Generalized Parton Distributions from Lattice QCD with Asymmetric Momentum Transfer: Unpolarized Quarks at Nonzero Skewness

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GPDs at Non-Zero Skewness

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Generalized parton distributions from lattice QCD with asymmetric momentum transfer: Unpolarized quarks

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GPDs – Skewness

- Internal structure of hadrons $\rightarrow$ simplest way: PDFs
- A generalization of PDFs: GPDs $\rightarrow$ more information than PDFs
- GPDs give access to:
  - Three-dimensional image of hadrons
  - Angular momentum of partons
  - Pressure and shear forces inside hadrons

GPDs – Skewness

- GPDs $\rightarrow$ off-foward hadron matrix element $\rightarrow P_i^\mu \neq P_f^\mu$:
    - $x \rightarrow$ fraction of the hadron's momentum carried by a parton
    - $t \equiv \Delta^2 \rightarrow$ square of the momentum transfer 4-vector Δ^μ
    - $\xi \rightarrow$ skewness, quantifies the fraction of momentum transferred in the longitudinal direction

GPDs – Amplitudes

- Vector matrix element for unpolarized spin 1/2 hadron:

$$F^\mu(z,P,\Delta) = \langle P_f,\lambda'|\bar\psi(-\tfrac{z}{2})\gamma^\mu\,\mathcal{W}(-\tfrac{z}{2},\tfrac{z}{2})\psi(\tfrac{z}{2})|P_i,\lambda\rangle$$

- Matrix element decomposition in $A_i \equiv A_i(z\cdot P, z\cdot\Delta, \Delta^2, z^2)$

$$\begin{aligned} F^\mu(z,P,\Delta) = \bar u(p_f,\lambda')\Big[&\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 \\ &+ \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\Big]u(p_i,\lambda) \end{aligned}$$

$$\sigma^{\mu z} \equiv \sigma^{\mu\rho}z_\rho, \quad \sigma^{\mu\Delta} \equiv \sigma^{\mu\rho}\Delta_\rho, \quad \sigma^{z\Delta} \equiv \sigma^{\rho\tau}z_\rho\Delta_\tau, \quad \sigma^{\alpha\beta} \equiv \tfrac{i}{2}[\gamma^\alpha,\gamma^\beta]$$

GPDs – Amplitudes

- $\xi = -\Delta_3/2P_3$
- Lorentz-invariant definition for twist-2 coordinate-space quasi-GPDs:

$$H_{\mathrm{LI}} = A_1 - 2\xi A_3$$

$$E_{\mathrm{LI}} = -A_1 + 2\xi A_3 + 2A_5 + 2P_3 z A_6 - 4\xi P_3 z A_8$$

GPDs – Amplitudes

- Only longitudinal momentum transfer: $\Delta = (\Delta_0, 0, 0, \Delta_3)$

GPDs – Longitudinal Momentum Transfer Case

- Only longitudinal momentum transfer: $\Delta = (\Delta_0, 0, 0, \Delta_3)$
- New decomposition F^{μ}:

$$F^{\mu,L}(z, P, \Delta_L) = \bar{u}(P_f, \lambda')\left[\frac{P^{\mu}}{m}A_1^L + mz^{\mu}A_2^L + im\sigma^{\mu z}A_4^L\right]u(P_i, \lambda)$$

- A_1^L, A_2^L, A_4^L are linear combinations of A_i, $i = 1, ..., 8$

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- New relation between GPDs and amplitudes

$$H_{\rm LI}^L \equiv \left(H + \frac{1}{2}\alpha\xi(H+E)\right)_{\rm LI} = A_1^L, \qquad \alpha = \frac{-(z\cdot\Delta)(z\cdot P)}{z^2P^2 - (z\cdot P)^2}$$

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- Small $\xi \implies$ small $\alpha \rightarrow H$ GPD approximately given A_1^L

