Lattice QCD Benchmark of Proton and Pion TMDs

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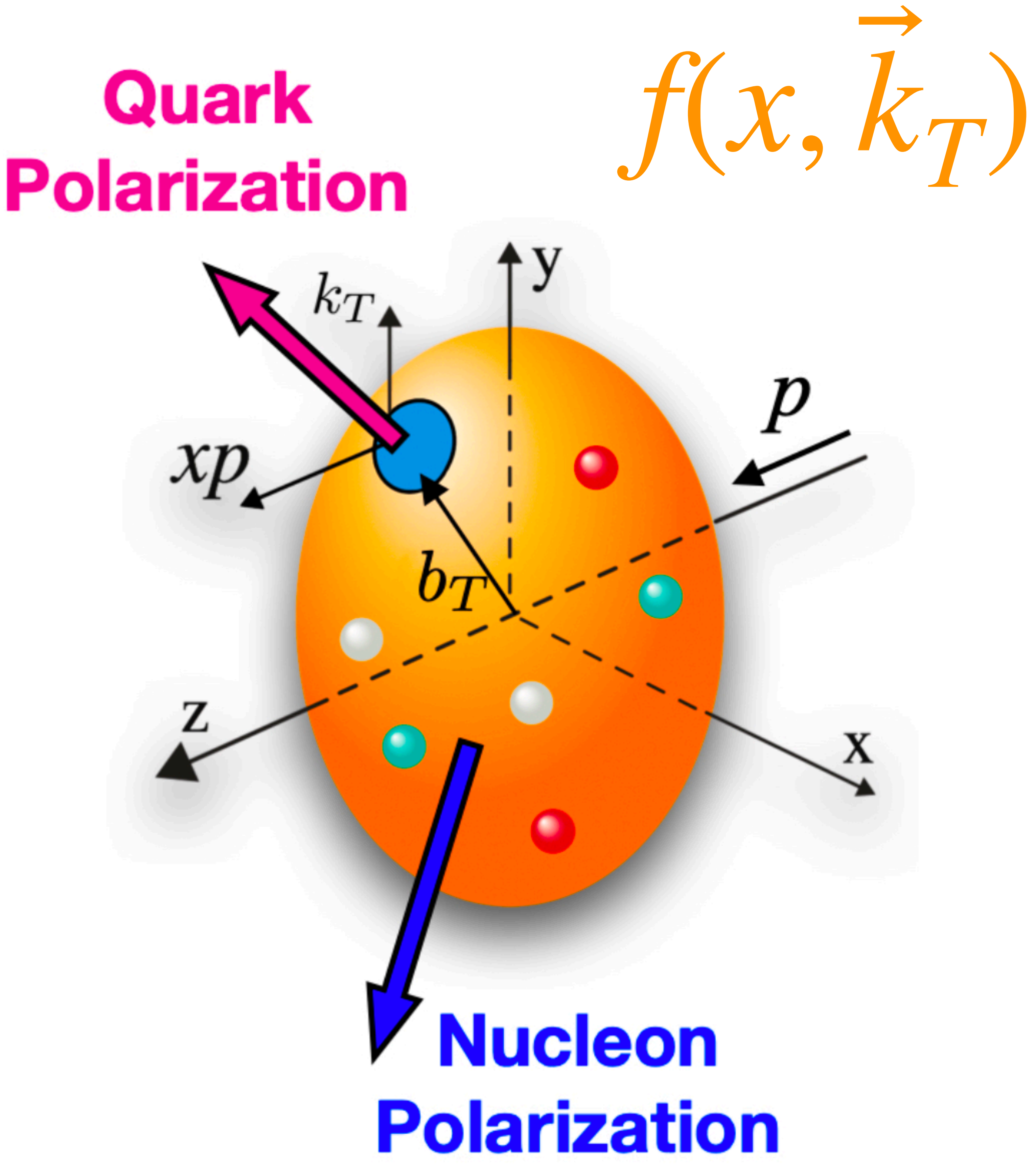
Transverse-momentum-dependent distributions



- Leading-power ("twist-2") quark TMDs

Nucleon Polarization	Quark Polarization: Un-Polarized (U)	Quark Polarization: Longitudinally Polarized (L)	Quark Polarization: Transversely Polarized (T)
U	f_1 = Unpolarized		$h_1^\perp$ = Boer-Mulders
L		g_1 = Helicity	$h_{1L}^\perp$ = Worm-gear
T	$f_{1T}^\perp$ = Sivers	$g_{1T}^\perp$ = Worm-gear	h_1 = Transversity $h_{1T}^\perp$ = Pretzelosity

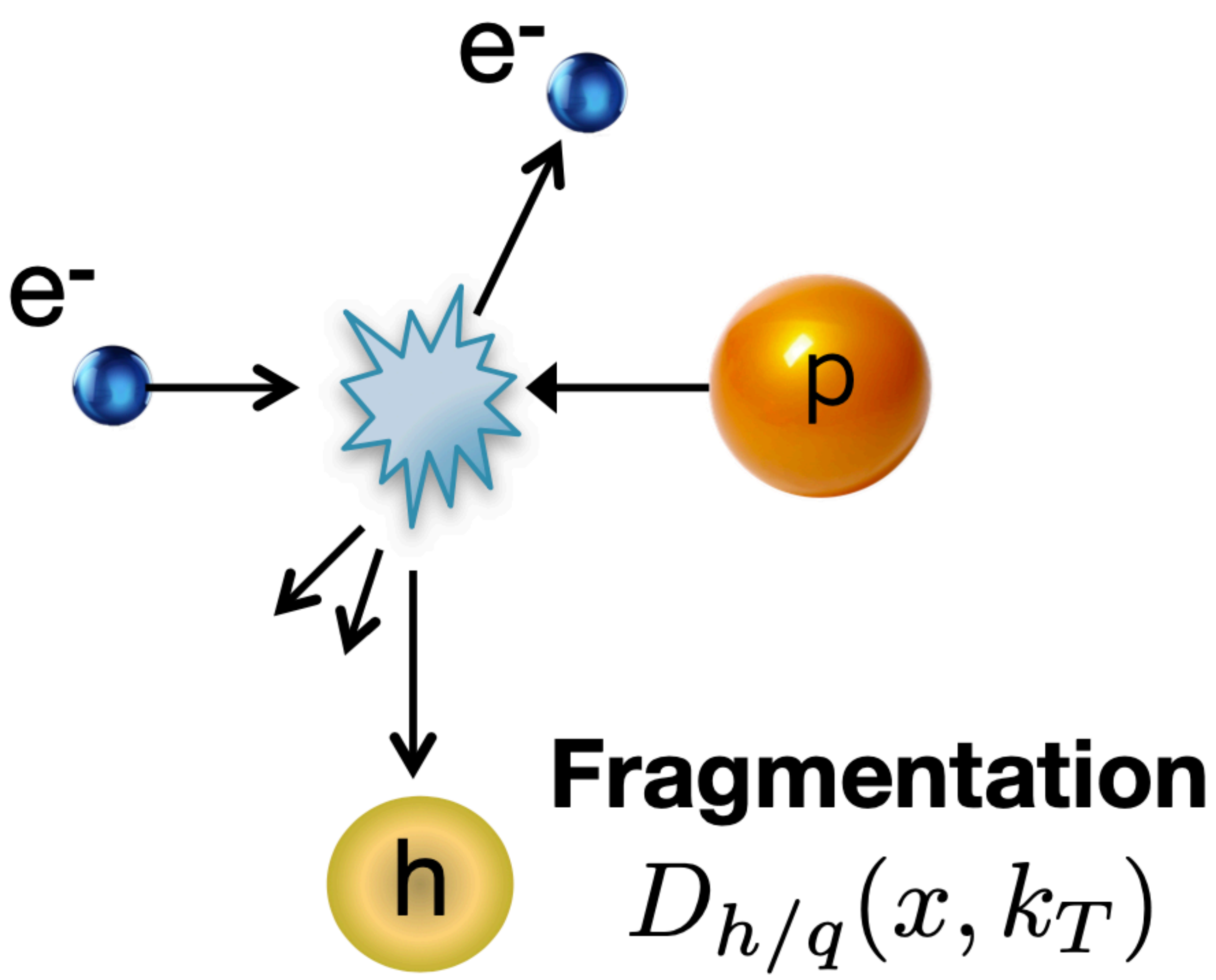
- TMD handbook, arXiv:2304.03302

- 3D image: longitudinal momentum fraction x and confined motion $\vec{k}_T$.
- Nucleon spin structure: orbital motion, Spin-orbit correlations…

TMDs from global analyses of experimental data

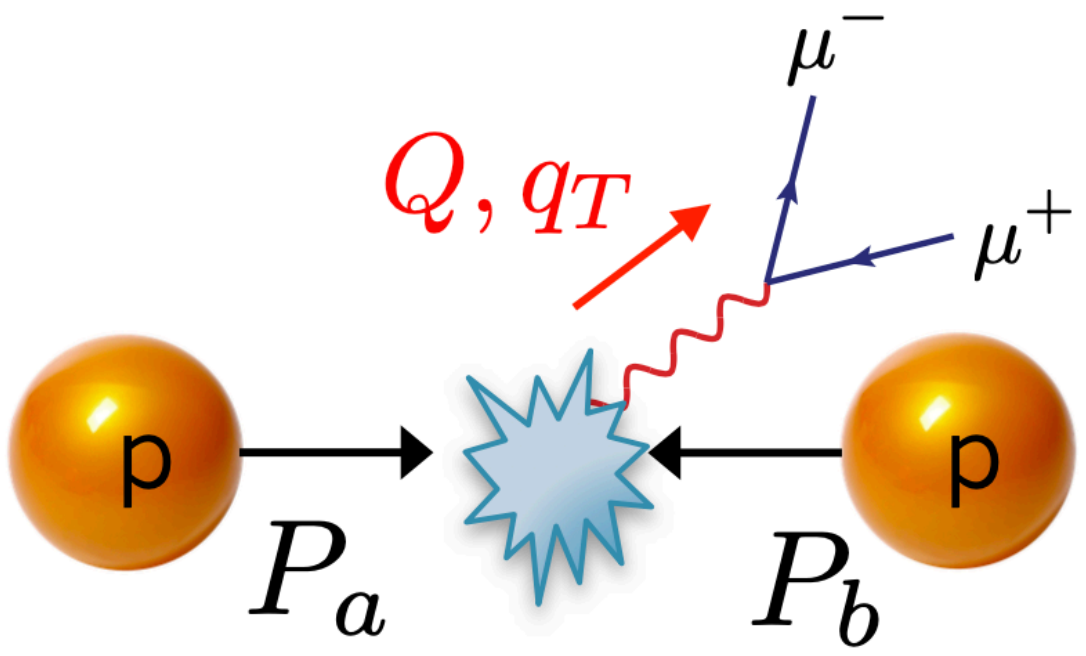
Semi-Inclusive DIS

$$\sigma \sim f_{q/P}(x, k_T) D_{h/q}(x, k_T)$$



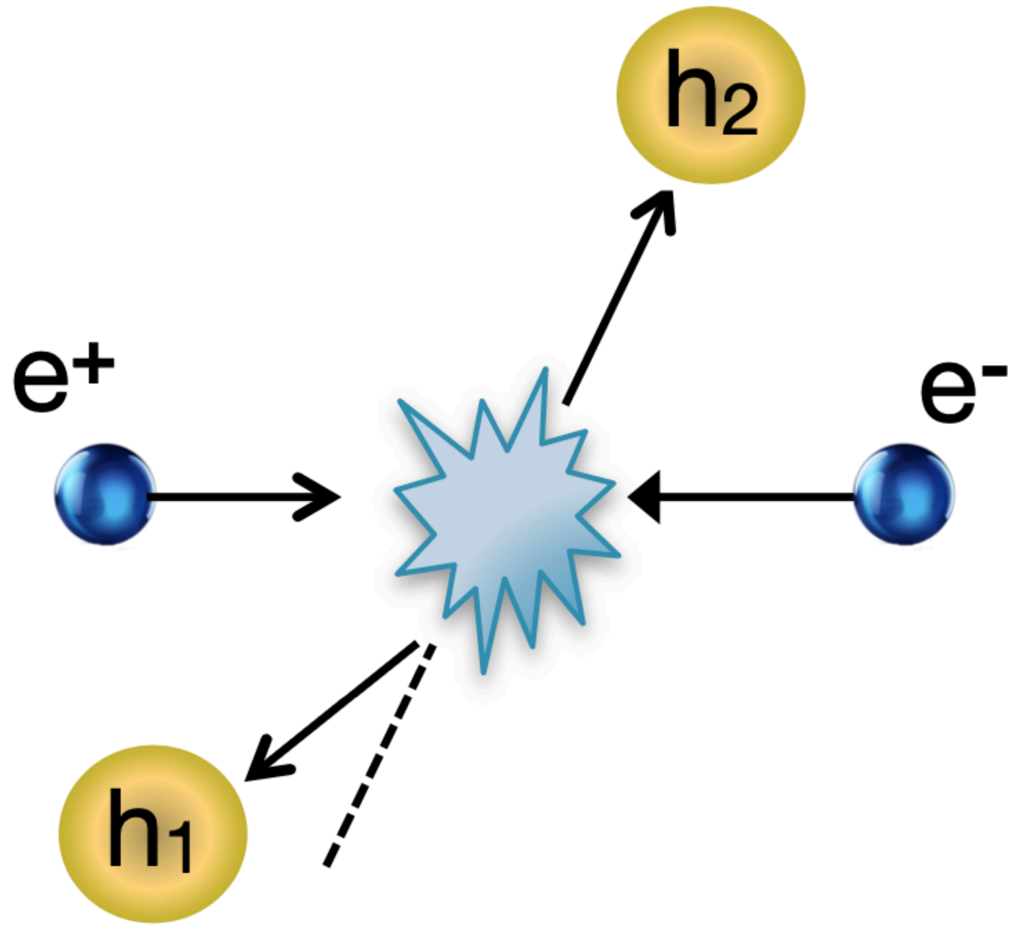
Drell-Yan

$$\sigma \sim f_{q/P}(x, k_T) f_{q/P}(x, k_T)$$



Dihadron in e+e-

$$\sigma \sim D_{h_1/q}(x, k_T) D_{h_2/q}(x, k_T)$$



$$\frac{d\sigma_{\rm DY}}{dQ\,dY\,dq_T^2} = H(Q,\mu) \int d^2\vec{b}_T e^{i\vec{q}_T\cdot\vec{b}_T} f_q(x_a, \vec{b}_T, \mu, \zeta_a)\, f_q(x_b, \vec{b}_T, \mu, \zeta_b)\left[1 + \mathcal{O}\left(\frac{q_T^2}{Q^2}\right)\right]$$

