Gluon PDFs from LaMET with Self-Renormalization

Alex NieMiera* (niemiera@msu.edu)
LaMET 2025, Center for Frontiers in Nuclear Science

MICHIGAN STATE UNIVERSITY

Center for Frontiers in Nuclear Science

Stony Brook University

Brookhaven National Laboratory

In collaboration with:

Huey-Wen Lin, Bill Good, Fei Yao

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Gluon PDFs in Lattice QCD

Amaroso, *et al*. [Snowmass] Acta Physica Polonica. B, 53, 12 (2023)
Achenbach, *et al*. [LRP] Nucl. Phys. A 1047:122874 (2024)
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Gluon PDFs are important for predicting processes like Higgs & J/ψ production; however, phenomenological extractions struggle at large x due to sparse data…

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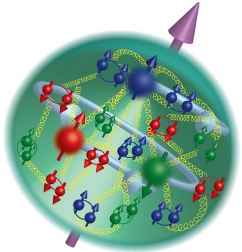

Image Credit: ANL

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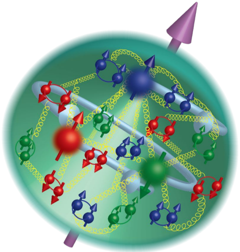

Image Credit: ANL

Lattice QCD is an *ab initio* method that can be used to obtain PDFs…

- While there are many approaches to extract PDFs from the lattice, we use large momentum effective theory (LaMET) in this work, where we can define PDFs as the Fourier transform of light front correlators,

$$xg(x) = \int \frac{d\xi^-}{2\pi P^+} e^{-ixP^+\xi^-} \langle P | F_a^{+i}(\xi^-) \mathcal{W}(\xi^-, 0) F_a^{+i}(0) | P \rangle$$

- The Wilson line $\mathcal{W}(\xi^-, 0)$ introduces linear divergences, which must be removed with a renormalization scheme to obtain finite, physically meaningful matrix elements.

Self Renormalization

LPC, J. Nuc. Phys B 115443 (2021)
Balitsky, *et. al.*, Phys. Rev. D.105.014008 (2022)

The self renormalization factor is defined as

$$Z_R(z, 1/a) = \exp\left[\frac{kz}{a\ln(a\Lambda_{\mathrm{QCD}})} + m_0 z + f(z)a^2 + \frac{5C_A}{3b_0}\ln\left(\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right) + \ln\left(1 + \frac{d}{\ln(1/a\Lambda_{\mathrm{QCD}})}\right)\right]$$

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The final two terms come from resummation of leading and sub-leading logarithmic divergences

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where we determine the free fitting parameters $k,\ \Lambda_{\mathrm{QCD}},\ m_0,\ d\ \text{and}\ f(z)$ through a fit across multiple lattice spacings using the parameterization,

$$\ln \tilde{h}^0_{\eta_s}(z, P_z = 0, 1/a) = \frac{kz}{a\ln(a/a\Lambda_{\mathrm{QCD}})} + f(z)a^2 + \frac{5C_A}{3b_0}\ln\left(\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\mathrm{QCD}})}\right) + \ln\left(1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right)$$

$$+ \begin{cases} \ln\left(C_{0,\mathrm{NLO}}(z,\mu)\right) + m_0 z, & z_0 \le z \le z_1 \\ g(z), & z_1 < z \end{cases}$$

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For the gluon, the Wilson coefficient is derived as

$$C_{0,\mathrm{NLO}}(z,\mu) = 1 + \frac{\alpha_s C_A}{4\pi}\left(\frac{5}{3}\ln\left[\frac{z^2\mu^2}{4e^{-2\gamma_E}}\right] + 3\right)$$

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$$+\begin{cases} \ln\left(C_{0,\rm NLO}(z,\mu)\right) + m_0 z, & z_0 \le z \le z_1 \\ g(z), & z_1 < z \end{cases}$$

In addition to the parameters above, we also fit g(z), the residual contribution containing intrinsic non-perturbative physics.

Bazavov, *et al.* [MILC], Phys. Rev. D 82:074501 (2010)
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Lüscher, JHEP 08 (2010) 071

Lattice Data Summary

- We use lattices with $N_f = 2 + 1 + 1$ highly improved staggered quarks generated by the MILC collaboration with Wilson-clover fermions in the valence sector.

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- Calculations are performed at lattice spacings of $a = \{0.151, 0.1207, 0.0888\}$ fm with valence pion masses of $M_\pi \approx 310,\ 690$ MeV.

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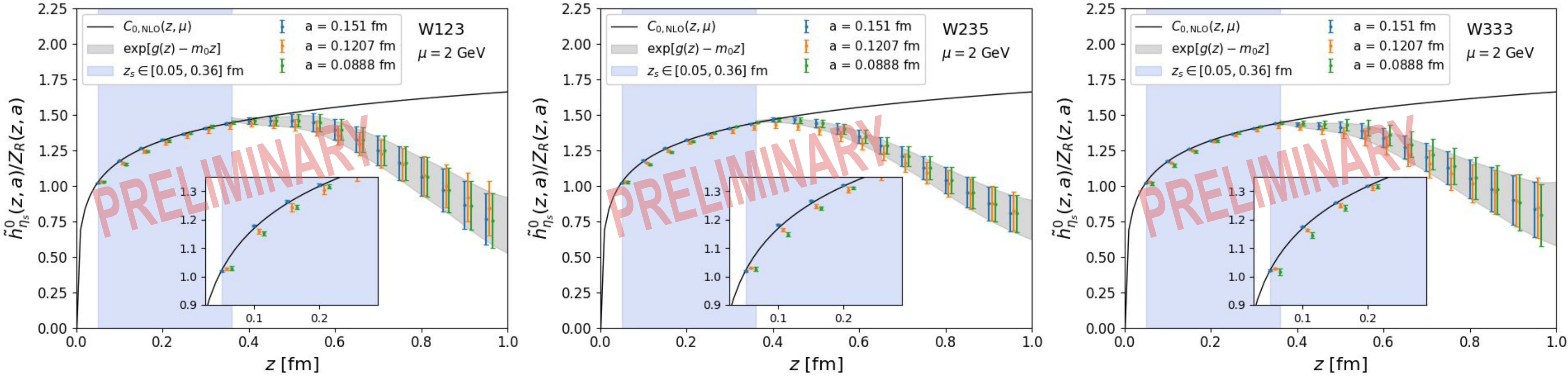
  - **W333** → fixed relative flow time of $t = 3a^2$ across all three lattices
  - **W123** → fixed physical flow time of $t = \{1a^2, 2a^2, 3a^2\}$ for lattices $a = \{0.151, 0.1207, 0.0888\}$ fm.
  - **W235** → fixed physical flow time of $t = \{2a^2, 3a^2, 5a^2\}$ for lattices $a = \{0.151, 0.1207, 0.0888\}$ fm.

The Renormalization Factors

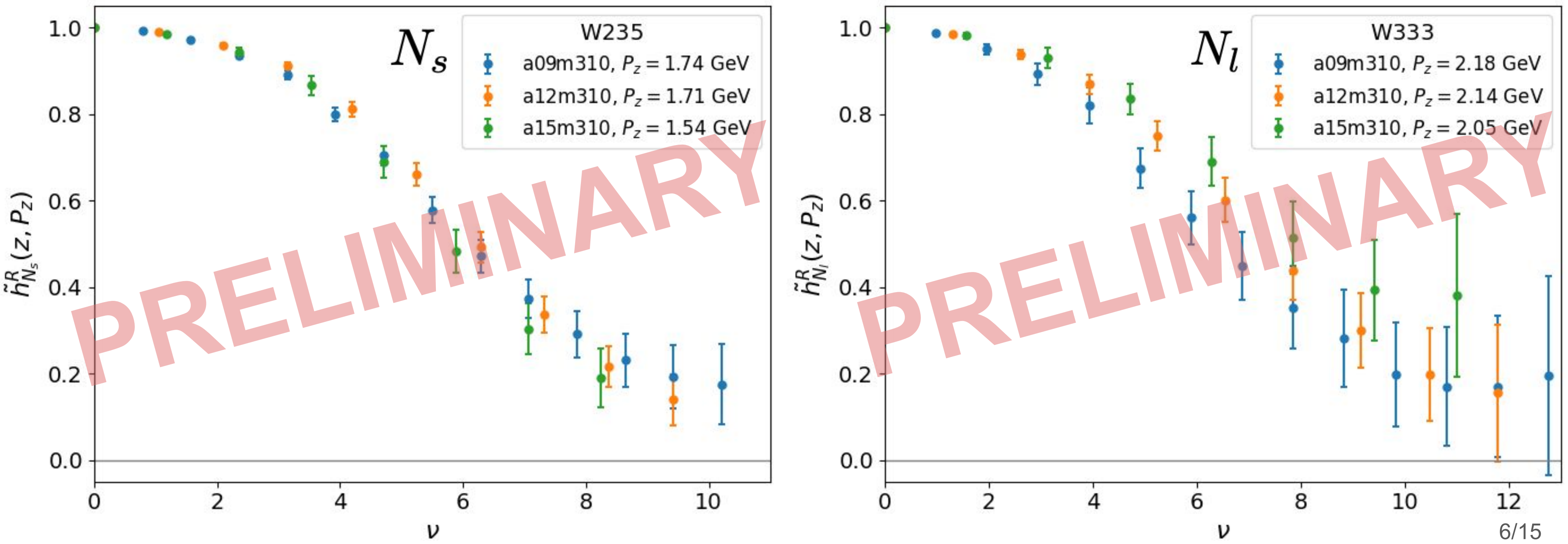
We find reasonable fitted parameters across all smearing schemes for our renormalization factors and similar values of χ^2/dof .

