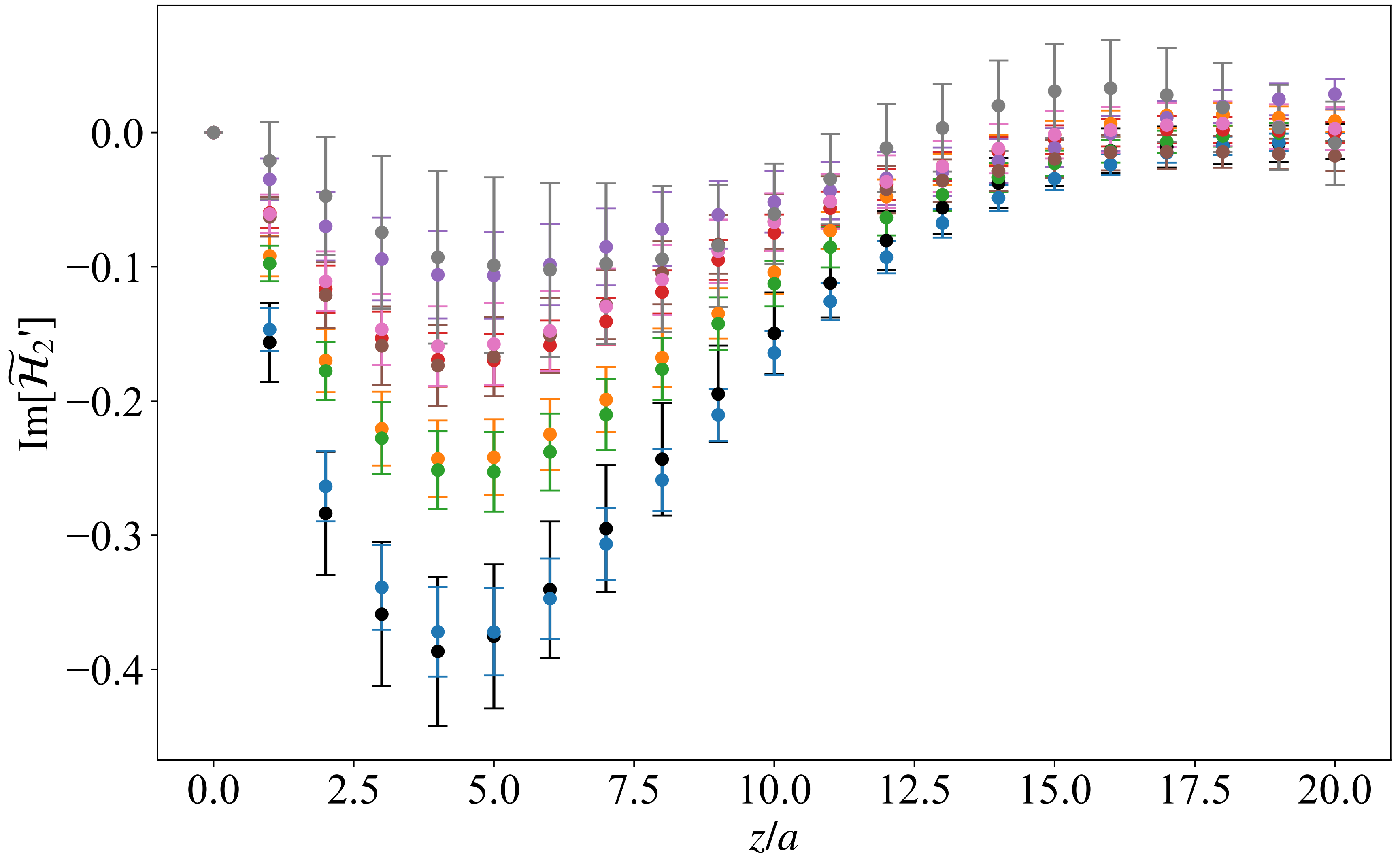
Twist-3 Quasi-GPDs

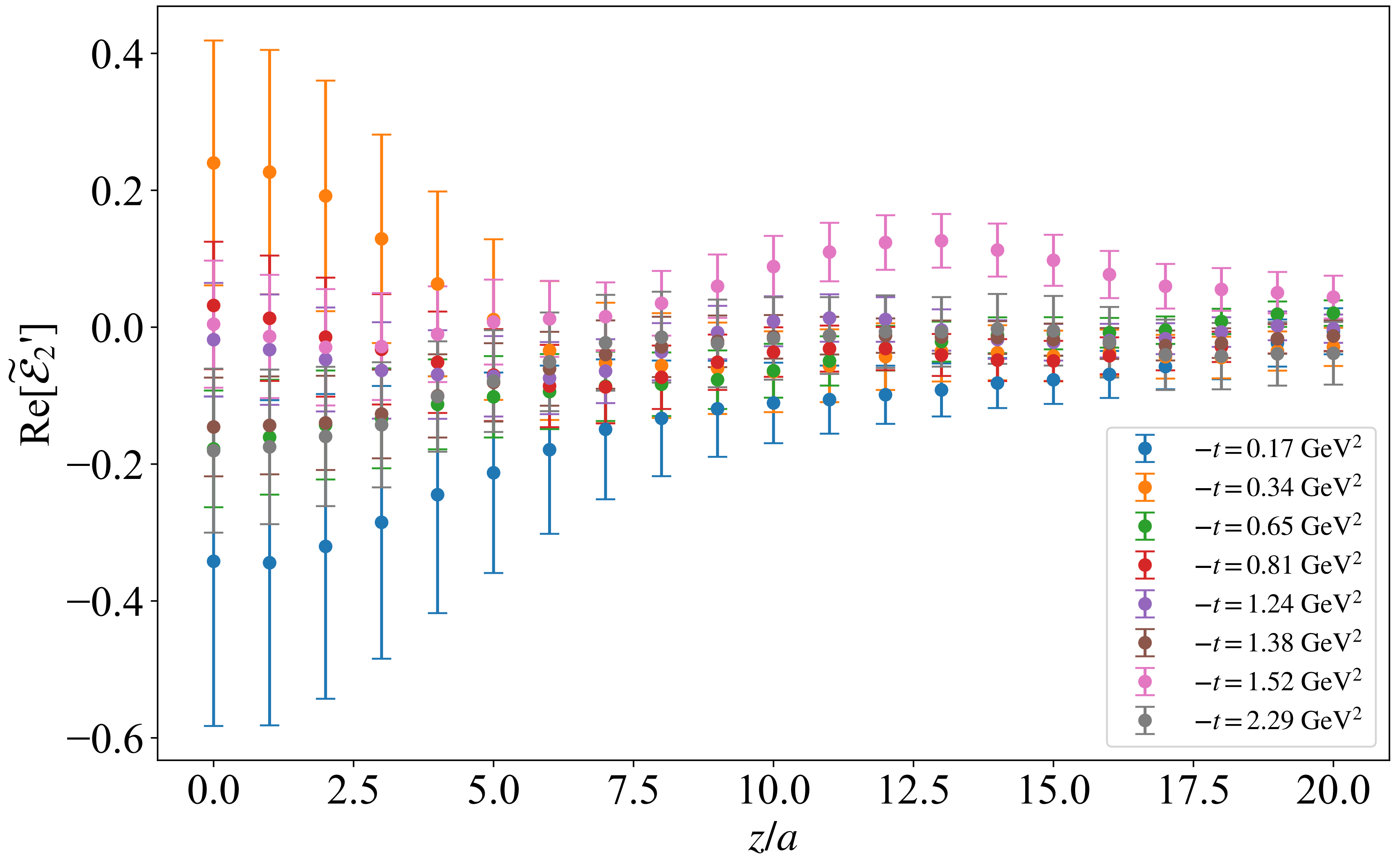
$$F^{[i\sigma^{+-}\gamma_5]} = \bar{u}(p_f,\lambda')\left[\frac{M}{P^+}\gamma^+\gamma_5\tilde{H}'_2 + \gamma_5\tilde{E}'_2\right]u(p_i,\lambda)$$

