Nucleon Parton Distribution Functions From Boosted Correlators in Coulomb Gauge

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In collaboration with ANL-BNL group

LaMET Meeting **2025/10**

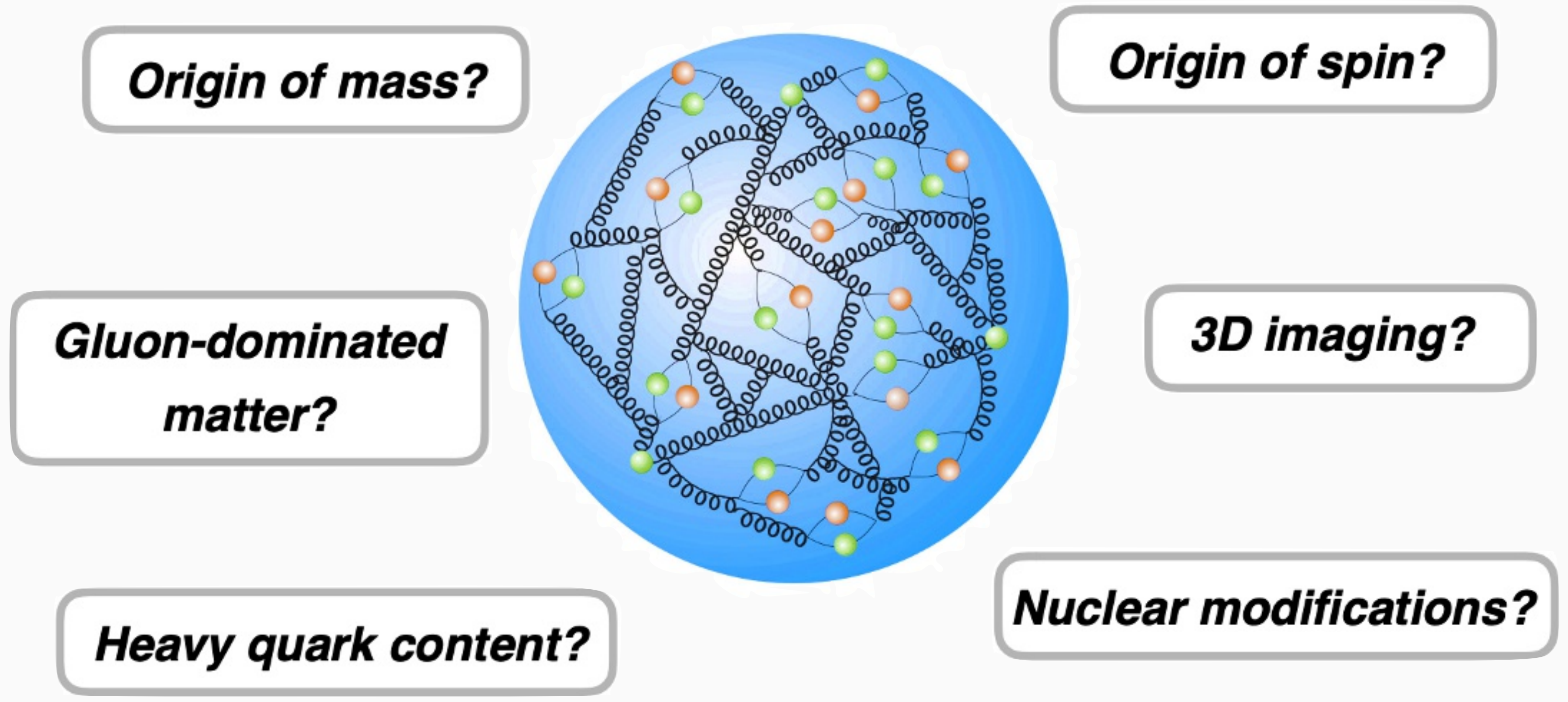
Contents

Parton Physics

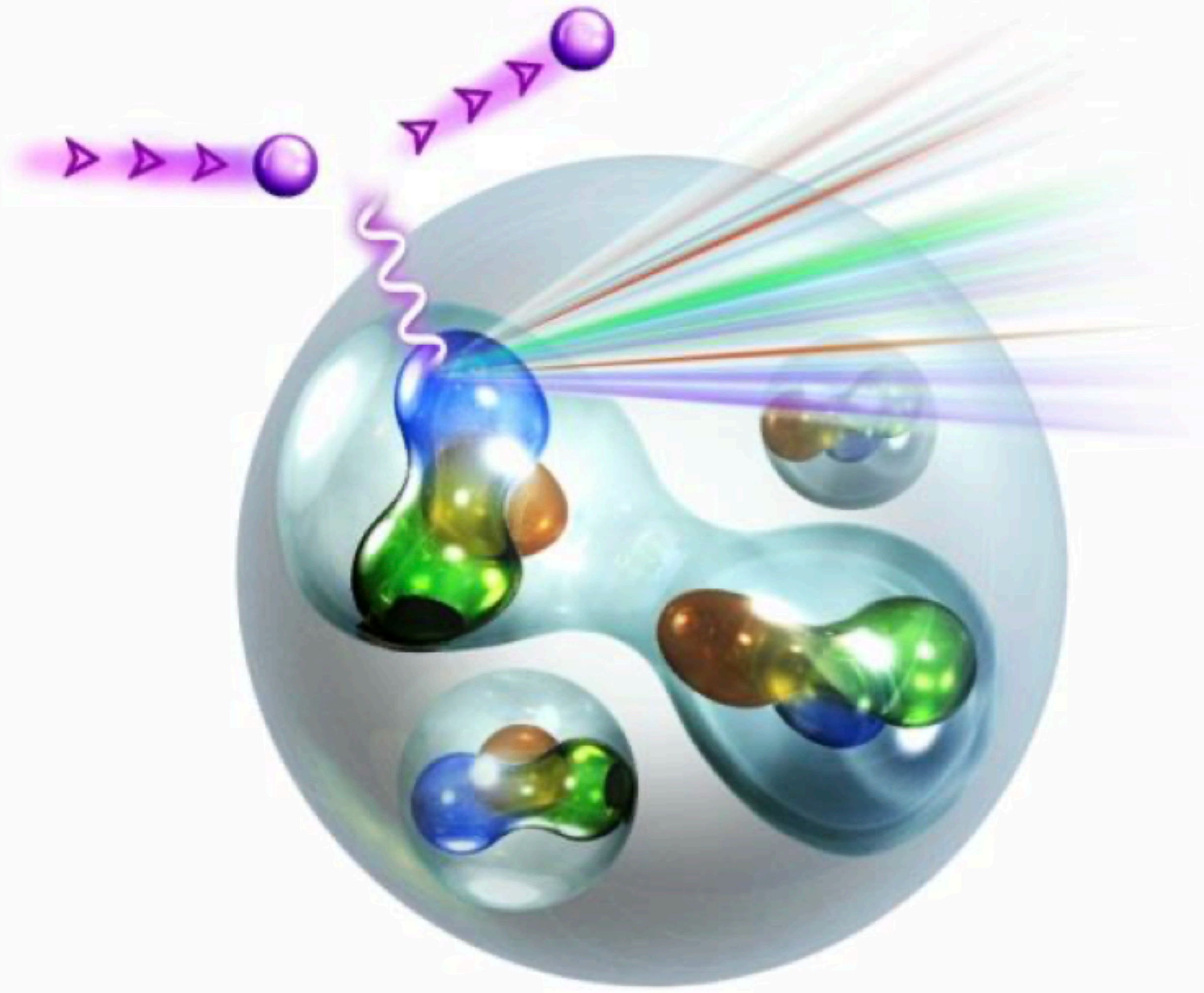
- Many experiments have been designed to probe the internal structure of nucleons.
- Our knowledge on nucleon is still limited:
  - Spin, mass …
  - How to describe a relativistic moving strong-coupled bound state?
- The language from Feynman: Parton Model in the infinite momentum frame
  R. P. Feynman, Conf. Proc. C 690905 (1969)
  - Quarks and gluons (partons) are "frozen" in the transverse plane;
  - During a high-energy collision, the struck parton appears like a free particle.

The many faces of the proton

QCD bound state of **quarks** and **gluons**

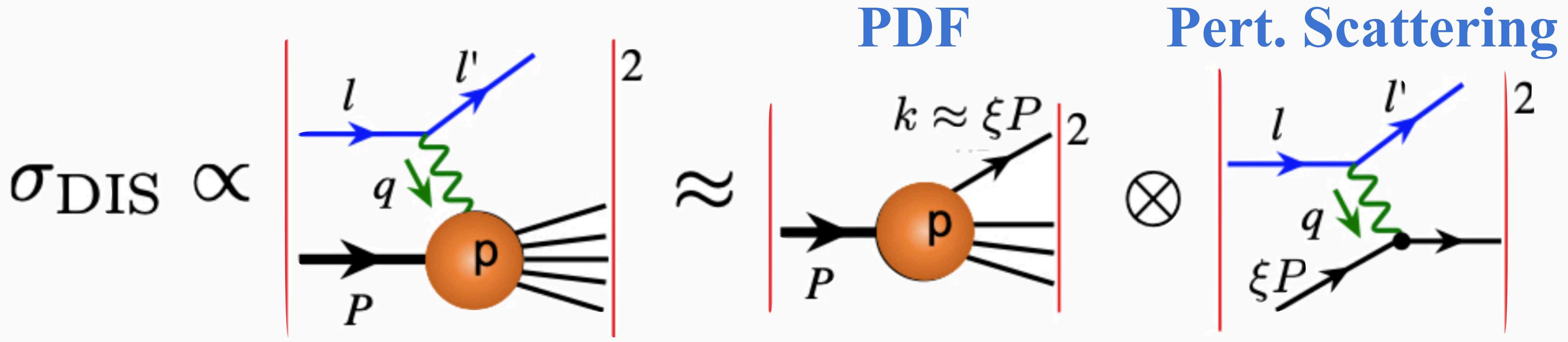


Cr. Juan Rojo



Cr. Dave Gaskell

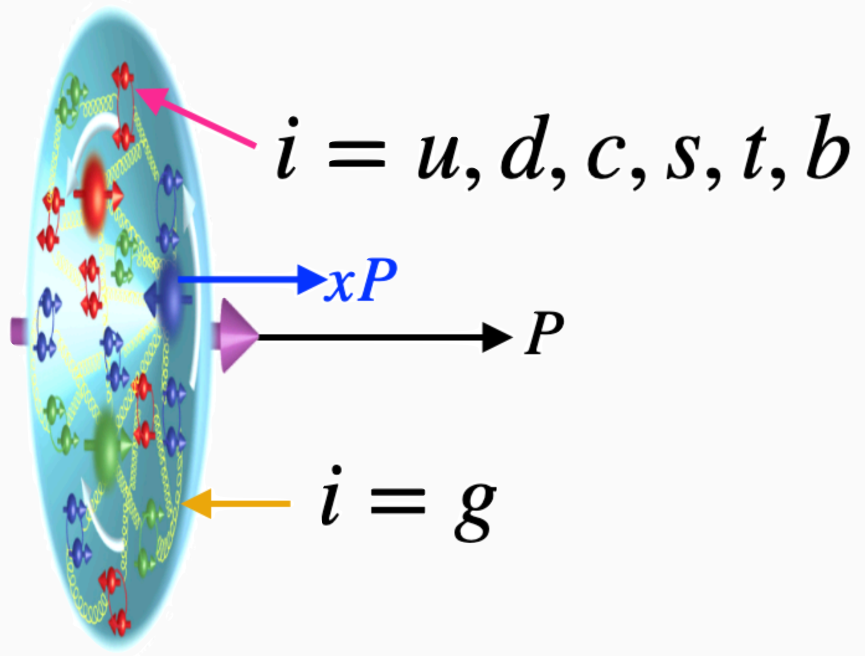
QCD Factorization



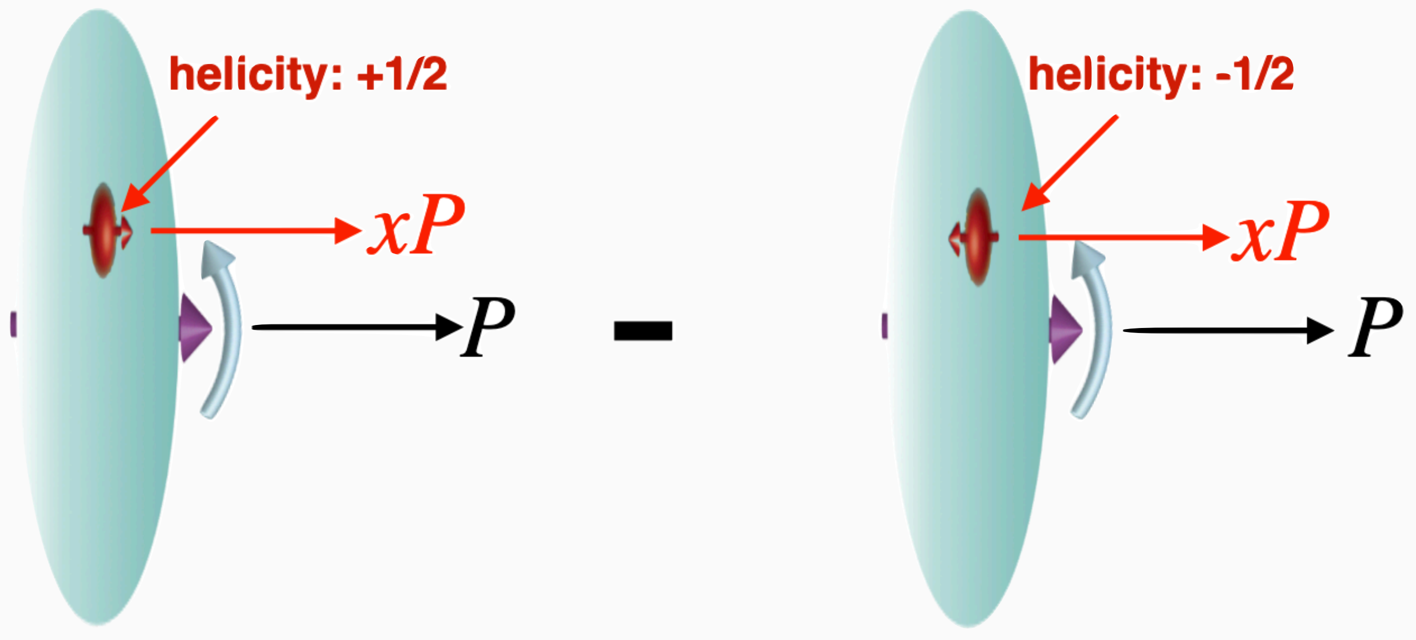
R. Boussarie, et al., ``TMD Handbook", [arXiv:2304.03302 [hep-ph]]