Parameter	W333	W123	W235
k	0.60(28)	1.04(56)	0.66(28)
d	~ 0.0	~ 0.0	~ 0.0
Λ_{QCD} (GeV)	0.295(84)	0.141(59)	0.141(55)
m_0 (GeV)	0.14(31)	0.45(52)	0.12(22)
χ^2/dof	0.75(48)	1.00(76)	0.82(62)



Renormalized Matrix Elements

Lattice spacing dependence seems to be removed from the renormalized matrix elements, regardless of smearing scheme used, and we find that everything is statistically equivalent…



Renormalized Matrix Elements - Additional Comparisons

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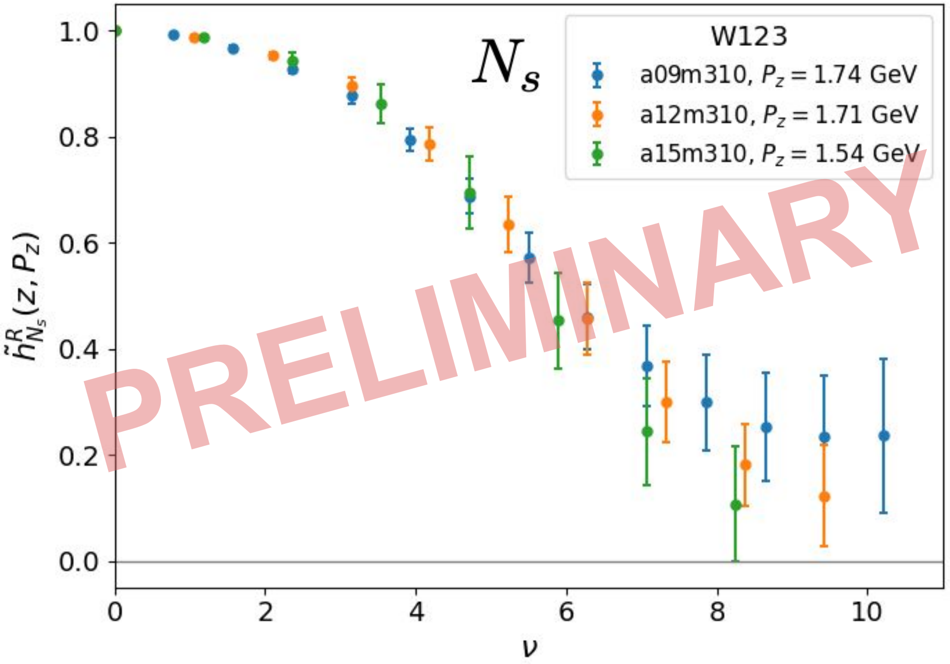
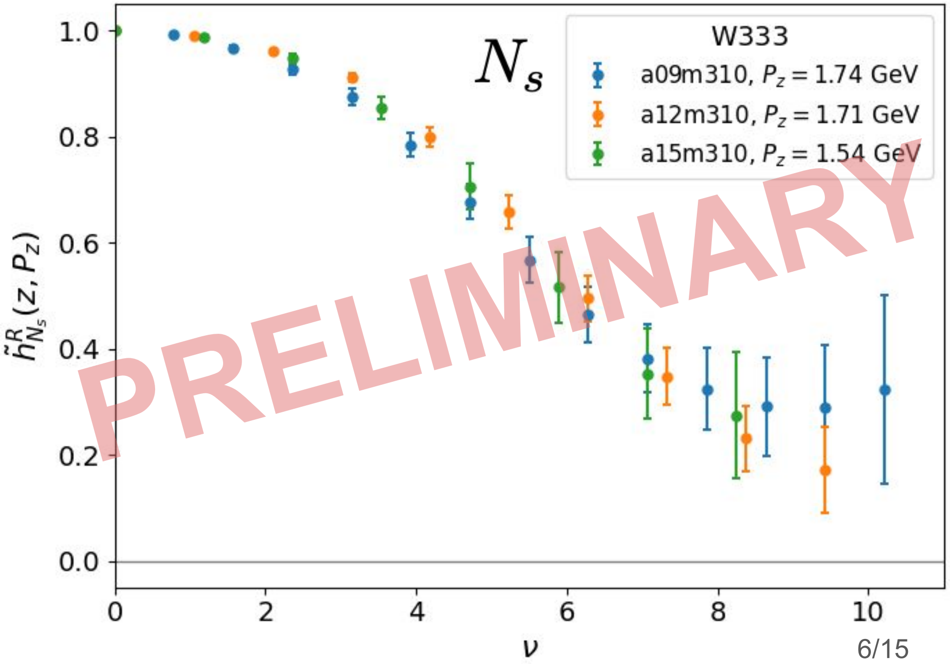
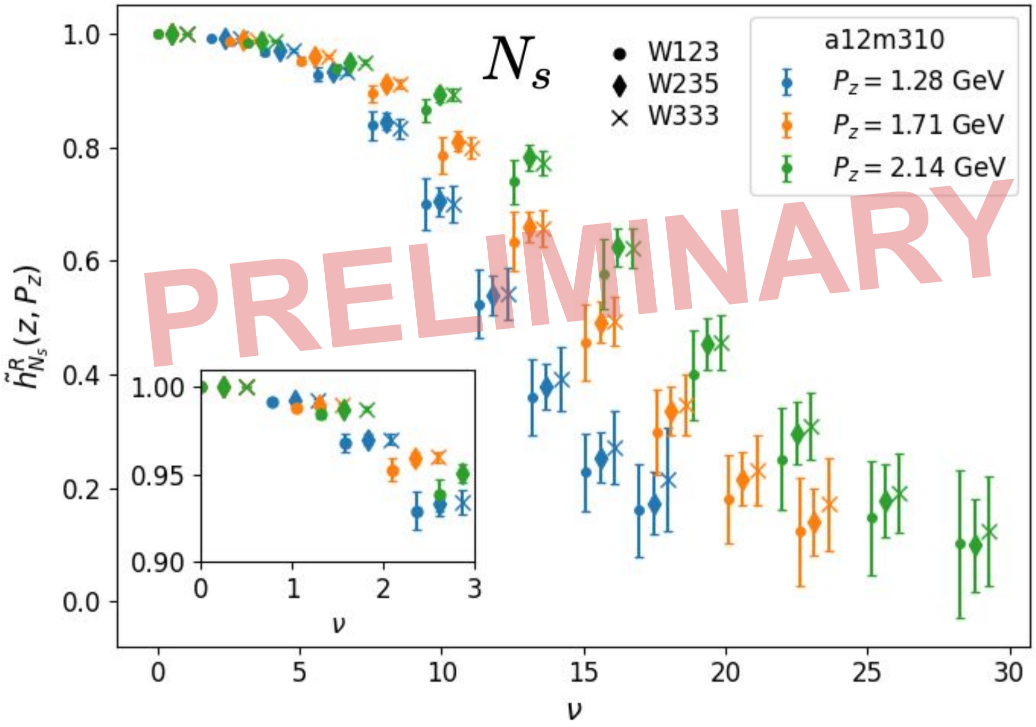




Smearing Analysis Comparison

We consider the heavier pion mass here for better control over statistical fluctuations…

For this representative case, W235 and W333 have identical smearing on the ensemble, but fixed physical and relative smearing for the renormalization factor.



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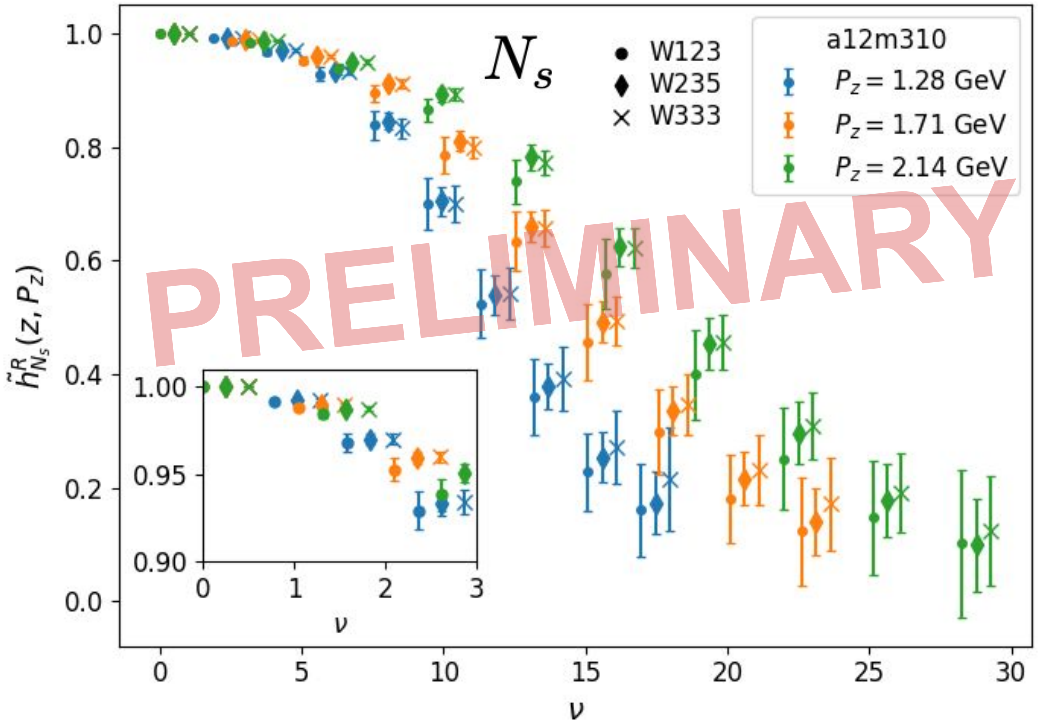