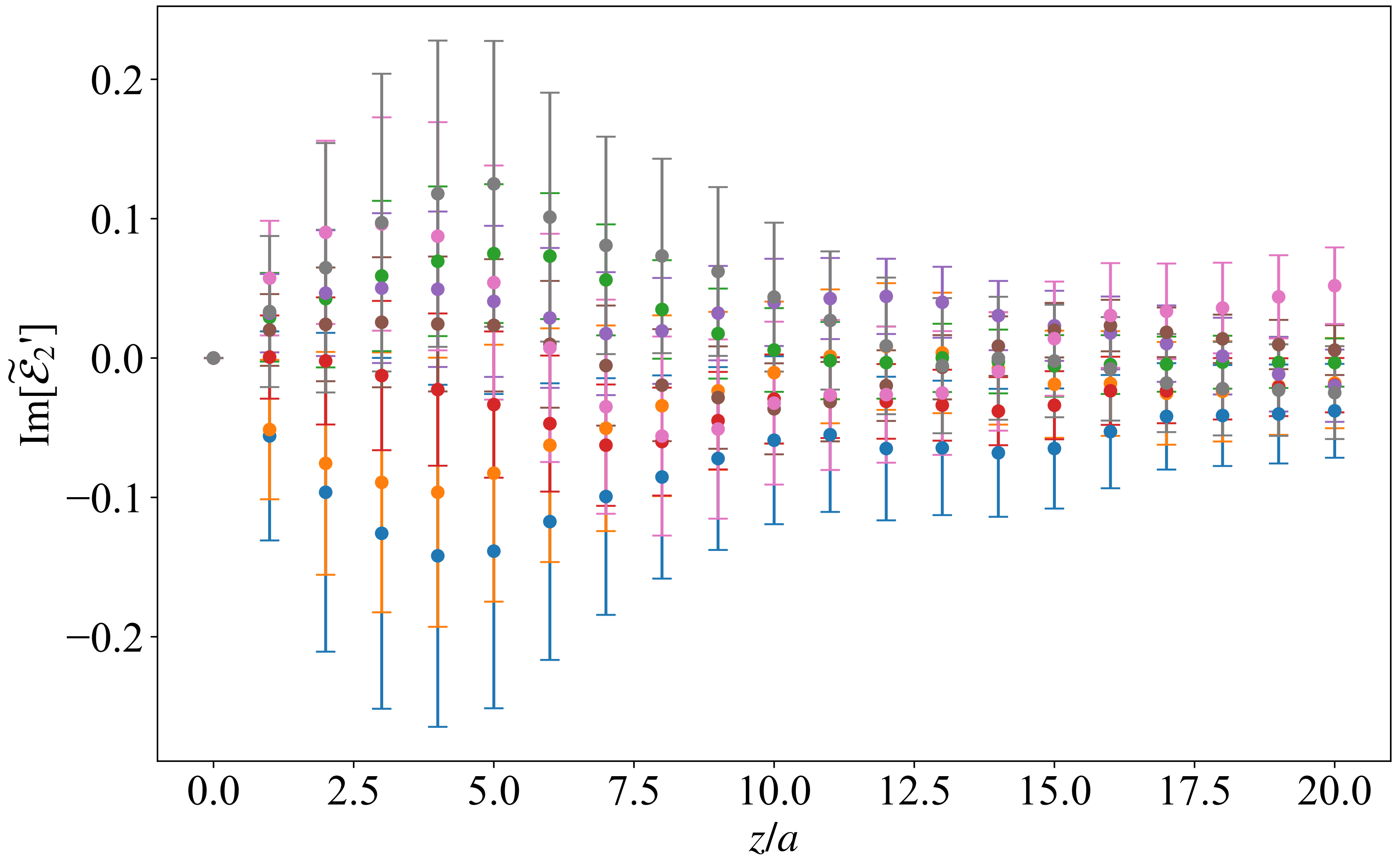
$$\tilde{\mathscr{H}}'_2 = \frac{E_f(E_f+E_i)-2P_3^2}{2m^2}A_{T4} + P_3 z A_{T5} + \frac{E_f(E_f+E_i)}{2}z^2 A_{T7} + A_{T10}$$

