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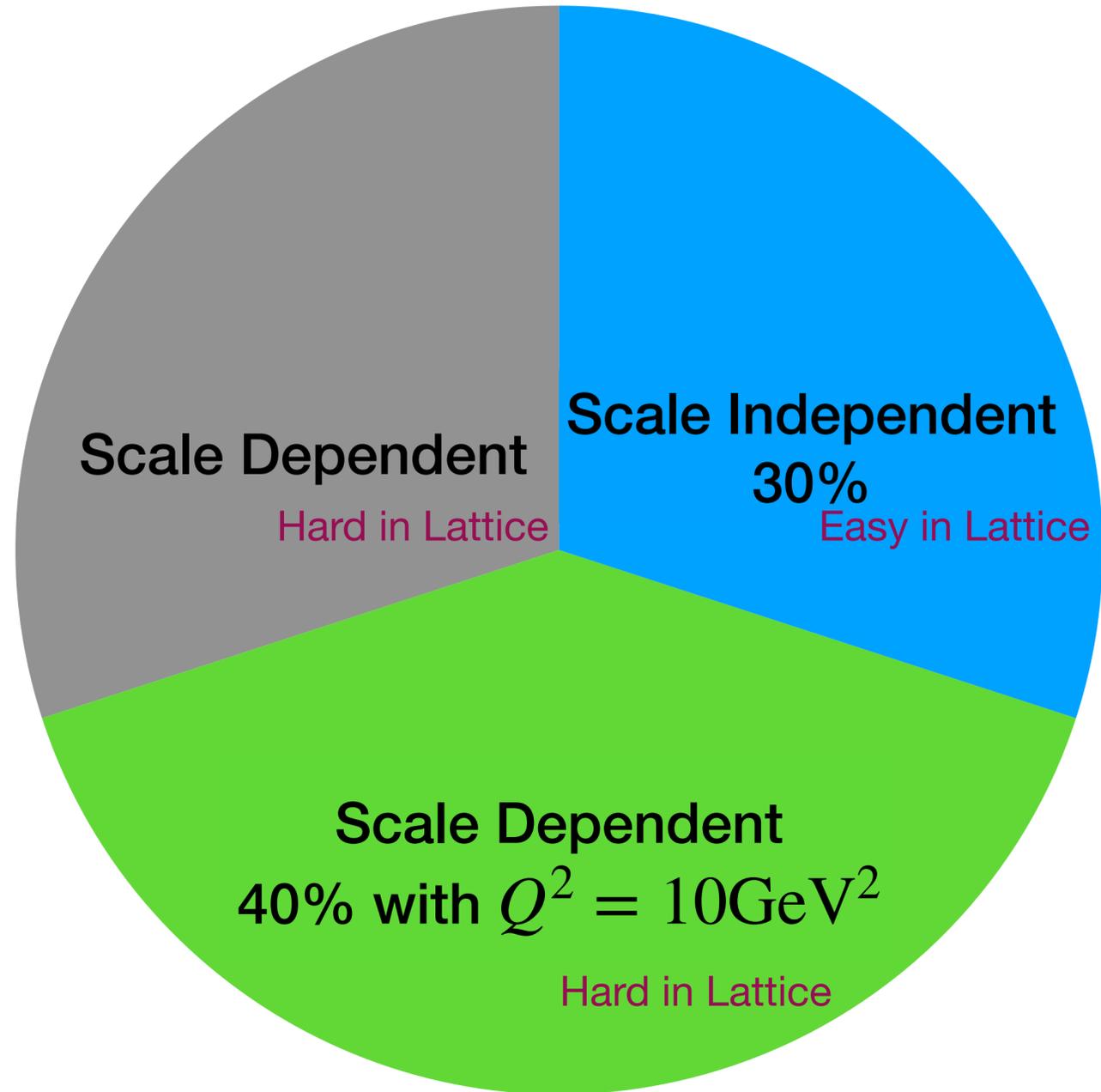
Total Gluon Helicity from Lattice

2025.10.12   9:30-9:50

Speaker: Dian-Jun Zhao

In Collaboration with Hong-Xin Dong, Liuming Liu, Zhuo-Yi Pang, Peng Sun, Yi-Bo Yang, Jianhui Zhang, Shi-Yi Zhong et. al.

Introduction on proton spin

Proton Spin

Pie chart regions:
- Scale Independent 30% — Easy in Lattice
- Scale Dependent — Hard in Lattice
- Scale Dependent 40% with $Q^2 = 10\text{GeV}^2$ — Hard in Lattice

Quark helicity in lattice QCD

$$\Delta q = \langle \text{PS} \,|\, \bar{q}\gamma_5\vec{\gamma} \cdot \vec{S}q \,|\, \text{PS} \rangle$$

$$\int d^3x (\bar{q}\gamma_\mu\gamma_5 q)(x) = 2m_f \int d^3x \vec{x}P(x) - 2i \int d^3x \vec{x}q(x)$$

$$\Delta u = 0.835(15), \qquad \Delta d = -0.435(15),$$
$$\Delta s = -0.095(15), \quad \Delta c \simeq 0.00$$

$$\boxed{\Delta\Sigma = \frac{1}{2}\sum \Delta q = 0.155(25)}$$

C. Adolph et al. Phys. Lett., B753:18–28, 2016.
Daniel de Florian et al. Phys. Rev. D, 80:034030, 2009.
Emanuele R. Nocera, et al. Nucl. Phys., B887:276–308, 2014.
Daniel de Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

Jian Liang, el al. Phys. Rev. D, 98(7):074505, 2018.
Huey-Wen Lin, et al. Phys. Rev. D, 98:094512, 2018.
C. Alexandrou, et al. Phys. Rev. Lett., 119(14):142002, 2017.
Ming Gong, et al. Phys. Rev., D95(11):114509, 2017.

Lattice interpretation of gluon helicity
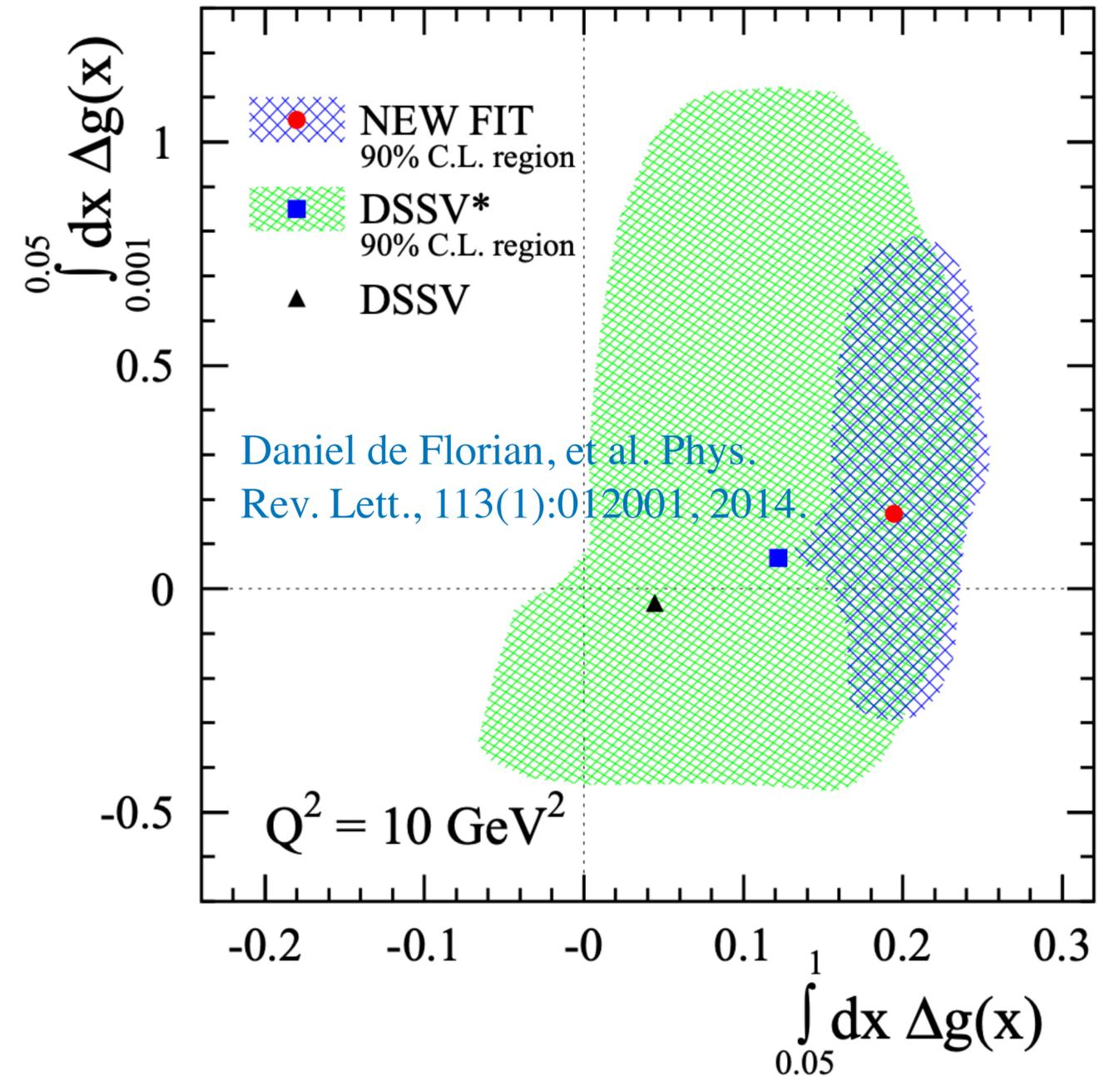
Gluon helicity in lattice QCD

$$\Delta G = \int dx \Delta g(x) = \int dx \frac{i}{2xP^+} \int \frac{d\varepsilon^-}{2\pi} e^{-ix\varepsilon^- P^+}$$

$$\langle \mathrm{PS} \, | \, F_a^{+\mu}(\varepsilon^-) \mathscr{L}_{ab}(\varepsilon^-, 0) \tilde{F}_{b,\mu}^+(0) \, | \, \mathrm{PS} \rangle$$

Integrating LCPDF

ΔG is difficult to calculate in LQCD because light-cone gauge (L.G.) $\rightarrow\!\!\!\!\times\!\!\!\rightarrow$ Euclidean space!