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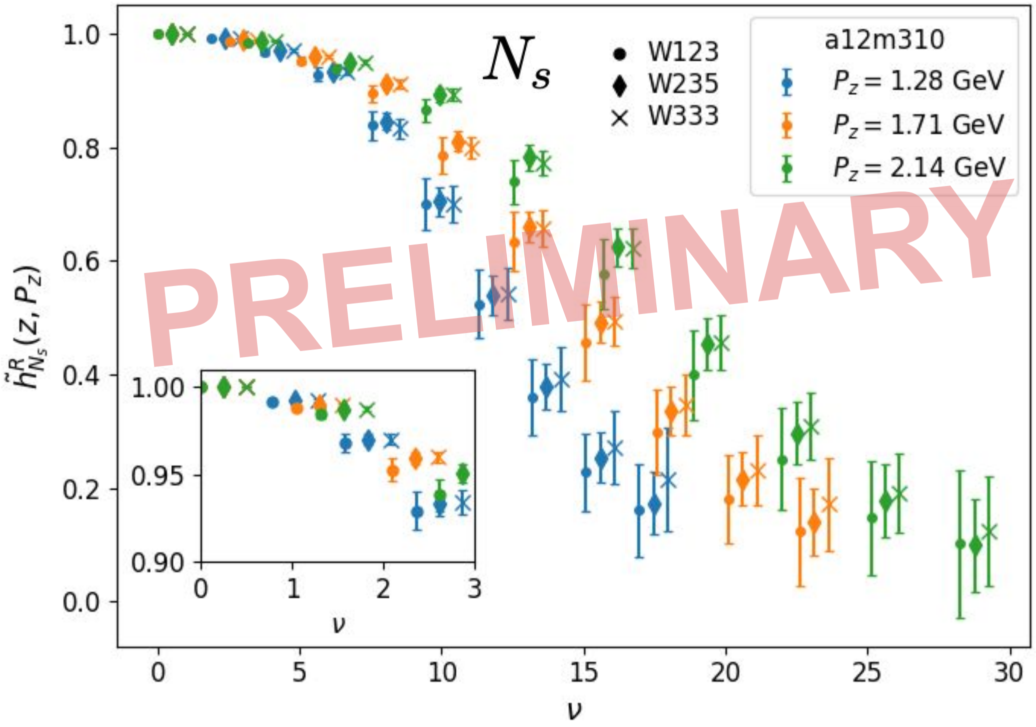
- At our current statistics, the smearing scheme seems to have minimal impact on the results…
- The self-renormalization procedure seems to accommodate both fixed physical and relative smearing, suggesting it might absorb smearing differences as lattice-spacing effects.



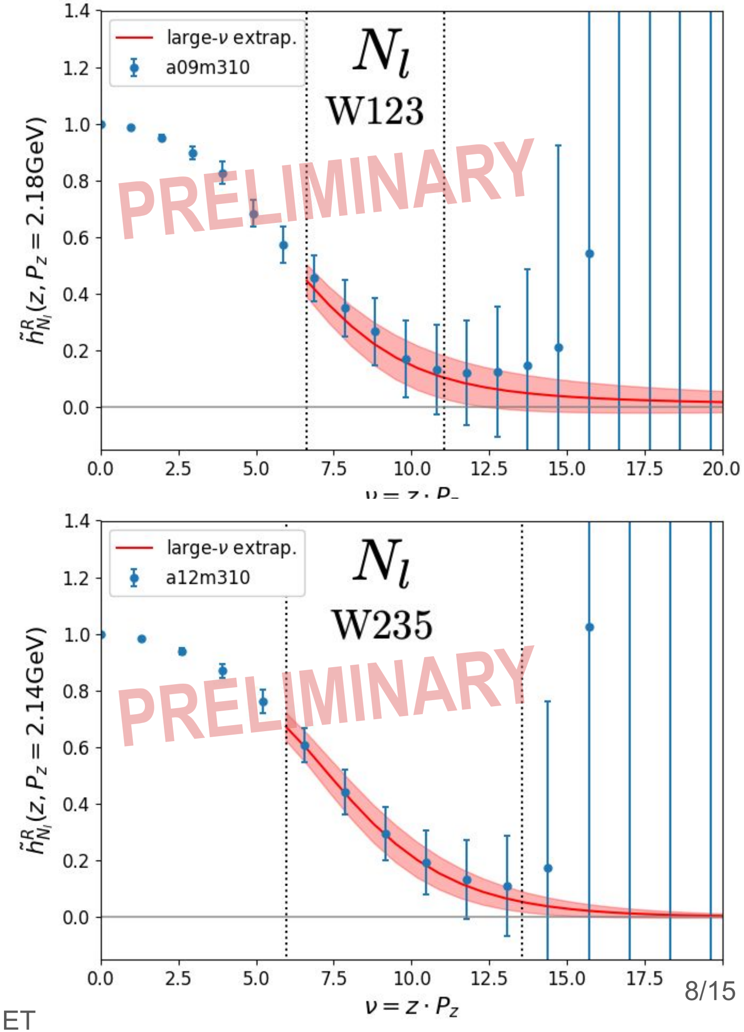
Large-ν Extrapolation

We apply a physically motivated extrapolation form to fit the renormalized matrix elements at intermediate - large distances with $z \gtrsim 0.6$ fm ,

$$\tilde{h}^R(z, P_z) \approx A \frac{e^{-m\nu}}{|\nu|^d}$$

Gao, et al. Phys. Rev. Lett. 128, 142003 (2022)

We're able to obtain reasonable extrapolations across all ensembles and pion masses for each smearing scheme (that's ~50 extrapolations!); however, future work should explore possible systematics more carefully.



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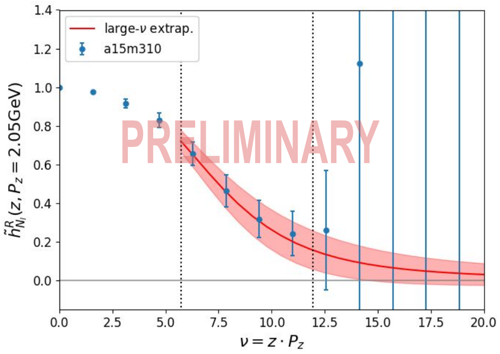

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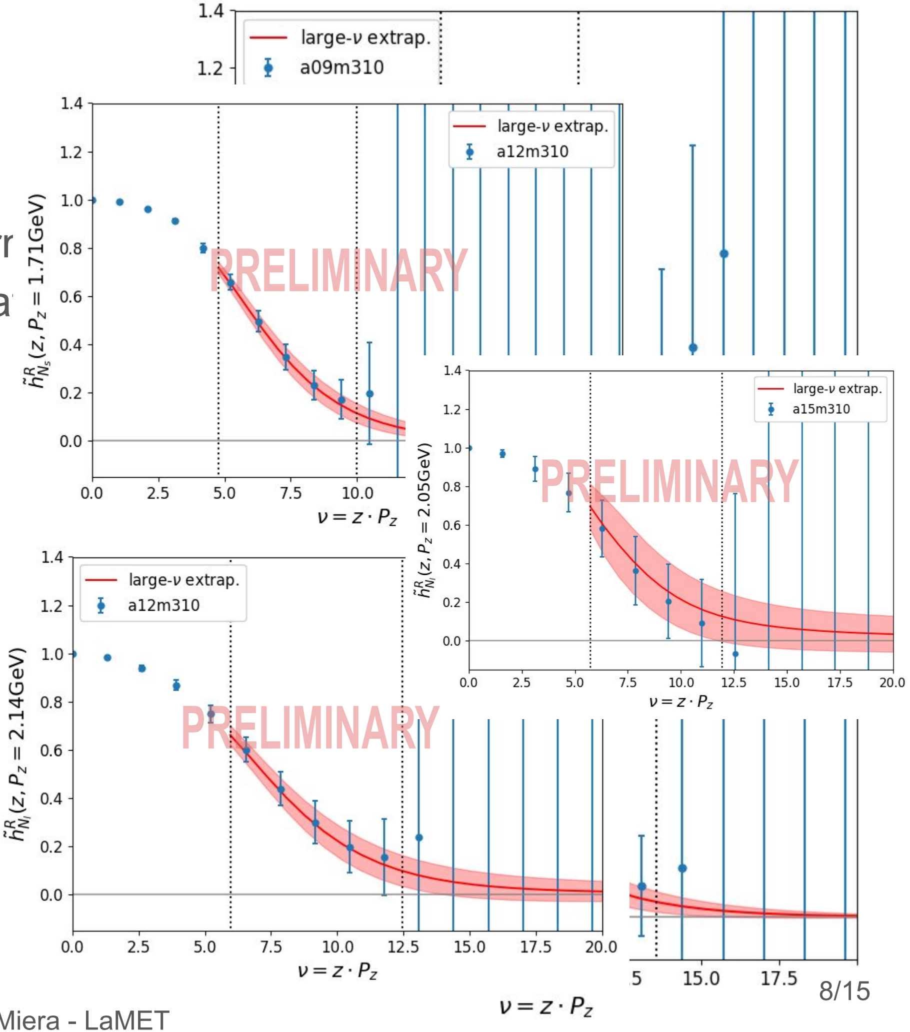