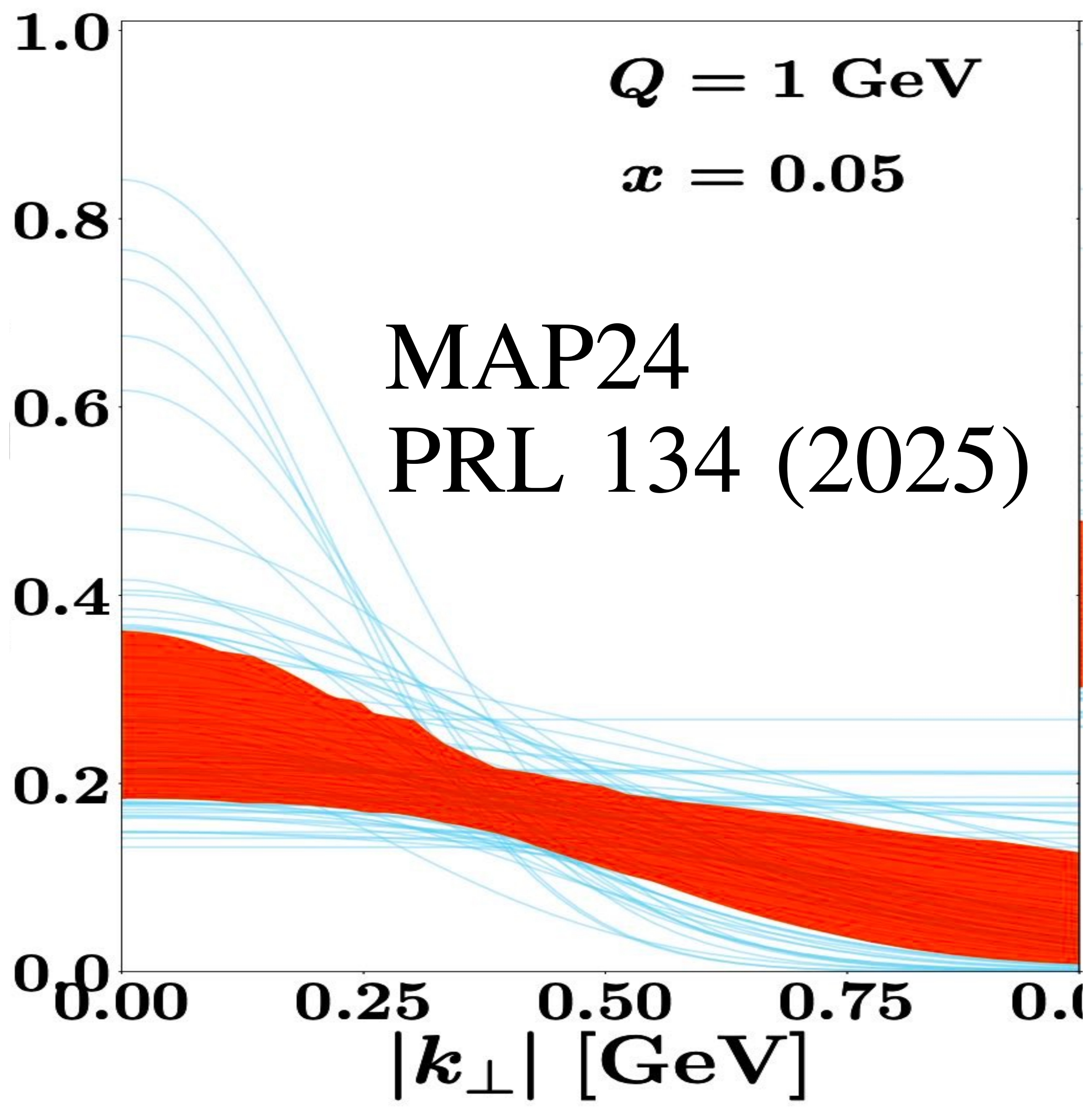
Perturbative hard kernels

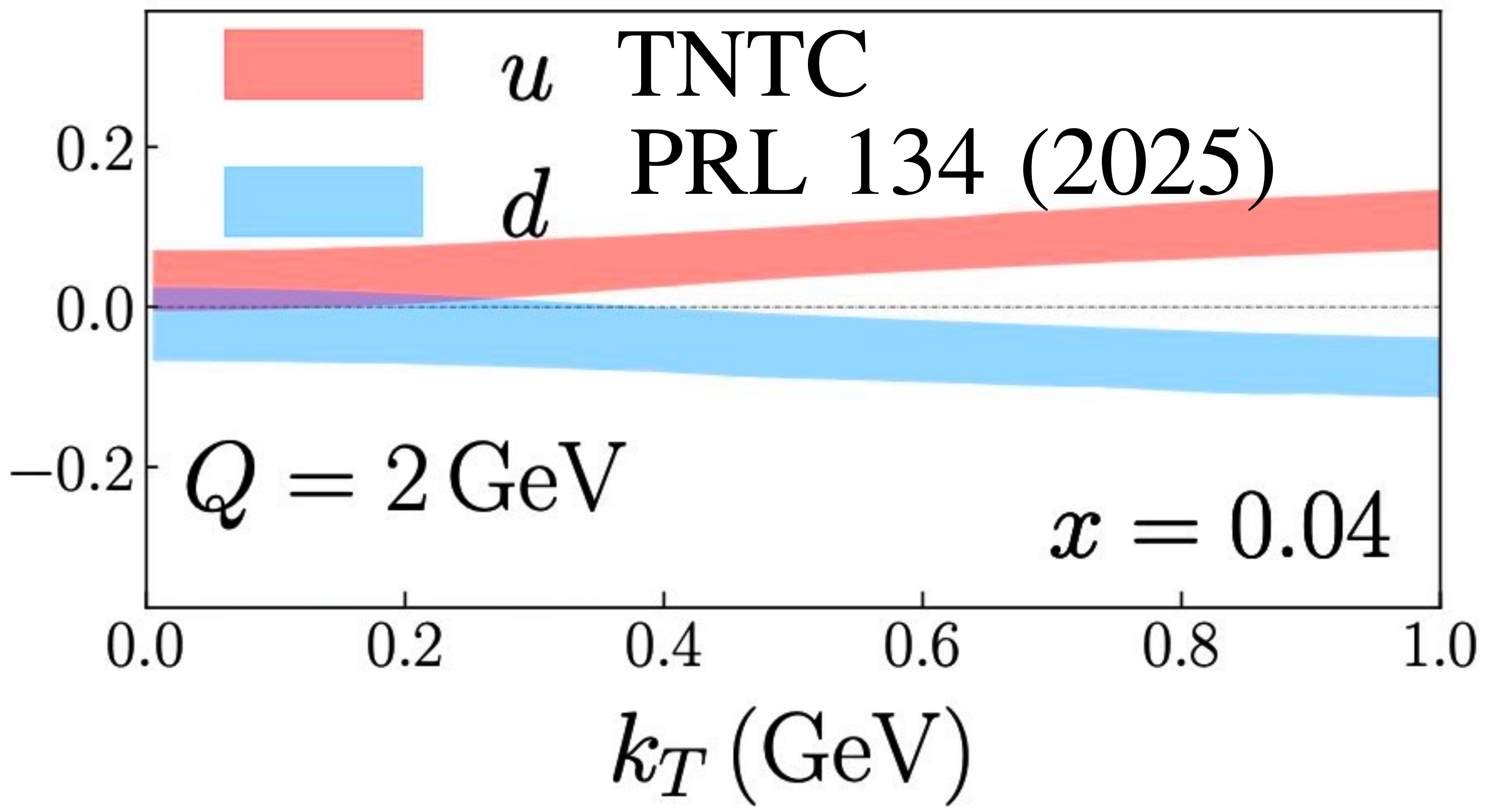
Nonperturbative TMDs

$$q_T^2 \ll Q^2$$

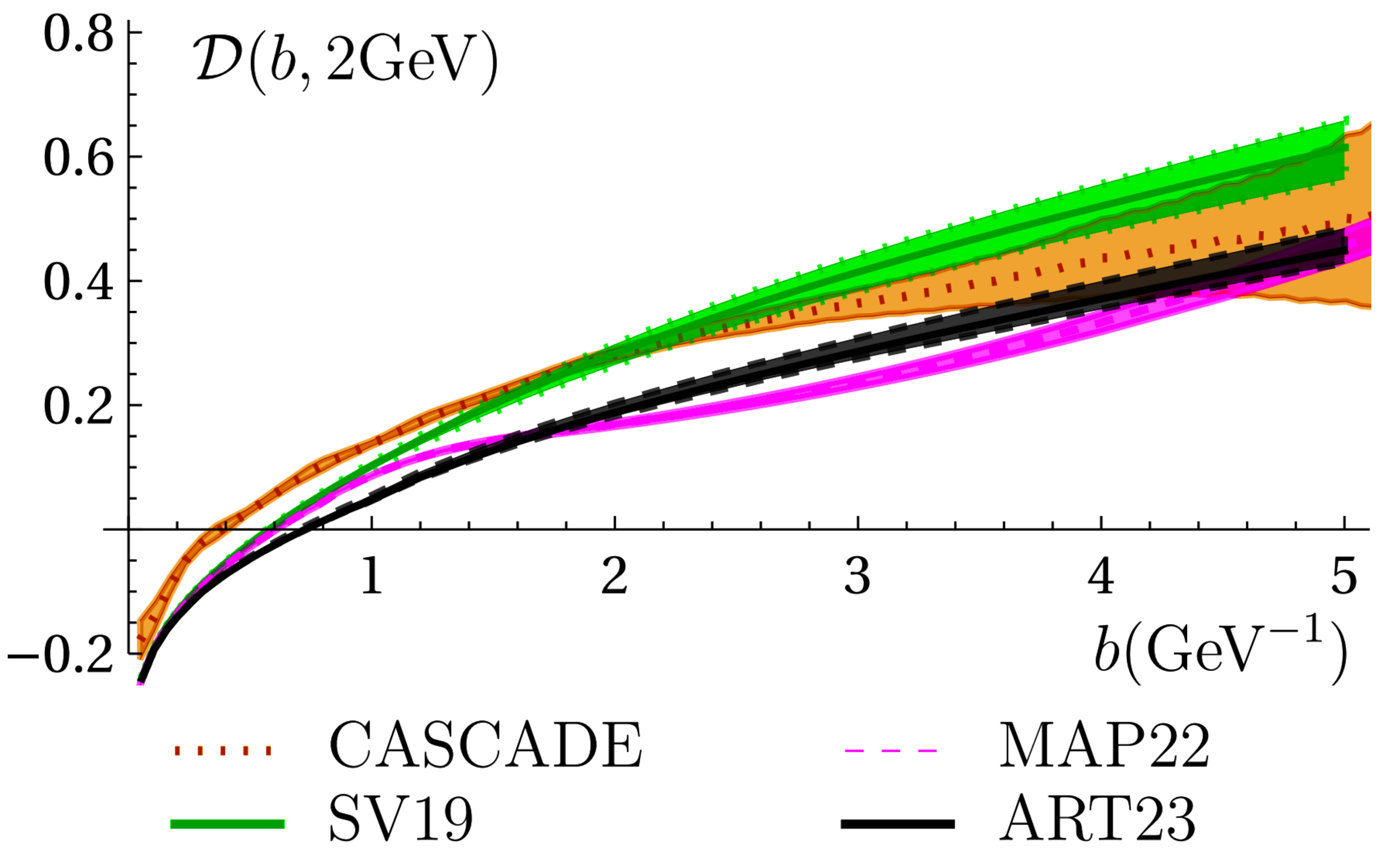
TMDs from global analyses of experimental data

- Helicity and unpolarized TMD ratio





- Collins-Soper kernel



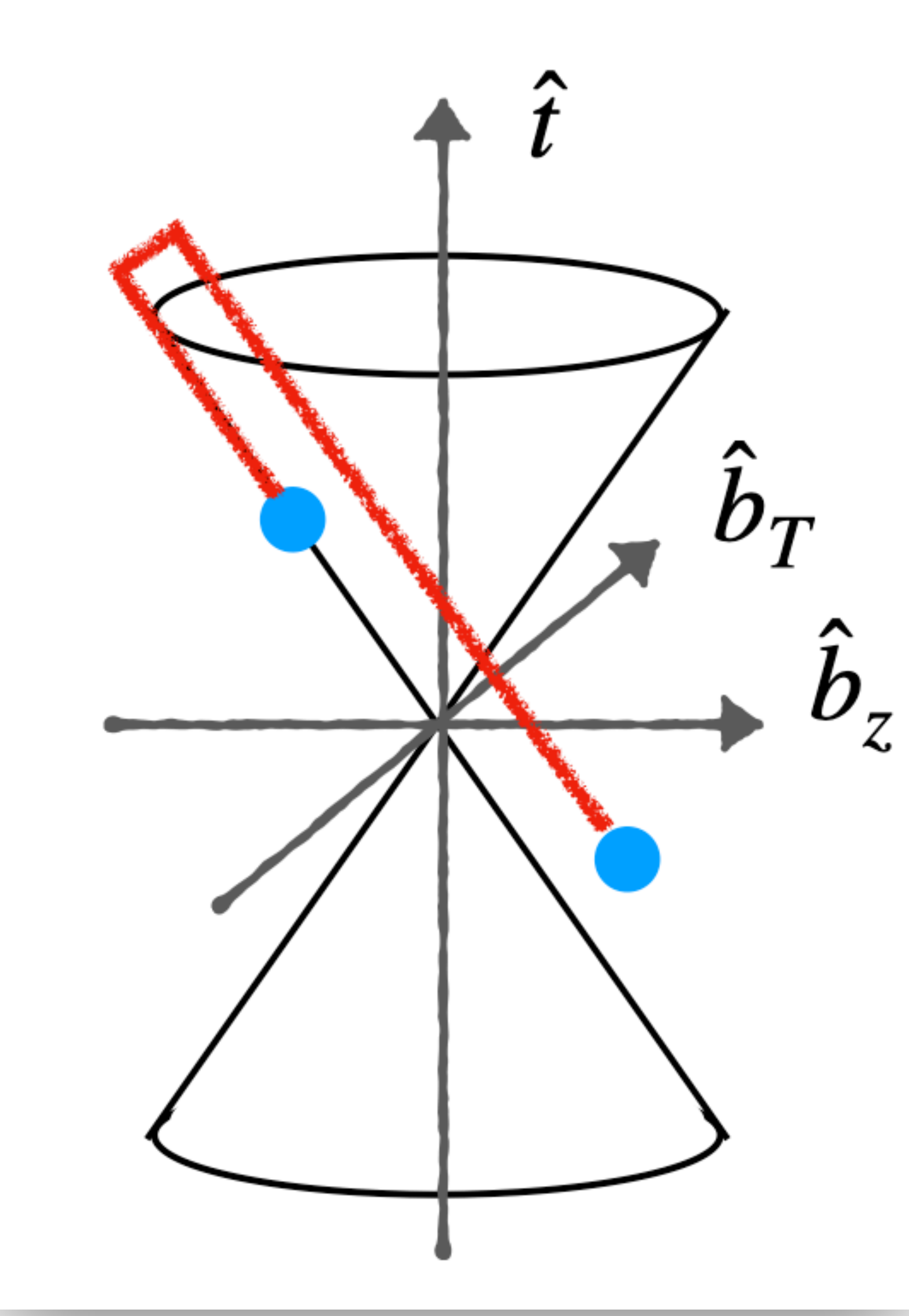
- Global fit is limited by sparse data.
- Rely on model assumptions producing noticeable inconsistencies.
- Complementary information from lattice QCD is essential.

The definition of TMDs

Beam function

$$f_q(x, \vec{b}_T, \mu, \zeta) = \lim_{\epsilon \to 0} Z_{\mathrm{UV}}(\epsilon, \mu, \zeta) \lim_{\tau \to 0} \frac{B_q(x, \vec{b}_T, \epsilon, \tau, \zeta)}{\sqrt{S_q(\vec{b}_T, \epsilon, \tau)}}$$

UV regulator **Rapidity regulator** **Soft function**



$$B_q(x, \vec{b}_T, \epsilon, \tau, \zeta) = \langle P, \lambda \,|\, \overline{\psi}(\frac{b^+}{2}, b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2}, 0) \,|\, P, \lambda \rangle$$

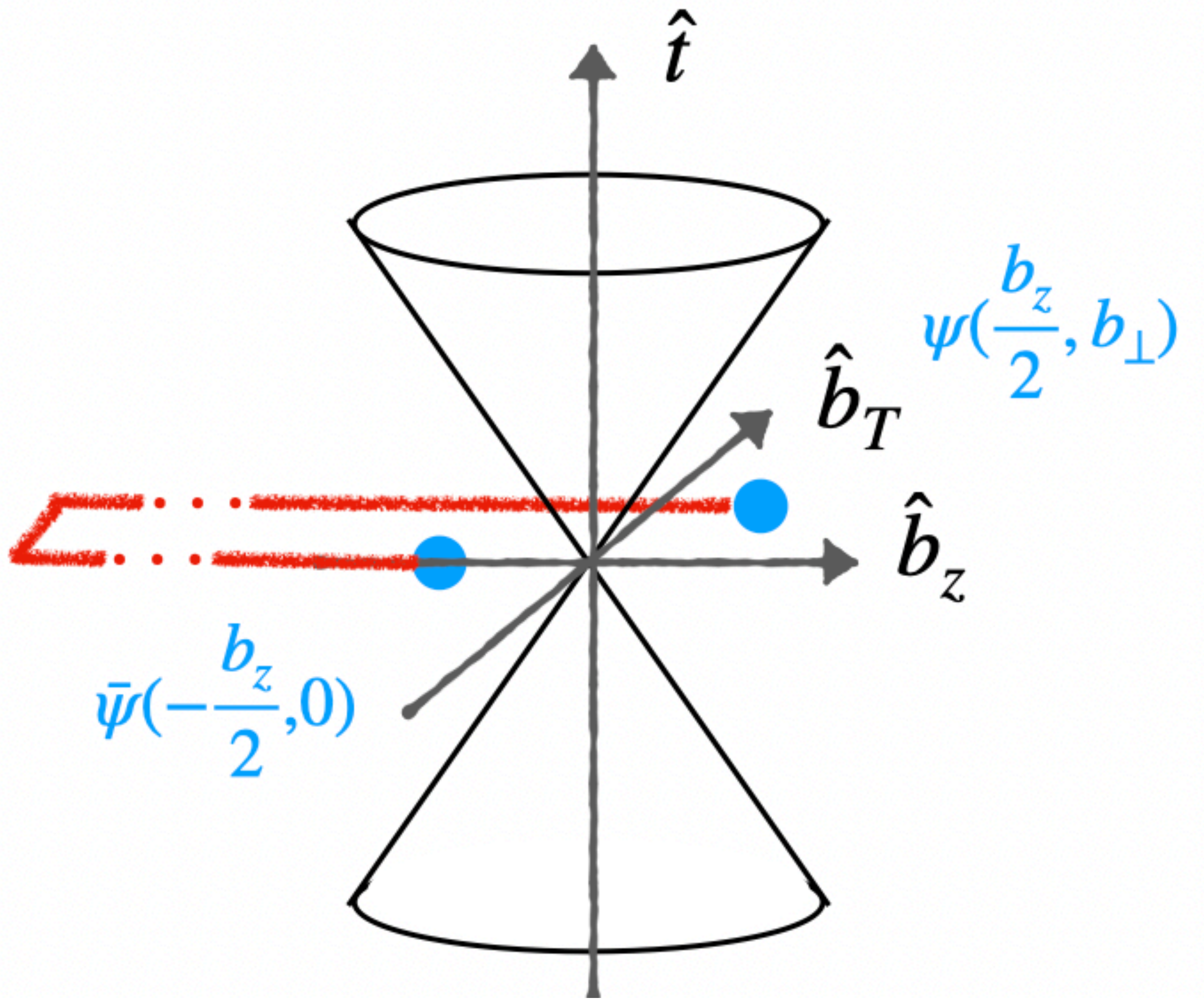
- λ: nucleon polarization
- Γ: quark polarization
- $W_{\sqsupset^+}$: light-cone staple-shaped Wilson link

✘ **Light-cone correlations: forbidden on Euclidean lattice**

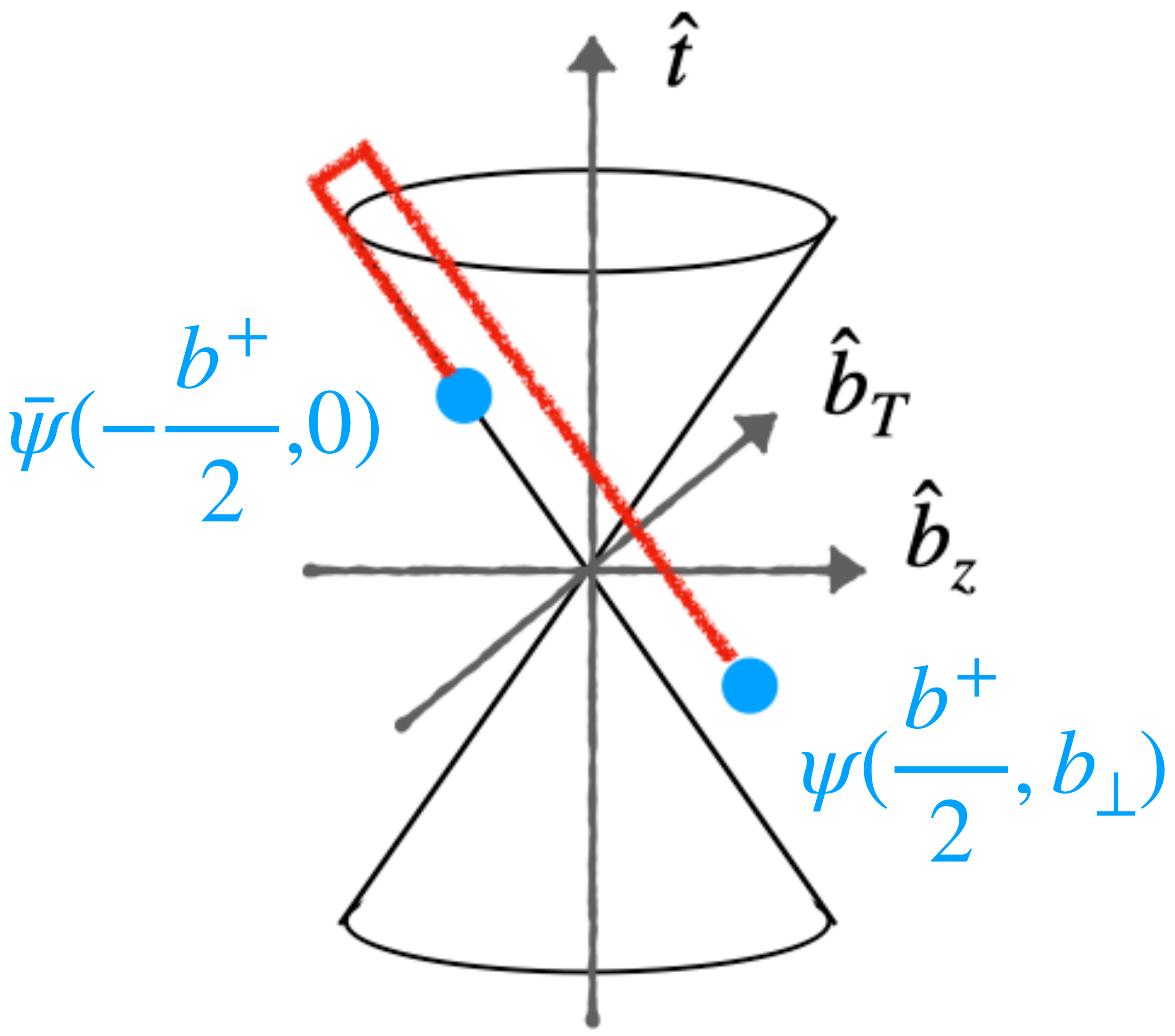
TMDs from lattice: quasi-TMDs

- Ji, Liu and Liu, NPB 955 (2020), PLB 811 (2020);
- A. Vladimirov, A. Schäfer Phys.Rev.D 101 (2020), 074517
- I. Stewart, Y. Zhao et al., JHEP 09 (2020) 099
- X. Ji et al., Phys.Rev.D 103 (2021) 7, 074005
- I. Stewart, Y. Zhao et al., JHEP 08 (2022) 084

- Equal-time correlators with spacial separations.
- Approaching light-cone TMDs in the large momentum limit (LaMET).



