Nucleon Form Factors with Domain Wall Fermions at Physical Pion Mass

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Large Momentum Effective Theory 2025
Stony Brook University

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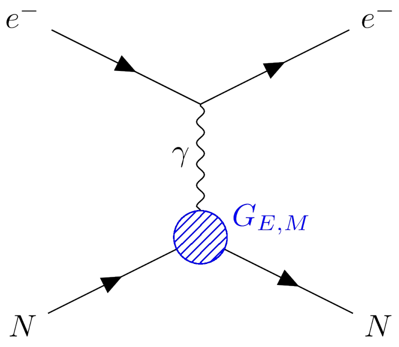
MICHIGAN STATE UNIVERSITY

EXCLAIM

NERSC

Motivation: Vector Form Factors

- Form factors encode response of the nucleon to external probes
- In terms of GPDs:

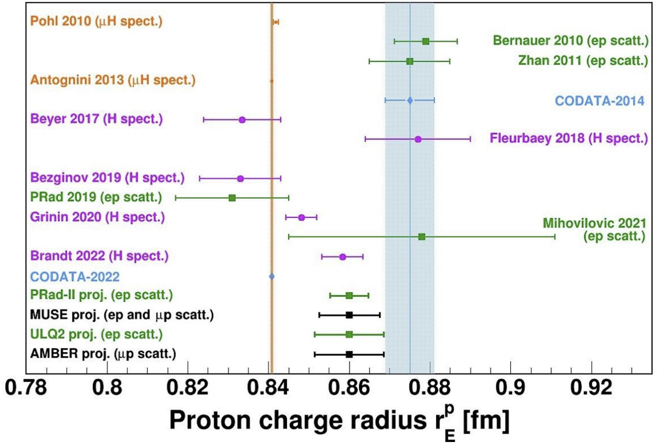


Dirac/Pauli Form Factors $\int_{-1}^{1} dx\, H^q(x,\xi,t) = F_1^q(t)$ $\quad \int_{-1}^{1} dx\, E^q(x,\xi,t) = F_2^q(t)$

Sachs Form Factors $G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M_N^2} F_2(Q^2)$ $\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$

- Give access to important structural information, for example:
  - Electromagnetic **radii** and **electric charge**

- **Proton Radius Puzzle:**
  - Early hydrogen spectroscopy and e-p scattering experiments indicate larger proton charge radius than for muonic-hydrogen experiments



Credit: A. Gasparian

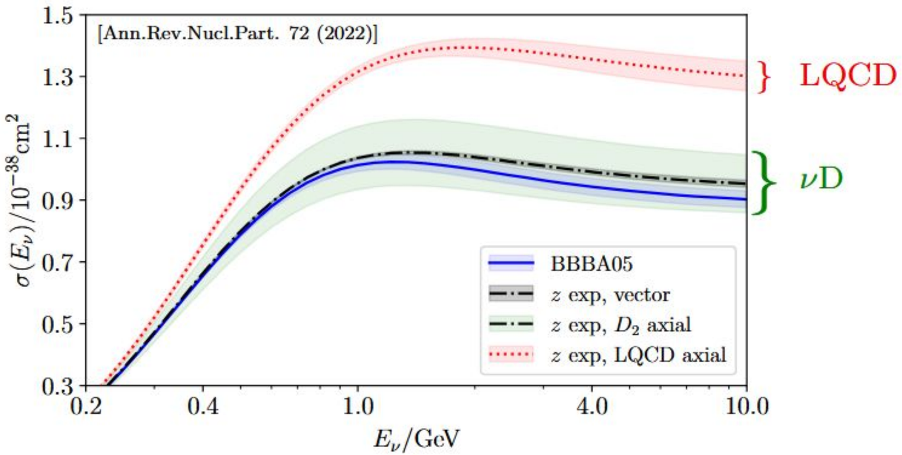
Motivation: Axial Form Factors

- The nucleon's axial and induced pseudoscalar form factor are also of interest

Axial FF $$\int_{-1}^{1} dx\, \tilde{H}^q(x,\xi,t) = G_A^q(t)$$

Induced pseudoscalar FF $$\int_{-1}^{1} dx\, \tilde{E}^q(x,\xi,t) = F_P^q(t)$$

- G_A is the dominant source of uncertainty in neutrino-nucleus scattering
    - Precision from LQCD form factors is leading experiment
    - But leads to vastly different expectations!



A. Meyer, et al., Ann.Rev.Nucl.Part.Sci. 72:205-232 (2022)

- These open questions present a strong motivator for studying the electromagnetic and axial form factors on the lattice as an ab initio study of QCD

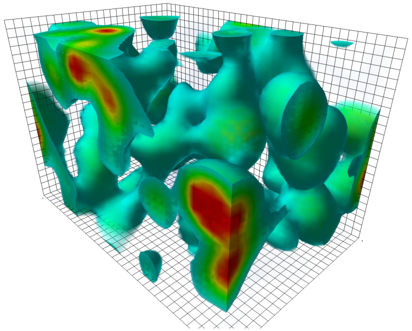
Outline

- **RBC/UKQCD lattice ensemble**
- **Isovector nucleon electric and magnetic form factors**
  - Extracting charges and form factors
  - Fitting the Q^2 dependence and obtaining radii
- **Isovector nucleon axial form factor (preliminary)**
  - Results for form factor, charge and radius
  - Implications of the Partially Conserved Axial Current relation
- **Summary and outlook**

RBC/UKQCD Ensemble Details

- We perform our analysis on a lattice ensemble from RBC/UKQCD with:
  - 2+1 flavours of dynamical quarks
  - Möbius Domain Wall fermion action
  - Iwasaki gauge action

T. Blum, et al. [RBC/UKQCD] PRD 93:074505 (2016)



Credit: D. Leinweber

Ensemble	$L^3 \times L_t$	L_s	a (fm)	m_π (MeV)	N_{cfg}	t_{sep}/a
48I	$48^3 \times 96$	24	0.114	139	130	$8, \ldots, 12$

- Introducing a 5th dimension to the lattice via the Domain Wall action leads to an increase in computational expense of fermion propagators
  - All-Mode-Averaging (AMA) is used to compensate this: *S. Syritsyn J. Phys.: Conf. Ser. 640 012054 (2015)*

$$\langle \mathcal{O} \rangle_{\text{imp}} = \langle \mathcal{O} \rangle_{N_{\text{approx}}} + \langle \Delta\mathcal{O} \rangle_{N_{\text{exact}}} \qquad \Delta\mathcal{O} = \mathcal{O}_{\text{exact}} - \mathcal{O}_{\text{approx}}$$

