First Gluon PDF from Large Momentum Effective Theory

William Good (goodwil9@msu.edu)
LaMET 2025
10.08.2025

In Collaboration with Fei Yao, and Huey-Wen Lin
Based on work: arXiv:2505.13321v2 (2025) (in review for PLB)

Introduction and Background

Amaroso, *et al.* **(Snowmass) Acta Physica Polonica. B, 53, 12 (2023)**
Aguilar, *et al.* **Eur. Phys. J. A 55, 190 (2019).**
Achenbach, *et al.* **(LRP) Nucl. Phys. A 1047:122874 (2024)**

Image credit: CERN

Introduction and Background

- Parton distribution functions (PDFs) provide the probability for a parton to carry a fraction x of the proton momentum, which is an important input for high energy scattering experiments

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- With the new perturbative calculations for our operator, I can now present the results for the first gluon PDF from LaMET

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Gluon PDFs from LQCD

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- Gluon correlators are notoriously challenging in lattice QCD because they come only from noisy disconnected diagrams
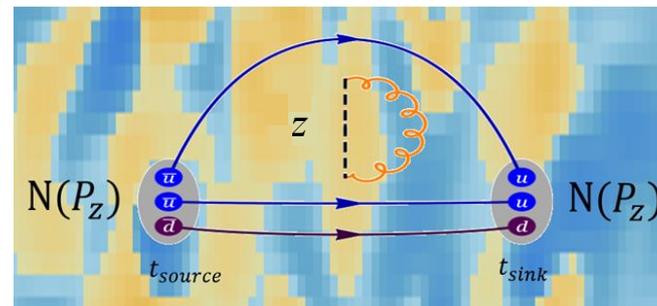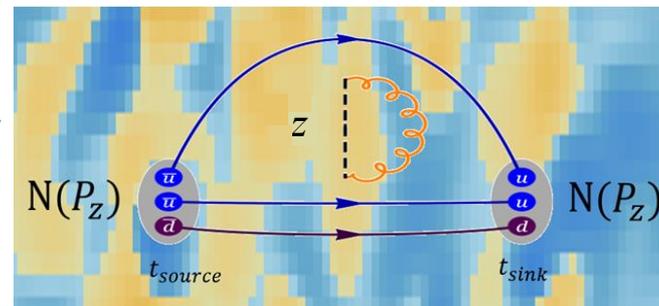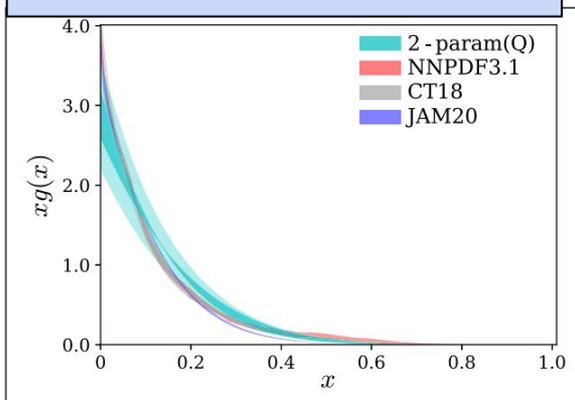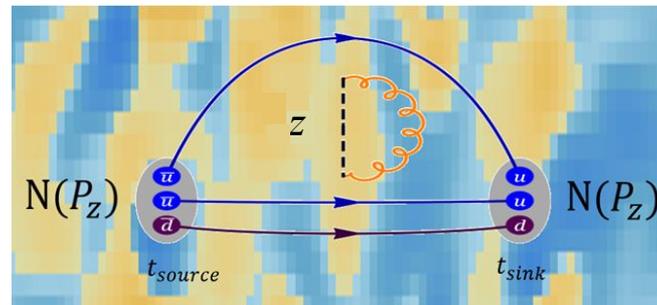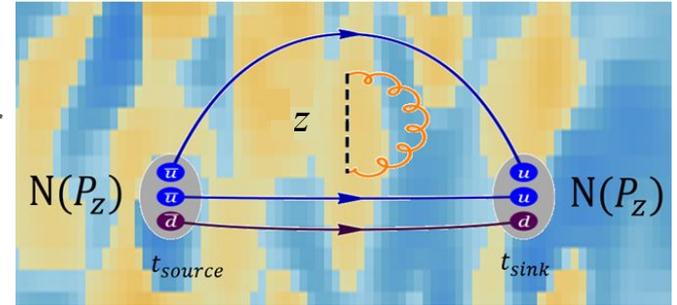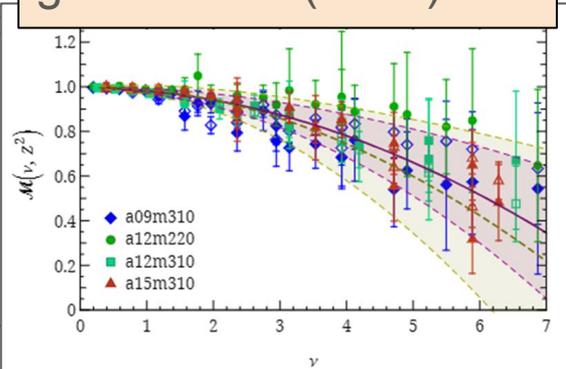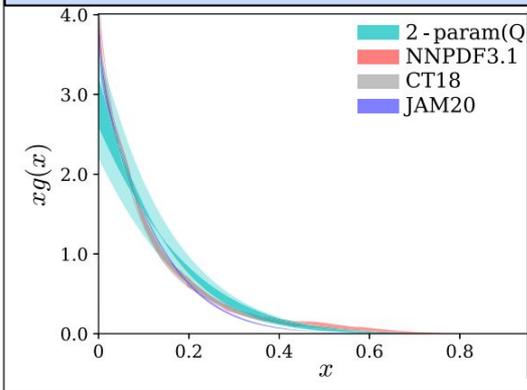
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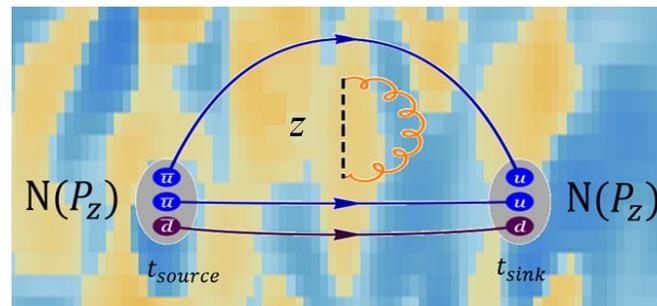
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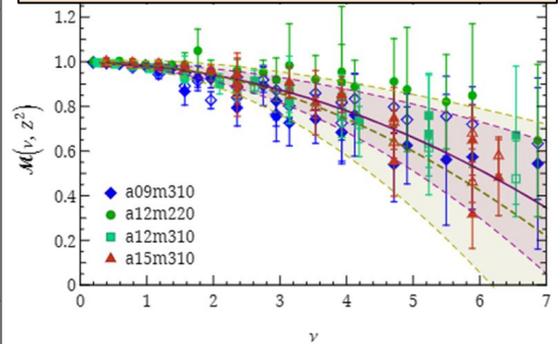
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[Hadstruc] First nucleon gluon PDF from pseudo-PDF (2021)
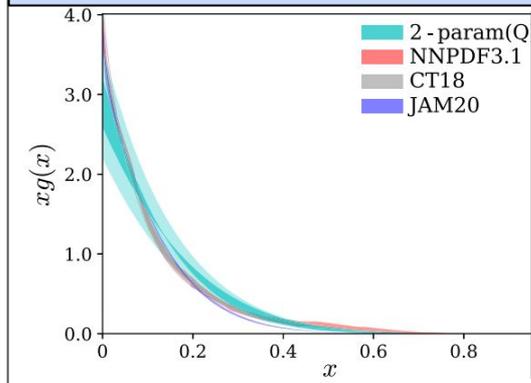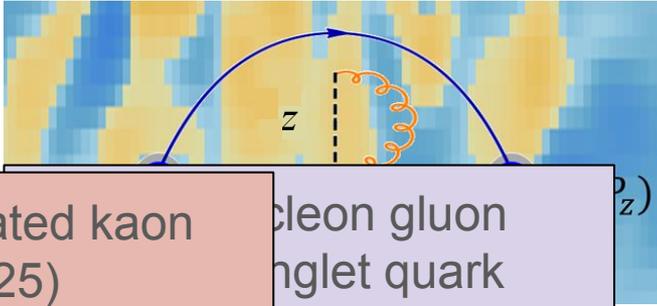
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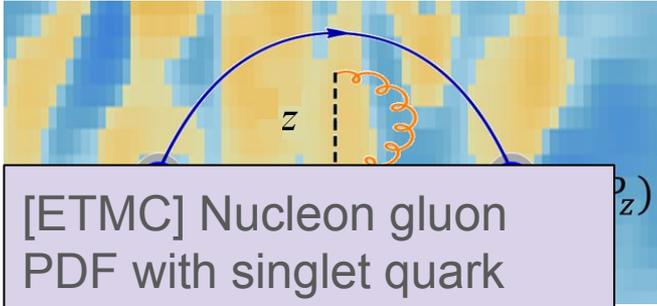
[Hadstruc] First nucleon gluon PDF from pseudo-PDF (2021)

[MSULat] First physical continuum nucleon gluon PDFs (2023)

PRD 108:014508(1), (2023)

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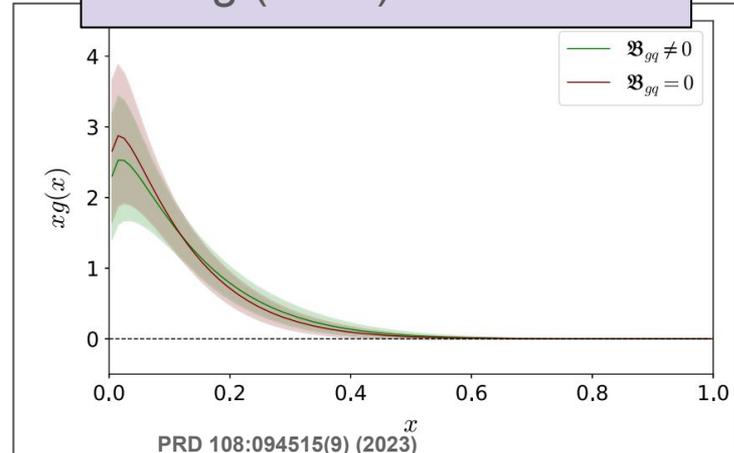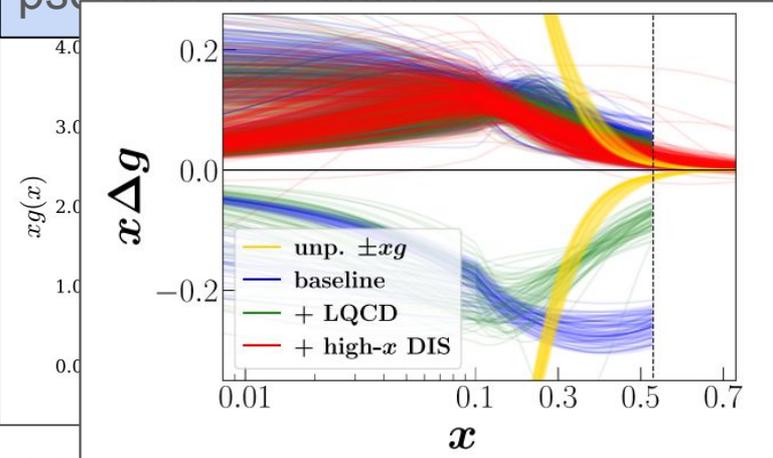