$P_z \to \infty$



Quasi TMD

$$\langle P | \overline{\psi}(\frac{b^z}{2}, b_\perp) \Gamma W_{\sqsupset^z} \psi(-\frac{b^z}{2}, 0) | P \rangle$$

$$= \langle P | \overline{\psi}(\frac{b^z}{2}, b_\perp) \Gamma \psi(-\frac{b^z}{2}, 0) |_{A^z=0} | P \rangle$$

Light-cone TMD

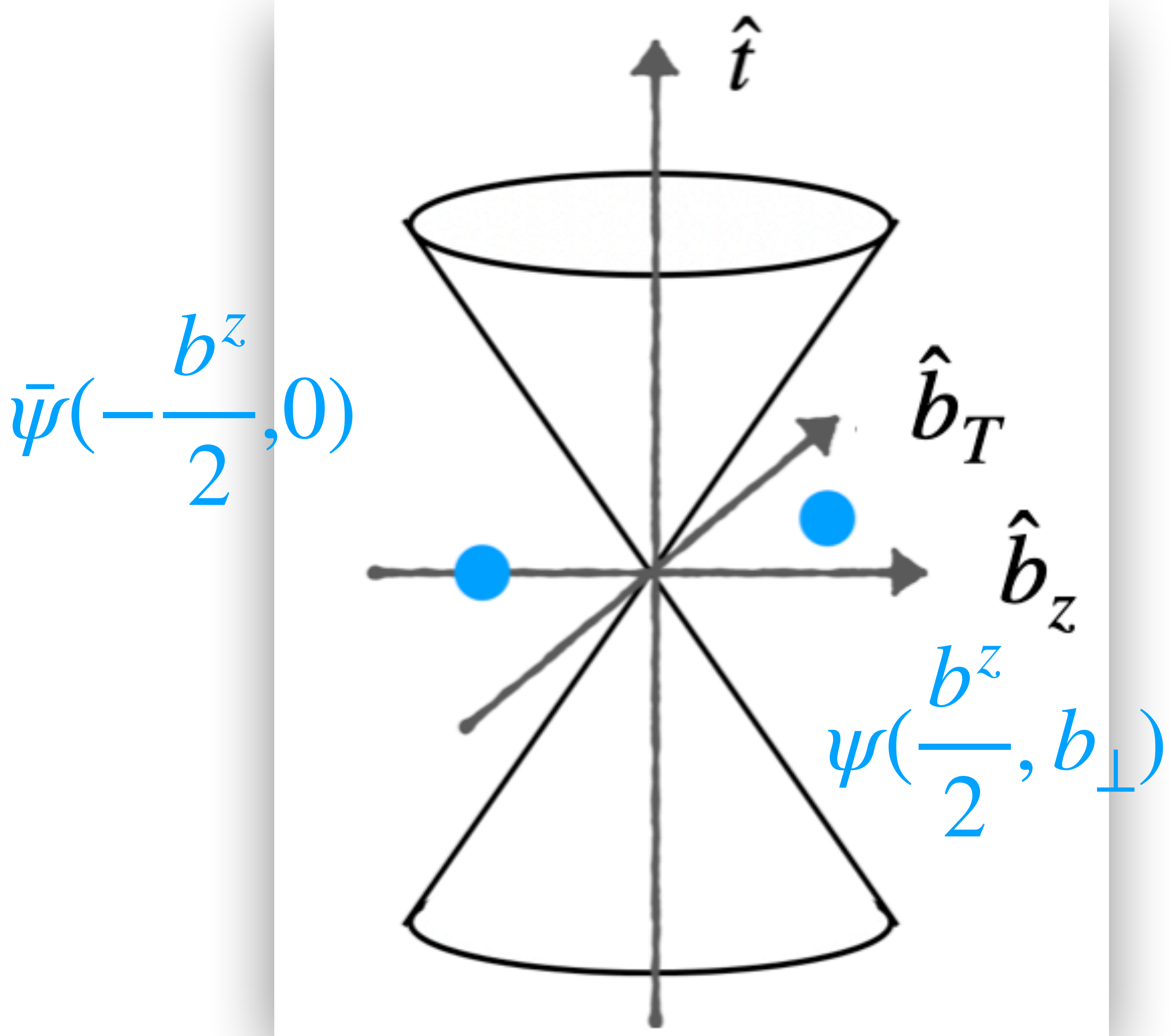
$$\langle P | \overline{\psi}(\frac{b^+}{2}, b_\perp) \Gamma W_{\sqsupset^+} \psi(-\frac{b^+}{2}, 0) | P \rangle$$

$$= \langle P | \overline{\psi}(\frac{b^+}{2}, b_\perp) \Gamma \psi(-\frac{b^+}{2}, 0) |_{A^+=0} | P \rangle$$

The universality class of quasi-TMDs

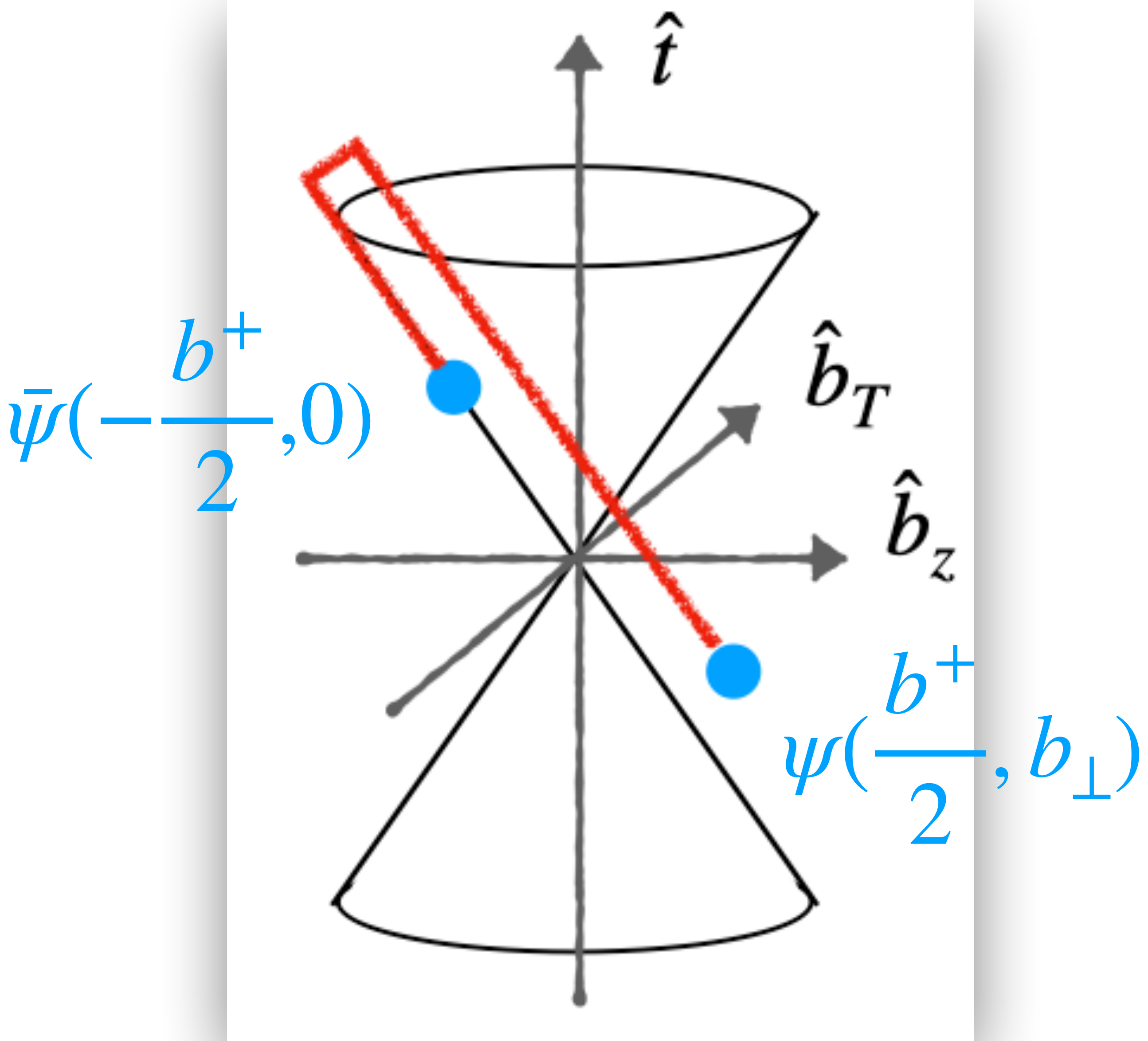
- Equal-time correlators in physical gauges: $A^z = 0$, $\vec{\nabla} \cdot \vec{A} = 0$, $A^t = 0$, ...

• XG, W.-Y. Liu, Y. Zhao, PRD 109 (2024) 9, 094506
• Y. Zhao, PRL 133 (2024) 24, 241904



$P_z \to \infty$

Physical gauge $\to A^+$ gauge



Quasi TMD

$$\langle P|\bar{\psi}(\frac{b_z}{2}, b_\perp)\Gamma\psi(-\frac{b_z}{2},0)\,|_{\vec{\nabla}\cdot\vec{A}=0}\,|P\rangle$$
$$|_{A^z=0}$$
$$|_{A^t=0} \cdots$$

Light-cone TMD

$$\langle P|\bar{\psi}(\frac{b^+}{2}, b_\perp)\Gamma W_{\sqsupset^+}\psi(-\frac{b^+}{2},0)|P\rangle$$
$$= \langle P|\bar{\psi}(\frac{b^+}{2}, b_\perp)\Gamma\psi(-\frac{b^+}{2},0)\,|_{A^+=0}\,|P\rangle$$

Large P_z expansion and perturbative matching

Quasi TMD
beam function

Collins-Soper kernel

$$\sqrt{S_I(\vec{b}_T,\mu)}\tilde{f}(x,\vec{b}_T,\mu,P_z) = H_f(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}}f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_zb_T)^2},\frac{\Lambda_{\rm QCD}^2}{(xP_z)^2}]\}$$

Soft-factor

Physical TMD

$P_z \lesssim a^{-1}$

- Relate to light-cone TMDs through **large momentum expansion (LaMET).**
- Multiplicative **perturbative matching** $H_f(\mu,xP_z)$ and **power corrections** are scheme dependent.

Collins-Soper kernel from CG quasi-TMD

perturbative correction

$$\gamma^{\overline{\mathrm{MS}}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)} \ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\mathrm{MS}}}(x,\mu,P_1,P_2)$$

$P_1 \to P_2$ quasi-TMDWF

Collins-Soper kernel from CG quasi-TMD

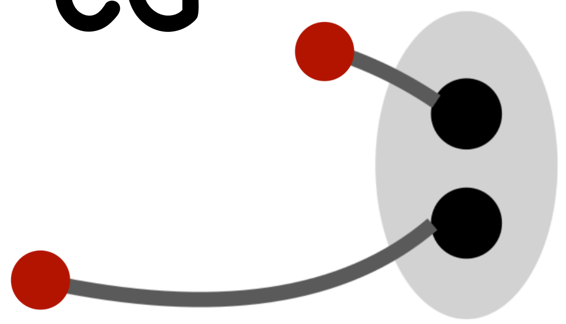
perturbative correction

$$\gamma^{\overline{\text{MS}}}(b_\perp,\mu) = \frac{1}{\ln(P_2/P_1)}\ln\left[\frac{\tilde{\phi}(x,b_\perp,P_2,\mu)}{\tilde{\phi}(x,b_\perp,P_1,\mu)}\right] + \delta\gamma^{\overline{\text{MS}}}(x,\mu,P_1,P_2)$$

$P_1 \to P_2$ quasi-TMDWF

- The CG quasi-TMD show advantage of: **Simplified renormalization** & **enhanced signal.**

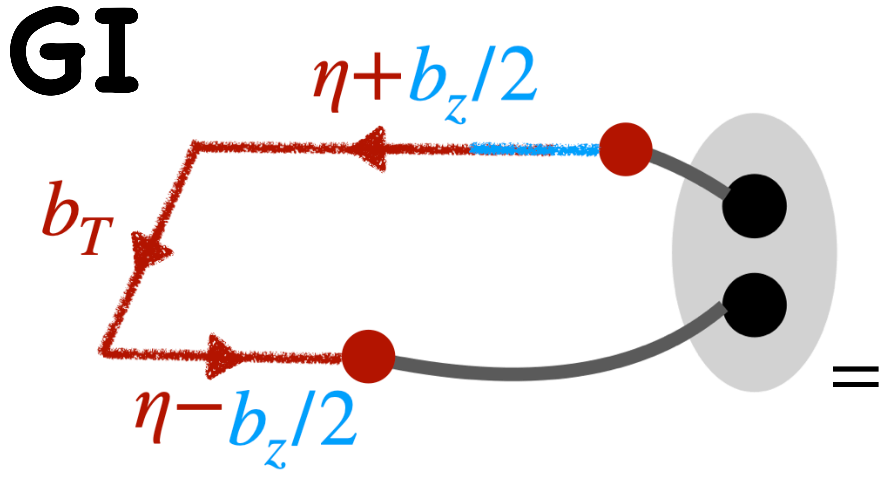
CG



$$[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_B$$

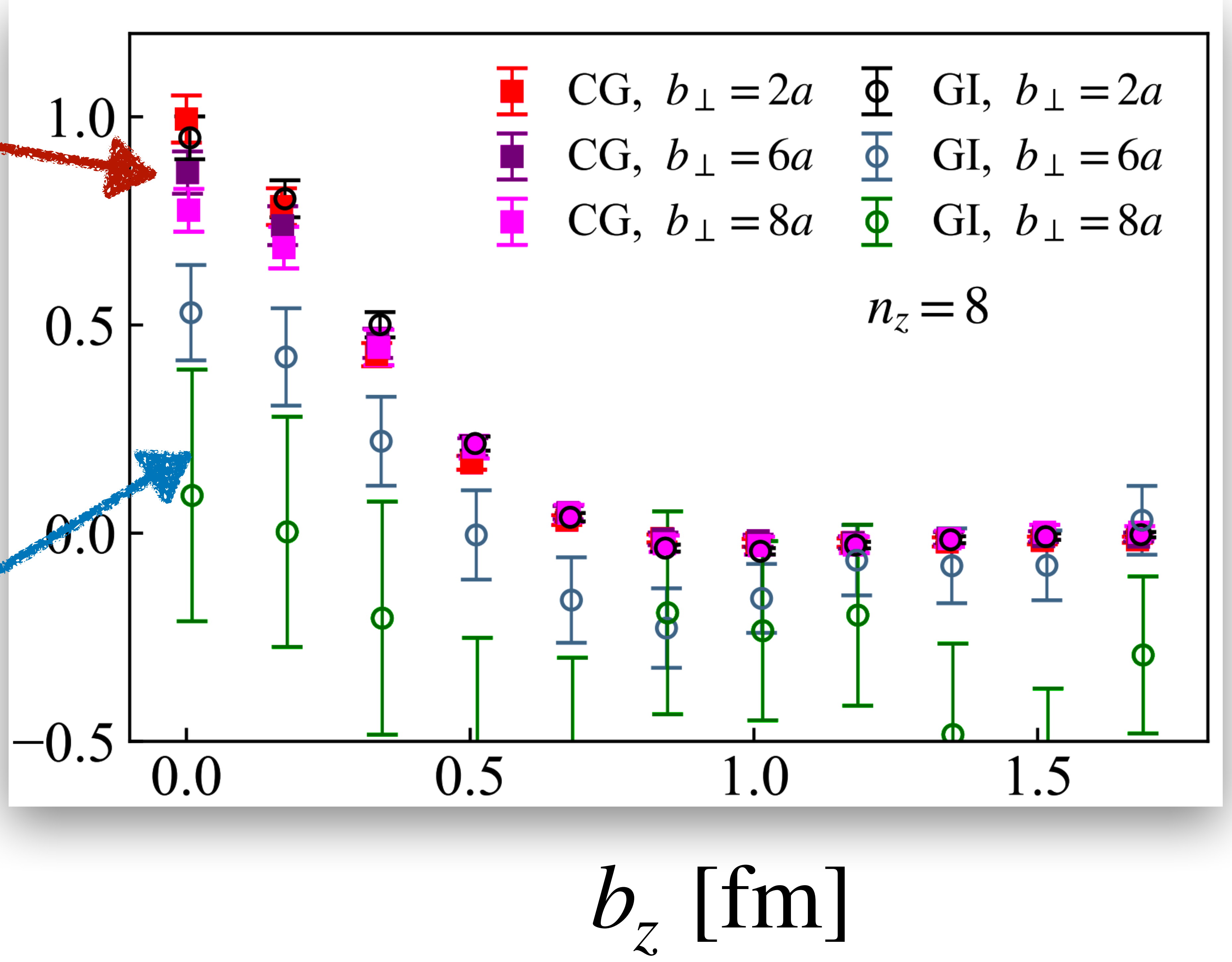
$$= Z(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_R$$