  - Per configuration we have $N_{\text{approx}} = 128$ and $N_{\text{exact}} = 4$

2-point correlators

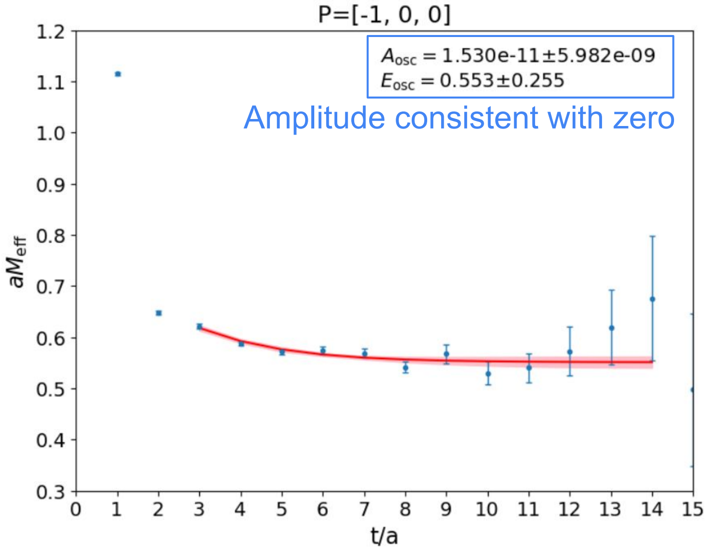
- We take a two-state fit form; for the 2-point correlators we take

$$C_{2\text{pt}}(p, t_{\text{sep}}) = |A_0(\vec{p})|^2 e^{-E_0(\vec{p})t_{\text{sep}}} + |A_1(\vec{p})|^2 e^{-E_1(\vec{p})t_{\text{sep}}}$$

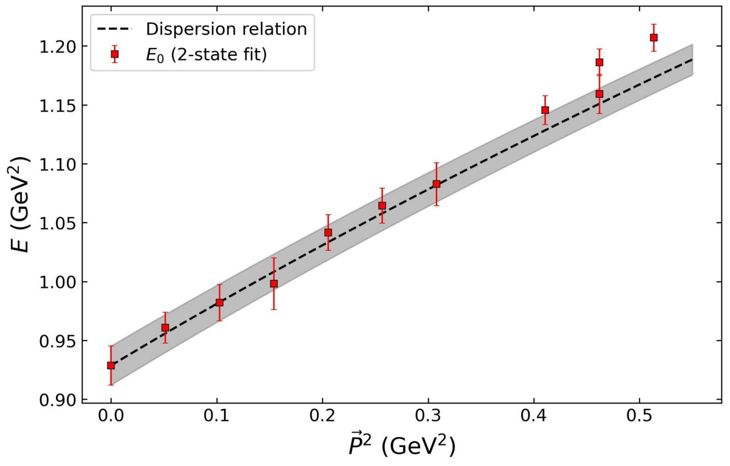
- Typically the Domain Wall action causes oscillations in the 2-point correlators

$$C_{2\text{pt}}(p, t_{\text{sep}}) = |A_0(\vec{p})|^2 e^{-E_0(\vec{p})t_{\text{sep}}} + |A_1(\vec{p})|^2 e^{-E_1(\vec{p})t_{\text{sep}}} + |A_{\text{osc}}(\vec{p})|^2 (-1)^{t_{\text{sep}}} e^{-E_{\text{osc}}(\vec{p})t_{\text{sep}}}$$

- We find negligible oscillatory signal:



- Proceeding with the traditional 2-point fit form we recover the continuum energy dispersion relation



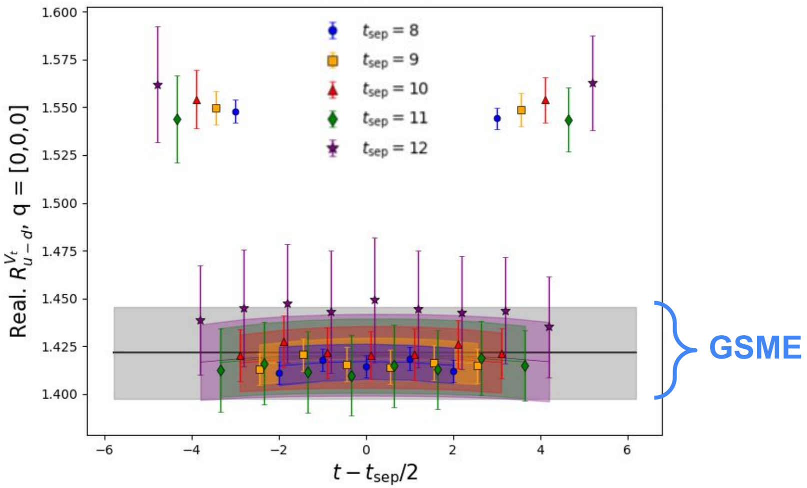
3-point correlators

- We also parameterize our 3-point correlators using a 2-state fit form:

$$C_{3\text{pt}}^{\mathcal{O}}(p,p',t,t_{\text{sep}}) = |A_0(\vec{p})||A_0(\vec{p}')|M_{00}^{\mathcal{O}}e^{-E_0(\vec{p})t}e^{-E_0(\vec{p}')(t_{\text{sep}}-t)} + |A_0(\vec{p})||A_1(\vec{p}')|M_{01}^{\mathcal{O}}e^{-E_0(\vec{p})t}e^{-E_1(\vec{p}')(t_{\text{sep}}-t)}$$
$$+|A_1(\vec{p})||A_0(\vec{p}')|M_{10}^{\mathcal{O}}e^{-E_1(\vec{p})t}e^{-E_0(\vec{p}')(t_{\text{sep}}-t)} + |A_1(\vec{p})||A_1(\vec{p}')|M_{11}^{\mathcal{O}}e^{-E_1(\vec{p})t}e^{-E_1(\vec{p}')(t_{\text{sep}}-t)}$$

where we define matrix elements $M_{ij}^{\mathcal{O}} = \langle i|\mathcal{O}|j\rangle$

- We perform simultaneous fits to correlators with $t_{\text{sep}}/a = \{8, 9, 10, 11, 12\}$ to control excited states
  - **Note:** A_i and E_i are held fixed from 2pt fits



- We construct the ratio

$$R^{\mathcal{O}}(p,p',t,t_{\text{sep}}) = \frac{C_{3\text{pt}}^{\mathcal{O}}(p,p',t,t_{\text{sep}})}{\sqrt{C_{2\text{pt}}(p,t_{\text{sep}})C_{2\text{pt}}(p',t_{\text{sep}})}}\sqrt{\frac{C_{2\text{pt}}(p,t_{\text{sep}}-t)C_{2\text{pt}}(p',t)}{C_{2\text{pt}}(p',t_{\text{sep}}-t)C_{2\text{pt}}(p,t)}}$$

- Taking limit of large times recovers the ground-state matrix element $\mathcal{R}^{\mathcal{O}} \equiv \lim_{\{t_{\text{sep}},\, t,\, t_{\text{sep}}-t\to\infty\}} R^{\mathcal{O}} = M_{00}^{\mathcal{O}}$

Extracting form factors

- Working from the correlator definitions, one can write

$$\mathcal{R}^{\mathcal{O}} = \frac{\mathrm{Tr}\left[\Gamma_{\mathrm{proj}}(-ip' + M_N)\Gamma^{\mathcal{O}}(p',p)(-ip + M_N)\right]}{2\sqrt{E_0(p')E_0(p)\mathrm{Tr}\left[\Gamma_{\mathrm{proj}}(-ip' + M_N)\right]\mathrm{Tr}\left[\Gamma_{\mathrm{proj}}(-ip + M_N)\right]}}$$

$$\Gamma_{\mathrm{proj}} = \frac{1}{2}(1+\gamma_4)(1 - i\gamma_3\gamma_5)$$

  - In the case of a vector current insertion, one has

$$\langle N(p',s')|V_\mu(q)|N(p,s)\rangle = \bar{u}(p',s')\underbrace{\left(\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q^\nu}{2M_N}F_2(Q^2)\right)}_{\Gamma^{\mathcal{O}}(p,p')}u(p,s)$$

$$V_\mu = \bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d$$

- Computing the required traces yields linear combinations of the form factors:

GSME = linear combination of FFs

$$\Re(\mathcal{R}_i)|_{i=1,2} = \frac{\varepsilon_{ij3}p_j}{\sqrt{2E_p(M_N + E_p)}}\left(F_1 + F_2\right)$$

$$\Re(\mathcal{R}_4) = \sqrt{\frac{M_N + E_p}{2E_p}}\left(F_1 - \frac{\vec{p}^2}{2M_N(E_p + M_N)}F_2\right)$$

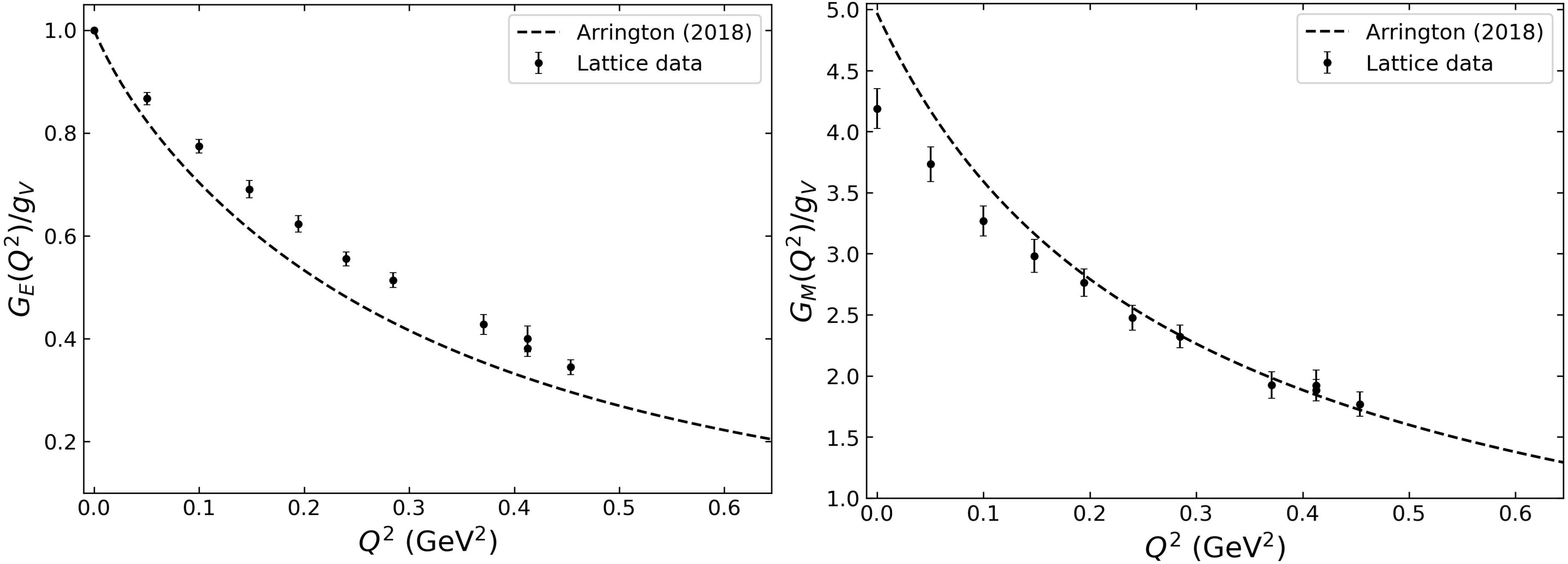
Degeneracy in Q^2 $\Rightarrow$ overdetermined system of equations

$Q^2 \neq 0 \Rightarrow F_1$ and F_2 extracted by fits to these equations

$Q^2 = 0 \Rightarrow$ directly extract the charge $g_V = F_1(0)$

Sachs Form Factor Results

- Extracting the form factors F_1 and F_2 and converting to the Sachs form factors we have:

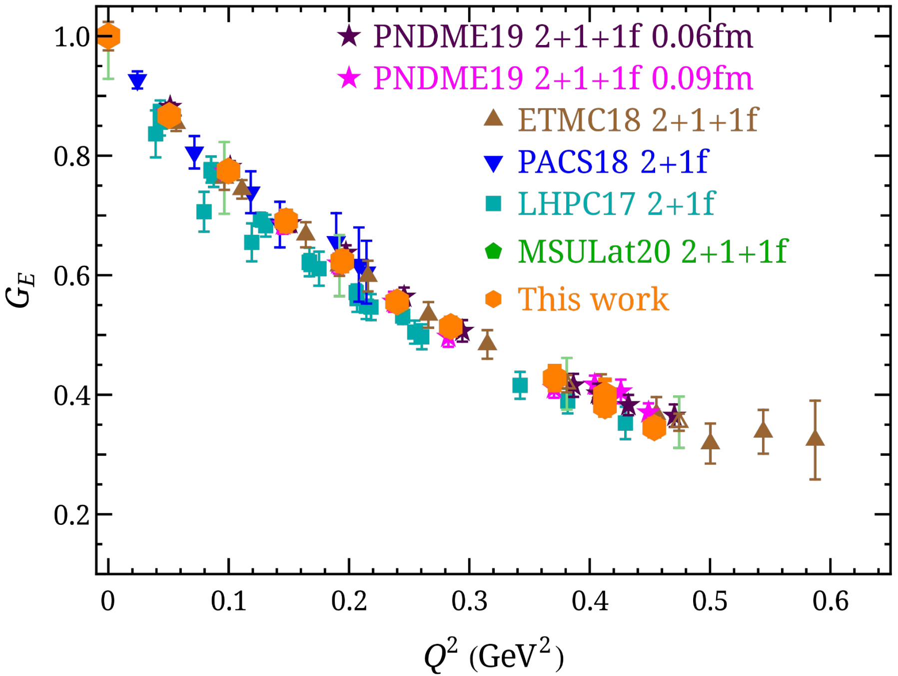


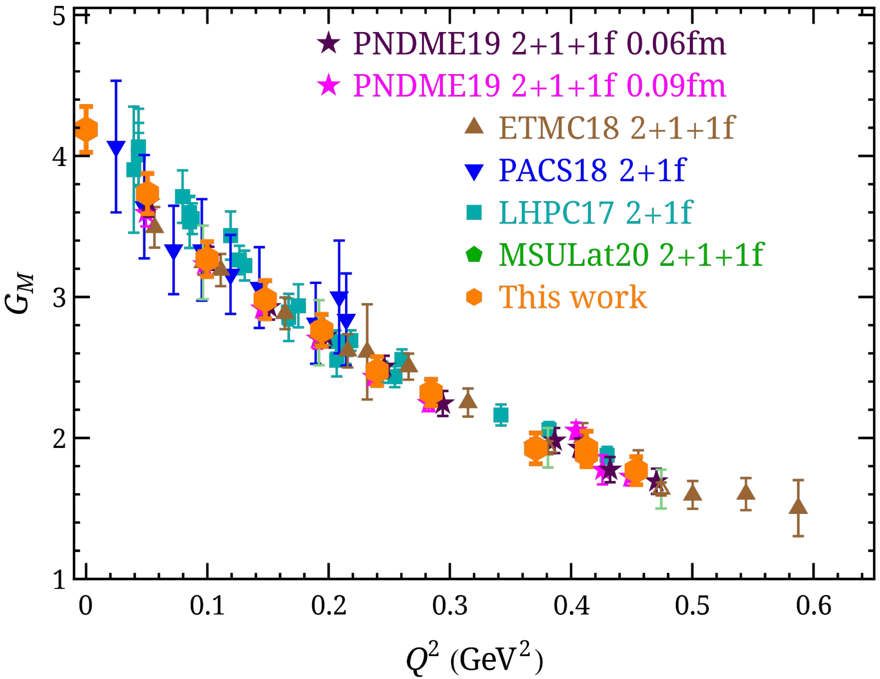
- These compare favourably with phenomenological results from parameterising the form factors of the proton and neutron where

Z. Ye, et al., Phys.Lett.B 777, 8-15 (2018)

$$G^{u-d}_{\{E,M\}} = G^{p}_{\{E,M\}} - G^{n}_{\{E,M\}}$$

Comparison of G_E and G_M with other groups





- We also find agreement with various other lattice groups using physical or close-to-physical pion masses
- Note, the value of G_M at $Q^2=0$ is obtained by considering a linear fit of the ratio G_M/G_E for the points with $n^2 = 1, \dots, 6$

Q^2 Dependence

- One can extract the associated radii by

$$G(Q^2) = G(0)\left(1 - \frac{\langle r^2\rangle}{3!}Q^2 + \frac{\langle r^4\rangle}{5!}Q^4 + \dots\right) \quad \Rightarrow \quad \langle r^2\rangle = -\frac{6}{G(0)}\frac{dG(Q^2)}{dQ^2}\bigg|_{Q^2=0}$$

- We consider two parameterizations of the Q^2 dependence:

Dipole

$$G(Q^2) = \frac{G(0)}{(1+Q^2/M_{\text{dip}}^2)^2}$$

z-expansion

$$G(Q^2) = \sum_{k=0}^{k_{\max}} a_k\, z(Q^2,t_0)^k \qquad z(Q^2,t_0) = \frac{\sqrt{t_{\text{cut}}+Q^2}-\sqrt{t_{\text{cut}}-t_0}}{\sqrt{t_{\text{cut}}+Q^2}+\sqrt{t_{\text{cut}}-t_0}}$$

- We take for simplicity $t_0 = 0$ and $t_{cut} = 4m_\pi^2$ is set by the $\pi\pi$ production threshold
  - With $t_0 = 0$, we set the value of $a_0 = G(Q^2{=}0)$

Series truncation

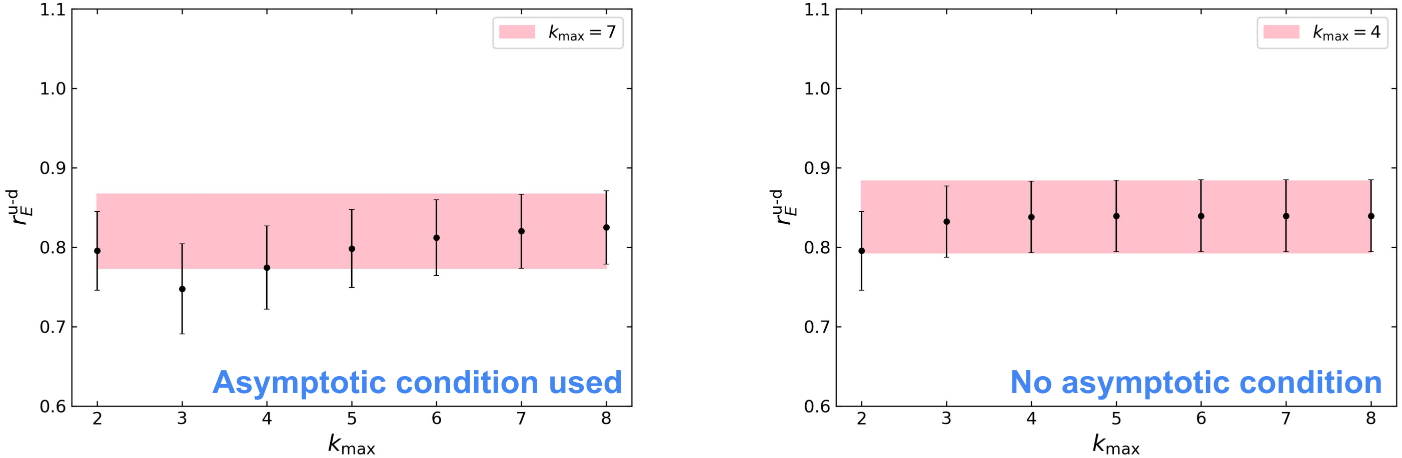
- k_{max} is taken such that additional terms in the expansion have negligible effect on extracted radii
- We first perform a fit with k_{max}=2 and then for higher k_{max} fits we take Gaussian priors of the form

$$\mathcal{N}(0,w), \quad \text{where } w = 5\cdot\max\{|a_0|,|a_1|\}$$

J. Green et al., Phys.Rev.D 95 11:114502 (2017)

Z-expansion convergence

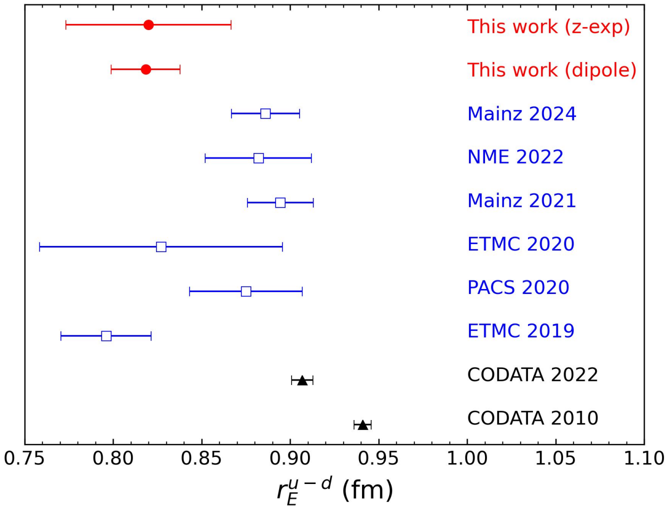
- Additional constraints come from the asymptotic behaviour of the form factors
  - From χPT the Sachs form factors follow (up to logs): $G_{\{E,M\}}(Q^2) \sim \frac{1}{Q^4}, \qquad Q^2 \to \infty$
    G. Lepage & S. Brodsky, Phys.Rev.D 22, 2157 (1980)
  - This leads to four sum rules for the z-expansion coefficients: $Q^k G(Q^2) \to 0, \qquad k = 0, 1, 2, 3$
  - However, this slows convergence: *C. Alexandrou, et al., Phys.Rev.D 100:014509 (2019)*

