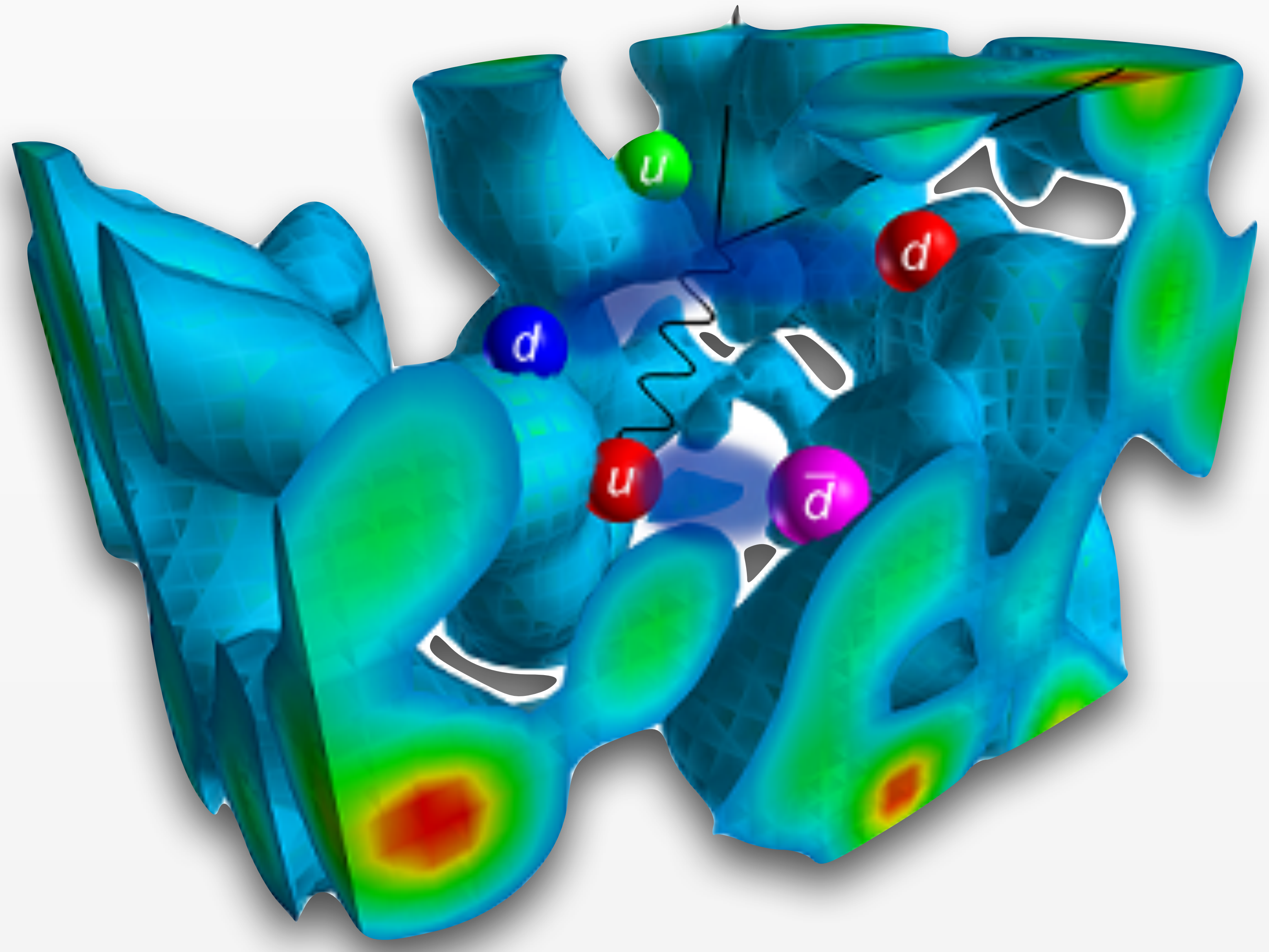
Lattice Parton Collaboration

Heavy meson light-cone distribution amplitudes from lattice QCD

Xue-Ying Han (IHEP, CAS)

H. F. Gao, Q. A. Zhang, W. Wang



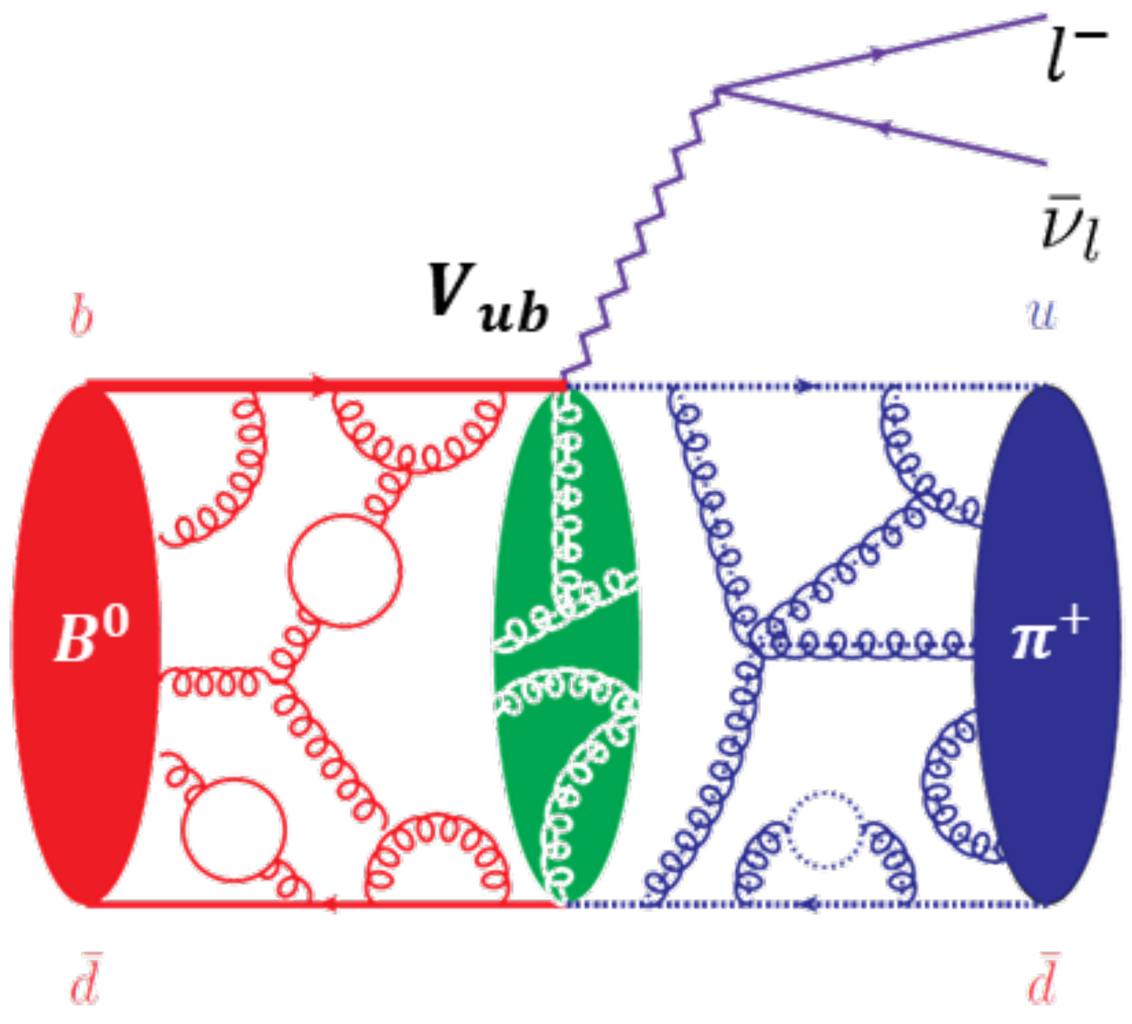
Oct 10, 2025 @ XIIth Meeting on Lattice Parton Physics from LaMET

Motivation & Overview

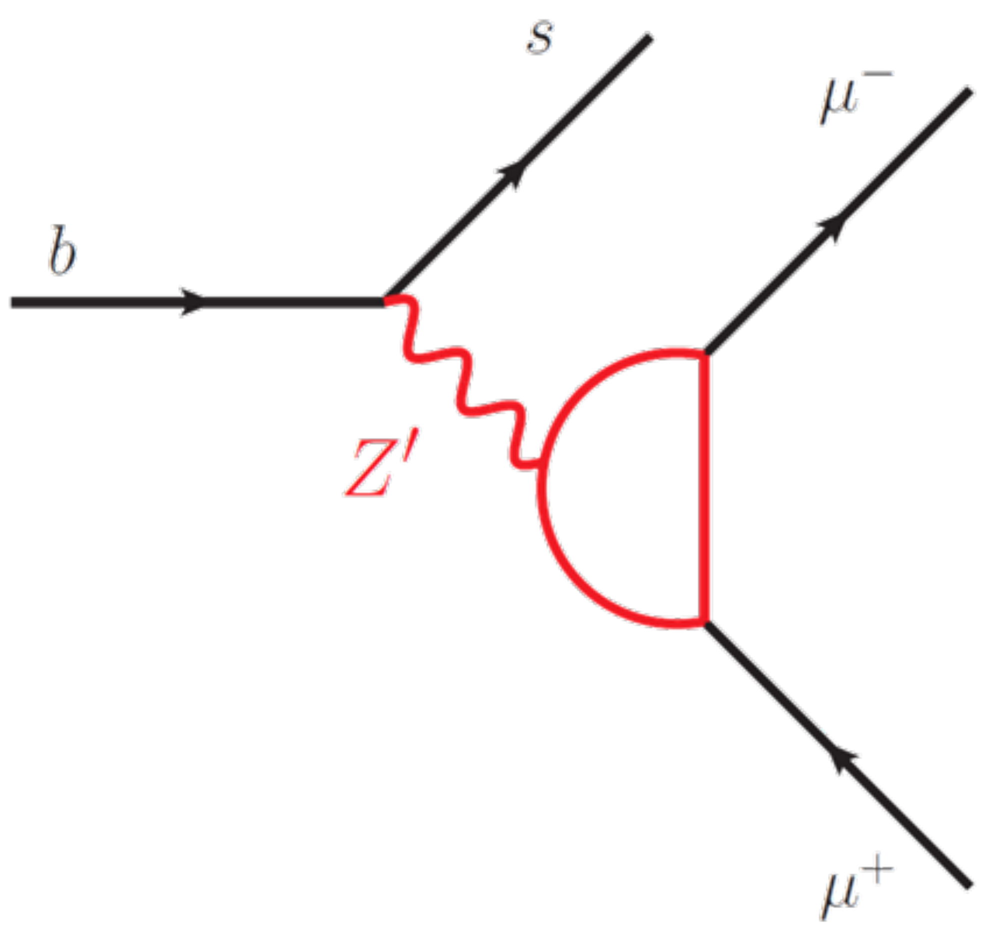
Why heavy meson structure matters?

🤔 Why heavy meson structure matters?

