Resumming Large Logarithms in the Lattice Calculation of Generalized Parton Distributions at Non-zero Skewness

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LaMET 2025

Stony Brook University, NY, 10/08/2025

QUARK-GLUON TOMOGRAPHY COLLABORATION

JHEP 207 (2025) 241, In collaboration with Jack Holligan, Huey-Wen Lin, and Yong Zhao

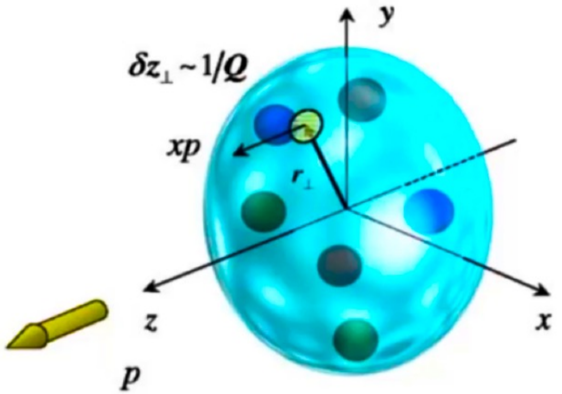
QUARK-GLUON TOMOGRAPHY COLLABORATION

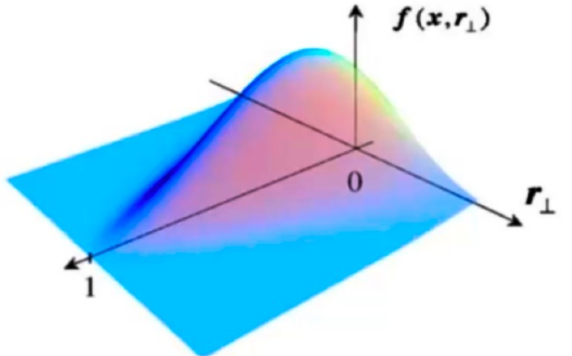
Generalized Parton Distributions

3D image of hadrons: $P^+ = \frac{p'^+ + p^+}{2}, t = \Delta^2 = (p'-p)^2, \xi = \frac{\Delta^+}{2P^+}$

$$F^q(x,\xi,t) = \int \frac{dz^-}{4\pi} e^{-ixP^+z^-} \langle p'|\bar{q}(z^-/2)\gamma^+ q(-z^-/2)|p\rangle$$

$$= \frac{1}{2P^+}\left[H^q(x,\xi,t)\,\bar{u}(p')\gamma^+ u(p) - E^q(x,\xi,t)\,\bar{u}(p')\frac{i\sigma^{+\alpha}\Delta_\alpha}{2m}u(p)\right]$$





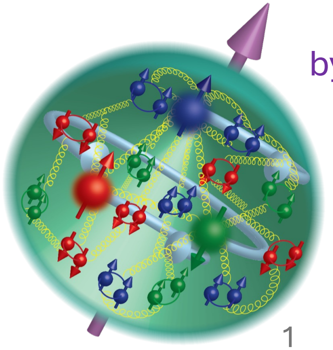
Belitsky and Radyushkin: Phys.Rept.(200

Connection to Gravitational Form Factors (Mass distribution):

- $\langle p'|T^{\mu\nu}|p\rangle = \bar{u}(p')\left[A(t)\frac{P^\mu P^\nu}{m} + B(t)\frac{P^{(\mu}\sigma^{\nu)\Delta}}{2m} + D(t)\frac{\Delta^\mu\Delta^\nu}{4m} + m\bar{c}g^{\mu\nu}\right]u(p)$
- $\int_{-1}^{1} dx\ xH(x,\xi,t) = A + \xi^2 D,\quad \int_{-1}^{1} dx\ xE(x,\xi,t) = B - \xi^2 D$

Connection to nucleon spin decomposition:

- Ji's sum rule: $J_P = J_{q,\bar{q}} + J_g, J_i = \lim_{t\to 0}\int_{-1}^{1} dx\ x[H_i(x,\xi,t) + E_i(x,\xi,t)]$

