Gluon Collins-Soper Kernel from lattice QCD

Speaker: Yang Fu (MIT)

Collaborators: Artur Avkhadiev (Argonne), Phiala Shanahan (MIT), Michael Wagman (Fermilab), Yong Zhao (Argonne)
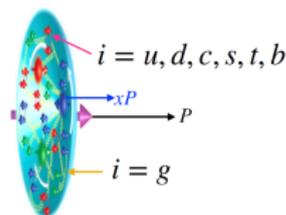
Oct 8 - 10, 2025

LaMET 2025, CFNS, Stony Brook University

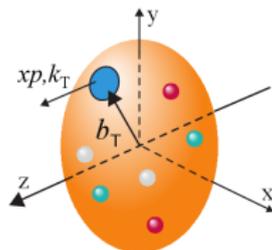- Parton distribution function (PDF): $f_{i/h}(x)$

- probability of finding a parton i in hadron h

  carrying momentum fraction $x \rightarrow$ longitudinal

- Transverse-momentum-dependent PDF (TMD PDF):

$$f_{i/h}(x, \vec{k}_{\mathrm{T}}), \quad \text{or coordinate-space} \quad f_{i/h}(x, \vec{b}_{\mathrm{T}}) = \int \mathrm{d}^2 \vec{k}_{\mathrm{T}} e^{i \vec{k}_{\mathrm{T}} \cdot \vec{b}_{\mathrm{T}}} f_i(x, \vec{k}_{\mathrm{T}})$$

- probability of finding parton i with fraction x and

  <u>transverse momentum $\vec{k}_{\mathrm{T}}$</u> $\rightarrow$ longitudinal and transverse
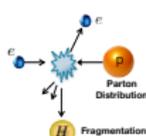
  (or the Fourier conjugate $\vec{b}_{\mathrm{T}}$)

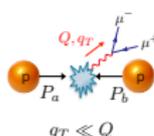$\Rightarrow$ Rich hadron 3D internal structure in TMD PDFs!

- TMD PDFs can be determined in various processes
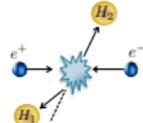
need ability to relate different energy scales

Semi-Inclusive DIS   Drell-Yan   Dihadron in e+e-

- Evolution of TMD PDFs:

1. UV renormalization scale μ 2. rapidity scale ζ

The evolution kernels are universal (independent of external hadron h)

$$f_{i/h}(x, b_{\mathrm{T}}, \mu, \zeta) = f_{i/h}(x, b_{\mathrm{T}}, \mu_0, \zeta_0)$$

$$\times \exp\left[\int_{\mu_0}^{\mu} \frac{\mathrm{d}\mu'}{\mu'} \underline{\gamma_\mu^i(\mu', \zeta_0)}\right] \exp\left[\frac{1}{2}\underline{\gamma_\zeta^i(\mu, b_{\mathrm{T}})} \ln \frac{\zeta}{\zeta_0}\right]$$

UV anomalous dimension   rapidity anomalous dimension
(Collins-Soper kernel)

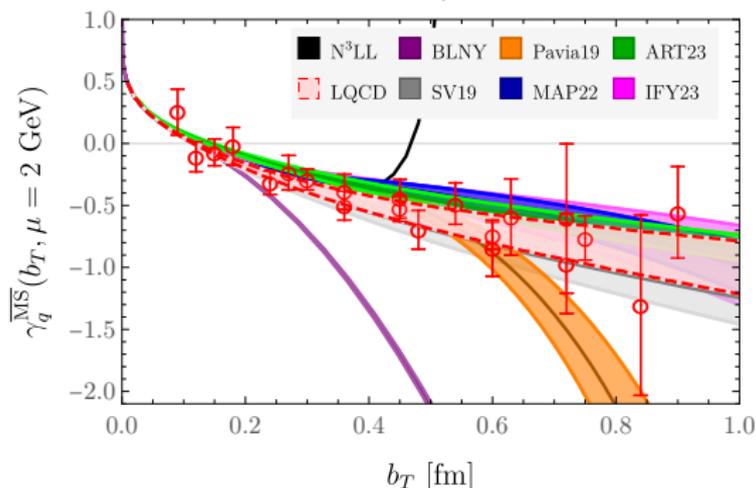- UV anomalous dimension γ_μ^i is perturbative as long as scales are large

But CS kernel γ_ζ^i is always nonperturbative for $b_{\mathrm{T}} \gtrsim \Lambda_{\mathrm{QCD}}^{-1}$

(even if the evolution variables are perturbative)

- Our group's LQCD calculation of quark CS kernel:

(A. Avkhadiev's talk at ~ 20 mins ago)

Avkhadiev, Shanahan, Wagman, Zhao, PRD 108 (2023) 11, 114505
PRL 132 (2024) 23, 231901

- First such calculation with systematic control of quark mass, operator mixing, and discretization effects

(Jin-Xin's talk for another syst. controlled calculation)

- Model-dependence in pheno. parameterizations is significant
lattice results are precise enough to discriminate between different models

What about gluon CS kernel?