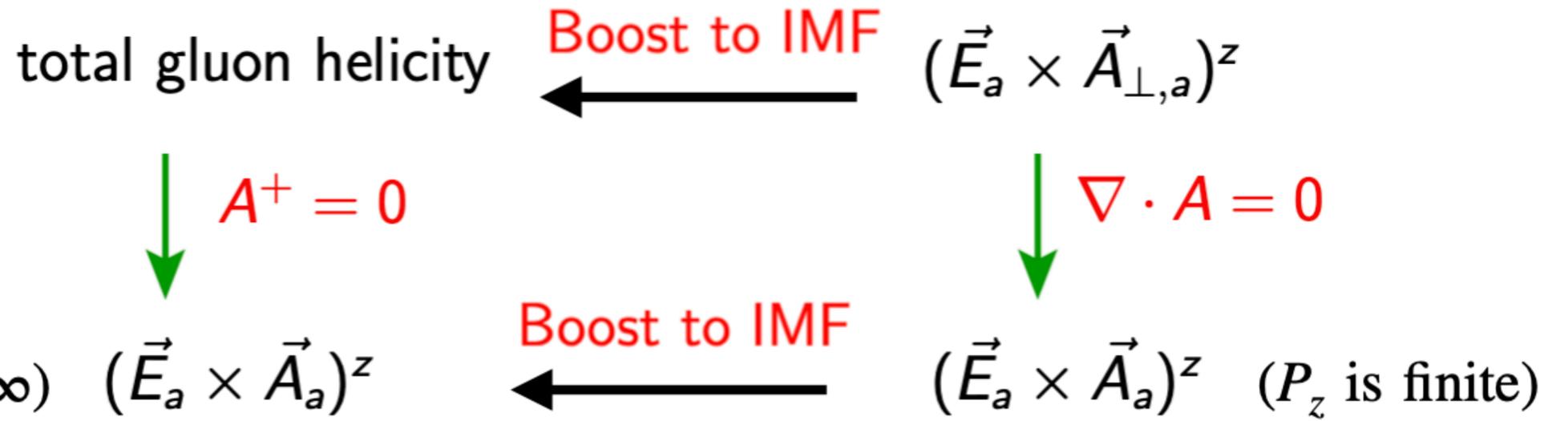Daniel de Florian, et al. Phys. Rev. Lett., 113(1):012001, 2014.

LaMET suggests that Coulomb gauge fixing (C.G.) condition become to L.G. when nucleon to IMF, then $\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\mathrm{C.G.}} = S_G|_{\mathrm{IMF}}$ with matching for their difference in UV behavior.

Xiangdong Ji et al. 10.1103/PhysRevLett.111.112002

Lattice interpretation of gluon helicity

Gluon helicity in lattice QCD

total gluon helicity $\xleftarrow{\text{Boost to IMF}}$ $(\vec{E}_a \times \vec{A}_{\perp,a})^z$

$\downarrow A^+ = 0$ $\qquad\qquad$ $\downarrow \nabla \cdot A = 0$

Yi-Bo Yang et al. 10.1103/
PhysRevLett.118.102001

$(P_z \to \infty)$ $(\vec{E}_a \times \vec{A}_a)^z$ $\xleftarrow{\text{Boost to IMF}}$ $(\vec{E}_a \times \vec{A}_a)^z$ $(P_z$ is finite$)$
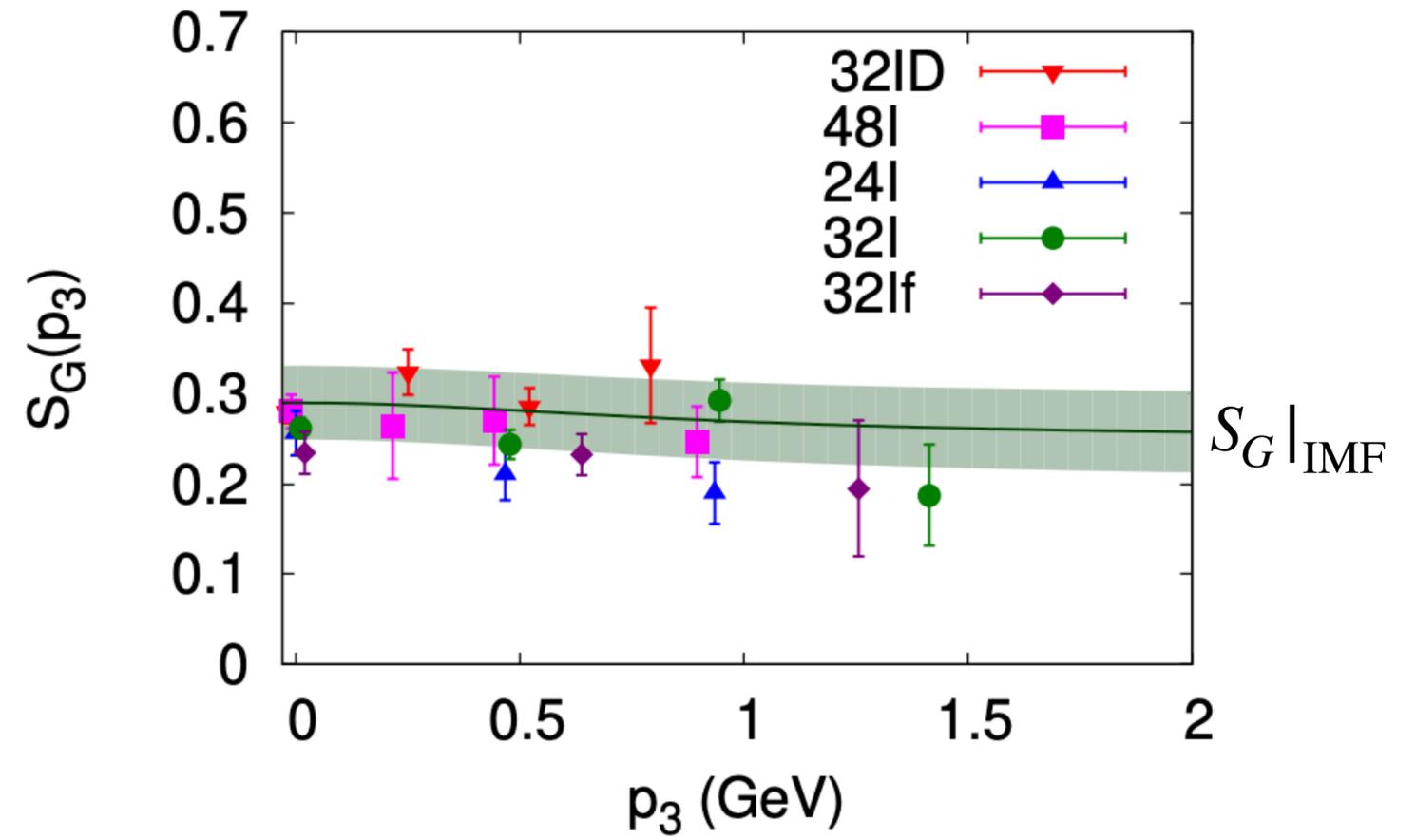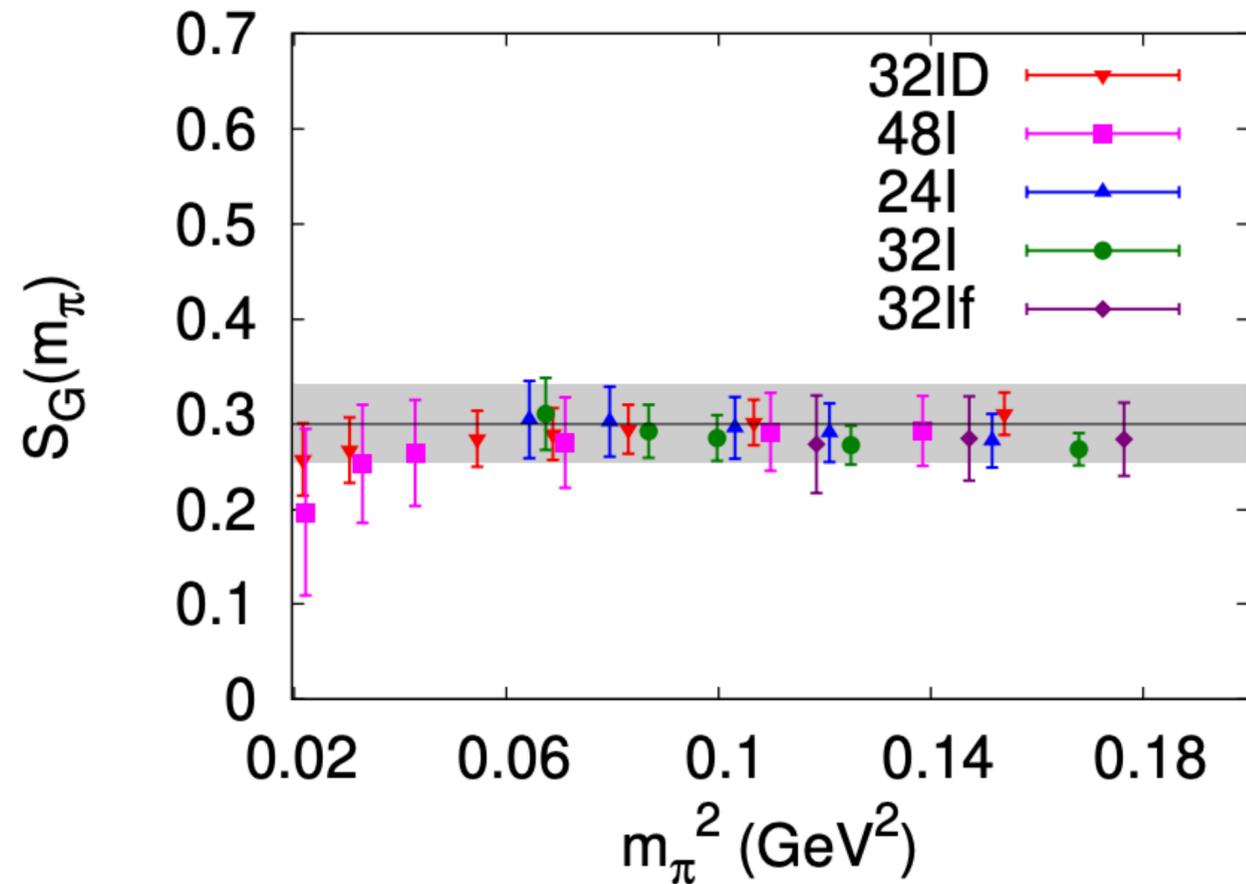
$S_G|_{\text{IMF}}$