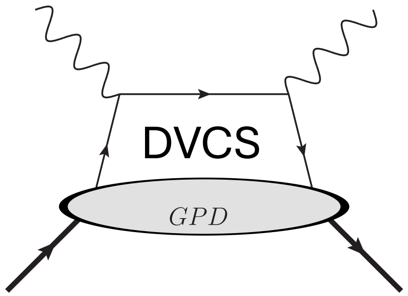


by ANL

Challenge in extracting GPD from experiment

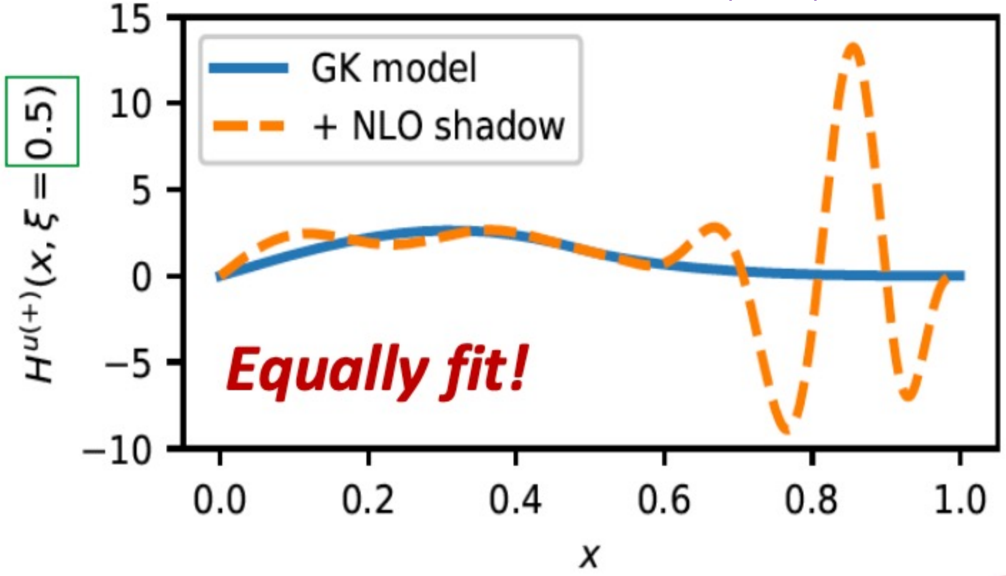
- Multi-dimensionality $F(x, \xi, t)$
- Contamination from B-H process
- x is always integrated over
  $iM \propto \int dx \frac{F(x,\xi,t)}{x-\xi+i\epsilon}$
- Mostly sensitive to $F(x \to \pm\xi, \xi, t)$
  - SDHEP has wider but still limited sensitivity Z.Yu & J.Qiu, PRL (2024)
- Shadow GPDs (degenerate solutions)
  - $\int dx \frac{\Delta F(x,\xi,t)}{x-\xi+i\epsilon} = 0$

Ji, PRD(1997)

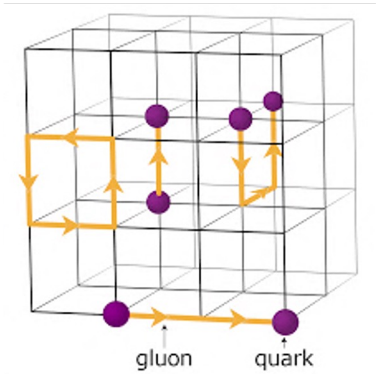


Hall A @JLAB
CLAS@JLAB
H1@HERA

Bertone et al. PRD (2021)

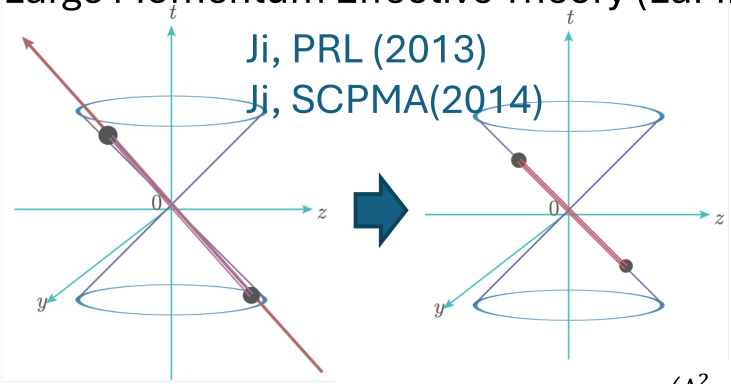


Inputs from Lattice QCD?



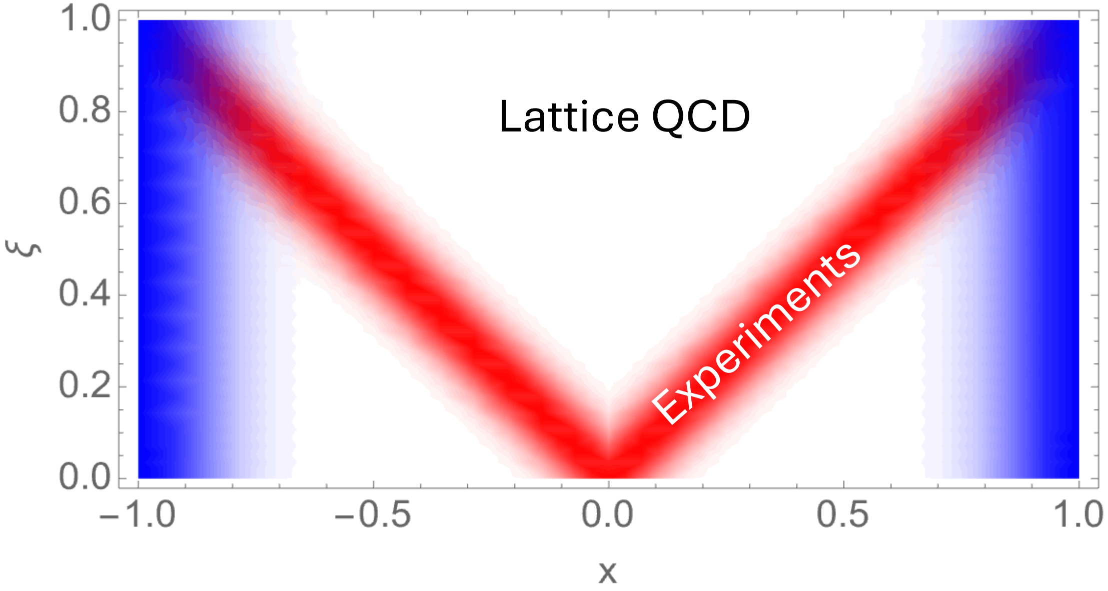
Large Momentum Effective Theory (LaMET)

