From quark to gluon Collins–Soper kernels with lattice QCD

and b_T-dependent LaMET matching

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in collaboration with

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[2402.06725]

+ 2 ongoing

Related talks:

1. Jin-Xin Tan, Thu 11:00 am
2. Yang Fu, Thu 1:55 pm
3. Wayne Morris, Thu 2:15 pm

XIIth Meeting on Lattice Parton Physics from Large Momentum Effective Theory (LaMET 2025)

Center for Frontiers in Nuclear Science, Stony Brook University

October 8–10, 2025

Center for Frontiers in Nuclear Science

Stony Brook University

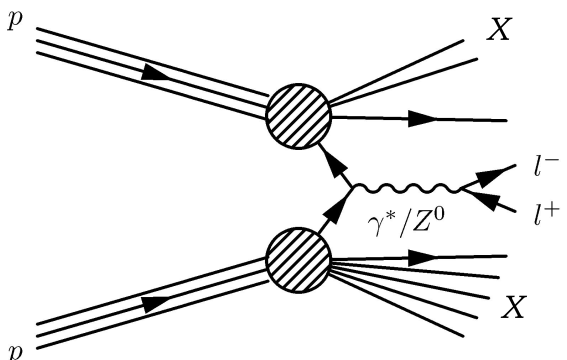
Talk Outline

1. *Review: TMDs and the Collins-Soper (CS) kernel*
2. *5-year progress in quark CS kernel (our group)*
3. *Takeaways for the gluon CS kernel*
4. *Takeaways for LaMET formalism*

Review:
TMDs and the Collins-Soper (CS) kernel

- TMD Factorization
- TMD evolution
- The CS kernel

TMD Factorization: Drell-Yan example



$$Q^2 = m_{l\bar{l}}^2$$

1. Collinear **(Y term)**: Λ_{QCD} << qT ~ Q

$$\frac{d\sigma}{dQ^2\, dY} = \sigma_0 \sum_{q,\bar{q}} H_{q\bar{q}}\,[Q/\mu]\, f_{q/p_a}(x_a,\mu) f_{\bar{q}/p_b}(x_b,\mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

2. TMD **(W term)**: qT ~ Λ_{QCD} << Q

$$\frac{d\sigma^{\rm W}}{dQ^2\, dY\, d^2\mathbf{q}_T} = \sigma_0 \sum_{q,\bar{q}} H_{q\bar{q}}\,[Q/\mu] \int_0^\infty d^2\mathbf{b}_T\, e^{i\mathbf{b}_T\cdot\mathbf{q}_T} f_{q/p_a}(x_a,\mathbf{b}_T,\mu,\zeta_a) f_{\bar{q}/p_b}(x_b,\mathbf{b}_T,\mu,\zeta_b) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}, \frac{q_T^2}{Q^2}\right)$$

Fourier transform
(large bT is nonperturbative)

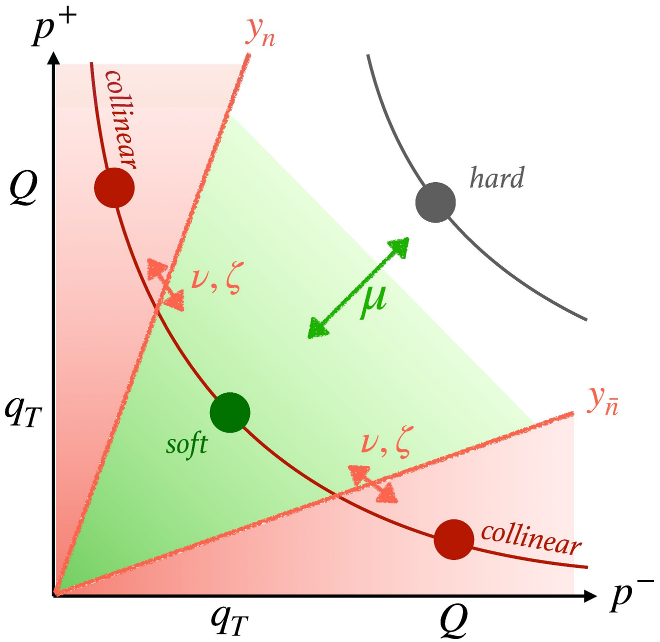
$$\zeta_a \zeta_b = Q^4$$

Holds for small qT
(~ large bT)

Collins-Soper scale ζ
- related to rapidity *Y*
- proportional to hadron momentum *P*

Artur Avkhadiev, Argonne National Lab

TMD evolution



RG equations of TMDs:

$$\mu \frac{\mathrm{d}}{\mathrm{d}\mu} \ln f_{i/h}^{\overline{\mathrm{MS}}}(x, b_T, \mu, \zeta) = [\gamma_\mu^{q/g}]^{\overline{\mathrm{MS}}}(\mu, \zeta)$$

$$\sqrt{\zeta} \frac{\partial}{\partial \sqrt{\zeta}} \ln f_{i/h}^{\overline{\mathrm{MS}}}(x, b_T, \mu, \zeta) = [\gamma_\zeta^{q/g}]^{\overline{\mathrm{MS}}}(\mu, b_T)$$

RG rapidity evolution:

$$f_{p/h}(x, b_T, \mu, \zeta) = f_{p/h}(x, b_T, \mu, \zeta_0) \exp\left[\frac{1}{2} \gamma_p(b_T, \mu) \ln \frac{\zeta}{\zeta_0}\right]$$

The Collins-Soper (CS) kernel

Computed as a ratio of TMDs at different ζ : Proportional to hadron momentum *P*

- **Before LaMET matching:**

$$\gamma_q(b_T,\mu) = \frac{2}{\ln(\zeta_1/\zeta_2)} \ln \frac{f_{q/h}(x,b_T,\mu,\zeta_1)}{f_{q/h}(x,b_T,\mu,\zeta_2)}$$

Encoded by light-like matrix elements

- **After LaMET matching:**

$$\gamma_q(b_T,\mu) = \frac{1}{\ln(P_1/P_2)} \ln \frac{C_q(xP_1^z,\mu)\tilde{f}_{q/h}(x,b_T,\mu,P_1)}{C_q(xP_2^z,\mu)\tilde{f}_{q/h}(x,b_T,\mu,P_2)} + \text{p. c.}$$

LaMET matching coefficients

Encoded by space-like matrix elements

Properties of CS kernel:

- Independent of hadronic state
- Differs for quarks and gluons
- Non-perturbative at large b_T for any μ

LaMET TMD matching:

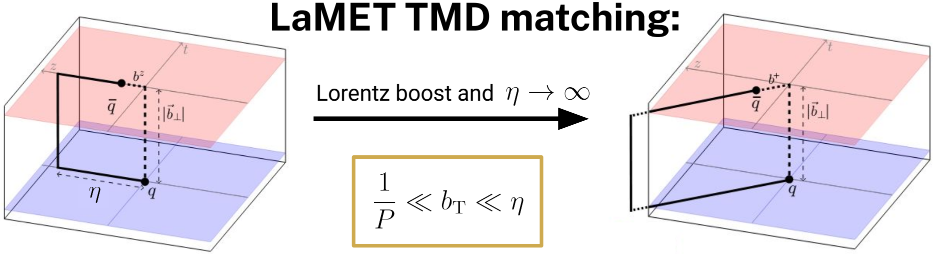
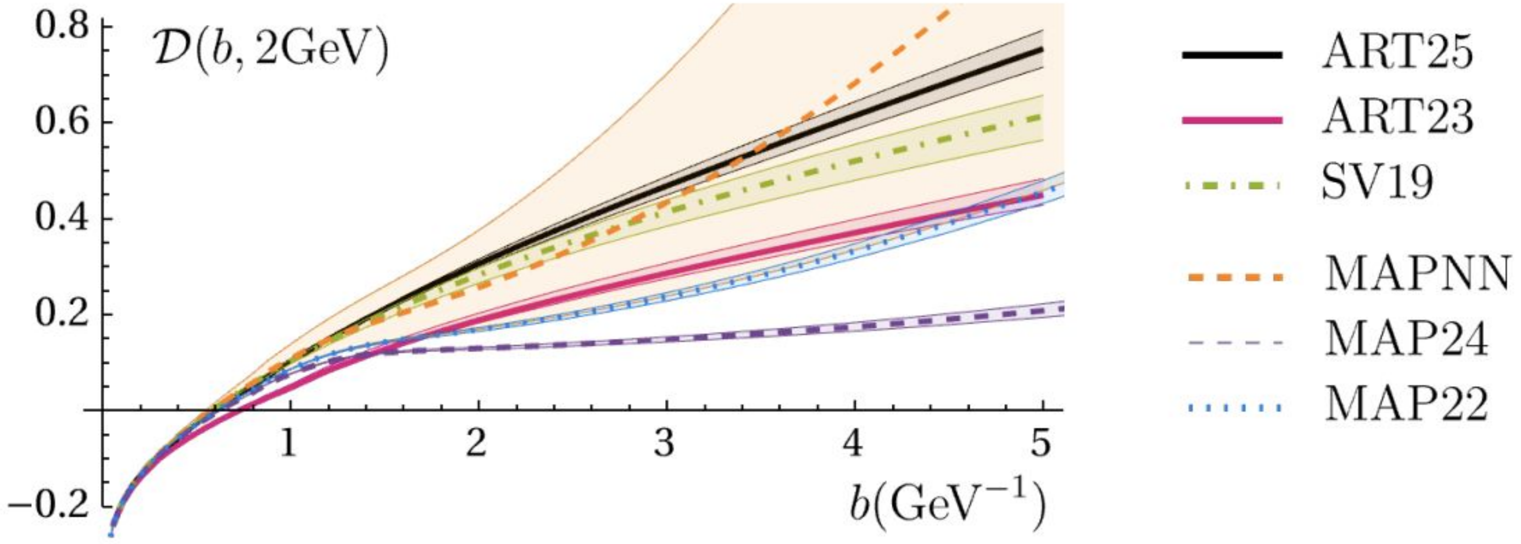


Fig. by Ebert, Stewart, Zhao, JHEP 1909 (2019)
(notation changed for consistency).

5-year progress in quark CS kernel (our group)

- Goal: joint TMD fits
- Results from lattice
- Preliminary joint fit

TMD global fits: significant uncertainty from nonperturbative modeling of CS kernel



Determination of unpolarized TMD distributions from the fit of Drell-Yan and SIDIS data at N4LL

V. Moos, I. Scimemi, A. Vladimirov, P. Zurita, PRD 112 (2025) 3, 034501 [2503.11201]

Shanahan, Wagman, Y.Zhao, PRD 102 (2020) 1, 014511 [2003.06063]

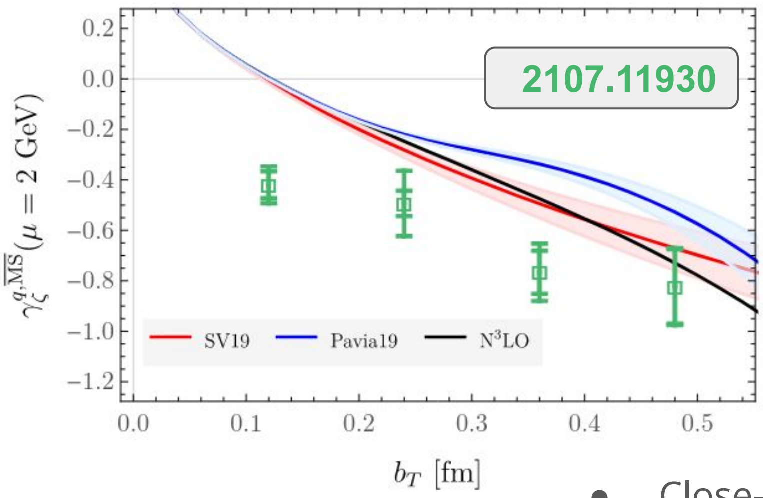
Shanahan, Wagman, Y.Zhao PRD 104 (2021), 2107.11930

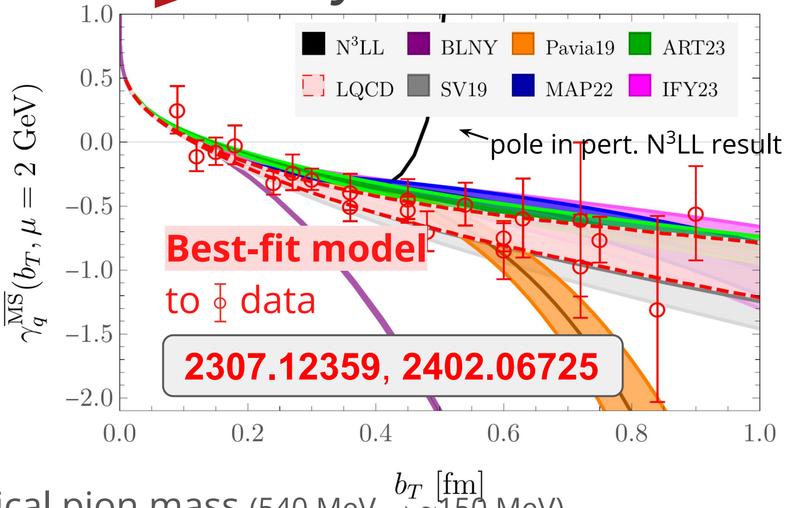
Avkhadiev, Shanahan, Wagman, Y.Zhao, PRD 108 (2023) 11, 114505 [2307.12359]

Avkhadiev, Shanahan, Wagman, Y.Zhao, PRL 132 (2024) 23, 231901 [2402.06725]

2020–2025: Significant progress in **quark CS kernel** calculations with lattice QCD (our group)

Proof of concept → ***Systematic control***





Improvements:

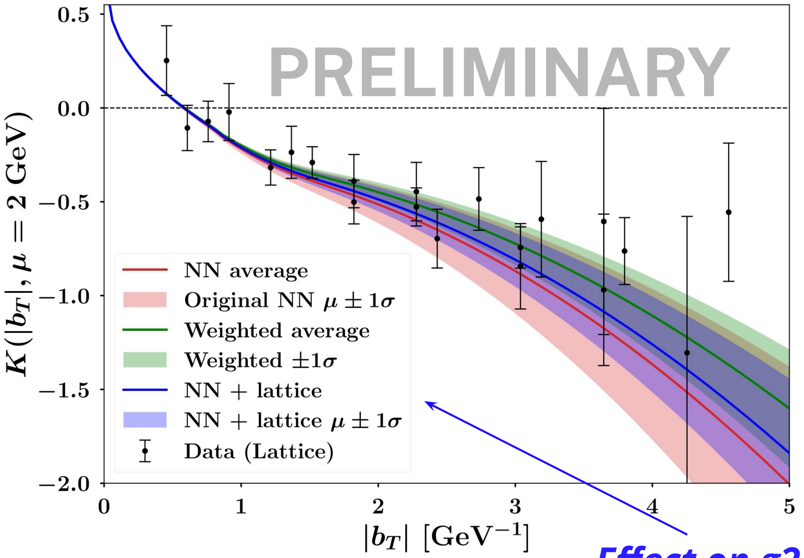
- Close-to-physical pion mass (540 MeV → ~150 MeV)
- Increased bT range (.48 fm → 0.90 fm)
- Quantified mixing from renormalization
- Improved LaMET matching (NLO → NNLL, renormalons, bT corrections)
- **Continuum extrapolation (+ model fit to extrapolated data)**