- Experimentally:

lack of data for gluon TMDs. But can expect in the near future from EIC

- Theoretically:

- perturbative region: 1-loop result is

$$\gamma_\zeta(\mu, b_T) = -\frac{\alpha_s}{\pi} \ln \frac{b_T^2 \mu^2}{4e^{-2\gamma_E}} \times \begin{cases} C_F, & \text{quark} \\ C_A, & \text{gluon} \end{cases} + \mathcal{O}(\alpha_s^2)$$

only differ by a group theory factor (C_A v.s. C_F), almost the same as quark

- non-perturbative region: nobody knows!

- This work: extend our calculation to the gluon CS kernel

it will be the first lattice prediction for future experiments

- Light-cone TMDs can be related to Quasi-TMDs via LaMET

Quasi-TMDs   pert. matching   light-cone TMDs   CS kernel

$$\frac{\tilde{f}(x, b_{\mathrm{T}}, \mu, P^z)}{\sqrt{S_r(b_{\mathrm{T}}, \mu)}} = H(\mu, xP^z) f(x, b_{\mathrm{T}}, \mu, \zeta) \exp\left[\frac{1}{2}\gamma_\zeta(b_{\mathrm{T}}, \mu) \ln \frac{(2xP^z)^2}{\zeta}\right]$$

Soft factor

$$+ \mathcal{O}\left[\frac{1}{(xP^z b_T)^2}, \frac{M^2}{(xP^z)^2}, \frac{\Lambda_{\mathrm{QCD}}^2}{(xP^z)^2} + (x \to 1-x)\right]$$

<div align="right">

Ji, Liu & Liu, PLB 811, 135946 (2020)

Ebert, Schindler, Stewart & Zhao, JHEP 04, 178 (2022)

</div>

- CS kernel extracted from ratio with different momenta P_1 and P_2

$$\gamma_\zeta(b_T, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\tilde{f}(x, b_{\mathrm{T}}, \mu, P_1^z)}{\tilde{f}(x, b_{\mathrm{T}}, \mu, P_2^z)}\right] + \delta\gamma_\zeta(x, \mu, P_1^z, P_2^z) + \text{p.c.}$$

with $\delta\gamma_\zeta(x, \mu, P_1^z, P_2^z)$ computed from pert. matching kernel

- Power corrections need to be under control $\to x$ away from 0 and 1
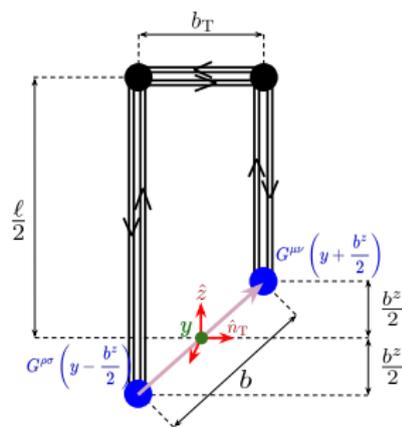
Staple-shaped Operator

- Operators for the gluon quasi-TMDs

$$\mathcal{O}_g^{\mu\nu,\rho\sigma}(b) = G^{\mu\nu}\left(\frac{b}{2}\right) W_{\sqsupset}^{\text{adj}}(b,l) G^{\rho\sigma}\left(-\frac{b}{2}\right)$$

- For the unpolarized case, four operators are

multiplicatively renormalizable (same as gluon PDF)

$$\mathcal{O}_g^{(1)} = \mathcal{O}_g^{0i,0i}, \quad \mathcal{O}_g^{(2)} = \mathcal{O}_g^{3i,3i}$$

$$\mathcal{O}_g^{(3)} = \frac{1}{2}(\mathcal{O}_g^{0i,3i} + \mathcal{O}_g^{3i,0i}), \quad \mathcal{O}_g^{(4)} = \mathcal{O}_g^{3\mu,3\mu}$$

Zhang et al, PRL 122, 142001 (2019)
Zhu et al, JHEP 02, 114 (2023)

⇒ renormalization cancelled in the ratio

- Symmetry properties: by Hermiticity and translation invariance

$$\mathcal{O}_g^{\mu\nu,\rho\nu}(b) = [\mathcal{O}_g^{\mu\nu,\rho\nu}(b)]^\dagger = \mathcal{O}_g^{\rho\nu,\mu\nu}(-b)$$

⇒ These operators are real and symmetric under $b \to -b$

- Two observables can be used to compute the CS kernel on lattice

- Quasi-beam functions from 2pt and 3pt functions

$$\tilde{B}(b^z, b_{\mathrm{T}}, \ell, P^z) = \langle h(P^z)|\mathcal{O}(b_\mu, 0, \ell)|h(P^z)\rangle$$