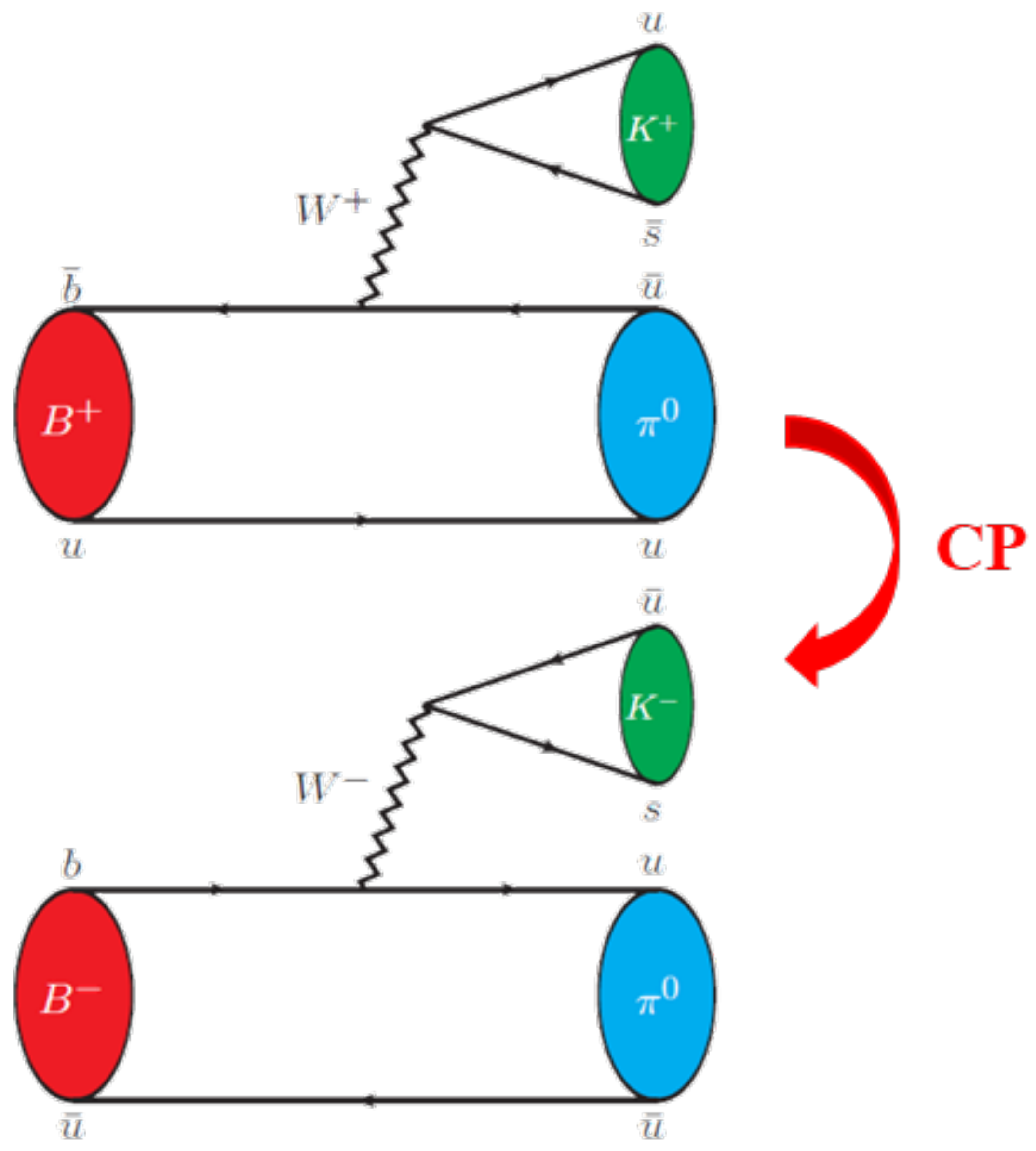
Heavy flavor physics is one of the frontier topics in particle physics



Precise test of Standard Model



Indirect searching for new physics



Study on CP violation

🤔 Why heavy meson structure matters?

V_{cb}, V_{ub} Puzzle

- $|V_{xb}|$ can be measured in semileptonic B decays with inclusive or exclusive processes
- " $|V_{xb}|$ Puzzle
  - From inclusive B decays

$$|V_{ub}^{\text{Incl.}}| = (4.19 \pm 0.17) \times 10^{-3}$$

$$|V_{cb}^{\text{Incl.}}| = (42.19 \pm 0.78) \times 10^{-3}$$

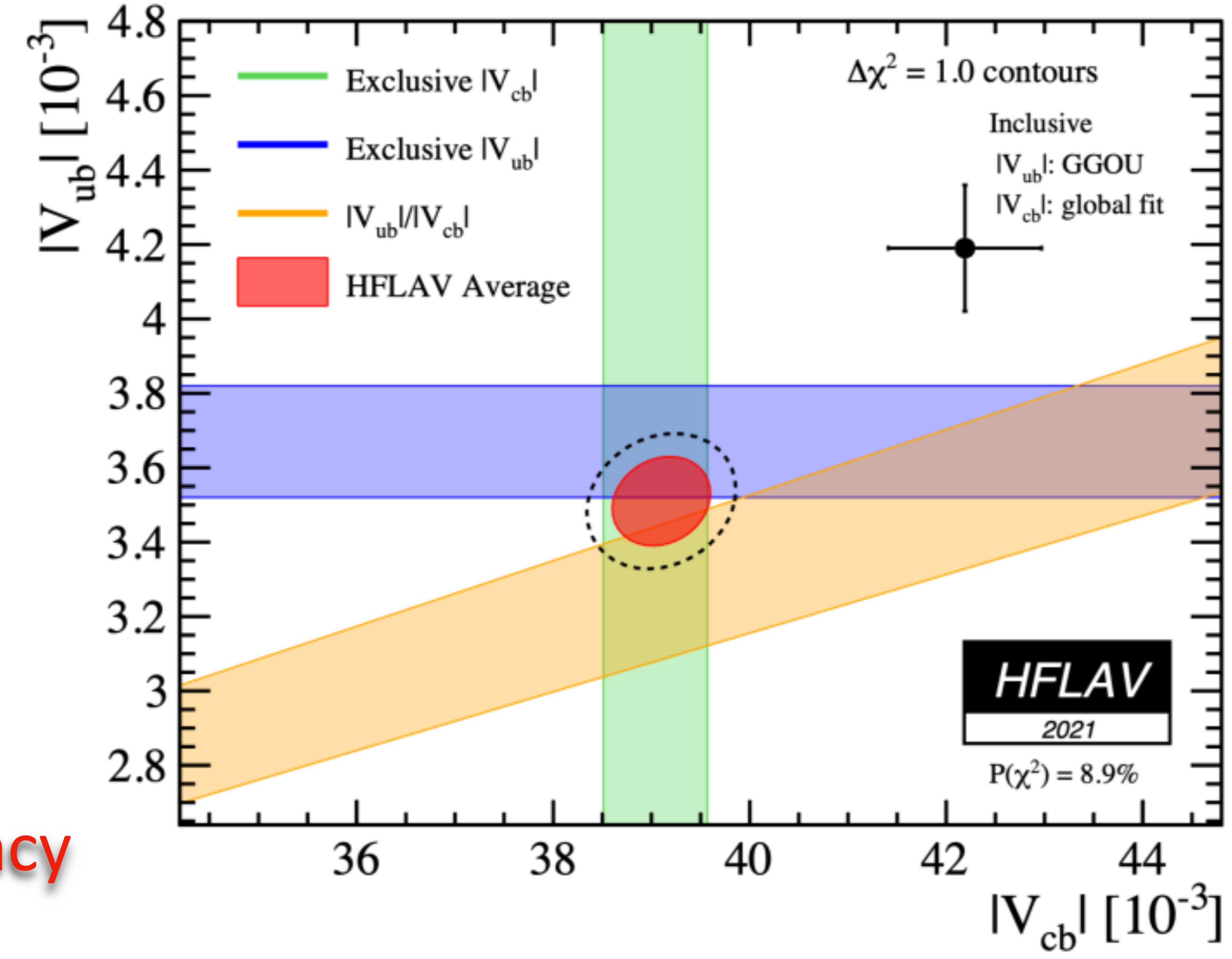
  - From exclusive B, B_s, Λ_b decays

$$|V_{ub}^{\text{Excl.}}| = (3.51 \pm 0.12) \times 10^{-3}$$

$$|V_{cb}^{\text{Excl.}}| = (39.10 \pm 0.50) \times 10^{-3}$$

3.3σ discrepancy

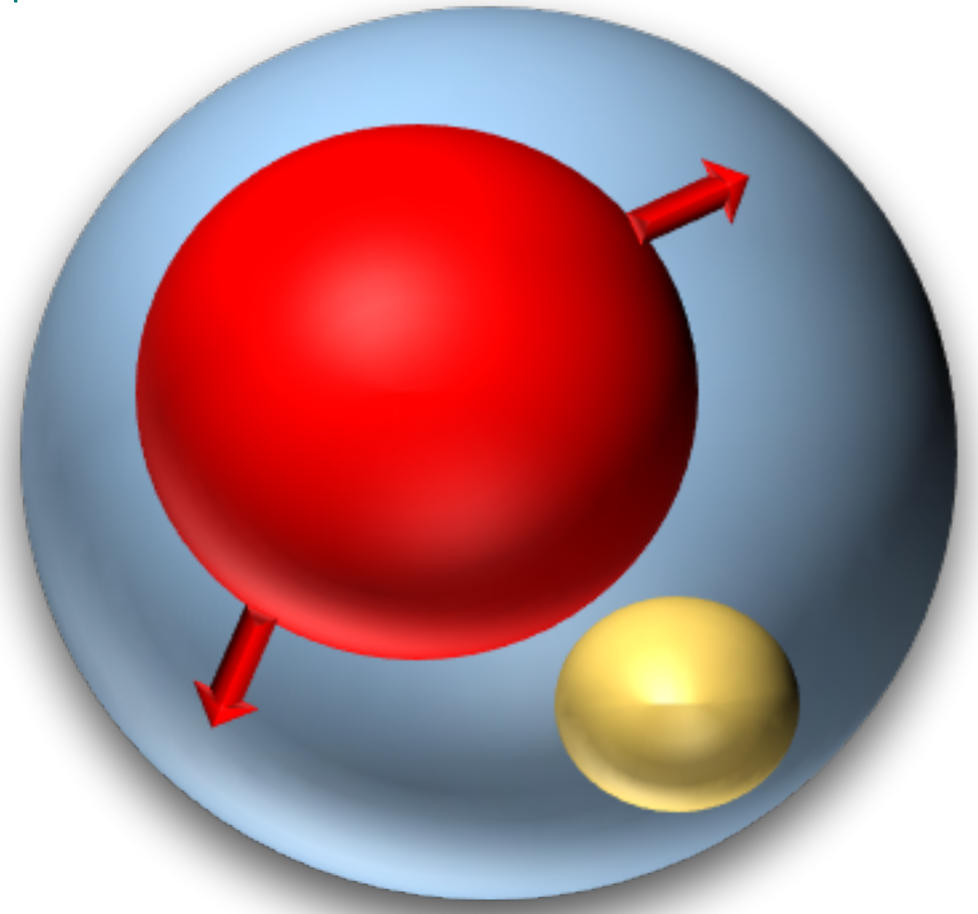
HFLAV, PRD 107, 052008 (2023)

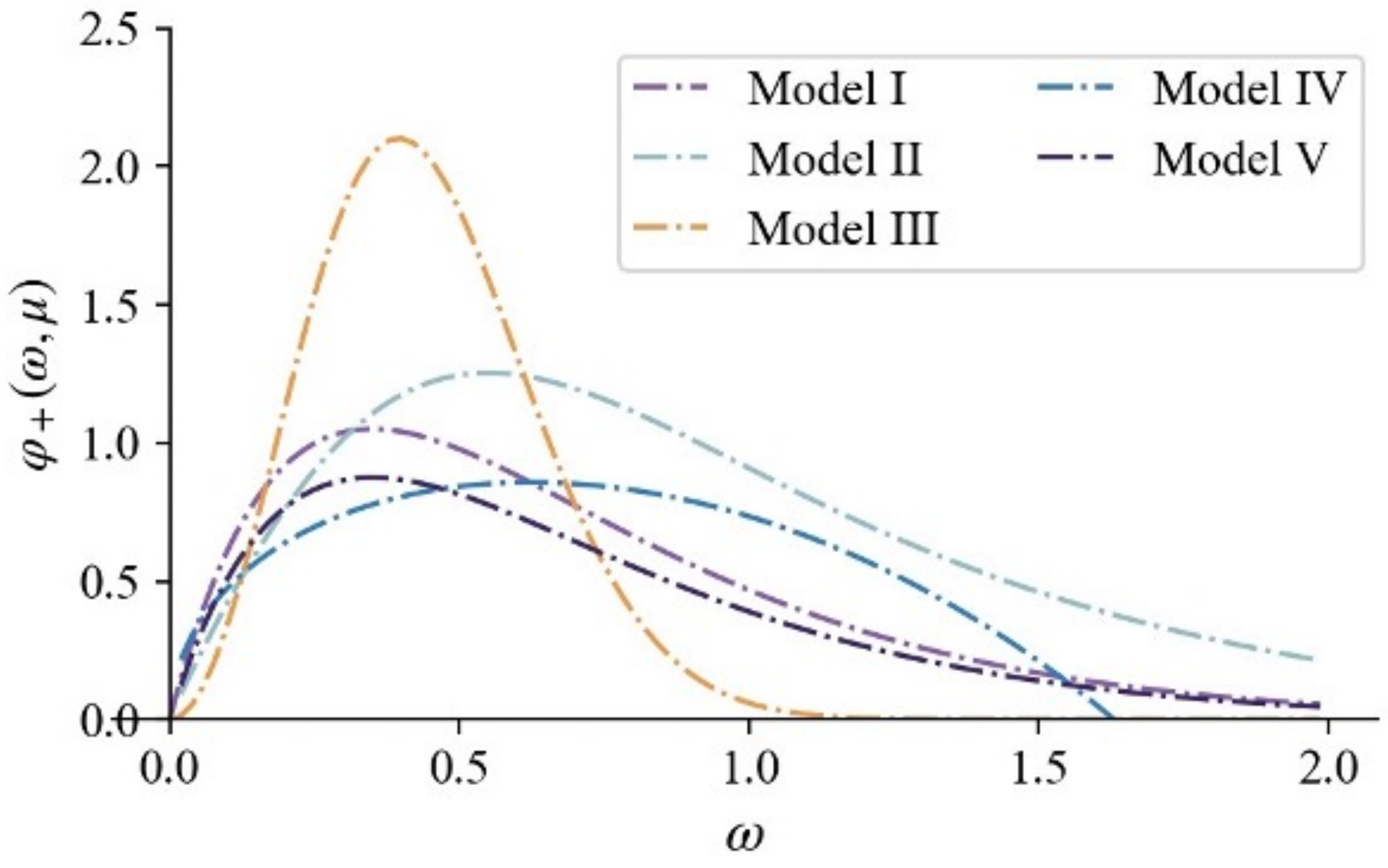


😨 Challenges in profiling the heavy mesons

- Limited understanding of the nonperturbative heavy meson LCDAs

$$\varphi^+(\omega,\mu) = \frac{1}{i\tilde{f}_{H_Q} m_{H_Q} n_+\cdot v}\int \frac{dt}{2\pi} e^{-i\omega t n_+\cdot v}\langle 0|\bar{q}(tn_+)\not{n}_+\gamma_5 W_c(tn_+,0)h_v(0)|H_Q(v)\rangle$$