Ji, PRL (2013)
Ji, SCPMA(2014)

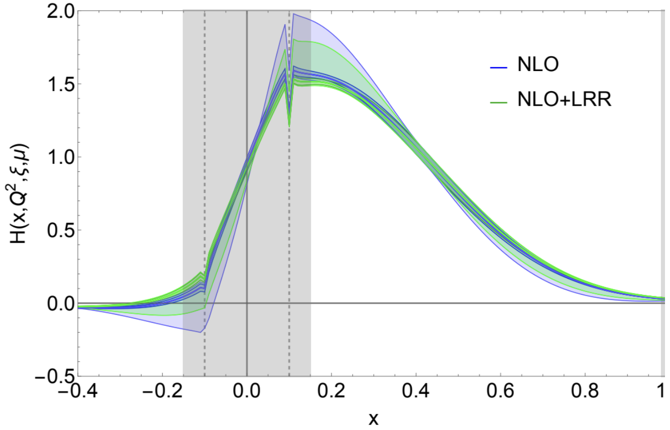


$$\tilde{F}(x, P_z) = C(x, y, \mu, P_z) \otimes F(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{x^2 P_z^2}, \frac{\Lambda_{QCD}^2}{\bar{x}^2 P_z^2}\right)$$

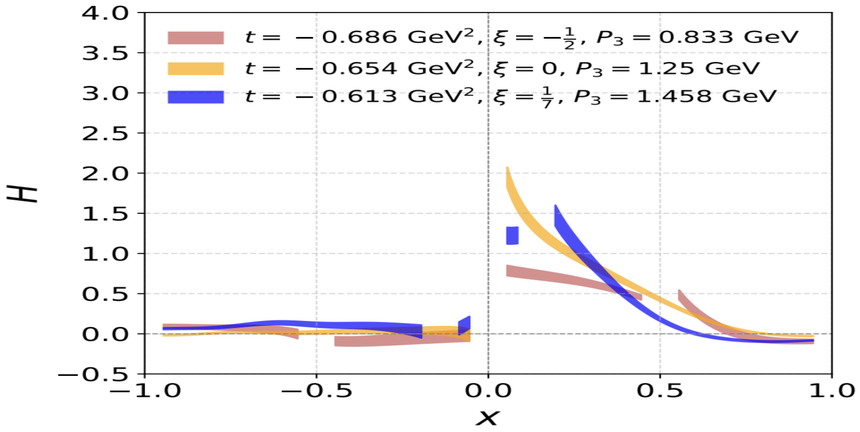
Quasi-Distribution

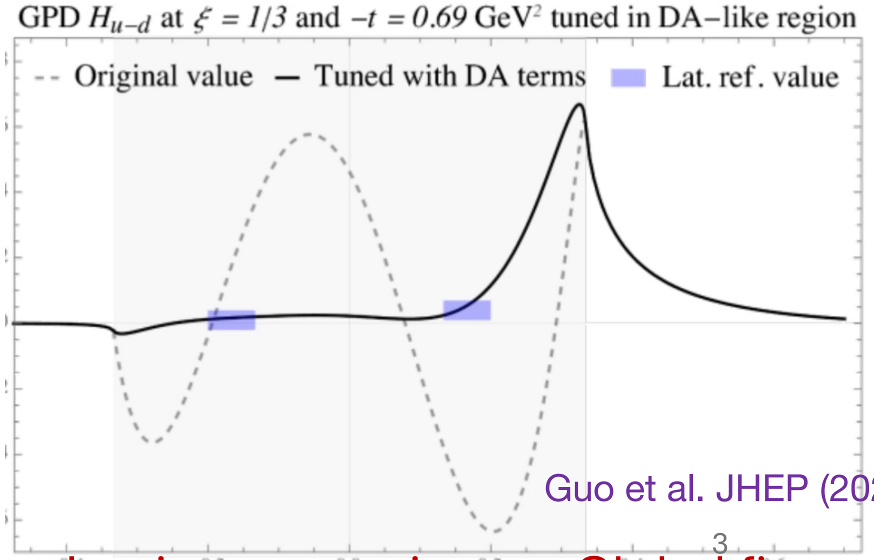


Holligan & Lin, PRD (2024)