Ongoing: joint fit to **lattice** + **experiment** [M.A.P. TMD]

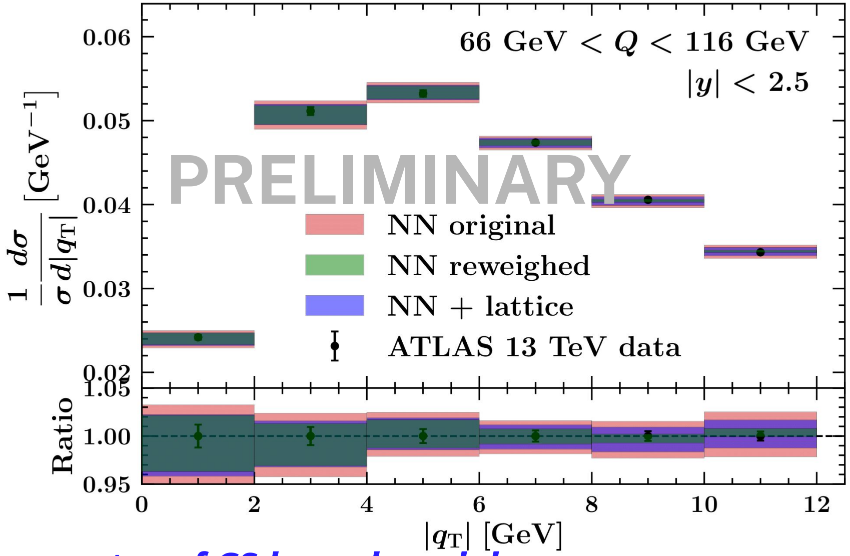
- **Experimental data [482 pts]** from full **TMD fit by MAP**
- **Lattice results [21 pts]** from the 3 ensembles
- Incorporate lattice results by (1) **reweighting** original replicas and (2) **refitting**:

A. Bacchetta, V. Bertone, C. Bissolotti, M. Cerutti, M. Radici, S. Rodini, L. Rossi, PRL 135 (2025) 2, 021904 [2502.04166]

Impact on CS kernel:



Impact on cross-section:



Effect on g2 parameter of CS kernel model: shift central value by ~5%, reduce uncertainty by ~30%

Takeaways for the gluon CS kernel

- Noise from Wilson lines
- Order of LaMET matching
- ***Choice of TMD observable***

} **See next talk (Yang Fu)**

+ Q&A and discussion

Calculation steps analogous for quark and gluon CS kernels:

X. Ji et. al., Phys. Lett. B 811 [1911.03840]

5. Continuum extrapolation

2. Fourier transform

1. Quasi-TMDs in position space

$$\gamma_\zeta^{(i)}(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln \frac{\int_{-\infty}^{\infty} \frac{dP_1^z b^z}{2\pi} e^{-ixP_1^z b^z} \lim_{\ell\to\infty} [\tilde{\mathcal{R}}_{i/h}^{(s)\ell}](b^z, b_T, P_1^z, \mu, a)}{\int_{-\infty}^{\infty} \frac{dP_2^z b^z}{2\pi} e^{-ixP_2^z b^z} \lim_{\ell\to\infty} [\tilde{\mathcal{R}}_{i/h}^{(s)\ell}](b^z, b_T, P_2^z, \mu, a)} + \delta\gamma_{(i)}(\mu, x, P_1^z, P_2^z) + \mathrm{p.c.}$$

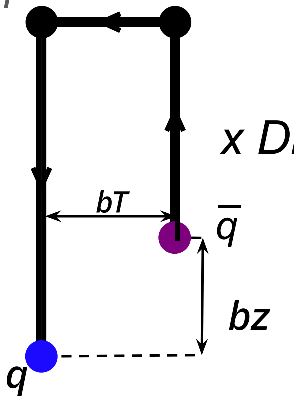
4. Power corrections

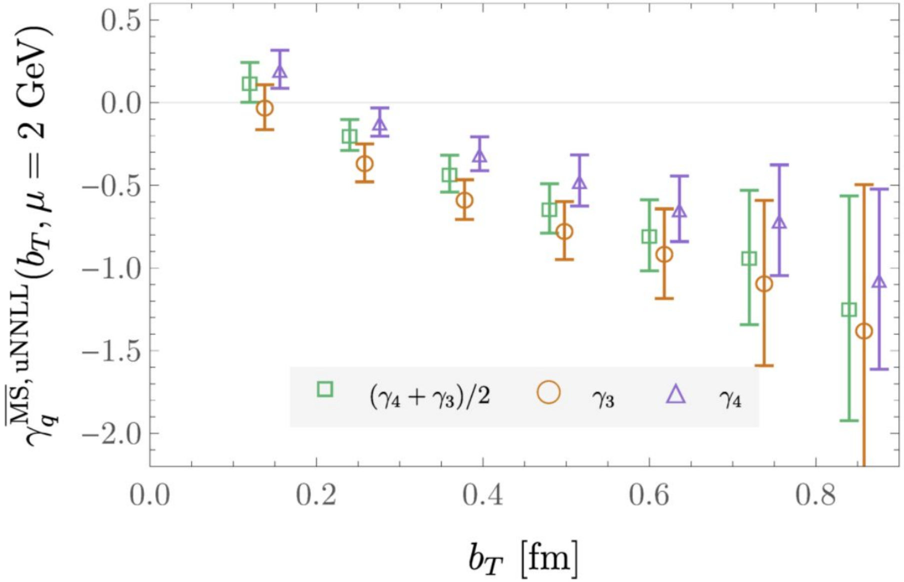
3. EFT matching

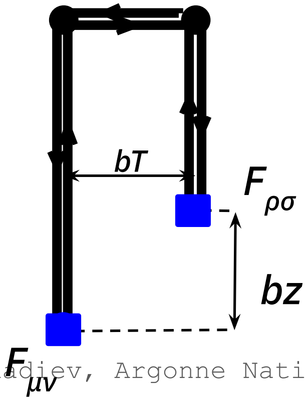
Takeaways for the gluon CS kernel calculation?

Choice of TMD observables

For quark CS kernel: *operator choice affects power corrections & stat. precision*







For gluon CS kernel, *choices of tensor components (μ,ν,ρ, σ) in operator may differ by power corrections & stat. precision*

Related talks:

1. William Good, Wed 9:00 am
2. Raza Sufian, Wed 2:15 pm
3. Yang Fu, Thu 12:30 pm

Takeaways for LaMET formalism

- Renormalon resummation
- ***bT*-dependent matching**

Renormalons in TMD matching

(numerical evidence)

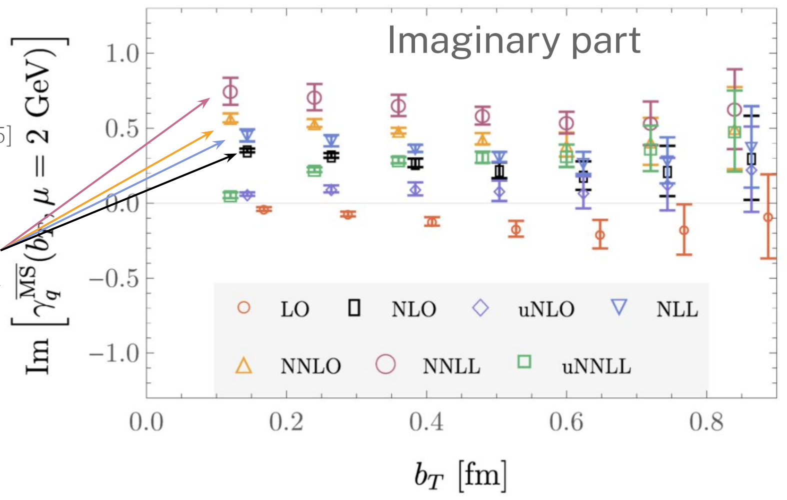
- The kernel is real-valued. M. A. Ebert et. al,, JHEP 09, 037, [1901.03685] Z.-F. Deng et. al, JHEP 09, [2207.07280]
- The kernel *estimate* with TMD WFs has an imag part, **increases w/ matching order:**
- Large *bT*: IR renormalon; use **leading renormalon resummation (LRR):** Y. Liu, Y. Su (LPC), JHEP02(204), [2311.06907]