$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085}\Big|_{\lambda_B}{}^{+0.019}_{-0.019}\Big|_{\sigma_1}{}^{+0.001}_{-0.062}\Big|_{\mu}{}^{+0.010}_{-0.004}\Big|_{M^2}{}^{+0.016}_{-0.017}\Big|_{s_0}{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

Model dependence

$$f^0_{B\to\pi}(0) = 0.122\times\Big[1\pm 0.07\Big|_{S_0^\pi}\pm 0.11\Big|_{\Lambda_q}\pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}{}^{+0.05}_{-0.06}\Big|_{M^2}\pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}{}^{+0.06}_{-0.10}\Big|_{\mu_h}\pm 0.04\Big|_{\mu}{}^{+1.36}_{-0.56}\Big|_{\lambda_B}{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big]$$

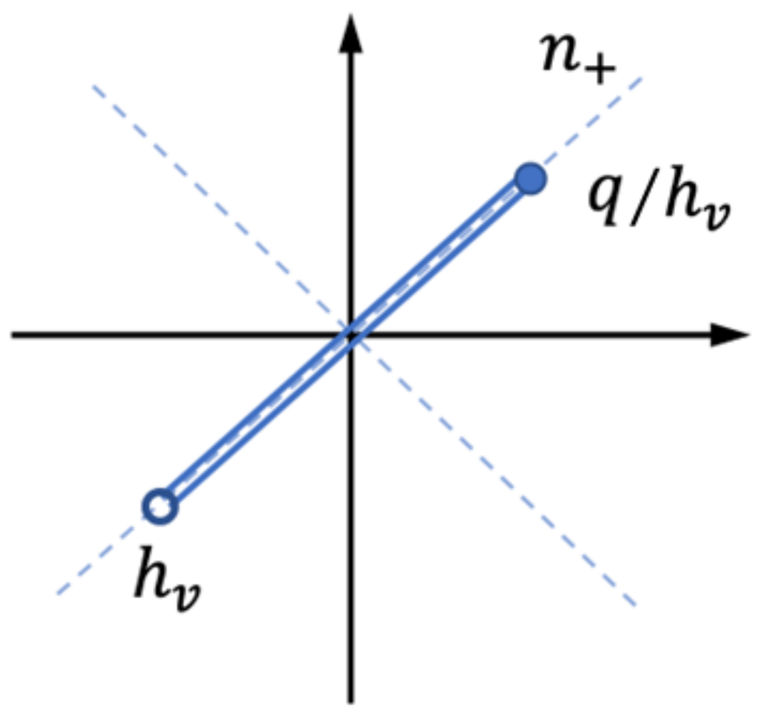
Inverse moment and log moment

Gao, Lu, Shen, Wang, Wei, 2020; Cui, Huang, Shen, Wang, 2023

Challenges in First-Principle determination

- Light-cone correlators containing HQET field

$$\langle 0 | \bar{q}\, W_c\, h_v | H_Q \rangle$$



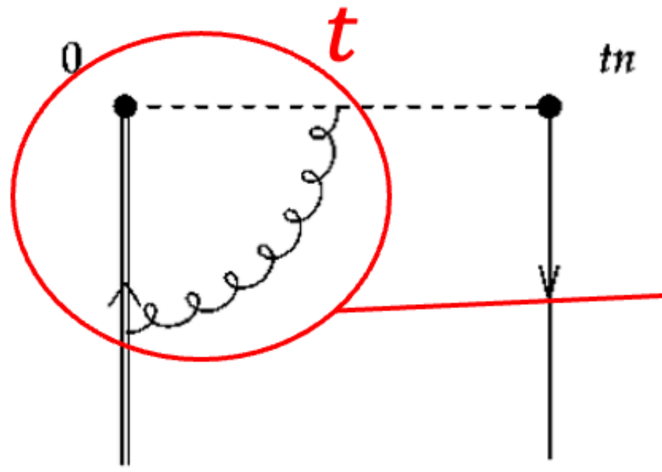
Challenge 1: light-cone correlators

- OPE: expansion into local operators matrix elements — QCD sum rules, lattice QCD
- LaMET: from equal-time correlation functions to light-cone variables — Lattice QCD

Challenge 2: cusp divergence

$$\sim \cosh\theta = \frac{n_+ \cdot v}{\sqrt{n_+^2}\sqrt{v^2}}$$

$$\langle 0|O_v^P(t)|H(p_H)\rangle = i\tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega\, e^{i\omega t n_+ \cdot v}\varphi^+(\omega;\mu),$$



Similar to Wilson line

$$O_v^{P,\mathrm{ren}}(t,\mu) = O_v^{P,\mathrm{bare}}(t) + \frac{\alpha_s C_F}{4\pi}\left\{\left(\frac{4}{\hat{\epsilon}^2} + \frac{4}{\hat{\epsilon}}\ln(it\mu)\right) O_v^{P,\mathrm{bare}}(t) - \frac{4}{\hat{\epsilon}}\int_0^1 du \frac{u}{1-u}\left[O_v^{P,\mathrm{bare}}(ut) - O_v^{P,\mathrm{bare}}(t)\right]\right\}$$

No local limit

OPE breaks down

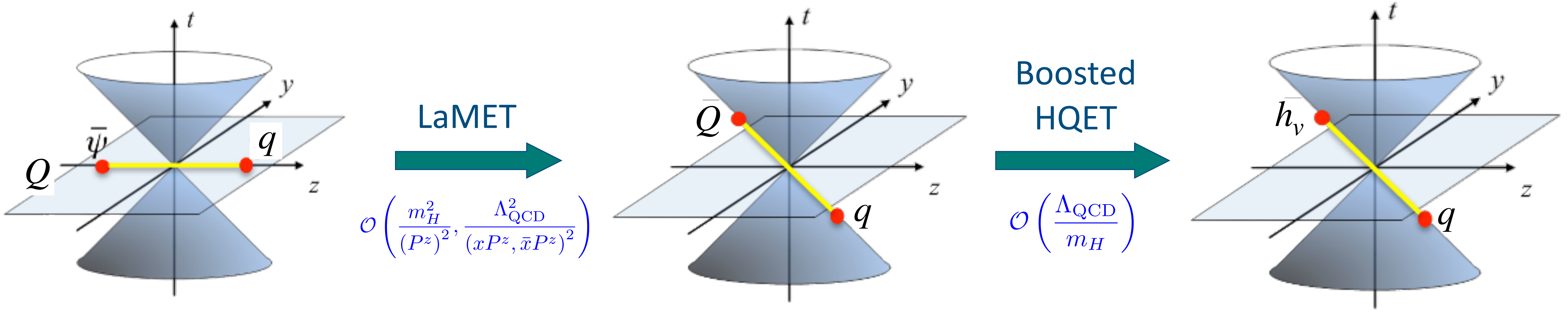
Take $z^2 \neq 0$, keep h_v; No $h_v \to$ QCD heavy quark.

Lattice boosted HQET

Theoretical Framework

Sequential effective theory

Sequential effective theory



Equal-time correlator

Light-cone correlator
QCD field

Light-cone correlator
HQET field

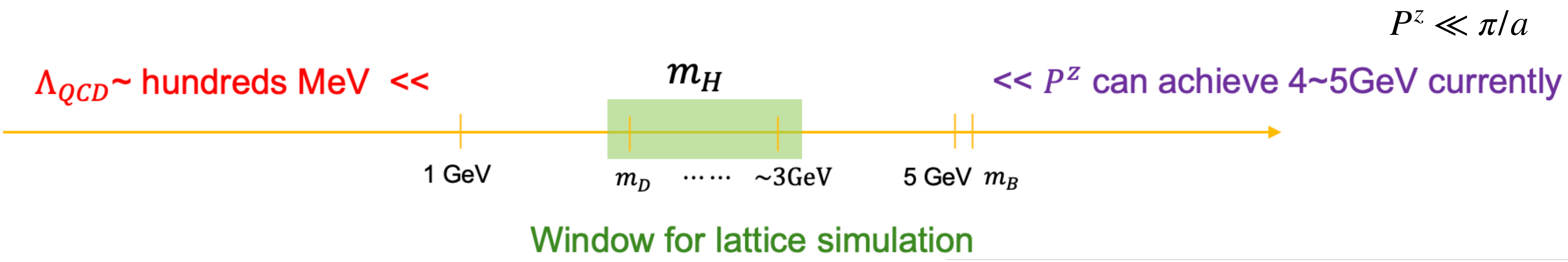
XYH, Hua, Ji, Lü, et al., 2024; **XYH**, Hua, Ji, Lü, et al., 2025

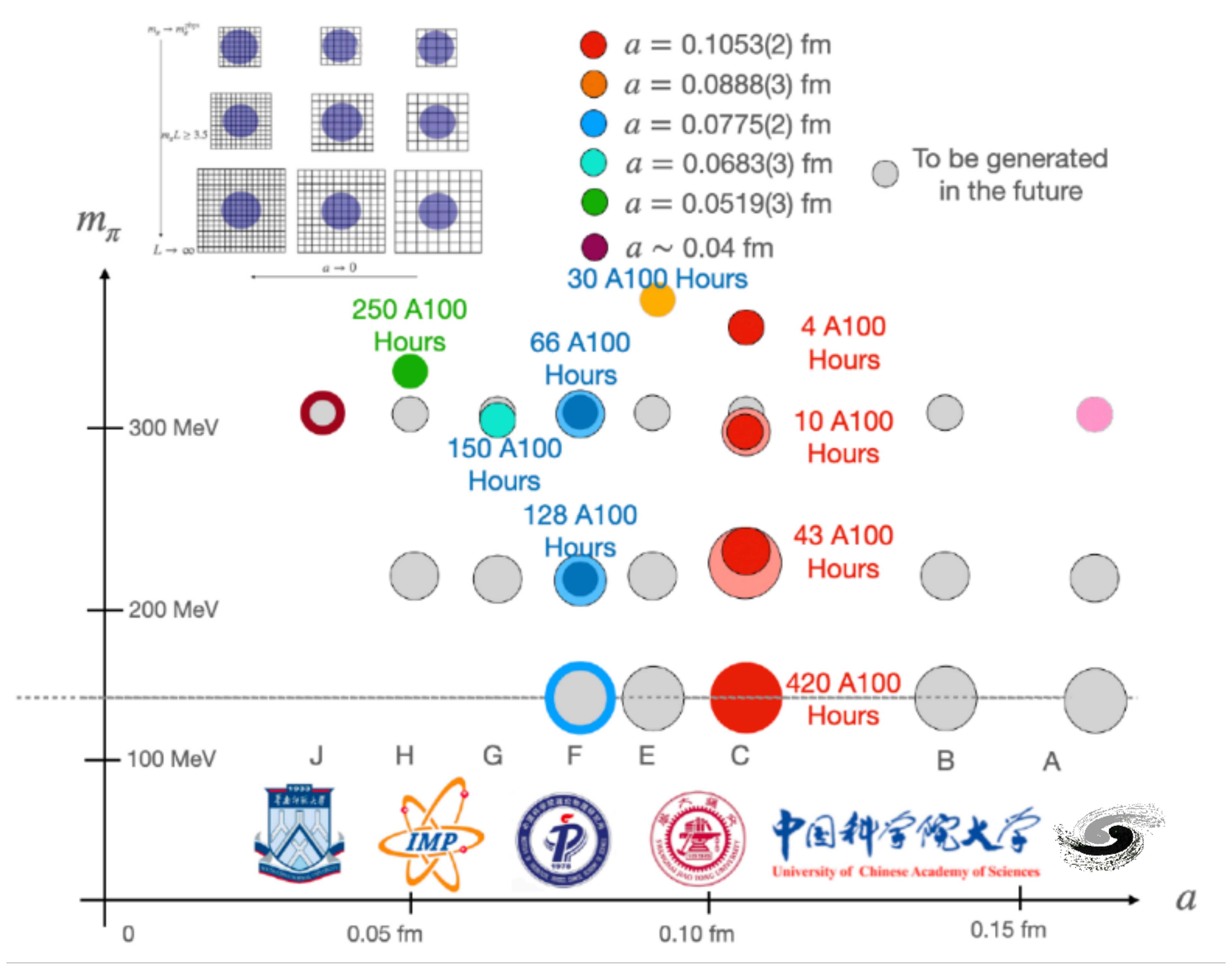
- 3 scales in the equal-time correlator
  - LaMET: $\Lambda_{\rm QCD}, m_Q \ll P^z$, integrate out P^z
  - Boosted HQET: $\Lambda_{\rm QCD} \ll m_Q$, integrate out m_Q