arxiv:2509.15931



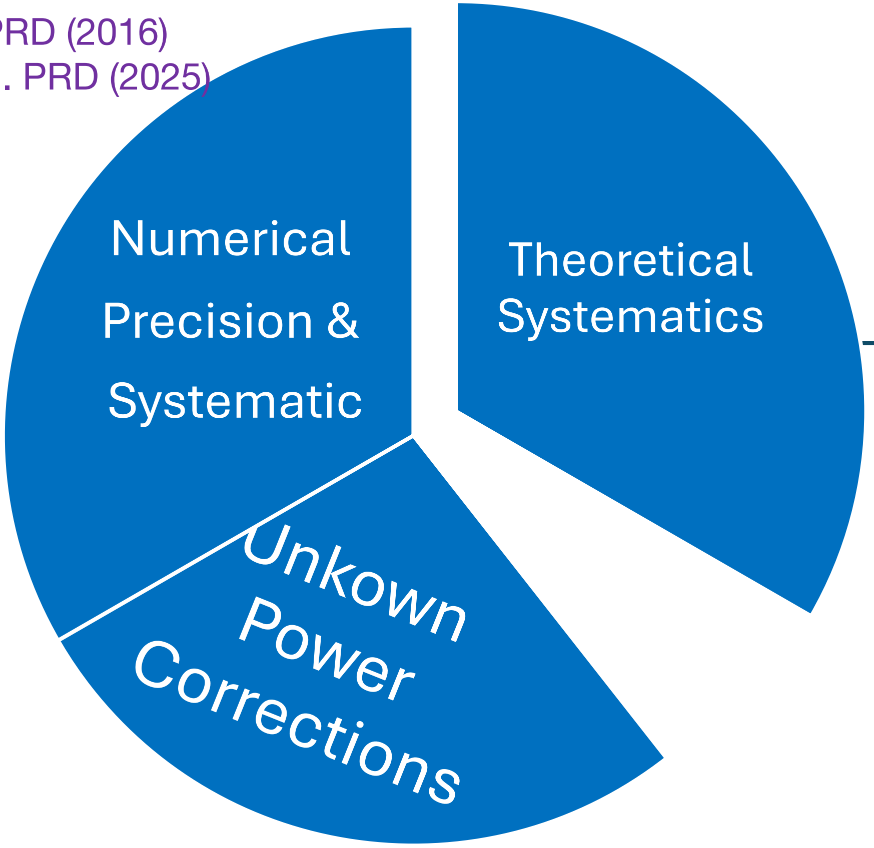


Guo et al. JHEP (2022)

Lattice constraints on Global fit

Precision control in LaMET

Bali et al. PRD (2016)
Zhang et al. PRD (2025)



Fourier Transformation

Asymptotic extrapolation
Ji et al. NPB (2021)

Renormalization

Hybrid Renormalization
Ji et al. NPB (2021)

$1/P_z$ correction

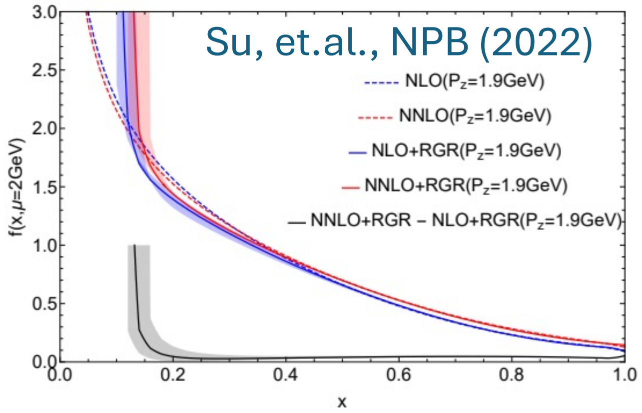
Leading Renormalon Resummation

Zhang et al. PLB (2023)

α_s^n correction

Higher-Order Pert Calculation
Large Log Resummation

Difficulties in off-forward matrix elements



- Forward matrix elements:
  - $\langle P|\bar{q}\Gamma W(0,z)q|P\rangle$
  - One single parton $p_z = xP_z$
  - One single scale $\mu_h = 2p_z$
  - Solved with one evolution equation: DGLAP equation
- Off-forward matrix elements (for DA, $|P'\rangle \to |\Omega\rangle,\ \xi \to 1$):
  - $\langle P|\bar{q}\Gamma W(0,z)q|P'\rangle$
  - Two partons $p_1^z = (x-\xi)P_z,\ \ p_2^z = (x+\xi)P_z$
  - Two distinct physical scales $\mu_{h_1} = 2p_1^z,\ \ \mu_{h_2} = 2p_2^z$
  - How to solve with one evolution equation?