Parton Distribution Functions (PDFs)

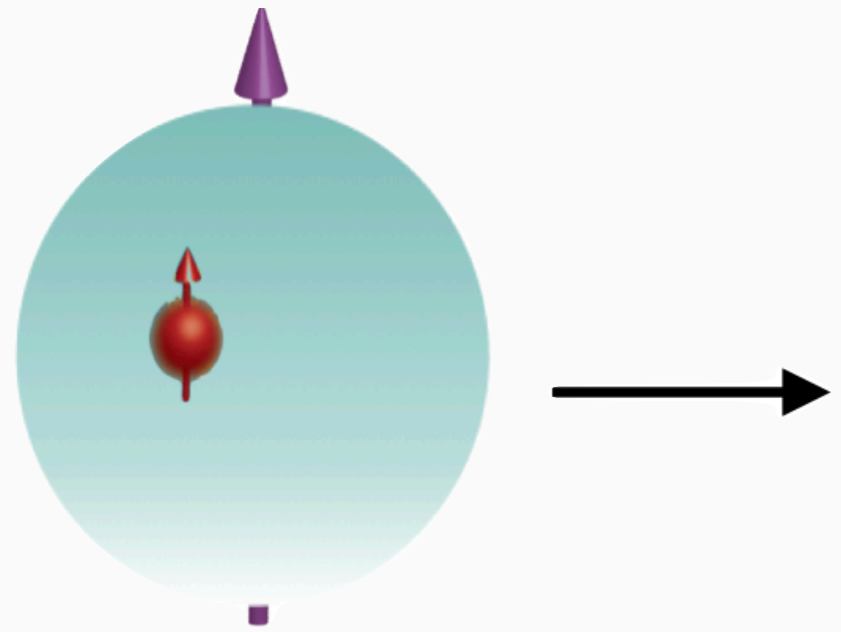
Unpolarized PDF $f(x, \mu)$



Helicity PDF $\Delta f(x, \mu)$



Transversity PDF $\delta f(x, \mu)$



Extract / Calculate PDFs

- **Phenomenology: global analysis of experimental data**
- **Lattice QCD: first-principles calculation**

Lattice QCD Calculation of PDFs

- As a first-principles non-perturbative method, Lattice QCD provides independent predictions of PDFs.
  - Mellin Moments
    - Up to $\langle x^3 \rangle$ *C. Alexandrou, et al., Phys. Rev. D 92 (2015); G. S. Bali, et al., Phys. Rev. D 98 (2018); …*
    - Smeared operators for higher moments *Z. Davoudi, M. J. Savage, Phys.Rev.D 86 (2012); …*
    - Gradient Flow for higher moments *A. Shindler, Phys. Rev.D 110 (2024); A. Francis, et al., PoS LATTICE2024, 336 (2025); …*
  - Large Momentum Effective Theory (LaMET) (quasi-PDF) *X. Ji, Phys.Rev.Lett. 110 (2013); X. Ji, et al., Rev.Mod.Phys. 93 (2021); X. Gao, et al., Phys. Rev. Lett. 128 (2022); …*
  - Short Distance Expansion
    - Pseudo PDF / Ioffe-time distribution *A. V. Radyushkin, Phys. Rev. D 96 (2017); C. Alexandrou, et al., Phys. Rev. D 98 (2018); …*
    - Current-current correlator *V. M. Braun, et al., Nucl. Phys. B 685 (2004); V. M. Braun, et al., Eur. Phys. J. C 55 (2008); R. S. Sufian, et al., Phys. Rev. D 102 (2020); …*
  - Operator Product Expansion (OPE)
    - Compton amplitude *A. J. Chambers, et al., Phys. Rev. Lett. 118 (2017); M. Gockeler, et al. [QCDSF], Phys. Rev. Lett. 92 (2004); …*
    - Heavy-quark Operator Product Expansion (HOPE) *W. Detmold, and C. J. David Lin, Phys. Rev. D 73 (2006); W. Detmold, et al. [HOPE], Phys. Rev. D 105 (2022); …*
  - Hadronic Tensor *K. F. Liu, Phys. Rev. D 62 (2000); K. F. Liu, and S. J. Dong, Phys. Rev. Lett. 72 (1994); …*

Large-Momentum Effective Theory(LaMET)

- **PDF is defined from a light-cone correlator in a hadron, which is Lorentz invariant.**

Parton model

$$f_\Gamma(x,\mu) = \int_{-\infty}^{\infty} \frac{d\lambda}{2\pi} e^{-i\lambda x} \frac{1}{2P^+} \left\langle P \left| \bar{\psi}\left(\xi^-\right) W\left(\xi^-,0\right) \Gamma \psi(0) \right| P \right\rangle \longleftrightarrow \left\langle |\vec{P}| = \infty \left| O(t=0) \right| |\vec{P}| = \infty \right\rangle$$

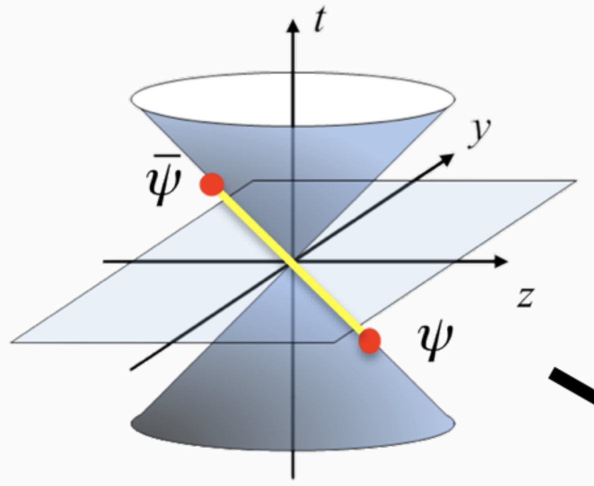
- **Define a quasi distribution with large-momentum states and time-independent operators.**

Different orders of limit, but pert.

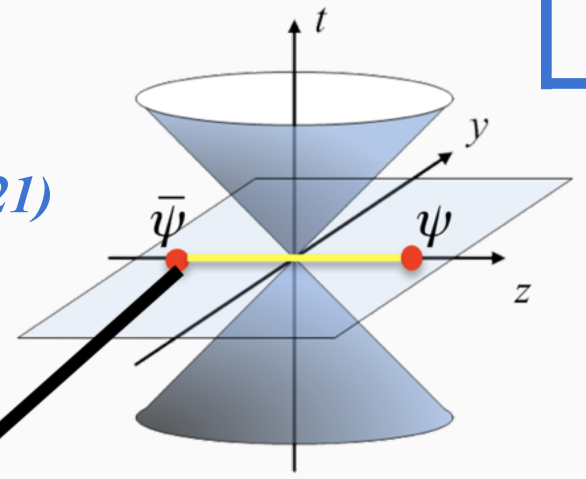
$$\tilde{f}^0_\Gamma(y,P^z,\mu) = P^z \int \frac{dz}{2\pi} e^{iz(yP^z)} \frac{1}{2P^t} \langle P | \bar{\psi}_0(z)\, W(z,0)\, \Gamma \psi_0(0) | P \rangle$$

$$\Lambda_{\rm QCD} \ll |\vec{P}| \ll \frac{\pi}{a}$$

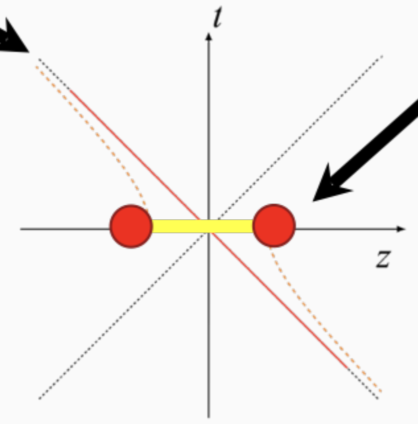
Large-momentum expansion



X. Ji, Phys.Rev.Lett. 110 (2013)
X. Ji, et al., Rev.Mod.Phys. 93 (2021)
X. Ji, Nucl. Phys. B 1007 (2024)



Light-cone correlation:
Cannot be directly calculated on the lattice



Equal-time correlation (for quasi distribution):
Directly calculable on the lattice

- **LaMET enables us to obtain the precision-controlled x-distribution of PDFs in $x \in [x_{\rm min}, x_{\rm max}]$.**

Pert. matching kernel — Power corrections

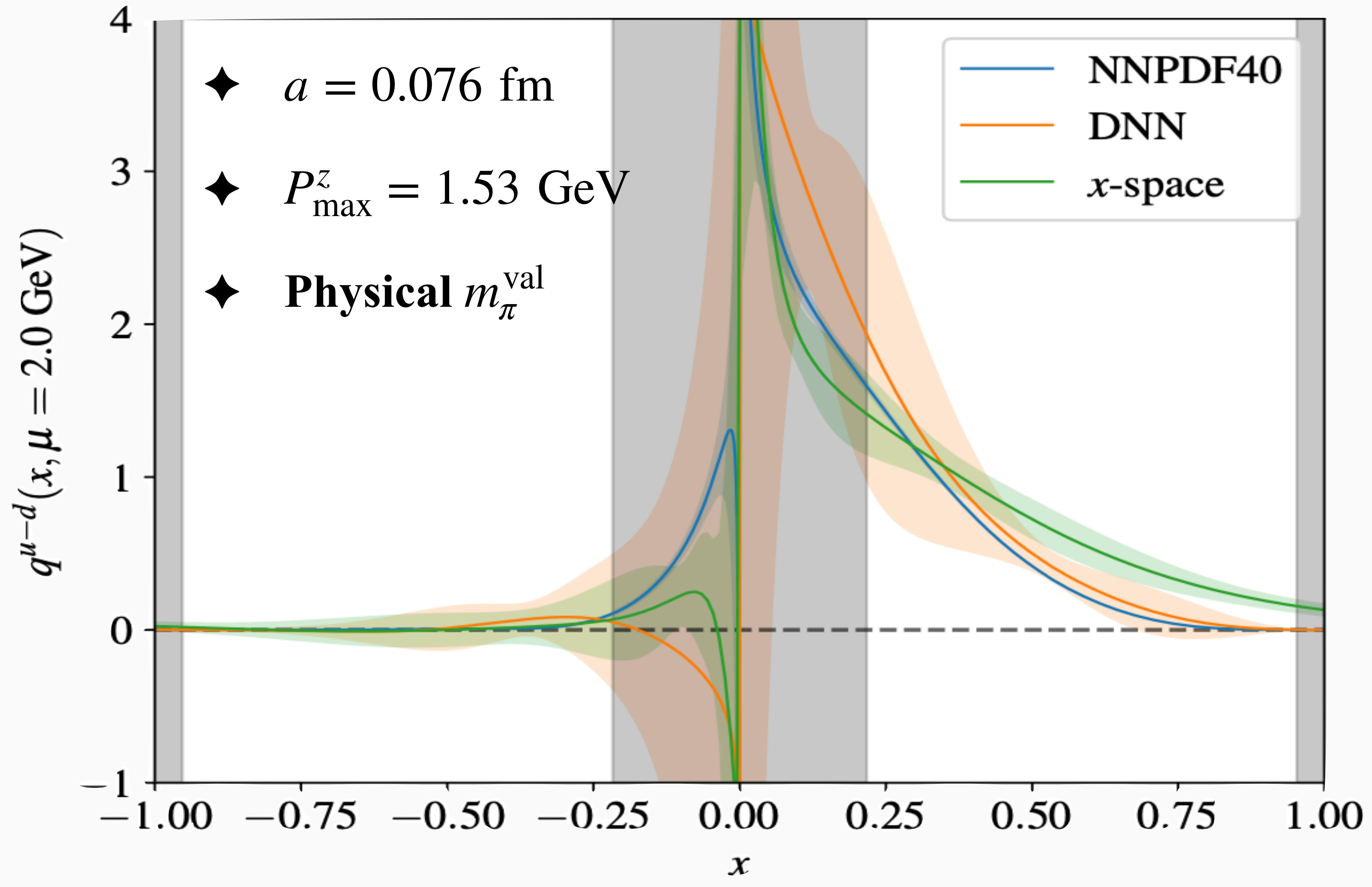
$$f(x,\mu) = C\left(\frac{y}{x}, \frac{P^z}{\mu}\right) \otimes \tilde{f}\left(y, \frac{P^z}{\mu}\right) + O\left(\frac{\Lambda^2_{\rm QCD}}{(xP^z)^2}, \frac{\Lambda^2_{\rm QCD}}{((1-x)P^z)^2}\right)$$

Nucleon PDFs from LaMET

- In recent years, a lot of improvements of renormalization and matching has been developed in LaMET;