Staple-shaped operator $\mathcal{O}$

- Quasi-TMD wavefunctions (WFs) from 2pt functions

$$\tilde{\psi}(b^z, b_{\mathrm{T}}, \ell, P^z) = \langle 0|\mathcal{O}(b_\mu, -P^z, \ell)|h(P^z)\rangle$$
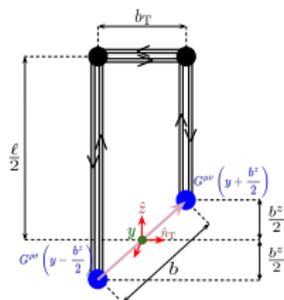

- For quark CS kernel, quasi-TMD WFs are used

- lower computational cost for 2pts

- For gluon CS kernel, we prefer quasi-beam functions

- 3pts can be computed by correlating 2pts with gluon operator

- No quark disconnected contractions

- Ensemble: clover-on-HISQ

  $L^3 \times T = 32^3 \times 48$, $a = 0.15$ fm, $m_\pi = 170$ MeV

  A. Bazavov et al. (MILC)
  PRD 87 (2013) 054505

  $N_{\text{cfg}} \times N_{\text{src}} = 1097 \times 2048 \approx 2M$ (and will be increased to $\sim 8M$)

- CS kernel is universal — independent of hadronic state

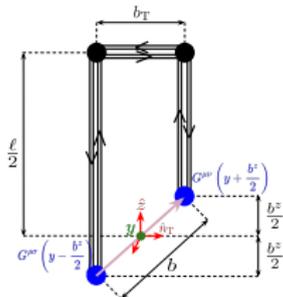 pion state is the primary target (suppressed power corrections $M^2/(xP^z)^2$)

  nucleon state is much noisier and requires different optimal smearing

- All multip. renormlizable operators calculated

  7 values of $\ell \in [1.05, 3.45]$ fm to control finite-ℓ effect

  5 values of $P^z = 0.77, 1.03, 1.29, 1.55, 1.81$ GeV

  $(n^z = 3 - 7)$

More about two-point correlator:

- Kinematically enhanced interpolating operators are used

  R. Zhang et al. PRD 112 (2025) L051502

- Details about momentum smearing

in order to have more measurements, a single smearing momentum is used

$$K^z = \frac{2\pi}{L} n^z \text{ with } n^z = 4.5$$

then do inversion from this source, and projected to different momentum

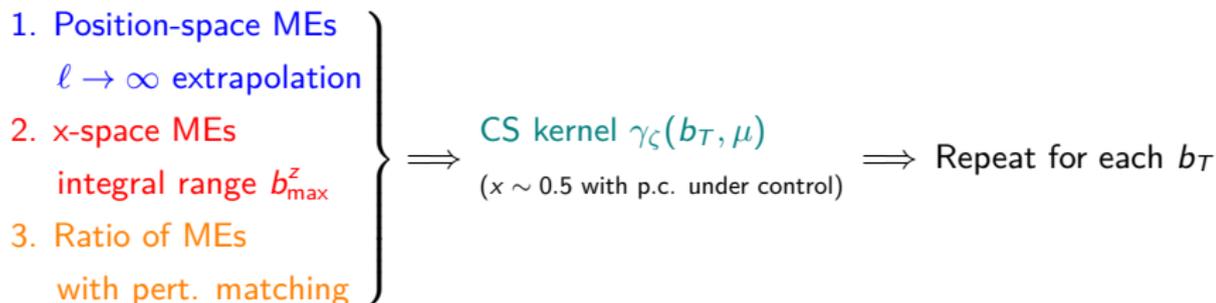- Best choice of smearing fraction for pion is around $\zeta \approx 0.8$

  G. S. Bali et al. 2016 and our test

- For larger momentum $n^z = 6, 7$, the fraction is close to optimal

- For smaller momentum $n^z \leq 5$, signal-to-noise problem is less significant

CS kernel from ratio:

$$\gamma_\zeta(b_T, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int \frac{\mathrm{d}b^z}{2\pi} \, e^{ixP_1^z b^z} N(P_1^z) \lim_{\ell \to \infty} \tilde{B}(b^z, b_T, \ell, P_1^z) \tilde{\Delta}^S(b_T, l)}{\int \frac{\mathrm{d}b^z}{2\pi} \, e^{ixP_2^z b^z} N(P_2^z) \lim_{\ell \to \infty} \tilde{B}(b^z, b_T, \ell, P_2^z) \tilde{\Delta}^S(b_T, l)} \right]$$

$$+ \, \delta\gamma_\zeta(x, \mu, P_1^z, P_2^z) + \text{p.c.}$$

Quasi-soft factor $\tilde{\Delta}^S(b_T, l)$ is a Wilson loop
to remove the linear divergence $\sim l + b_T$

1. Position-space MEs
   $\ell \to \infty$ extrapolation
2. x-space MEs
   integral range $b_{\max}^z$
3. Ratio of MEs
   with pert. matching

$\Longrightarrow$ CS kernel $\gamma_\zeta(b_T, \mu)$
($x \sim 0.5$ with p.c. under control) $\Longrightarrow$ Repeat for each b_T

1-loop matching for gluon available in
Schindler et al., JHEP 08, 084 (2022)
Zhu et. al, JHEP 02, 114 (2023)