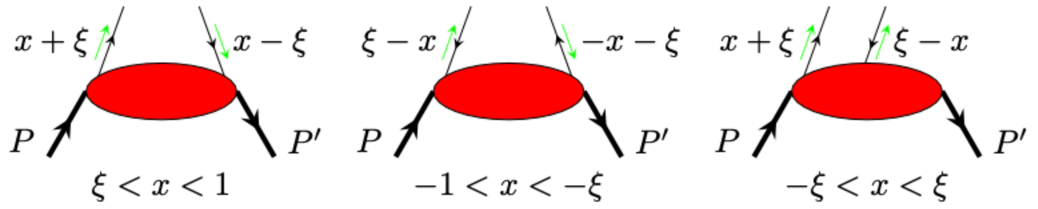
QUARK-GLUON TOMOGRAPHY COLLABORATION

$\xi \neq 0$ Matching kernel for GPD

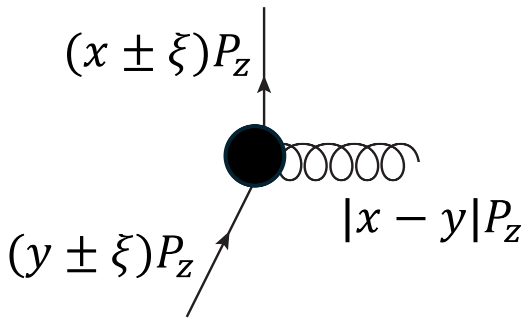
Ji, Yao & Zhang JHEP (2023)

$$
\begin{aligned}
C(x, y, \xi, \mu, P_z) &= \delta(x-y) \\
&+ \frac{\alpha_s C_F}{4\pi}\left[\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right)\left(\ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right) - 1\right)\right. \\
&+ \left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right)\left(\ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right) - 1\right) \\
&+ \left.\left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right)\left(\ln\left(\frac{4(x-y)^2 P_z^2}{\mu^2}\right) - 1\right)\right]
\end{aligned}
$$

Logs in GPD matching



- Outgoing quark/Incoming antiquark momentum
  - $\left(\frac{|\xi+x|}{2\xi(\xi+y)} + \frac{|\xi+x|}{(\xi+y)(y-x)}\right) \ln\left(\frac{4(\xi+x)^2 P_z^2}{\mu^2}\right)$
  - Suppressed when $(\xi + x) \to 0$ except for $x \to y$ or $\xi \to 0$
- Incoming quark/Outgoing antiquark momentum
  - $\left(\frac{|\xi-x|}{2\xi(\xi-y)} + \frac{|\xi-x|}{(\xi-y)(x-y)}\right) \ln\left(\frac{4(\xi-x)^2 P_z^2}{\mu^2}\right)$
  - Suppressed when $(\xi - x) \to 0$ except for $x \to y$ or $\xi \to 0$



- Gluon emission momentum
  - $\left(\left(\frac{\xi+x}{\xi+y} + \frac{\xi-x}{\xi-y}\right)\frac{1}{|x-y|} - \frac{|x-y|}{\xi^2-y^2}\right) \ln\left(\frac{4(y-x)^2 P_z^2}{\mu^2}\right)$
  - Enhanced when $x \to y$

In the ERBL region, logarithms are important only in the threshold limit

$x \to y$

Verified at two-loop

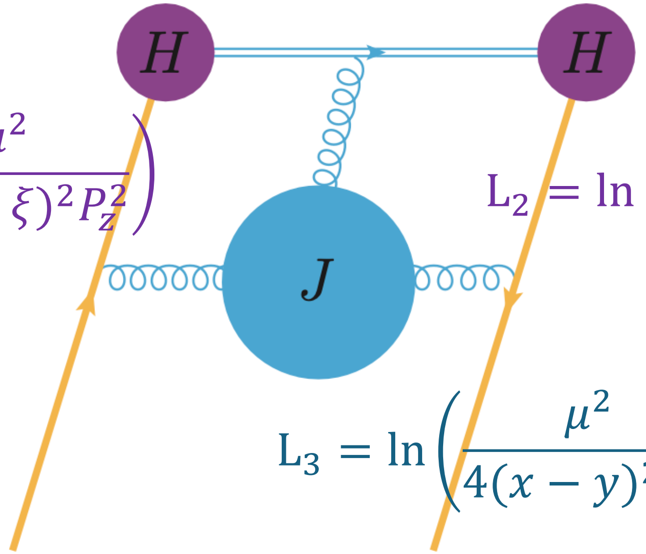
arxiv:2504.09367

Further Factorization when $x \to y$

Becher, Neubert & Pecjak JHEP(2007)