GI



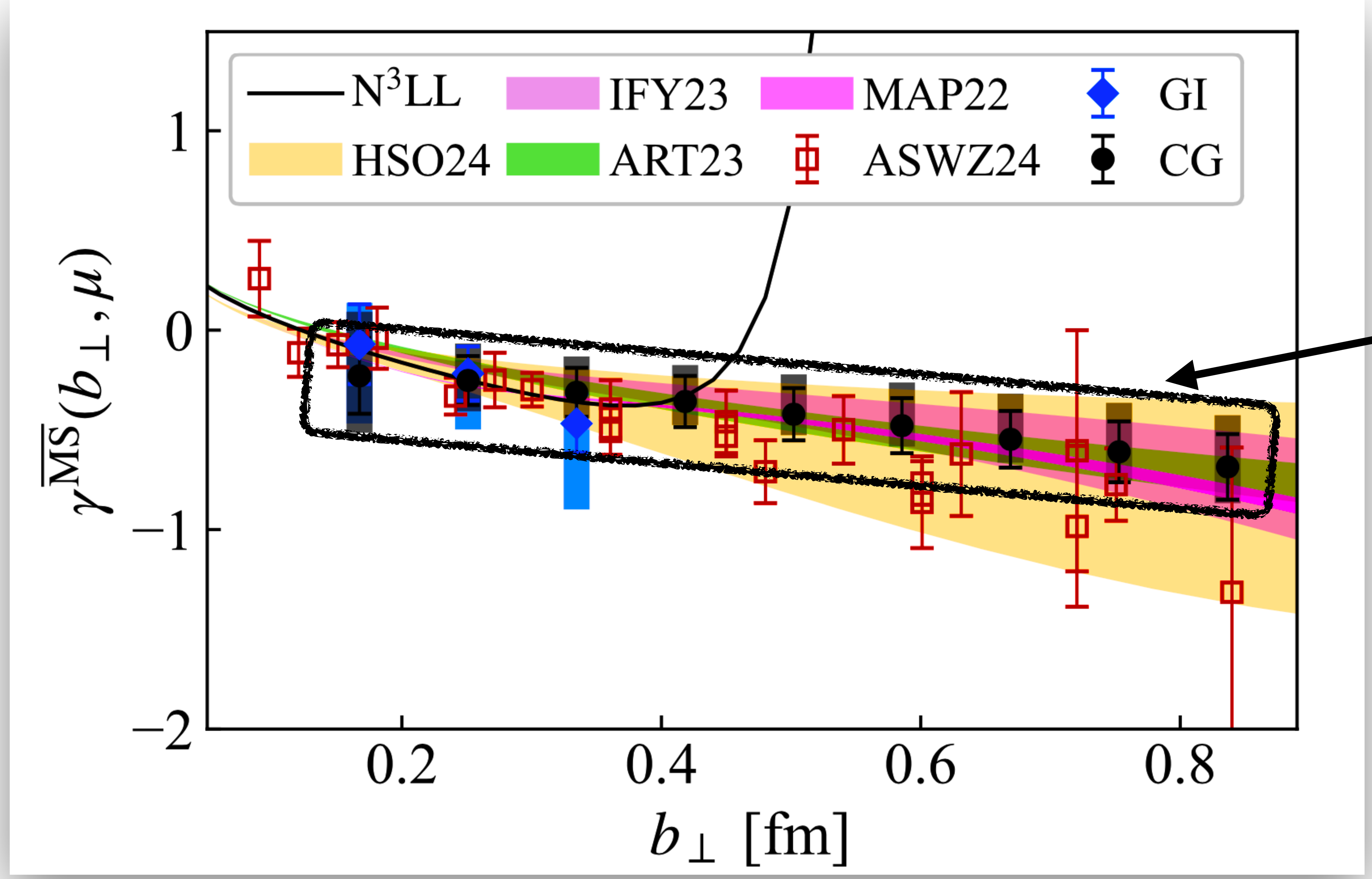
$$[\bar{\psi}(-\frac{\vec{b}}{2})W_{\sqsupset z}\Gamma\psi(\frac{\vec{b}}{2})]_B$$

$$= e^{-\delta m(a)L_{\sqsupset z}}Z(a)[\bar{\psi}(-\frac{\vec{b}}{2})\Gamma\psi(\frac{\vec{b}}{2})]_R$$



Collins-Soper kernel from CG quasi-TMD

- CS kernel $\gamma^{\overline{\mathrm{MS}}}(b_\perp, \mu)$



- Consistent with most recent global fits and GI quasi-TMDs.
- No significant signal decay as $b_\perp$ grows.

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, PLB 852 (2024) 138617

Ratio of TMDPDFs from quasi-TMD beam functions

Quasi TMD beam function

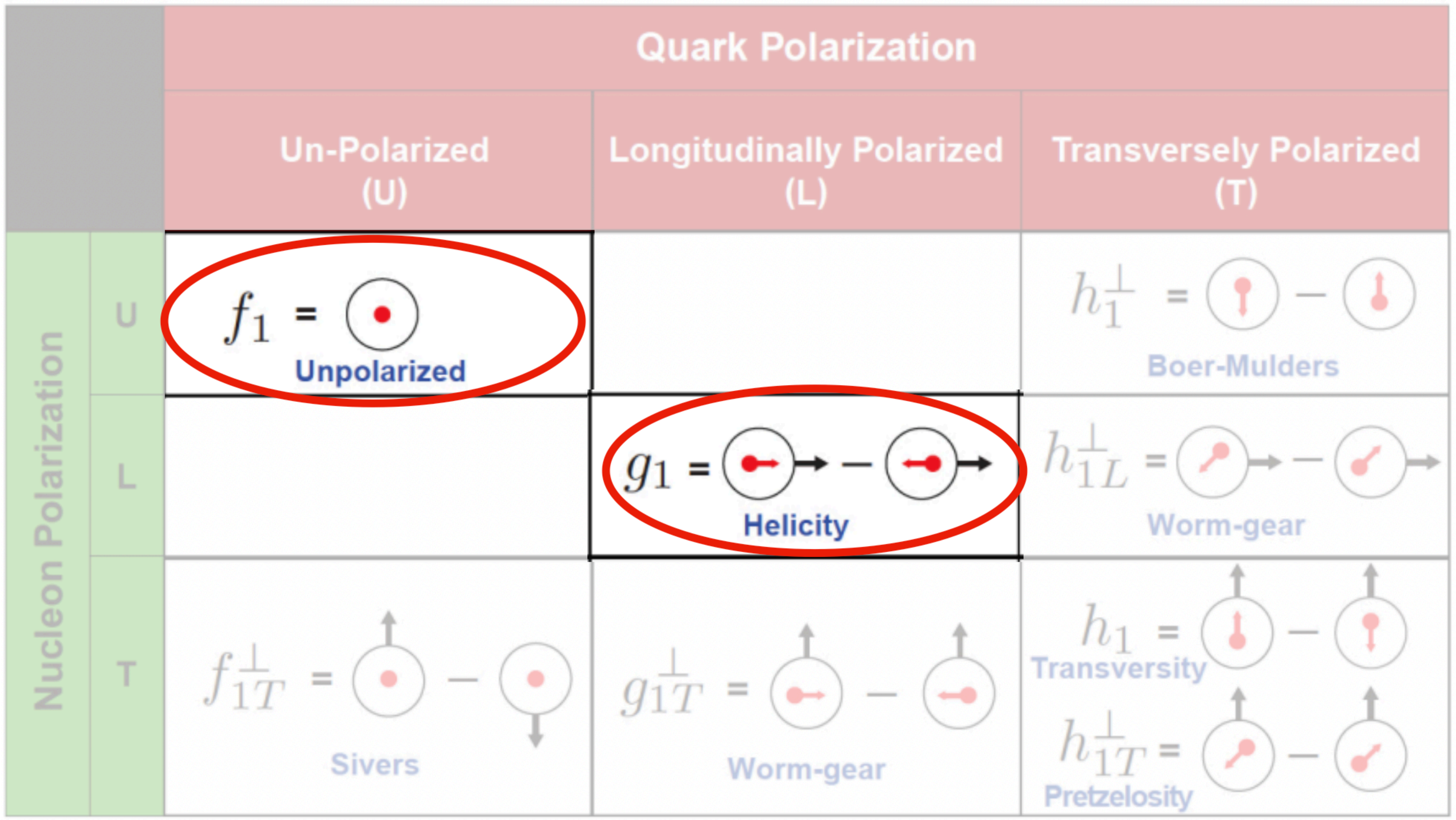
Physical TMD

$$\sqrt{S_I(\vec{b}_T,\mu)}\tilde{f}(x,\vec{b}_T,\mu,P_z) = H_f(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}} f(x,\vec{b}_T,\mu,\zeta)\{1+\mathcal{O}[\frac{1}{(xP_z b_T)^2},\frac{\Lambda^2_{\rm QCD}}{(xP_z)^2}]\}$$

$$\frac{\tilde{f}_1(x,b_T,P_z,\mu)}{\tilde{f}_2(x,b_T,P_z,\mu)} = \frac{f_1(x,b_T,\zeta,\mu)}{f_2(x,b_T,\zeta,\mu)} + \mathrm{p.c.}$$

- Ratios cancel soft factor & perturbative corrections & scale evolution.
- Renormalization-group-invariant (RGI).
- Valid to all orders in perturbation theory.

Unpolarized and helicity TMDPDFs from lattice



Quasi-TMD Beam functions

$$\tilde{h}(b_T, b_z, P_z, \mu)$$

$$= \langle \lambda; P_z | \bar{\psi}(b_T, \frac{b_z}{2}) \Gamma \psi(0, -\frac{b_z}{2}) |_{\vec{\nabla}\cdot\vec{A}=0} | \lambda; P_z \rangle$$

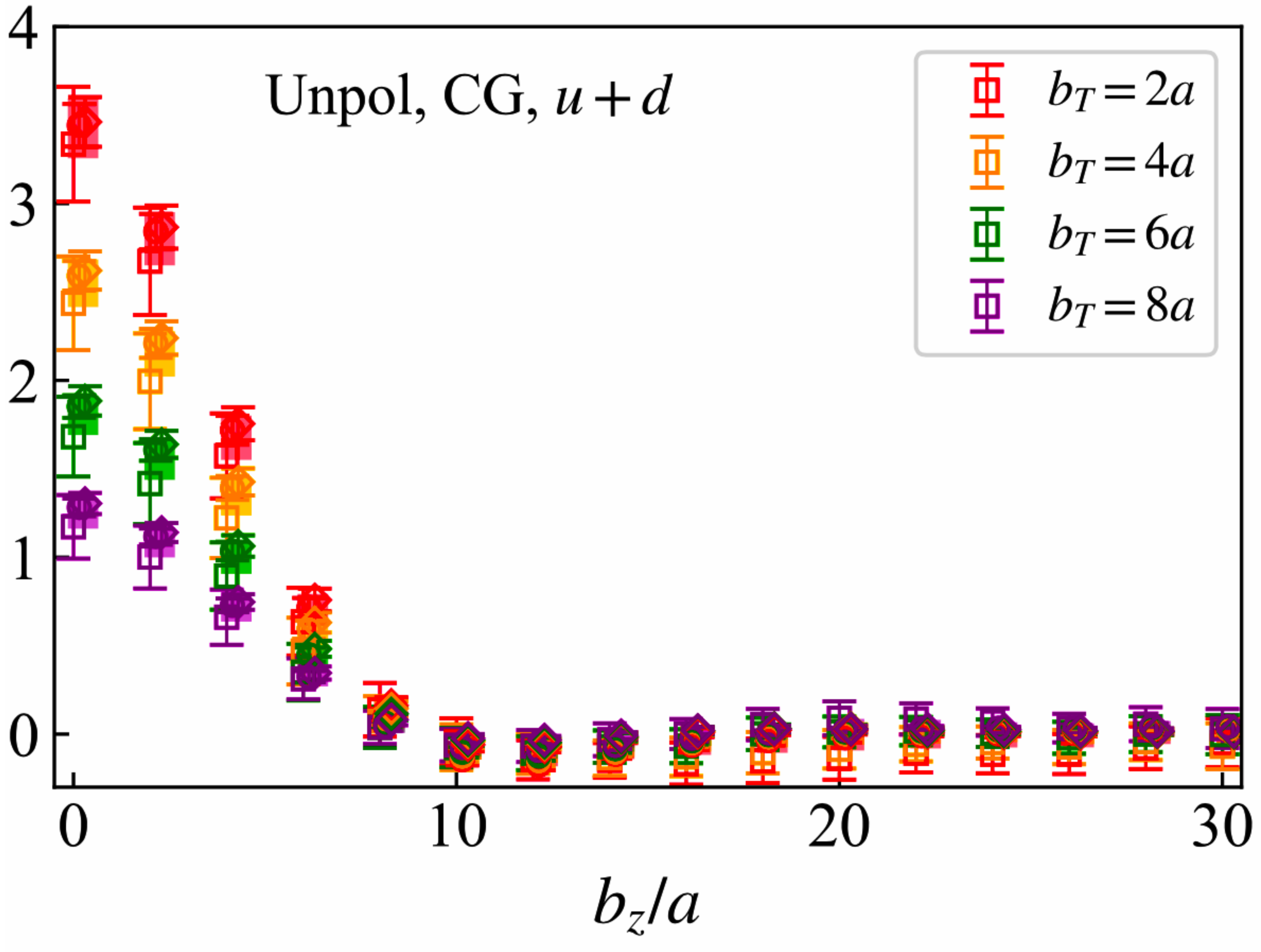
- Unpolarized TMDs: $\Gamma = \gamma^t$, $\psi = u - d$ and $u + d$ (disconnected diagrams ignored)
- Helicity TMDs: $\Gamma = \gamma^z \gamma^5$, $\psi = u - d$

Lattice setup:

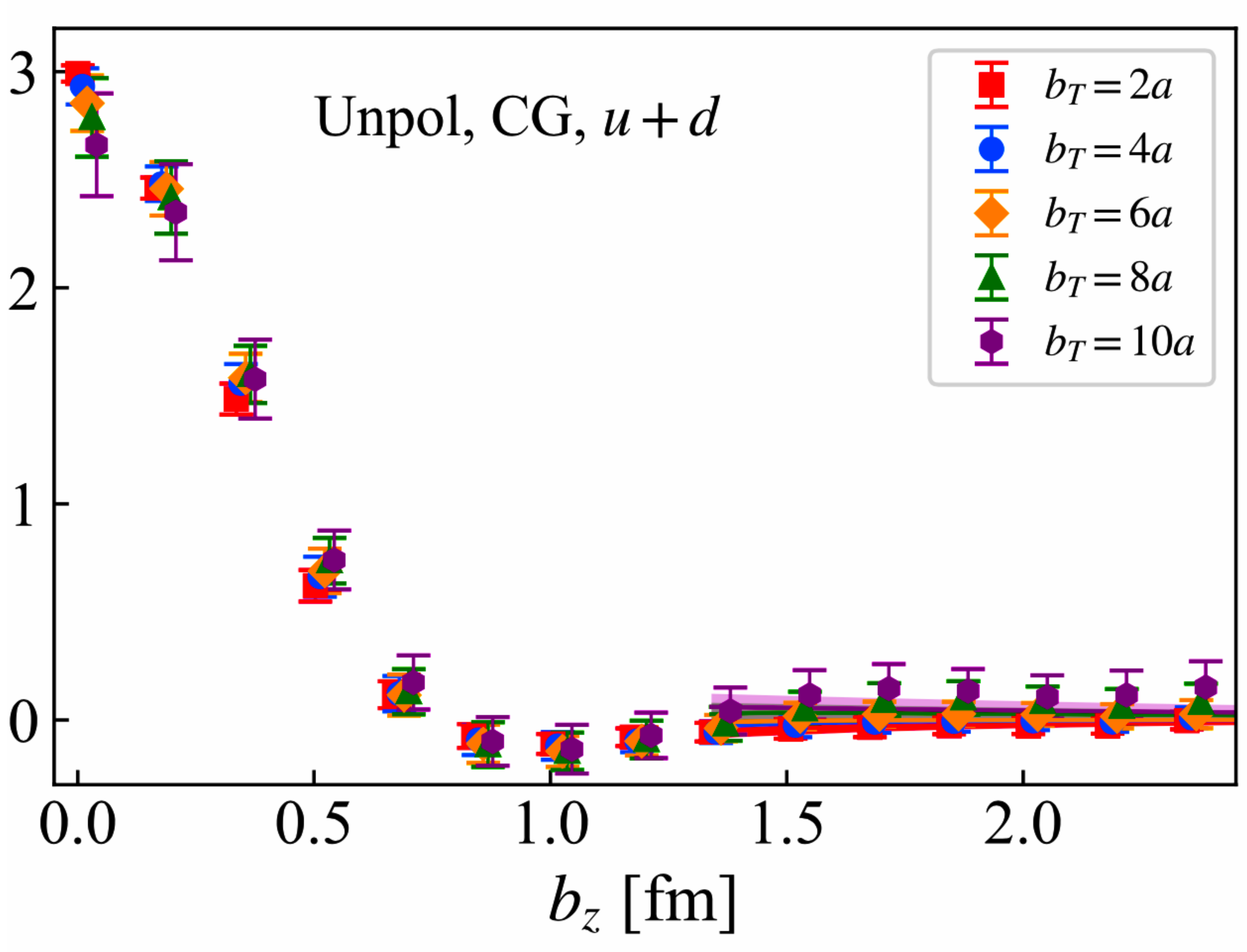
- 2+1 flavor Domain-wall (chiral) fermion discretization.
- Physical quark masses, $64^3 \times 128$ lattice with spacing $a = 0.084$ fm.
- Nucleon momentum up to $P_z = 1.62$ GeV, b_T up to 1 fm.