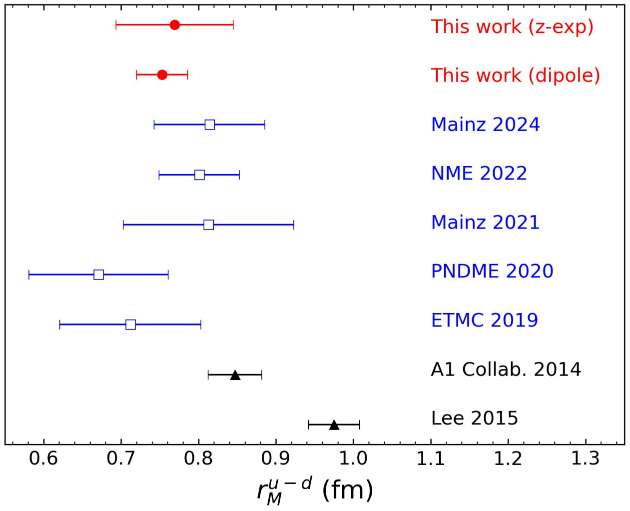


- Results with and without the asymptotic condition are in good agreement; recall the radius depends on the *small* Q^2 behaviour

- We use fits involving the additional asymptotic constraints as a point of comparison only

Electric and Magnetic Radii Results





- Note: experimental results for the proton radii are converted to isovector nucleon radii using the PDG neutron radius:

$$\langle r_i^2 \rangle^{u-d} = \langle r_i^2 \rangle^p - \langle r_i^2 \rangle^n$$

S. Navas, et al. (Particle Data Group), Phys. Rev. D 110, 030001 (2024) and 2025 update

- We observe slight tension in the isovector nucleon charge radius with the CODATA 2022 result
- We find general agreement with the A1 Collaboration's ep scattering result - note this is in tension with Lee et al.'s analysis of world ep scattering data

Axial Form Factor

- Likewise, we also extract the axial form factor G_A using:

$$\langle N(p',s')\,|\,A_\mu(q)\,|\,N(p,s)\rangle = \bar{u}(p',s')\left[\gamma_\mu\, G_A(Q^2) + \frac{q^\mu}{2M_N}\, F_P(Q^2)\right]\gamma_5 u(p,s) \qquad A_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d$$

- The relations between GSMEs and form factors are:

$$\Im(\mathcal{R}_i)|_{i=1,2} = \frac{1}{\sqrt{2E_p(E_p+M_N)}}\left[-\frac{q_i q_3}{2M_N}F_P\right]$$

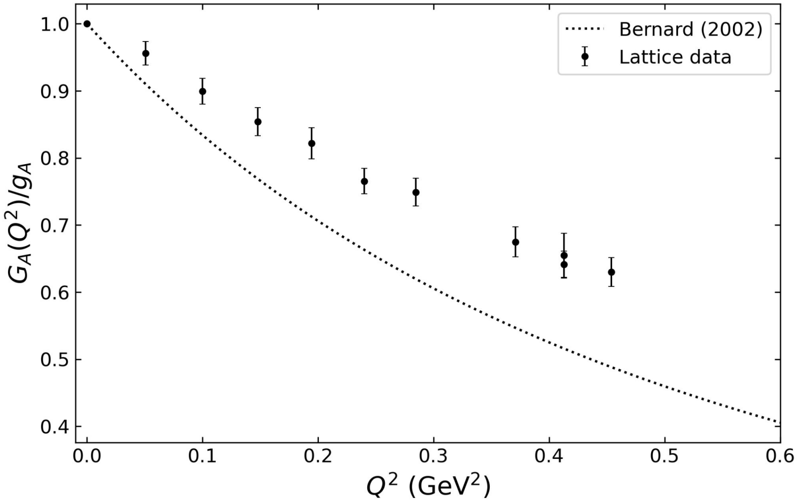
$$\Im(\mathcal{R}_3) = \frac{1}{\sqrt{2E_p(E_p+M_N)}}\left[-\frac{q_3^2}{2M_N}F_P + (M_N+E_p)G_A\right]$$

We directly extract the axial charge g_A from $\mathcal{R}_3$ at $Q^2=0$

$$\Re(\mathcal{R}_4) = \frac{q_3}{\sqrt{2E_p(E_p+M_N)}}\left[\frac{M_N-E_p}{2M_N}F_P + G_A\right]$$

Temporal component of the current is susceptible to excited states and has small ground-state signal - **not used**

Y.-C. Jang, et al., Phys. Rev. Lett. 124 (2020)
C. Alexandrou et al., Phys.Rev.D 111, 5:054505 (2025)



V. Bernard, et al., J.Phys.G 28, R1-R35 (2002)

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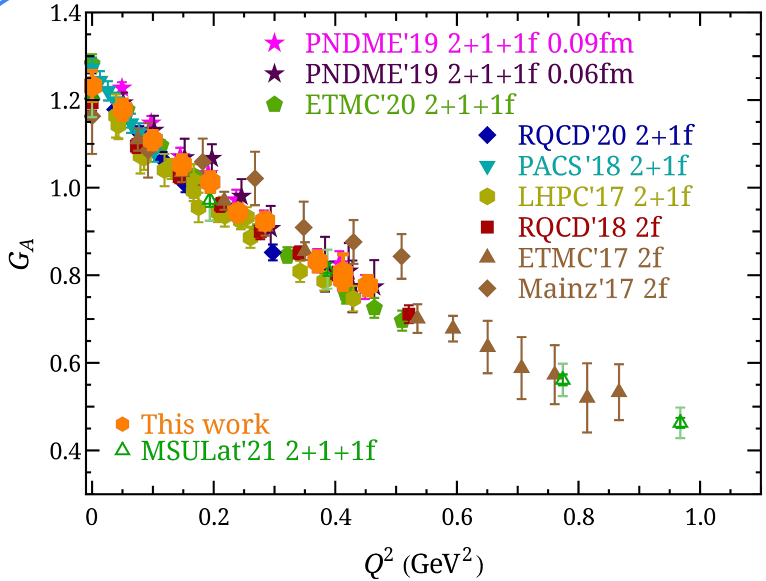


Y.-C. Jang, et al., Phys. Rev. Lett. 124 (2020)
C. Alexandrou et al., Phys.Rev.D 111, 5:054505 (2025)

Partially Conserved Axial Current

- A consequence of the axial Ward identity is the PCAC relation: $m_q G_P(Q^2) = m_N G_A(Q^2) - \frac{Q^2}{4m_N} F_P(Q^2)$