[Hadstruc] First nucleon gluon PDF from pseudo-PDF (2021)

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[ETMC] Nucleon gluon PDF with singlet quark mixing (2023)

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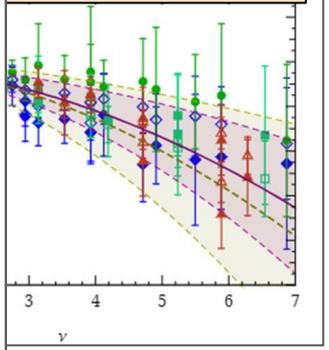- This limits the long distance signal important for ~~gluon PDFs~~

[Hadstruc+JAM] Gluon helicity PDFs (2024)

PRD 109:036031(3), (2024)
PRL 133:161901(16), (2024)

[Hadstruc] First physical ~~um nucleon~~ ~~pseudo-PDF~~ (2023)

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PRD 104:094516(9) (2021)

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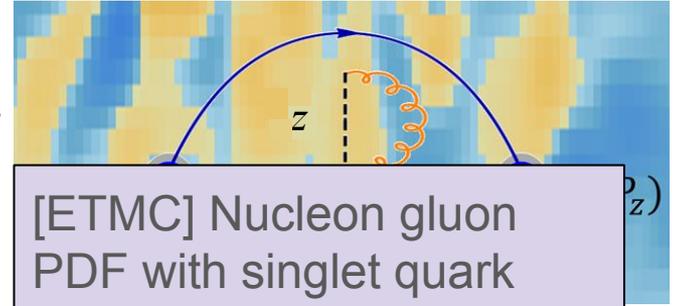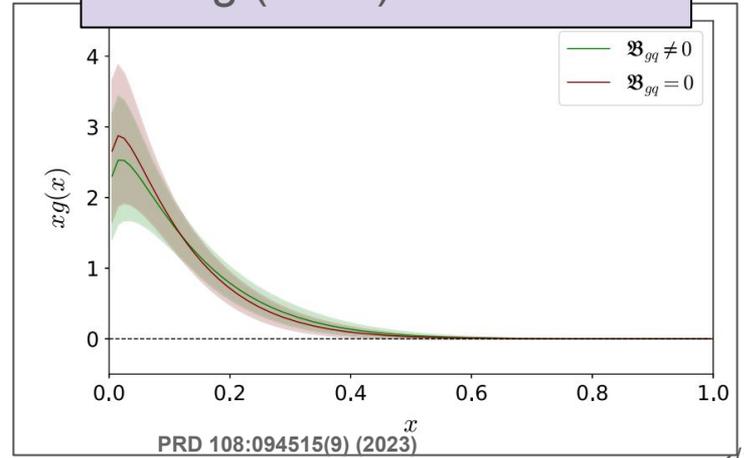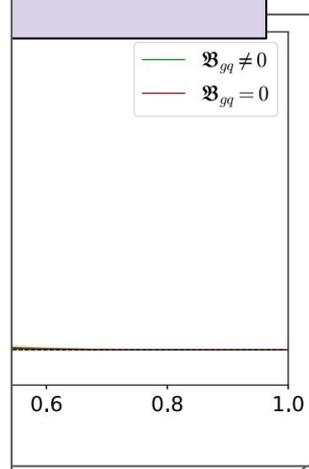PRD 109:036031(3), (2024)
PRL 133:161901(16), (2024)

[MSULat] Updated kaon gluon PDF (2025)
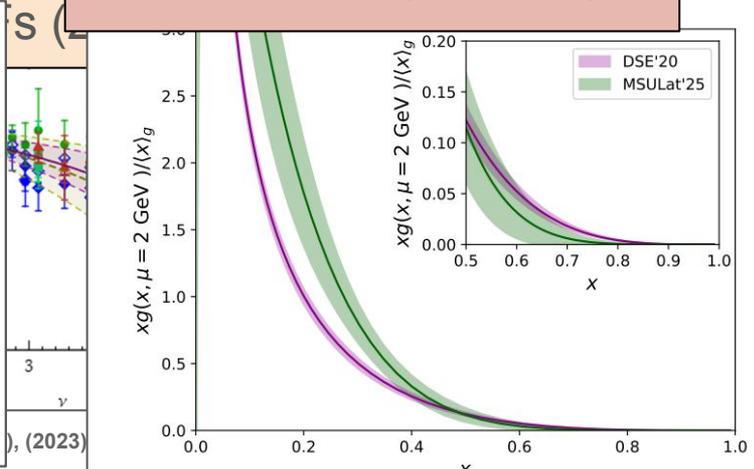arXiv:2506.03002 (in review for PRD)
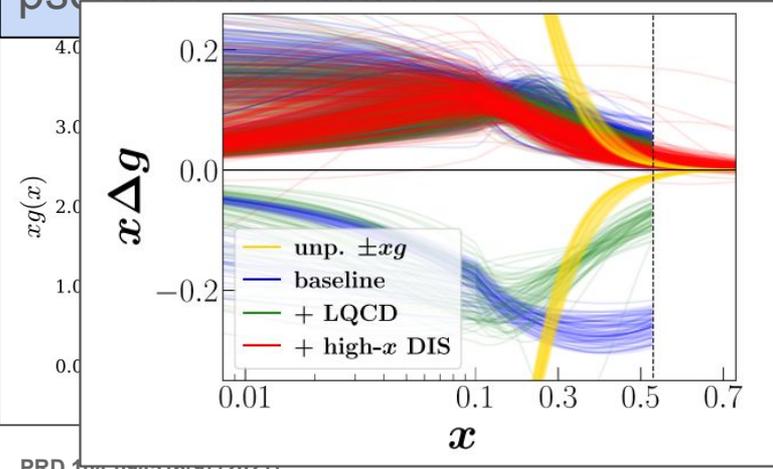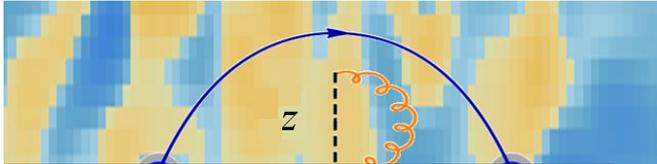
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[Hadstruc+JAM] Gluon helicity PDFs (2024)
PRD 109:0360
PRL 133:16190

[MSULAT+JAM] Pion gluon data in a global fit
arXiv:2507.22730 (in review for PRL)

[Had... gluo... pseud...

...ated kaon ...25) ...w for PRD)

...cleon gluon ...nglet quark ...)

PRD 104.09451?(9) (2021)

Large Momentum Effective Theory

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Large Momentum Effective Theory

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- The gluon PDF has some freedom of operator choice. Until this work, these were the only two operators which had perturbative details for hybrid renormalization described explicitly in the literature:

Zhang, et al., PRL 121:142001 (2019)
Wang, et al. PRD 100:074509 (2019)
Yao, et al. JHEP 11(2023)021

$$O^{(1)}(z) = F^{zi}(z)W(z,0)F_i^z(0)$$
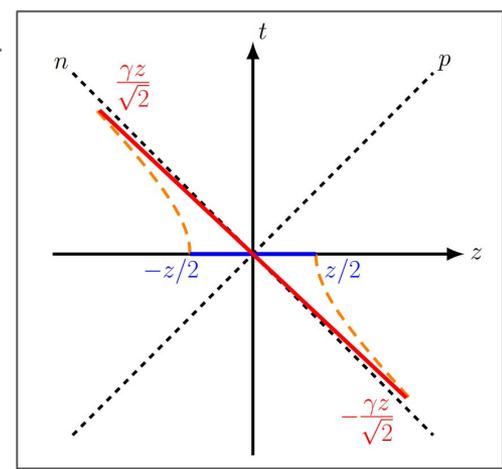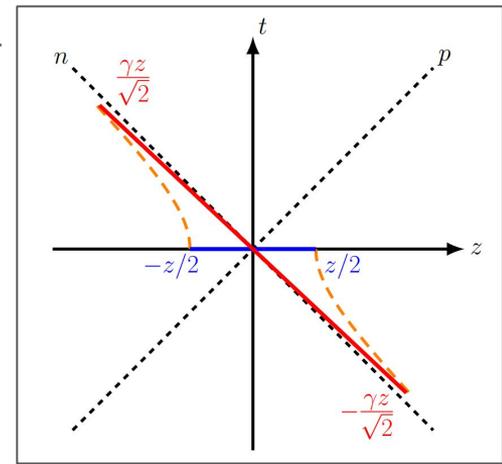
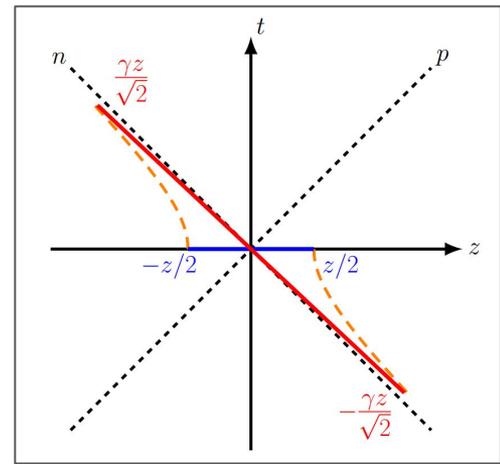$$O^{(2)}(z) = F^{z\mu}(z)W(z,0)F_\mu^z(0)$$

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$$O^{(1)}(z) = F^{zi}(z)W(z,0)F_i^z(0) \qquad O^{(2)}(z) = F^{z\mu}(z)W(z,0)F_\mu^z(0)$$

- Previous pseudo-PDF studies have used this operator, which has better signal

$$O^{(3)}(z) = F^{ti}(z)W(z,0)F_i^t(0) - F^{ij}(z)W(z,0)F_{ij}(0)$$

Balitsky, et al., PLB 808:135621 (2020)

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Light Nucleon Results for Hybrid-Renormalization