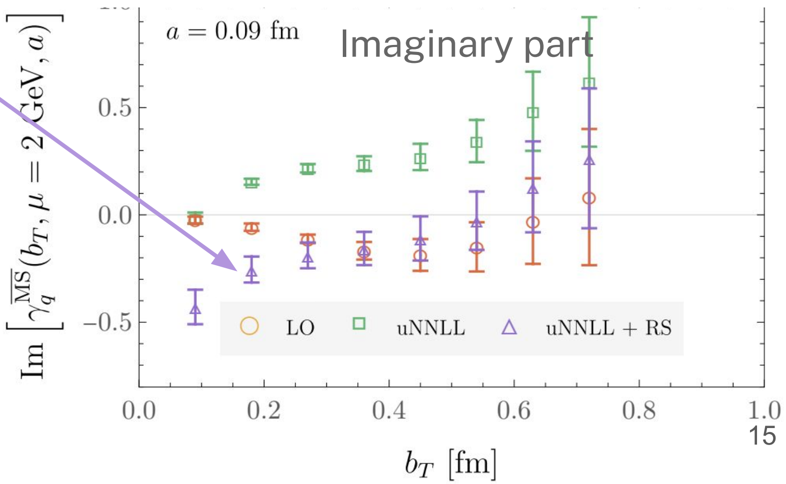
$$C^{\mathrm{LRR}}(p^z,\mu) = C(p^z,\mu) - R(p^z,\mu) + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{p^z}\right)$$

$$R(p^z,\mu) = iN_m \frac{\mu}{p^z}\sum_{n=0}^{\infty}\beta_0^n \alpha_s^{n+1}(\mu)n!$$

N_m = 0.552 for 4 flavors; β_0: lowest-order β-function

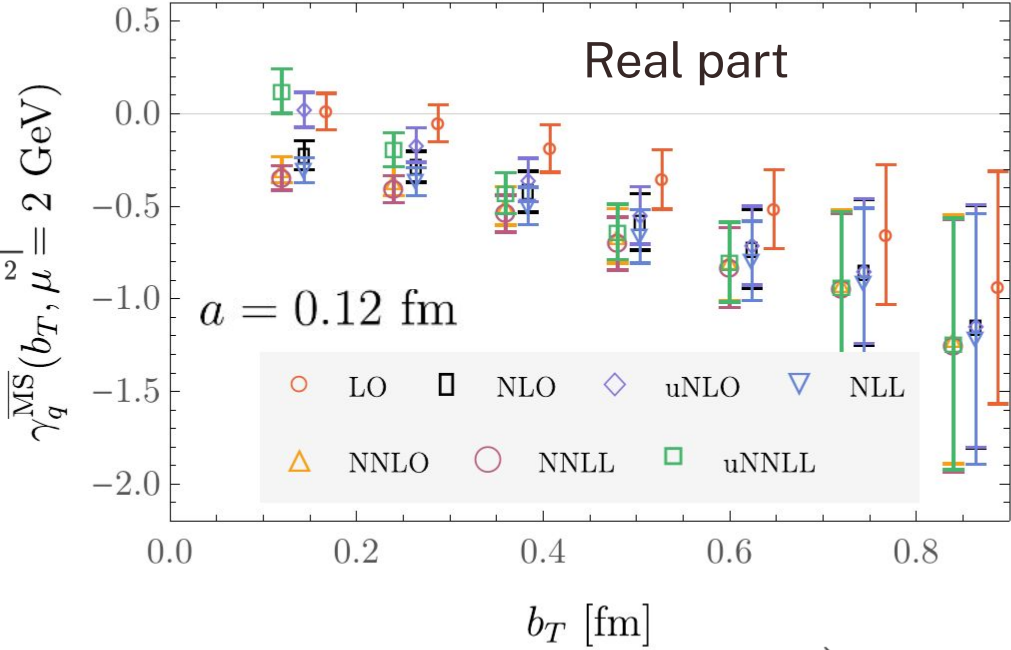
- At small *bT*: due to bT-dep. power corrections, use **unexpanded matching** *(next slide)*





bT-dependence of power corrections (numerical evidence) [1/3]

- TMD LaMET matching needs **qT << Pz**
- Lattice results at small bT have **qT~Pz =>** power corrections $\frac{1}{(xP^z b_T)^2}$
- Full matching in this regime is
  (1) ***bT*-dependent** and
  (2) **convolutional** in ***x***:



$$\tilde{\phi}(x, b_T, \mu, P^z) = \int_0^1 \mathrm{d}y\, H_\phi(y, x, b_T, \mu, P^z)\phi\Big(y, b_T, \mu, \zeta = 2(xP^z)^2, \bar{\zeta} = 2((1-x)P^z)^2\Big)$$

[2307.12359], [2504.04625]

bT-dependence of power corrections (numerical evidence) [2/3]

- Full ***bT*-dependent convolutional** matching: $H_\phi(y, x, \mu, P^z)$

- So far: isolate the *bT*-dependent **multiplicative** part:

Avkhadiev, Shanahan, Wagman, Y.Zhao
PRD 108 (2023) 11, 114505 [2307.12359]
D. Bollweg, X. Gao, J. He, S. Mukherjee, Y. Zhao
[2504.04625]

$$H_\phi(y, x, b_T, \mu, P^z) \xrightarrow{P^z b_T \gg 1} \delta(y-x)\left(\mathbf{C}_\phi^{(\mathrm{u})}(xP^z, b_T, \mu)\mathbf{C}_\phi^{(\mathrm{u})}((1-x)P^z, b_T, \mu)\right)$$

$$\cancel{+\delta H_\phi(y, x, b_T, \mu, P^z)}$$

Drop convolutional piece (for now)

- Defines the **"unexpanded" matching** in this work:

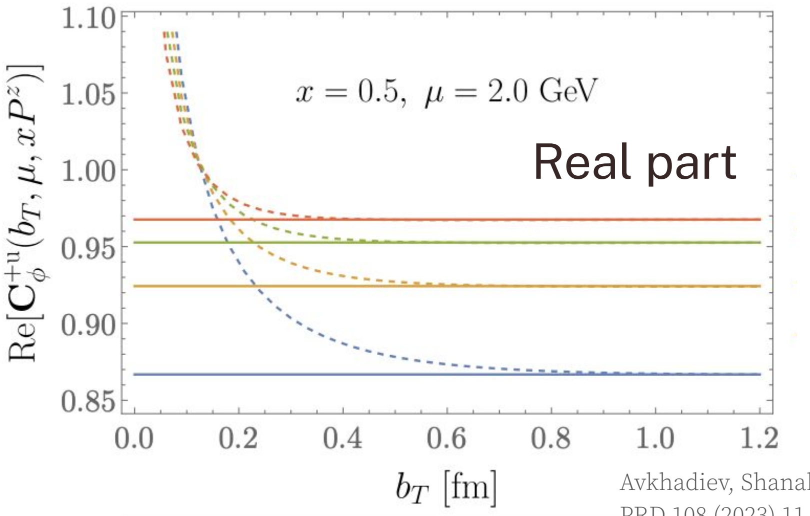
$$\mathbf{C}_\phi^{(\mathrm{u})}(xP^z, b_T, \mu) = \boxed{C_\phi(xP^z, \mu)} + \delta C_\phi(xP^z, b_T, \mu)$$