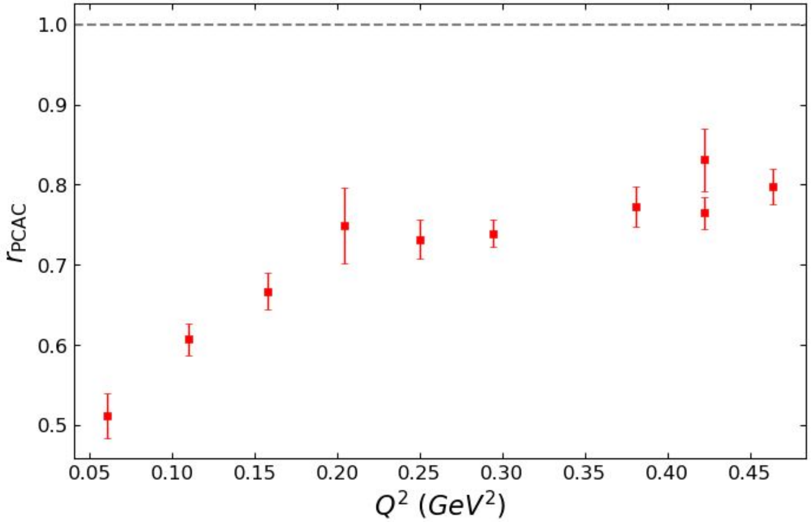
- Pseudoscalar form factor defined by

$$\langle N(p', s')|P(q)|N(p, s)\rangle = \bar{u}(p', s') G_P(Q^2)\gamma_5 u(p, s)$$

$$P = \bar{u}\gamma_5 u - \bar{d}\gamma_5 d$$

- PCAC is violated if

$$r_{\mathrm{PCAC}} = \frac{m_q G_P(Q^2) + \frac{Q^2}{4m_N^2} F_P(Q^2)}{m_N G_A(Q^2)} \neq 1$$



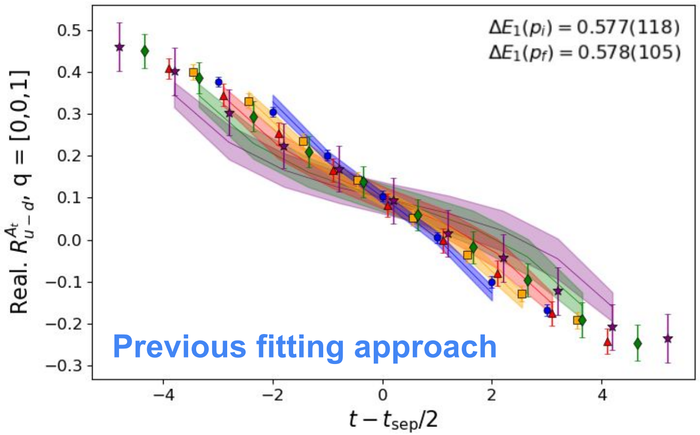
- χPT+LQCD ⇒ low-lying excited-state effects from $N\pi$ states lead to observed violation:
  - Q^2-dependence of the PCAC violation is driven by loop diagrams contributing to the induced pseudoscalar form factors

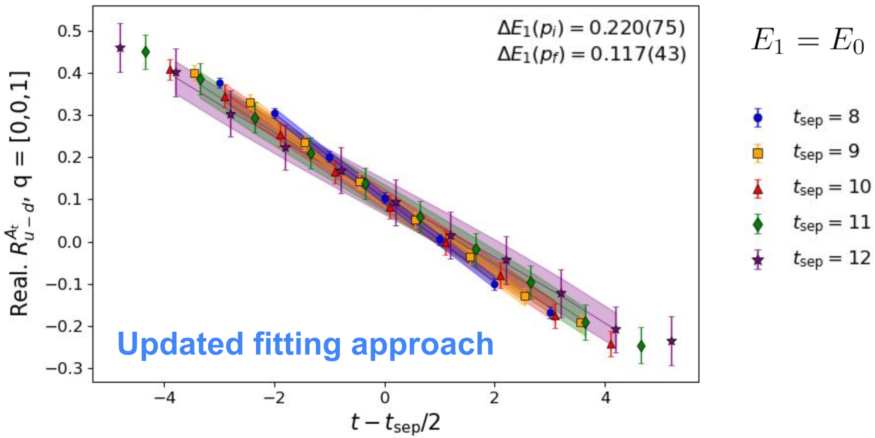
O. Bär, Phys. Rev. D, 99:054506 (2019)

R. Gupta, et al., Phys.Rev.D. 96:114503 (2017)

Excited state effects

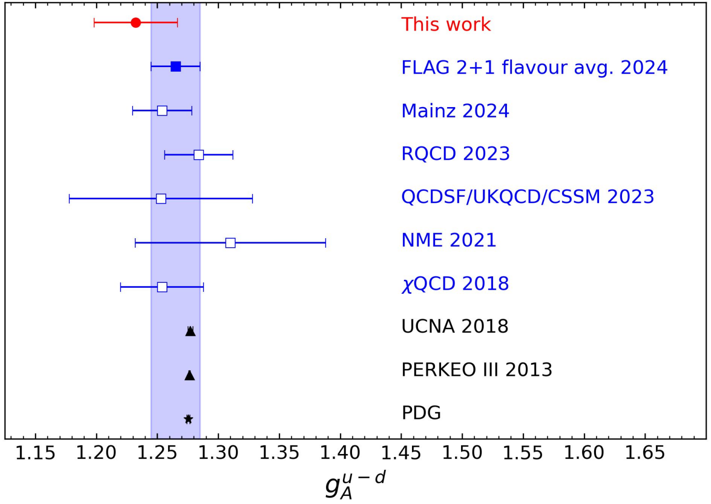
- We aim to explore alternative correlator fitting schemes to control these excited state contaminations
  - One such scheme is to perform fits to $C_{3\text{pt}}^{A_4}$ with the excited state energies left as free parameters *C. Alexandrou et al., Phys.Rev.D 111, 5:054505 (2025)*
  - Preliminary results indicate better fits to the 3pt correlators *and* smaller excited state energies, in agreement with similar calculations *Y.-C. Jang, et al., Phys. Rev. Lett. 124 (2020)*

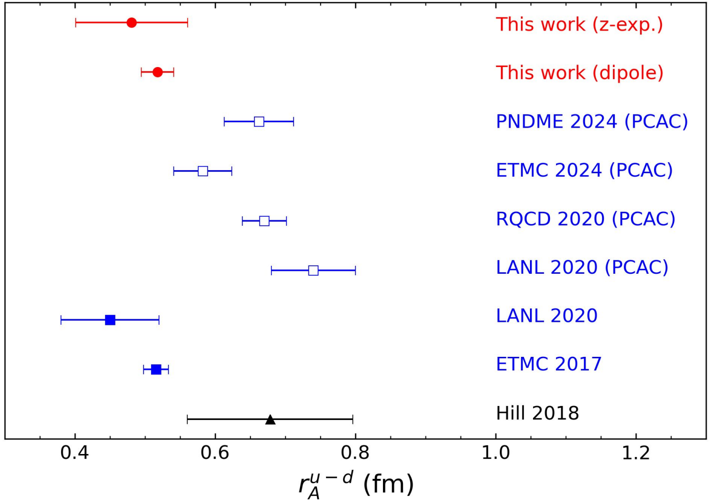




$$E_1 = E_0 + \Delta E_1$$

Axial Charge and Radius





- Our axial charge is in agreement with the FLAG average, though slightly below experiment
- The axial radius is also small, and lies in agreement with other LQCD analyses which did not account for PCAC violation (those not marked with a PCAC label)
- We expect both of our results to undergo positive shifts after PCAC restoration c.f. LANL (2020)

Summary and outlook

- I presented results for the Sachs E&M form factors, with our preferred estimate for the associated radii being the result from using the z-expansion with $k_{max}=4$
  - We observe small radii in the context of experiment
  - We are currently studying the dependence on t_0 and this may shift the results slightly