$\mathcal{O}_g^{(1)} = \mathcal{O}_g^{0i,0i}$, rel. error $\sim 10\%$
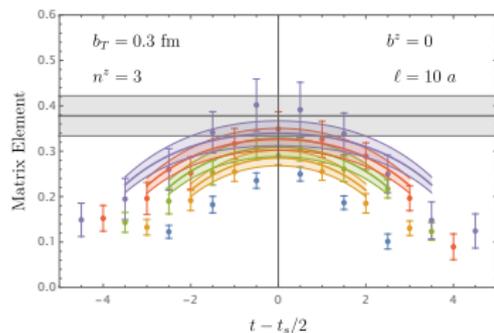
$\mathcal{O}_g^{(2)} = \mathcal{O}_g^{3i,3i}$, rel. error $\sim 40\%$
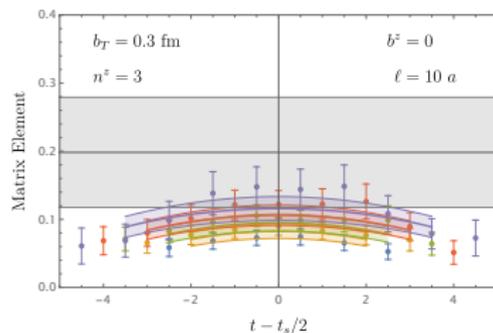
$\mathcal{O}_g^{(3)} = \frac{1}{2}(\mathcal{O}_g^{0i,3i} + \mathcal{O}_g^{3i,0i})$, very noisy
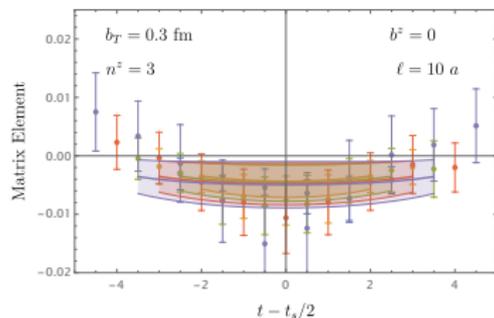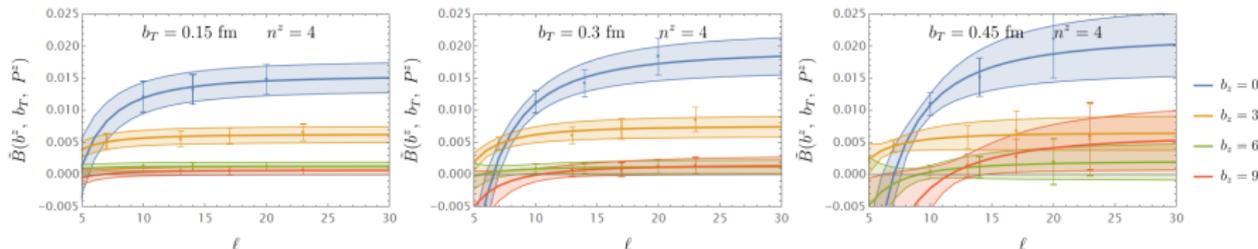
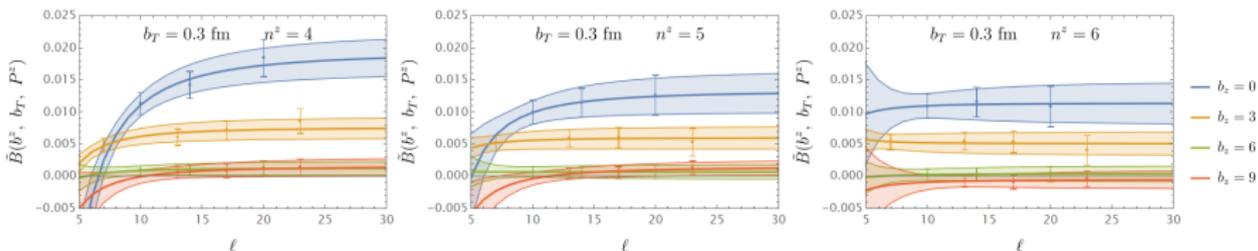$\mathcal{O}_g^{(4)} = \mathcal{O}_g^{3\mu,3\mu}$, rel. error $\sim 20\%$
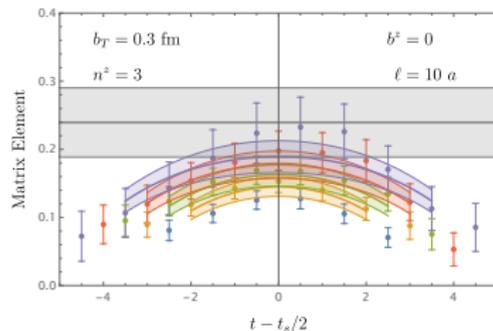
Results shown below are using $\mathcal{O}_g^{(1)} = \mathcal{O}_g^{0i,0i}$ (most precise)