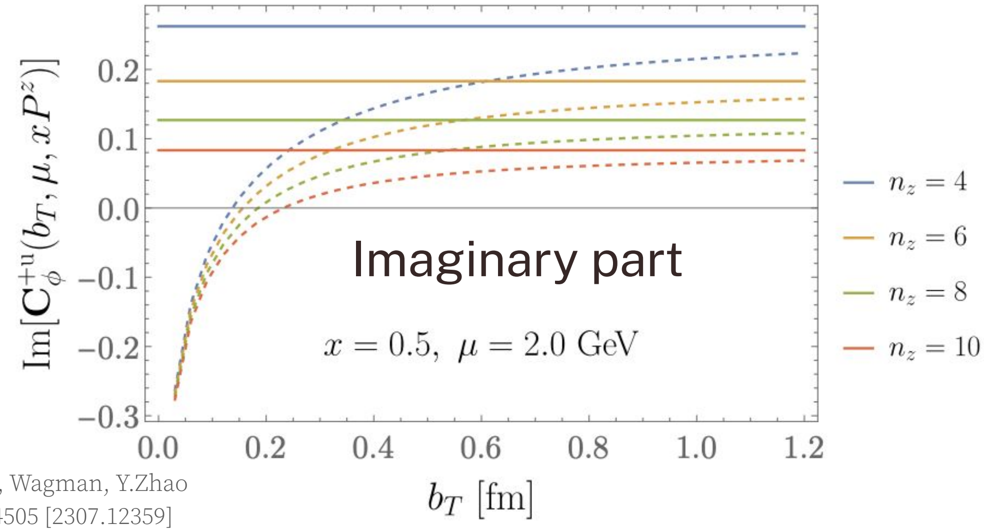
Usual, bT-independent LaMET matching

bT-dependence of power corrections (numerical evidence) [3/3]

- This "unexpanded" matching mitigates *bT*-dependent power corrections
  [(--)**dashed** = unexpanded, (–)**solid** = bT-independent]

- Higher bT sensitivity in the imaginary part of estimate — **explains imaginary part at small bT after renormalon subtraction.**





Avkhadiev, Shanahan, Wagman, Y.Zhao
PRD 108 (2023) 11, 114505 [2307.12359]

Summary and outlook

1. Nonperturbative input to quark, gluon CS kernels crucial to constrain TMDs

2. Quark CS kernel: **global joint TMD fit underway** incorporating lattice results

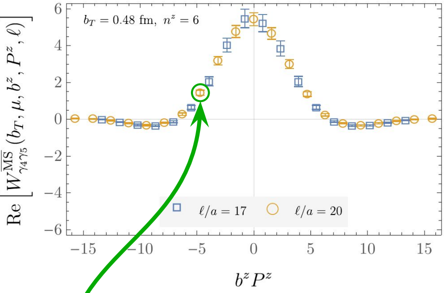
3. Takeaways for gluon CS kernel:
   - Challenging **statistical noise** *[⇒ develop & apply Coulomb-Gauge (CG) methods]*
   - Important **choice of TMD observable** *[noise vs. power corrections]*
   - Need for **higher-order matching** *[⇒ higher-order CG matching]*

4. Takeaways for LaMET formalism:
   need *bT*-dependent convolutional matching.

Q&A

Steps of quark CS kernel calculation:

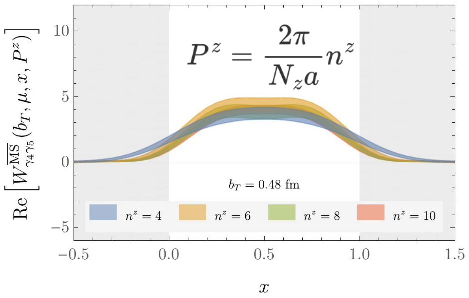
Calculate position-space MEs



$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_1^z)$

Each point is a separate matrix element calculation

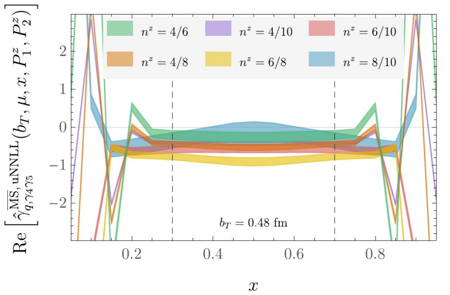
Fourier transform (FT) to momentum-space MEs



$\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z)$

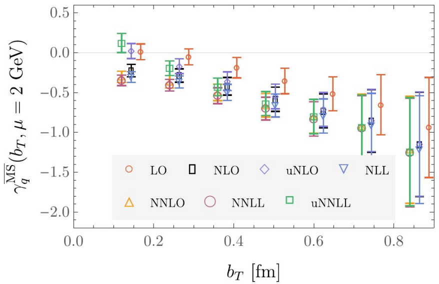
$\sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_1^z)$

Form ratios of MEs + match + fit in x

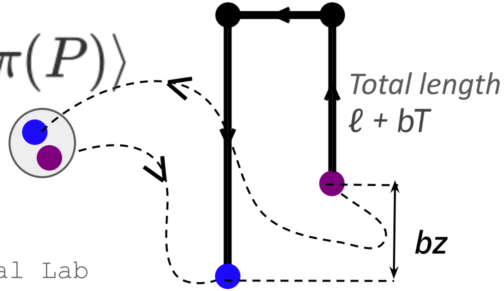


$$\gamma_q(b_T,\mu) = \lim_{a\to 0} \frac{1}{\ln(P_1^z/P_2^z)} \ln \left[\frac{\int \mathrm{d}b^z e^{ib^z x P_1^z} P_1^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_1^z)}{\int \mathrm{d}b^z e^{ib^z x P_2^z} P_2^z N_\Gamma(P_1^z) \sum_{\Gamma'} Z_{\Gamma\Gamma'}(\mu) \lim_{\ell\to\infty} W_{\mathcal{O}}^{\Gamma'}(b^z, b_\mathrm{T}, \ell, P_2^z)} \right] + \delta\gamma_q(\mu, P_1^z, P_2^z) + \mathrm{p.c.}$$

...repeat for each bT (... and for each *a*)

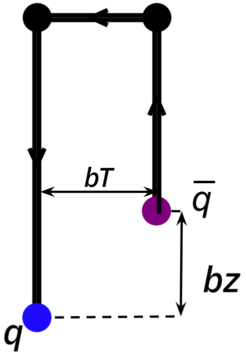


$$\tilde{\phi}_\Gamma = \langle 0 | O_\Gamma | \pi(P) \rangle$$



Takeaways for the gluon CS kernel calculation?

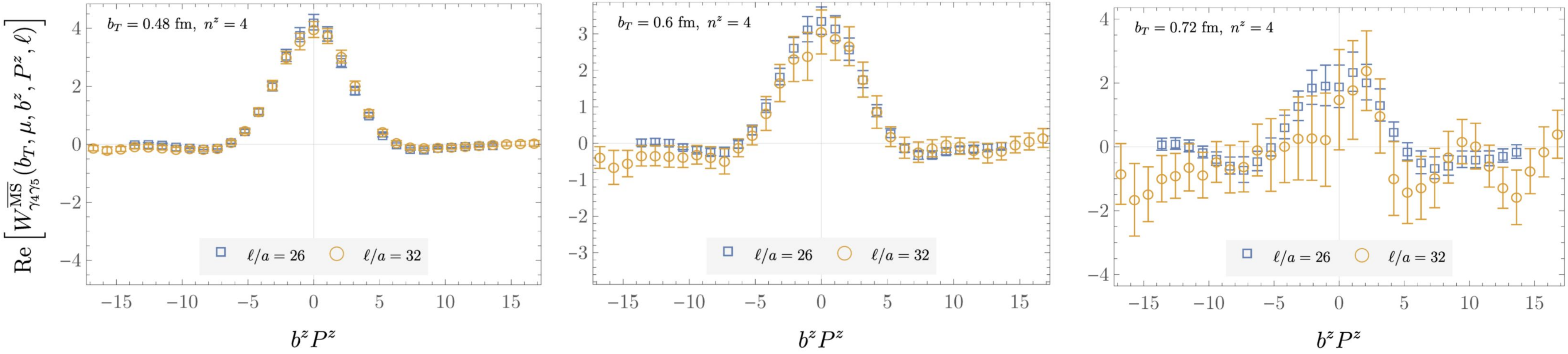
Statistical noise from Wilson lines



For quark CS kernel:

- Statistical noise grows with length of Wilson line.
- Compare *l=26* vs *l=32* with increasing *bT*:

Increasing bT

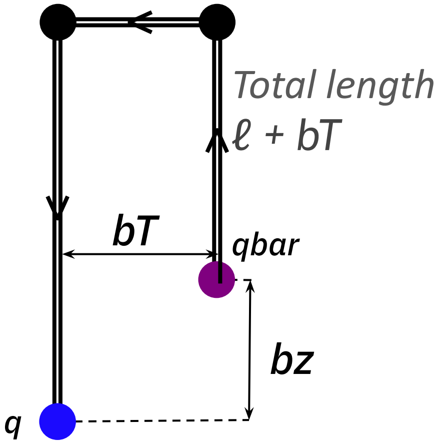


For gluon CS kernel, expect even greater challenge from statistical noise.

Coulomb-Gauge (CG) methods promising, but need to understand systematics.

Lattice QCD matrix elements (MEs) for quark CS kernel

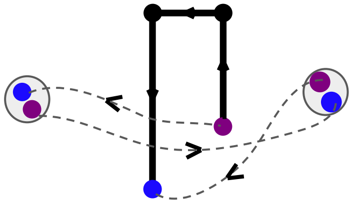
- Defined by operators $\mathcal{O}_\Gamma(b_T, b^z, y, \ell)$ sith staple-shaped Wilson lines.
- Calculated for each *Pz, bT, bz, ℓ* — expensive!



- **2 equivalent options for MEs:**

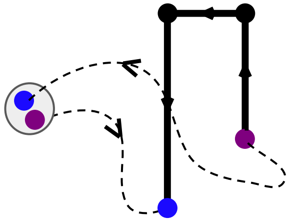
1. TMD Beam function

$$\tilde{B}_\Gamma = \langle \pi(P) | O_\Gamma | \pi(P) \rangle$$



2. TMD Wavefunction (WF)

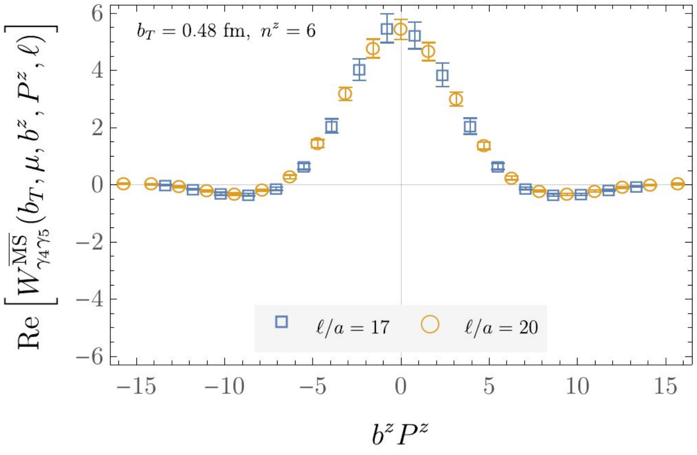
$$\tilde{\phi}_\Gamma = \langle 0 | O_\Gamma | \pi(P) \rangle$$



Choose #2: less fermion propagators ⇒ cheaper

- **Linear divergences** in MEs $\sim \ell + b_T$
- Subtract the divergences in **quasi-TMD WF ratios**

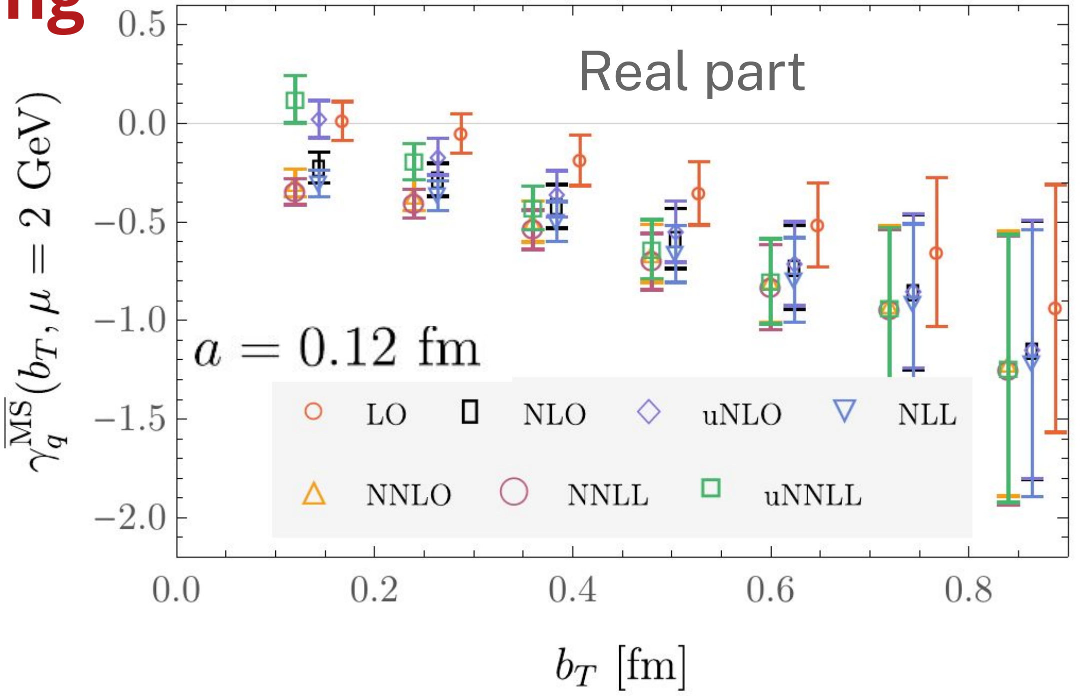
$$W_\Gamma^{(0)}(b_T, b^z, P^z, \ell) = \frac{\tilde{\phi}_\Gamma(b_T, b^z, P^z, \ell)}{\tilde{\phi}_{\gamma^4\gamma^5}(b_T, 0, 0, \ell)}$$



Order of LaMET matching

For quark CS kernel:

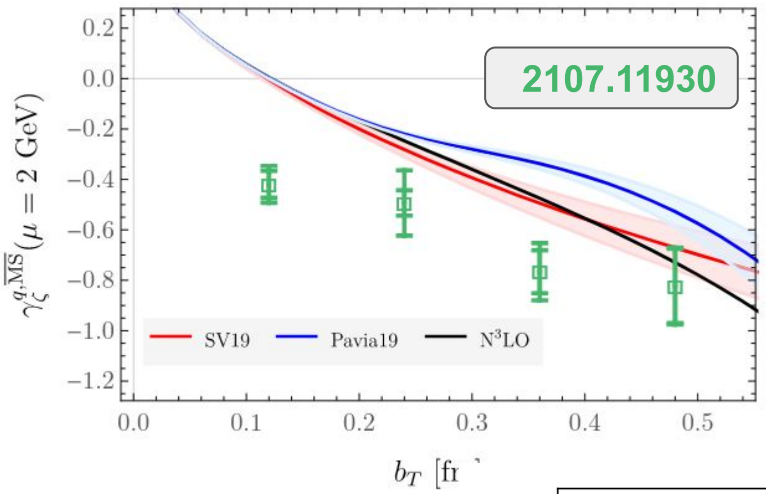
- Can access momenta up to Pz ~ 2 GeV.
- Results shift with **LO** → **NLO** in matching
- Need NNLO + resummations to confirm convergence within uncert.