$$\Delta G \propto \langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}} = S_G|_{\text{IMF}} = 0.251(47)(16)$$

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Lattice interpretation of gluon helicity

Potential problems with ΔG extraction using $\langle \vec{E} \times \vec{A} \rangle_{\text{C.G.}}$

1. Should be non-perturbative renormalization + perturbative matching;

2. Inconsistency of $\int dx\, g(x) \neq C_{gg}\Delta G + C_{gq}\Delta\Sigma + \text{h.t.} \ldots$

Intrinsic momentum scale in matching shall be parton momentum, not proton momentum!
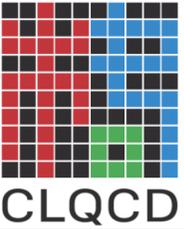
Gluon helicity ΔG from topological current

We propose a scheme that relates ΔG to **local topological current** $K^{\mu}_{\text{C.G.}}$. This scheme solves all of these problems and doesn't require lattice perturbative theory.

$$\text{Topological Current } K^{\mu}(x) = \epsilon^{\mu\nu\rho\sigma}\text{Tr}[A_{\nu}F_{\rho\sigma} - 2ig_s A_{\nu}A_{\rho}A_{\sigma}/3](x)$$

Target Three-PT $\langle \text{PS}_{\text{Proj.i}} | K^{t/i} | \text{PS}_{\text{Proj.i}} \rangle_{\text{C.G.}} \propto S^{t/i}\Delta G + \text{h.t.}$

Lattice Setup

CLQCD	Sea Type	$N_L^3 \times N_T$	Lattice Spacing(fm)	$m_\pi^{(s)}$(MeV)	Valence Type	Ncfg	Nsource
C24P29	Clover (2+1) + TITLS	$24^3 \times 72$	0.1052	292.3(1.0)	Clover	880	72
C48P23		$48^3 \times 96$		224.1(1.2)		380	Volume
E32P29		$32^3 \times 64$	0.0897	287.3(2.5)		890	64
F32P30		$32^3 \times 96$	0.0775	300.4(1.2)		780	96

Ensemble used for renormalization has same UV properties as ensemble used for bare matrix element calculations.

$\langle \vec{E} \times \vec{A} \rangle^{\text{Bare.}}$ with optimization smear methods

Yi-Bo Yang et al. 10.1103/PhysRevLett.118.102001

Appropriate smearing scheme can improve SNR of bare matrix elements.
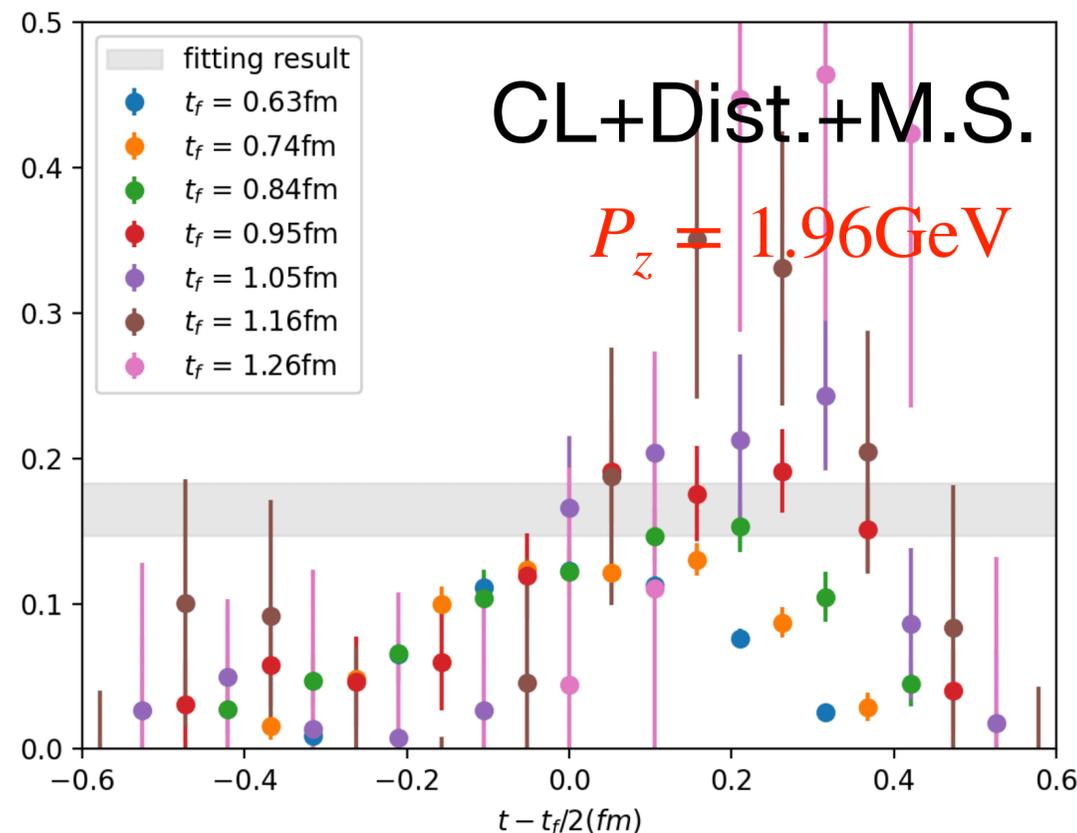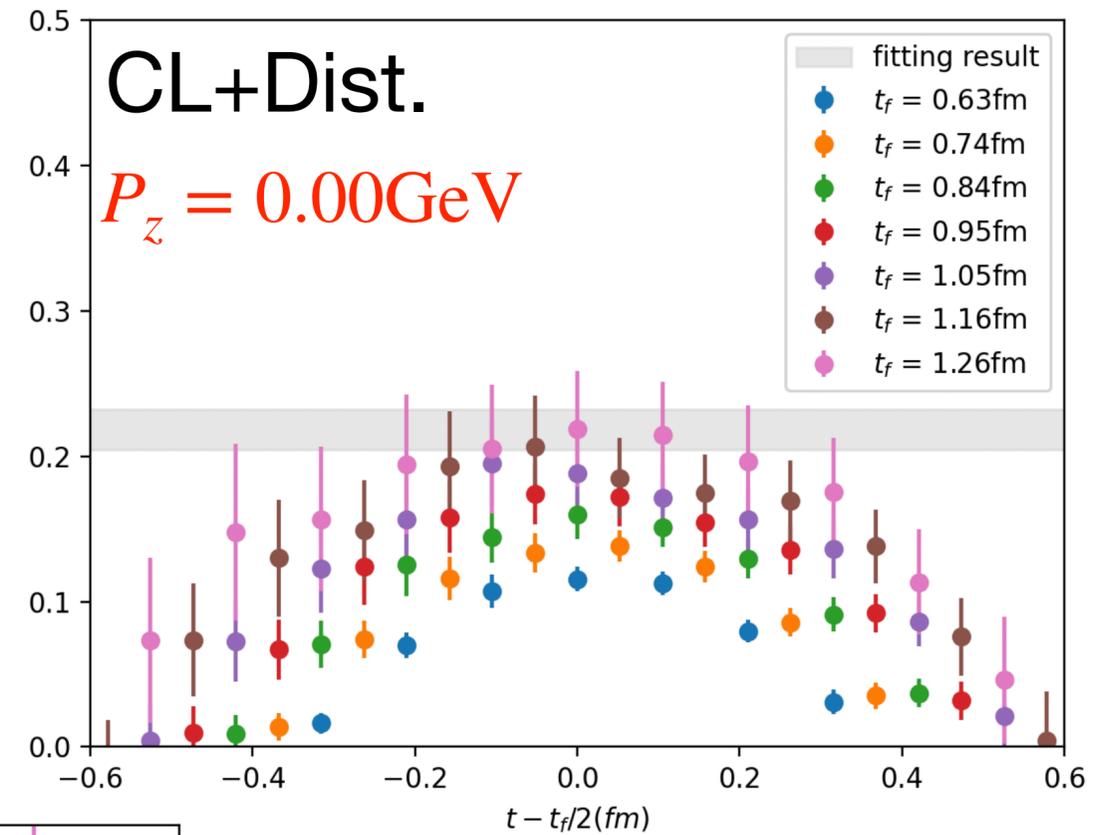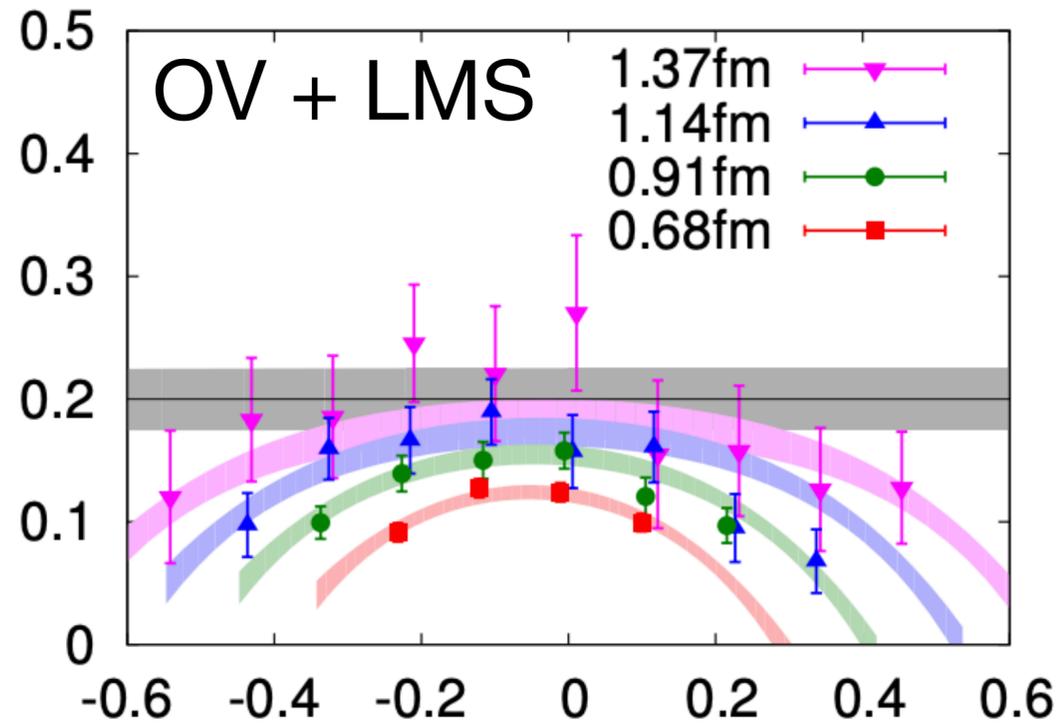
Fitting Form:

$$\frac{C_3(t_f, t)}{C_2(t_f)} = \langle \vec{E} \times \vec{A} \rangle^{\text{Bare.}} + A_1 e^{-\Delta E(t_f - t)} + A_2 e^{-\Delta E t}$$

Dist. for small p_z, Dist. + M.S. for large p_z.

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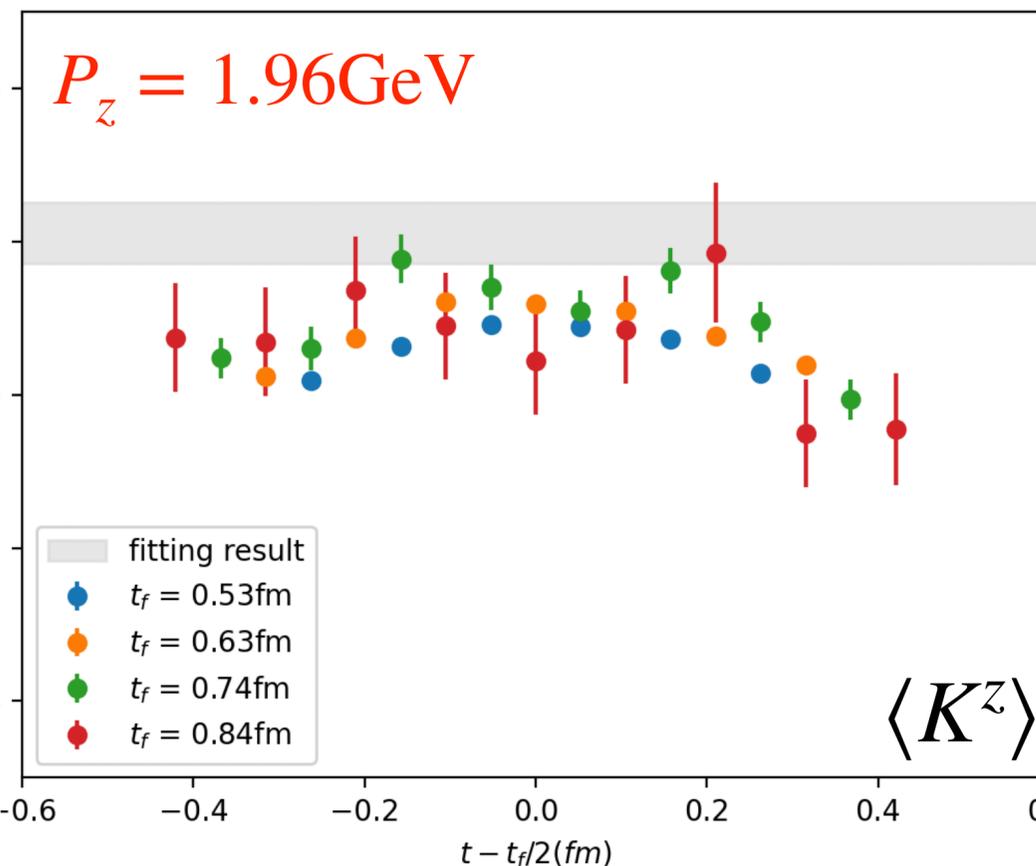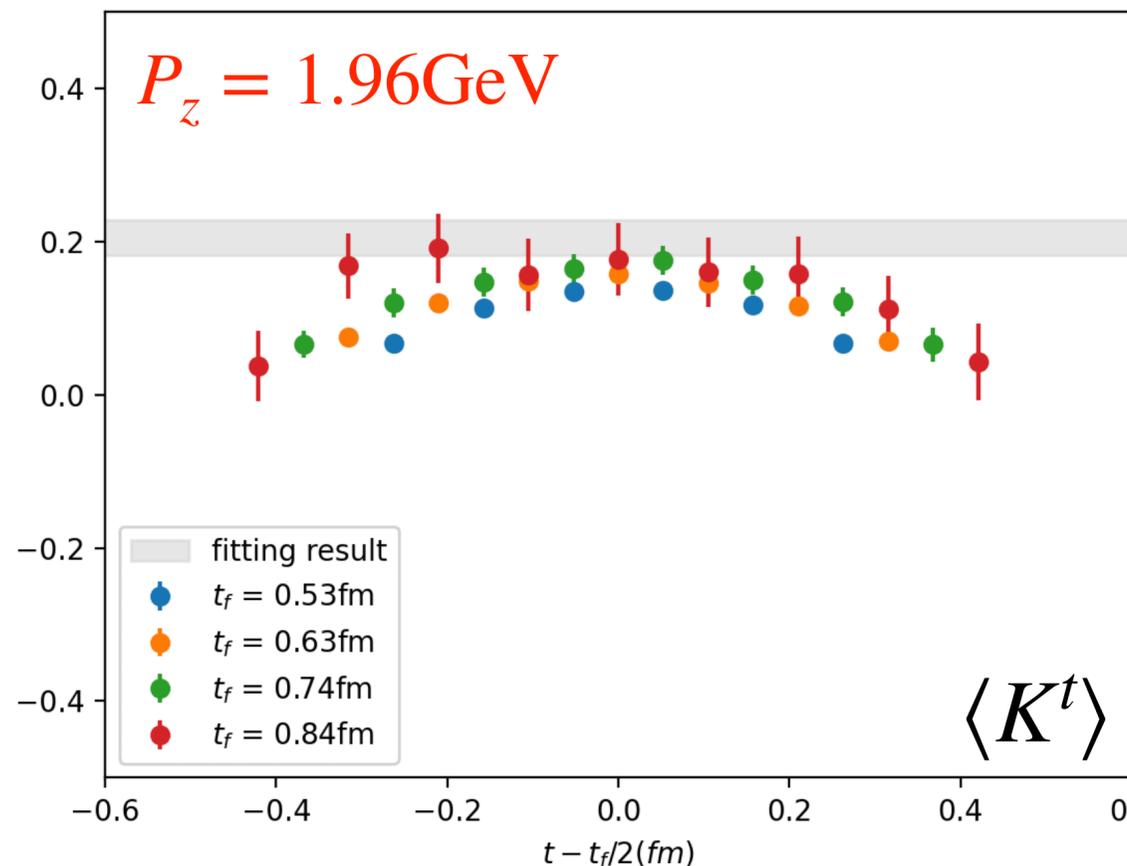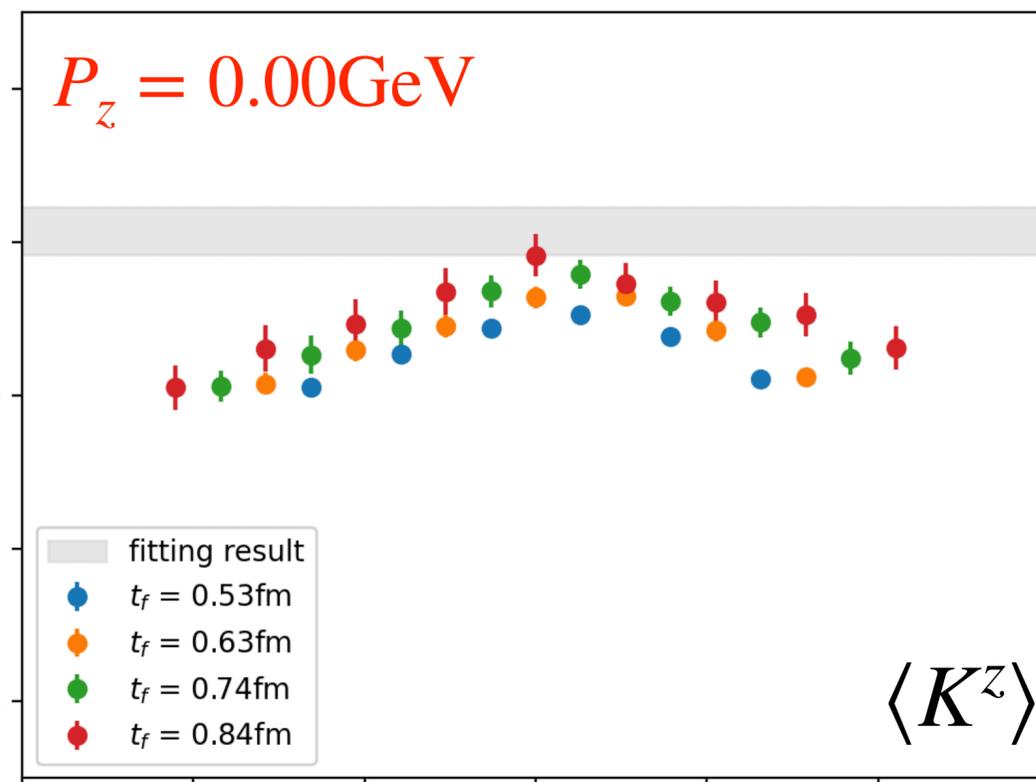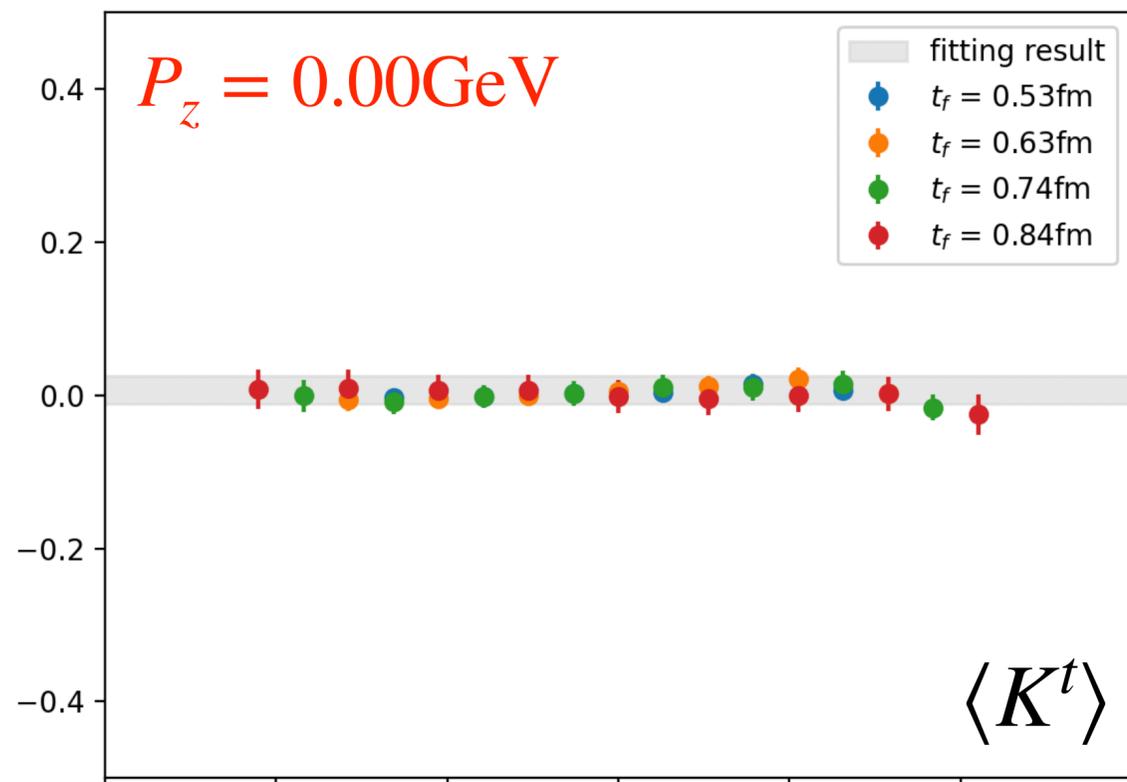# Performance of B.M.E. $\langle K^\mu \rangle^{\text{Bare.}}$ under different momentum