→ Total hierarchy

$\Lambda_{\rm QCD} \ll m_Q \ll P^z$

Sequential effective theory





- Heavy quark flavor symmetry ⇒ The HQET measurement is independent of heavy quark mass at leading power of $1/m_Q$.
- Determination by charm, application in beauty.

SET for heavy meson LCDAs

Quasi DA —LaMET→ QCD LCDA —bHQET→ HQET LCDA

- Step I: matching in LaMET

$$\tilde{\phi}(x,P^z) = \int_0^1 dy C\left(x,y,\frac{\mu}{P^z}\right)\phi(y,\mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z,\bar{x}P^z)^2}\right)$$

matching kernel @ NLO:

$$C\left(x,y,\frac{\mu}{P^z}\right) = \delta(x-y) + C_B^{(1)}\left(x,y,\frac{\mu}{P^z}\right) - C_{CT}^{(1)}(x,y) + \mathcal{O}(\alpha_s^2),$$

$$C_B^{(1)}\left(x,y,\frac{\mu}{P^z}\right) = \frac{\alpha_s C_F}{2\pi}\begin{cases}[H_1(x,y)]_+ & x<0<y \\ [H_2(x,y,P^z/\mu)]_+ & 0<x<y \\ \left[H_2\left(1-x,1-y,\frac{P^z}{\mu}\right)\right]_+ & y<x<1 \\ [H_1(1-x,1-y)]_+ & y<1<x\end{cases},$$

$$C_{CT}^{(1)} = -\frac{3\alpha_s C_F}{4\pi}\left[\frac{2\mathrm{Si}[(x-y)z_s P^z]}{\pi(x-y)}\right]_+,$$

XYH, Zhang, et al., PRD111, 034503, (2025)

- Step II: matching in bHQET

$$\varphi^+(\omega,\mu) = \begin{cases}\varphi^+_{\rm peak}(\omega,\mu), & \omega\sim\Lambda_{\rm QCD} \\ \varphi^+_{\rm tail}(\omega,\mu). & \omega\sim m_H\end{cases}$$

with:

$$\varphi^+_{\rm peak}(\omega,\mu) = \frac{1}{m_H}\frac{f_H}{\tilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H)$$

$$\varphi^+_{\rm tail}(\omega,\mu) = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right],$$

Beneke, Finauri, Vos, Wei, 2023
Ishaq, Jia, Xiong, Yang, et al., 2020
Lee, Neubert, et al., 2005

NPR of nonlocal operators

Linear divergence + non-perturbative effects

$$\left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_B = e^{\delta m|z|} Z(a) \left[\bar{\psi}(z)\,\Gamma W(z,0)\,\psi(0)\right]_R$$

Log divergence

[Ji, Liu, Schäfer, et al. 2020]

$$\langle O_\Gamma(z)\rangle = \Gamma\left(1+\gamma g^2 \log(z^2/a^2) - m_{-1}\frac{z}{a} + \ldots\ldots\right)$$

$$\ln\tilde{h}^{\pi}_{B}(z, P_z=0, 1/a) = \frac{kz}{a\ln[a\Lambda_{\rm QCD}]} + f(z)a^2 + \frac{3C_F}{b_0}\ln\left[\frac{\ln(1/a\Lambda_{\rm QCD})}{\ln(\mu/\Lambda_{\rm QCD})}\right] + \frac{1}{2}\ln\left[1+\frac{d}{\ln[a\Lambda_{\rm QCD}]}\right]^2 + \begin{cases} \ln\left[C_{0,\rm NLO}(z,\mu)\right] + m_0 z & \text{if } z_0 \le z \le z_1 \\ g(z) & \text{if } z_1 < z \end{cases}$$

[Huo, Su, Gui, et al. 2021]

	⬅ v1	⬅ v2	➡ Current work
Data	Very limited	Limited	**Sufficient**
Short distance	Ratio scheme	Ratio scheme	Ratio scheme
Large distance	Ratio scheme	RGR+LRR	**Self renormalization**

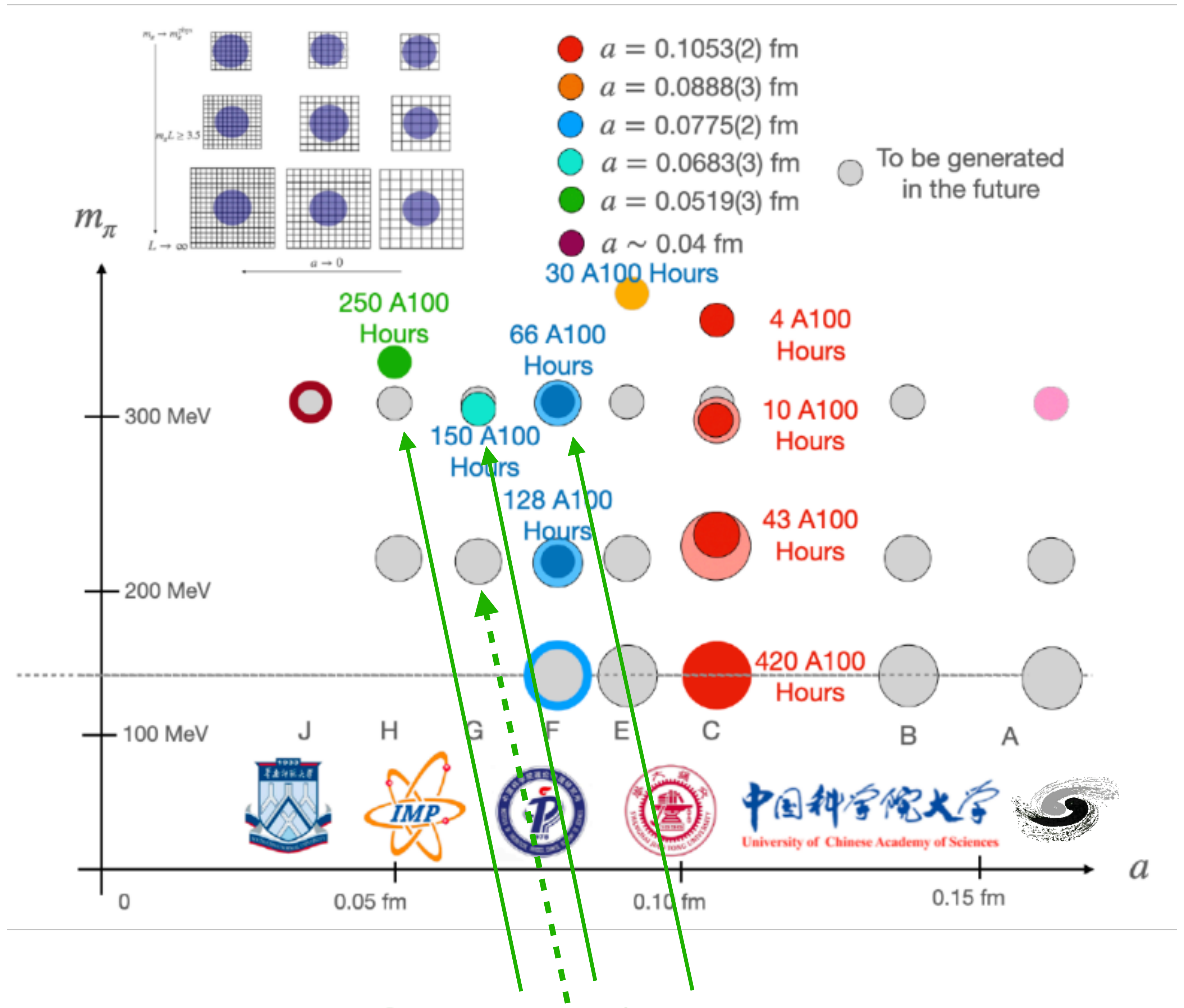
[**XYH**, Hua, Ji, Lü, et al., 2024;
XYH, Hua, Ji, Lü, et al., 2025]

Lattice Implementation

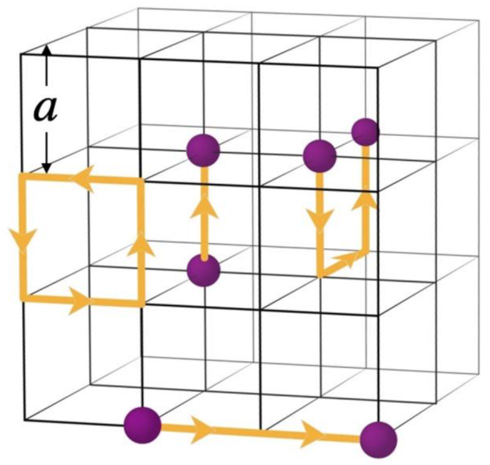
The most advanced lattice simulation

Lattice Setup

	Previous work	Towards precision calculation
Action	Clover fermion (tadpole improved)	
Improvement	—	HYP smear for Wilson line; Kinematically-enhanced interpolators
Lattice spacing	a=0.05187fm	a=0.0775fm, 0.06826fm, 0.05187fm
Pion mass	$m_\pi \simeq 300\,\mathrm{MeV}$	$m_\pi \simeq 300\,\mathrm{MeV},\ 210\,\mathrm{MeV}$ (Ongoing)
NPR	Simplified hybrid scheme	Hybrid scheme with self-renormalization
Pz extrapolation	—	Infinite momentum extrapolation
Statistics	~5k	60k~120k