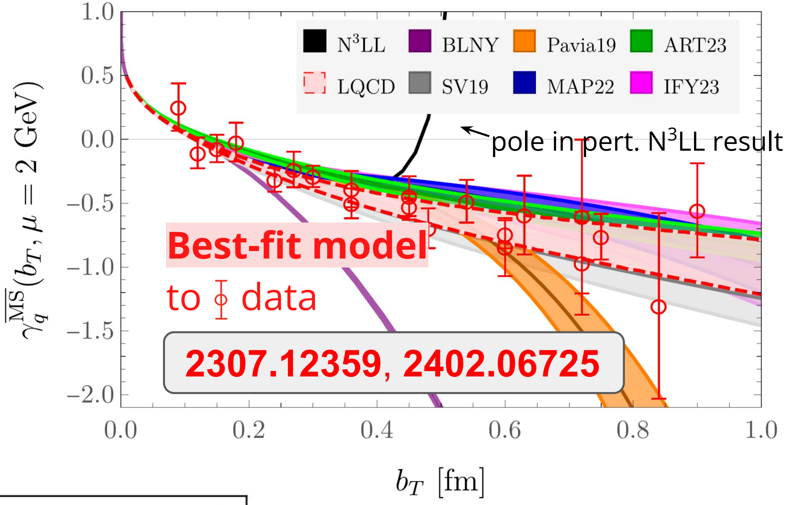


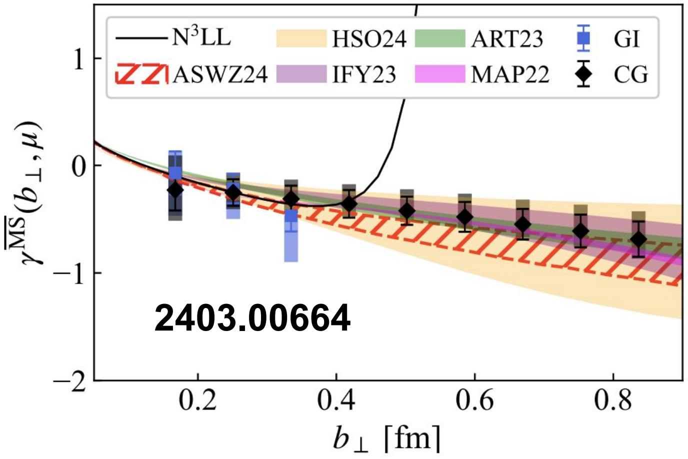
For gluon CS kernel: *up to NLO matching calculated; no matching yet for Coulomb-Gauge (CG) method.*

Observable	Collaboration	Power corrections	Matching	Fourier Transform	Operator Mixing	Continuum extrapolation
Quasi-Beam Functions	**SWZ 20** PRD 102 (2020)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.219$ *quenched*	LO	**Yes**	✔ (RI'-MOM)	✗
	SWZ 21 PRD 104 (2021)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.129$	**NLO**	**Yes**	✔ (RI'-MOM)	✗
Quasi-TMD Wavefunctions (also 2403.00664) Related talks: Jin-Xin Tan, Thu 11:00 am	**LPC 20** PRL 125 (2020)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.067$	LO	N/A	✔ (RI'-MOM)	✗
	PKU/ETMC 21 PRL 128 (2022)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.063$	LO	N/A	✗	✗
	LPC 22 JHEP 08 (2023)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.067$	**NLO**	**Yes**	✗	✗
	ASWZ 23/24 PRD 108 (2023) PRL 132 (2024)	$\frac{(m_\pi^{\text{val}})^2}{(P^z_{\text{max}})^2} = 0.007$	**NNLL**	**Yes**	✔ (RIx-MOM)	✔ **(2024)**
Mellin Moments of Quasi-TMDs	**SVZES 21** JHEP 08 (2021) (also 2302.06502)	$\frac{(m_\pi^{\text{val}})^2}{(P^+_{\text{max}})^2} = 0.035$	**NLO**	N/A	✗	✗

Progress in systematic control and precision







Highly encouraging results for quark CS kernel with CG calculations: extends bT range from ~0.4 fm to >1 fm on just 64 configurations (vs. ~500x3 configurations above on the right)

Calculations of quark vs. gluon CS kernel differ by operator and matrix element

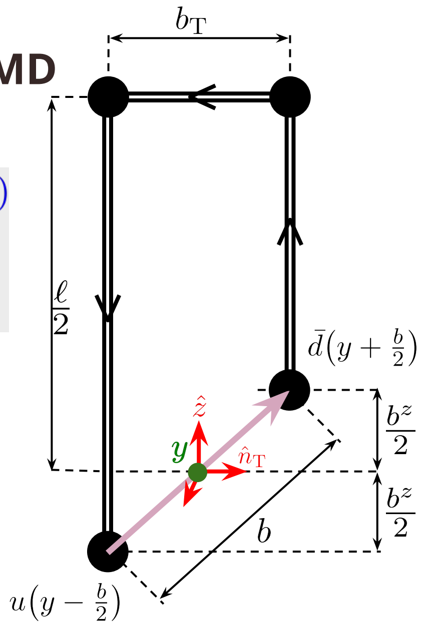
Quark CS kernel — completed

Computed using **quasi-TMD wavefunctions (WFs)**:

$$W_\Gamma(b_T, b^z, \ell, P^z) \; (\Gamma \in \{\gamma^4\gamma^5, \gamma^5\}) = \frac{\langle 0|\mathcal{O}_\Gamma(b_T, b^z, \ell, P^z)\,|\pi(P^z)\rangle}{\langle 0|\mathcal{O}_{\gamma^4\gamma^5}(b_T, 0, \ell, 0)\,|\pi(0)\rangle}$$

Pz=0, bz=0 matrix elt to subtract divergences linear in ℓ

Account for renormalization-induced mixing between Γ structures



Gluon CS kernel — ongoing

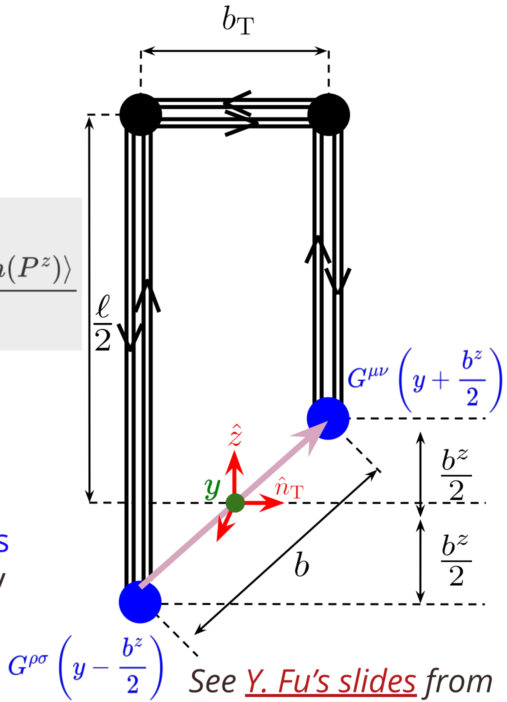
Computed using **quasi-TMD beam functions**:

$$P^\mu P^\nu \tilde{B}^{\rho\sigma}_{g/h}(b^z, b_T, P^z, \ell) = \frac{\langle h(P^z)|\mathcal{O}_g^{\mu\nu\rho\sigma}(b_T, b^z, 0, \ell)\,|h(P^z)\rangle}{\sqrt{Z_A(b_T, \ell)}}$$