Position-space matrix elements: ℓ-dependence

- stronger at large b_T

- suppressed at large P^z

extrapolate to $\ell \to \infty$ using $\tilde{B}(\ell) = \tilde{B}(\ell \to \infty) + c/\ell^2$ for each b_T and P^z

- MEs as function of $b^z P^z$

Numerically real and symmetric after averaging over all staple orientations

Before $\ell \to \infty$ extrapolation
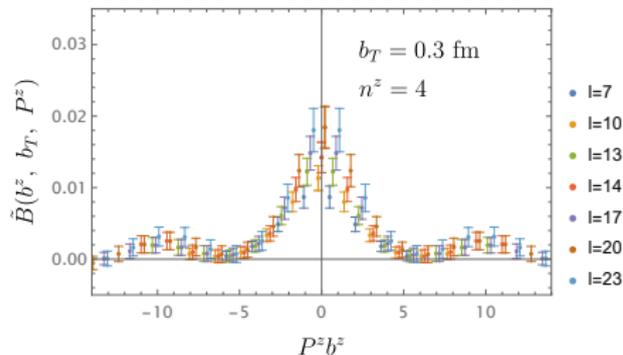
after extrapolation
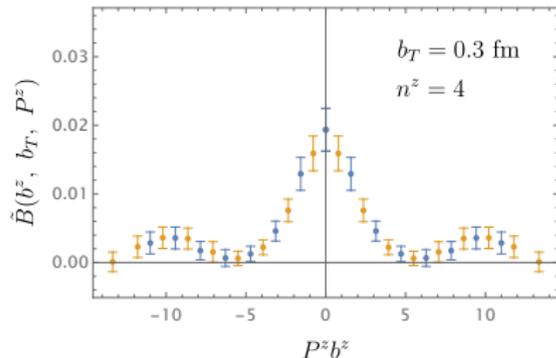
- Small P^z need longer staple length $\ell \sim 20a = 3$ fm

- Most of the contribution is from the region of $b^z P^z \lesssim 5$
tails are not very important within current precision (will show later)

- Comparison of quark and gluon cases

- $b_T = 0.3$ fm with similar staple length $\ell_{max} \sim 3$ fm

- Small $P^z = 3 \times \dfrac{2\pi}{L} = 0.77$ GeV

Quark: few % errors with 1.7×10^4 meas   Gluon: 10% errors with 2×10^6 meas
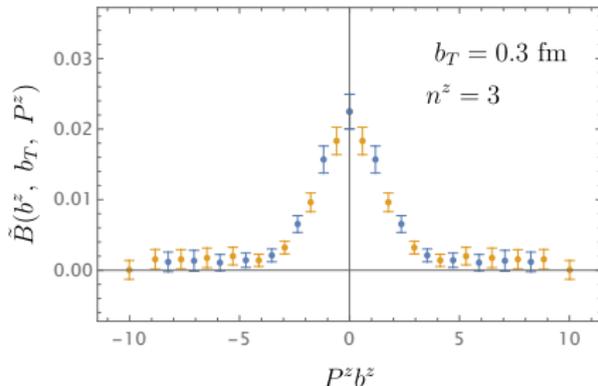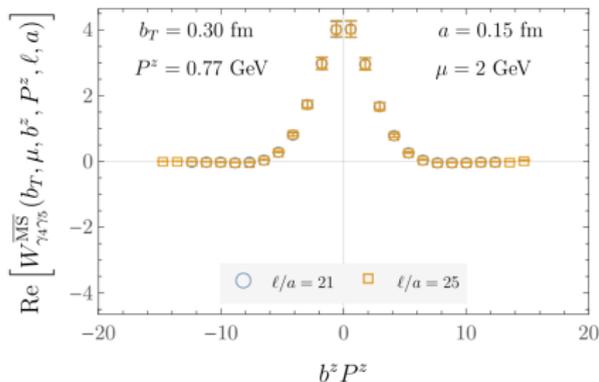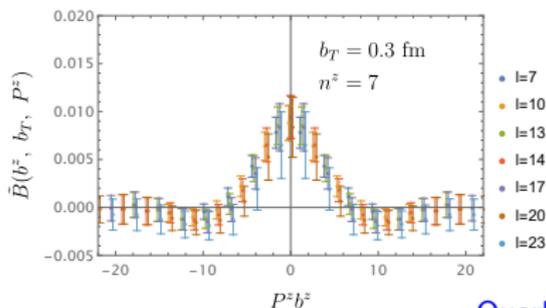