GPDs – Amplitudes and Matrix Elements

- Amplitudes are found from:

$$\text{Matrix element} \rightarrow \Pi_\mu(\Gamma_\nu) = K \,\mathrm{Tr}\left[\Gamma_\nu \left(\frac{-i\not{P}_f + m}{2m}\right) \tilde{F}^\mu \left(\frac{-i\not{P}_i + m}{2m}\right)\right]$$

$$\Gamma_0 = \frac{1}{4}(1+\gamma_0)\,, \quad \Gamma_k = \frac{1}{4}(1+\gamma_0)\, i\gamma_5\gamma_k\,, \quad k = 1,2,3$$

$$K = \frac{2m^2}{\sqrt{E_f E_i (E_f + m)(E_i + m)}}$$

$$\tilde{F}^\mu(z,P,\Delta) = \Big[\frac{P^\mu}{m}A_1 + mz^\mu A_2 + \frac{\Delta^\mu}{m}A_3 + im\sigma^{\mu z}A_4 + \frac{i\sigma^{\mu\Delta}}{m}A_5 + \frac{P^\mu i\sigma^{z\Delta}}{m}A_6 + mz^\mu i\sigma^{z\Delta}A_7 + \frac{\Delta^\mu i\sigma^{z\Delta}}{m}A_8\Big]$$

$$\tilde{F}^{\mu,L}(z,P,\Delta_L) = \left[\frac{P^\mu}{m}A_1^L + mz^\mu A_2^L + im\sigma^{\mu z}A_4^L\right]$$

GPDs – Lattice QCD

- Compute nucleon $\Pi_\mu(\Gamma_\nu)$ from Lattice QCD
  - Each $\Pi_\mu(\Gamma_\nu)$ is computed for kinematical setup P_f and P_i

GPDs – Asymmetric frame

- Asymetric frame:

$$\vec{P}_f = \left(0, 0, P_f^3\right)$$

$$\vec{P}_i = \vec{P}_f - \vec{\Delta} = \left(-\Delta^1, -\Delta^2, P_f^3 - \Delta^3\right)$$

GPDs – Kinematical Setup

- Lattice kinematic setup

ξ	t (GeV^2)	$(\Delta_1, \Delta_2, \Delta_3)$	N_{meas}	P_{i3}	P_{f3}
$-1/2$	-0.445	$(0, 0, \pm 2)$	30720	± 1	± 3
	-0.686	$(1, 0, \pm 2)$	30720		
	-0.914	$(1, 1, \pm 2)$	61440		
	-1.337	$(2, 0, \pm 2)$	30720		
	-1.534	$(1, 2, \pm 2)$	122880		
	-2.253	$(3, 0, \pm 2)$	30720		
$-1/5$	-0.089	$(0, 0, \pm 1)$	30720	± 2	± 3
	-0.295	$(1, 0, \pm 1)$	30720		
	-0.493	$(1, 1, \pm 1)$	61440		
	-0.865	$(2, 0, \pm 1)$	30720		
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$1/7$	-0.061	$(0, 0, \mp 1)$	30720	± 4	± 3
	-0.203	$(1, 0, \mp 1)$	30720		
	-0.343	$(1, 1, \mp 1)$	61440		
	-0.613	$(2, 0, \mp 1)$	30720		
	-0.745	$(1, 2, \mp 1)$	122880		
	-1.248	$(3, 0, \mp 1)$	30720		
$1/5$	-0.089	$(0, 0, \mp 1)$	6080	± 3	± 2
	-0.366	$(1, 1, \mp 1)$	12160		
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- Same t only when $\Delta_T = 0$: $\xi = \pm\frac{1}{5}$, $t = -0.089$ GeV^2

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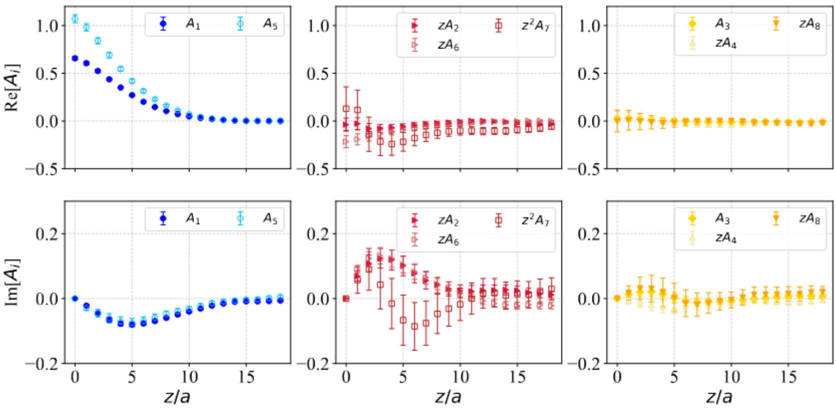
- Same t only when $\Delta_T = 0$: $\xi = \pm\frac{1}{5}$, $t = -0.089$ GeV^2
- Problem in studying ξ dependence of results $\rightarrow$ solution: use similar values of t