Square root of adjoint Wilson loop to subtract divergences linear in ℓ

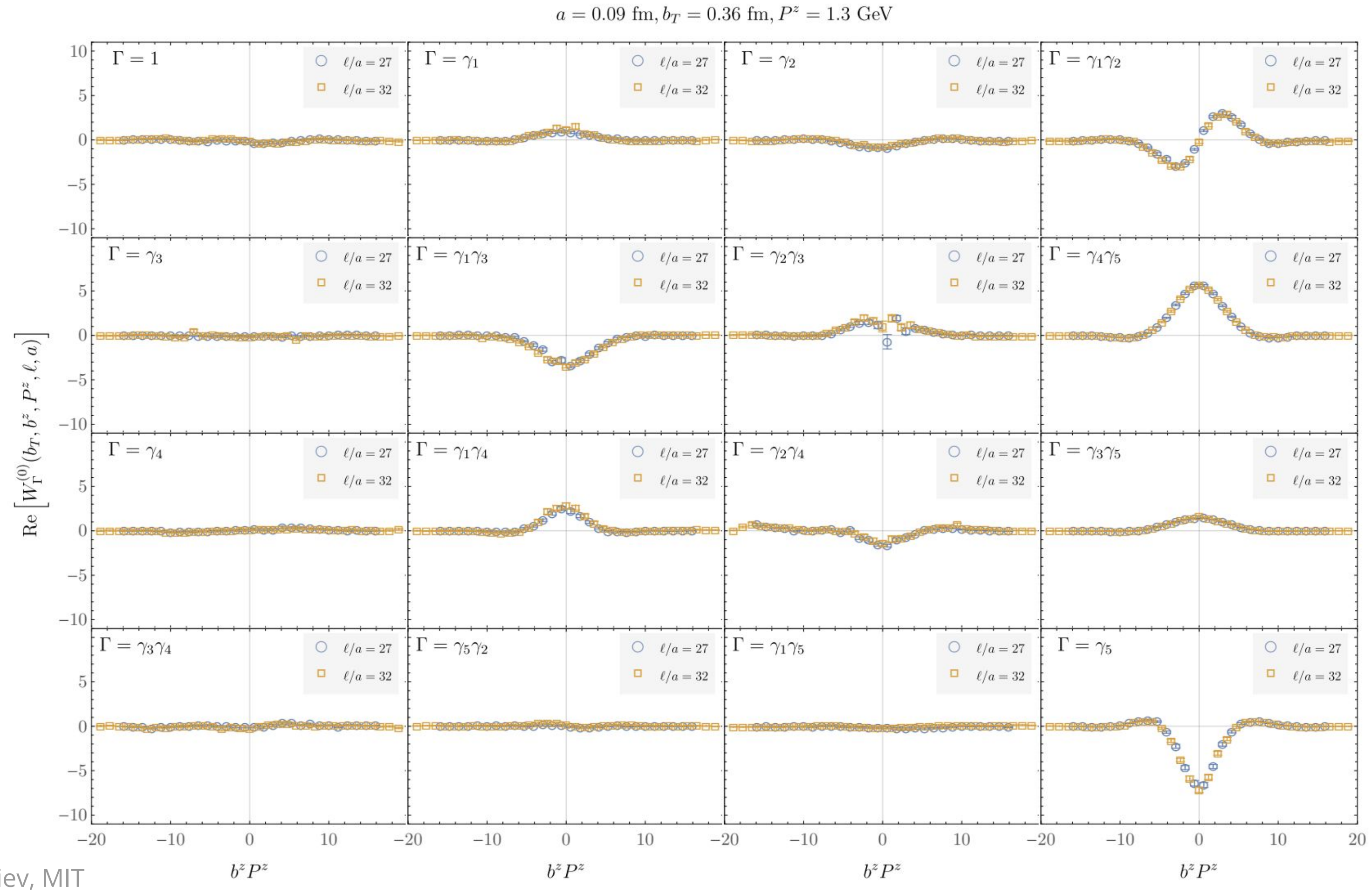
Can choose Lorentz indices that lead to multiplicatively renormalizable operators

J-H. Zhang, X. Ji, Schäfer, W. Wang, S. Zhao, PRL 122 (2019) [1808.10824]



See Y. Fu's slides from Lattice 2024

MEs with all 16 Dirac structures calculated



Mixing effects quantified with RIxMOM

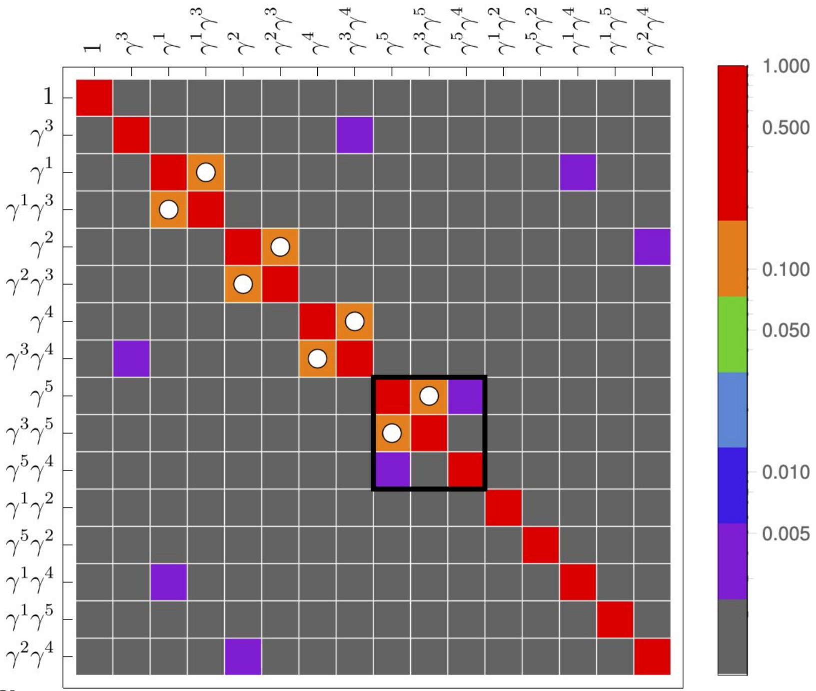
- Calculation of mixing effects in RIxMOM independent of staple geometry.

$$W_{\Gamma}^{\overline{\mathrm{MS}}}(b_T, \mu, b^z, P^z, \ell) = \sum_{\Gamma'} Z_{\Gamma\Gamma'}^{\overline{\mathrm{MS}}}(\mu)\, W_{\Gamma}^{(0)}(b_T, b^z, P^z, \ell)$$

- Full 16x16 mixing matrix computed

$$\mathcal{M}_{\Gamma\Gamma'}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}}, \xi_{\mathrm{R}}, a) \equiv \frac{\mathrm{Abs}[Z_{\Gamma\Gamma'}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}}, \xi_{\mathrm{R}}, a)]}{\frac{1}{16}\sum_{\Gamma}\mathrm{Abs}[Z_{\Gamma\Gamma}^{\mathrm{RI/xMOM}}(p_{\mathrm{R}}, \xi_{\mathrm{R}}, a)]}$$

- Dominant mixings consistent with lattice perturbation theory at 1-loop.*



$$p_{\mathrm{R}}^{\mu} = \frac{2\pi}{L} \times (0, 0, 10, 0),\ \xi = 0.24\ \mathrm{fm}$$

X. Ji, et. al, PRL 120 (2018), [1706.08962]
J. Green et. al, PRL 121 (2018), [1707.07152]
J. Green et. al, PRD 101 (2020), [2002.09408]
*M. Constantinou et al., PRD 99 (2019), [1901.03862]
Y. Ji et. al., PRD 104 (2021), [2104.13345]
C. Alexandrou et al., [2305.11824]

Mixing reduced at finer lattice spacings, as expected