- More than two orders of magnitude stats are needed for a similar precision

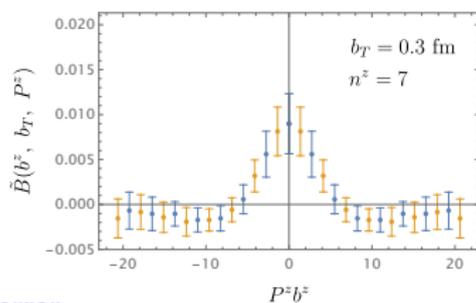- Large $P^z = 7 \times \dfrac{2\pi}{L} = 1.81$ GeV

less staple length dependence, $\ell \sim 1.5$ fm is sufficiently long
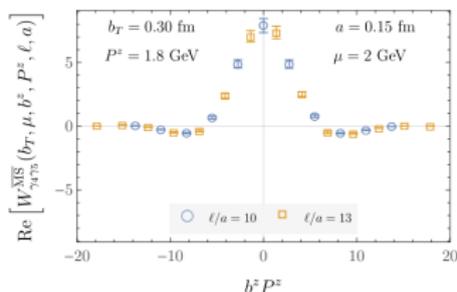
Gluon: before extrapolation   after extrapolation, 30% error

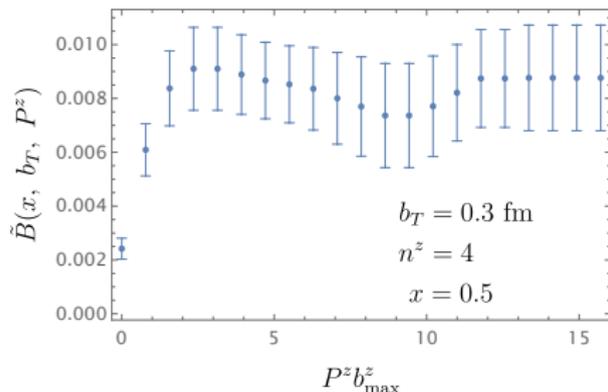

Quark: $\sim 10\%$ error

- At large momentum the gluon signal is getting significantly worse

- Fourier transform to x-space

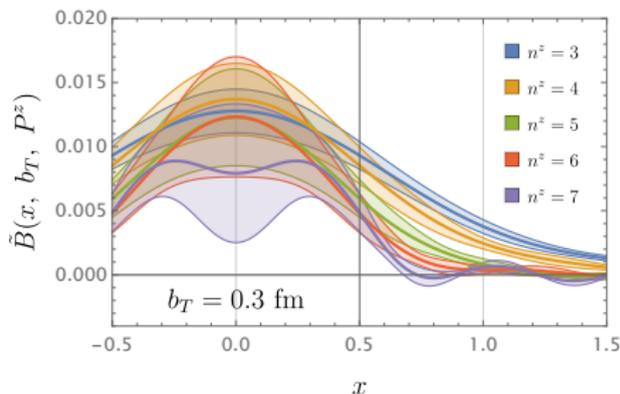$$\tilde{B}(x, b_T, P^z) = \int_{|b^z| < b_{max}^z} \frac{db^z}{2\pi} e^{ix P^z b^z} N(P^z) \tilde{B}(b^z, b_T, P^z)$$

- Dependence on b_{max}^z

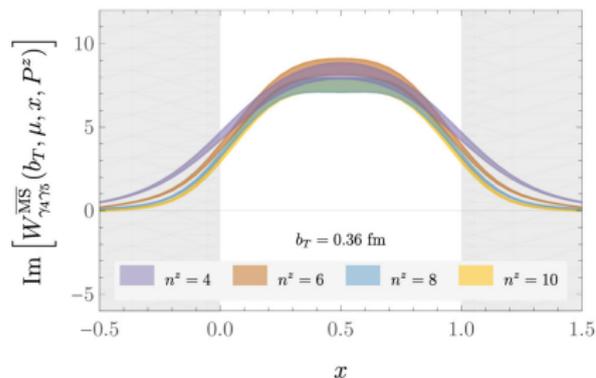Fourier transformation is saturated for
$P^z b_{max}^z \gtrsim 5$ with errors

$b_T = 0.3$ fm
$n^z = 4$
$x = 0.5$

- MEs as a function of x

tails outside physical range $x \in [-1, 1]$
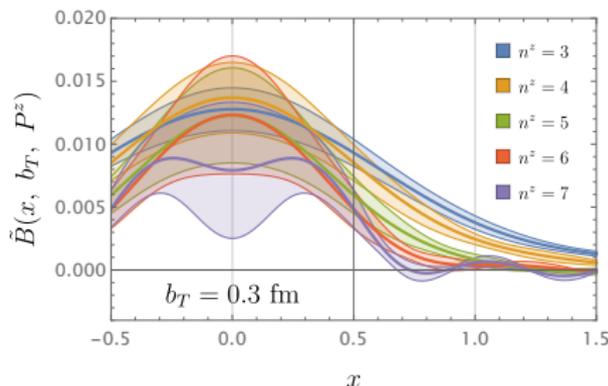are reduced as P^z increases

$b_T = 0.3$ fm

- Quark and gluon x-space MEs have different symmetries

- Quark (with pion state)