Numerical results – Bare Amplitudes

- Bare amplitudes; $t = -0.493$ GeV^2; after $-z$,$+z$ average:



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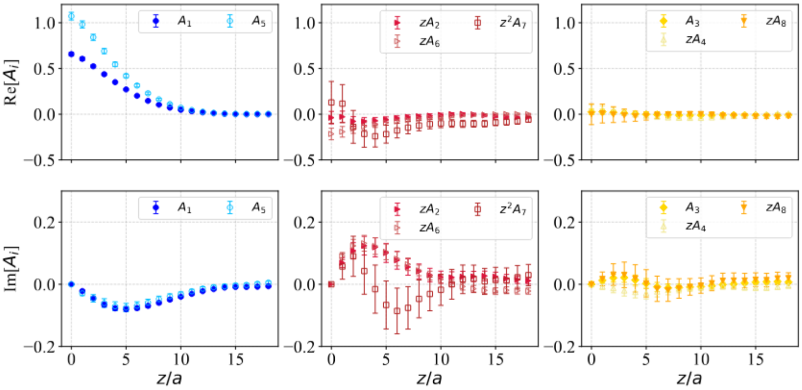
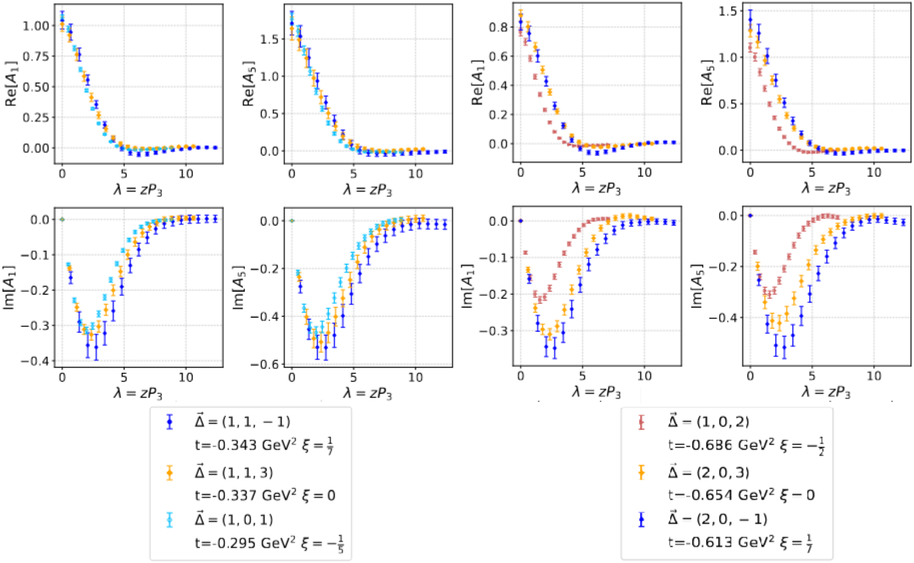


- A_1, A_5 $\rightarrow$ main contributions
- zA_2, zA_6, z^2A_7 $\rightarrow$ suppressed
- A_3, zA_4, zA_8:
  - $\xi = 0 \implies A_3 = zA_4 = zA_8 = 0$
  - No such condition at $\xi \neq 0$ $\rightarrow$ consistent with zero for this kinematic setup

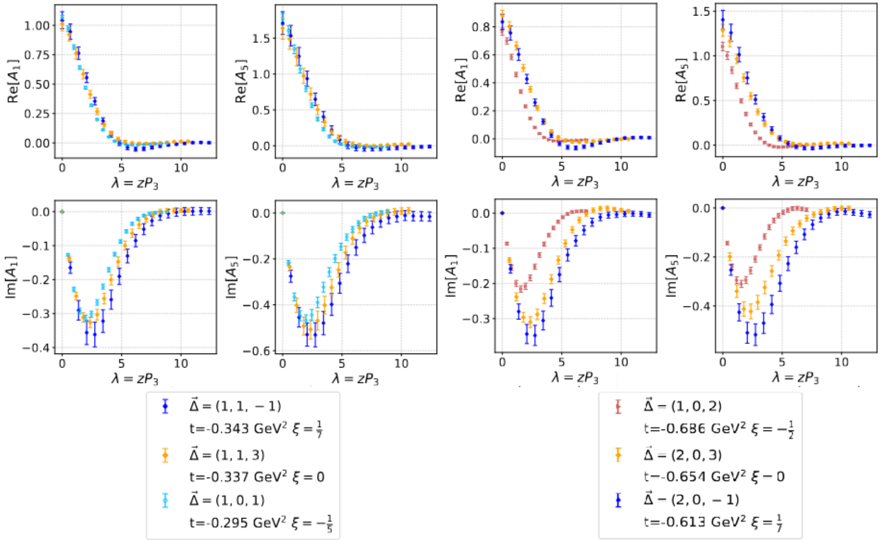
Numerical results – Bare Amplitudes

- Amplitudes and skewness (values of t are as similar as possible)