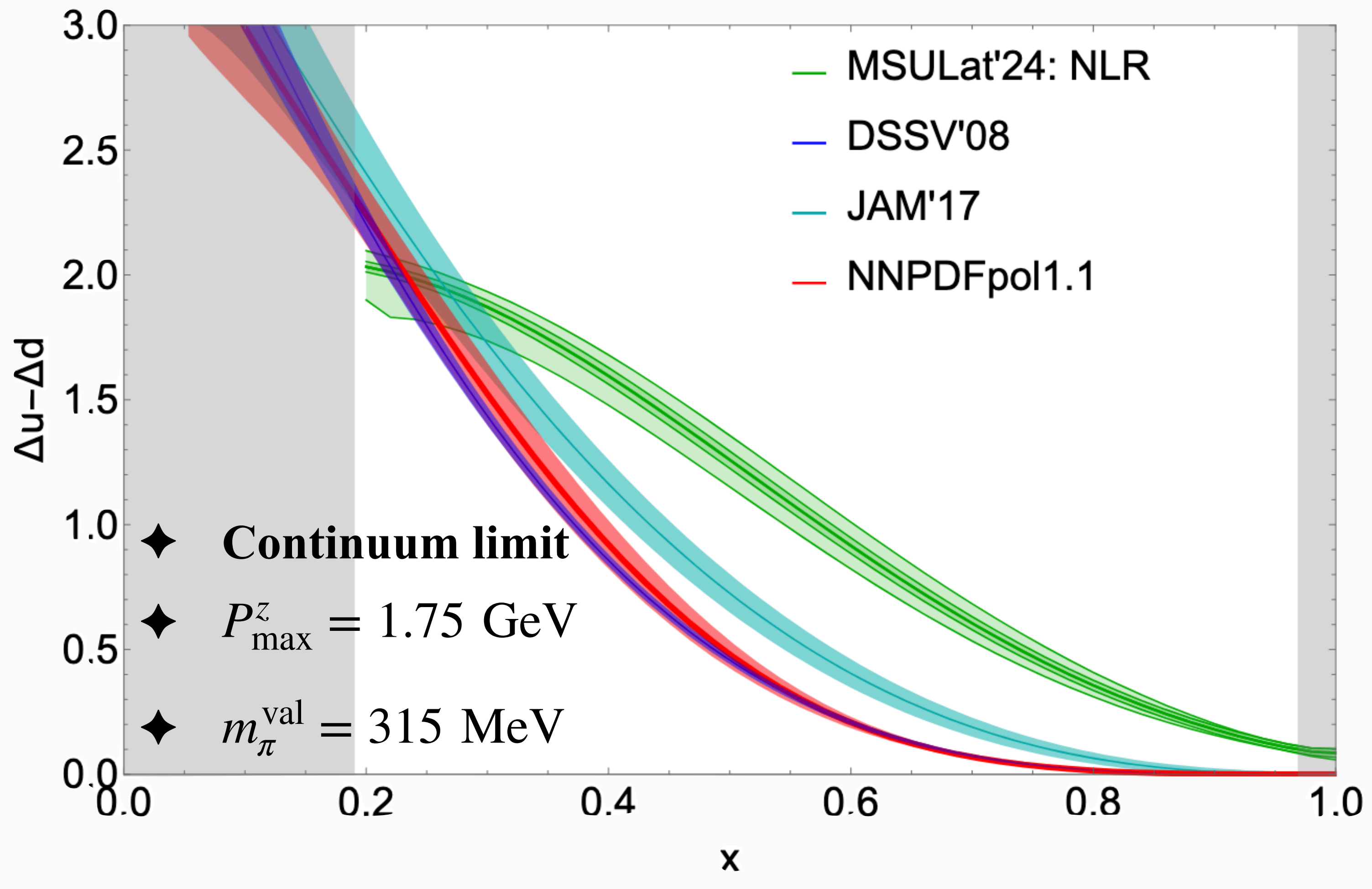
Y. Su, et al., Nucl. Phys. B 991 (2023);
R. Zhang, et al., Phys. Lett. B 844 (2023);
X. Ji, et al., 2410.12910 [hep-ph]

Unpolarized isovector PDF



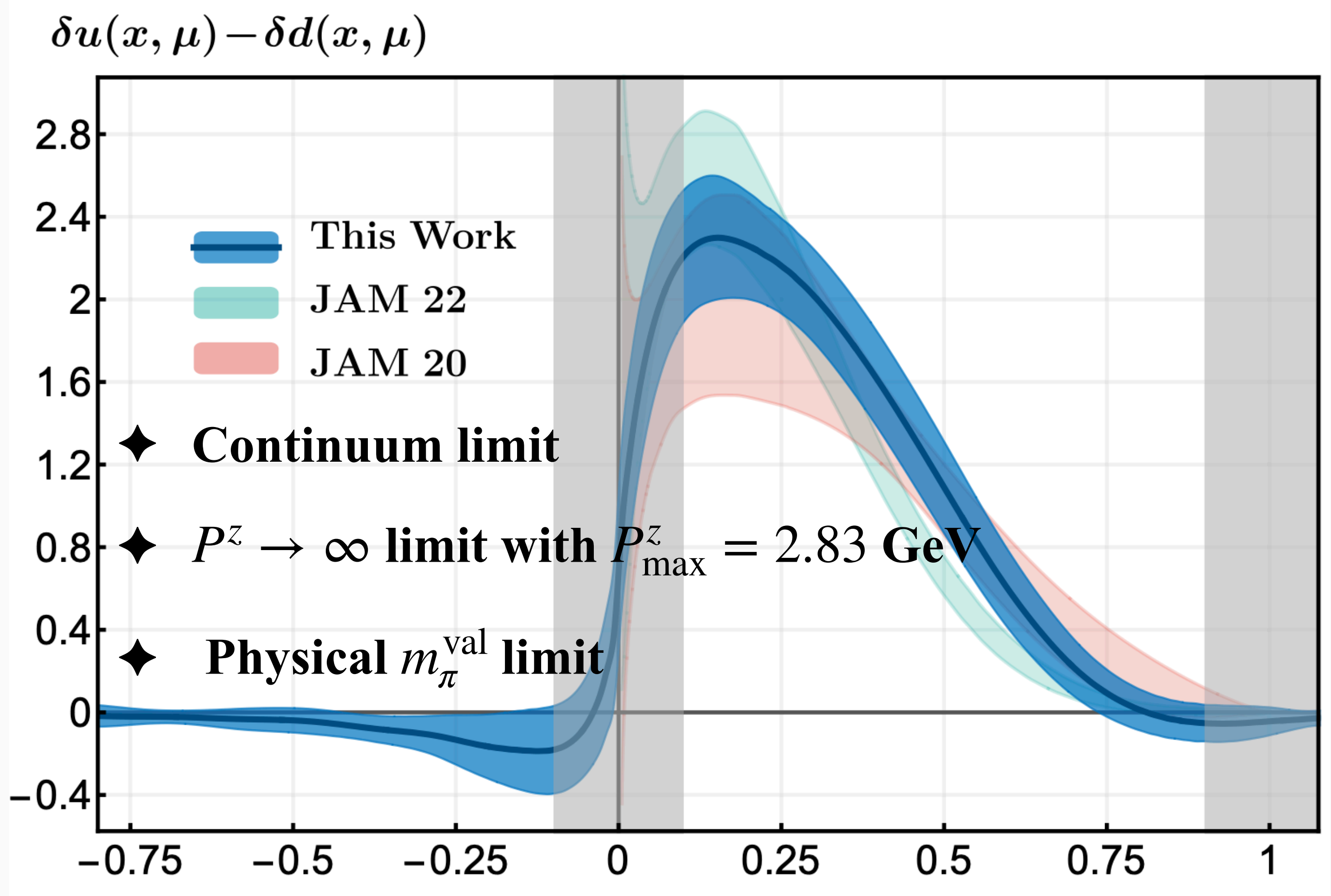
X. Gao, et al., Phys. Rev. D 107 (2023)

Helicity isovector PDF



J. Holligan and H. W. Lin, Phys. Lett. B 854 (2024)

Transversity isovector PDF



F. Yao et al. [LPC], Phys. Rev. Lett. 131 (2023)

- Existing calculations of the nucleon PDFs still deviate from the global analyses, which is possibly due to the systematics from:
  - Hadron momentum is not large enough;
  - The renormalization scheme for the imaginary part of quasi-PDFs could be optimized (to be discussed later);
  - Other lattice systematics, like excited-state contamination (especially for the imaginary part in coordinate space) …

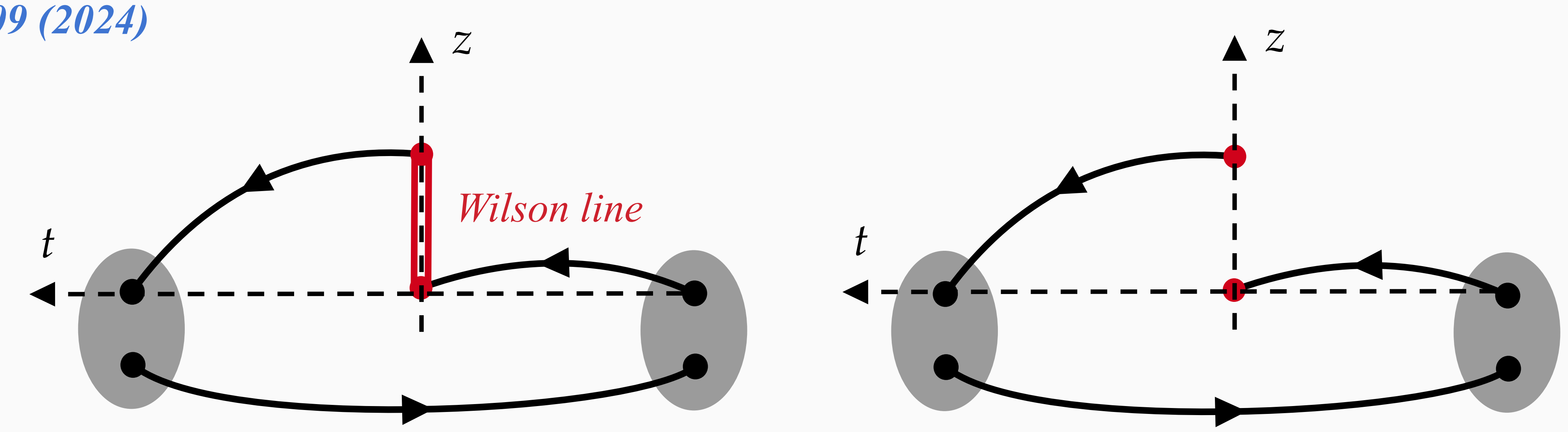
} Will be addressed in this work

C. Alexandrou et al., Phys.Rev.D 103 (2021)

Coulomb Gauge Method

- **Define a quasi distribution in CG without Wilson line:** *X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)* *Y. Zhao, PRL 133 (2024)*

$$\tilde{f}^{0}_{\rm CG}(y,P^z,\mu) = P^z \int \frac{dz}{2\pi} e^{iz(yP^z)} \frac{1}{2P^t} \langle P|\bar{\psi}_0(z)\Gamma\psi_0(0)\Big|_{\vec{\nabla}\cdot\vec{A}=0}|P\rangle$$

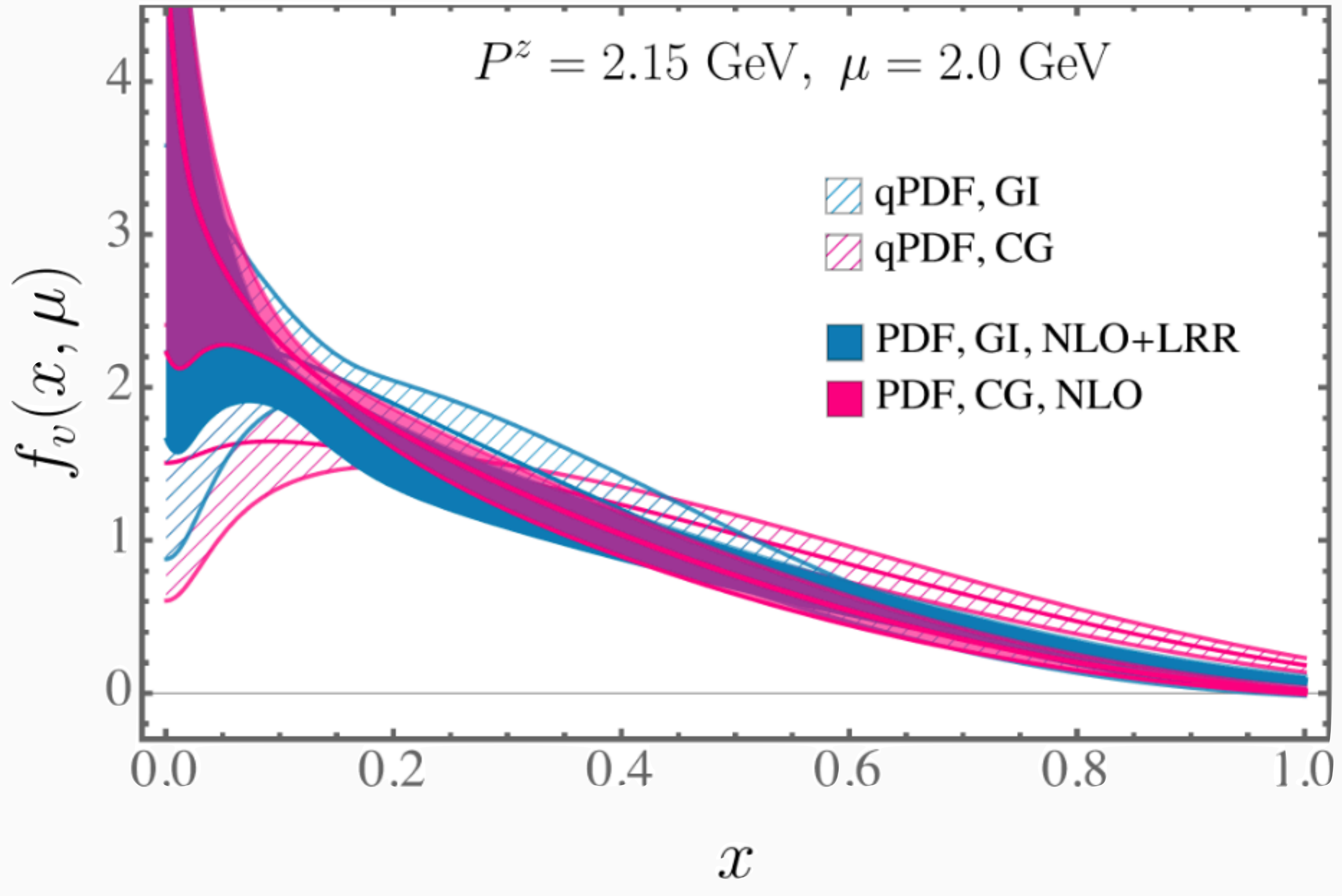


Gauge Invariant (GI) **Coulomb Gauge (CG)**

- **Why choose CG?**
  - *X. Ji, Y. S. Liu, Y. Liu, J. H. Zhang and Y. Zhao, RMP 93 (2021)*
  - $\vec{\nabla}\cdot\vec{A}=0$ **becomes** $A^+=0$ **in the infinite boost, so the quasi distribution in CG belongs to the universality class in LaMET;**
  - **No linear divergence / linear renormalon;**
  - **Simplified renormalization** $\bar{\psi}_0(z)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z)\Gamma\psi(0)\right]$**;**
  - **Larger off-axis momenta (3D rotational symmetry) can be accessed;**

- **In this work, we will explore the CG method to test its efficacy in nucleon PDFs.**