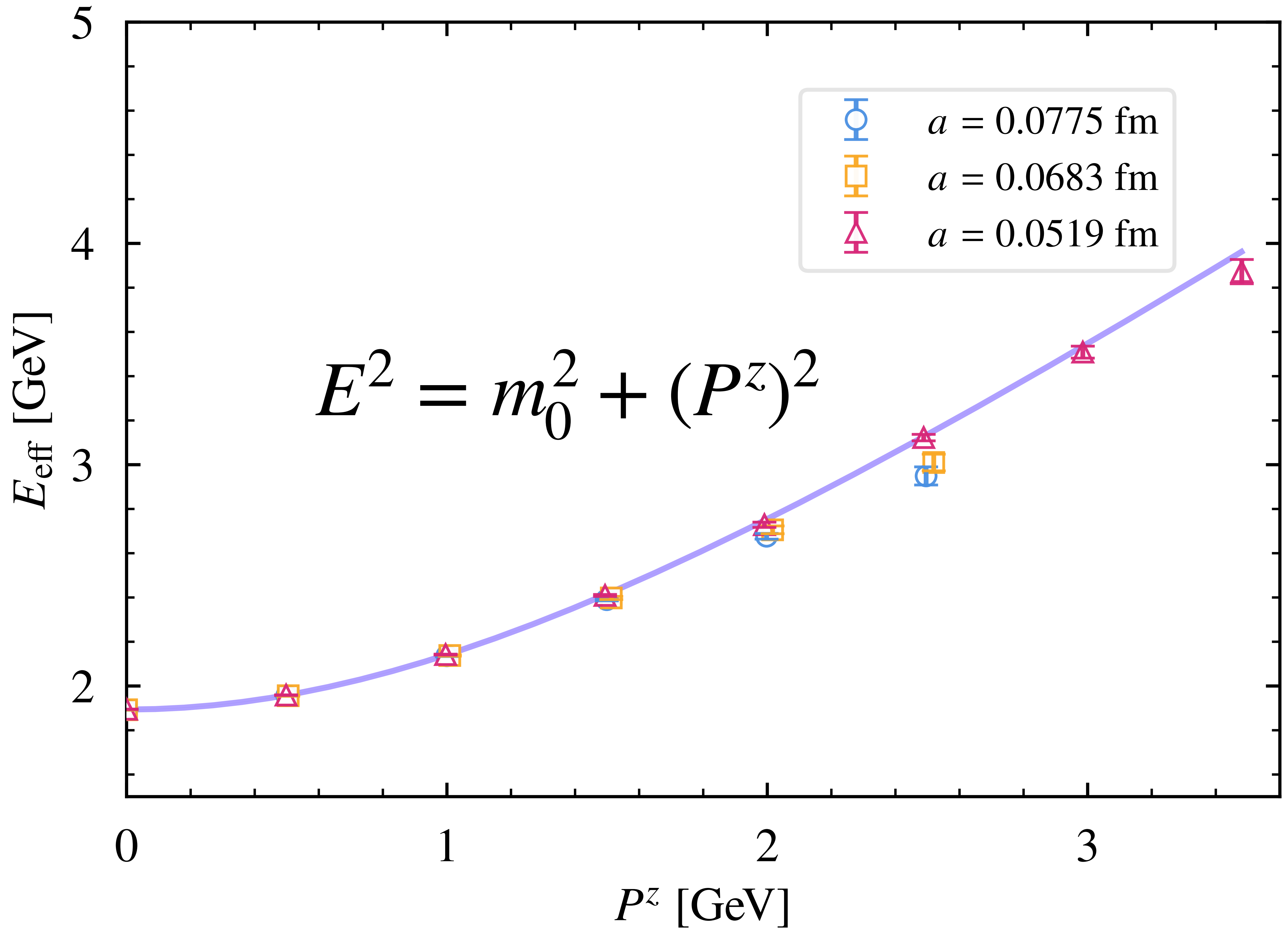
Ensembles we use

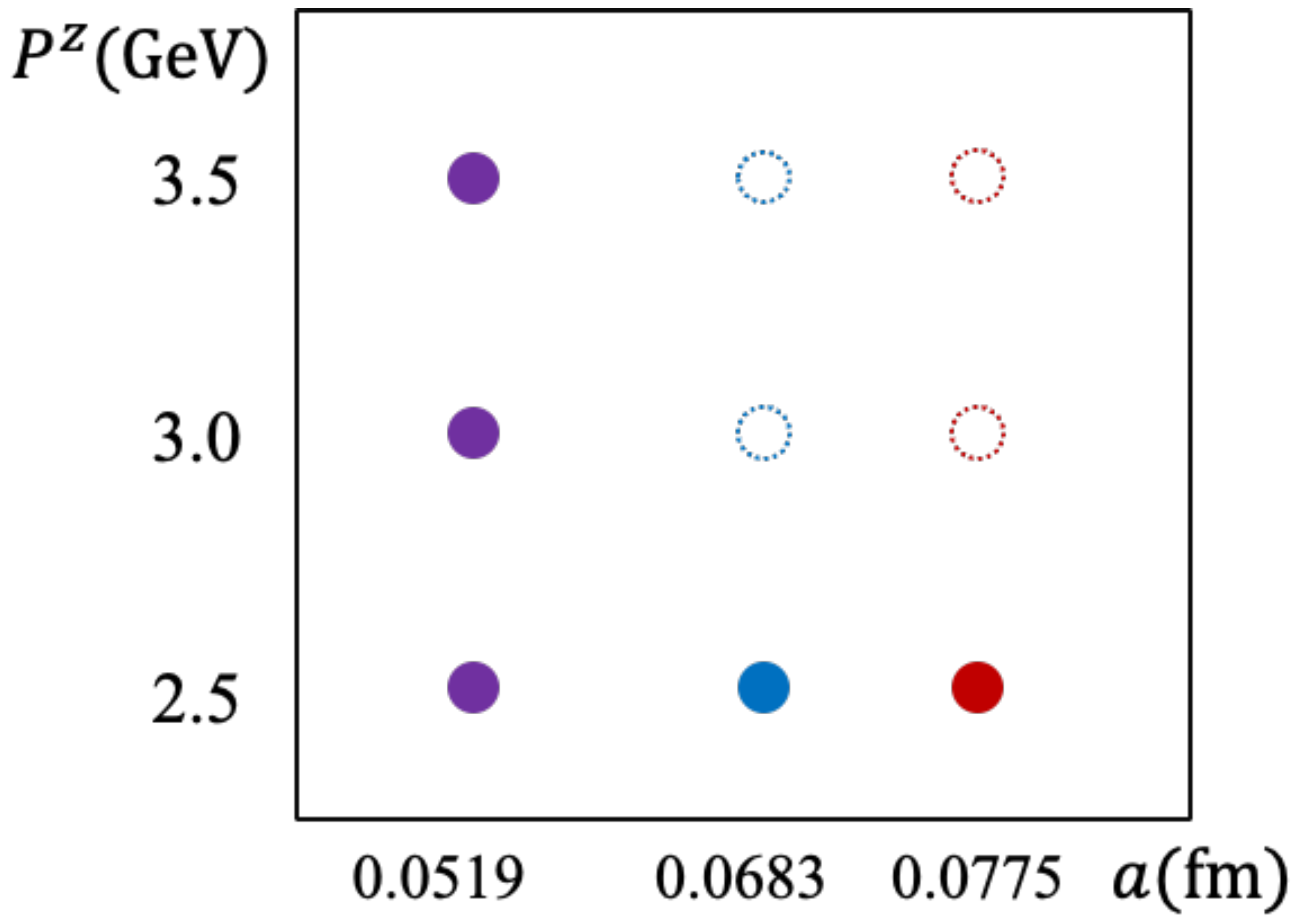


[**XYH**, Hua, Ji, Lü, et al., 2024;
XYH, Hua, Ji, Lü, et al., 2025]

Dispersion relation and setup selection

Since the dispersion relation works well for $a\,p < 1$, we keep the 5 setups (solid dots) for further analysis.