Fitting Form:

$$\frac{C_3(t_f, t)}{C_2(t_f)} = \langle K^\mu \rangle^{\text{Bare.}} + A_1 e^{-\Delta E(t_f - t)}$$

$$+ A_2 e^{-\Delta E t}$$

Almost all of $\langle K^\mu \rangle$'s signal comes from $\epsilon^{\mu\nu\rho\sigma}\text{Tr}[A_\nu F_{\rho\sigma}]$.

Relationship between B.M.E. $\langle K^\mu \rangle^{\text{Bare.}}$

5HYP+Coulomb gauge fixing

$$\langle K^\mu \rangle = S^\mu \Delta G^{\text{Bare.}}/E$$

$$|\Delta G^{\text{Bare.}}_{a=0.105\text{fm}}| = 0.210(10)$$

IMF

Bare Matrix Element

$p(GeV)$

$|\langle K^t \rangle|$
$|\langle K^z \rangle|$
$|\langle K^y \rangle|$

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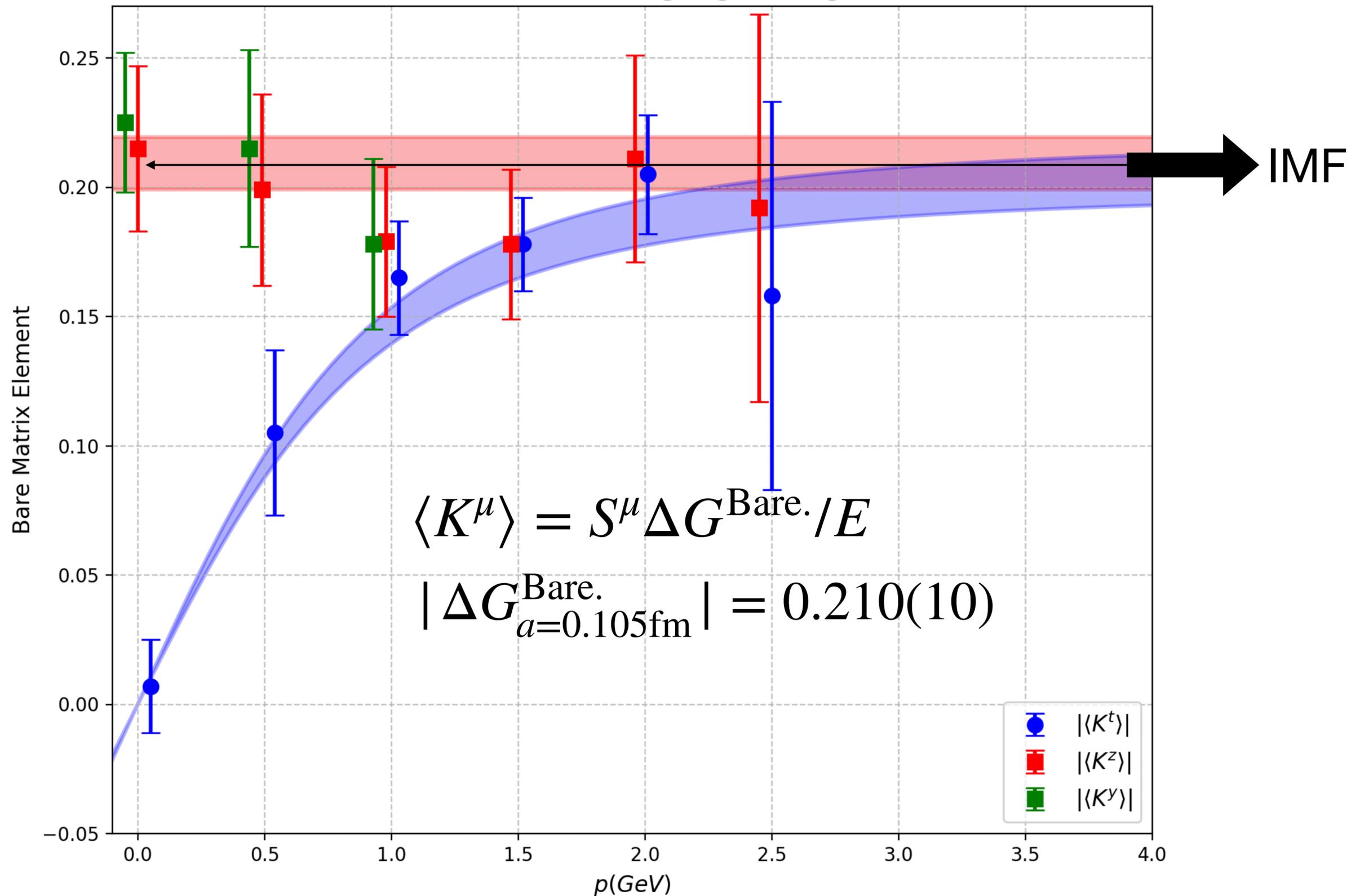
Renormalization of topological current K^μ

RI/MOM is non-perturbative renormalization scheme on LQCD.