$O^{(1)}(z)$

$P_z=0.427$ GeV
$P_z=0.855$ GeV
$P_z=1.28$ GeV
$P_z=1.71$ GeV
$P_z=2.14$ GeV

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$z_s = 0.24$ fm

N_l W3 — Even worse decay and divergences

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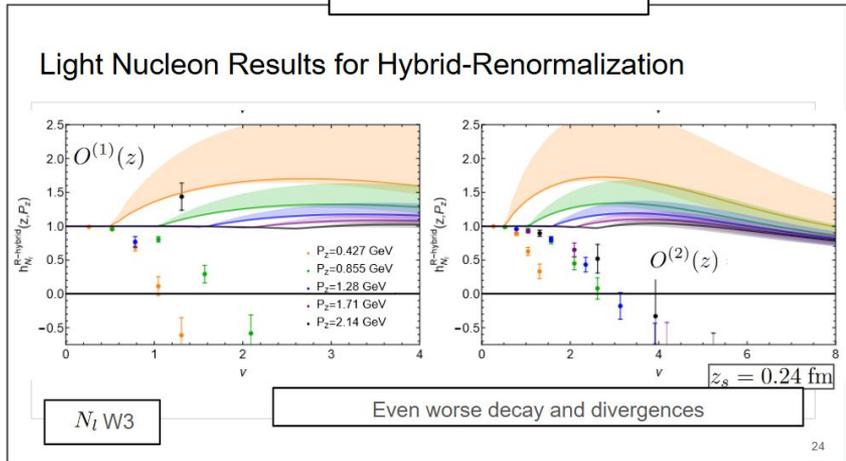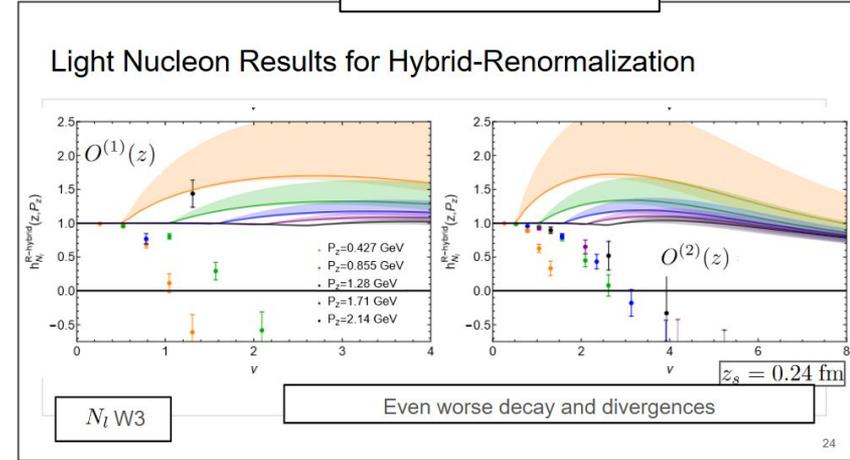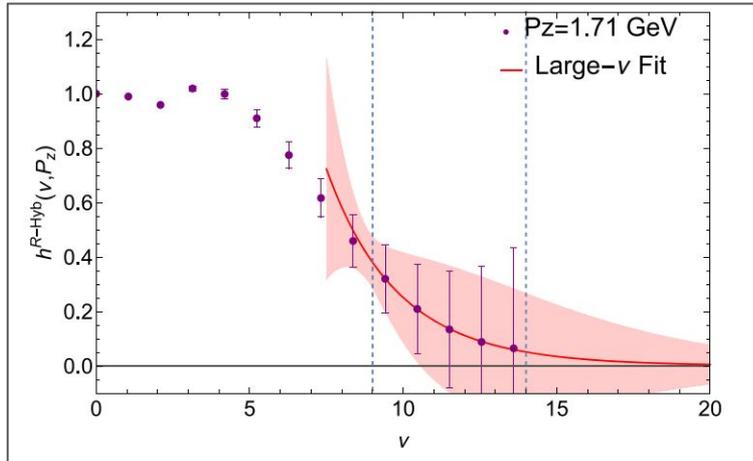
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- Large–v Fit

Quasi-PDF

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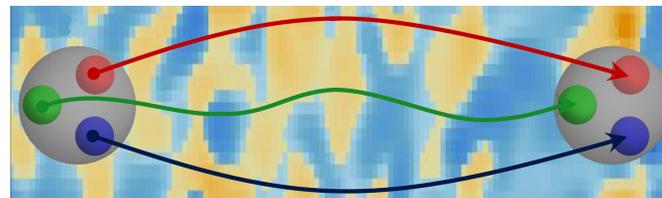- We took the information from the pseudo-PDF matching paper for $O^{(3)}(z)$ to derive the Wilson coefficient and momentum space matching

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- With these two pieces of information, we can properly renormalize our data for this operator and match to the light-cone!

Lattice Information

Follana *et al*. PRD 75:054502, 2007.
Bazavov, *et al*. [MILC], PRD 82:074501 2010.
Bazavov, *et al*. [MILC], PRD 87:054505 2013.
Bazavov, *et al*. [F Lattice and MILC] PRD 98:074512 2018

Lattice Information

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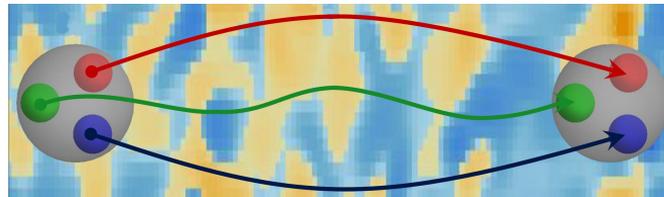
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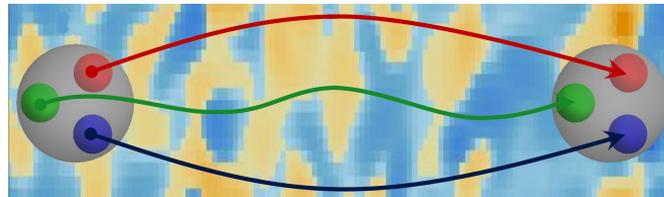Bazavov, *et al*. [F Lattice and MILC] PRD 98:074512 2018

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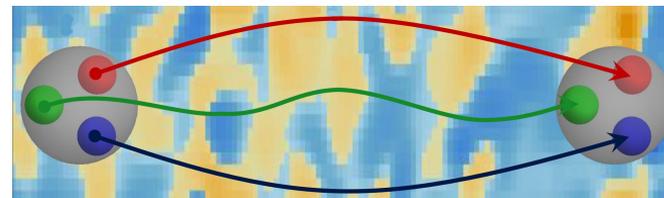
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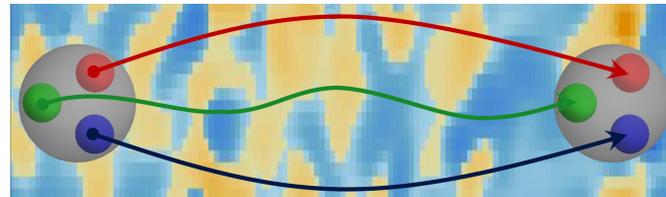Bazavov, *et al*. [F Lattice and MILC] PRD 98:074512 2018

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- Two gauge link smearing sets for the 3-point (3pt) correlators: 5 steps of hypercubic smearing (HYP5) and Wilson flow with time t=3a² (W3)

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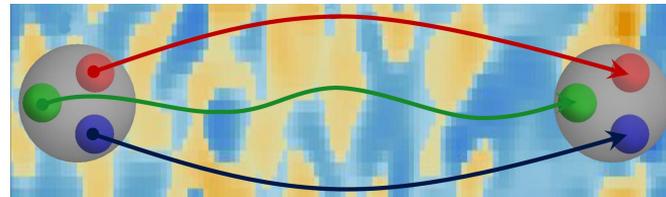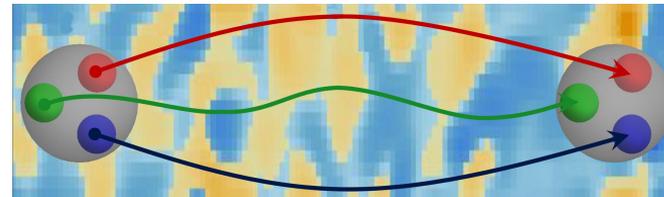Bazavov, *et al*. [F Lattice and MILC] PRD 98:074512 2018

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- We use lattice spacing $a \approx 0.12$ fm, and tune valence pion masses to $M_\pi \approx 310$ and 690 MeV with volume $24^3 \times 64$

- ~1.2M 2-point (2pt) correlator measurements over 1013 configurations

- Gaussian momentum smearing to get signal up to 2.14 GeV boost momentum

- Two gauge link smearing sets for the 3-point (3pt) correlators: 5 steps of hypercubic smearing (HYP5) and Wilson flow with time t=3a² (W3)

- We look at the "strange" nucleon (N_s), light nucleon (N_l), strange pion (η_s), and the light pion (π) for renormalization and focus on the nucleons for the final results

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- $\delta m + m_0$ can be fit by matching to the Wilson coefficients for the given operator:

$$(\delta m + m_0)z + I_0 \approx \ln\left[\frac{\mathcal{H}(z,\mu)}{h^B(z,0)}\right]$$