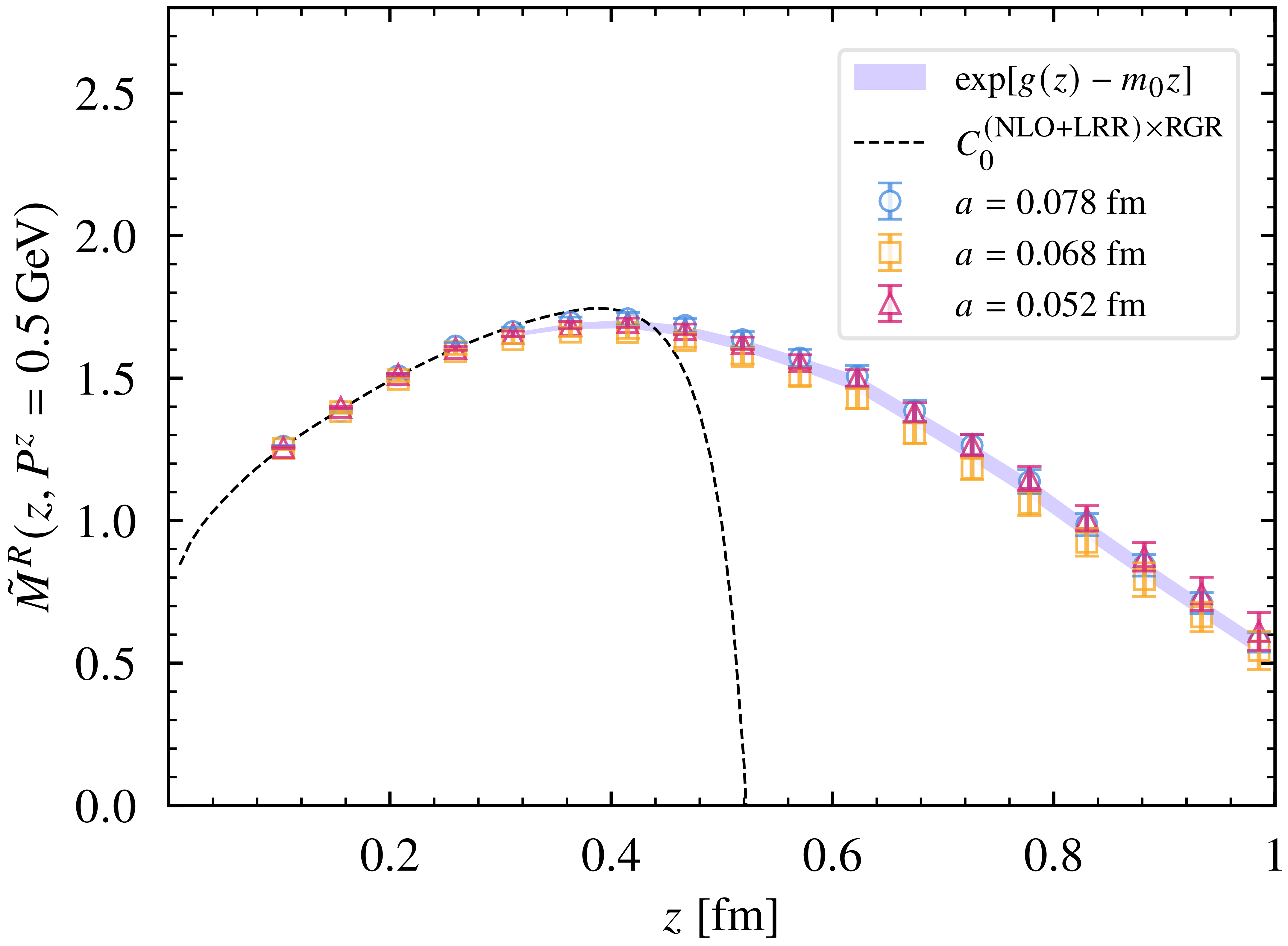


Solid dots: chosen setups

Open circles: excluded setups

Self-renormalization approach

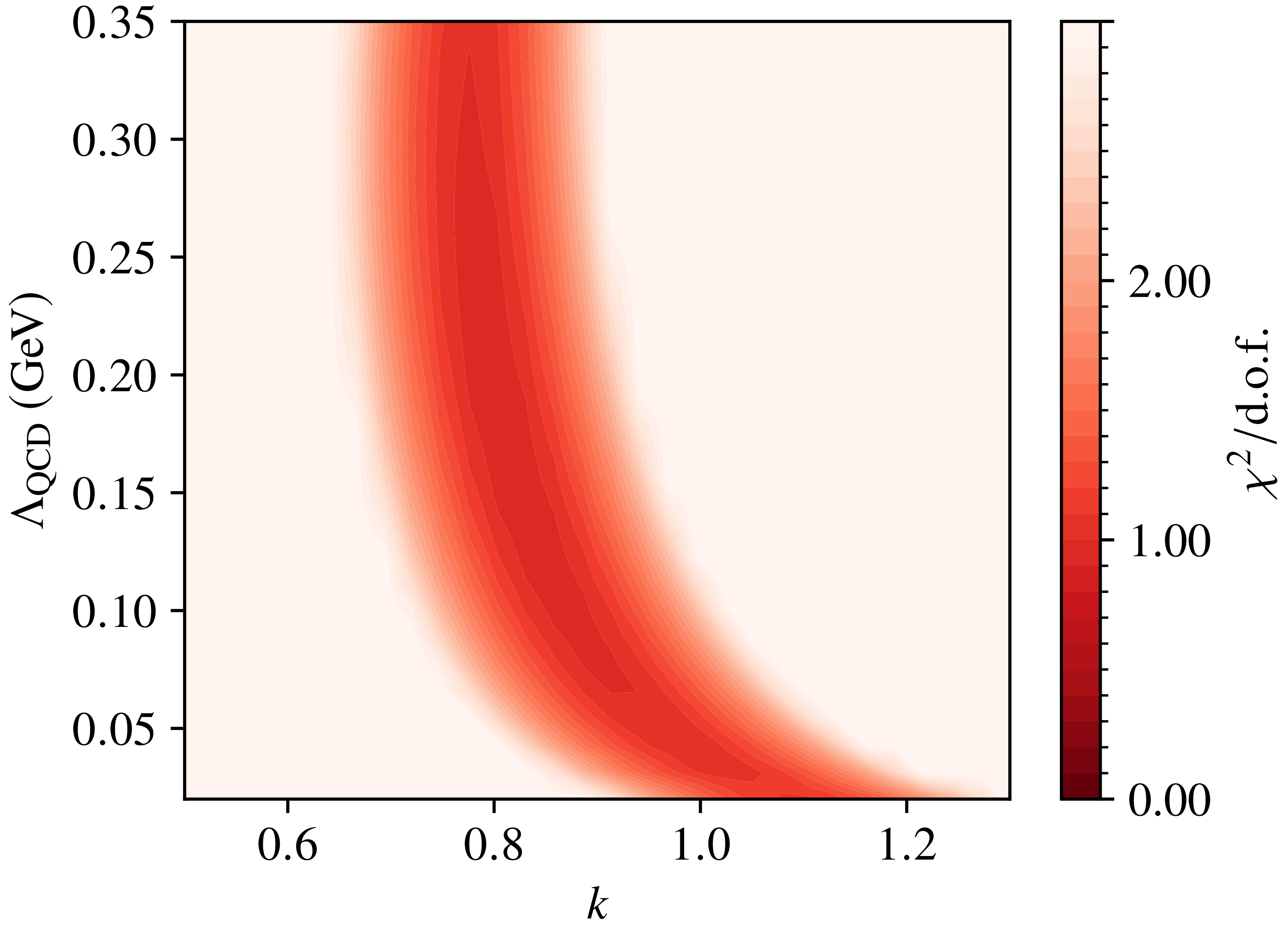
- Renormalized matrix element



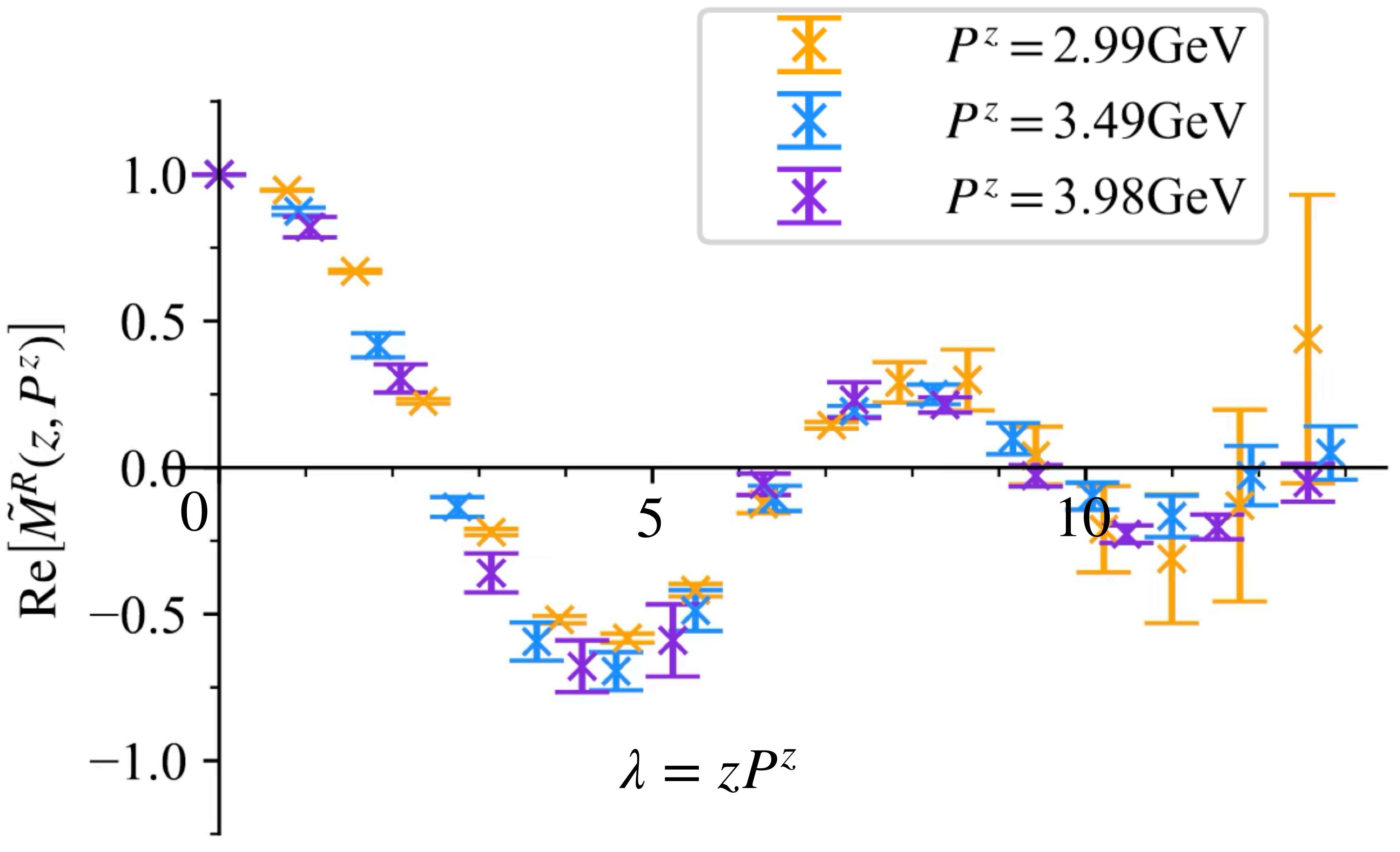
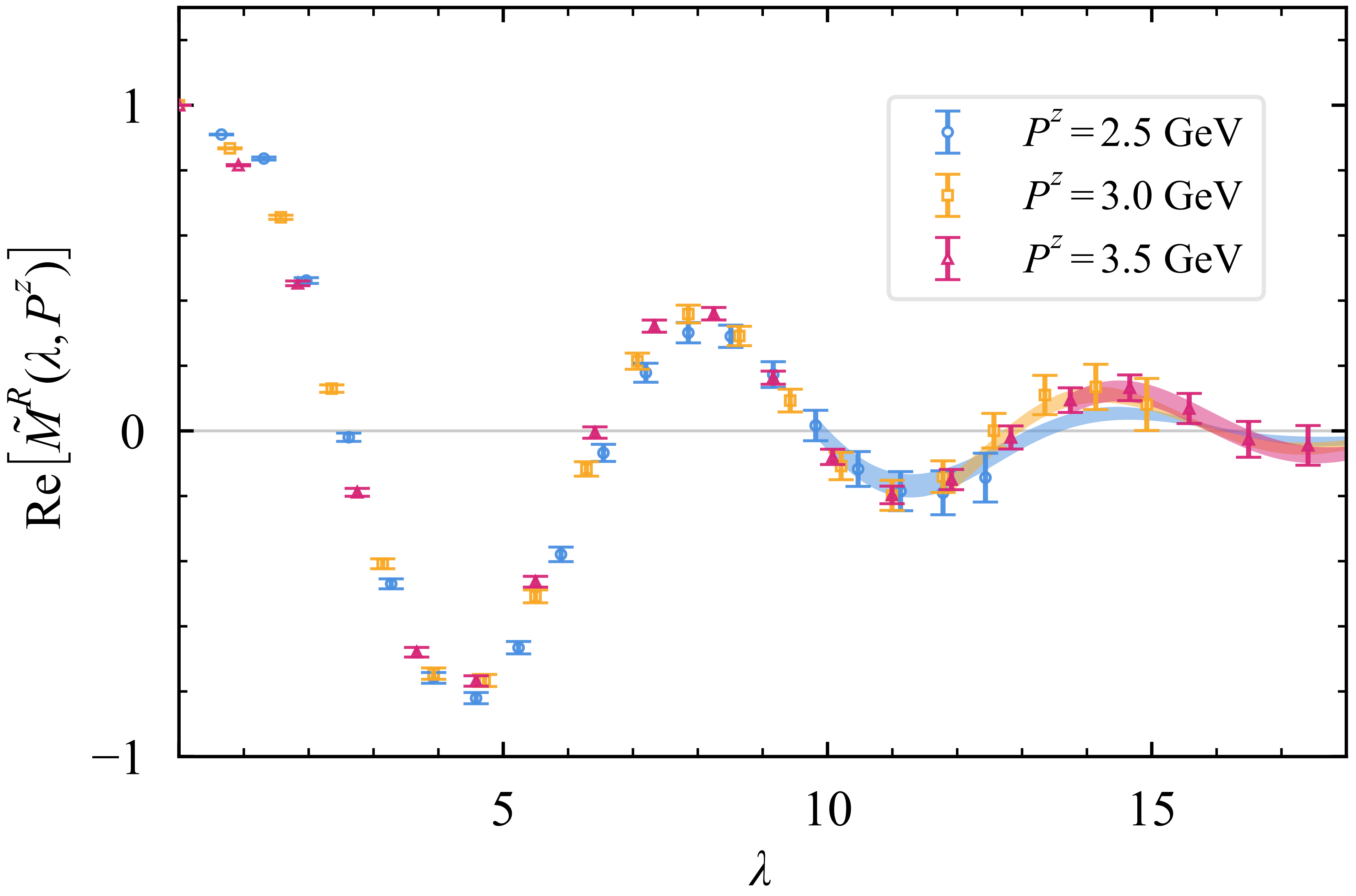
- Parameters in renormalization factor

Params.	Value
k	0.819(40)
Λ_{QCD}(GeV)	0.176(72)
m_0(GeV)	0.231(94)
d	-0.18(10)

- Goodness-of-fit scan in $(k, \Lambda_{\text{QCD}})$ plane



Renormalized matrix elements

Previous work	Towards precision calculation
	

- Obtain a more precise results at large λ
- Multi lattice spacings to perform the continuum extrapolation

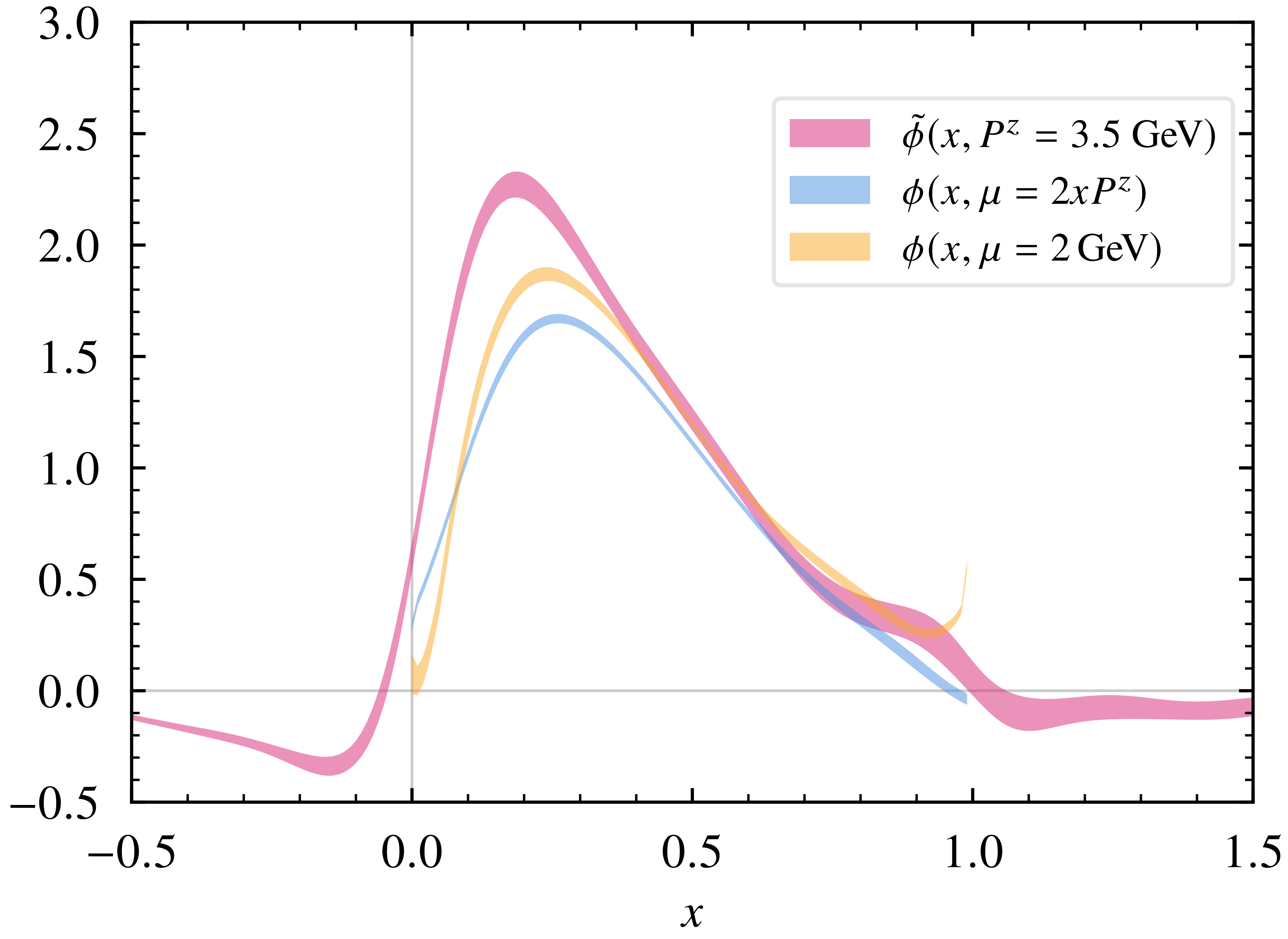
LaMET matching

- LaMET inverse matching

$$\tilde{\phi}(x, P^z) = \int_0^1 C\left(x, y, \frac{\mu}{P^z}\right) \phi(y, \mu) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(xP^z, \bar{x}P^z)^2}\right)$$