Pion valence PDF in CG v.s. GI

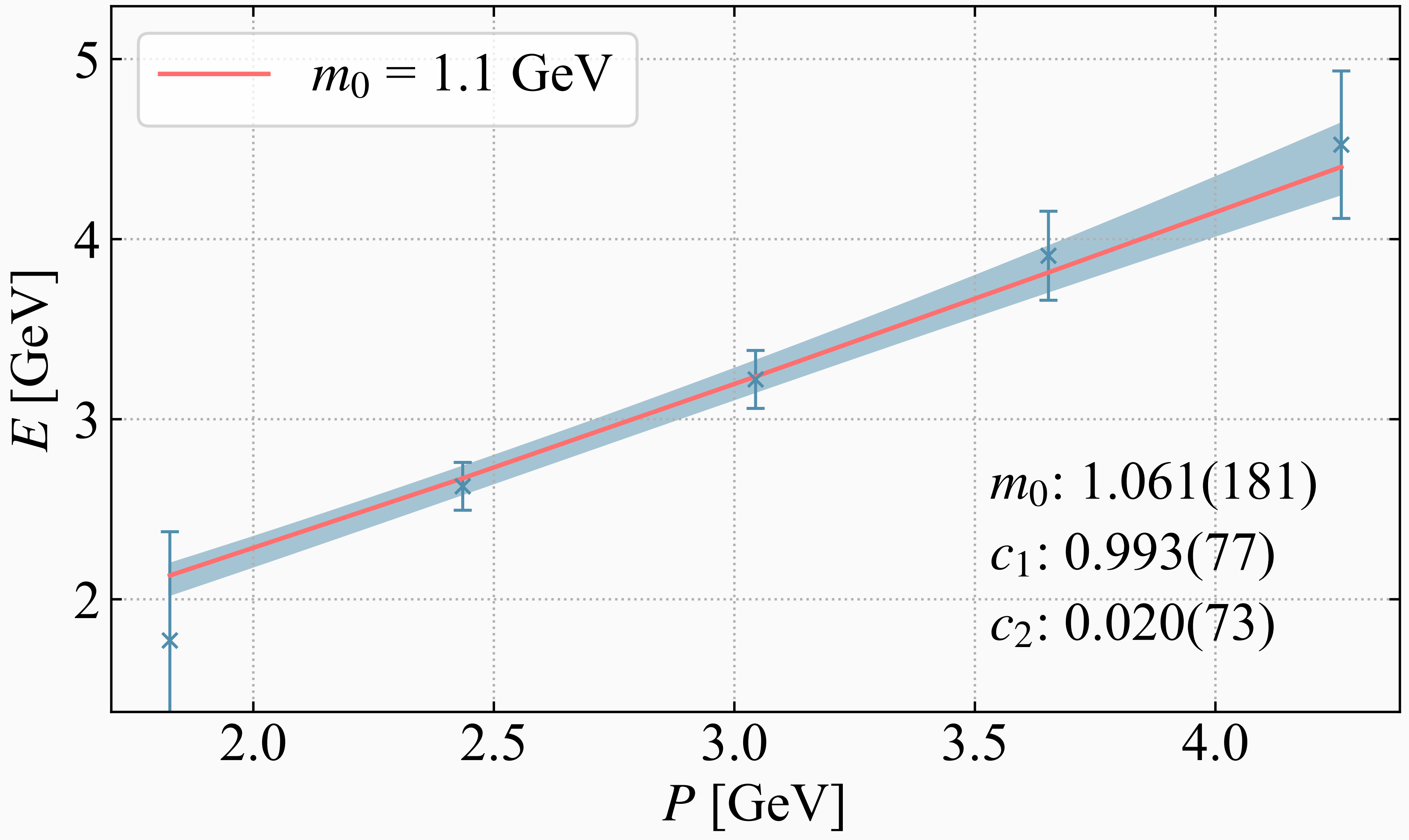


X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)

The results in CG and GI are consistent with the same lattice setup.

Lattice Setup for Nucleon Calculation

Dispersion relation: $E^2 = m_0^2 + c_1\ P^2 + c_2\ a^2 P^4$



- **2+1 flavor HISQ ensemble by HotQCD with volume** $L_s \times L_t = 48^3 \times 64$**;**

  A. Bazavov, et al. [HotQCD], Phys.Rev.D 90 (2014)

- **Lattice spacing is** $a = 0.06$ fm**;**
- **Pion mass of sea quark:** $m_\pi^{\mathrm{sea}} = 160$ MeV**;**
- **Pion mass of valence quark:** $m_\pi^{\mathrm{val}} = 300$ MeV**;**
- **Off-axis (**$\vec{n} = (1,1,0)$**) hadron momenta:** 2.43 GeV and 3.04 GeV**;**
- **Statistics for each lattice correlator:** 553 (configs) × 176 (inversions) × 2 ($\pm z$ directions) = 194,656**;**
- **Gauge fixing criterion: variation of functional satisfies** $\delta F / F < 10^{-8}$.

Ground State Fit

- **Ratio of three-point and two-point correlators**

$$R\left(t_{\rm sep},\tau\right)=\frac{C_{\rm 3pt}(t_{\rm sep},\tau)}{C_{\rm 2pt}(t_{\rm sep})}=\frac{\sum_{n,m} z_n O_{nm} z_m^{\dagger}\cdot e^{-E_n\left(t_{\rm sep}-\tau\right)}e^{-E_m\tau}}{\sum_n z_n z_n^{\dagger}\cdot\left(e^{-E_n t_{\rm sep}}+e^{-E_n(L_t-t_{\rm sep})}\right)}\xrightarrow{t_{\rm sep},\ \tau,\ (L_t-t_{\rm sep})\to\infty} O_{00}$$

- **Feynman-Hellmann (FH) inspired Method** *C. Bouchard, et al., Phys. Rev. D 96 (2017)*

Cancellation of excited-state contamination.

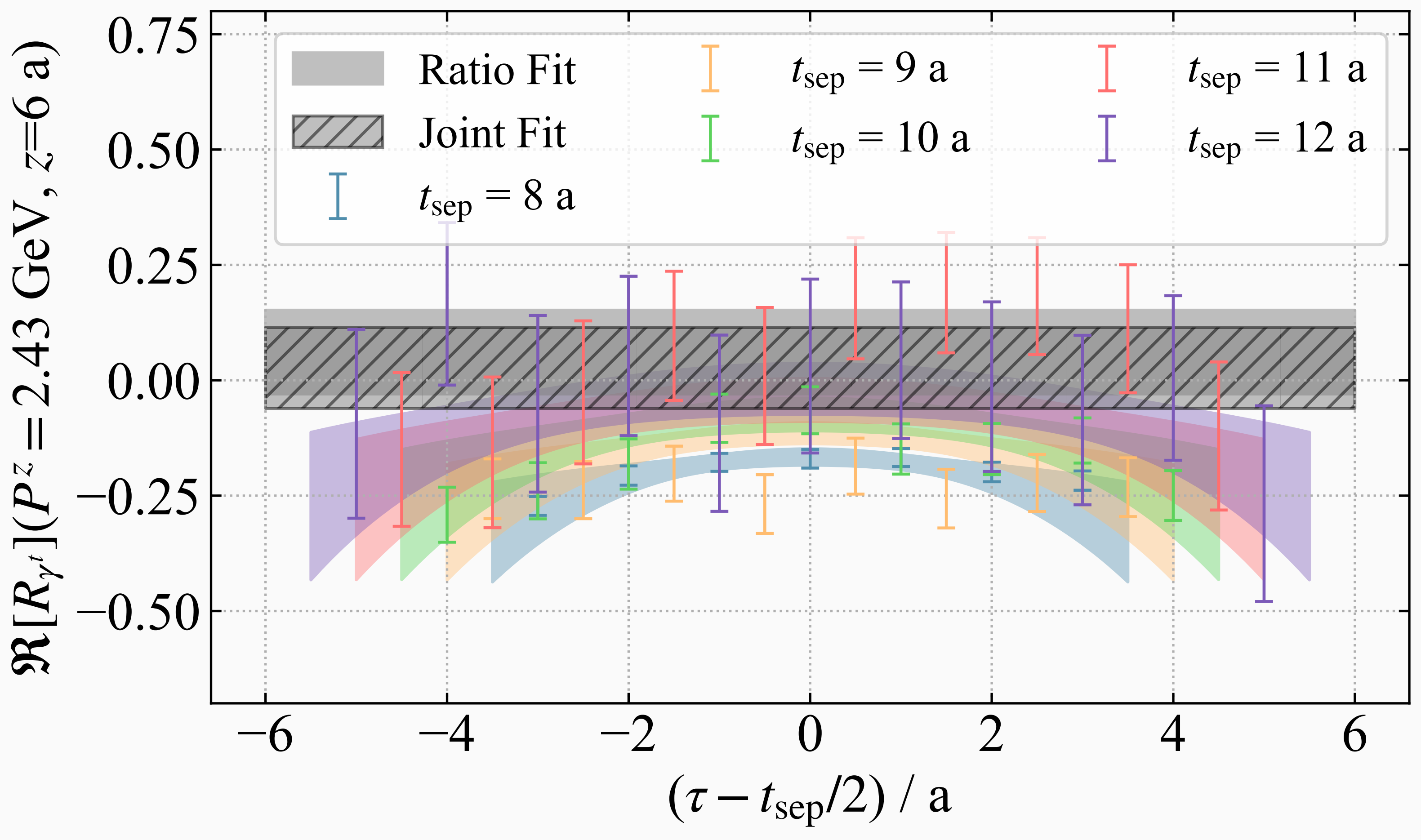
JH, et al., Phys. Rev. C 105 (2022)

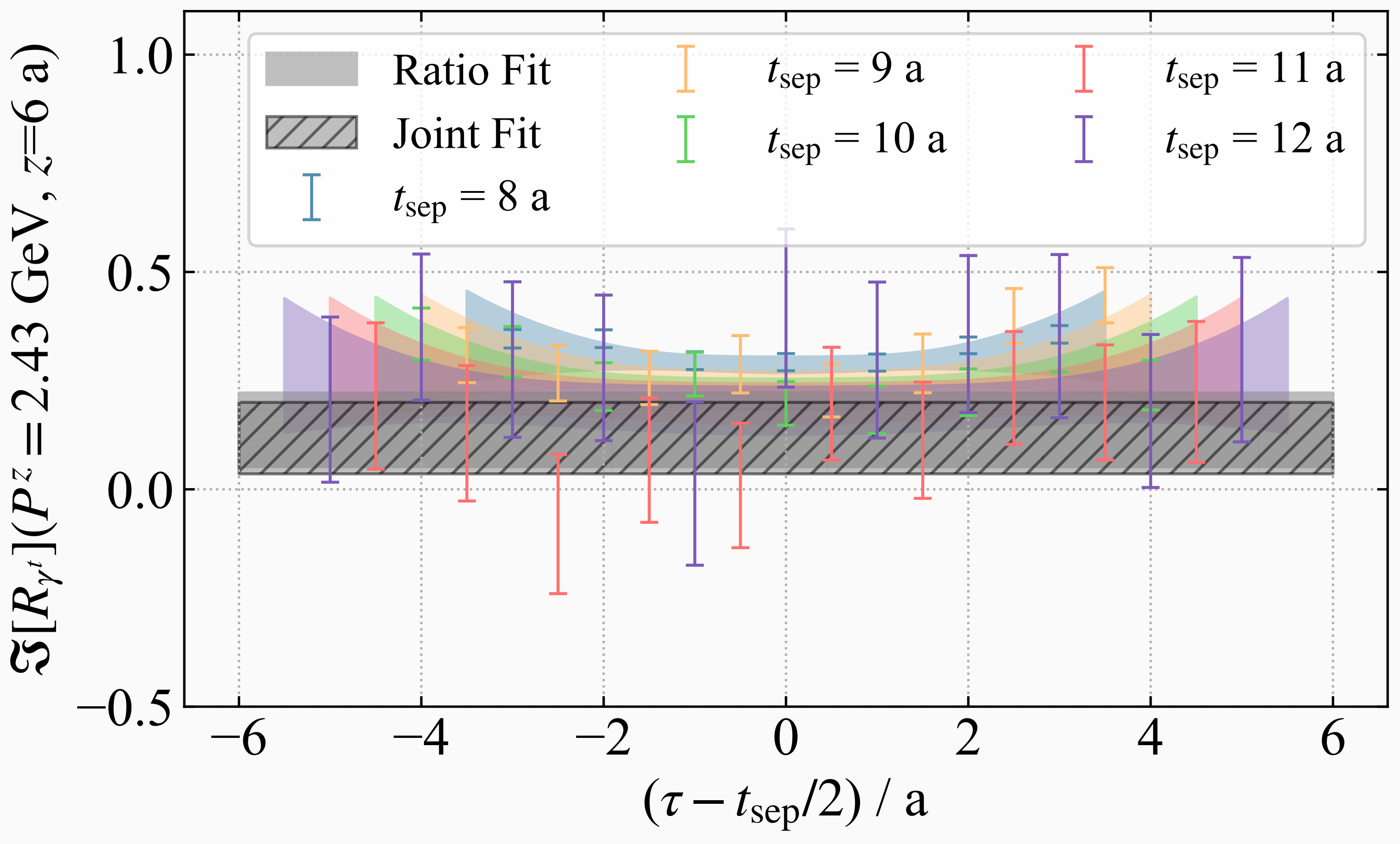
C. Bouchard, et al., Phys. Rev. D 96 (2017)