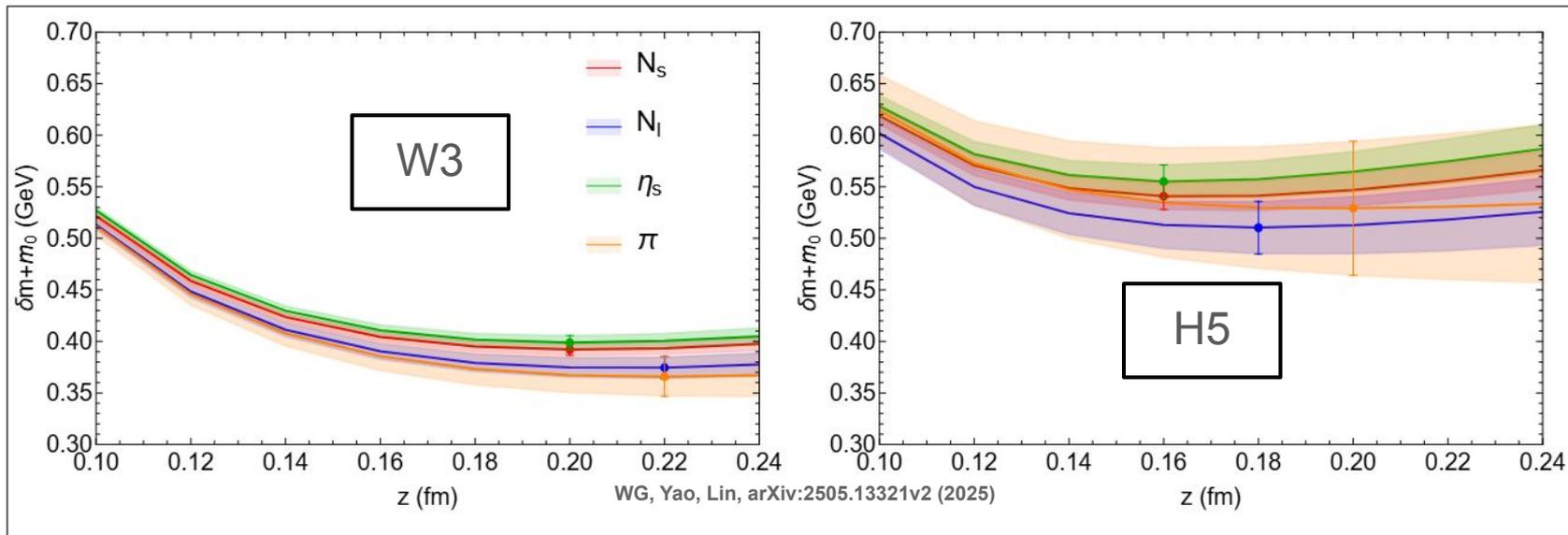# $\delta m + m_0$ Fit Results

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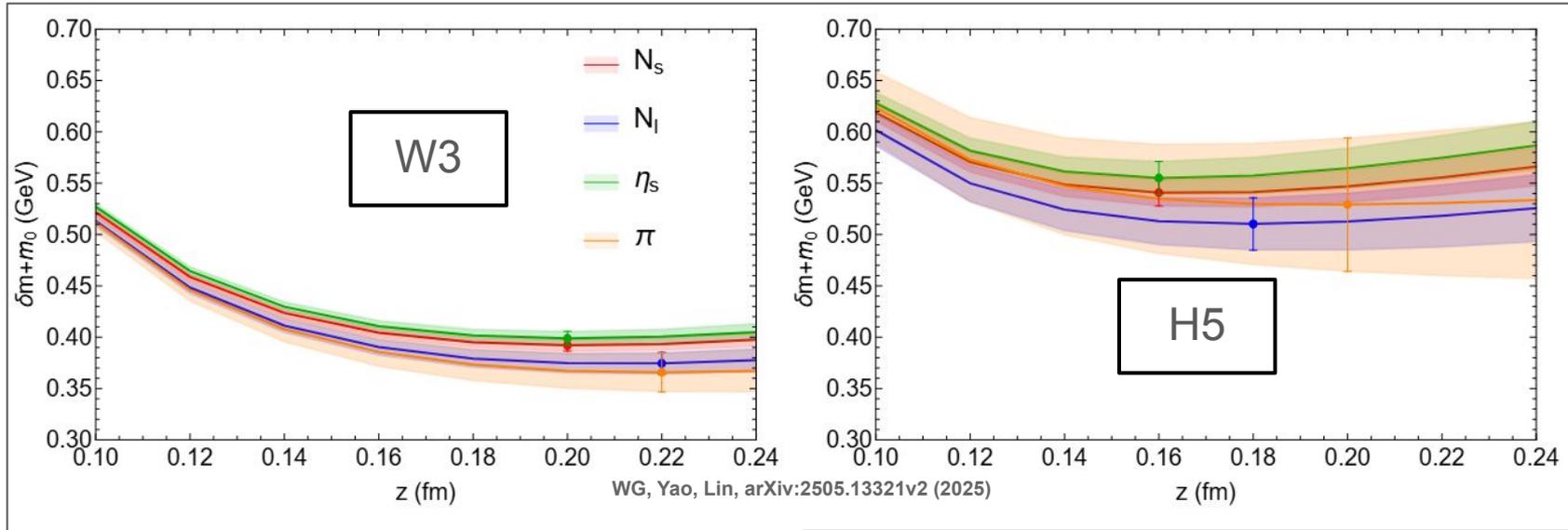
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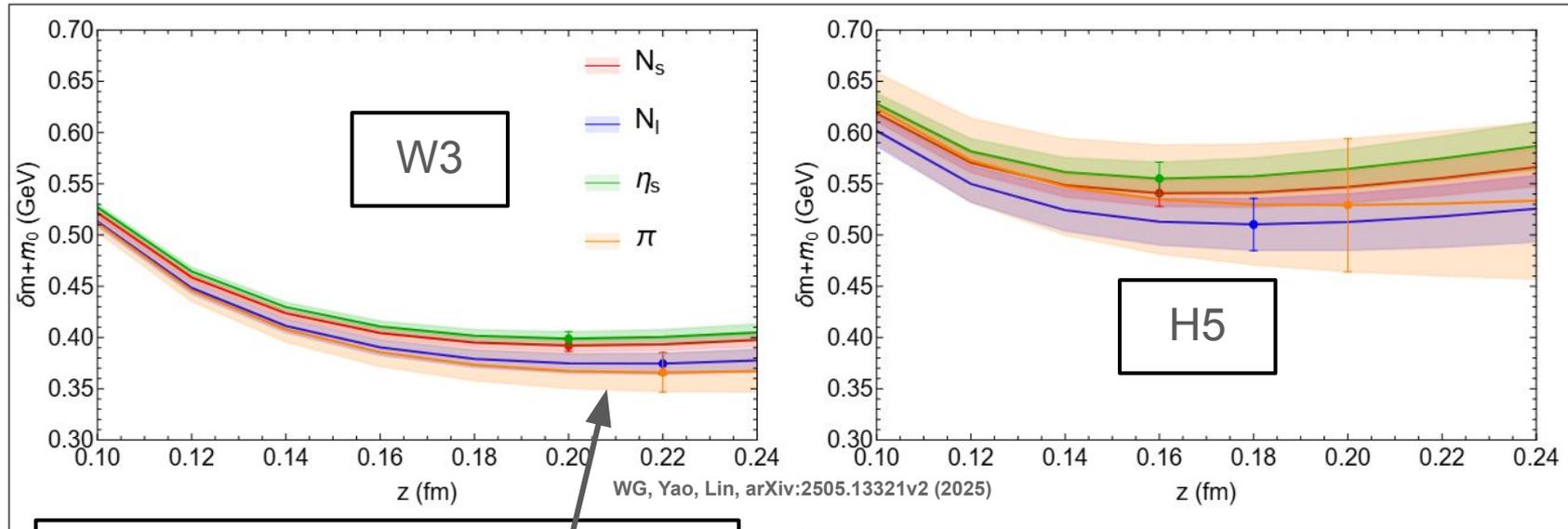
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8/15

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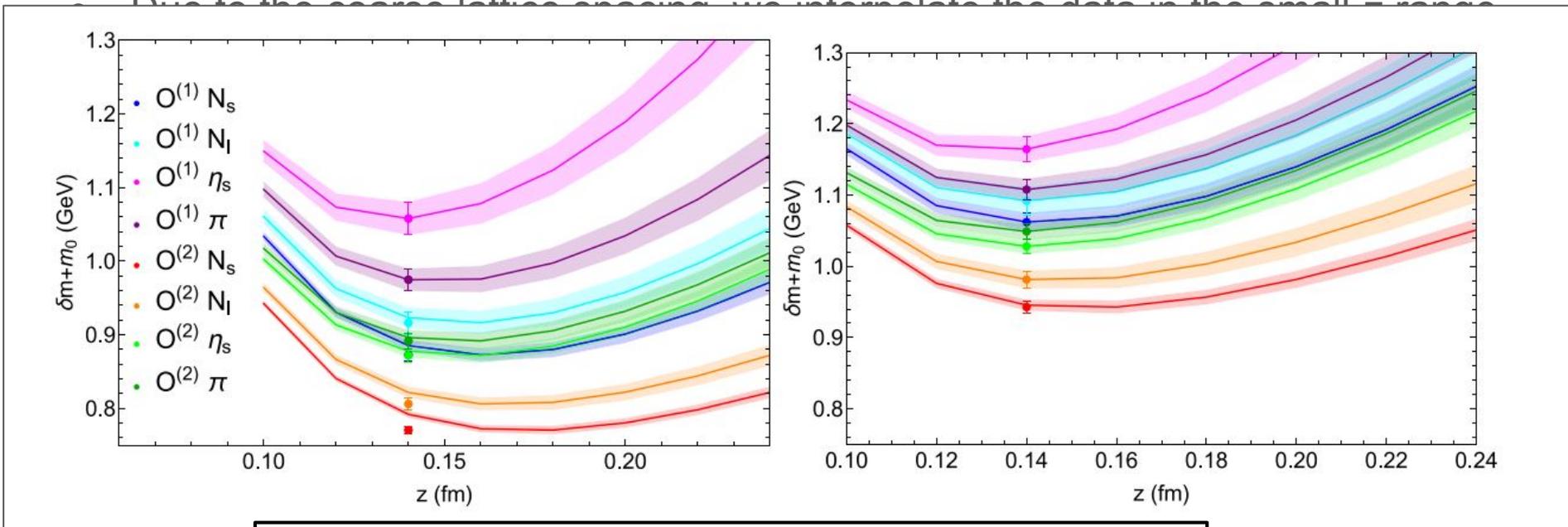

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Statistical consistency across different hadrons, is a good sign!

Different linear divergences for different smearing amounts is expected

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This is in strong contrast to the fits for the other operators in our previous study, which seem to have some hadron dependent contaminations

Statistica... different b...

...ces for different smearing amounts is expected
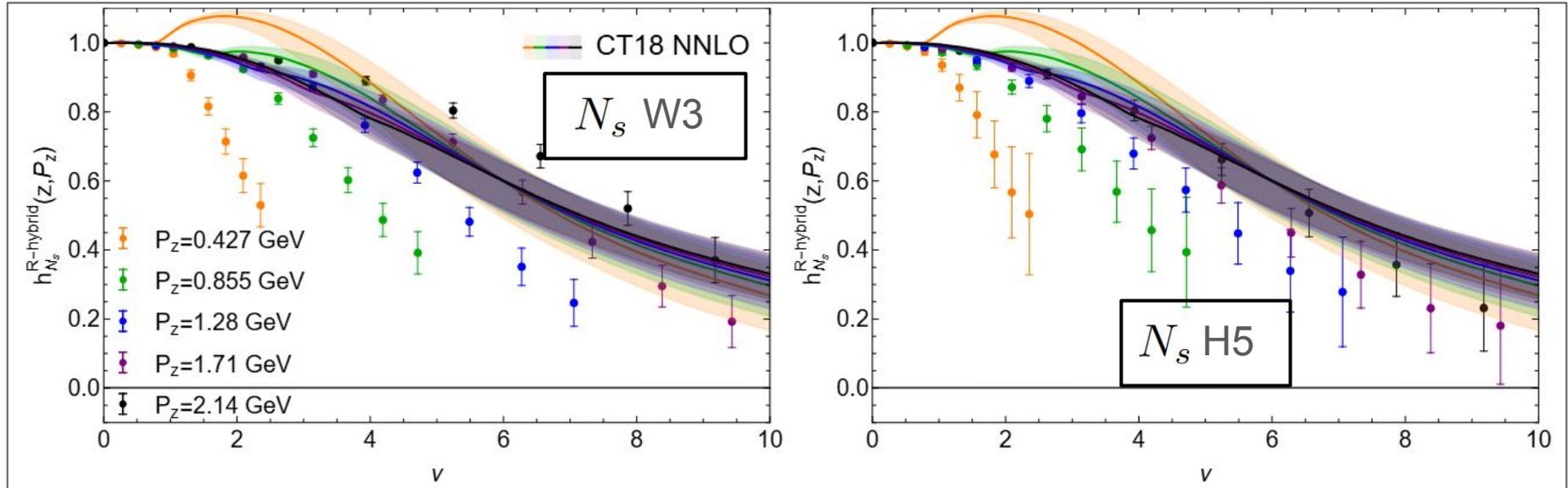
Renormalized Nucleon Gluon Matrix Elements