- Gluon

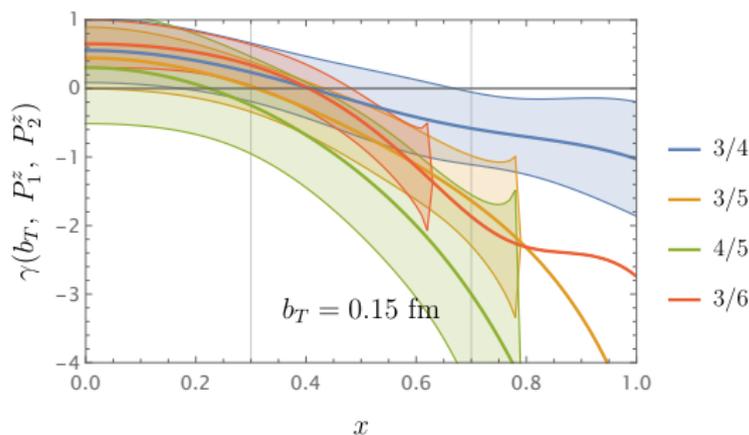
Symmetric under $x \to 1 - x$

momentum fraction of two quarks

Symmetric under $x \to -x$

gluons are their own anti-particles

- For the gluon, the situation is even worse since the ratio is not taken around the peak, but at $x \sim 0.5$.

- CS kernel from ratio:

$$\gamma_\zeta(b_T, \mu) = \frac{1}{\ln(P_1^z/P_2^z)} \ln\left[\frac{\tilde{B}(x, b_T, P_1^z)}{\tilde{B}(x, b_T, P_2^z)}\right] + \delta\gamma_\zeta(x, \mu, P_1^z, P_2^z) + \text{p.c.}$$
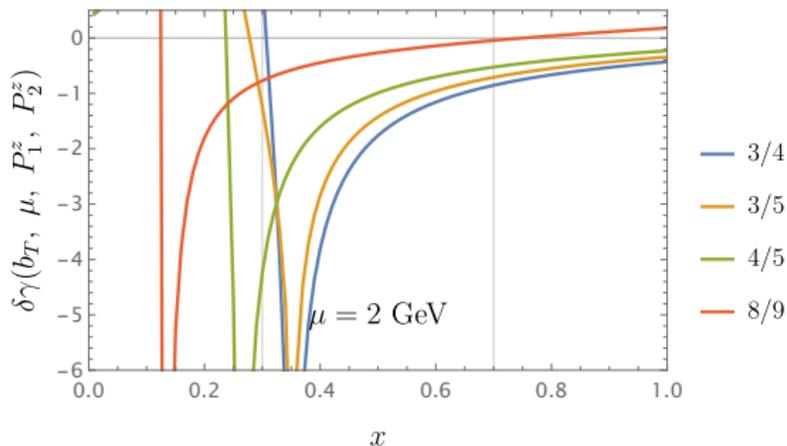
currently just LO matching

didn't plot all the combinations as large P^z
is too noisy and might not even be positive

- Still residual x-dependence: need higher-order matching & larger P^z

NLO matching

- The NLO matching has a singularity near $x \sim 0.5$ for small P_1^z/P_2^z pairs

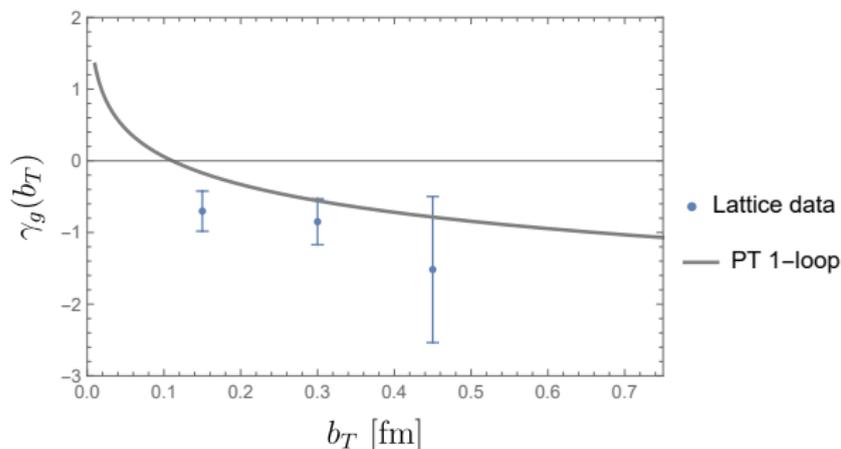Schindler et al., JHEP 08, 084 (2022)

Zhu et. al, JHEP 02, 114 (2023)

hence also need larger momentum pair, which requires more measurements

- Higher-order matching will be useful and can confirm convergence

- Our (very) preliminary result of the gluon CS kernel

- Comparable with 1-loop perturbative calculation $\rightarrow$ proof of concept

- We'll improve the result and have one more point at $b_T = 4a = 0.6$ fm with more stats