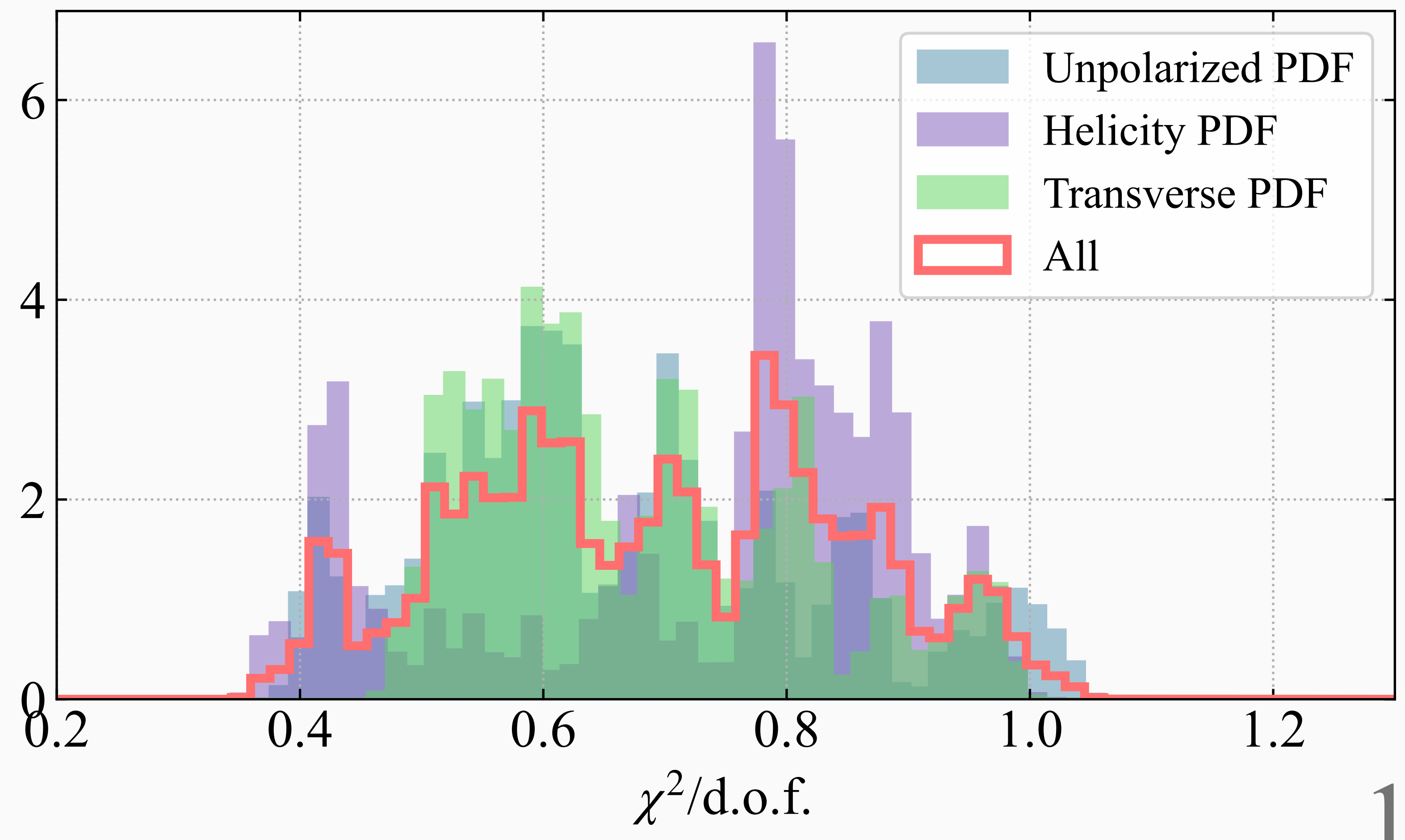
$$\mathrm{FH}\left(t_{\rm sep},\tau_{\rm cut},dt\right)\equiv\frac{\sum_{t=\tau_{\rm cut}}^{t=t_{\rm sep}+dt-\tau_{\rm cut}} R\left(t_{\rm sep}+dt,t\right)-\sum_{t=\tau_{\rm cut}}^{t=t_{\rm sep}-\tau_{\rm cut}} R\left(t_{\rm sep},t\right)}{dt}\xrightarrow{t_{\rm sep},\ \tau,\ (L_t-t_{\rm sep})\to\infty} O_{00}$$

The ratio fit and joint fit are consistent.

$\chi^2/\mathrm{d.o.f.}\sim 1$ for all joint fits







Non-perturbative Renormalization

- **Because of the absence of Wilson line, the CG correlation is free from linear divergence, the renormalized operator can be defined as** $\bar{\psi}_0(z)\Gamma\psi_0(0) = Z_\psi(a)\left[\bar{\psi}(z)\Gamma\psi(0)\right]$ **with** $z \neq 0$**;**

  X. Gao, W. Y. Liu and Y. Zhao, PRD 109 (2024)

- **Thus, we adopt the hybrid scheme as below, which does not introduce extra IR effects in the non-perturbative region:**

  X. Ji, et al., Nucl. Phys. B 964 (2021)

$$\tilde{h}_\Gamma\left(z, P^z, z_s\right) = \frac{\tilde{h}^0_\Gamma(0,0;a)}{\tilde{h}^0_\Gamma(0,P^z;a)}\frac{\tilde{h}^0_\Gamma(z,P^z,a)}{\tilde{h}^0_\Gamma(z,0,a)}\theta\left(z_s - |z|\right) + \frac{\tilde{h}^0_\Gamma(0,0;a)}{\tilde{h}^0_\Gamma(0,P^z;a)}\frac{\tilde{h}^0_\Gamma(z,P^z,a)}{\tilde{h}^0_\Gamma\left(z_s,0,a\right)}\theta\left(|z| - z_s\right),$$ **where** $a \ll z_s \ll 1/\Lambda_{\rm QCD}$

  No normalization at $z = 0$**, because it introduces extra discretization effects at large** z

- **Note that** $\tilde{h}^0_\Gamma(z,0,a)$ **is real and can partially cancel the discretization effects and power corrections in the real part of** $\tilde{h}^0_\Gamma(z,P^z,a)$**. However, such a cancellation is not guaranteed for the imaginary part of** $\tilde{h}^0_\Gamma(z,P^z,a)$**;**

- **Thus, we propose to renormalize imaginary part separately, for example,**

$$\Im[\tilde{h}_\Gamma]\left(z, P^z, \mu\right) = \Im[\tilde{h}^0_\Gamma](z, P^z, a)/Z_\psi^{\overline{\rm MS}}(a,\mu)$$

- **The scheme dependence will be cancelled by the corresponding matching kernel that relates the quasi-PDF to the PDF.**

Fourier Transform

- The CG quasi-PDF matrix elements at large $\lambda = zP^z \gtrsim 8$ are consistent with zero, while the error bars remain constant;
- Because of the finite correlation length, the exponential decay starts to dominate in the sub-asymptotic region ($0.5 \text{ fm} \lesssim z \lesssim 1.2 \text{ fm}$);

J. W. Chen, et al., 2505.14619

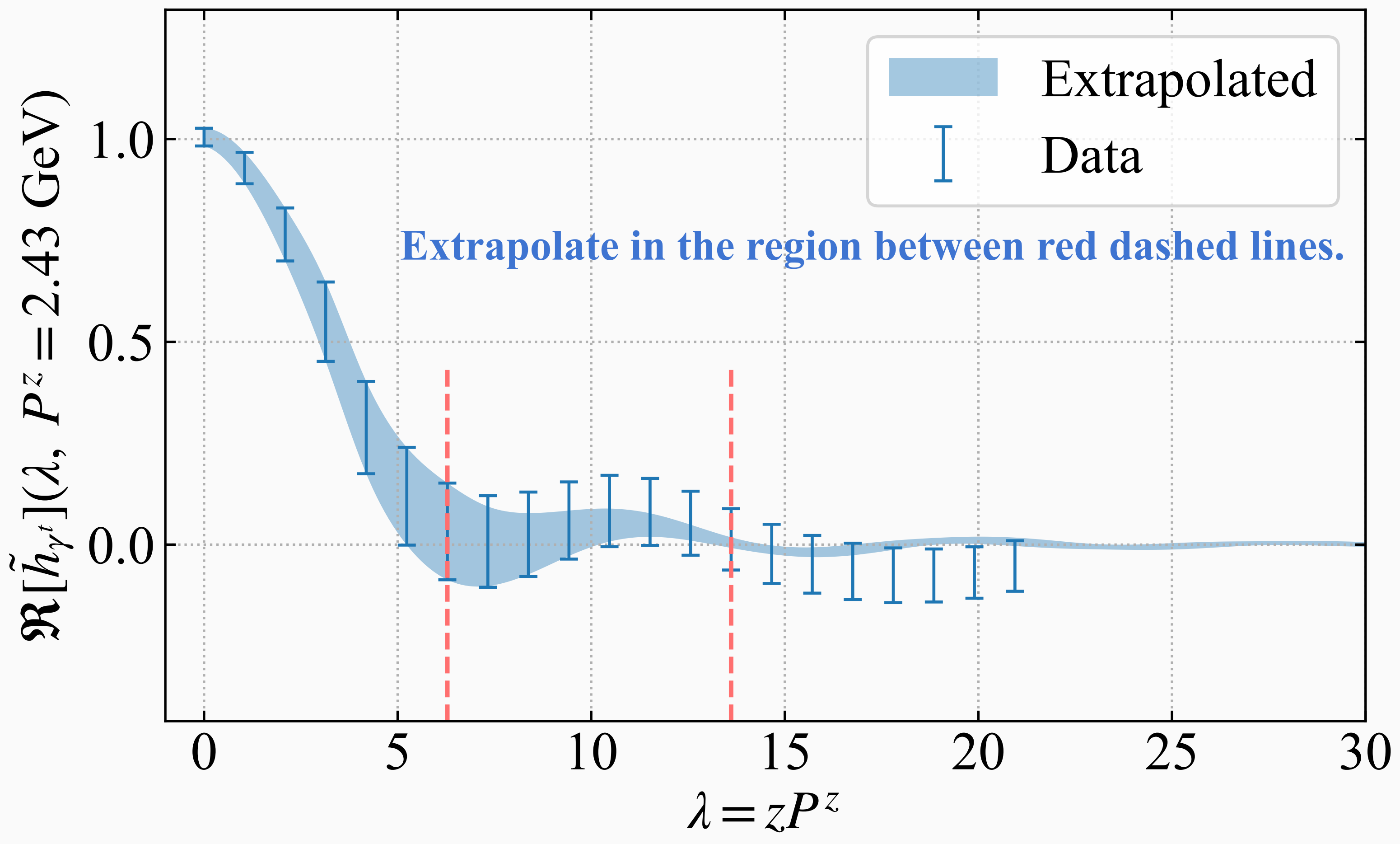
- An asymptotic fit is applied within the region between red dashed lines with fit function:

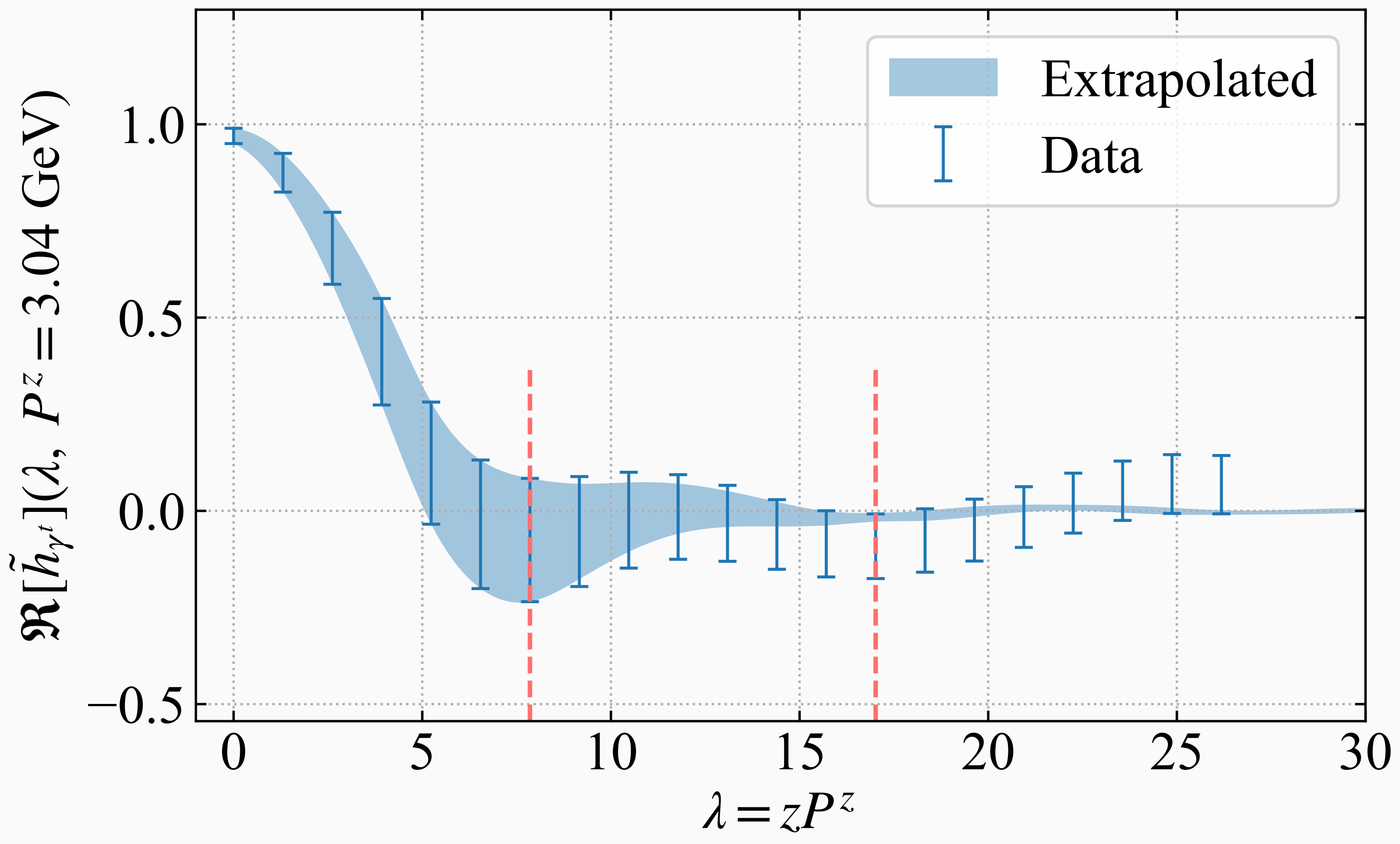
$$\tilde{h}^{\text{fit}}(\lambda) = Ae^{-m\lambda}\sin(a\lambda + b)/\lambda^d$$

- Take $\tilde{h}^{\text{ext}} = w \cdot \tilde{h}^{\text{data}} + (1-w) \cdot \tilde{h}^{\text{fit}}$, where the weight $w(z)$ linearly decays from 1 to 0 within the fit range to make error bands smooth;
- A conservative upper bound of model uncertainty: $\delta f\left(x, P, \lambda_L\right) < 4N_x \left|\tilde{h}\left(z, P; \lambda_L\right)\right|_{\max} /(\pi x) \lesssim 0.075$ at $x = 0.5$.