- All three logarithms are important only in the threshold limit
  - $x - y \to 0$, soft gluon emission
- Matching QCD to Soft-Collinear Effective Theory
  - Matrix elements of collinear fileds
    - GPD in SCET
  - Matching for collinear modes
    - Sudakov factor H
      - Outgoing q (Incoming $\bar{q}$) component
      - Incoming q (outgoing $\bar{q}$) component
  - Soft modes
    - Jet function J

$$C(x, y, \xi, P_z) \to J(|x-y|P_z) \otimes H((x \pm \xi)P_z)$$



$$L_1 = \ln\left(\frac{\mu^2}{4(x+\xi)^2 P_z^2}\right)$$

$$L_2 = \ln\left(\frac{\mu^2}{4(x-\xi)^2 P_z^2}\right)$$

$$L_3 = \ln\left(\frac{\mu^2}{4(x-y)^2 P_z^2}\right)$$

Threshold log: soft gluon

Ji, Liu & Su JHEP (2023)

Separating all three scales

- $C(x \to y, \xi, \mu, P) \approx H(|\xi + x|P, \mu) H(|\xi - x|P, \mu) J(|x - y|P, \mu)$
- $H\left(L_z^{\pm} = \ln\left(\frac{2xP}{\mu}\right)^2 + i\pi \,\mathrm{sgn}(zx), \mu\right) = 1 + \frac{C_F \alpha_s(\mu)}{4\pi}\left[-\frac{1}{2}(L_z^{\pm})^2 + L_z^{\pm} - 2 - \frac{5\pi^2}{12}\right]$

Vladimirov & Schafer PRD (2020)
Ji & Liu PRD (2022)

- $J\left(l_z = \ln\frac{z^2\mu^2 e^{2\gamma_E}}{4}, \mu\right) = 1 + \frac{\alpha_s C_F}{2\pi}\left(\frac{1}{2}l_z^2 + l_z + \frac{\pi^2}{12} + 2\right)$

Ji, Liu & Su JHEP (2023)

- Double logarithm come from soft and colinear divergences
- Cancellation of $\ln^2 \mu^2$ between H and J happens at all orders

Correcting the matching kernel

- Resummed Sudakov factor: $H = |H|e^{i\hat{A}}$

$$H(p,\mu,\pm) = |H|(p,\mu_h)e^{S(\mu_h,\mu)-a_H(\mu_h,\mu)}e^{iA_\pm(\mu_h)\mp i\frac{\pi}{2}a_\Gamma(\mu_h,\mu)}\left(\frac{\mu_h}{2p}\right)^{a_\Gamma(\mu_h,\mu)}$$

- Resummed Jet function:

Becher, Neubert & Pecjak JHEP(2007)

$$J(\Delta,\mu) = e^{[-2S(\mu_i,\mu)+a_J(\mu_i,\mu)]}\tilde{J}_z(l_z = -2\partial_\eta, \alpha_s(\mu_i))\left[\frac{\sin(\eta\pi/2)}{|\Delta|}\left(\frac{2|\Delta|}{\mu_i}\right)^\eta\right]_* \frac{\Gamma(1-\eta)e^{-\eta\gamma_E}}{\pi}\Bigg|_{\eta=2a_\Gamma(\mu_i,\mu)}$$

- $C_{TR} = (H \otimes J)_{TR} \otimes (H \otimes J)^{-1}_{NLO} \otimes C_{NLO}$
- Inverse matching:

$$C^{-1}_{TR} = C^{-1}_{NLO} \otimes (H \otimes J)_{NLO} \otimes (H \otimes J)^{-1}_{TR}$$

What are the scale choices of $\mu_{1,2}$ and μ_i?

Scale choices of resummation

- Hard scale:
  - $H(|\xi + x|P, \mu)$: quark momentum $\mu_{h_1} = 2|\xi + x|P$
  - $H(|\xi - x|P, \mu)$: quark momentum $\mu_{h_2} = 2|\xi - x|P$
- Semi-hard scale:
  - $J(|y - x|P, \mu)$: gluon momentum $\mu_i = 2|y - x|P$?
  - This scale choice is not applicable because $\mu_i \to 0$ hits the Landau Pole for any given x! But they're integrable. Becher, Neubert & Pecjak JHEP(2007)
  - The semi-hard scale is related to the analyticity of GPD
- Actual semi-hard scale choice turns out to be
  - $2|x - x_0|P_z$, where x_0 is the closest non-smooth point in GPD
  - Could be $2|x \pm 1|P_z$ or $2|x \pm \xi|P_z$

2-step resummation in DGLAP region

If we set $\mu = 2|x|P_z$, the logarithm in DGLAP region are important only in threshold limit