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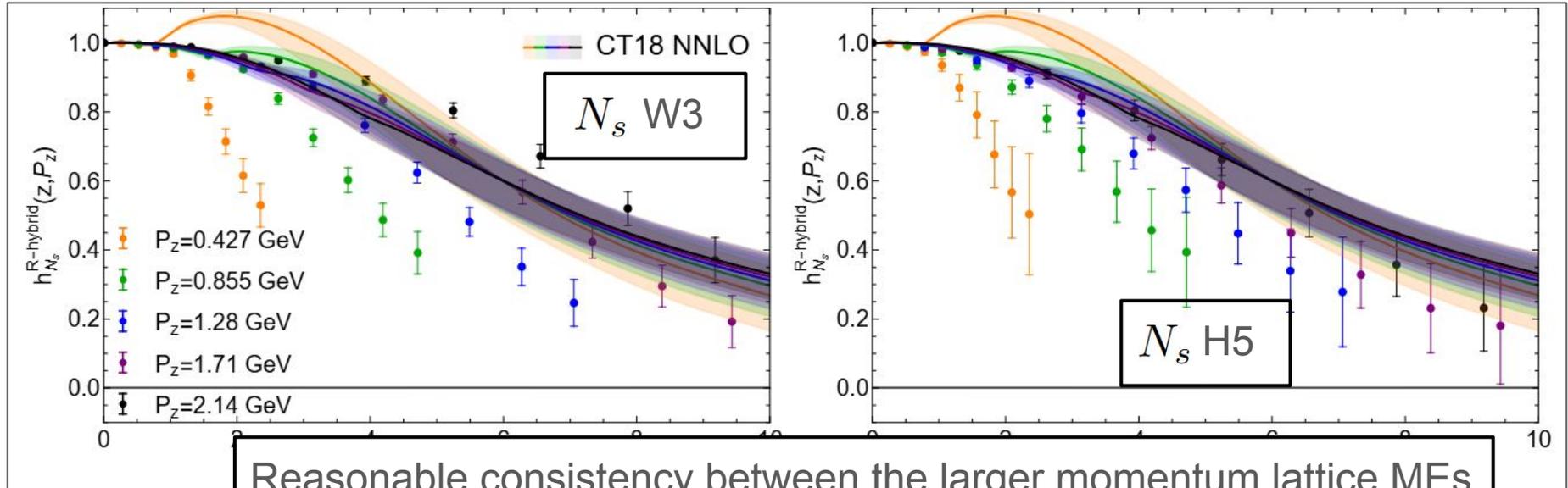
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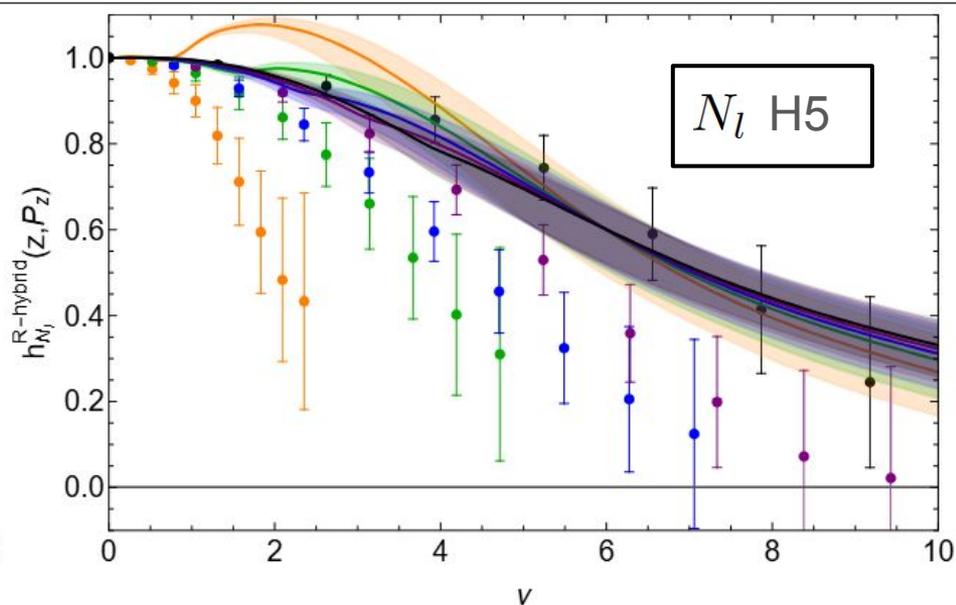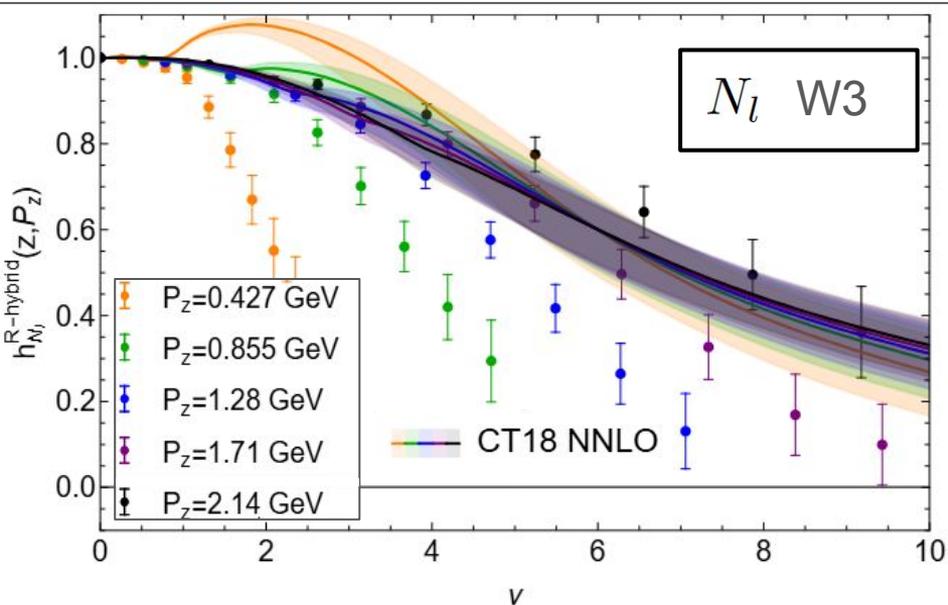
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N_s W3

N_s H5

Reasonable consistency between the larger momentum lattice MEs and the pheno! Not concerned about low momentum MEs

Light Nucleon MEs Are Similar, Just a Bit Noisier

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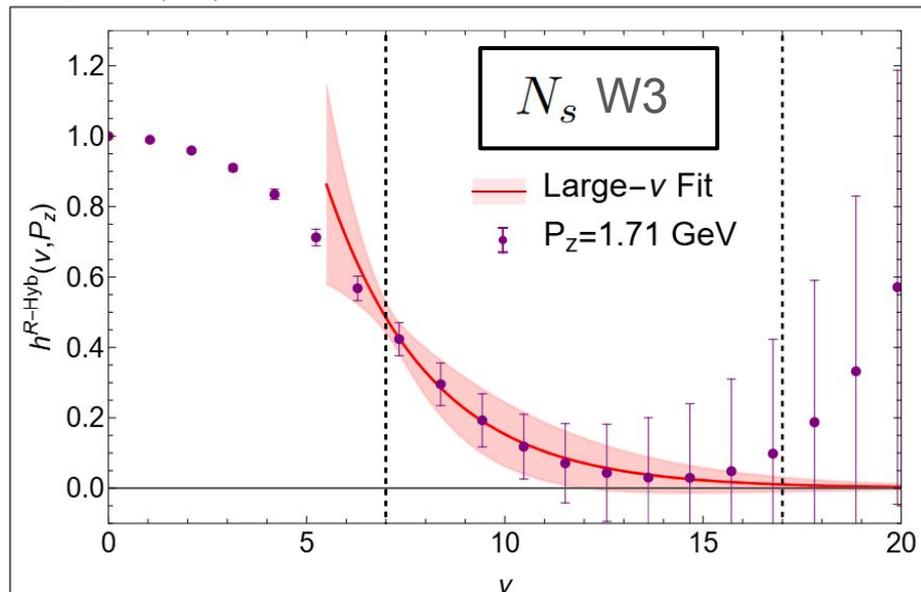
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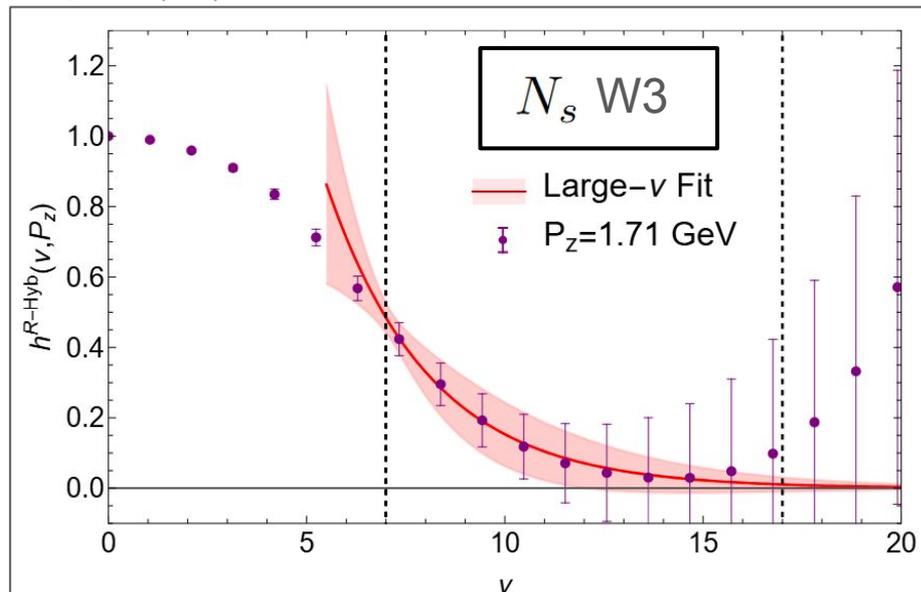