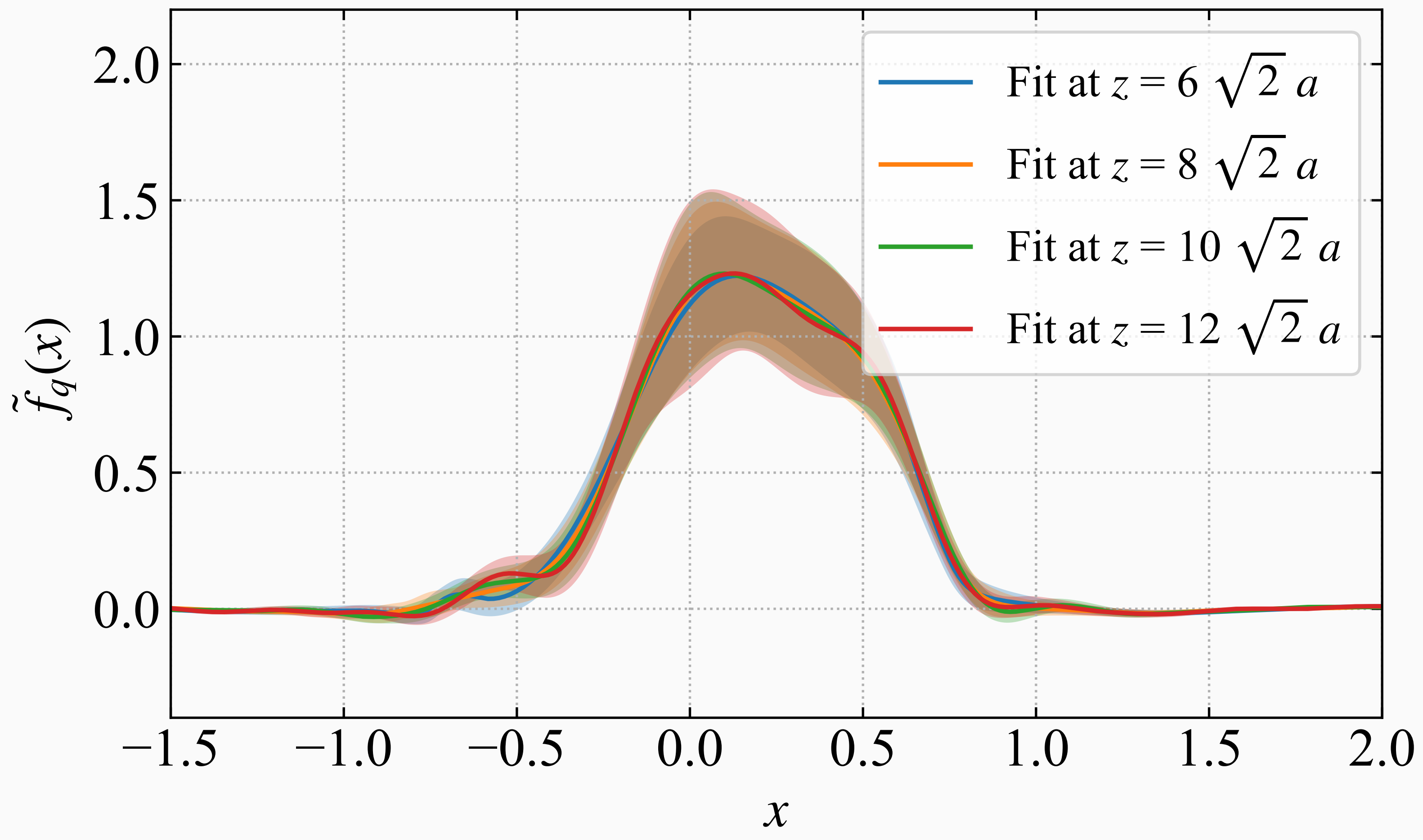
J. W. Chen, et al., 2505.14619
X. Gao, et al., Phys. Rev. Lett. 128 (2022)

Unpolarized quasi-PDF in the coordinate space



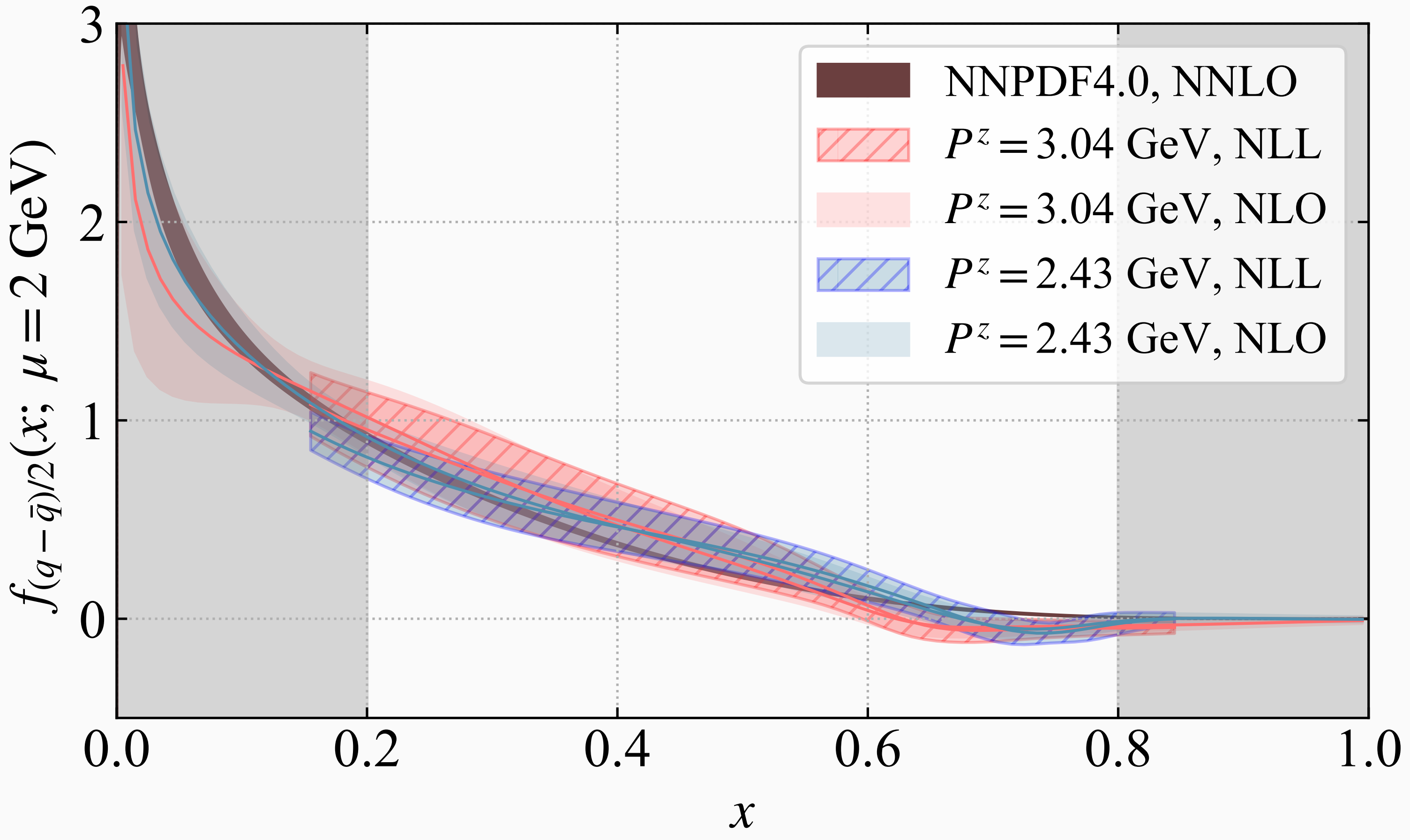


Different starting points of asymptotic fit



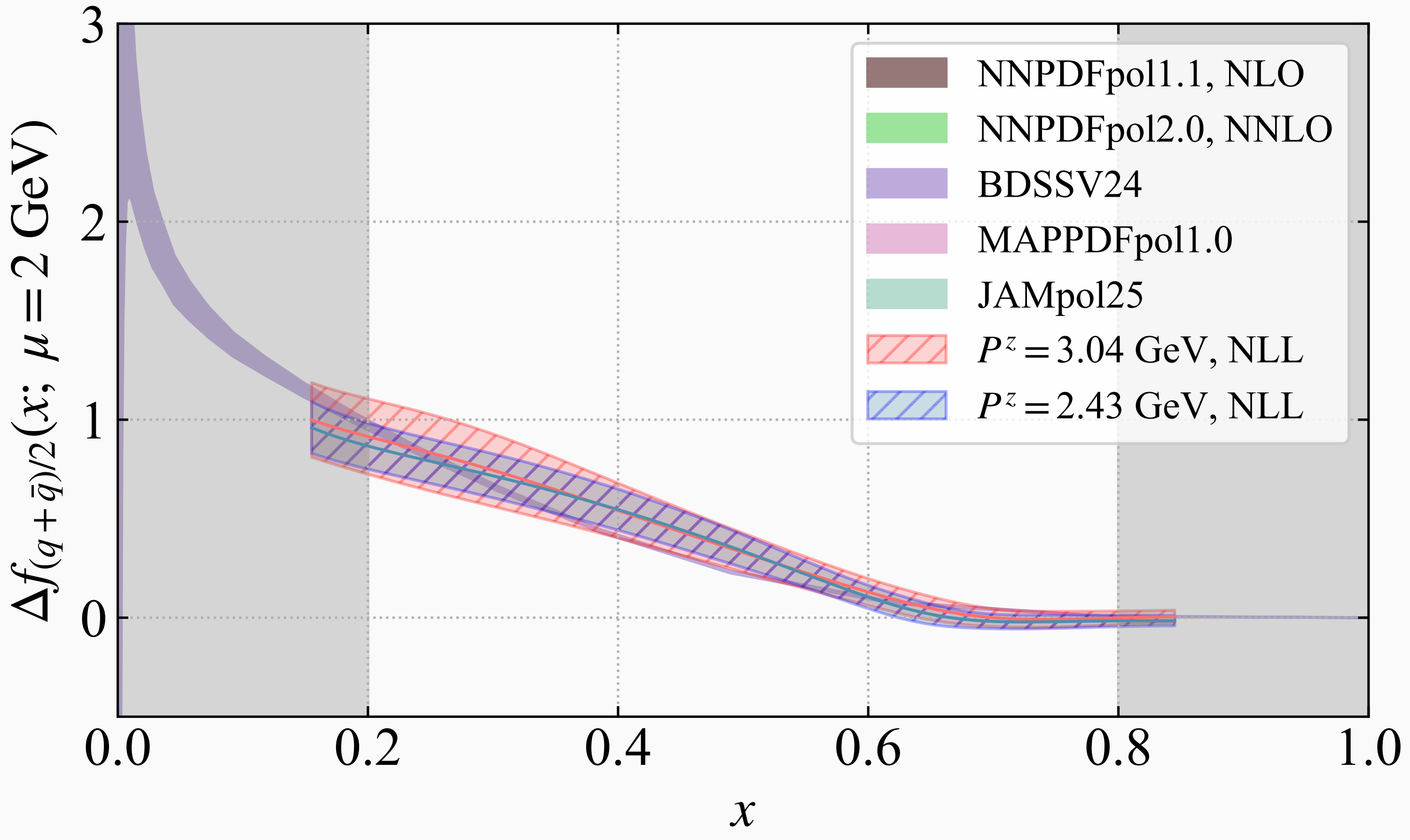
Nucleon PDFs From the Real Part

Unpolarized PDF $(q-\bar{q})/2$



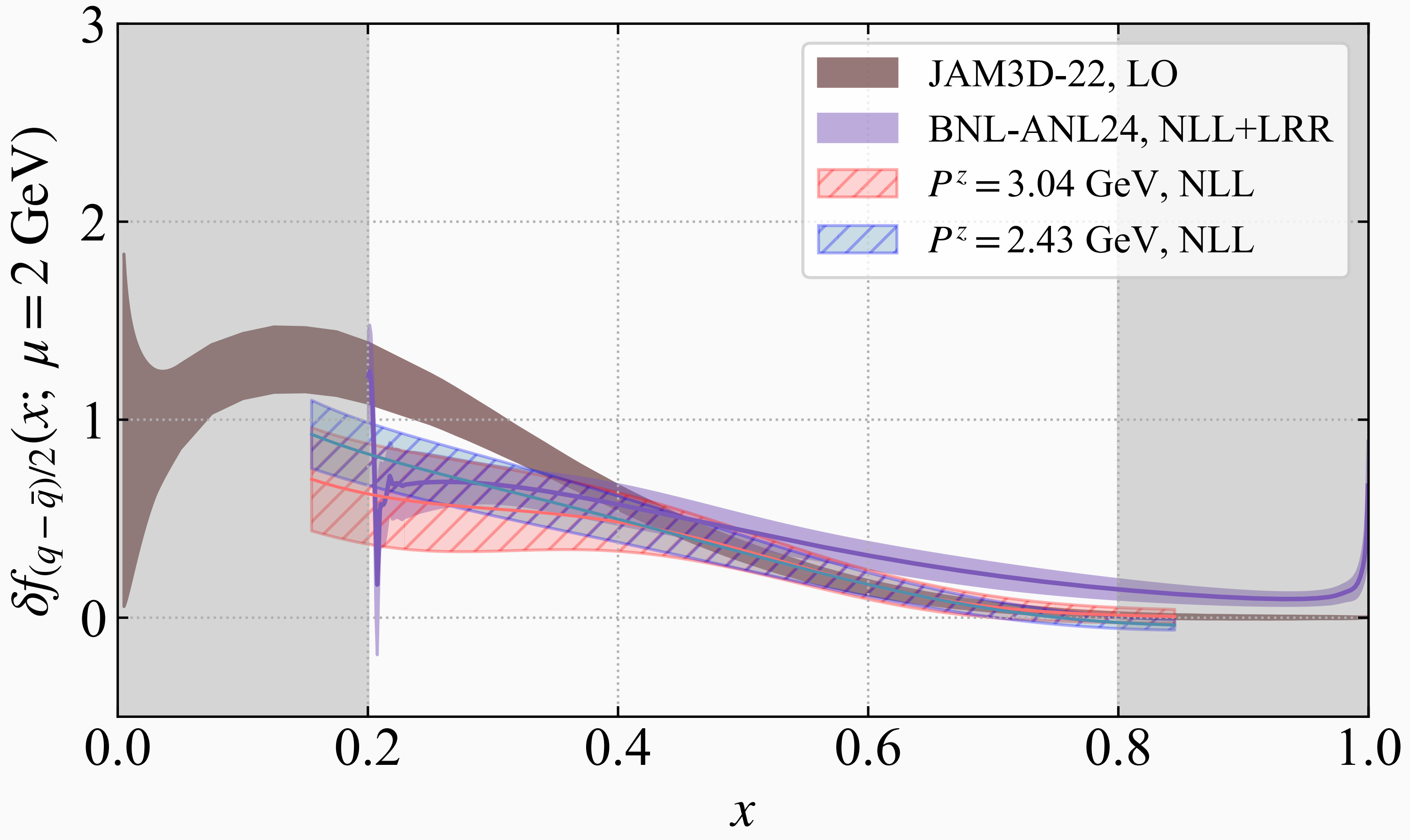
NNPDF4.0: R. D. Ball, et al. [NNPDF], EPJC 82 (2022)

Helicity PDF $(q+\bar{q})/2$



NNPDFpol1.1: E. R. Nocera et al. [NNPDF], NPB 887 (2014)
NNPDFpol2.0: J. Cruz-Martinez et al. [NNPDF], JHEP 07 (2025)
BDSSV24: I. Borsa et al., PRL 133 (2024)
MAPPDFpol1.0: V. Bertone et al. [MAP], PLB 865 (2025)
JAMpol25: C. Cocuzza et al. [JAM], 2506.13616