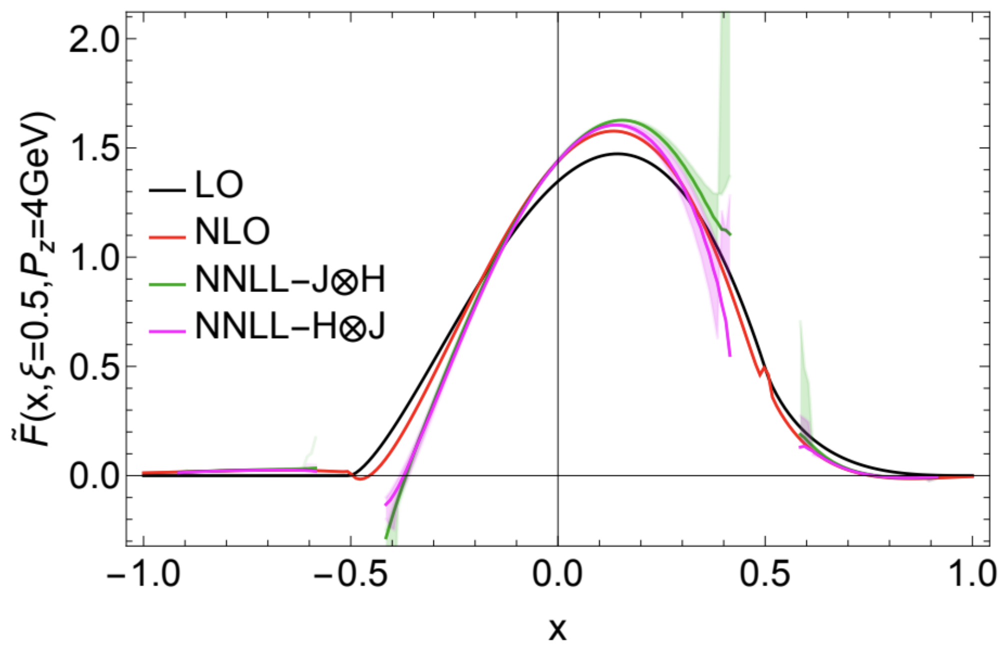
1. First evaluate the full kernel $C^{NLO}(x, y, \xi, P, \mu_h)$ at $\mu_h = 2|x|P_z$
2. Then resum logarithm in threshold limit at $\mu_h = 2|x|P_z$
   - $H(\mu_h, x, \xi, P) = H\left(\mu_{h_1}, |\xi + x|P\right) H\left(\mu_{h_2}, |\xi - x|P\right) e^{S(\mu_{h_1}, \mu_h) + S(\mu_{h_2}, \mu_h)}\ldots$
   - $J(\mu_h, x, y, P) = J(\mu_i, |x - y|P)\, e^{-2S(\mu_i, \mu_h)}\ldots$
3. Finally use the full evolution to evolve C^{TR} to scale μ
   - $C(x, y, \xi, P, \mu) = C^{TR}(x, y, \xi, P, \mu_h)\widehat{K}(\mu_h, \mu)$

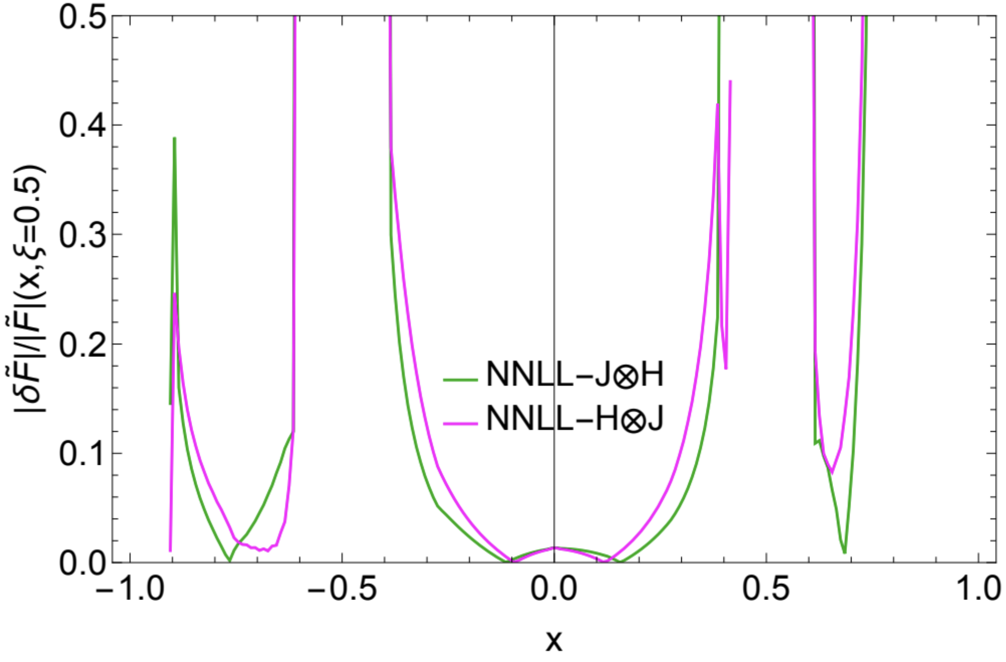
Test on A GPD model

- Double-distribution:

$$H(x,\xi)=\theta(x>-\xi)\frac{2+\lambda}{4\xi^3}\left(\frac{x+\xi}{1+\xi}\right)^{\lambda}[\xi^2-x+\lambda\xi(1-x)]$$
$$-\theta(x>\xi)\frac{2+\lambda}{4\xi^3}\left(\frac{x-\xi}{1-\xi}\right)^{\lambda}[\xi^2-x-\lambda\xi(1-x)],$$