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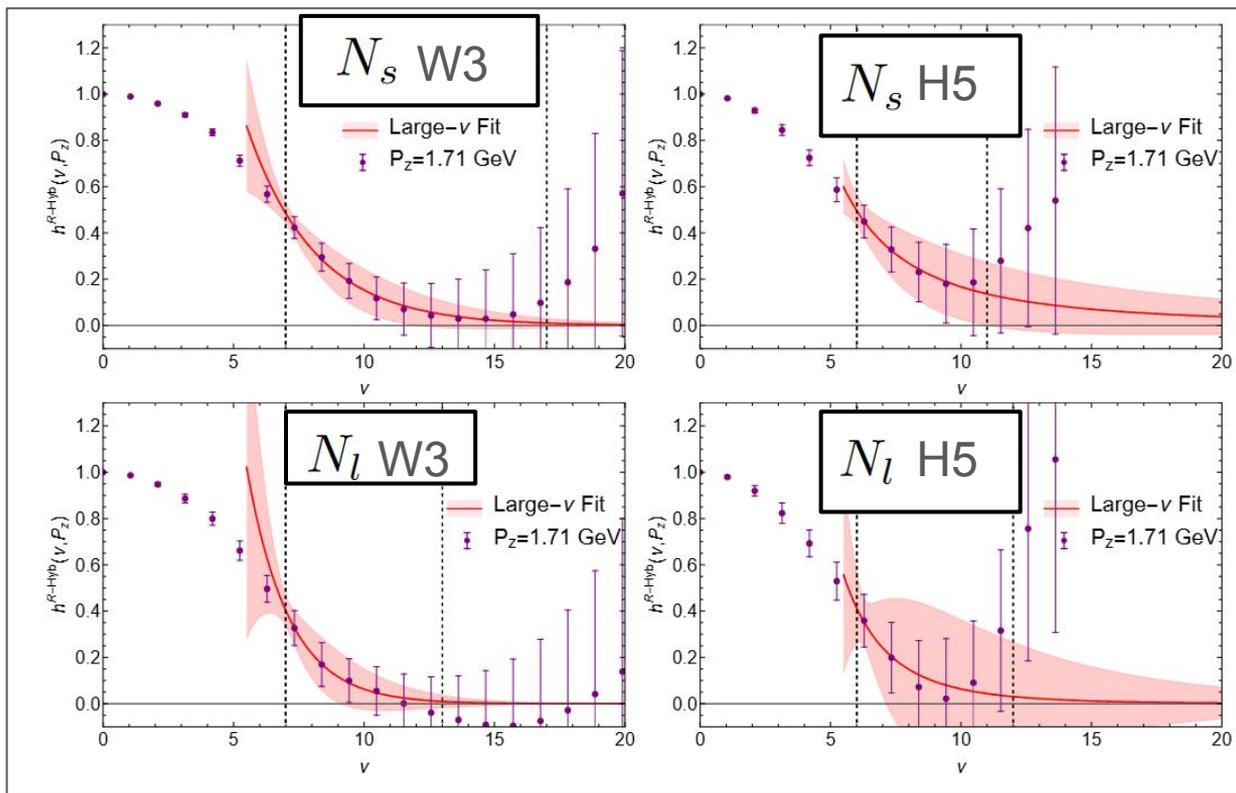
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- The extrapolation fits the data fairly well and provides long distance matrix elements with much smaller error than the lattice
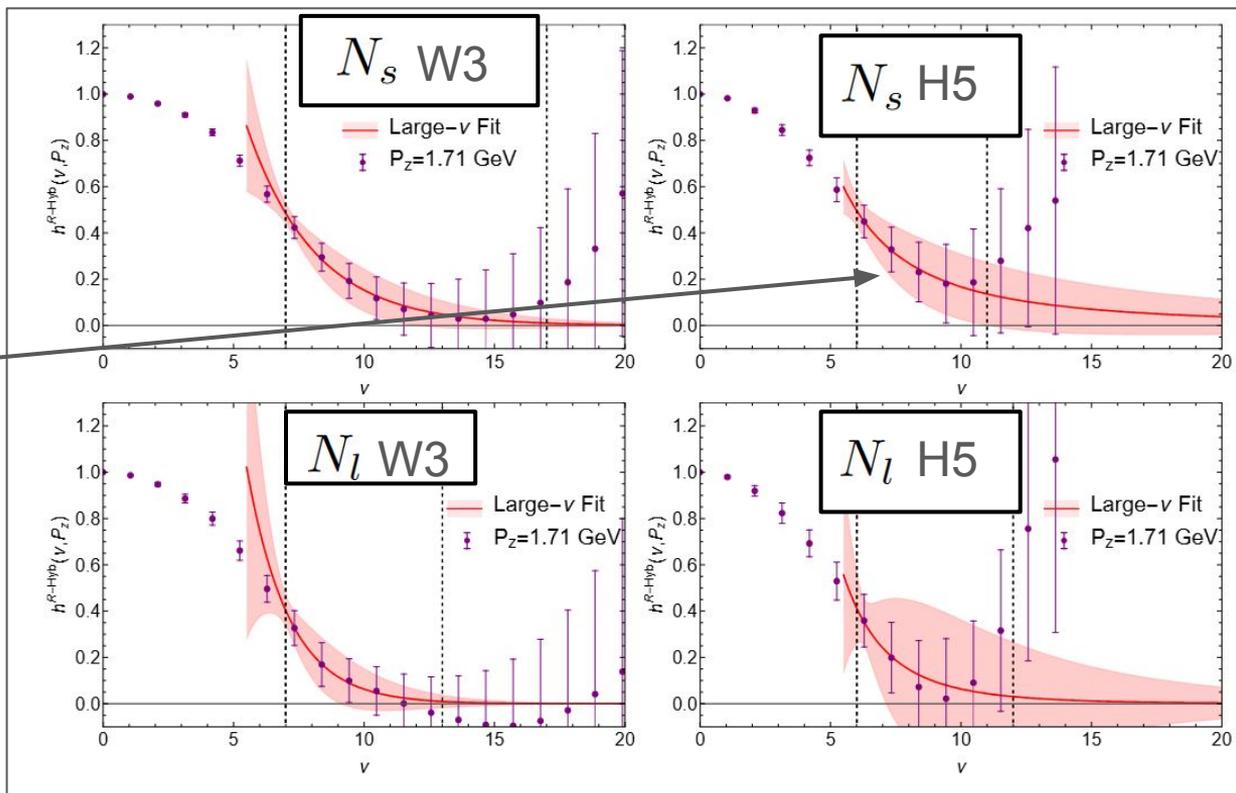

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Extrapolation on Other Data Sets

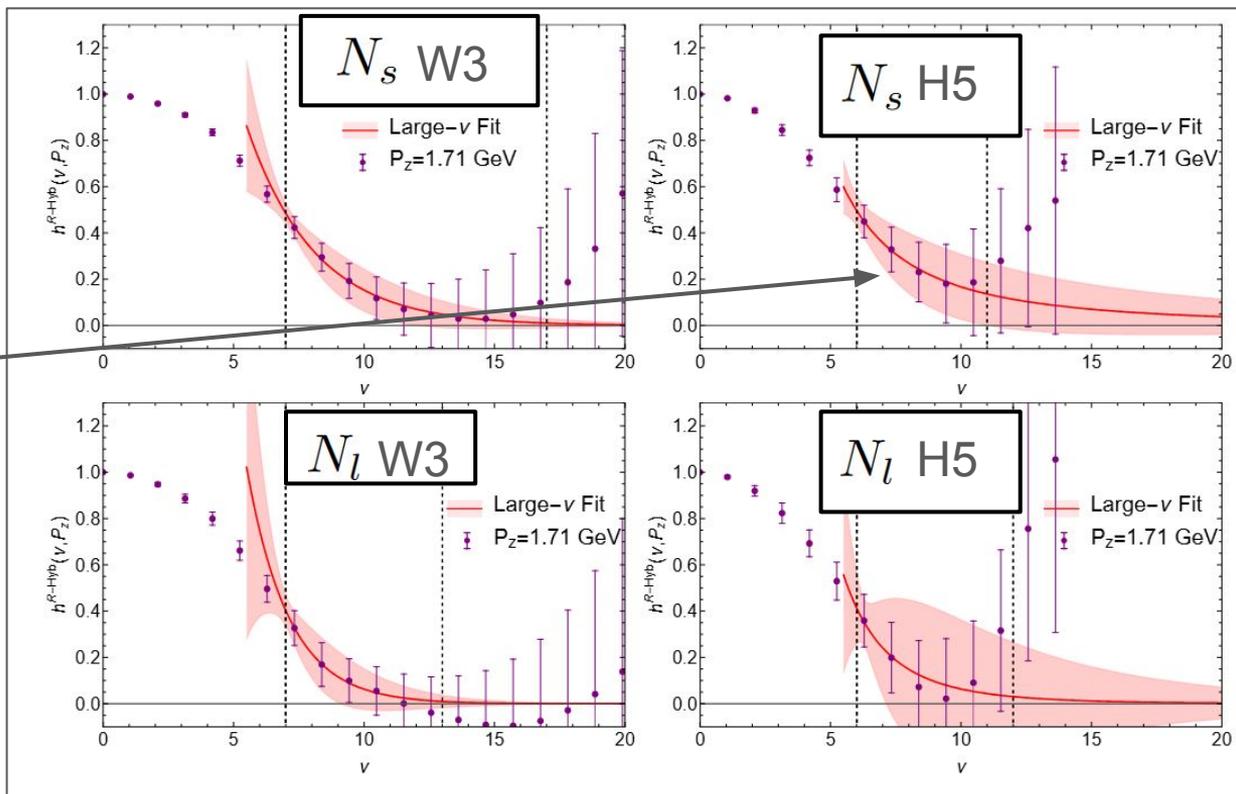- The extrapolation works reasonably well on all 4 data sets,

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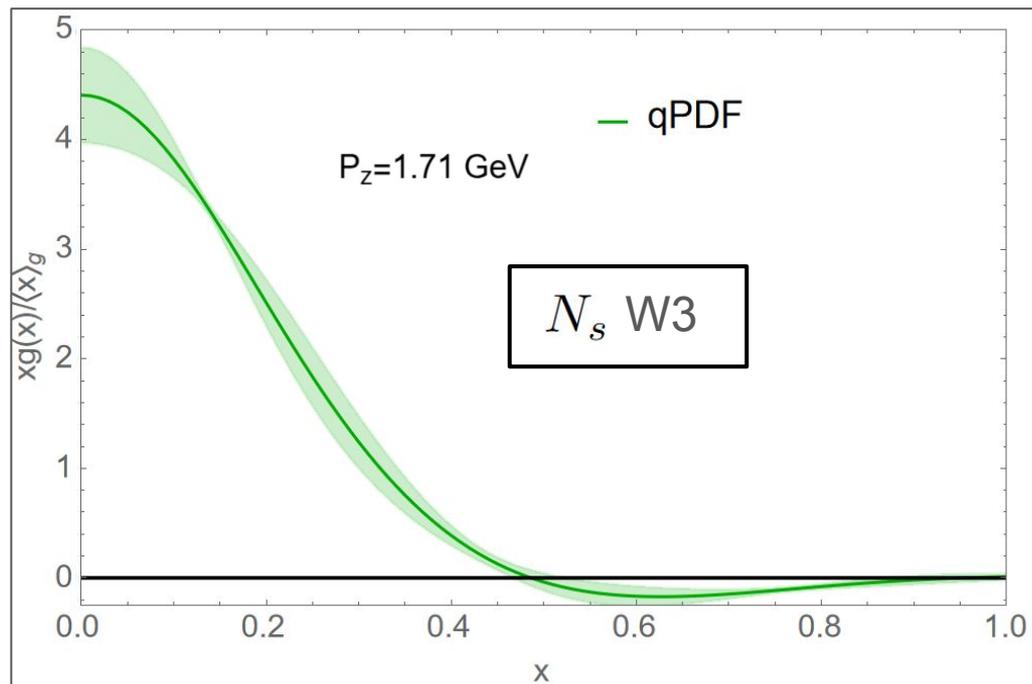
Extrapolation on Other Data Sets

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- There may be some systematic effects from the fit ranges, which should be explored and controlled further