RI/MOM on lattice   Zhuo-Yi Pang et al. [10.1007/JHEP07(2024)222](#)

$$\langle PS | \{\widehat{K^\rho, A^\rho}\} | PS \rangle^{\text{tree.}} = Z^{\text{RI}}_{\{1,2\}1} \langle PS | K^\rho | PS \rangle^{\text{lat.}} + Z^{\text{RI}}_{\{1,2\}2} \langle PS | A^\rho | PS \rangle^{\text{lat.}}$$

Coupled

$|\text{PS}\rangle$: Parton state (Gluon or Quark) with momentum and polarization; $A^\rho = \bar{q}\gamma_5\gamma^\rho q$: Axial-vector current operator

$$\mathcal{O}^{\text{X.},g} \equiv \langle g | \mathcal{O} | g \rangle^{\text{X.}} \qquad \mathcal{O}^{\text{X.},q} \equiv \langle q | \mathcal{O} | q \rangle^{\text{X.}} \qquad A^{\text{lat.},g} \to 0, \text{ also } K^{\text{tree.},q} = A^{\text{tree.},g} = 0$$

then $\quad Z^{\text{RI}}_{11} = \dfrac{K^{\text{tree.,g}}}{K^{\text{lat.,g}}}, \qquad\qquad Z^{\text{RI}}_{12} = -\dfrac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}} Z^{\text{RI}}_{11} = \mathcal{O}(\alpha_s) \times Z^{\text{RI}}_{11}$

$\quad Z^{\text{RI}}_{21} = 0, \qquad\qquad\qquad Z^{\text{RI}}_{22} = \dfrac{A^{\text{tree.,q}}}{A^{\text{lat.,q}}} = Z^{\text{RI}}_A \longleftarrow$ Zhi-Cheng Hu et al. [10.1103/PhysRevD.109.054507](#)

$$Z^{\overline{\text{MS}}}_A = Z^{\text{RI}}_A = 0.857 \text{ under C48P23}$$

$$\boxed{\Delta G^{\text{RI}} = Z^{\text{RI}}_{11}\left(\Delta G^B - \frac{K^{\text{lat.,q}}}{A^{\text{lat.,q}}}\Delta\Sigma^B\right), \qquad \Delta\Sigma^{\text{RI}} = Z^{\text{RI}}_A \Delta\Sigma^B}$$

Renormalization of topological current K^μ

RI/MOM to $\overline{\text{MS}}$

$$\Delta G^{\overline{\text{MS}}} = R_{11}\Delta G^{\text{RI}} + R_{12}\Delta\Sigma^{\text{RI}},$$

$$\Delta\Sigma^{\overline{\text{MS}}} = R_{21}\Delta G^{\text{RI}} + R_{22}\Delta\Sigma^{\text{RI}}$$

$$\longrightarrow$$

$$\Delta G^{\overline{\text{MS}}} \approx R_{11}Z_{11}^{\text{RI}}\Delta G^{\text{B}}$$

$$\Delta\Sigma^{\overline{\text{MS}}} \approx Z_A^{\overline{\text{MS}}}\Delta\Sigma^{\text{B}}$$

$R_{22} = 1$: quark helicity is scale independent.

If RI/MOM scale is high, then R_{12} & $R_{21} \approx 0$.

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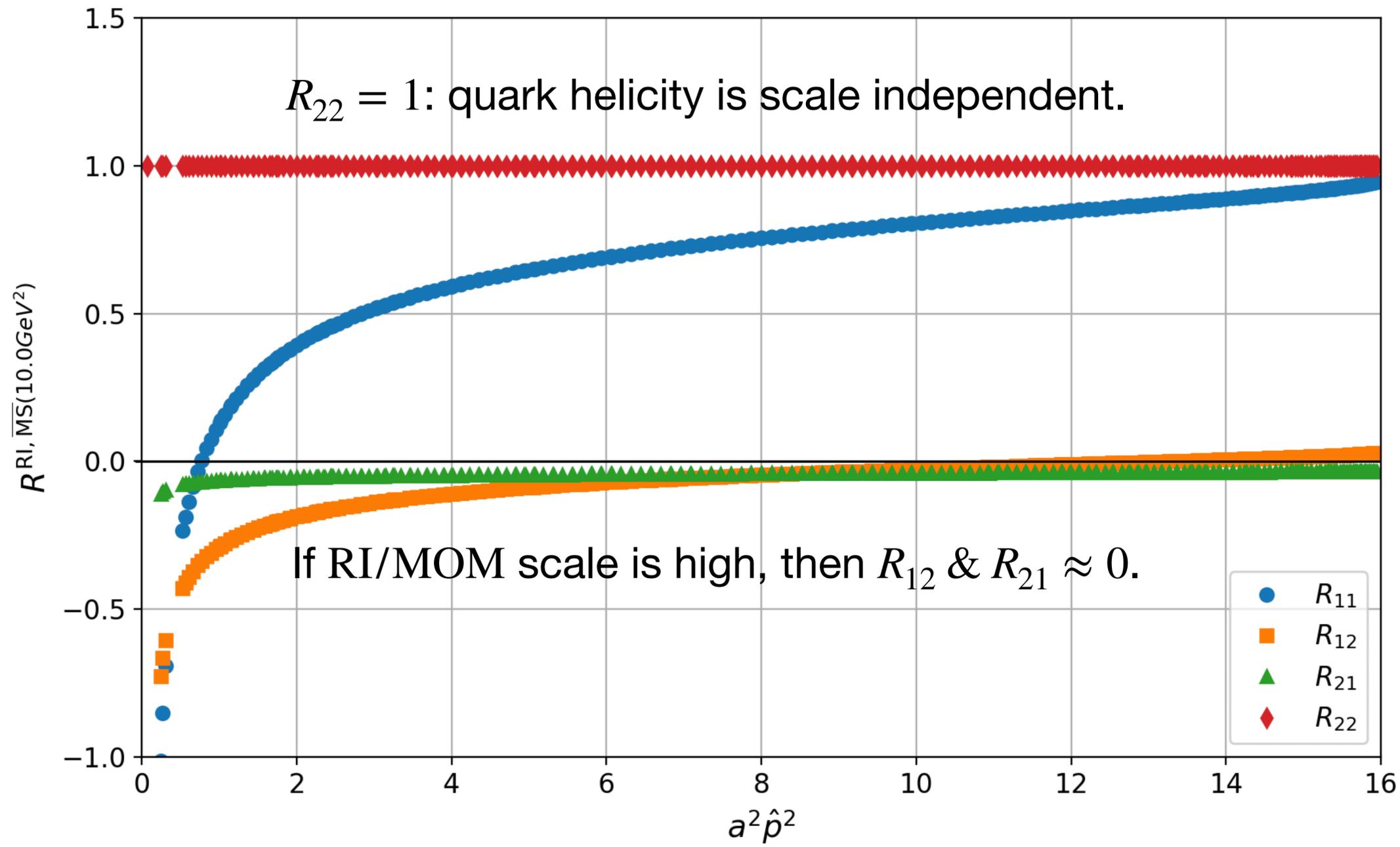
Pure gluon contribution scheme under RI/MOM

By definition of RI/MOM: $Z_{11}^{\text{RI}} = \dfrac{K^{\text{tree.,g}}}{K^{\text{lat.,g}}} = \dfrac{Z_{g,\text{diag/off}}}{\text{Tr}[S_g^{-1}\langle AKA\rangle^{\text{lat.}}S_g^{-1}(K^{\text{tree,g}})^{-1}]}$

where $Z_{g,\text{diag}} = (S_{g,\mu\mu}\hat{p}^2)^{-1}$, $Z_{g,\text{off}} = (S_{g,\mu\nu}\dfrac{\hat{p}^4}{\hat{p}_\mu\hat{p}_\nu})^{-1}$ and $K^{\text{tree,g}} = \langle A_\mu(p)\,|\,K^\rho\,|\,A_\nu(p)\rangle^{\text{tree}} = i\epsilon^{\rho\mu\sigma\nu}\epsilon_\mu^*\hat{p}_\sigma\epsilon_\nu$

Discretization error of RI/MOM is proportional to high-order correction of each momentum component, such as gluon propagator:

$$S_g^{\text{lat.}}(p) = S_g^{\text{cont.}}(p)\Big(1 + c_0\frac{\sum_\mu (a\hat{p}_\mu)^4}{\sum_\mu ((a\hat{p}_\mu)^2)^2} + \mathcal{O}(a^4)\Big)$$

Select **diagonal** momentum mode $p_{xyzt} = N \times (1111)$ and **CDER** to generate Z_{11}^{RI}:

$$Z_{11}^{-1,\text{RI}} = \frac{\hat{p}^2\langle A_\mu K^\rho A_\nu\rangle^{\text{lat.}}}{i\epsilon^{\rho\mu\sigma\nu}\hat{p}_\sigma(\langle A_\mu A_\mu\rangle + \langle A_\nu A_\nu\rangle)/2} = \frac{\hat{p}^4\langle A_\mu K^\rho A_\nu\rangle^{\text{lat.}}}{i\epsilon^{\rho\mu\sigma\nu}\hat{p}_\sigma\hat{p}_\mu\hat{p}_\nu\langle A_\mu A_\nu\rangle}$$