Quasi-TMD beam functions

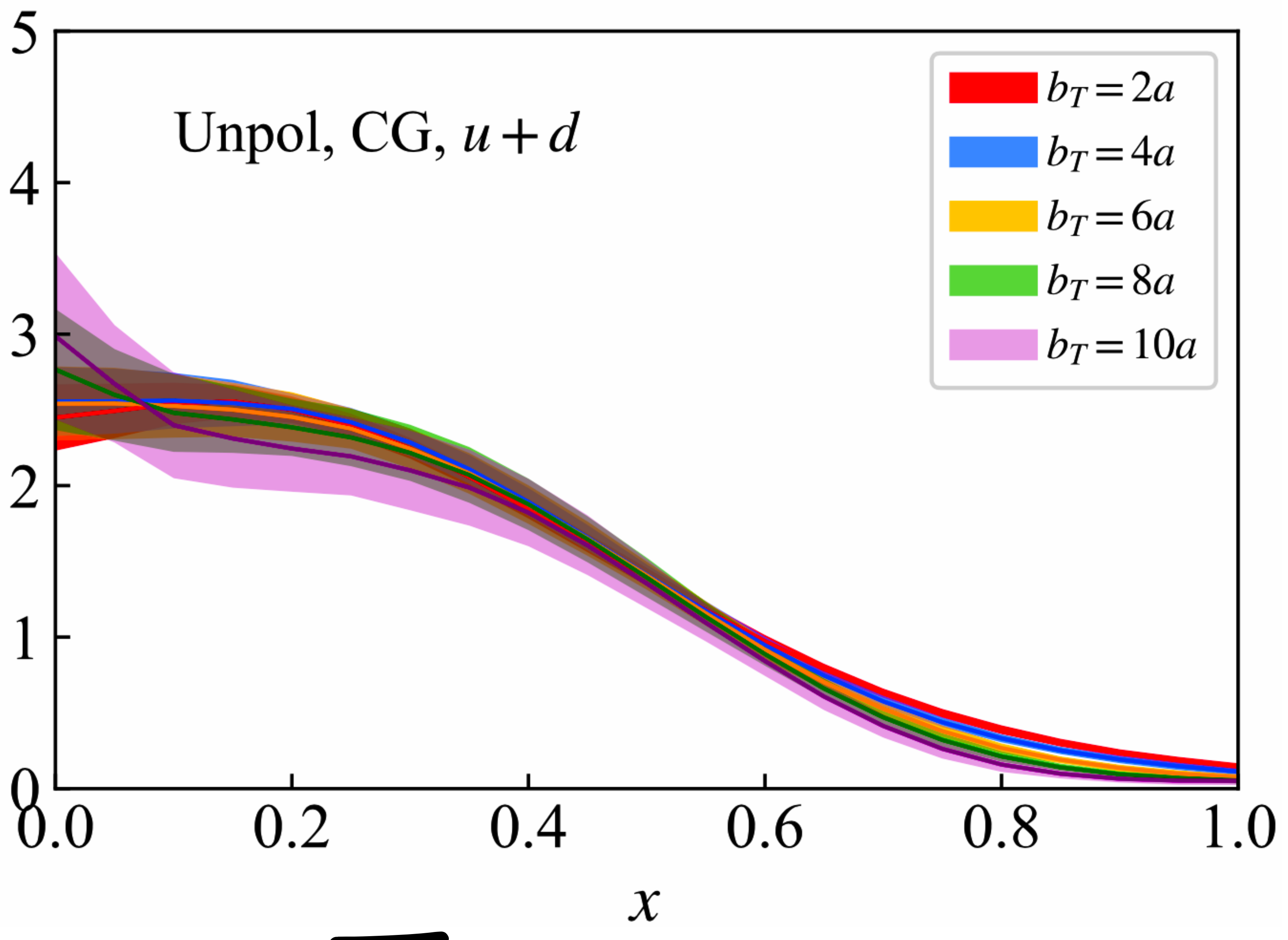
- Bare $\tilde{h}(b_T, b_z, P_z, a)$



- $\tilde{h}(b_T, b_z, P_z, \mu)$



- Quasi-TMD $\tilde{f}_1(x, b_T, P_z)$



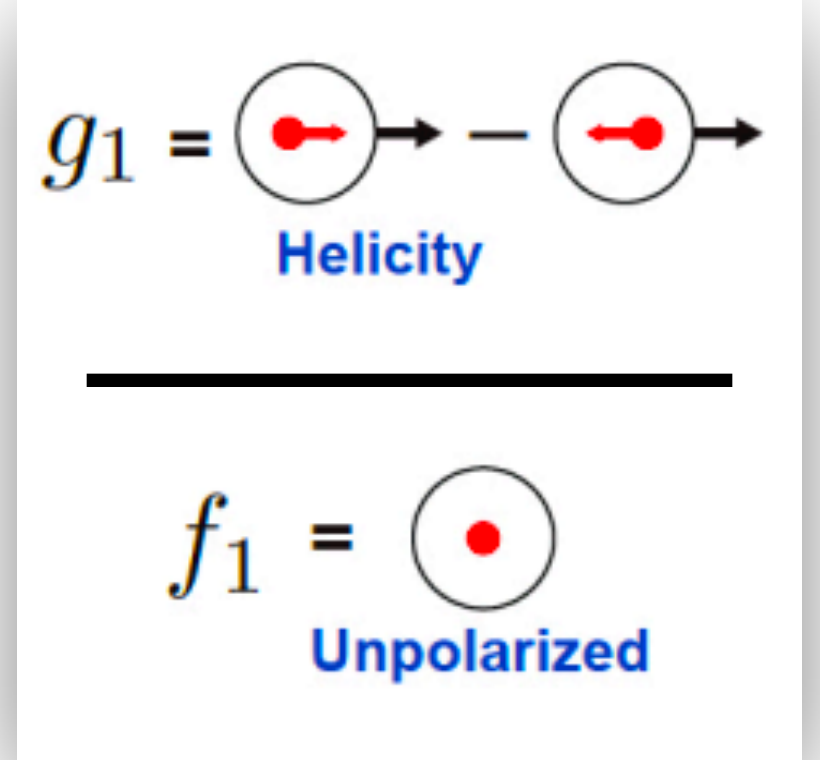
Renormailzation

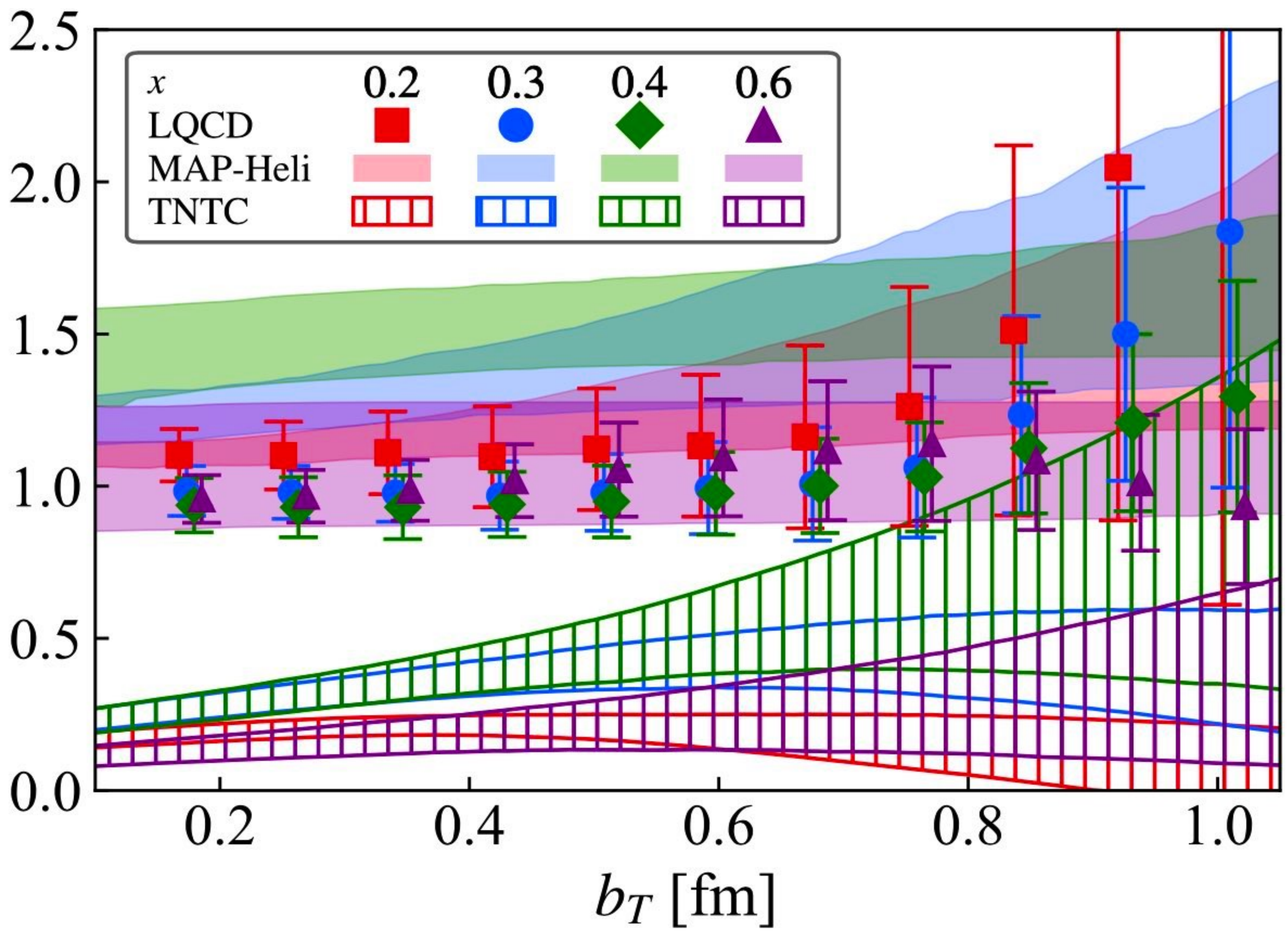
$b_z \xrightarrow{\text{F.T.}} x$

Ratios between $u-d$ heli. and unpol. TMDPDFs

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, arXiv: 2505.18430

$$\frac{g_{1L}^{\Delta u^+ - \Delta d^+}(x, b_T)}{f_1^{u_v - d_v}(x, b_T)} \frac{1}{g_A} = \frac{\tilde{g}_{1L}^{\Delta u^+ - \Delta d^+}(x, b_T)}{\tilde{f}_1^{u_v - d_v}(x, b_T)} \frac{1}{g_A}$$



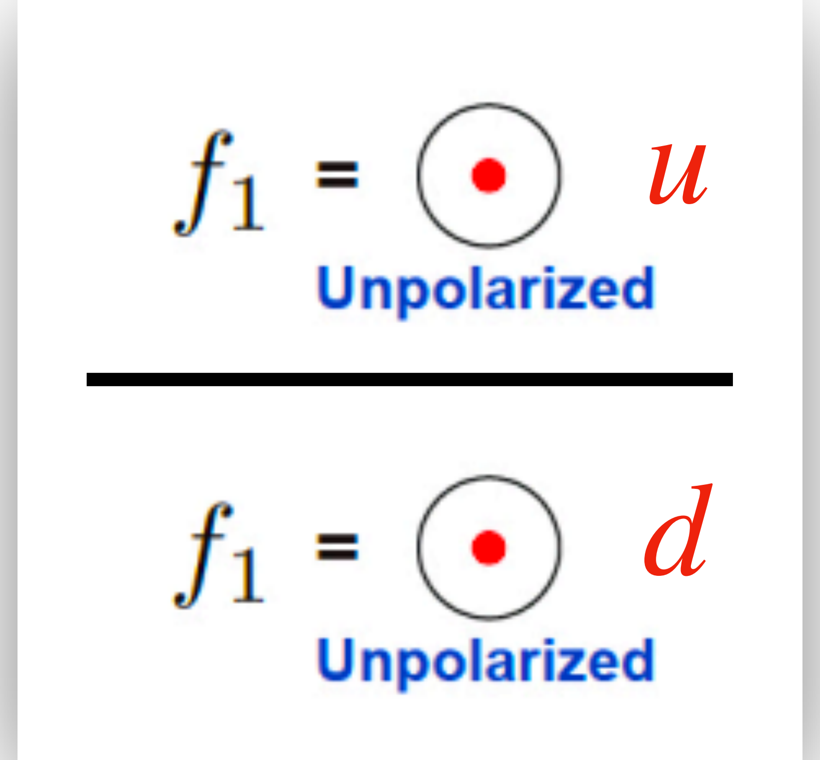


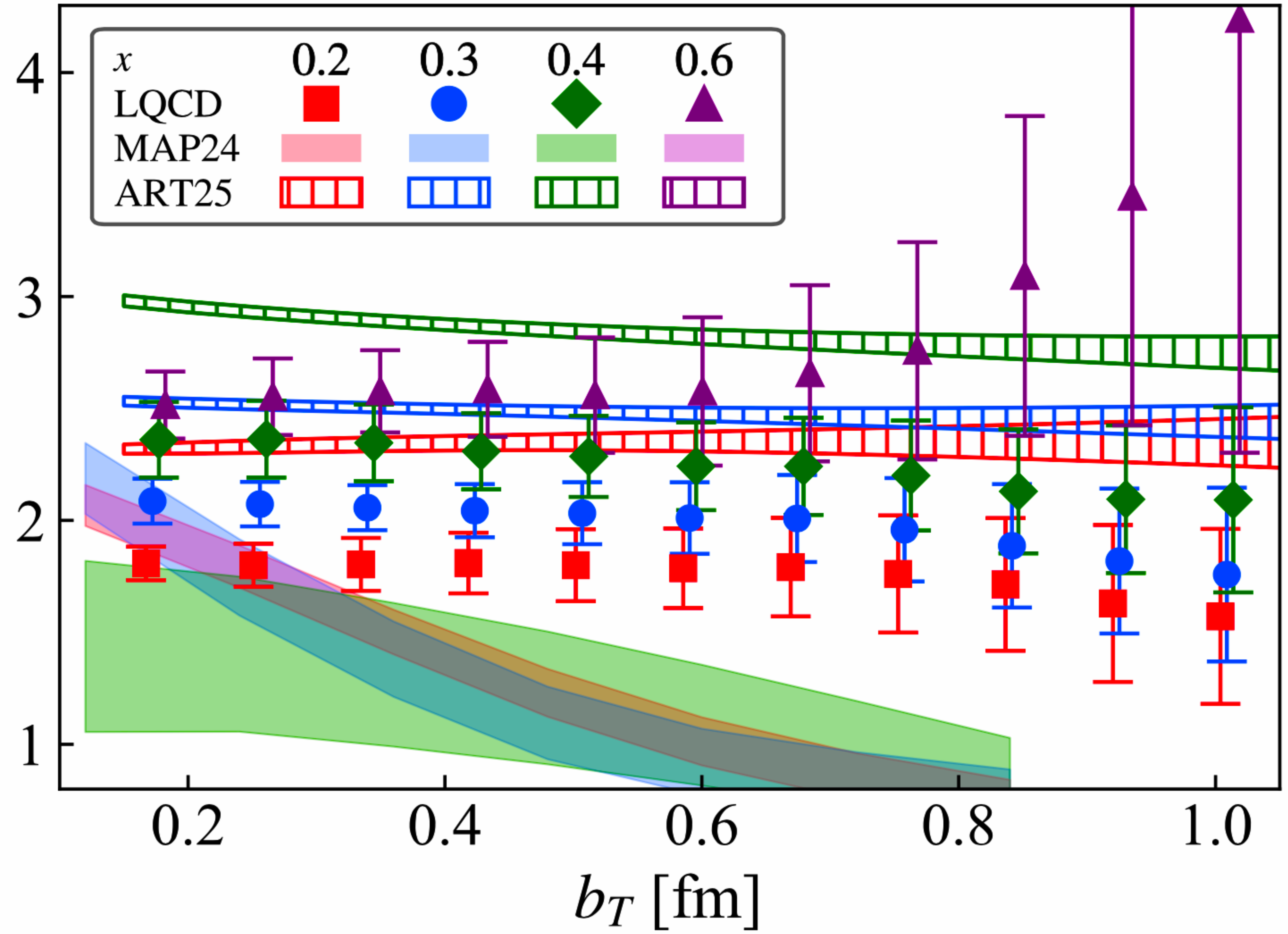
- No strong dependence on b_T: longitudinal spin polarization has limited impact on the intrinsic transverse motion of quark inside nucleon.

Ratios between valence $u-$ and $d-$unpolarized TMDs

• D. Bollweg, **XG**, S. Mukherjee, Y. Zhao, arXiv: 2505.18430

$$\frac{f_1^{u_v}(x, b_T)}{f_1^{d_v}(x, b_T)} = \frac{\tilde{f}_1^{u_v}(x, b_T)}{\tilde{f}_1^{d_v}(x, b_T)}$$





- Weak b_T dependence also observed.
- Lattice results could provide a first-principles benchmark for global fit in less constrained regions.