Transversity PDF $(q-\bar{q})/2$



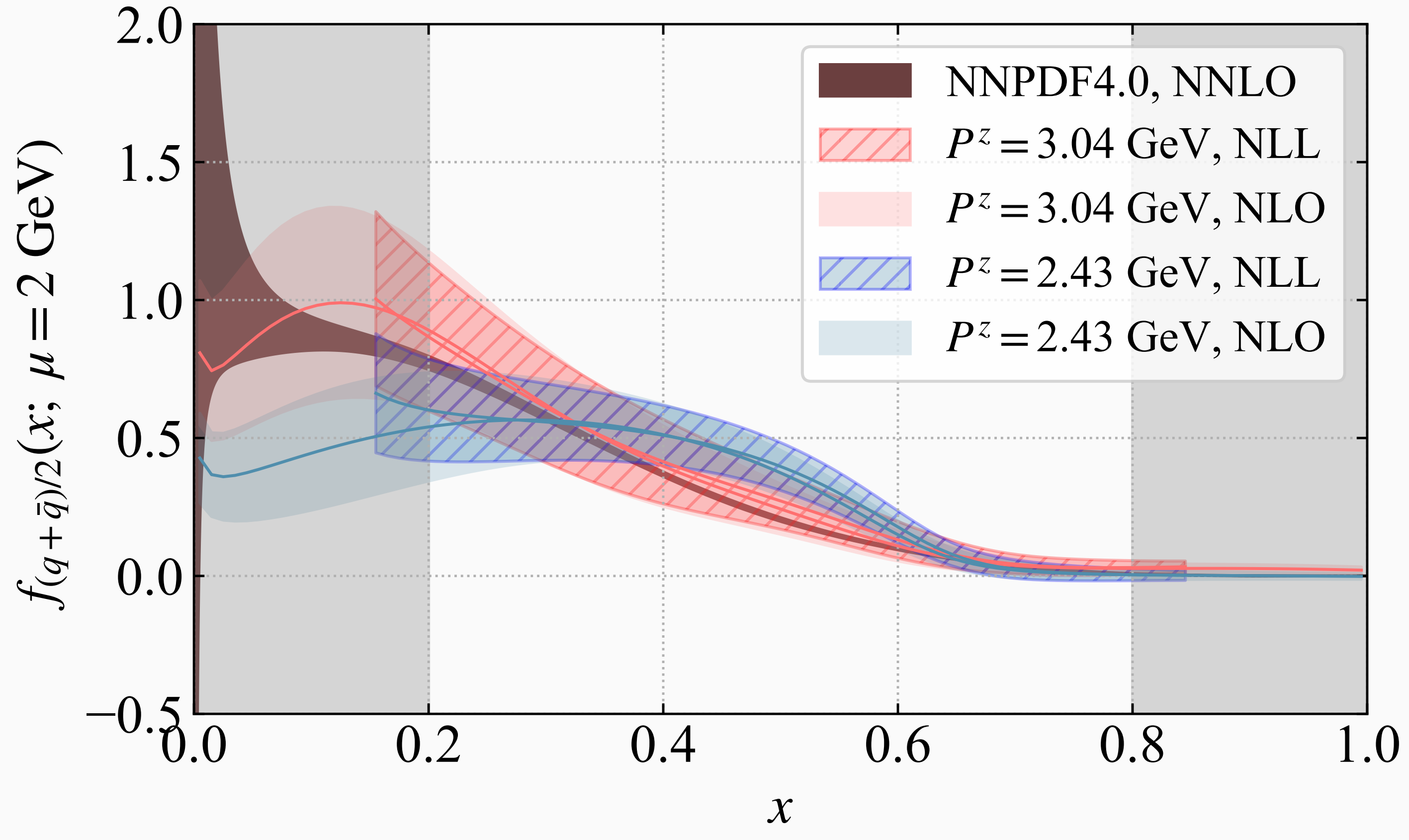
JAM3D-22: L. Gamberg, et al. [JAM], PRD 106 (2022)
BNL-ANL24: X. Gao, et al., PRD 109 (2024)

- **Comparing with the global fit results, CG method gives a consistent prediction on the valence part of all three nucleon PDFs, which provides encouraging evidence for the efficacy of the CG method;**
- **The hadron momentum is large enough to see the convergence in P^z;**
- **The NLO and NLL with RGR results show an aligned behavior at moderate x, where LaMET can make reliable prediction;**

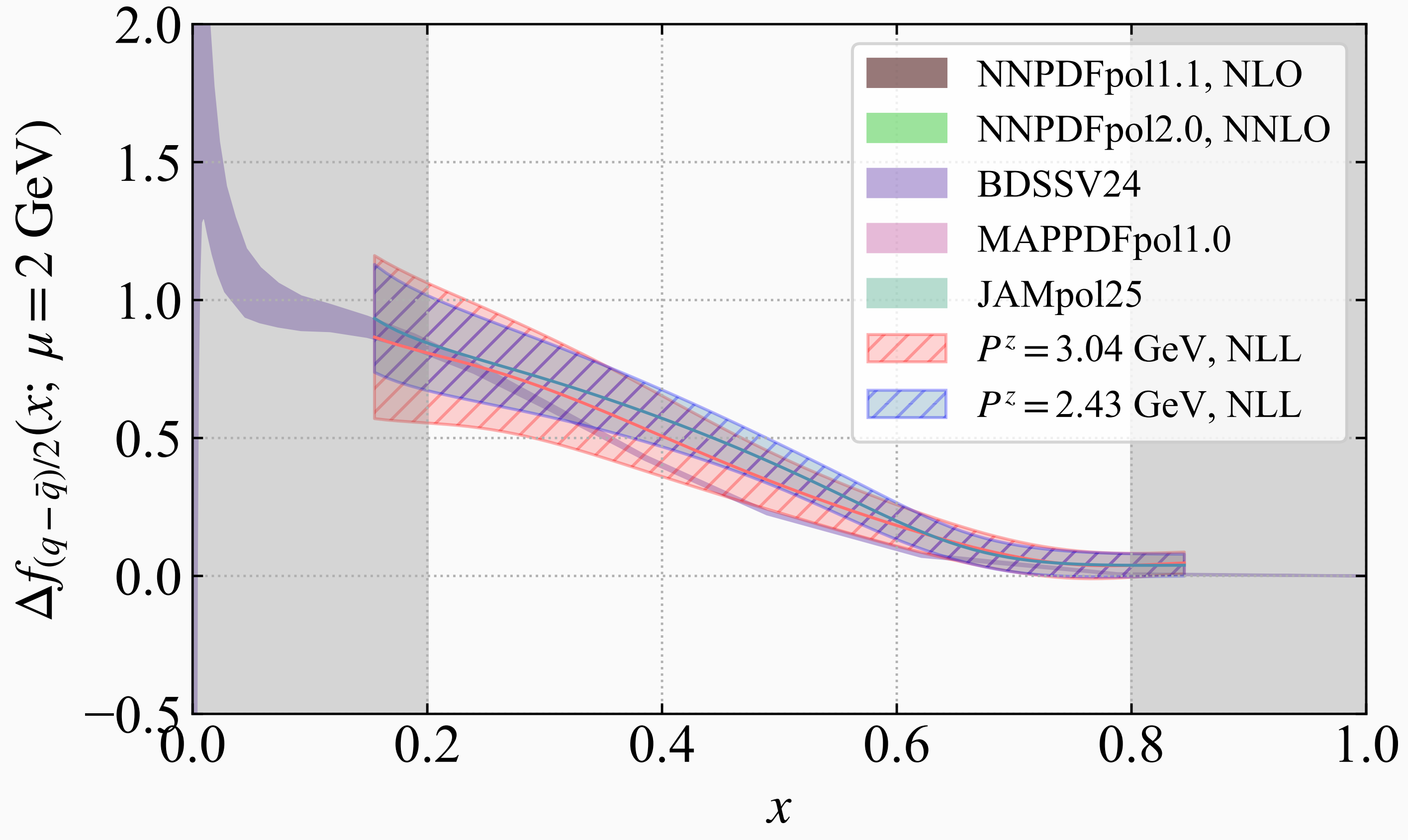
Nucleon PDFs From the Imaginary Part

Unpolarized PDF $(q+\bar{q})/2$ | **Helicity PDF $(q-\bar{q})/2$** | **Transversity PDF $(q+\bar{q})/2$**

$Z_\psi \cdot f$ from lattice v.s. f from literature



NNPDF4.0: R. D. Ball, et al. [NNPDF], EPJC 82 (2022)



NNPDFpol1.1: E. R. Nocera et al. [NNPDF], NPB 887 (2014)
NNPDFpol2.0: J. Cruz-Martinez et al. [NNPDF], JHEP 07 (2025)
BDSSV24: I. Borsa et al., PRL 133 (2024)
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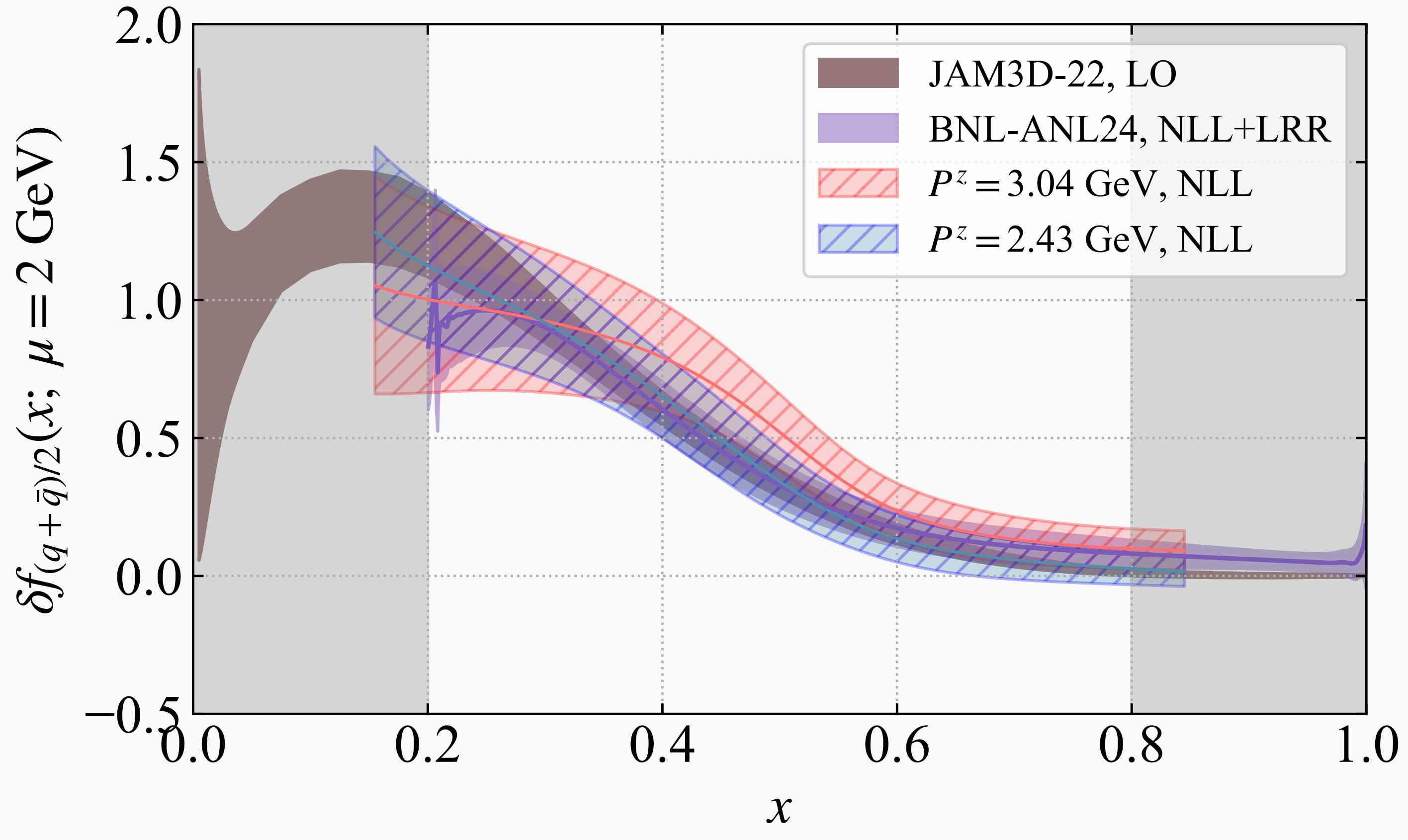
JAMpol25: C. Cocuzza et al. [JAM], 2506.13616