Cluster Decomposition of $Z_{11}^{\overline{\text{MS}}}$

$Z_{11}^{\overline{\text{MS}}} \equiv R_{11} Z_{11}^{\text{RI}}$ contributed by pure gluons is difficult to see signal, so we use **CDER** scheme to enhance signal-to-noise ratio. Yi-Bo Yang et al. 10.1103/PhysRevD.98.074506

$|Z_{11}^{-1,\overline{\text{MS}}(10.0GeV^2)}|$
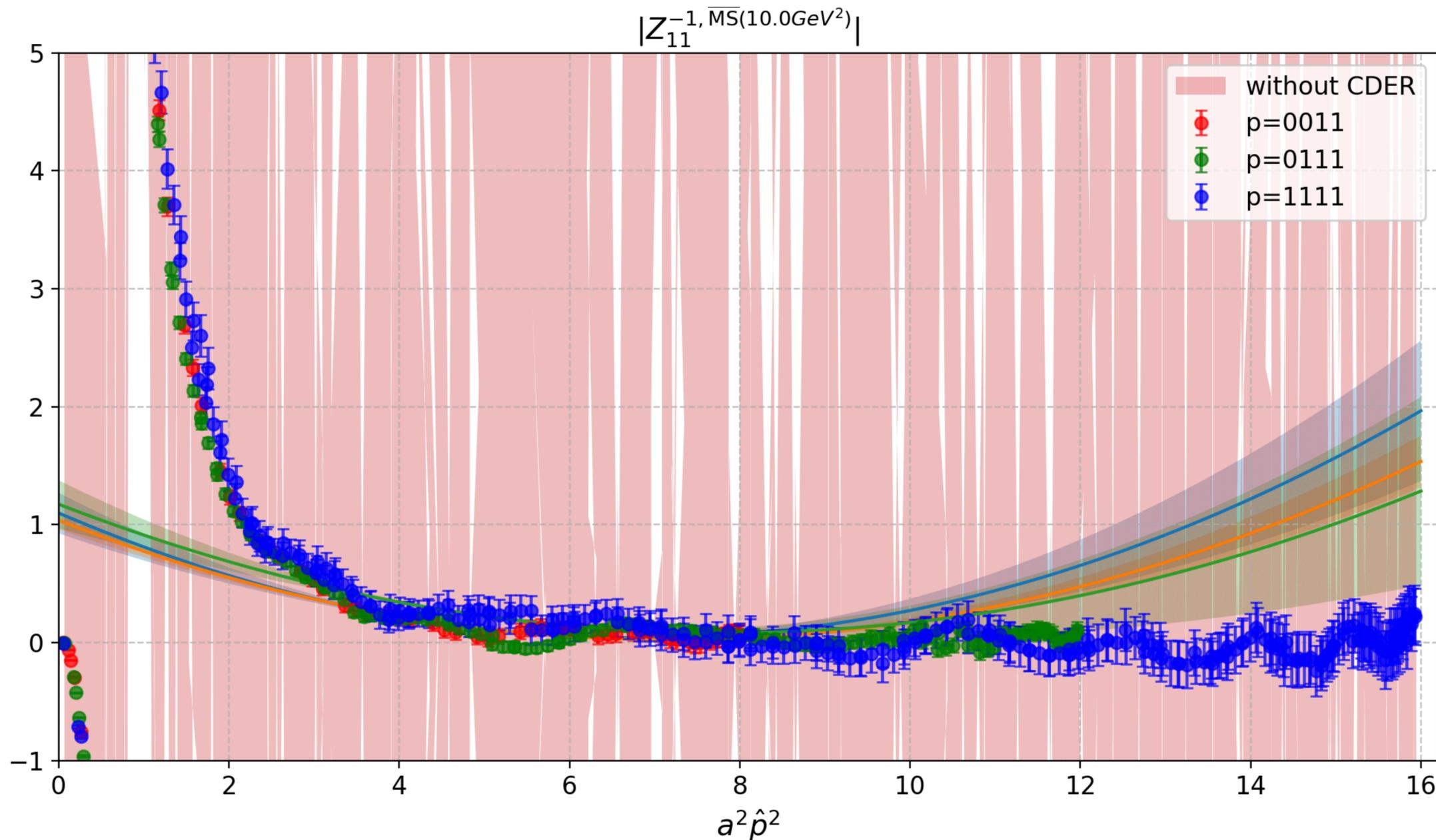
$Z_{11}^{\overline{\text{MS}}}$ in different momentum modes are self-consistent.

$$|Z_{11,a=0.105\text{fm}}^{\overline{\text{MS}},10\text{GeV}^2}| = 0.86(18)$$

$$\Delta G^{\overline{\text{MS}}} \approx Z_{11}^{\overline{\text{MS}}} \Delta G^B$$

$$\Delta G_{a=0.105\text{fm}}^{\overline{\text{MS}},10\text{GeV}^2} \approx 0.179(40)$$

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Summary and Outlook

Summary

1. **Distillation + Momentum smear** (for B.M.E.) and **CDER** (for Renorm.) scheme.

2. After non-perturbative matching, $\Delta G_{a=0.105\text{fm}}^{\overline{\text{MS}},10\text{GeV}^2} \approx 0.18$, which accounts for 36% of proton spin, which is latest result from scheme designed entirely for LQCD.

Outlook

1. Need to calculate dependence on off-diagonal renormalization under RI/MOM.

2. Continuous extrapolation and systematic error analysis is needed.

3. Lattice methodology for O.A.M calculations is required to give accurate results for proton spin components.

THANKS FOR LISTENING!

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Backup

Derivation on RI/MOM helicity renormalization

$$S_q(p) = \sum_x e^{-ip\cdot x}\langle\psi(x)\bar\psi(0)\rangle$$

$$\langle PS|\{K^\rho, J^\rho\}|PS\rangle^{\text{tree.}} = Z^{\text{RI}}_{\{1,2\}1}\langle PS|K^\rho|PS\rangle^{\text{lat.}} + Z^{\text{RI}}_{\{1,2\}2}\langle PS|J^\rho|PS\rangle^{\text{lat.}}$$

$$\langle q\mathcal{O}q\rangle = \sum_{x,y} e^{-i(p_1\cdot x - p_2\cdot y)}\langle\psi(x)\mathcal{O}(0)\bar\psi(y)\rangle$$

$$J^{\text{lat.},g} \to 0, \text{ also } K^{\text{tree.},q} = 0 \qquad \text{then } Z^{\text{RI}}_{11} = \frac{K^{\text{tree.},g}}{K^{\text{lat.},g}}, \qquad Z^{\text{RI}}_{12} = \frac{K^{\text{tree.},q}}{J^{\text{lat.},q}} - \frac{K^{\text{lat.},q}K^{\text{tree.},g}}{J^{\text{lat.},q}K^{\text{lat.},g}} = -\frac{K^{\text{lat.},q}}{J^{\text{lat.},q}}Z^{\text{RI}}_{11} \propto \alpha_s Z^{\text{RI}}_{11}$$

$$J^{\text{lat.},g} \to 0, \text{ also } J^{\text{tree.},g} = 0 \qquad \text{then } Z^{\text{RI}}_{21} = 0, \qquad Z^{\text{RI}}_{22} = \frac{J^{\text{tree.},q}}{J^{\text{lat.},q}}$$

$$K^{-1,\text{lat.},g} = \frac{Z_g}{\text{Tr}[S_g^{-1}\langle gKg\rangle^{\text{lat.}}S_g^{-1}]} \text{ where } Z_g = (S_g p^2)^{-1} \quad \text{then } K^{\text{lat.},g} = \frac{p^2\langle gKg\rangle^{\text{lat.}}}{\langle S_g\rangle} = \frac{p^2\text{Im}\{\langle A_\mu K^\rho A_\nu\rangle^{\text{lat.}}\}}{\langle S_g\rangle}$$

$$\mathcal{O}^{-1,\text{lat.},q} = \frac{Z_q}{\frac{1}{12}\text{Tr}[S_q^{-1}\langle q\mathcal{O}q\rangle^{\text{lat.}}S_q^{-1}]} \text{ where } Z_q = S_q^{-1}\not p \quad \text{then } \frac{K^{\text{lat.},q}}{J^{\text{lat.},q}} = \frac{\text{Tr}[S_q^{-1}\langle qKq\rangle^{\text{lat.}}S_q^{-1}]}{\text{Tr}[S_q^{-1}\langle qJq\rangle^{\text{lat.}}S_q^{-1}]} \qquad \begin{array}{l} K^{\text{tree.},g} = ip^\sigma \\[6pt] J^{\text{tree.},q} = \gamma^\mu\gamma_5\delta \end{array}$$