- Presented preliminary results on the axial charge and radius
  - We identify violation of the PCAC relation and observe sensitivity to low-lying excited states in the temporal component of the axial 3-point correlator
  - These were previously missed in the extraction of our results and so we aim to compare these initial results with an analysis with PCAC restored as a next step

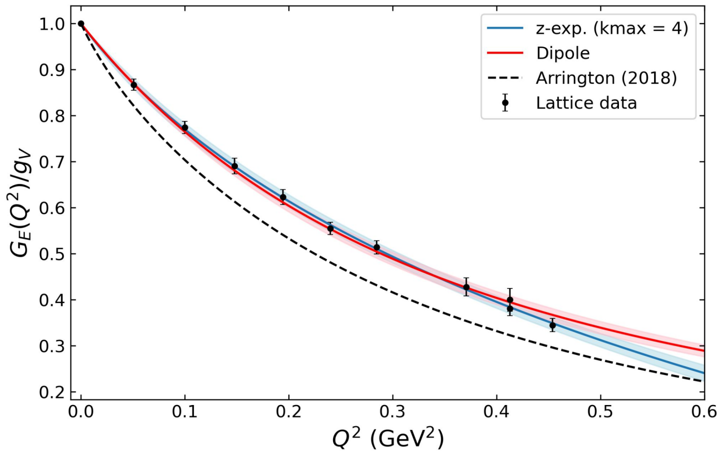
- As a final note, we envisage strengthening this analysis with a finer lattice ensemble (a~0.09 fm) also at physical pion mass from RBC/UKQCD

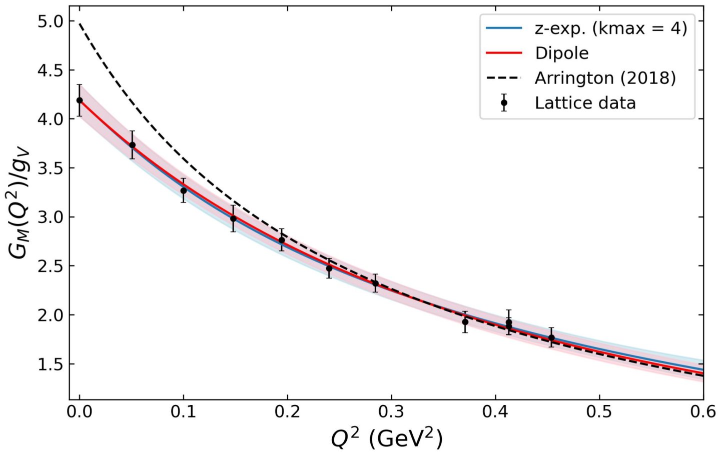
Thank you!

Backup Slides

Sachs Form Factor Fit results

- Taking our results for both the dipole and z-expansions we thus have:





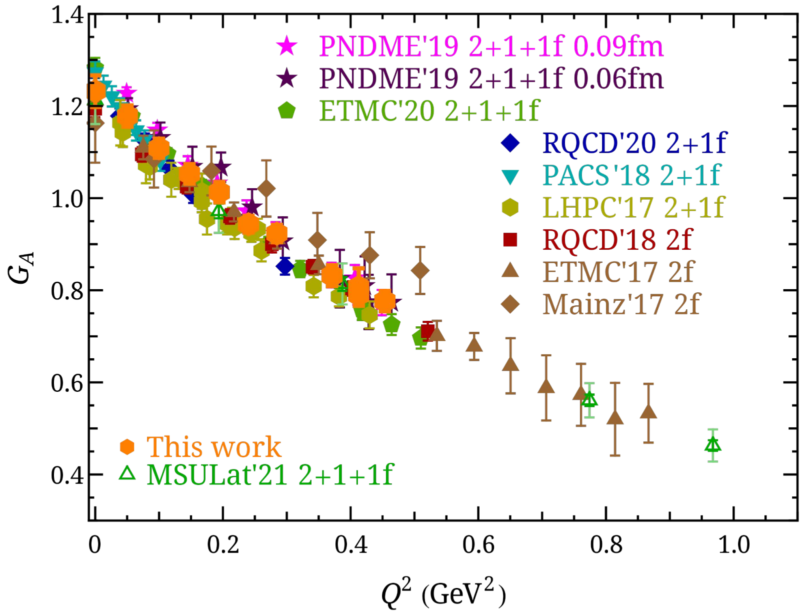
- The radii are extracted using:

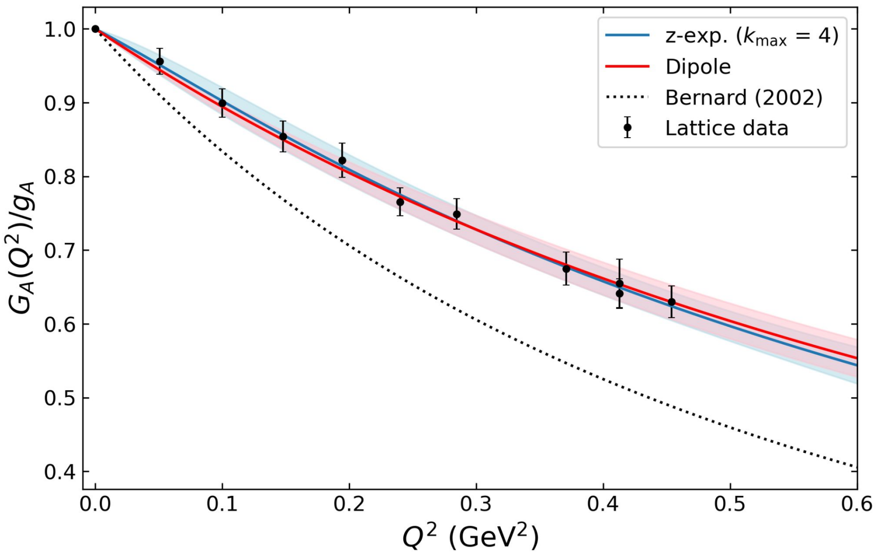
$$\langle r_i^2 \rangle_{\text{dipole}} = \frac{12}{M_i^2} \qquad \langle r_i^2 \rangle_{\text{z-exp.}} = -\frac{3a_1}{2a_0 t_{\text{cut}}}$$

- Our final results are then:

$(r_E^{u-d})_{\text{dipole}}$ [fm]	$(r_E^{u-d})_{\text{z-exp.}}$ [fm]	$(r_M^{u-d})_{\text{dipole}}$ [fm]	$(r_M^{u-d})_{\text{z-exp.}}$ [fm]
0.818(19)	0.840(58)	0.753(33)	0.768(78)