- Test on moderate $\xi=0.5$ to check the effects in both DGLAP and ERBL region, choosing $P_z=4$ GeV for better numerical stability



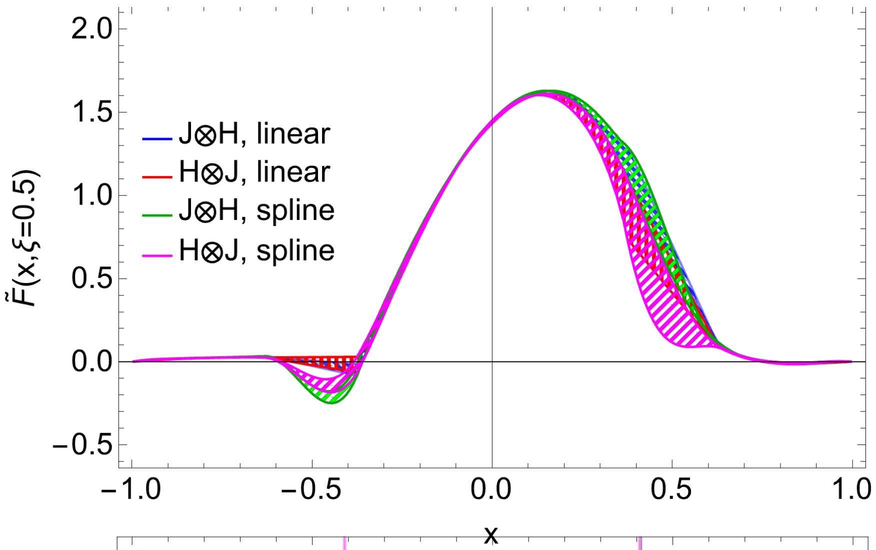


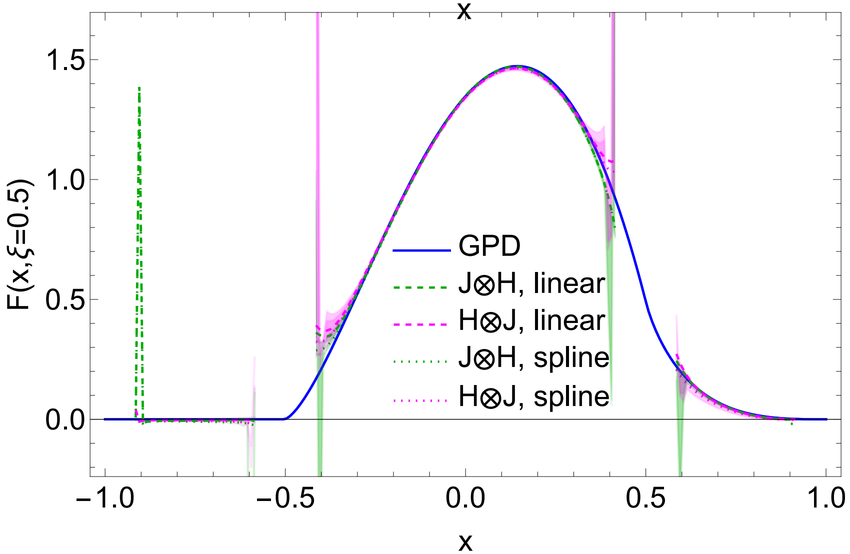
Applicable range of LaMET

- Scale variation blows up near
  - $x = \pm(1 - 0.1)$
  - $x = \pm\xi \pm 0.1$

  for $P_z = 4$ GeV.

- Test inverse matching: Interpolation between truncated points
  - Spline interpolation
  - Linear interpolation
- Difference between interpolations can be thought as NP effects or power corrections





Summary

- There are 3 different logs in quasi-GPD matching, cannot resum in the traditional approach by solving 1 single evolution equation
- We demonstrate ALL the logarithms are only important in the threshold limit $x \to y$
- The matching can be further factorized in the threshold limit into 3 different quantities, each containing 1 single logarithm
- The resummed results clearly demonstrate the LaMET calculation is reliable when $|x \pm 1| > \delta$ and $|x \pm \xi| > \delta$ with δ suppressed by the hadron momentum
- The results are self-consistent, and LaMET matching is insensitive to NP regions.

Puzzles for future study

- What about points $x = \pm\xi$?

- Are neighborhoods of $x = \pm\xi$ protected by continuity?

Thank you for listening!