Numerical results – Bare Amplitudes

- Faster decay: $|\xi| = 1/5$ (largest $|\xi|$), $t = -0.295$ GeV^2 (largest t)
- Faster decay: $|\xi| = 1/2$ (largest $|\xi|$), $t = -0.686$ GeV^2 (lowest t)



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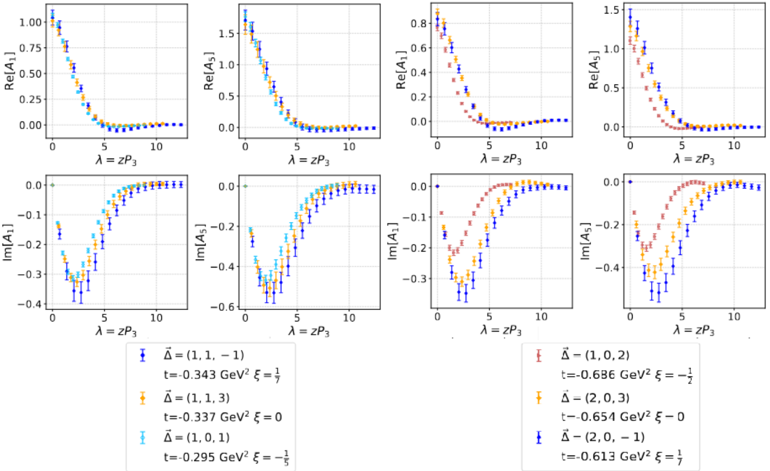
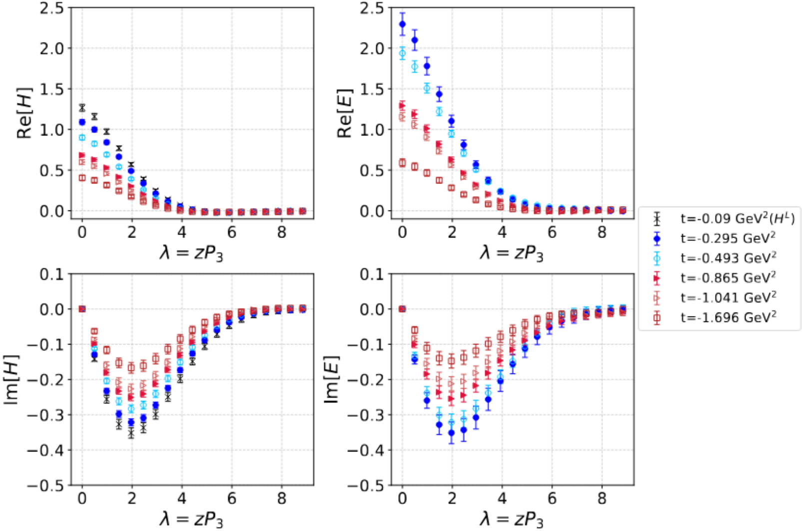
- Faster decay: $|\xi| = 1/2$ (largest $|\xi|$), $t = -0.686$ GeV^2 (lowest t)
- Faster decays cannot be attributed to the values of t
- ξ is the variable that plays a role in the rate of decay of the amplitudes



Numerical results – Bare Quasi-GPDs in Coordinate Space

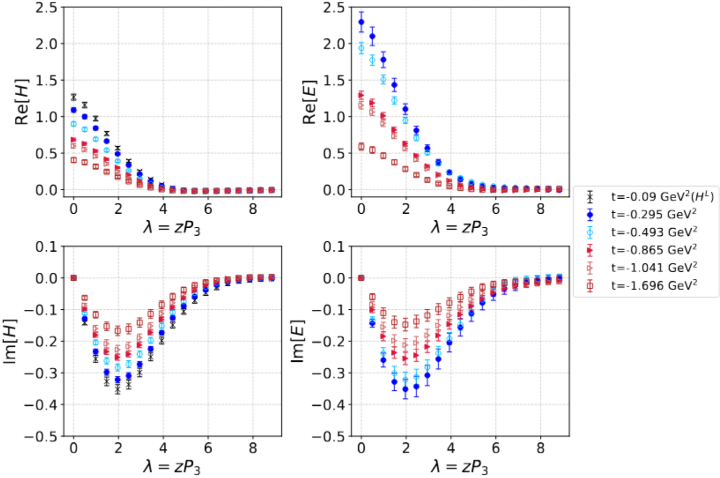
- Magnitude of GPD increases as t increases