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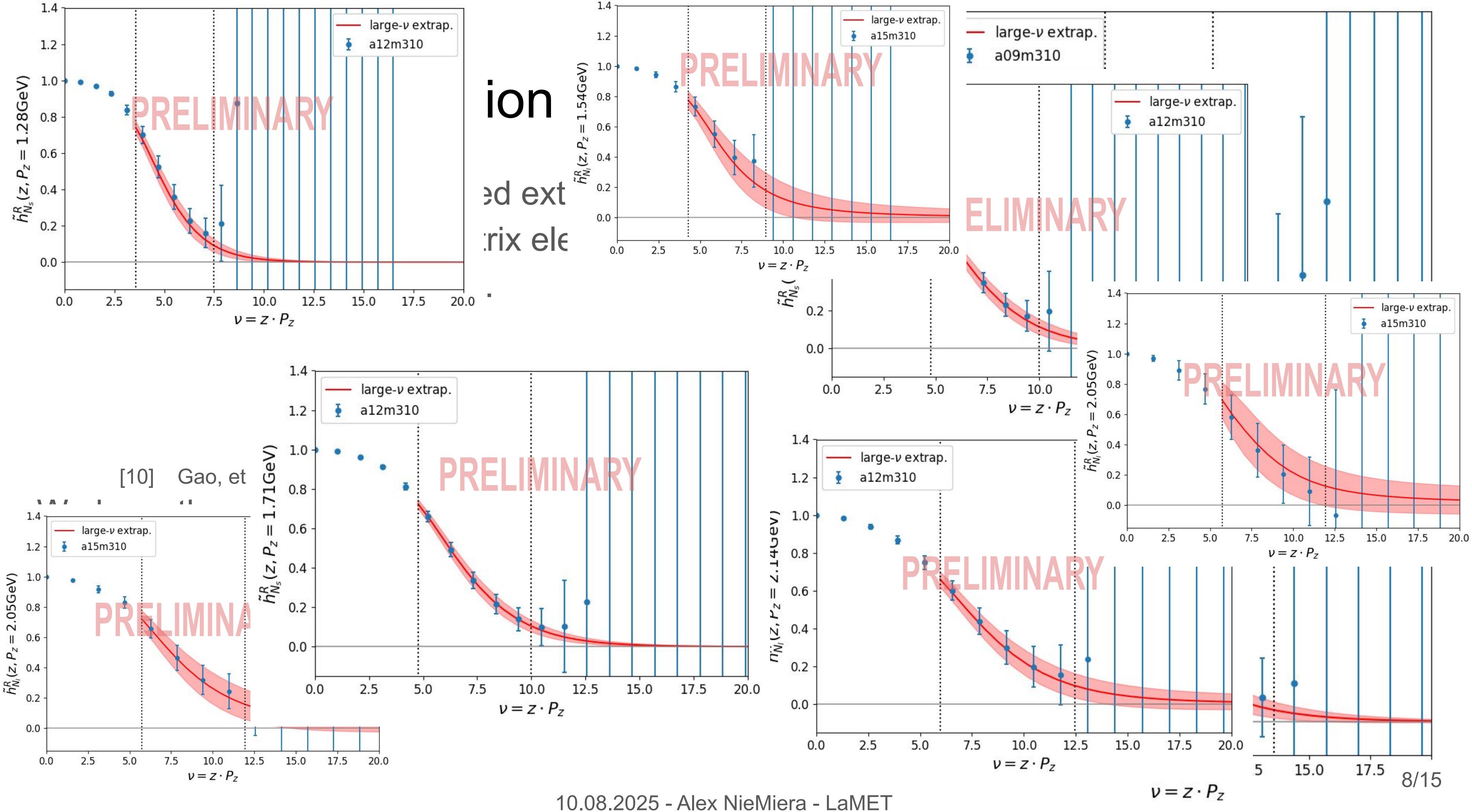

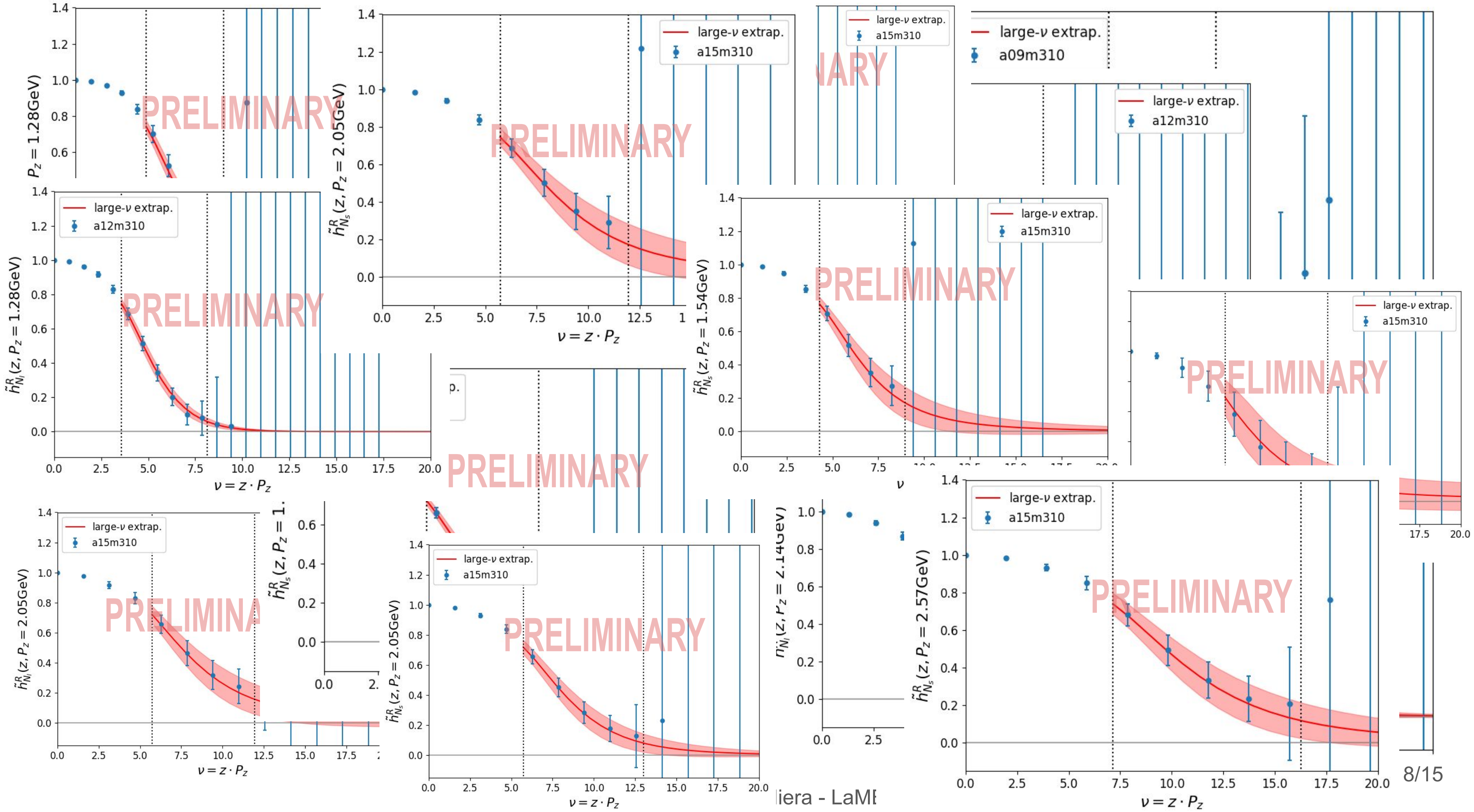
PRELIMINARY
large-ν extrap.
a15m310
a12m310
a09m310
$P_z = 1.28$GeV
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$\tilde{h}^R_{N_l}(z, P_z = 1.28\text{GeV})$
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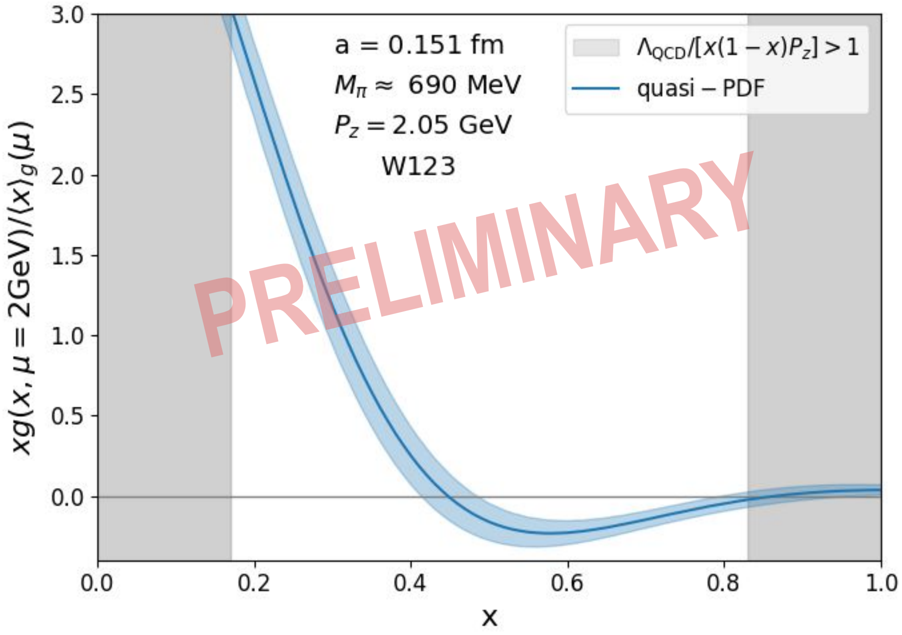
Quasi-PDF Matching

Combining the interpolated short distance lattice data with the long distance extrapolation gives a smooth function to obtain the quasi-PDFs