Quasi-PDF and Light Cone Matching
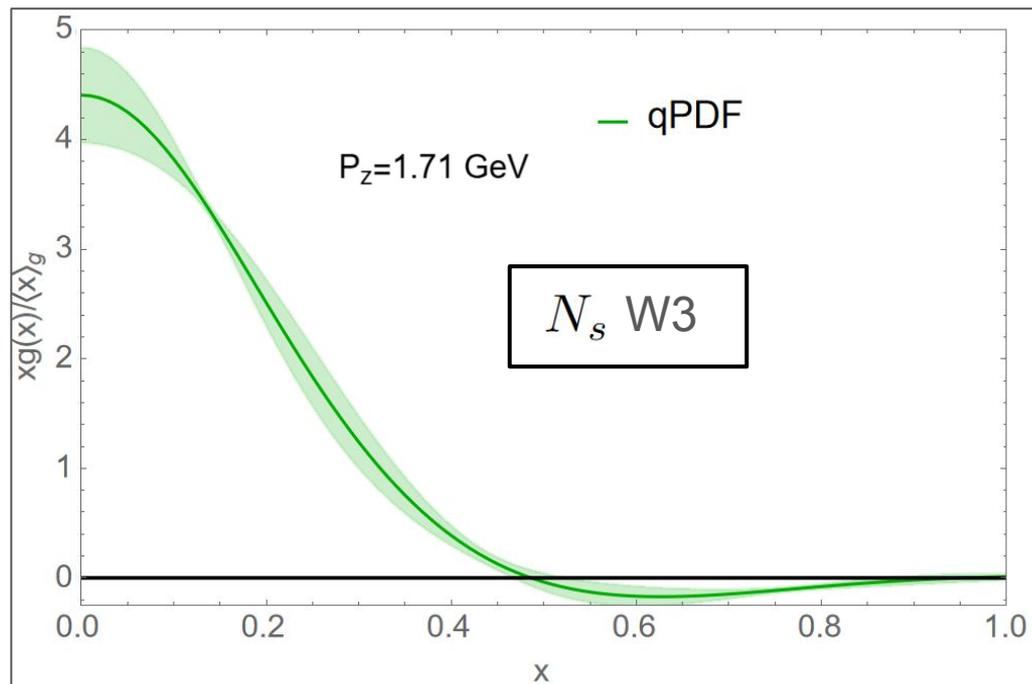
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$$+ \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{(xP_z)^2}, \frac{\Lambda_{QCD}^2}{((1-x)P_z)^2}\right)$$

P_z=1.71 GeV

qPDF

N_s W3

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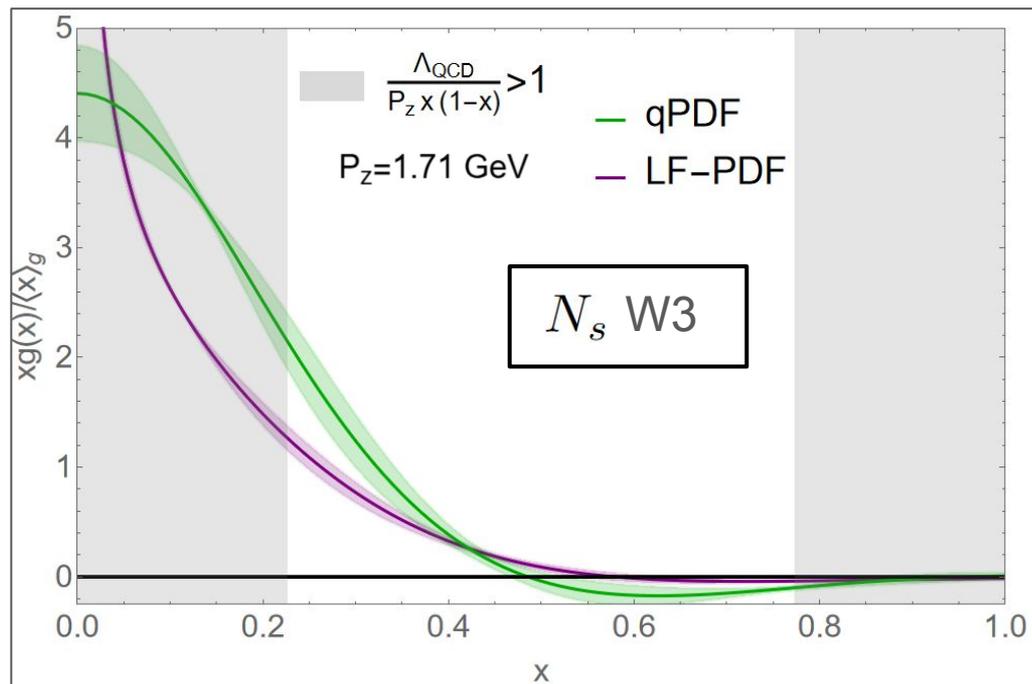
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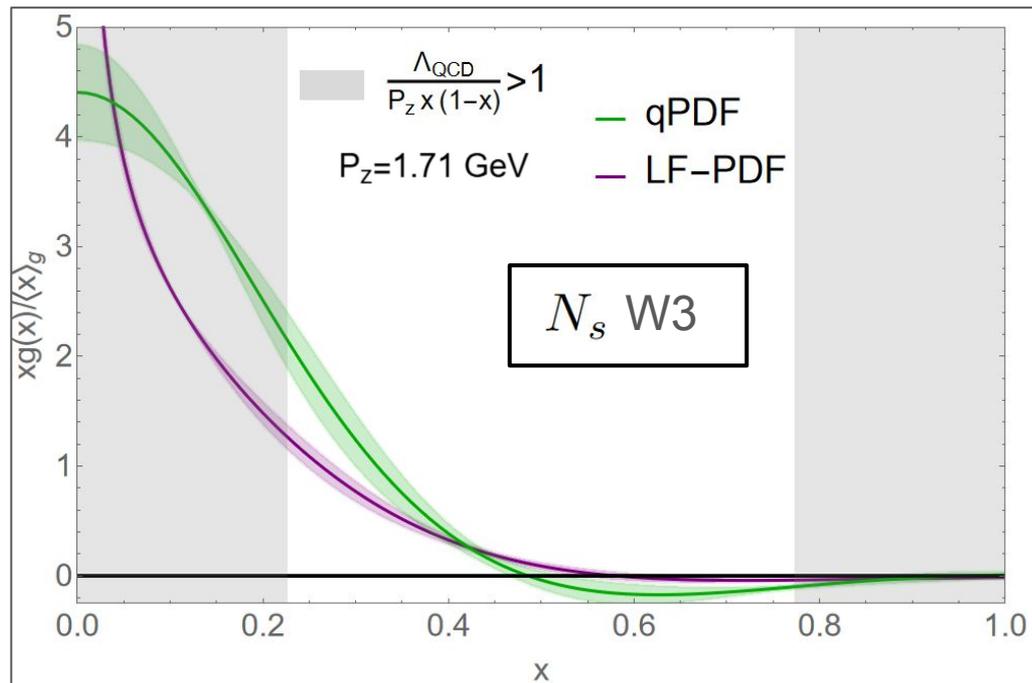

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- We find that the quasi-PDF has some negative region, but it is almost completely washed out by matching

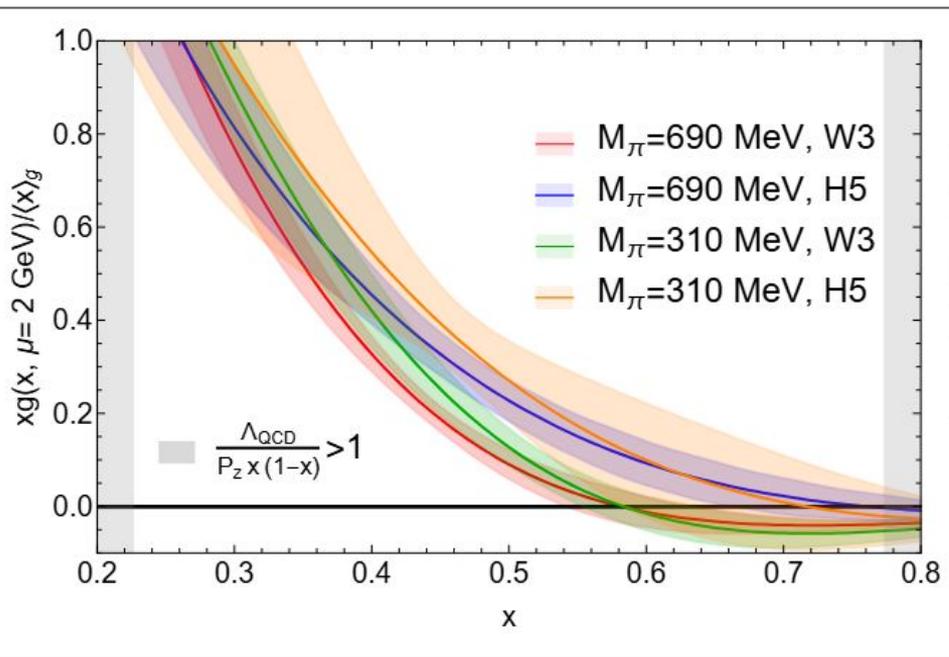Λ_{QCD} = 300 MeV, as a conservative estimate