Numerical results – Bare Quasi-GPDs in Coordinate Space

- Magnitude of GPD increases as t increases
- H and H^L have identical behaviours, if small α, $H^L \approx H$

$$H^L_{\rm LI} \equiv \left(H + \frac{1}{2}\alpha\xi(H+E)\right)_{\rm LI} = A_1^L, \qquad \alpha = \frac{-(z\cdot\Delta)(z\cdot P)}{z^2P^2-(z\cdot P)^2}$$



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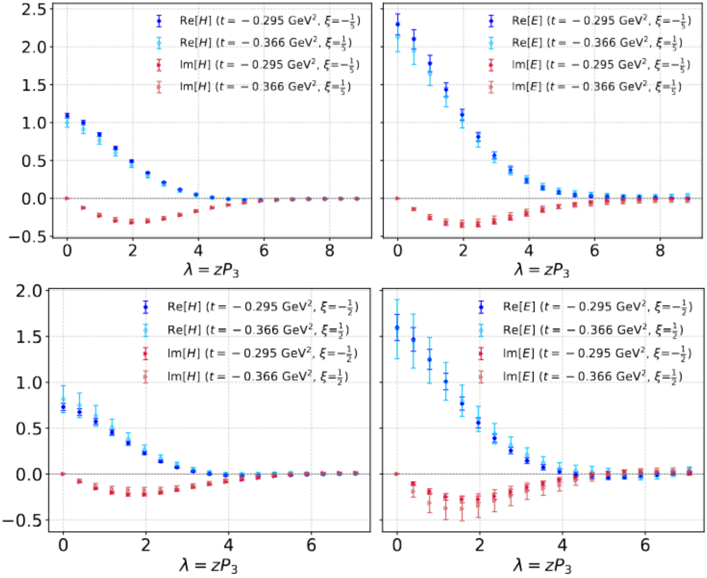
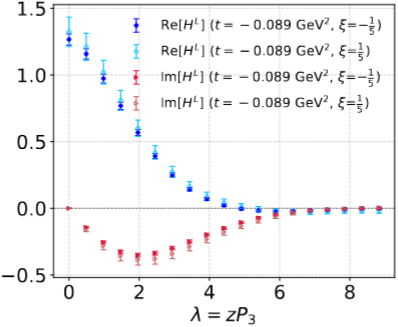
- Symmetry of GPDs under $\xi \longleftrightarrow -\xi$ for different but similar t



Numerical results – Bare Quasi-GPDs in Coordinate Space

- Symmetry of GPDs under $\xi \longleftrightarrow -\xi$ for same t



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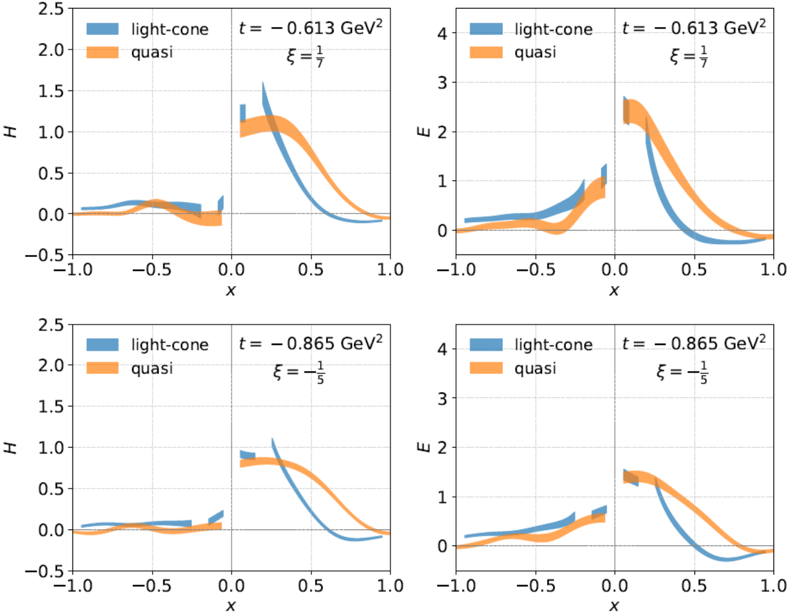
- RI/MOM and BG
  - Used in previous $\xi = 0$ studies
  - Focus of this work was on $\xi \neq 0$, not improving these methods
  - Improvement planned in follow-up work