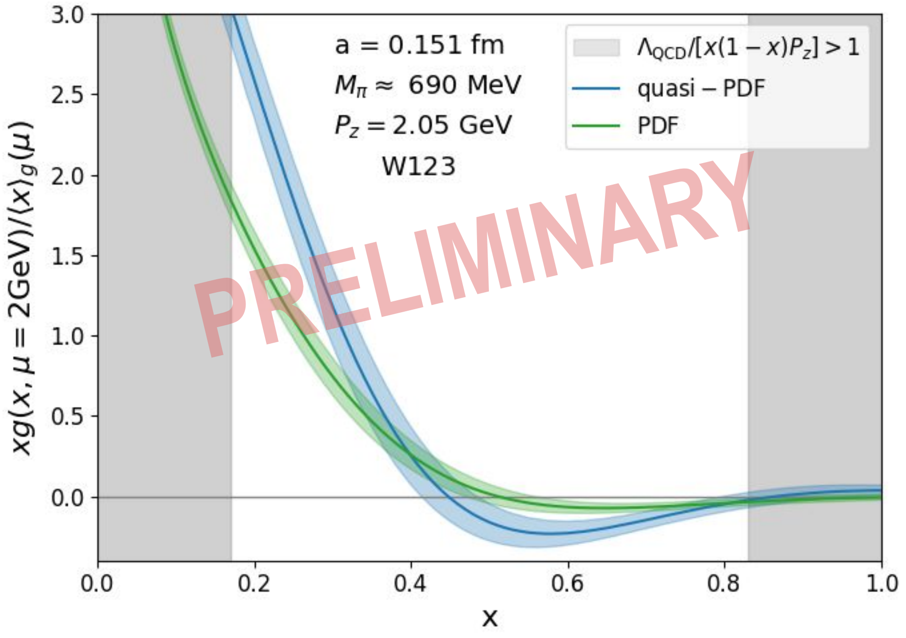
Quasi-PDF Matching

Additionally, the matching procedure seems to get rid of some of the negativity in the quasi-PDF.

Combining the interpolated short distance lattice data with the long distance extrapolation gives a smooth function to obtain the quasi-PDFs

$$x\tilde{g}(x, P_z, \mu) = \int_{-1}^{+1} dy\, F_{gg}(x, y, \mu) y g(y, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P_z)^2}\right)$$

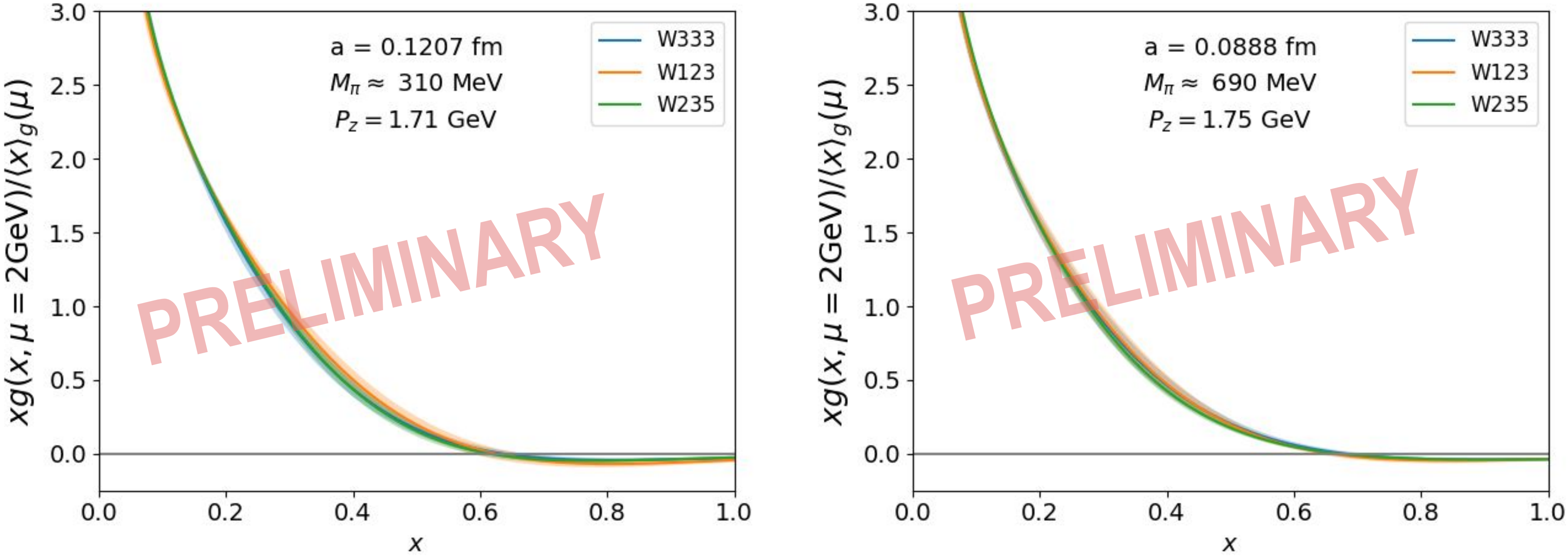
After matching, we obtain our gluon PDFs!



We use $\Lambda_{\rm QCD} = 300$ MeV

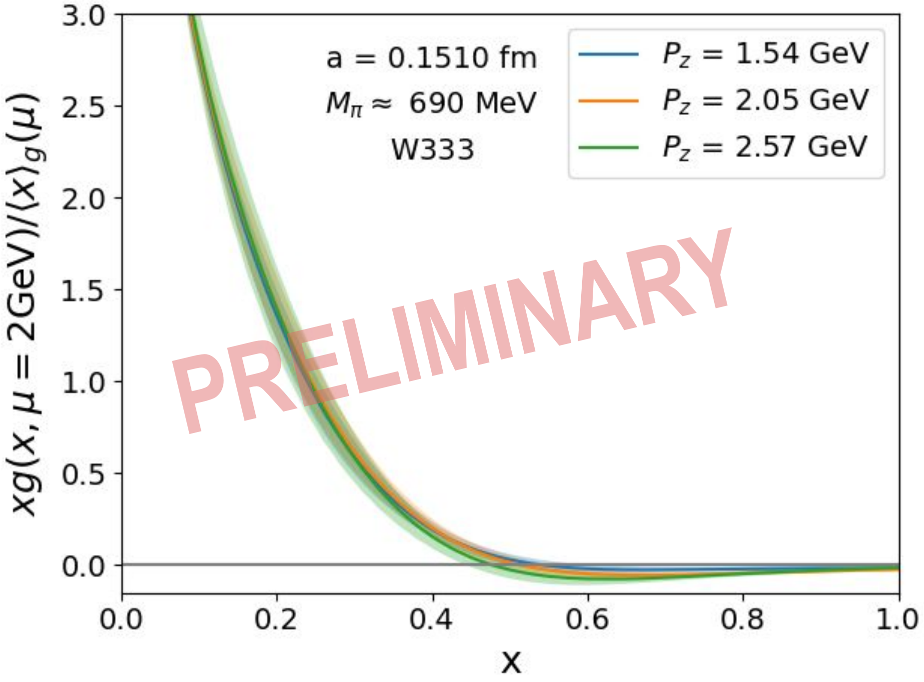
PDF Dependencies: Smearing Scheme

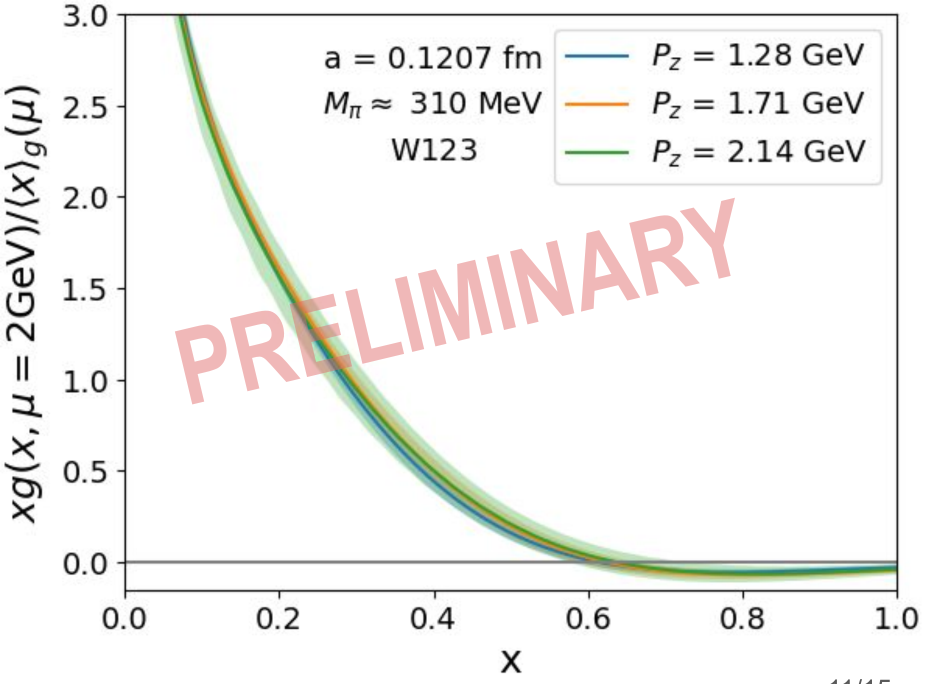
As seen at the matrix element level, the PDFs are strikingly similar across smearing scheme!