Light-Cone PDFs

Anderson, *et al.* arXiv 2501.00665 (2024)
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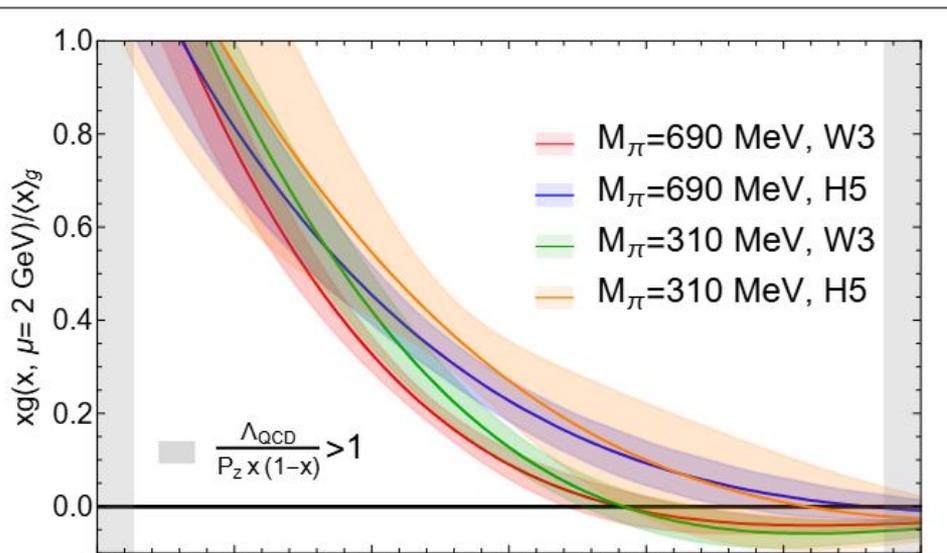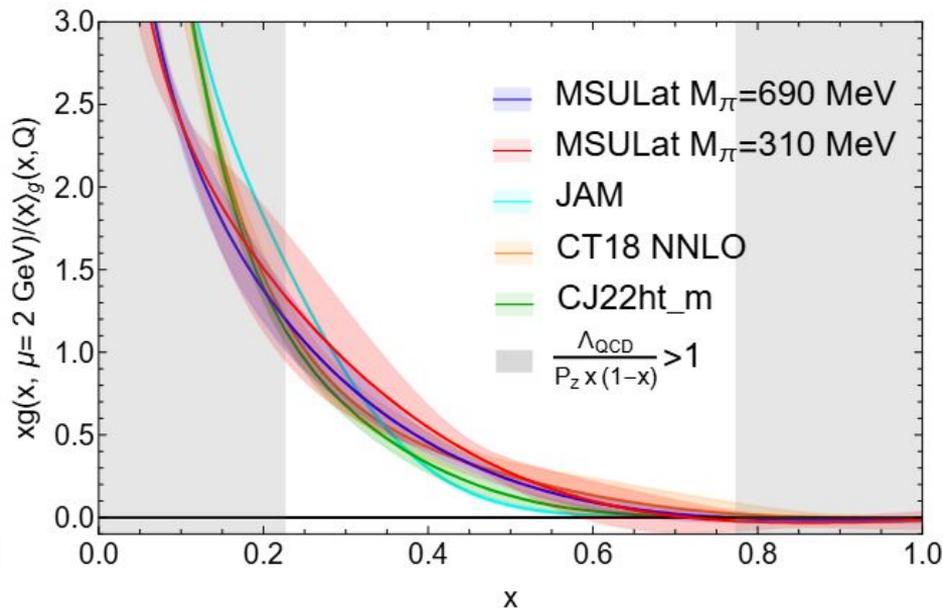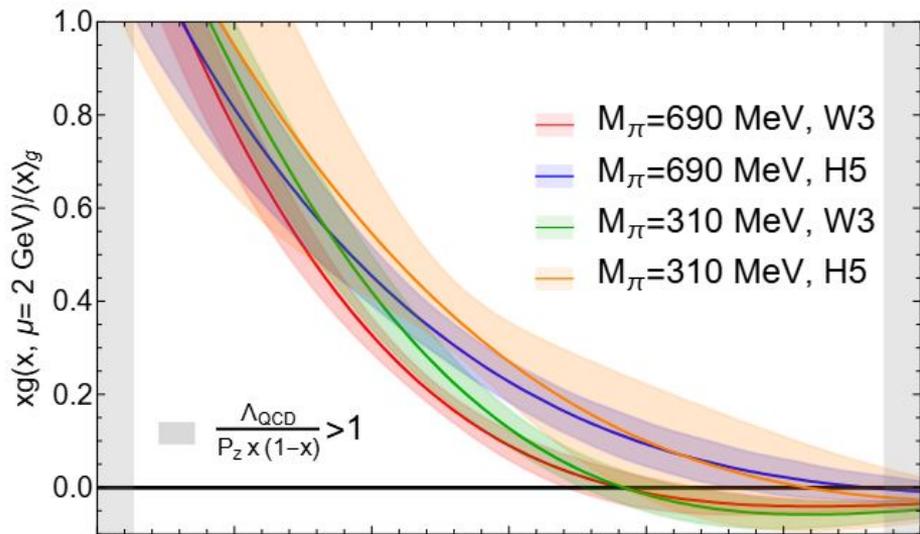
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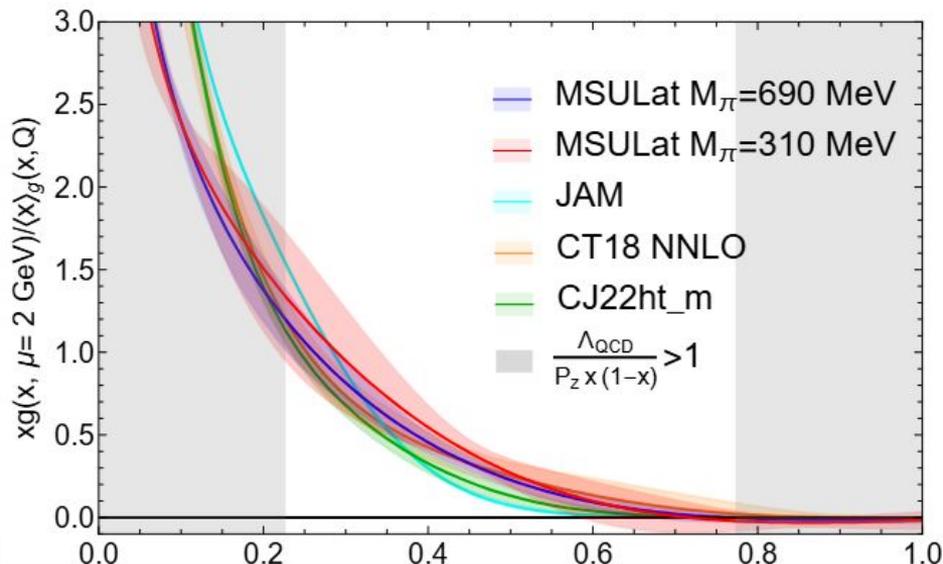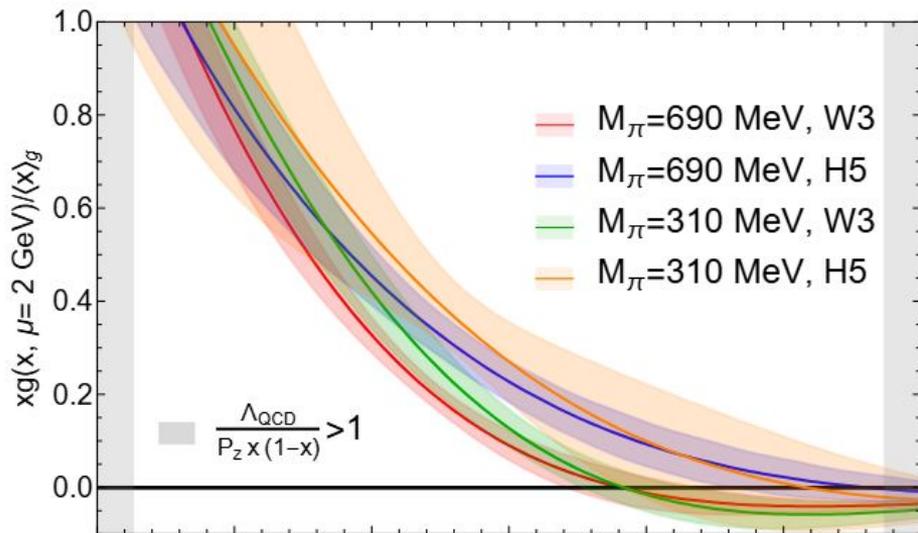

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The HYP5 PDFs are most consistent with the CT18 PDF, and only a standard deviation or two from the other PDFs

Outlook and Conclusion

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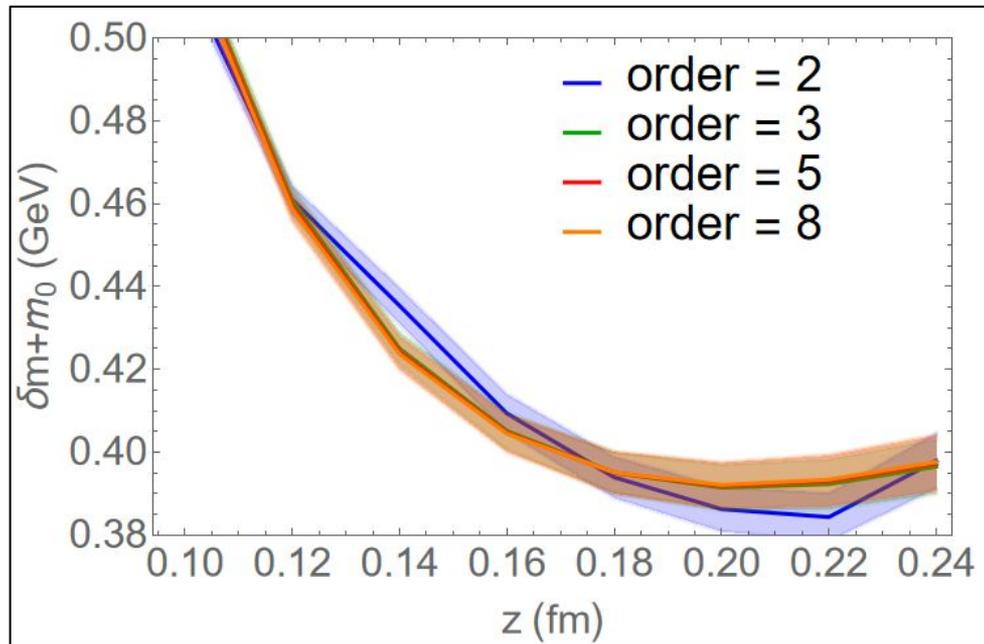- Reducing statistical uncertainties is still a major challenge, as well

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- Alex will show her results for a similar study with various lattice spacings with better control of discretization effects using self-renormalization

Backup Slides

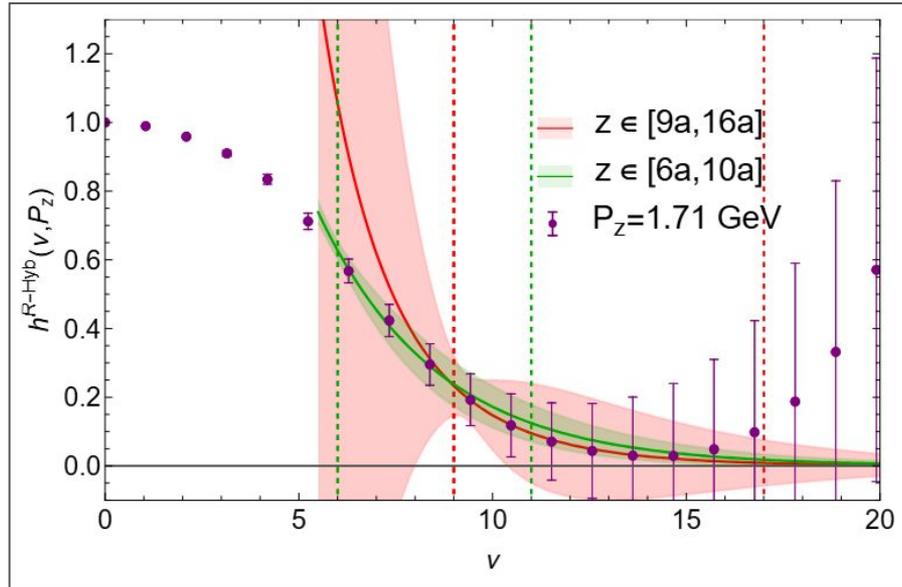# $\delta m + m_0$ Interpolation Order Dependence

- We checked how the interpolation affects the $\delta m + m_0$ fit, and we see convergence by order cubic interpolation

N_s W3

Extrapolation Fit Range Dependence (1)

- The errors on the extrapolation are quite sensitive to the fit range, while the central values are statistically indistinguishable
- Here we pick a very short range vs a much longer range for the extrapolations

Extrapolation Fit Range Dependence

- These errors do have noticeable effects on the final PDF, as well

- We think this could be a significant source of systematic uncertainty at the current level of statistics, so in the future, we would like to consider model averaging or some more sophisticated method to do the Extrapolations

- Currently, we just use an intermediate fit range