$$\boxed{\begin{array}{l} \langle A_y K_x A_t\rangle|_{p=1010}, \langle A_z K_x A_t\rangle|_{p=1100}, \langle A_x K_y A_t\rangle|_{p=0110}, \langle A_z K_y A_t\rangle|_{p=1100}, \langle A_x K_z A_t\rangle|_{p=0110},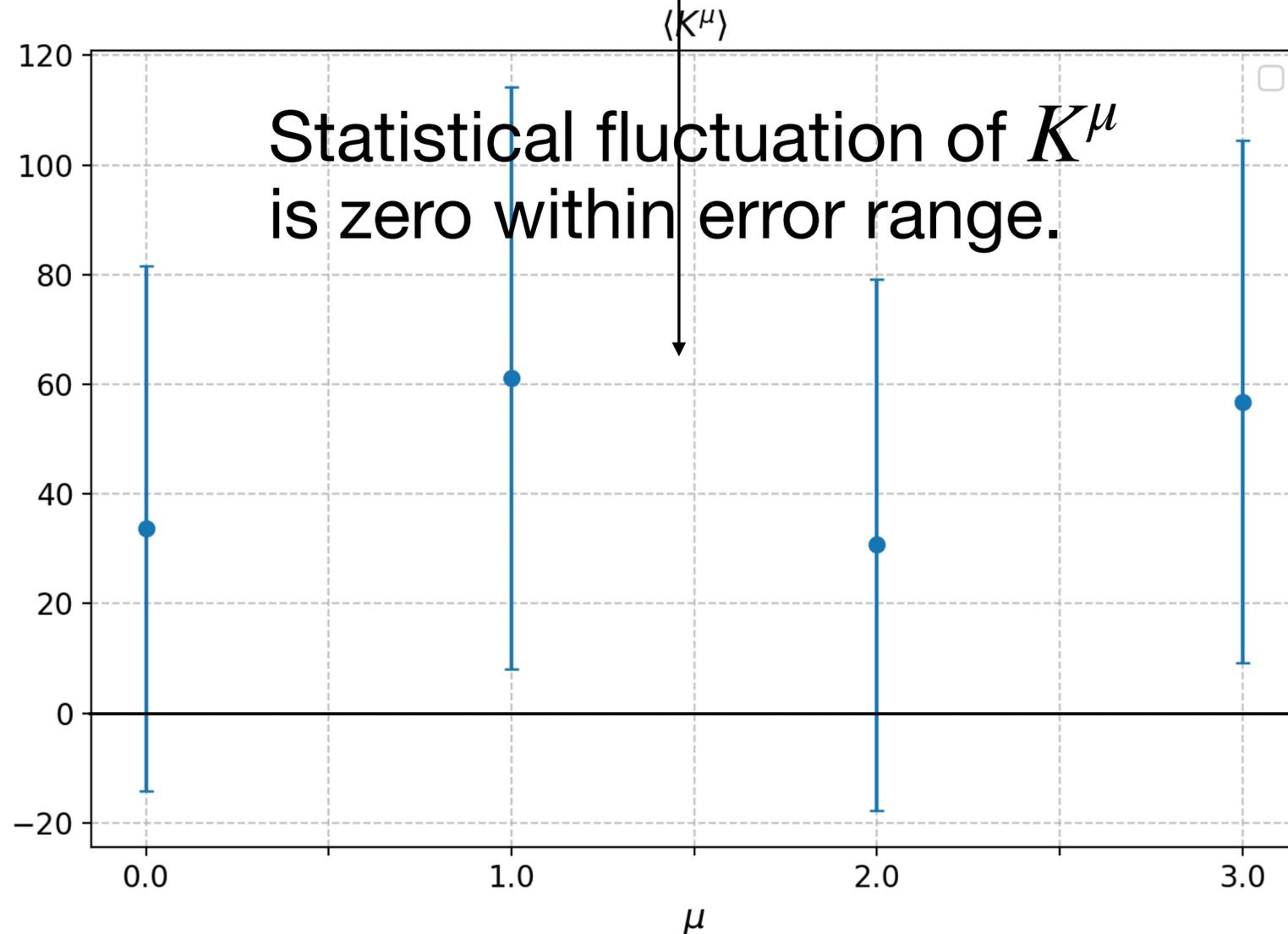 \langle A_y K_z A_t\rangle|_{p=1010} \\[8pt] \langle A_x K_t A_y\rangle|_{p=0011}, \langle A_x K_t A_z\rangle|_{p=0101}, \langle A_y K_t A_z\rangle|_{p=1001}, \langle A_y K_x A_z\rangle|_{p=1001}, \langle A_x K_y A_z\rangle|_{p=0101}, \langle A_x K_z A_y\rangle|_{p=0011} \end{array}}$$

All of these 12 Three-PT calculate Z^{RI}_{11}, reduce error to $1/\sqrt{12}$ of each configuration.

Notes on Z_{11} Statistical Fluctuations

$$Z_{11}^{-1,\mathrm{RI}}(\mu_R^2) = \frac{p^2 \langle \mathrm{Im}\{K^\rho \mathrm{Tr}[A_\mu(p)A_\nu(-p)]\}\rangle}{\epsilon^{\rho\mu\sigma\nu}p_\sigma\langle S_g\rangle} \Big|_{p^2=\mu_R^2,\mu\neq\nu\neq\rho\neq\sigma,p_\mu=p_\nu=0,p_\rho\neq0}$$

$$= \frac{p^2 \langle \mathrm{Im}\{(K^\rho - \langle K^\rho\rangle)(\mathrm{Tr}[A_\mu(p)A_\nu(-p)] - \langle\mathrm{Tr}[A_\mu(p)A_\nu(-p)]\rangle)\}\rangle}{\epsilon^{\rho\mu\sigma\nu}p_\sigma\langle S_g\rangle} \Big|_{p^2=\mu_R^2,\mu\neq\nu\neq\rho\neq\sigma,p_\mu=p_\nu=0,p_\rho\neq0}$$

$\langle K^\mu\rangle$

Statistical fluctuation of K^μ
is zero within error range.

$$S_{\mu\nu}^b(p^2) = \frac{2}{(N_c^2 - 1)V}\mathrm{Tr}[A_\mu(p)A_\nu(-p)]$$

Transverse off-diagonal mode
$\mathrm{Tr}[A_\mu(p)A_\nu(-p)]$, so the vacuum
expectation value is originally 0.

B2

Cluster Decomposition Error Reduction

Consider 3PT $\{\text{Tr}[AA]\mathcal{O}\}(p)$

Cut operator $\mathcal{O}_{\text{cut}}^{R_{sO}}(x) = \displaystyle\int_{|r|<R_{sO}} d^4 r \mathcal{O}(x+r)$

$$\mathcal{O}(p) = \mathscr{F}\big(\mathcal{O}(x)\big), \quad f(p, R_{sO}) = \mathscr{F}\big(f(x, R_{sO})\big) = \mathscr{F}\big(\theta(R_{sO} - |x|)\big)$$

$$\mathscr{F}\big(\mathcal{O} * f\big) = \mathcal{O}(p) \cdot f(p, R_{sO}), \quad \mathcal{O}_{\text{cut}}(x) = \mathscr{F}^{-1}\mathscr{F}\big(\mathcal{O} * f\big) = \int_{||r||<R_{sO}} \mathcal{O}(x)$$

Cut gauge potential $A(x)$ for 3PT

$$B(x) = A(x)\mathcal{O}_{\text{cut}}(x) \qquad B(-p) = \mathscr{F}^{-1}\big(B(x)\big) \times V, \quad A(p) = \mathscr{F}\big(A(x)\big)$$

$$g(x, R_{sA}) = \theta(R_{sA} - |x|) \qquad \{\text{Tr}[AA]\mathcal{O}\}_{\text{cut}}(p) = \mathscr{F}\big\{\mathscr{F}^{-1}\big(A(p) \cdot B(-p)\big) \cdot g(x, R_{sA})\big\}$$

$1-$ loop **approximation of** ΔG**'s off-diagonal elements**

$$\Delta G^{\overline{\text{MS}}} = R_{11}\Delta G^{\text{RI}} + R_{12}\Delta\Sigma^{\text{RI}},$$

$$\Delta\Sigma^{\overline{\text{MS}}} = R_{21}\Delta G^{\text{RI}} + R_{22}\Delta\Sigma^{\text{RI}}$$

$$R_{12} = \frac{R_{12}}{R_{11}} = \frac{K^{1-\text{loop.},q}}{A^{1-\text{loop.},q}} \approx \frac{K^{\text{lat.,q}}}{A^{\text{lat.,q}}}$$

Define $Z_{11}^{\overline{\text{MS}}} \equiv R_{11}Z_{11}^{\text{RI}}$

$$\Delta G^{\overline{\text{MS}}} = R_{11}Z_{11}^{\text{RI}}\left(\Delta G^{B} - \frac{K^{\text{lat.,q}}}{J^{\text{lat.,q}}}\Delta\Sigma^{B}\right) + R_{12}\Delta\Sigma^{\text{RI}}$$

$$= \left\{ Z_{11}^{\overline{\text{MS}}}\left(\Delta G^{B} - R_{12}\Delta\Sigma^{B}\right) + R_{12}Z_{A}^{\text{RI}}\Delta\Sigma^{B} \right\}\big|_{1-\text{loop}}$$

$$= Z_{11}^{\overline{\text{MS}}}\Delta G^{B} + (Z_{A}^{\text{RI}} - Z_{11}^{\overline{\text{MS}}})R_{12}\Delta\Sigma^{B}$$

$$\simeq Z_{11}^{\overline{\text{MS}}}\Delta G^{B}$$

$$\boxed{\begin{array}{l} \Delta G^{\overline{\text{MS}}} = R_{11}\Delta G^{\text{RI}} + R_{12}Z_{A}^{\overline{\text{MS}}}\Delta\Sigma^{B} \overset{1-\text{loop}}{=} Z_{11}^{\overline{\text{MS}}}\Delta G^{B} + (Z_{A}^{\overline{\text{MS}}} - Z_{11}^{\overline{\text{MS}}})R_{12}\Delta\Sigma^{B} \approx Z_{11}^{\overline{\text{MS}}}\Delta G^{B} \\[2mm] \Delta\Sigma^{\overline{\text{MS}}} = R_{21}\Delta G^{\text{RI}} + R_{22}Z_{A}^{\overline{\text{MS}}}\Delta\Sigma^{B} \overset{1-\text{loop}}{=} R_{21}Z_{11}^{\text{RI}}\Delta G^{B} + (Z_{A}^{\overline{\text{MS}}} - R_{12}R_{21}Z_{11}^{\text{RI}})\Delta\Sigma^{B} \approx Z_{A}^{\overline{\text{MS}}}\Delta\Sigma^{B} \end{array}}$$