PDF Dependencies: Momentum

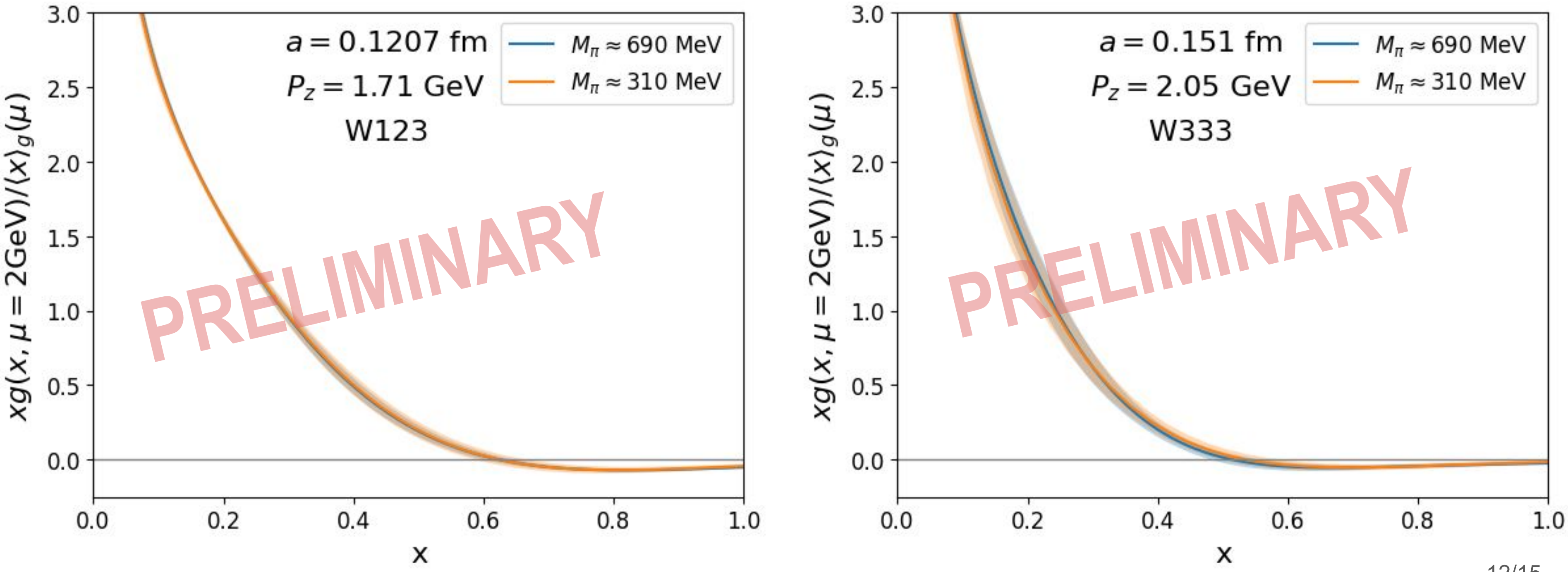
The momentum dependence is also statistically equivalent across all lattice combinations…





PDF Dependencies: Pion Mass

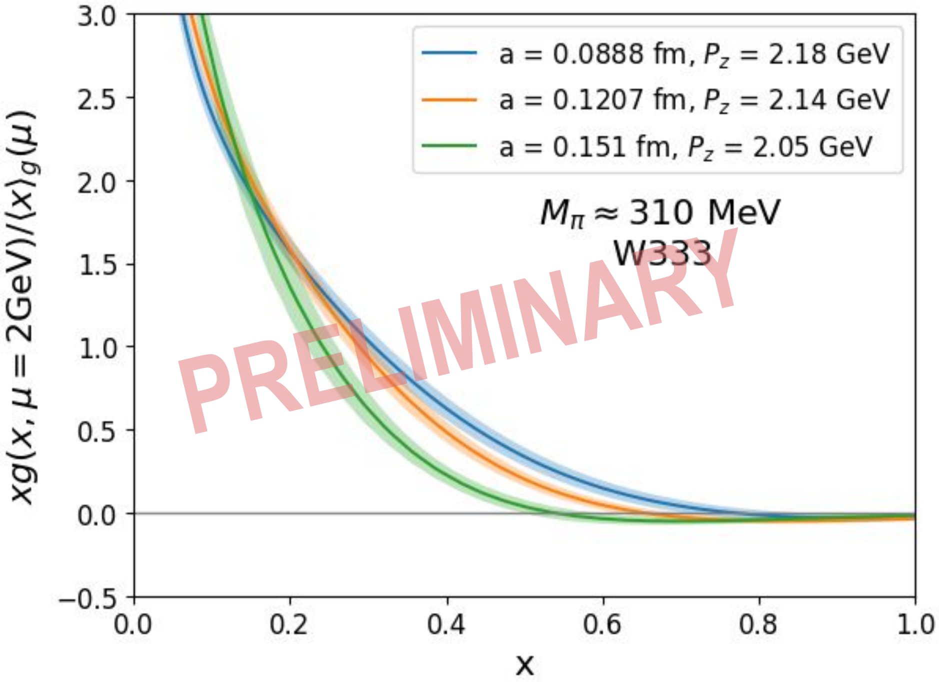
Additionally, we have very little dependence on the pion mass…

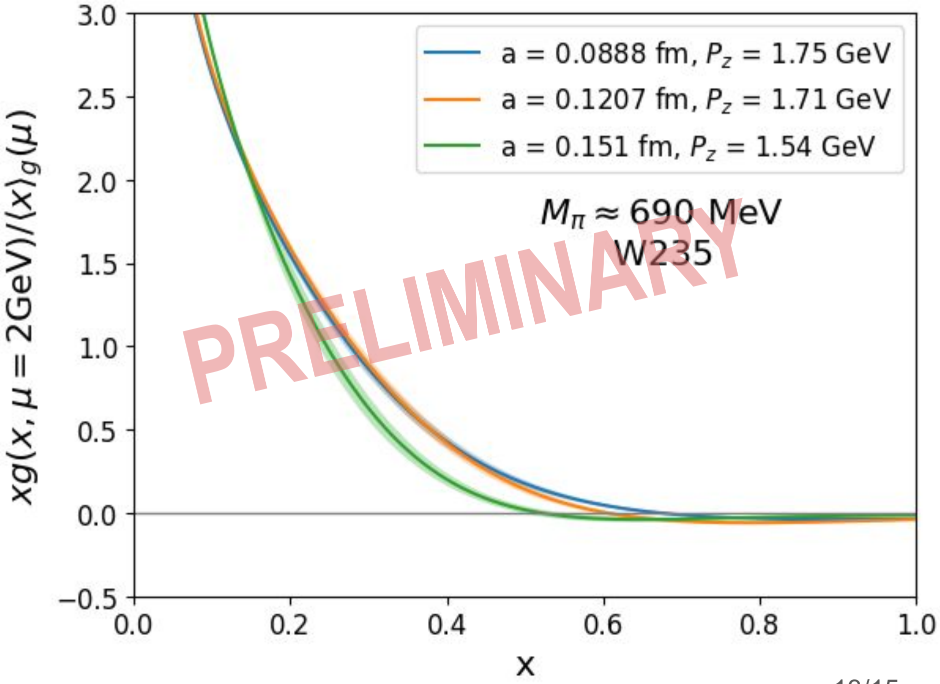


PDF Dependencies: Lattice Spacing

We hope to replace our coarsest lattice spacing with something finer in future work…

Lattice spacing seems to be the only systematic not under full control yet…



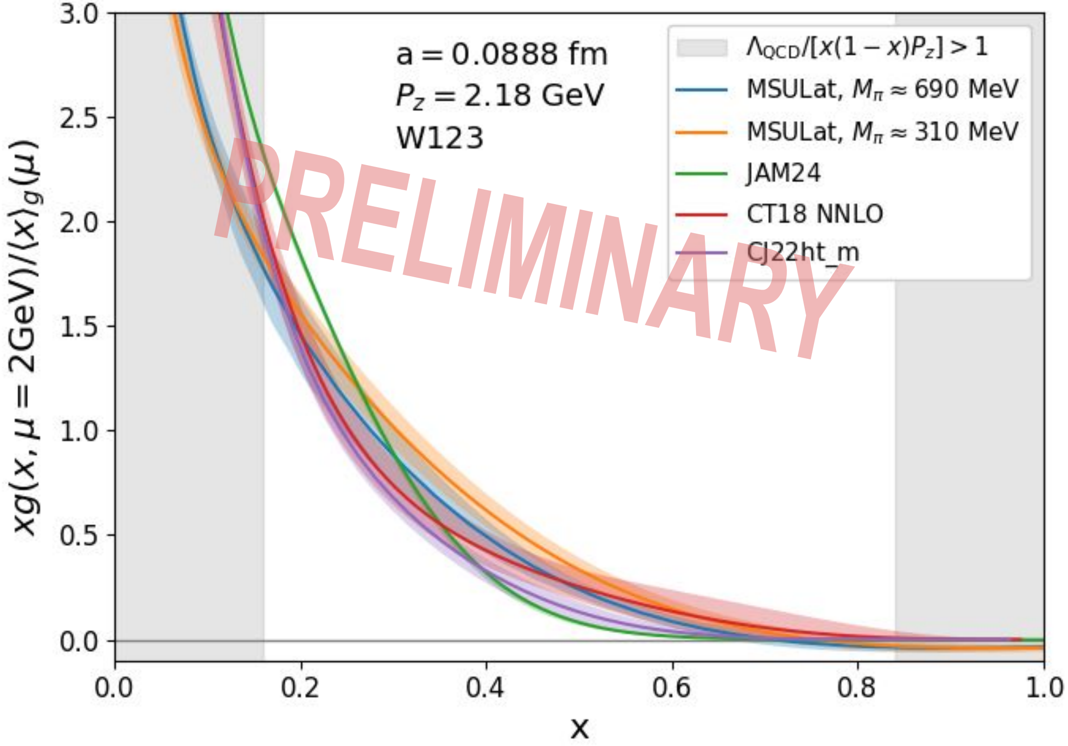


Gluon PDFs vs. Pheno PDFs

Anderson, et al. arXiv 2501.00665 (2024)
Hou, et al. (CTEQ) PRD 103(1):014013 (2021)
Cerutti, et al. PRD 111:094013 (2025)

Comparing the PDFs extracted from our most physical lattice spacing and largest momentum, we find overall good agreement with the selected pheno PDFs…

- We hope this work might inspire the community to further explore possible renormalization schemes that absorb smearing.