$$\phi(y) = \tilde{\phi}(y) - \int dx\, C^{(1)}(y, x)\, \tilde{\phi}(x) + \mathcal{O}(\alpha_s^2)$$

[Liu, Wang, Xu, Zhang, Zhao, 2019]



Quasi DA & QCD LCDA

Fix order: $\mu = 2yP^z$

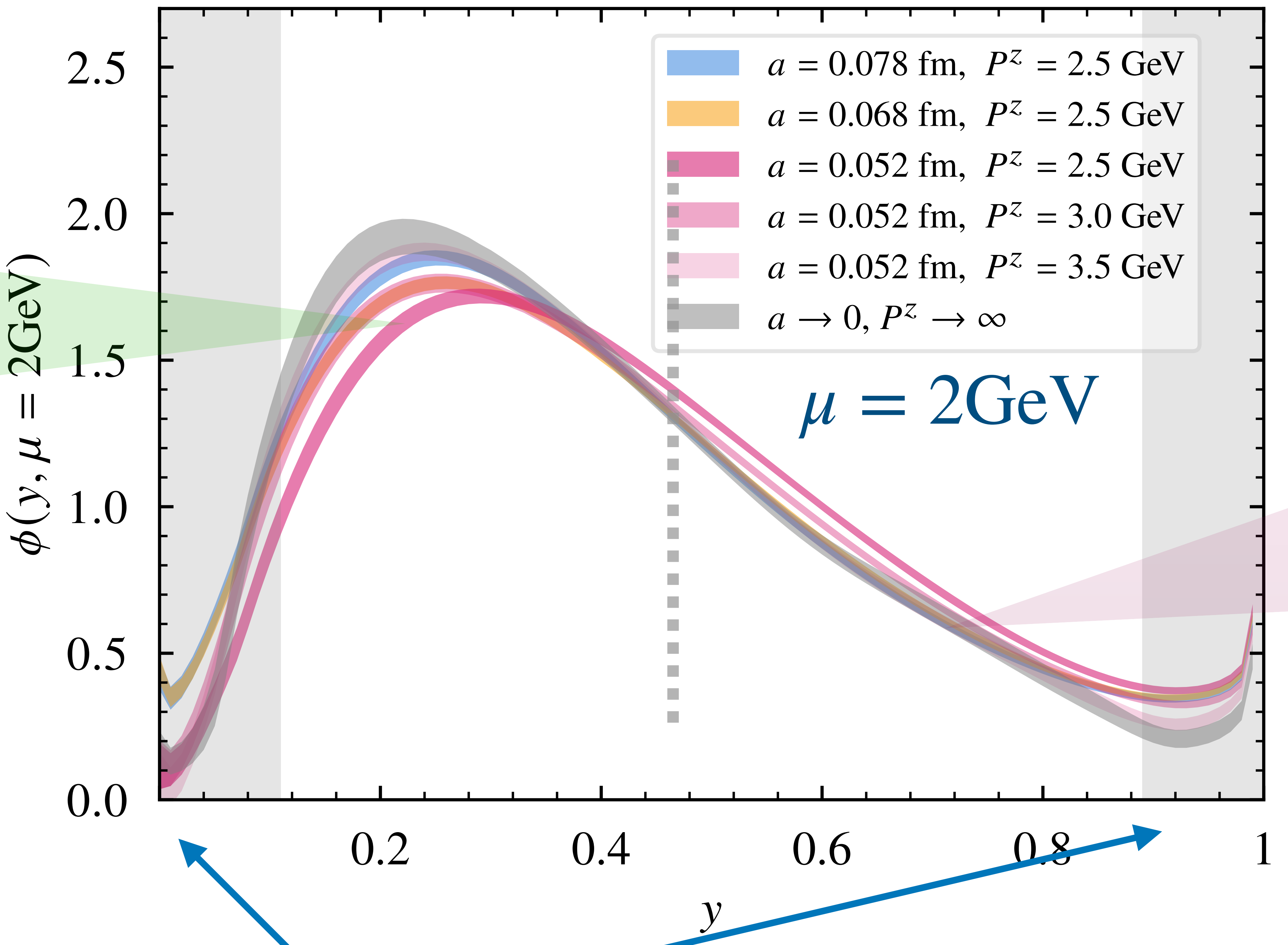
RGR: $\phi(y,\ 2yP^z) \xrightarrow{\text{ERBL}} \phi(y,\ 2\text{GeV})$

QCD LCDA of D meson

Peak region: $y \sim \frac{\Lambda_{\text{QCD}}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA

Beneke, Finauri, Vos, Wei, 2023
Ishaq, Jia, Xiong, Yang, et al., 2020



Tail region: $y \sim 1$

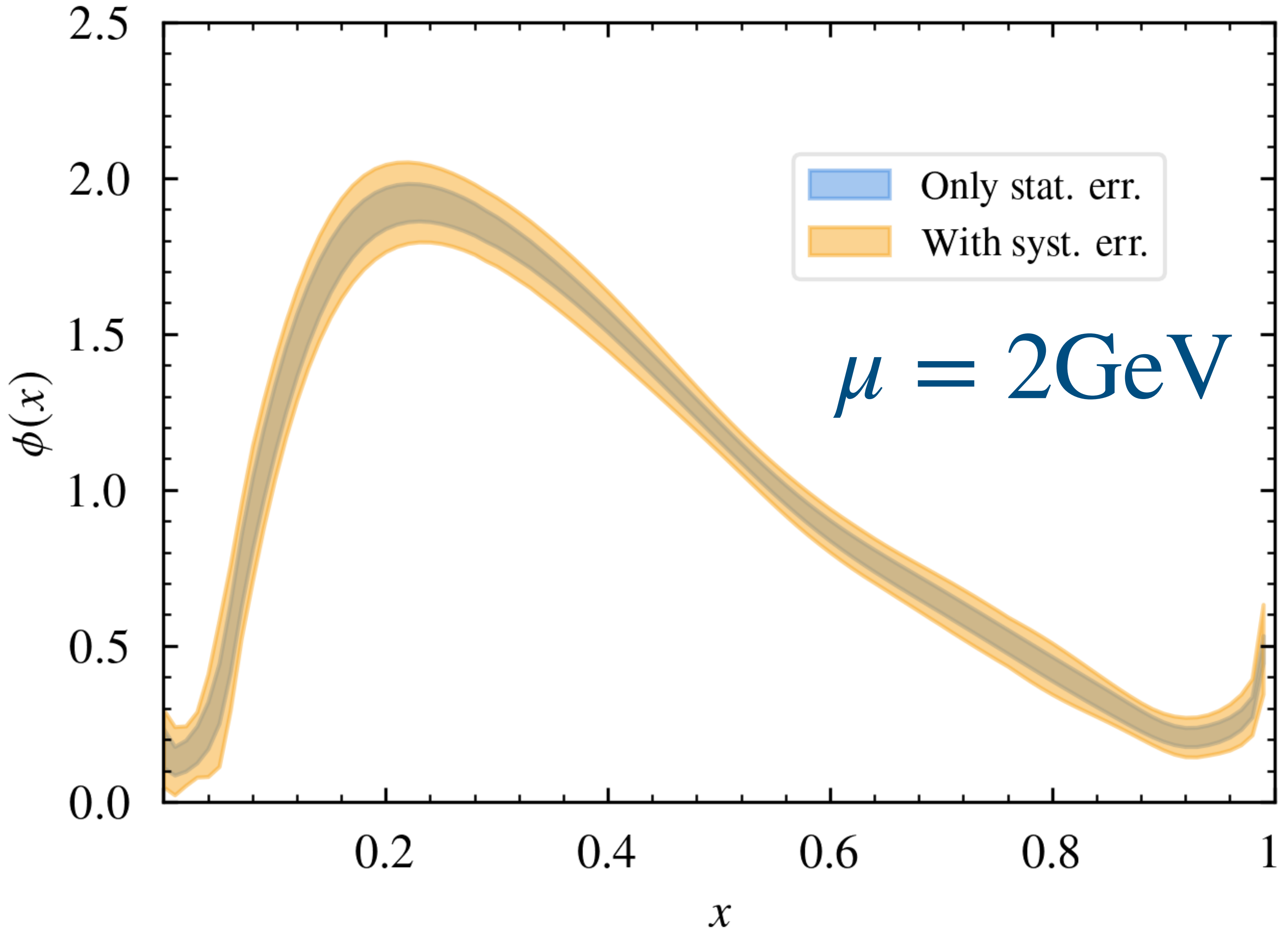
- Contain only hard-collinear physics;
- Suppressed in LCDA.

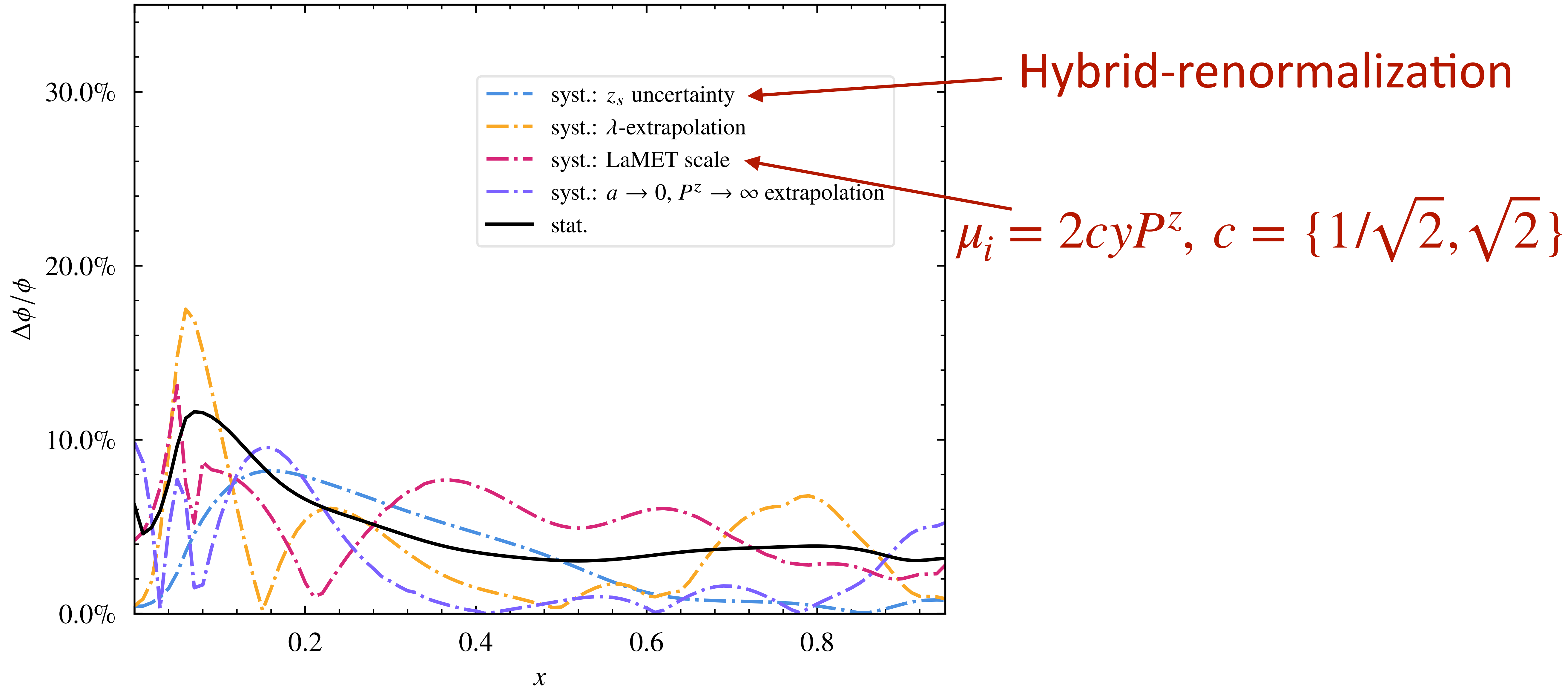
End-point region

- LaMET matching kernel suffer large power corrections
- Lattice predictions fail