- ❖ Good signal to noise ratio
- ❖ Dominant contribution from A_{T10}





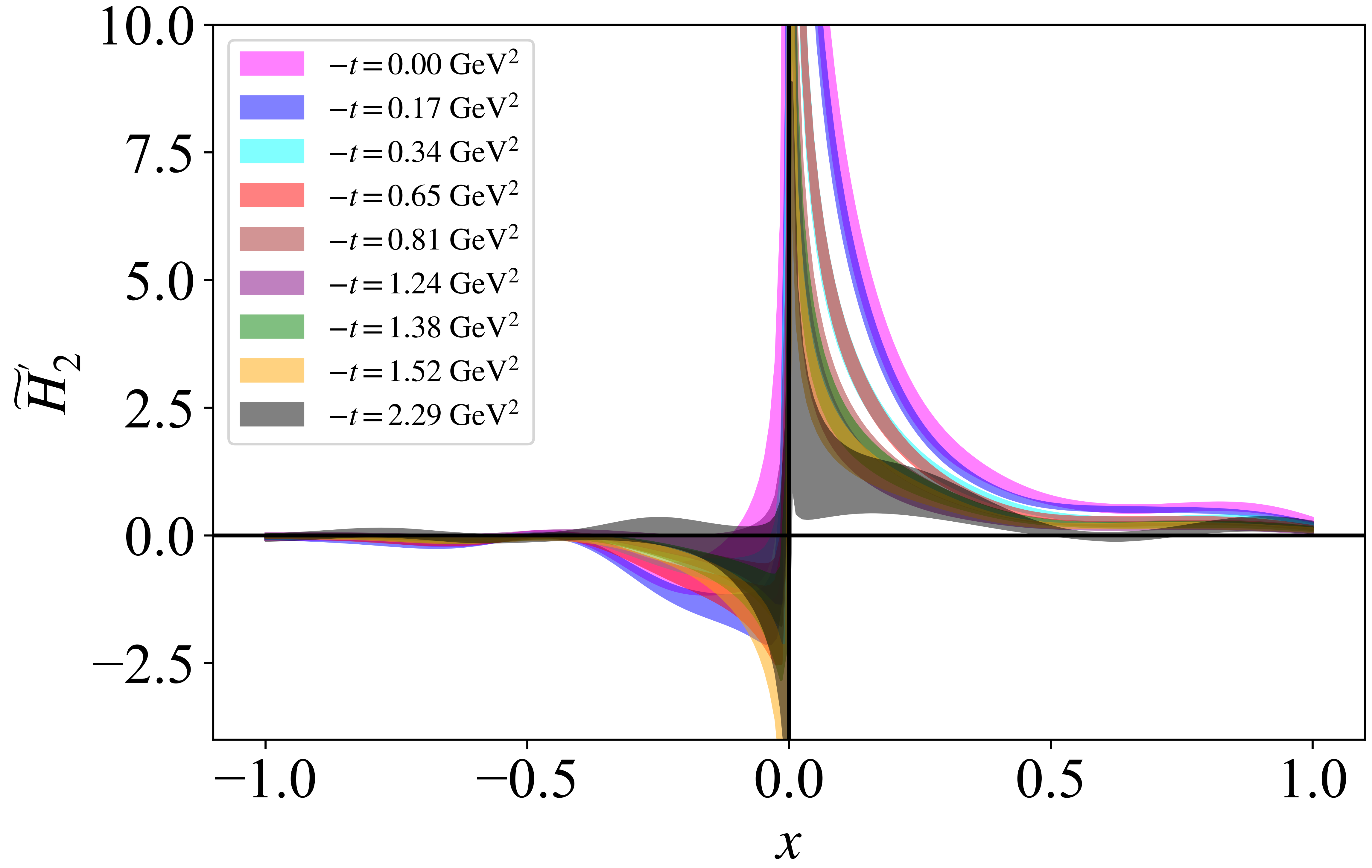




$$\tilde{\mathscr{E}}'_2 = \sqrt{\frac{E_f(E_f+E_i)}{2}}\,z A_{T1}$$

- ❖ Zero when $\xi = 0$

Light-Cone GPDs



- ❖ Good signal to noise ratio
- ❖ Dense range of $-t$ accessible!
  - ❖ $-t = 2.29\ \mathrm{GeV}^2$ not reliable
    - ❖ Free in calculations

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Thank You!!!

Acknowledgements

- ❖ U.S. Department of Energy, Office of Nuclear Physics, Early Career Award under Grant No. DE-SC0020405
- ❖ This research used resources of the National Energy Research Scientific Computing Center, a DOE Office of Science User Facility supported by the Office of Science of the U.S. Department of Energy under Contract No. DE-AC02-05CH11231 using NERSC award ALCC-ERCAP0025953.
- ❖ PLGrid Infrastructure by Prometheus in Cracow
- ❖ Poznan Supercomputing and Networking Center by Eagle
- ❖ Interdisciplinary Centre for Mathematical and Computational Modeling of the Warsaw University by Okeanos
- ❖ Academic Computer Center in Gdańsk by Tryton