Conclusions & Outlook

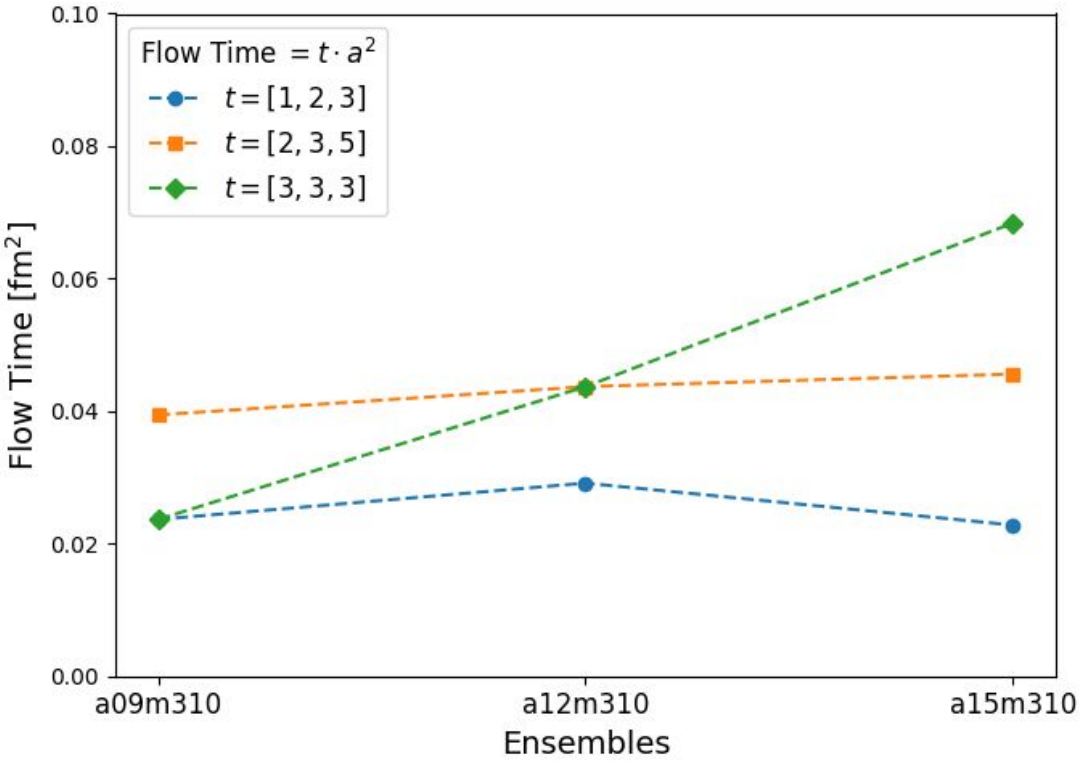
- We have calculated gluon PDFs from LaMET using self-renormalization!
- A smearing study with fixed physical and relative schemes shows minimal impact on the self-renormalization factor and resulting PDFs.
- At the current statistical level, lattice spacing effects are the dominant systematic; however, this can be controlled with a continuum extrapolation.
- Future work could explore the renormalization scale dependence and include leading renormalon and renormalization-group resummations…

Thank you for your attention!

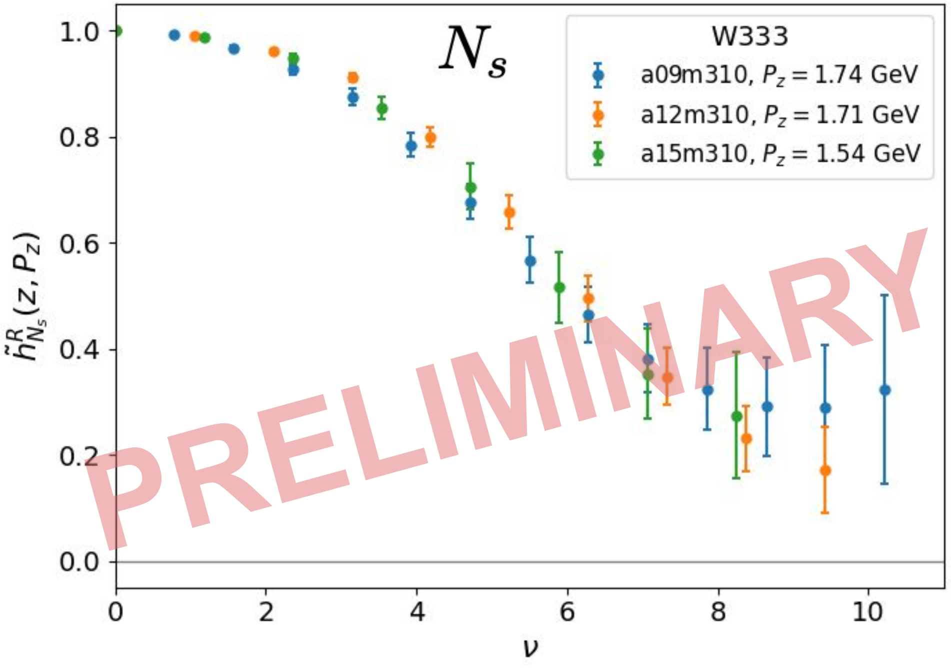
Questions?