Larger b_T

- Use sqrt of Wilson loop as the quasi-soft factor to remove linear divergence

But it's no longer numerically positive at large b_T and ℓ
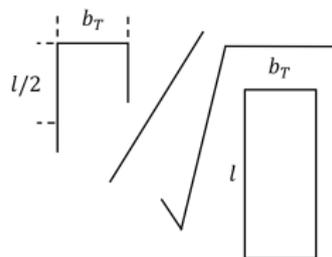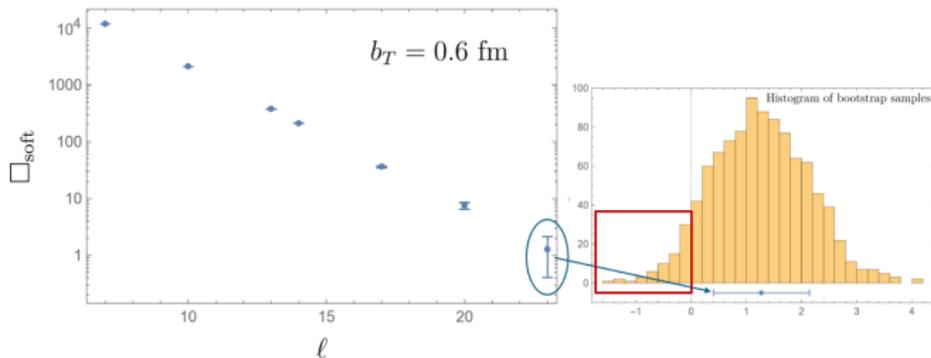
"staple" divided by sqrt of Wilson loop to remove linear div.

- Either use more configurations, or extrapolate to large ℓ using a function that guarantees the result is positive

$\Rightarrow$ in any case this indicates gluonic observables are noisy

- Coulomb gauge method may help!

(Y. Zhao, PRL 2024 & Jinchen and Xiang's talks in the morning session)

CS kernel from quark to gluon:

- In contrast to quark TMDs, gluon TMDs are almost unknown
  (both experiments and lattice QCD)

- With current statistics we're able to obtain the CS kernel up to 0.45 fm,
  results are comparable with 1-loop perturbation theory

- We'll have more two-points, and also try Coulomb gauge in the future,
  gluon operators can be calculated separately and are cheap

Thank you!

Backup Slides

- The $\mathcal{O}_g^{(1)} = \mathcal{O}_g^{0i,0i}$ with trace subtraction has less ℓ-dependence
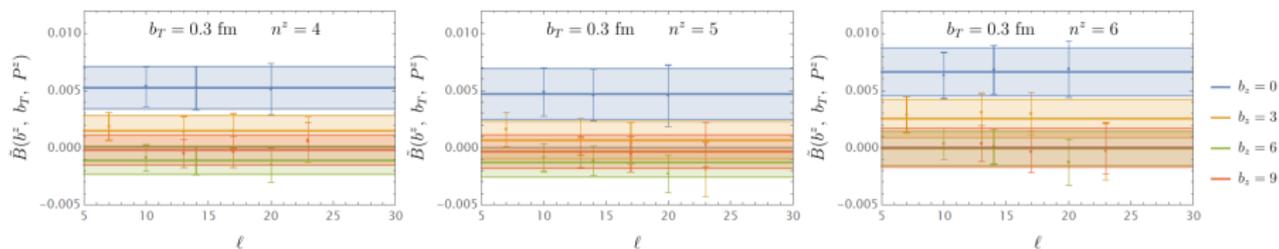
bands are results of fitting to a constant

- No visible ℓ-dependence even at small momentum ($n^z = 4$)

- However they are unfortunately too noisy to use

Rapidity divergence

- Regulators such as dimensional regularization only regulate UV divergences rapidity divergences arise in soft and collinear need a dedicated regulator
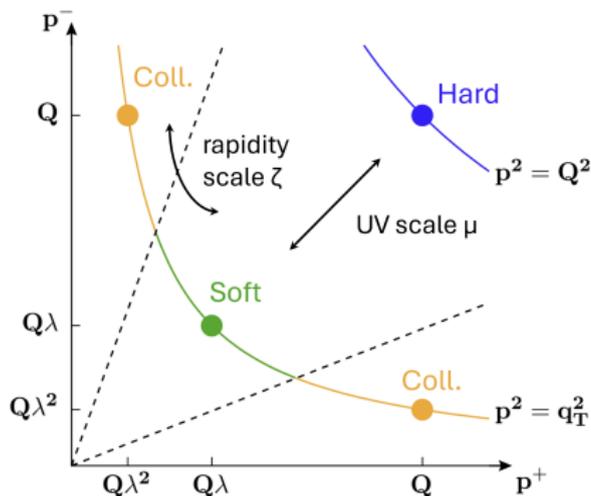
Ebert, Stewart & Zhao, JHEP 09, 037 (2019)

- A concrete example

$$I_{\text{div}} = \int \mathrm{d}p^+ \mathrm{d}p^- \frac{f(p^+p^-)}{(p^+p^-)^{1+\epsilon}} = \frac{1}{2} \int \frac{\mathrm{d}(p^-/p^+)}{p^-/p^+} \int \mathrm{d}(p^+p^-) \frac{f(p^+p^-)}{(p^+p^-)^{1+\epsilon}}$$