Axial Form Factor Results





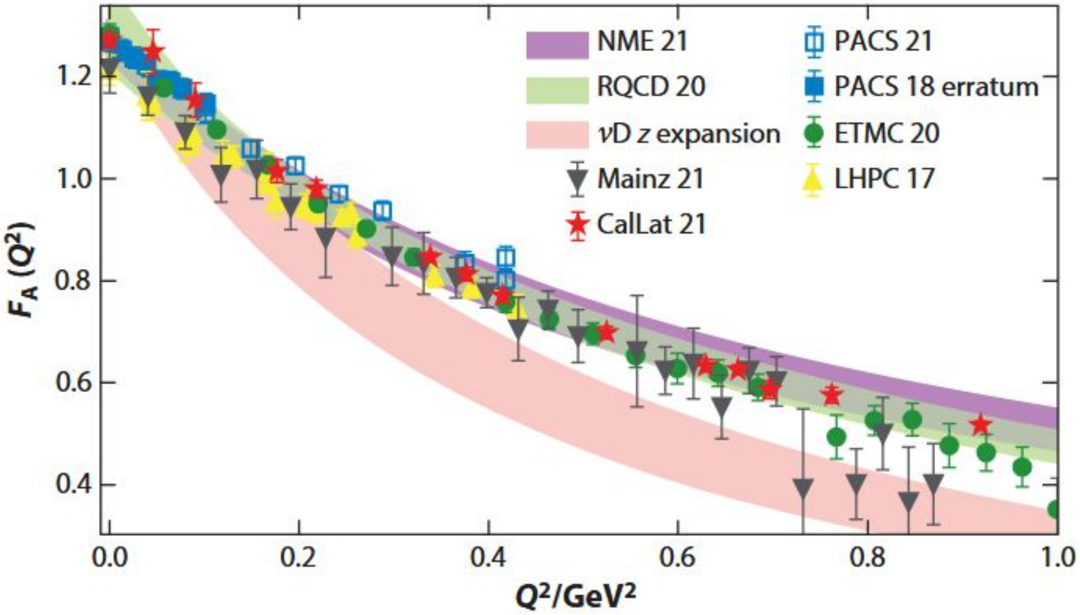
- Our results for $\mathrm{G_A}$ compare well with contemporary lattice results and, as is typically found, sit well above the naive dipole expectation with $\mathrm{M_A} = 1.026\ \mathrm{GeV}$ *V. Bernard, et al., J.Phys.G 28, R1-R35 (2002)*

- We obtain for the axial charge and radius:

g_A^{u-d}	$(r_A^{u-d})_{\text{dipole}}$ [fm]	$(r_A^{u-d})_{\text{z-exp.}}$ [fm]
$1.232(34)$	$0.518(23)$	$0.481(80)$

Axial form factor: deviations from experiment

LQCD systematically deviates from experiment in the large Q^2 region

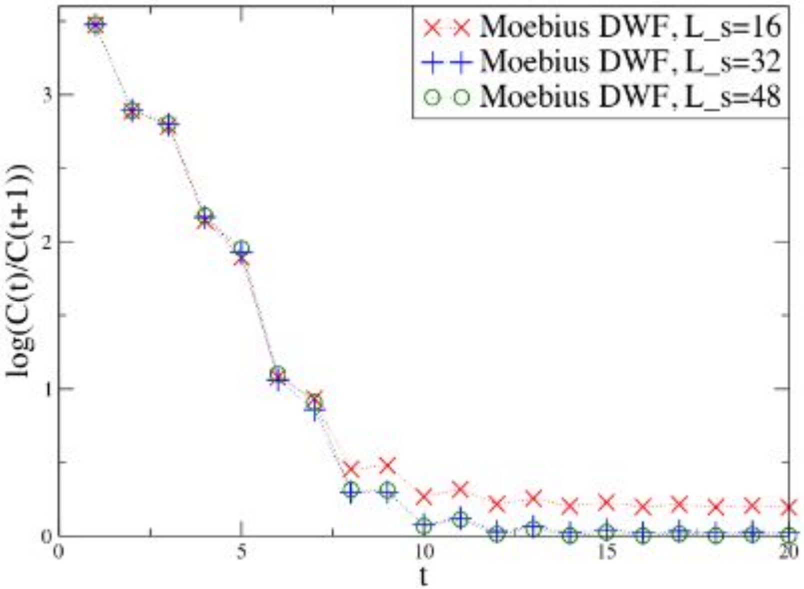


A. Meyer, et al., Ann.Rev.Nucl.Part.Sci. 72:205-232 (2022)

Oscillations in hadronic correlators

Example of oscillations present in other Domain Wall correlators

R. Sufian, et al., arXiv:1603.01591



Impact of PCAC restoration

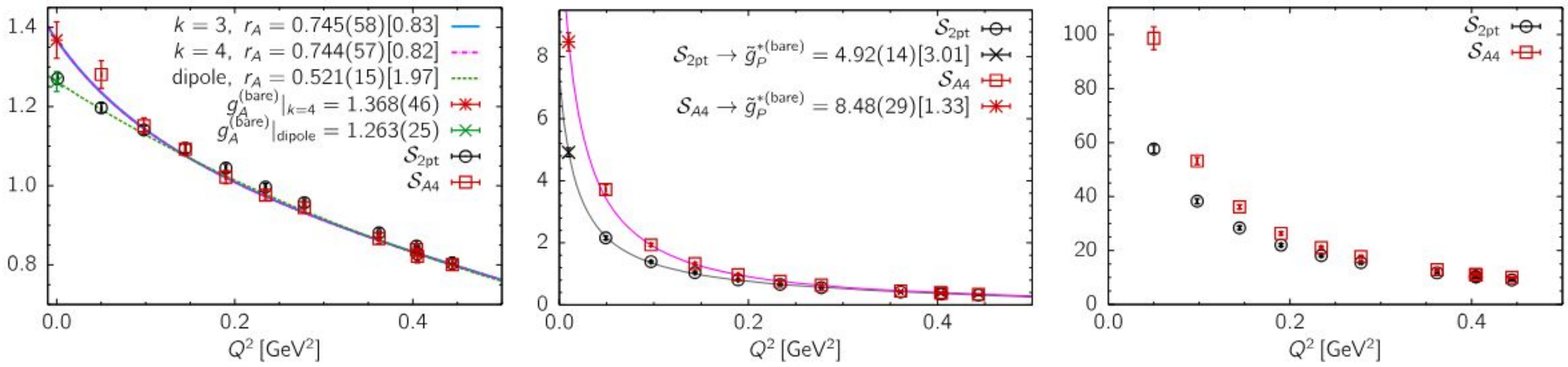


FIG. 4. Comparison of (left panel) G_A, (middle panel) $\tilde{G}_P \times (m_\mu/2M)$, and (right panel) G_P obtained using the two strategies $\mathcal{S}_{A4}$ and $\mathcal{S}_{2pt}$. The lines show the dipole and z^k-expansion fits to G_A from $\mathcal{S}_{A4}$ and the PPD ansatz to $\tilde{G}_P$.

- The axial form factor is underestimated at small Q^2
- gA shifts slightly but this induces a more pronounced change in the slope of the form factor so the radius changes more significantly

Correlated form factor fits

- In presenting the results from the form factor extraction, we use uncorrelated fit results

$$\mathcal{F} = \sum_{\alpha\beta} (A_{\alpha i} F_i - R_\alpha)\, C^{-1}_{\alpha\beta}\, (A_{\beta j} F_j - R_\beta)$$

- We are currently investigating shrinkage as a way of stabilising the inversions of the covariance matrix for the GSMEs

$$C_s = (1-s)C + s \times \mathrm{diag}(C), \qquad s \in [0,1]$$