Towards TMDPDF with intrinsic soft factor

Soft-factor

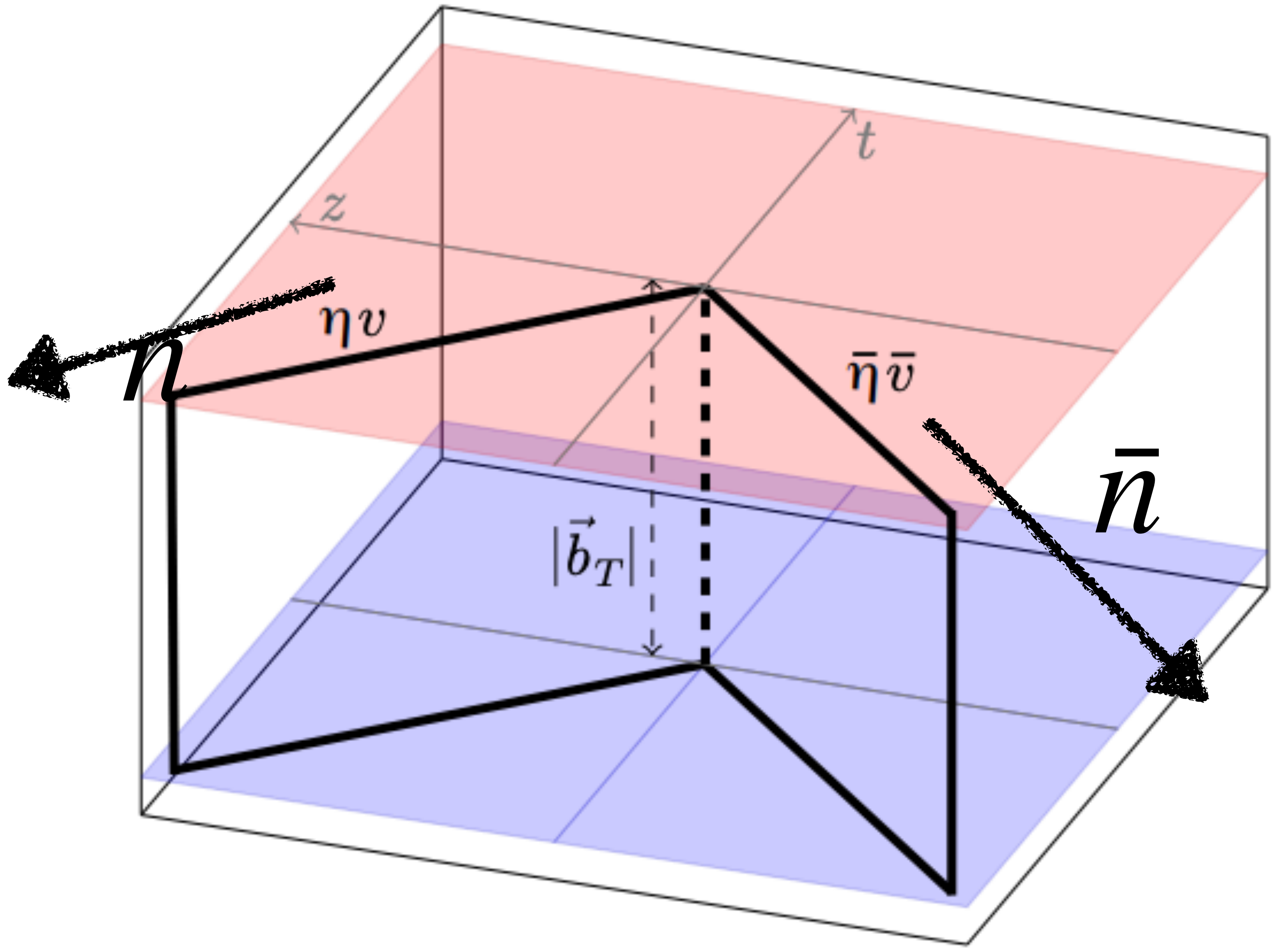
Quasi TMD beam function

Collins-Soper kernel

$$\sqrt{S_I(\vec{b}_T,\mu)}\tilde{f}(x,\vec{b}_T,\mu,P_z) = H_f(\mu,xP_z)e^{\frac{1}{2}\gamma_\zeta(\mu,b_T)\ln\frac{(2xP_z)^2}{\zeta}}f(x,\vec{b}_T,\mu,\zeta)$$

Physical TMD

- Soft factor

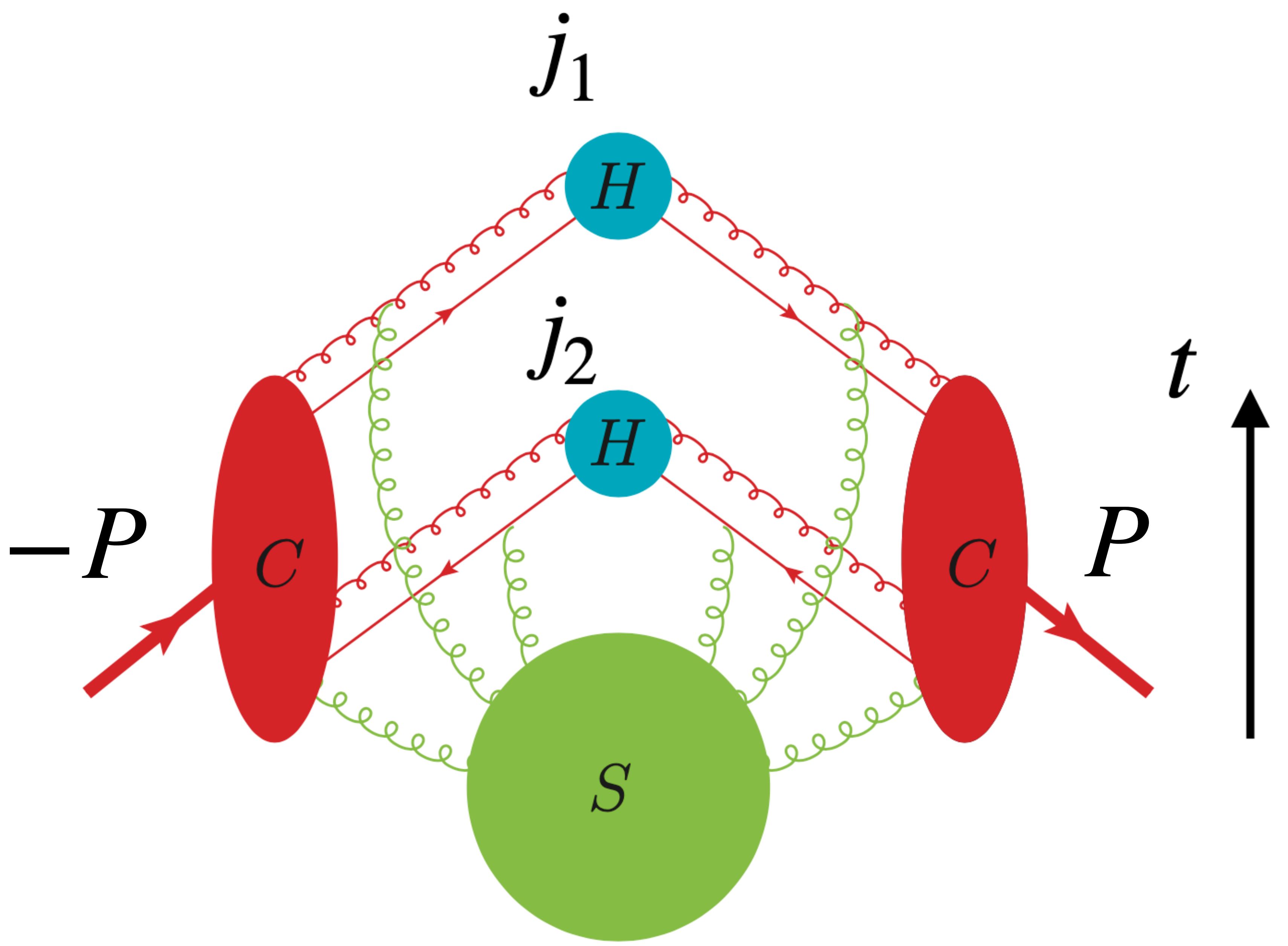


- Operator involves two light-cone directions.
- Can't be derived through hadron boost along one direction.

Towards TMDPDF with intrinsic soft factor

- X. Ji, et al., Nucl.Phys.B 955 (2020) 115054
- Z.-F. Deng, et al., JHEP 09 (2022) 046

- Large momentum form factor



Boosted pion in opposite direction

$$\langle \pi(-P) | j_1(b_T) j_2(0) | \pi(P) \rangle = \int dx_1 dx_2 H_{\text{sud}}(x_1, x_2, P_z, \mu) \phi^\dagger(x_1, b_T, P_z, \mu) \phi(x_2, b_T, P_z, \mu)$$

Pion light-cone TMDWF

quasi-TMDWF

$$\sqrt{S_I(\vec{b}_T, \mu)} \tilde{\phi}(x, \vec{b}_T, \mu, P_z) = H_\phi(x, \bar{x}, P_z, \mu) \phi(x, \vec{b}_T, \mu, \zeta)$$

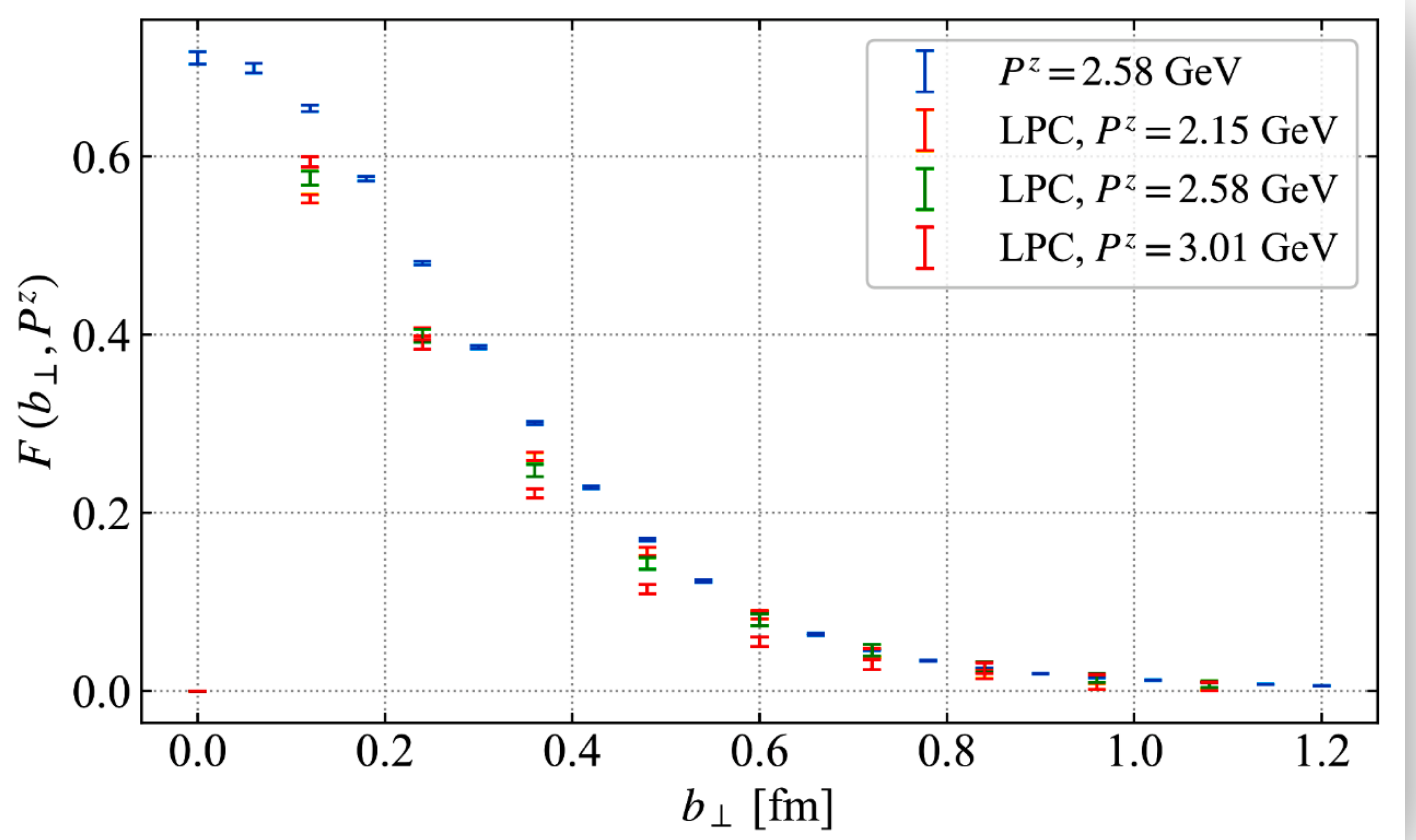
Soft factor

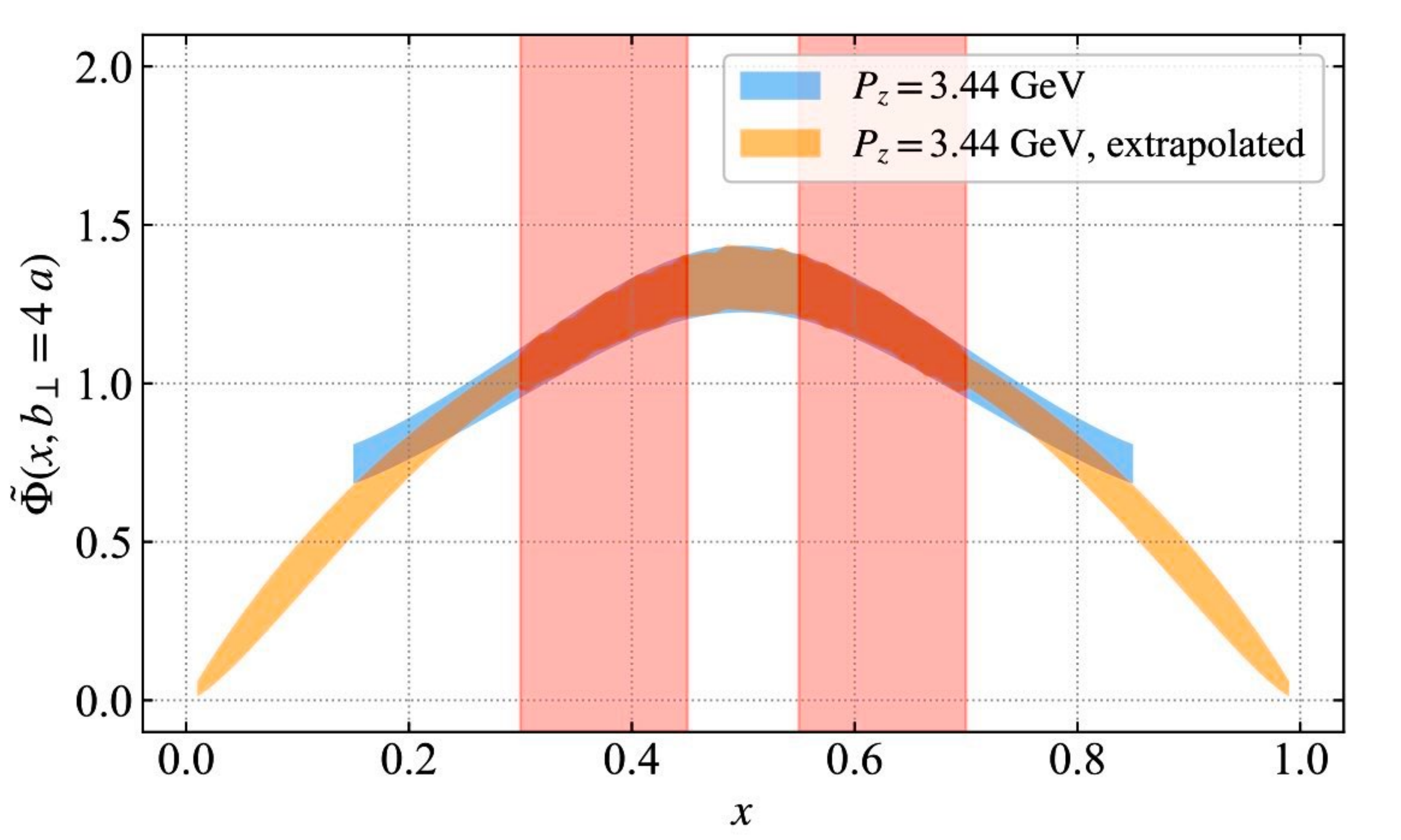
Form factor and TMD wave function from lattice

$$\langle \pi(-P) | j_1(b_T) j_2(0) | \pi(P) \rangle = S_I(b_T, \mu) \int dx_1 dx_2 H_{\text{sud}}(x_1, x_2, P_z, \mu) \tilde{\Phi}^\dagger(x, \vec{b}_T, \mu, P_z) \tilde{\Phi}(x, \vec{b}_T, \mu, P_z)$$

- Large momentum form factor

$$\tilde{\Phi}(x, \vec{b}_T, \mu, P_z) \equiv \tilde{\phi}(x, \vec{b}_T, \mu, P_z) / H_\phi(x, \bar{x}, P_z, \mu)$$





Wilson-Clover, $m_\pi = 300$ MeV,
$48^3 \times 64$, a = 0.06 fm

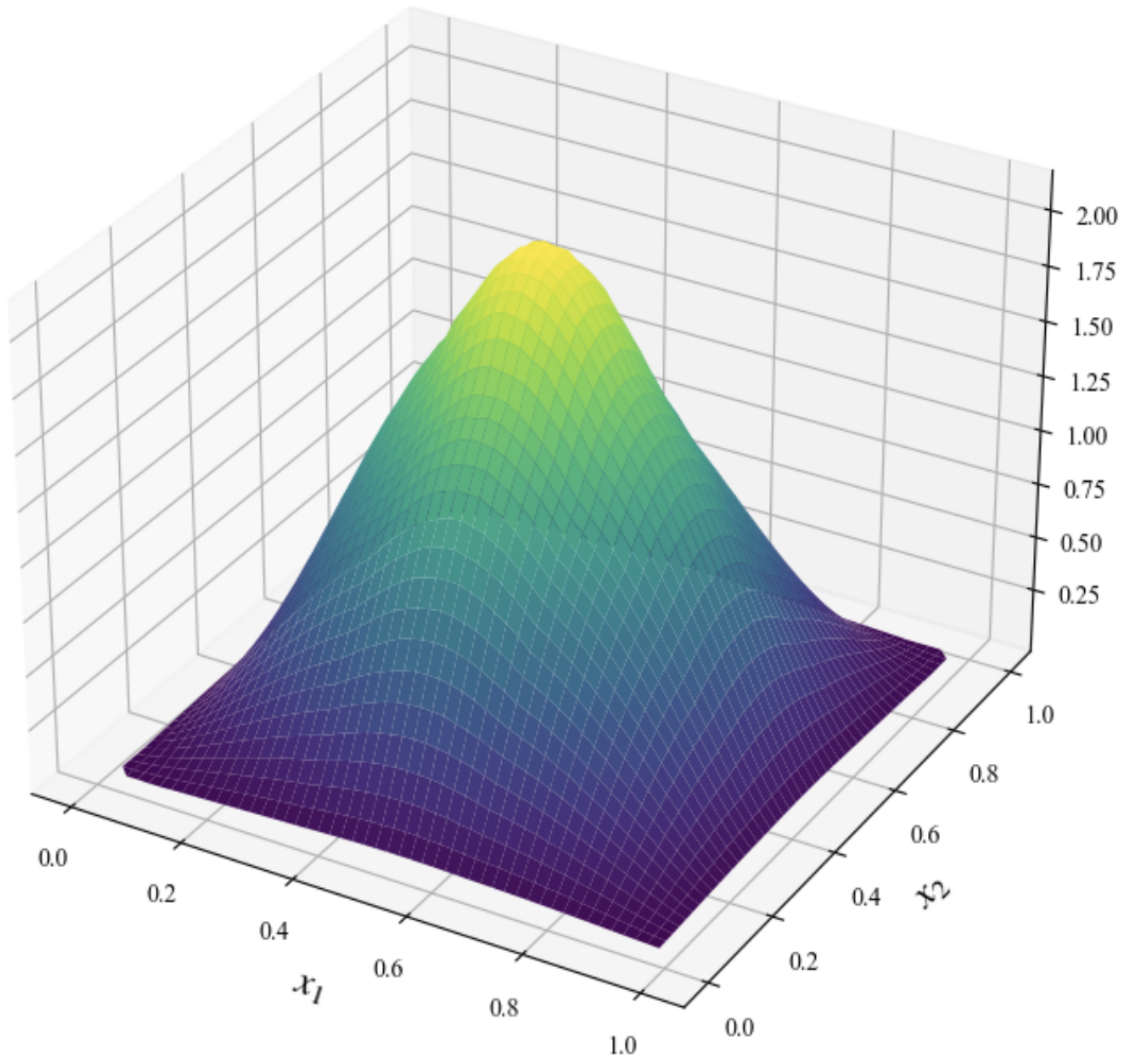
- quasi-TMD matching kernel (NLL) divergent at end-point region.
- $cx^\alpha(1-x)^\alpha$ parametrization is used.