Numerical results – GPDs

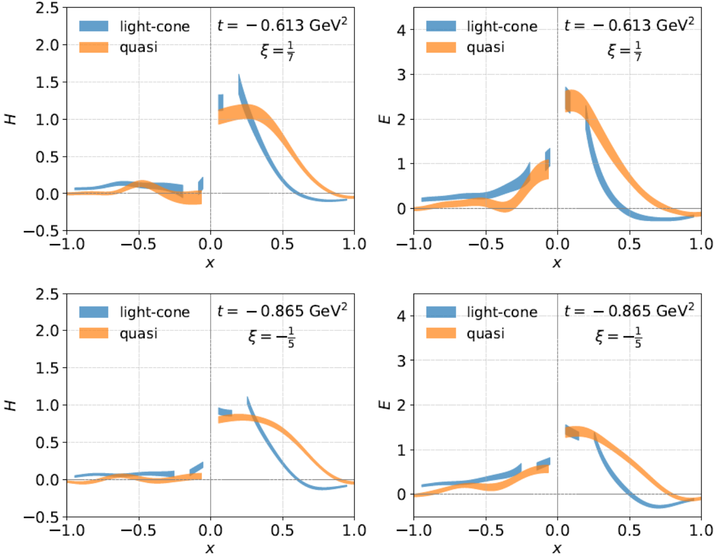
- Matching to the infinite momentum frame $\rightarrow$ large momentum effective field theory
- Quasi-GPDs and light-cone GPDs differ at UV scale
- Difference at the UV scale:
    - Computed by perturbation theory
    - Up to higher-twist corrections
    - One-loop order is chosen in this work

Numerical results – Light-Cone GPDs



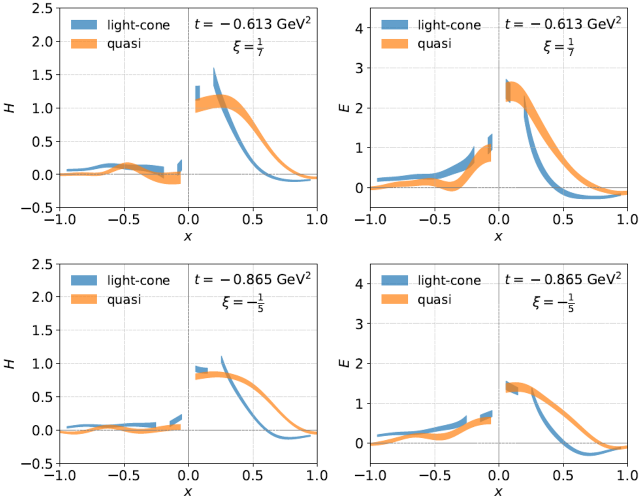
Numerical results – Light-Cone GPDs

- 5 Missing regions:
    - Backus-Gilbert → unreliable quasi-GPD region near $x = 0$



Numerical results – Light-Cone GPDs

- 5 Missing regions:
  - Backus-Gilbert $\rightarrow$ unreliable quasi-GPD region near $x = 0$
  - Breaking down of perturbation theory at $x \rightarrow \pm 1$ and $x \rightarrow \pm\xi$
  - Different matching in ERBL/DGLAP region boundary: $x \rightarrow \pm\xi$



Conclusions

- Generalization of the previous zero-skewness GPD paper
    - Decomposition of bilocal matrix element into amplitudes
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  - Find coordinate-space quasi-GPDs
- Purely longitudinal transfer yields H and E combination; here it effectively gives H.
- $\xi \to -\xi$ symmetry confirmation
- Discontinuities induced matching and x-space reconstruction $\to$ 5 unreliable regions

Conclusions – Path forward

- Path forward:
  - Improve matching:
    - Larger momentum boosts $\rightarrow$ suppress higher-twist effects
    - Combining quasi- and pseudo-GPDs with artificial neural networks – see talk on 10th of October at 12:15 by Min-Huan Chu

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  - Broaden kinematics coverage of GPDs
  - More statistics to reduce statistical fluctuations