QCD LCDA of D meson

- Analysis of systematic errors





- Fit Gegenbauer moments directly

$$\phi(x) = 6x(1-x)\left[1 + \sum_n^\infty a_n C_n^{3/2}(2x-1)\right]$$

n	a_1	a_2	a_3	a_4	a_5	a_6
2	-0.397(18)	0.118(10)				
4	-0.412(20)	0.134(16)	-0.016(11)	0.005(8)		
6	-0.391(22)	0.111(18)	0.026(20)	-0.019(15)	0.024(9)	-0.004(7)
8	-0.376(24)	0.109(19)	0.041(22)	-0.034(18)	0.041(11)	-0.012(9)

HQET LCDA

- $\omega \sim \Lambda_{\rm QCD}$

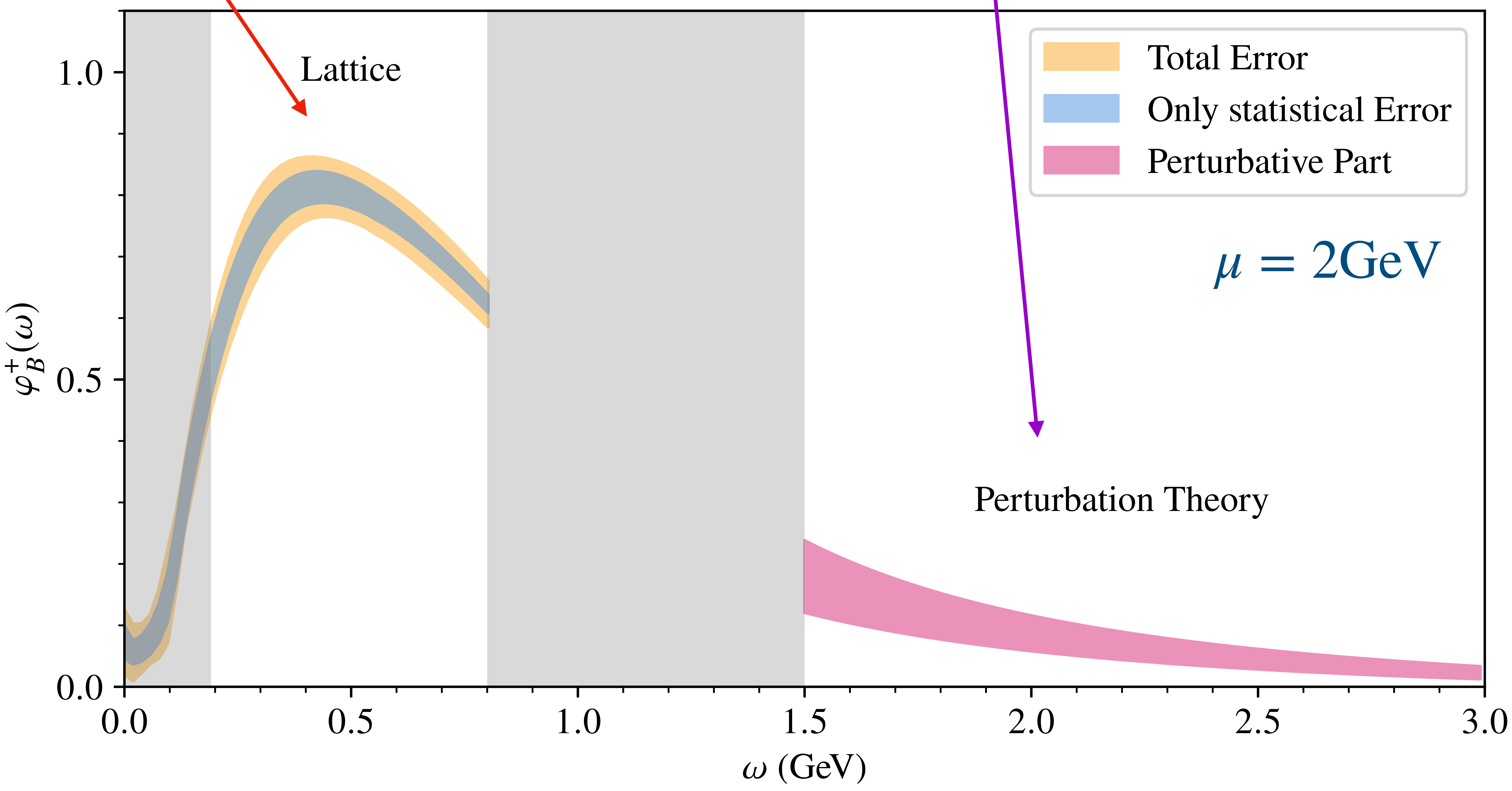
$$\varphi^+_{\rm peak}(\omega,\mu) = \frac{1}{m_H}\frac{f_H}{\tilde{f}_H}\frac{1}{\mathcal{J}_{\rm peak}}\phi(y,\mu;m_H)$$

- $\omega \gg \Lambda_{\rm QCD}$

$$\varphi^+_{\rm tail} = \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2} - \ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}}{3\omega}\left(2 - \ln\frac{\omega}{\mu}\right)\right]$$

[Lee, Neubert, 2005]

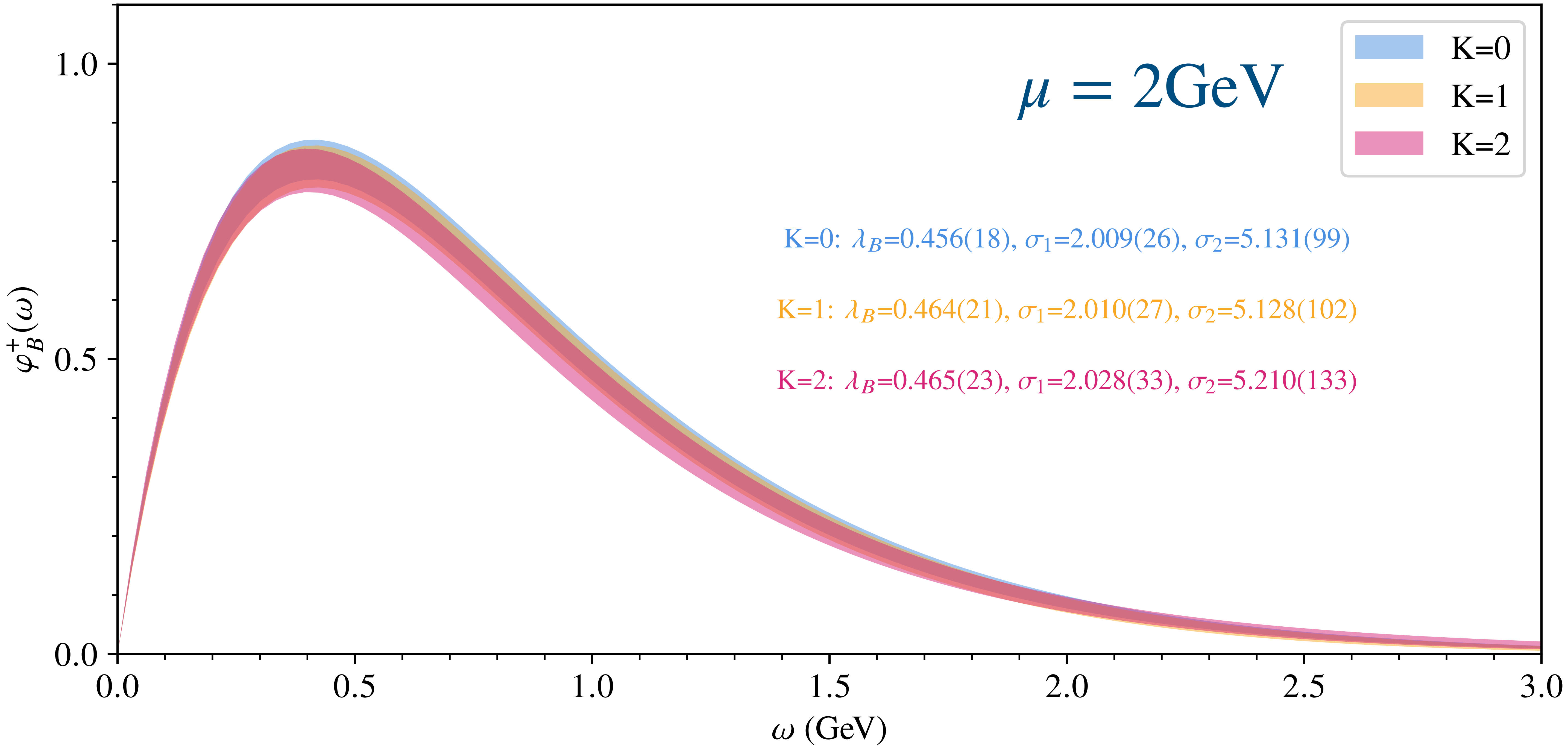
$\mu = 2\text{GeV}$



Latest model-based fitting

- Expansion in generalized Laguerre polynomials [Feldmann, Lüghausen, Dyk et al. 2022]

$$\varphi^+(\omega,\mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^{K} \frac{a_k(\mu)}{1+k} L_k^{(1)}(2\omega/\omega_0)$$



Inverse moment

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega}\varphi^+(\omega,\mu)$$

Log moment

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu)\int_0^\infty \frac{d\omega}{\omega} \ln\left(\frac{\mu}{\omega}\right)^n \varphi^+(\omega,\mu)$$

$$\mu = 2\text{GeV},\ \lambda_B = 0.464(21),\ \sigma_1 = 2.010(27),\ \sigma_2 = 5.128(102),\ \sigma_3 = 15.118(360)$$

1-loop running $\mu = 1\text{GeV},\ \lambda_B = 0.416(19),\ \sigma_1 = 1.426(26)$

Inverse moments

- λ_B and $\sigma_B^{(1)}$ at $\mu = 1\mathrm{GeV}$

		λ_B	$\sigma_B^{(1)}$
Our results	*K=0*	0.408(16)	1.425(26)
	K=1	0.416(19)	1.426(26)
	K=2	0.416(20)	1.445(33)
Experiment	Belle 2018	>0.24	-
Other theoretical approach	Khodjamirian, Mandal, Mannel (2020)	0.383(153)	-
	Gao, Lu, Shen, Wang, Wei (2020)	$0.343^{+0.064}_{-0.079}$	-
	Lee, Neubert (2005)	0.48(11)	1.6(2)
	Braun, Ivanov, Korchemsky (2004)	0.46(11)	1.4(4)
	Grozin, Neubert (1997)	0.35(15)	-
	Mandal, Nandi, Ray (2024)	0.338(68)	-

🚀 Summary & Outlook

🌱 **Sequential Effective Theory (SET)** bridges lattice correlators and heavy-meson distribution amplitudes (and parton distributions).

🌱 **Factorization formula for LCDAs** within SET is established → systematic and high-precision lattice QCD studies now feasible.

🌱 **SET for shape-function framework** was proposed recently → will be validated in future lattice QCD calculations. [Wang, Xu, Zhang, Zhao, 2025]

Thank you for your attention!