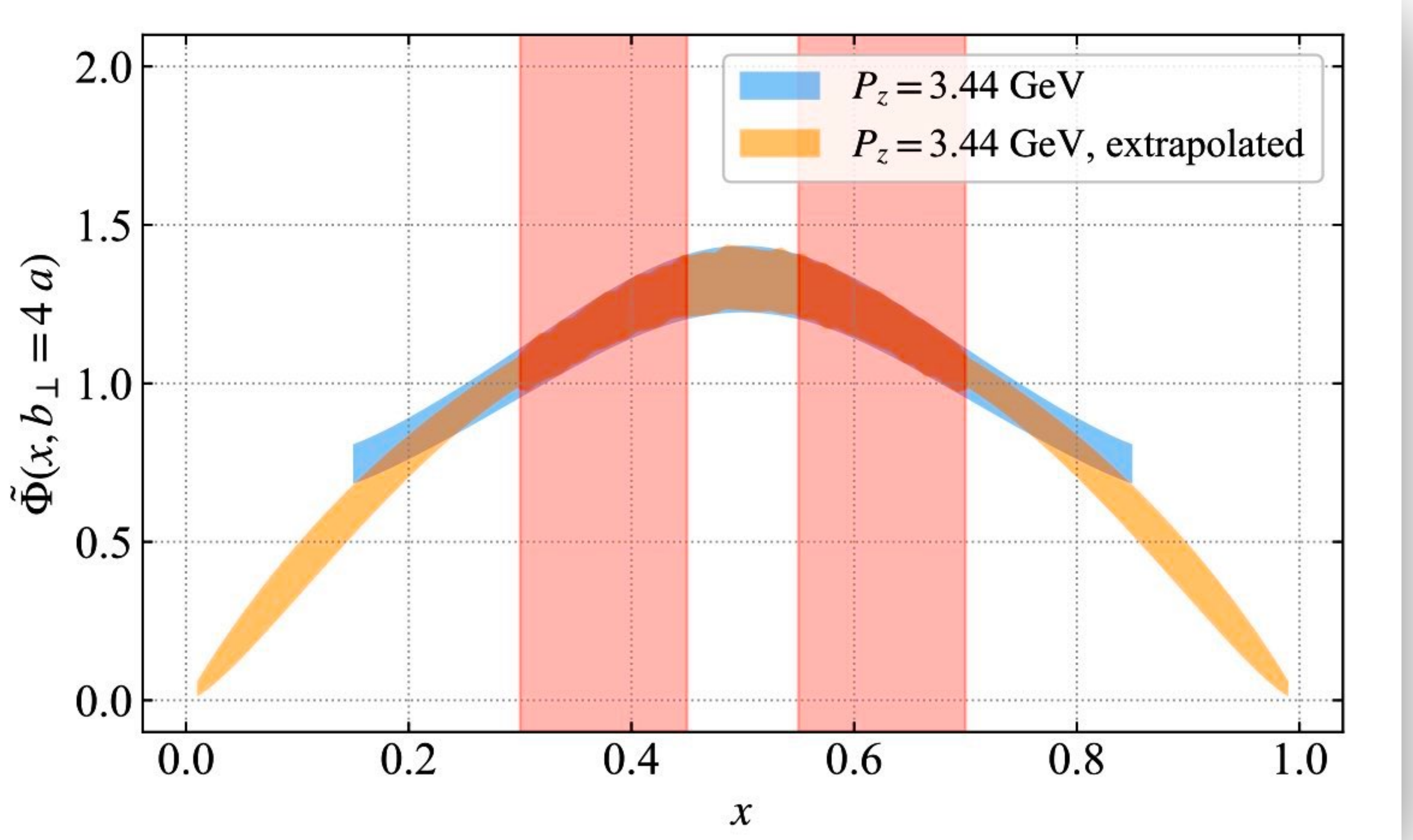
Form factor and TMD wave function from lattice

$$\langle \pi(-P)\,|\, j_1(b_T) j_2(0)\,|\,\pi(P)\rangle = S_I(b_T,\mu)\int dx_1 dx_2 H_{\rm sud}(x_1,x_2,P_z,\mu)\tilde{\Phi}^\dagger(x,\vec{b}_T,\mu,P_z)\tilde{\Phi}(x,\vec{b}_T,\mu,P_z)$$

- Sudakov factor

$$\tilde{\Phi}(x,\vec{b}_T,\mu,P_z) \equiv \tilde{\phi}(x,\vec{b}_T,\mu,P_z)/H_\phi(x,\bar{x},P_z,\mu)$$

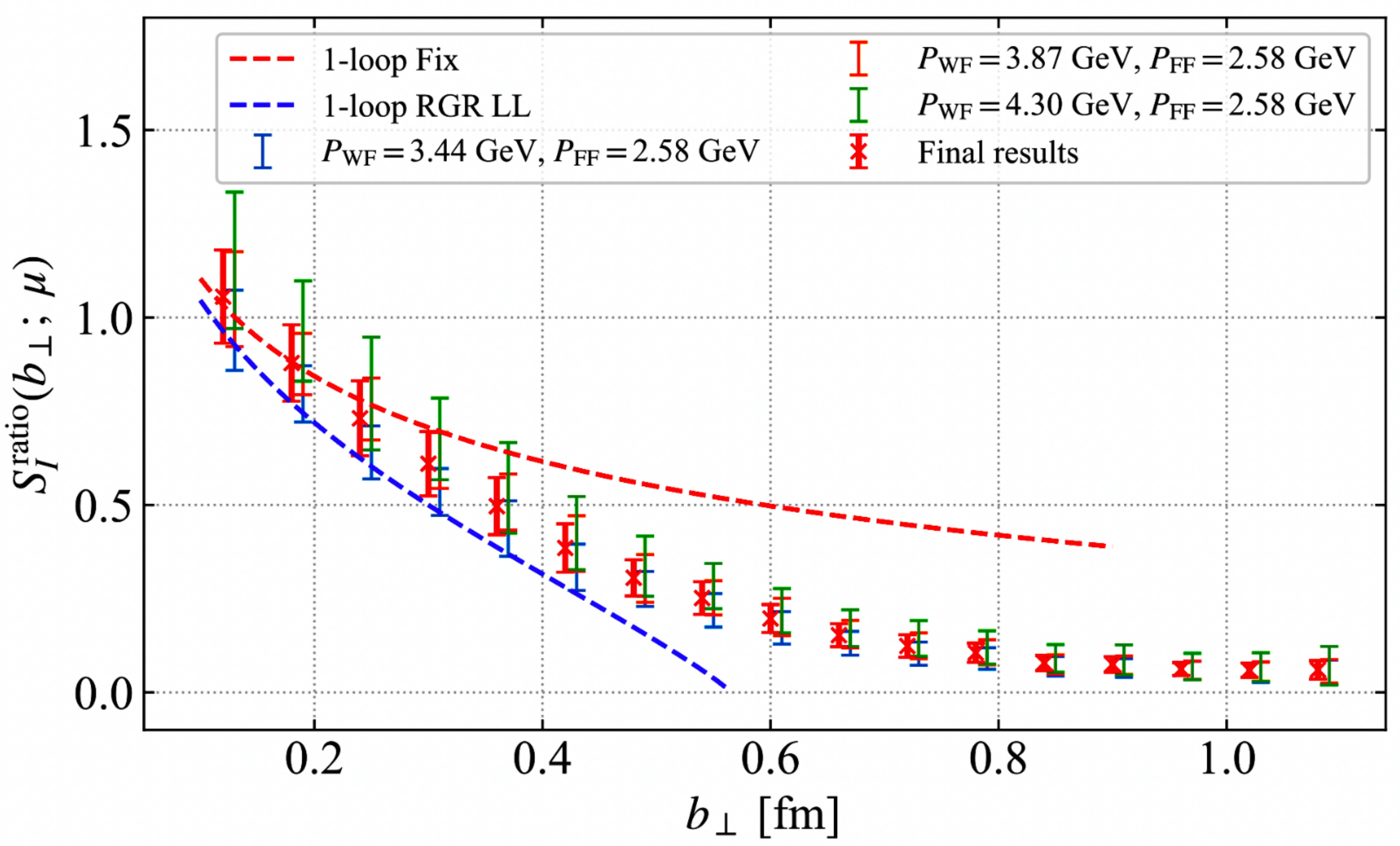




- End-point region contribution is highly suppressed by $H_{\rm sud}(x_1,x_2,P_z,\mu)$, reducing end-point uncertainty.

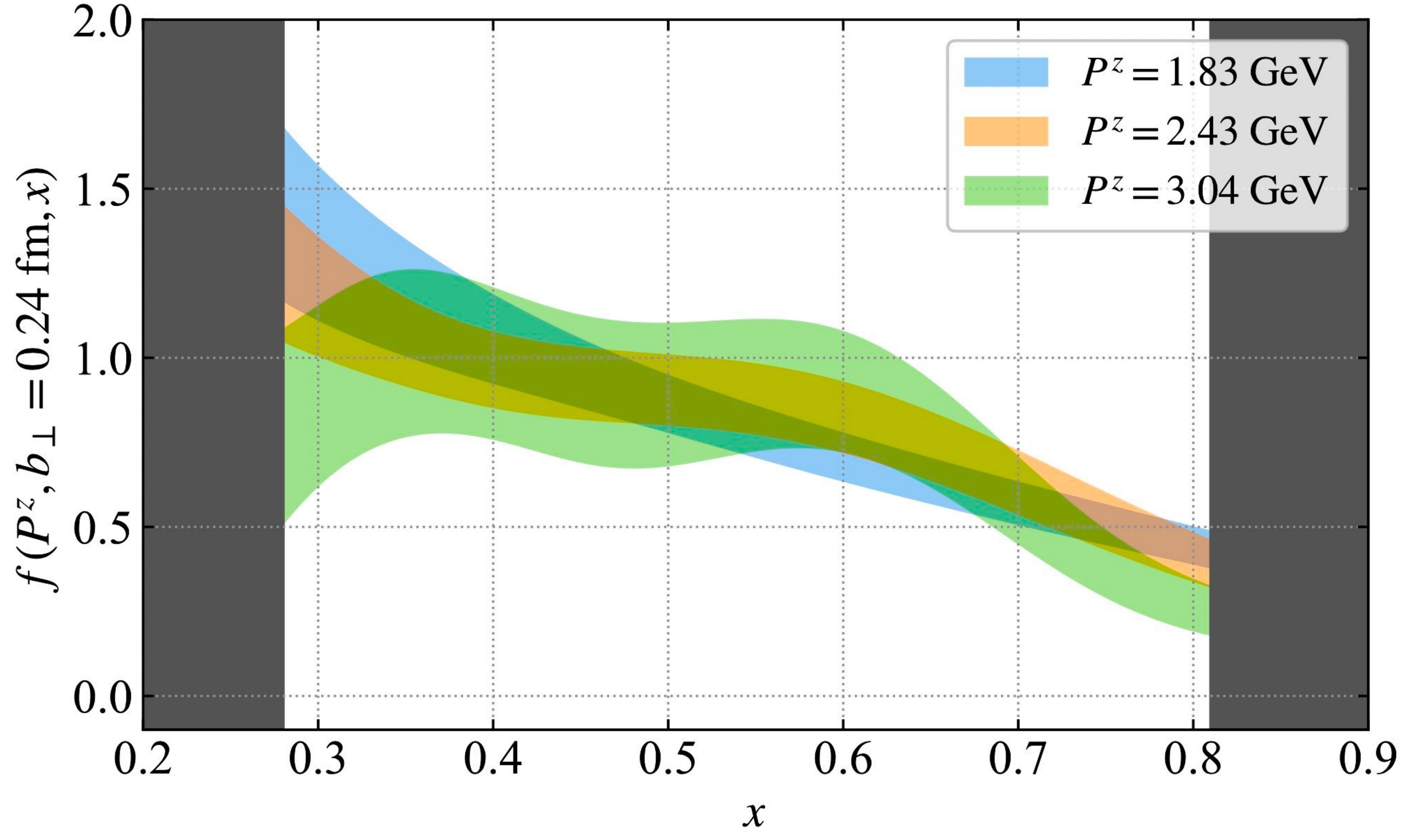
- quasi-TMD matching kernel (NLL) divergent at end-point region.
- $cx^\alpha(1-x)^\alpha$ parametrization is used.