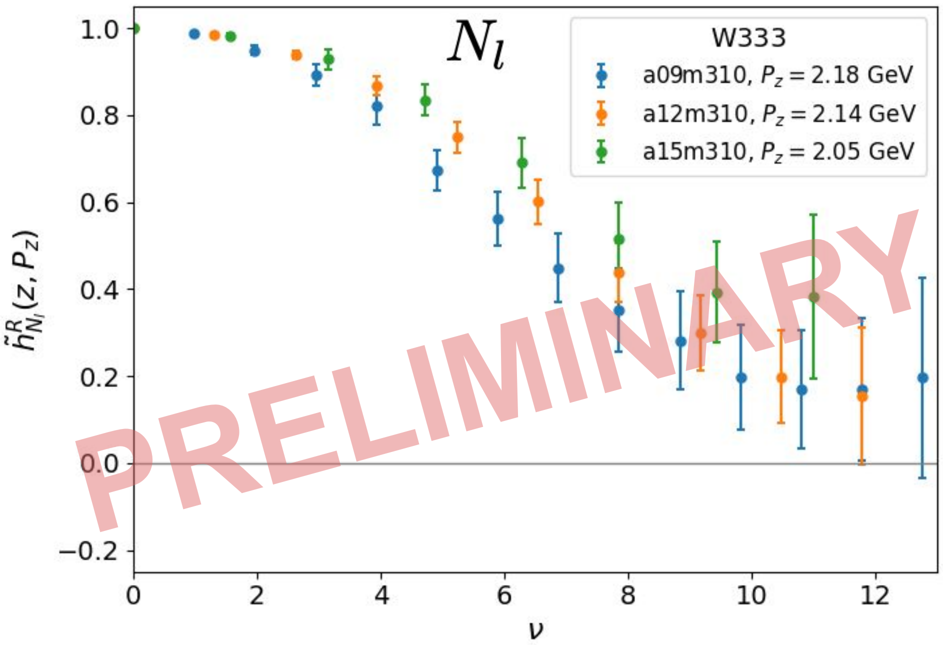
Backup Slides

Fixed Physical vs. Relative Smearing

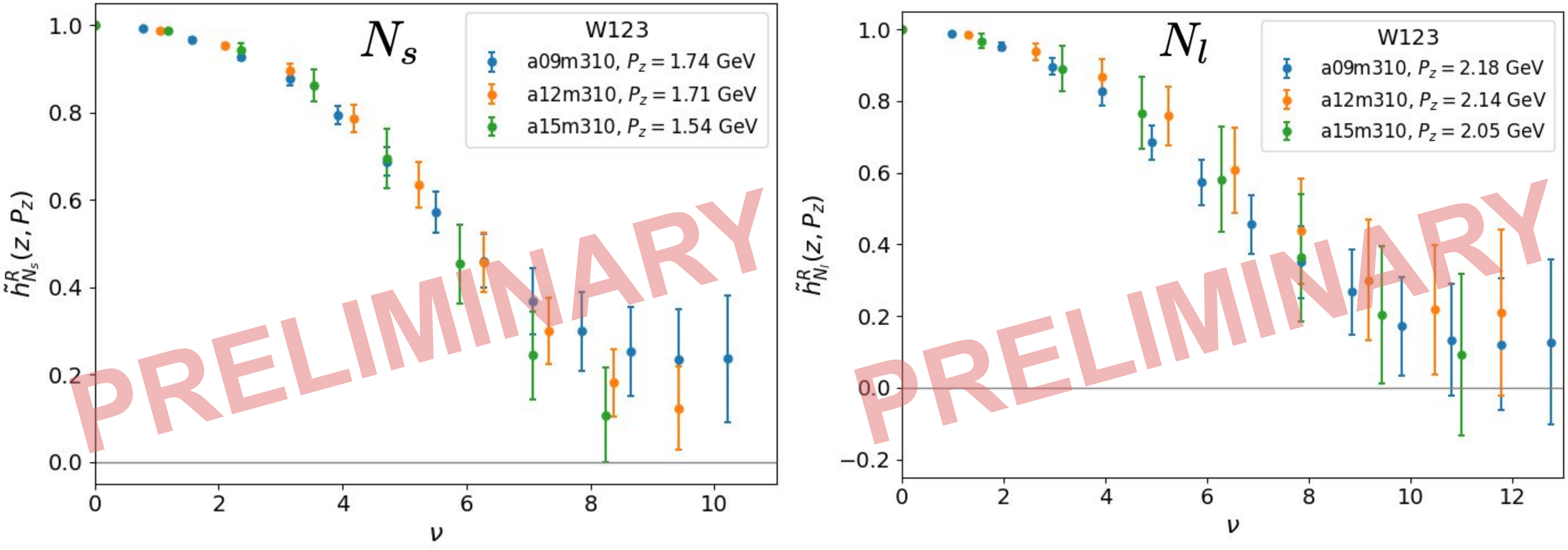


Renormalized Matrix Elements: W333

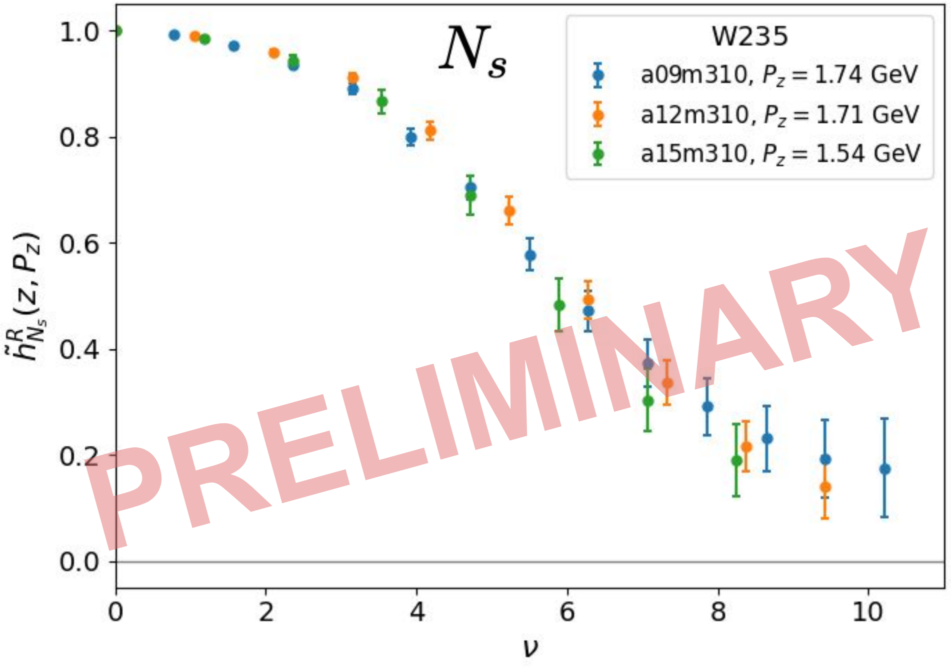


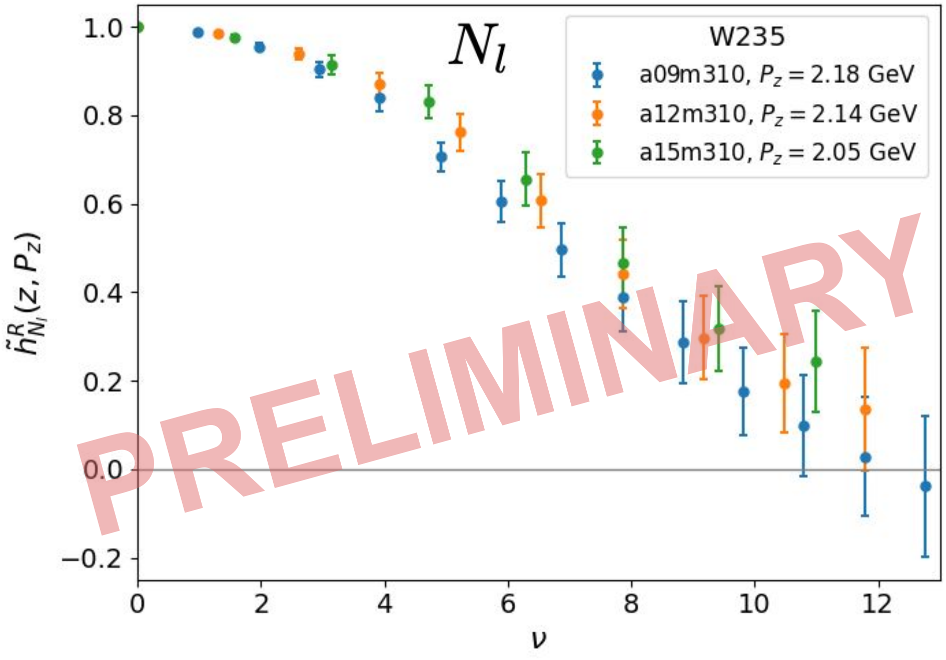


Renormalized Matrix Elements: W123



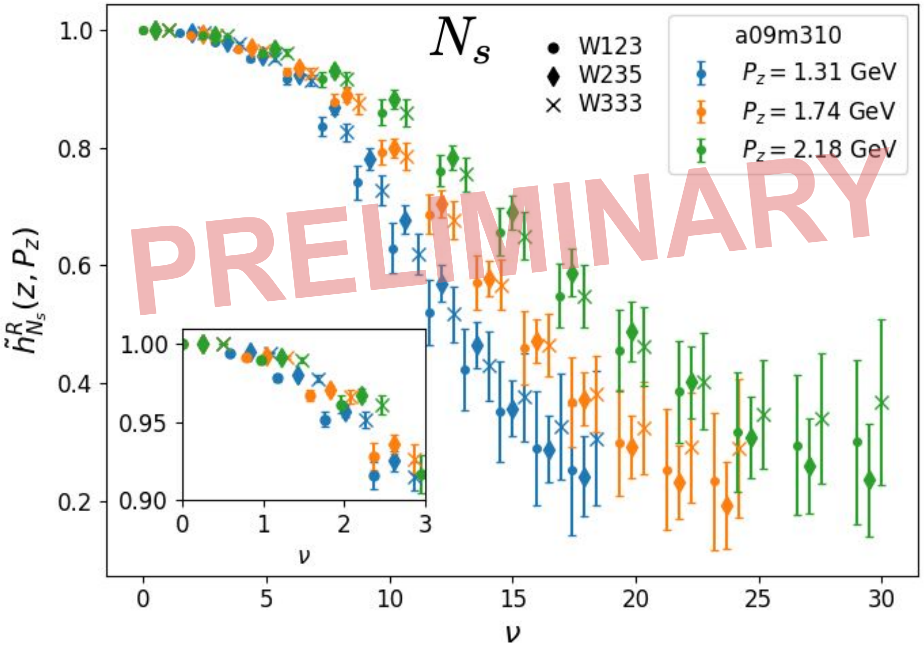
Renormalized Matrix Elements: W235

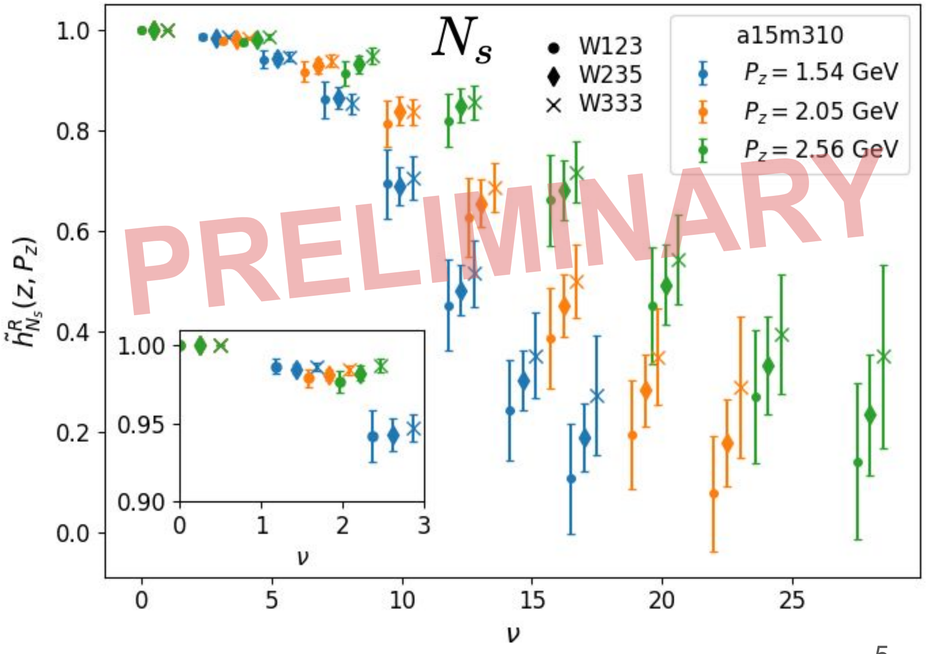




Additional Smearing Analysis Comparison

Additional comparisons of fixed physical and relative smearing schemes for the a09m310 and a15m310 ensembles across select momenta…





Hybrid Self Renormalization vs. Hybrid Renormalization