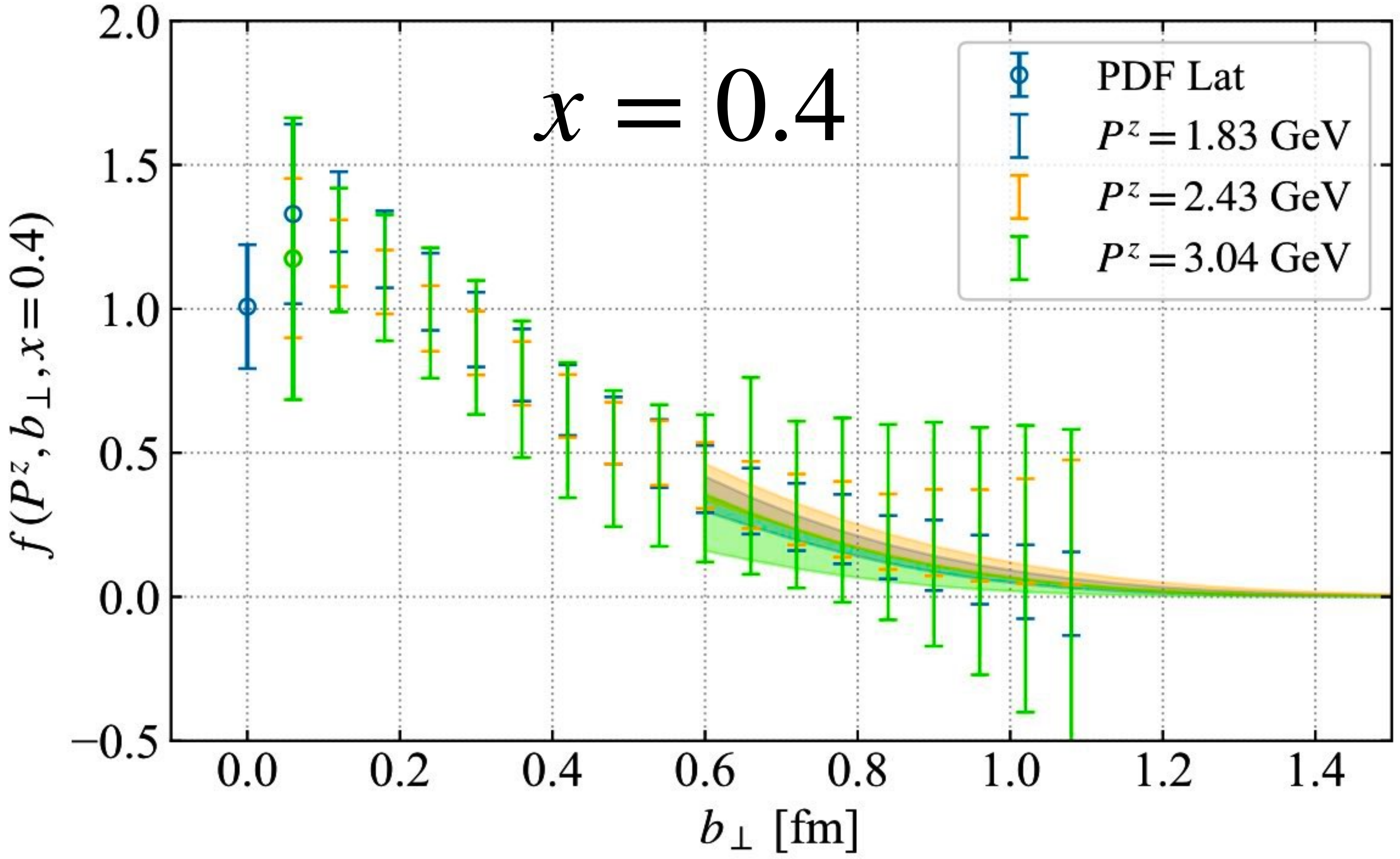
Intrinsic soft factor



• D. Bollweg, **XG**, J. He, et al., Phys.Rev.D 112 (2025) 3, 034501

Pion unpolarized TMDPDF

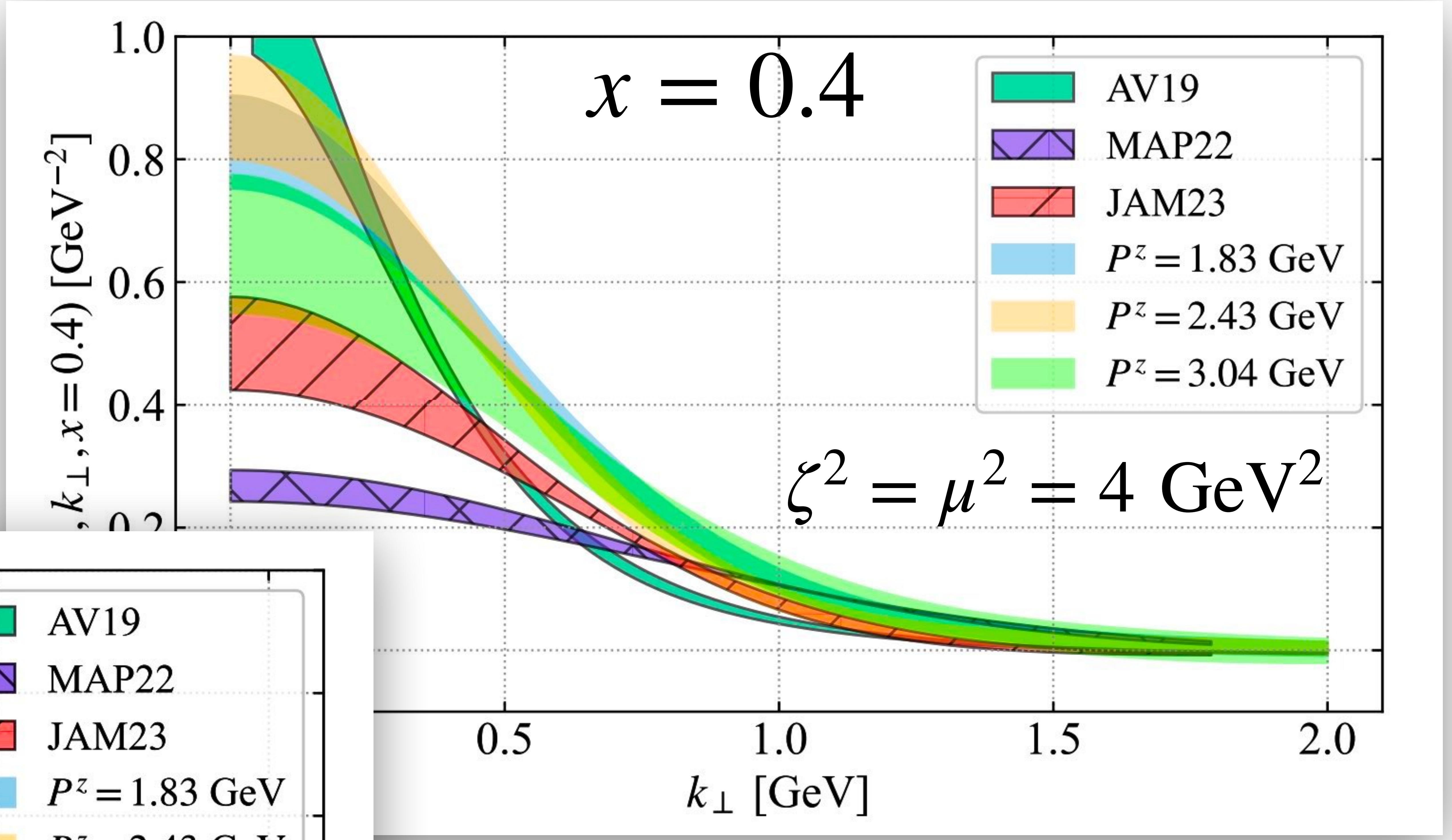


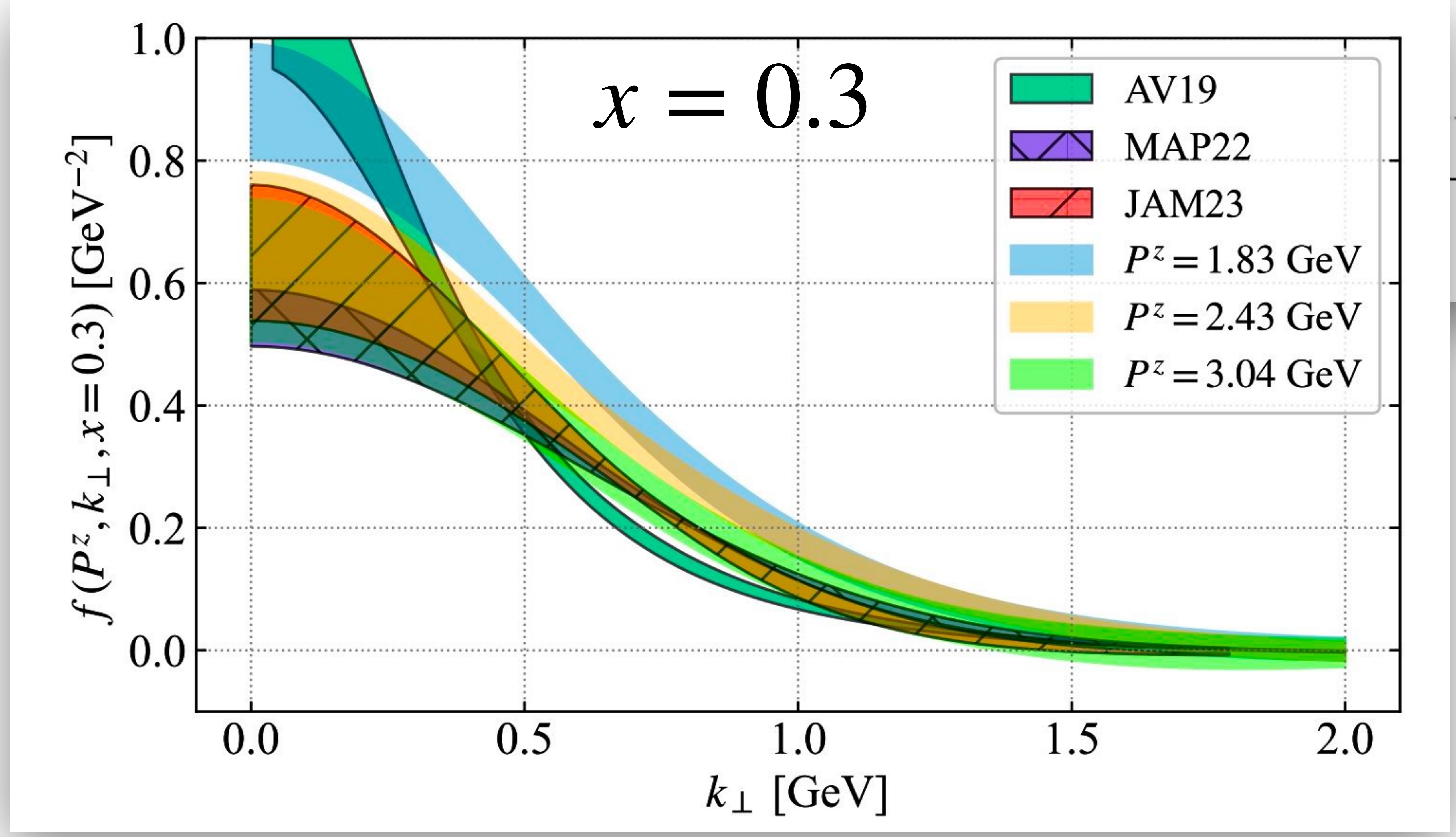


- Results from different momenta are consistent in the moderate range of x.
- Reasonable signal at large $b_\perp$ support a model extrapolation $\sim e^{-mb_\perp^2}$ and F.T. to $k_\perp$.

- D. Bollweg, **XG**, J. He, et al., Phys.Rev.D 112 (2025) 3, 034501

Pion unpolarized TMDPDF



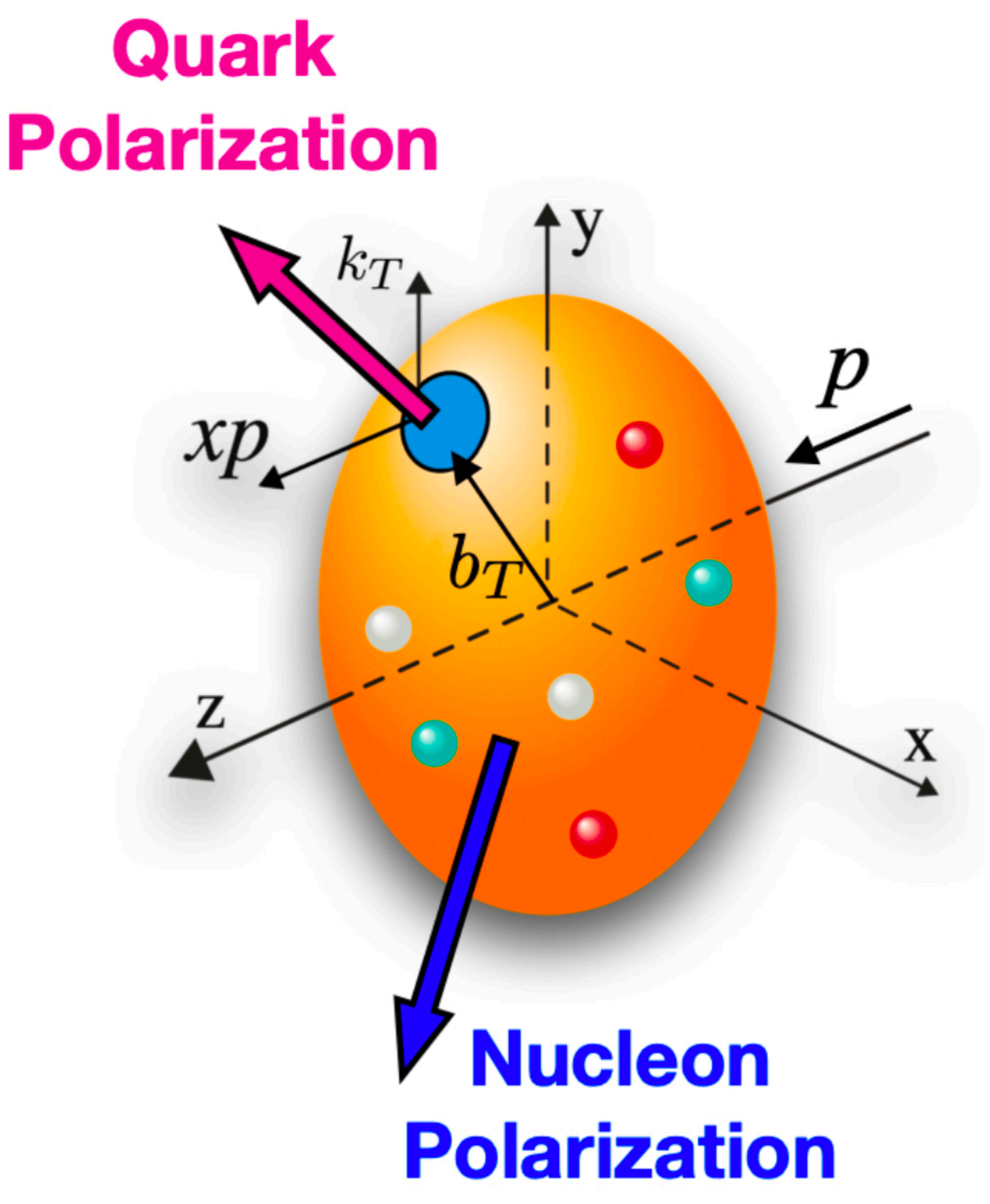


- Comparable with recent global fit.

- D. Bollweg, **XG**, J. He, et al., Phys.Rev.D 112 (2025) 3, 034501

Summary & outlook

- TMDs can be extracted from a universality class of quasi-TMD correlators from lattice under the framework of LaMET.
- The CG quasi-TMDs show advantage of simplified renormalization and enhanced signal.
- We calculated the ratios of nucleon TMDs with spin and flavor dependence from quasi-TMD beam functions.
- When combined with the soft factor, we determined the pion unpolarized TMDPDF.

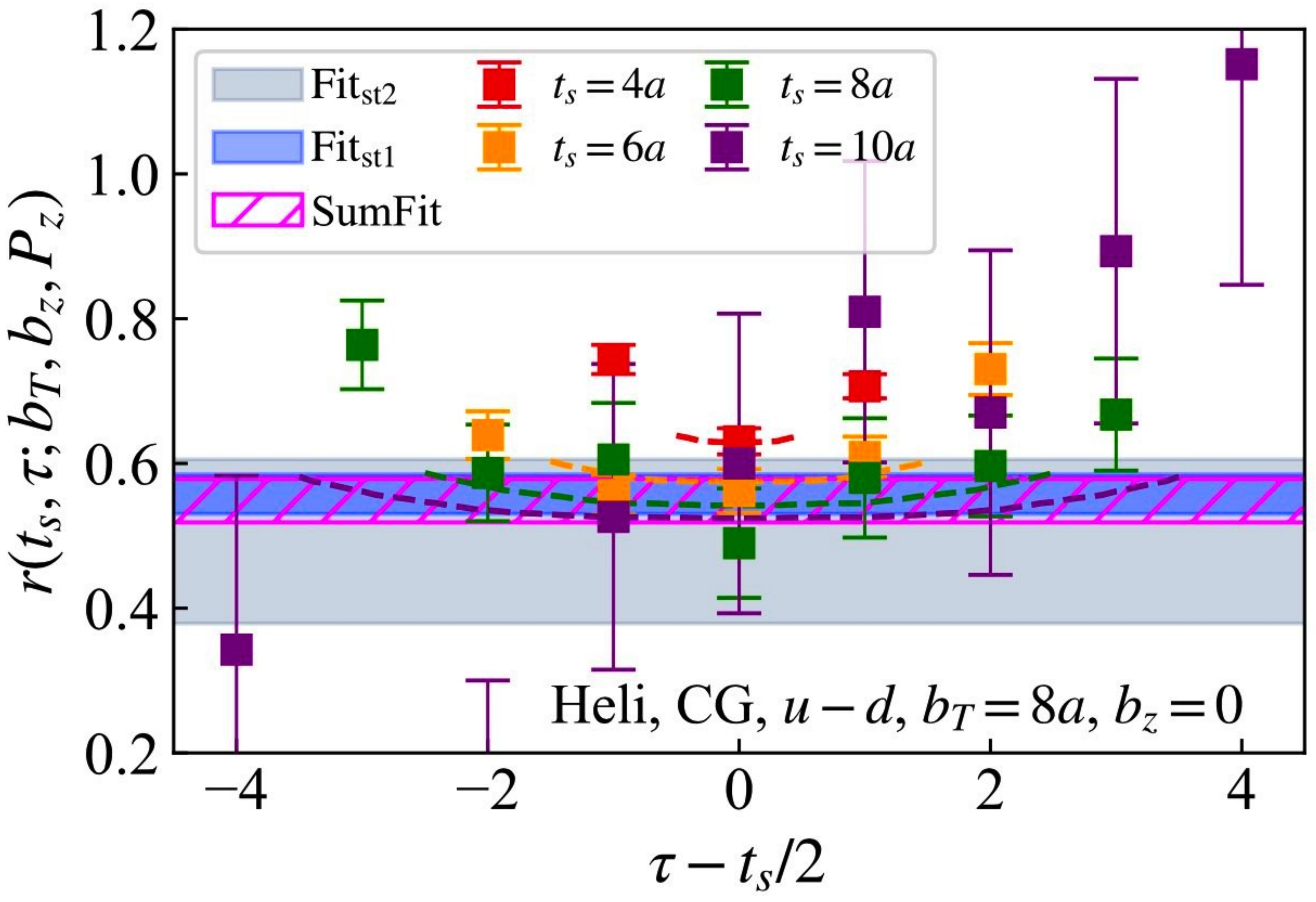


Thanks for your attention!

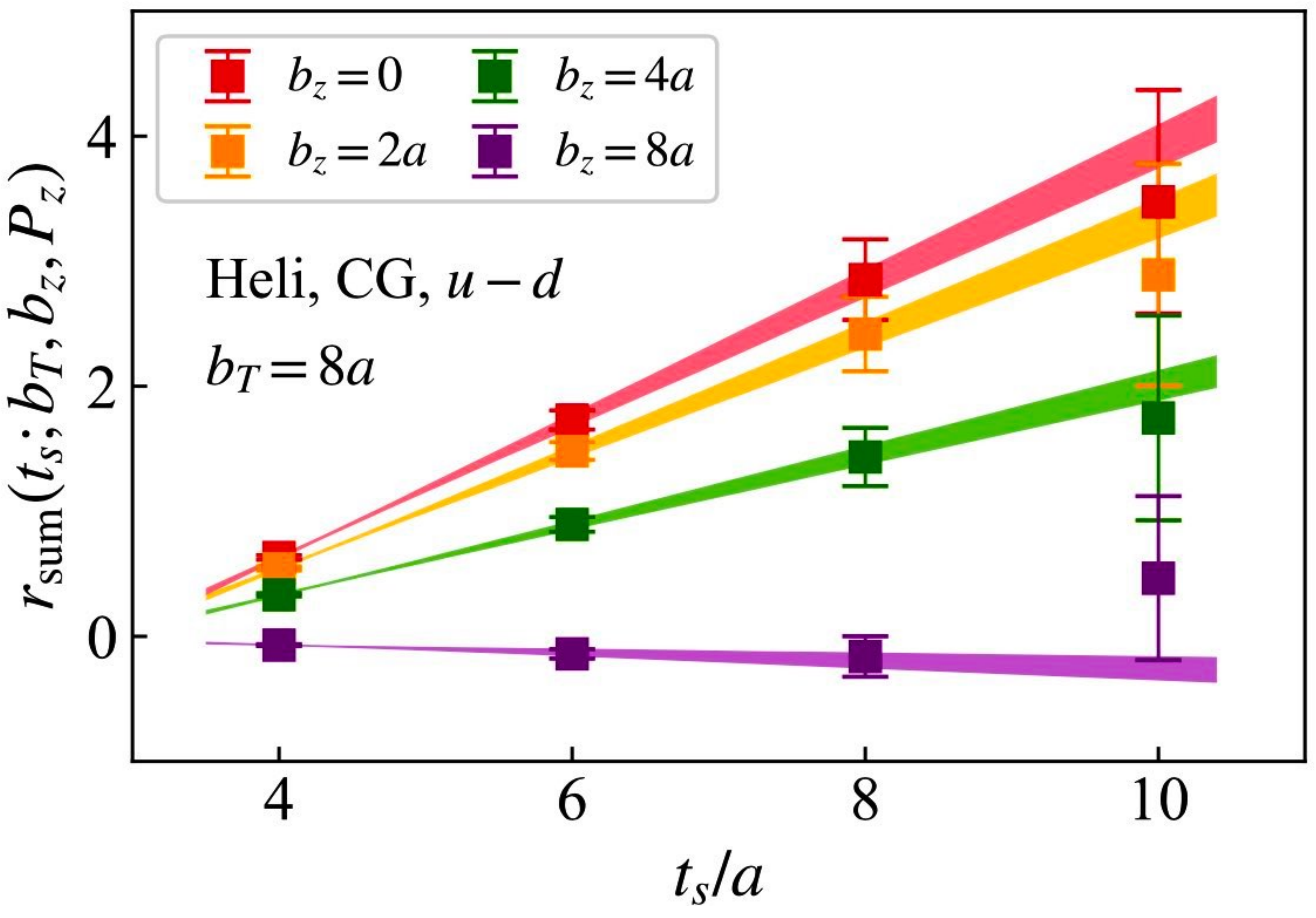
Back up

Bare matrix elements of quasi-TMD beam function

- Two-state fit



- Summation fit



Comparison between GI and CG approach

$$\frac{g_{1L}^{\Delta u^+ - \Delta d^+}(x, b_T)}{f_1^{u_v - d_v}(x, b_T)} \frac{1}{g_A} = \frac{\tilde{g}_{1L}^{\Delta u^+ - \Delta d^+}(x, b_T)}{\tilde{f}_1^{u_v - d_v}(x, b_T)} \frac{1}{g_A}$$