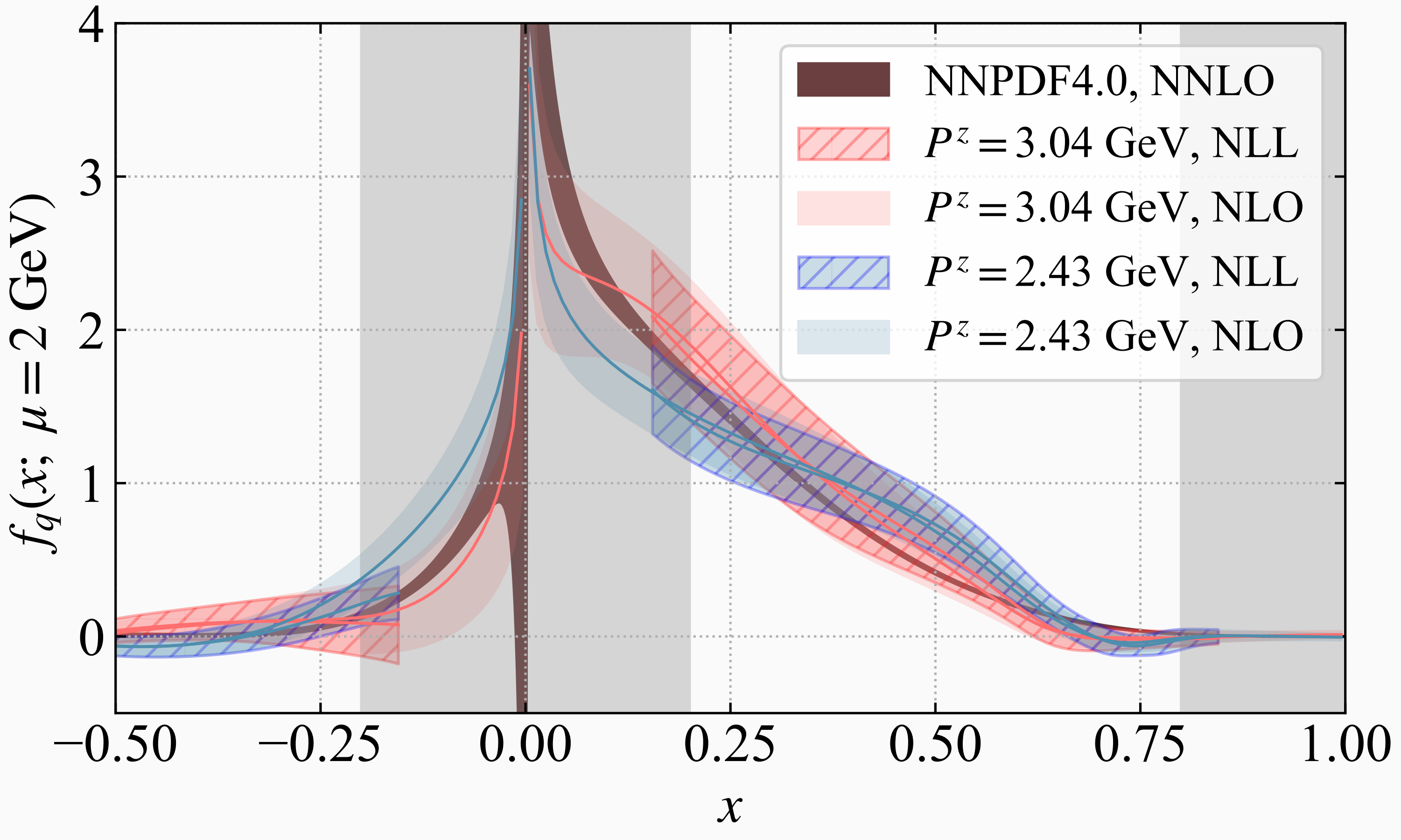


JAM3D-22: L. Gamberg, et al. [JAM], PRD 106 (2022)
BNL-ANL24: X. Gao, et al., PRD 109 (2024)

- **$Z_\psi \cdot f$ from lattice are all consistent with the global fits, but Z_ψ still needs to be determined independently;**
- **For the imaginary part, the discrepancy between two different momenta is larger than that for the real part;**

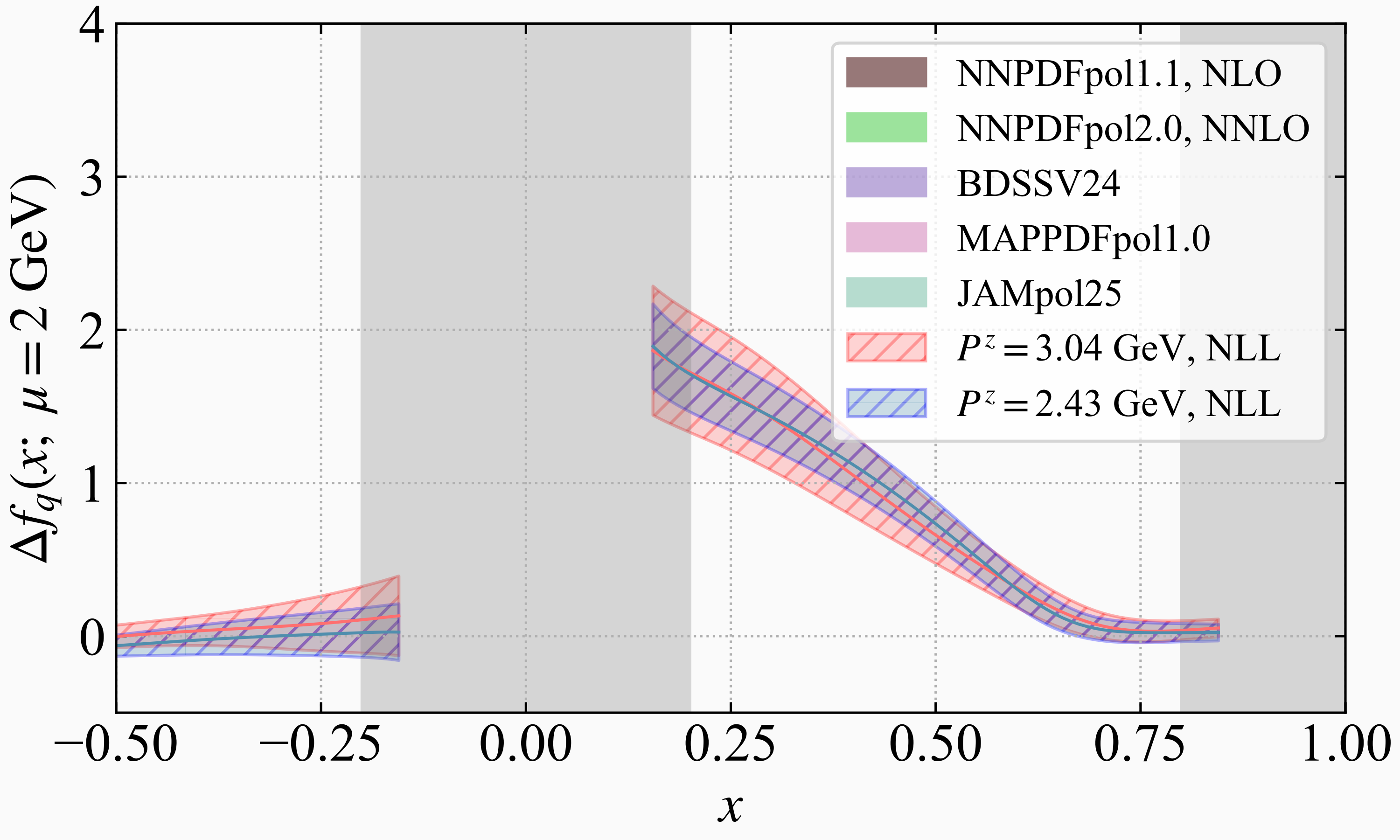
Nucleon PDFs

Unpolarized PDF



NNPDF4.0: R. D. Ball, et al. [NNPDF], EPJC 82 (2022)

Helicity PDF



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NNPDFpol2.0: J. Cruz-Martinez et al. [NNPDF], JHEP 07 (2025)
BDSSV24: I. Borsa et al., PRL 133 (2024)
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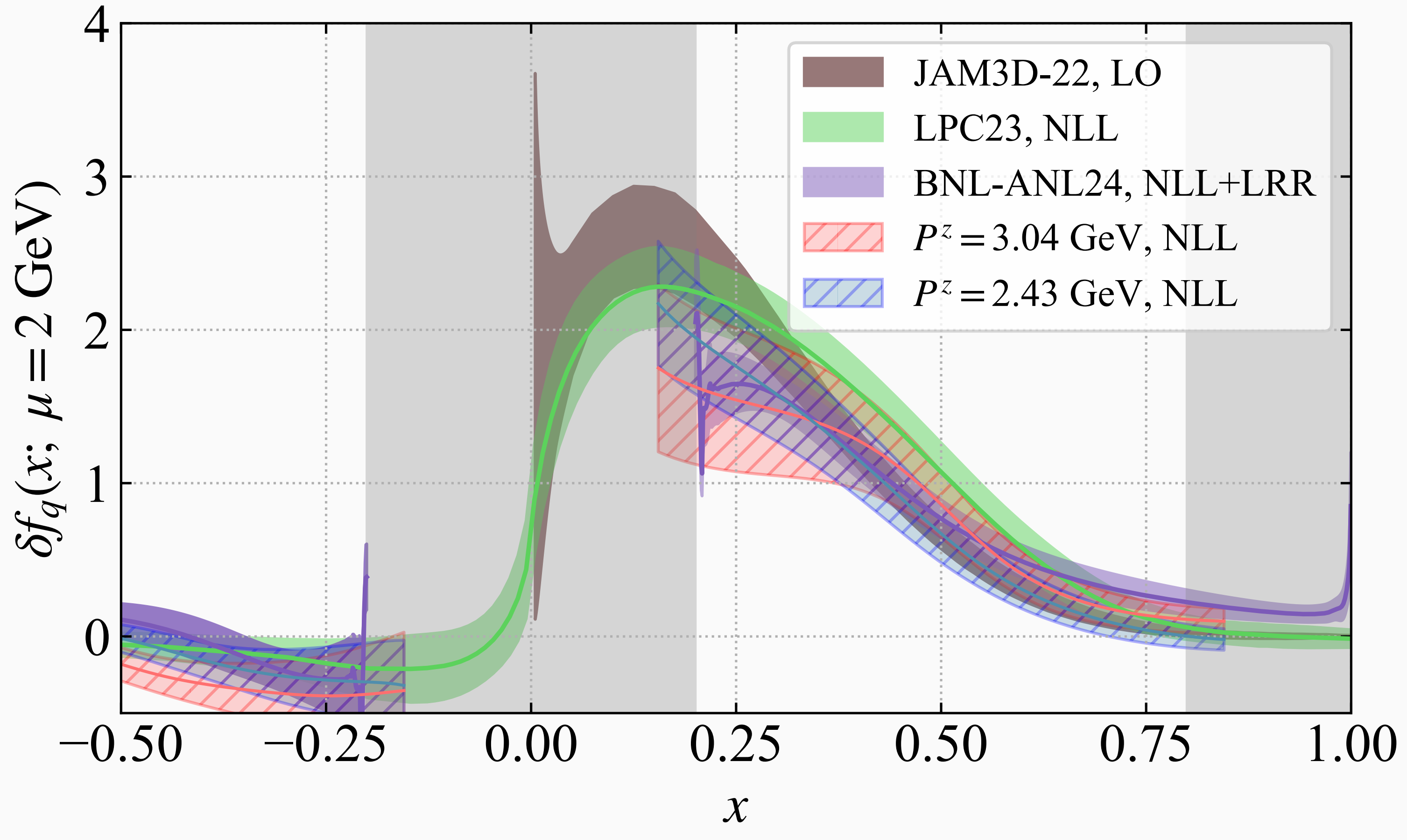
JAMpol25: C. Cocuzza et al. [JAM], 2506.13616

Transversity PDF



JAM3D-22: L. Gamberg, et al. [JAM], PRD 106 (2022)
LPC23: F. Yao, et al. [LPC], PRL 131 (2023)
BNL-ANL24: X. Gao, et al., PRD 109 (2024)

- **Assuming $Z_\psi = 1$, we combine the contributions of real and imaginary parts to obtain the PDFs;**

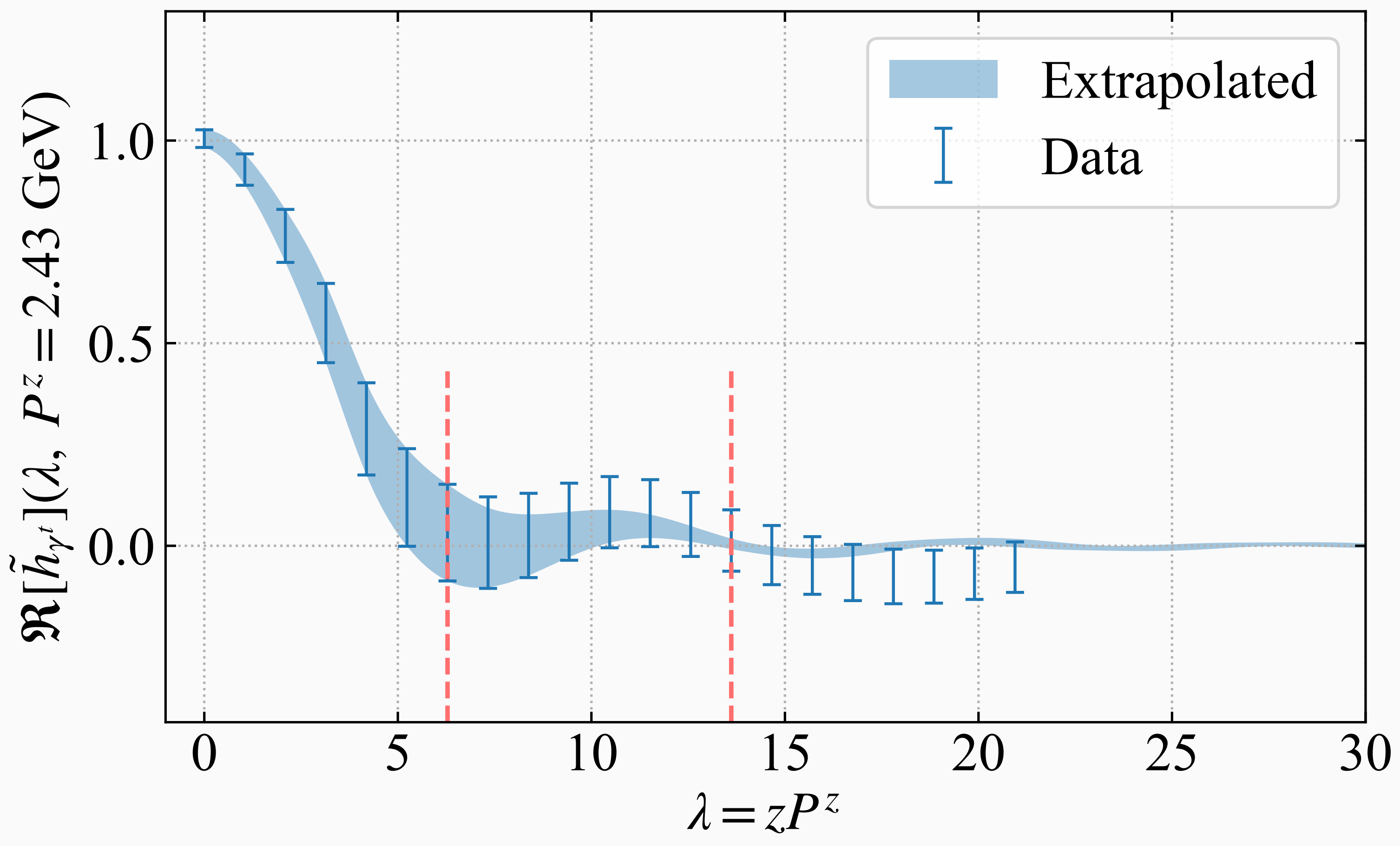
Summary

- This work presents the first lattice calculation of the nucleon PDFs for all three polarizations using the CG method;
- Our results of real part contribution in all three PDFs show encouraging agreement with existing results at moderate x region;
- We propose to use different strategies in the renormalization of the real and imaginary part of quasi-PDFs;
- The imaginary part contribution will be improved in the future work, on both renormalization and excited state contamination;
- This work also serves as an examination of universality in LaMET.

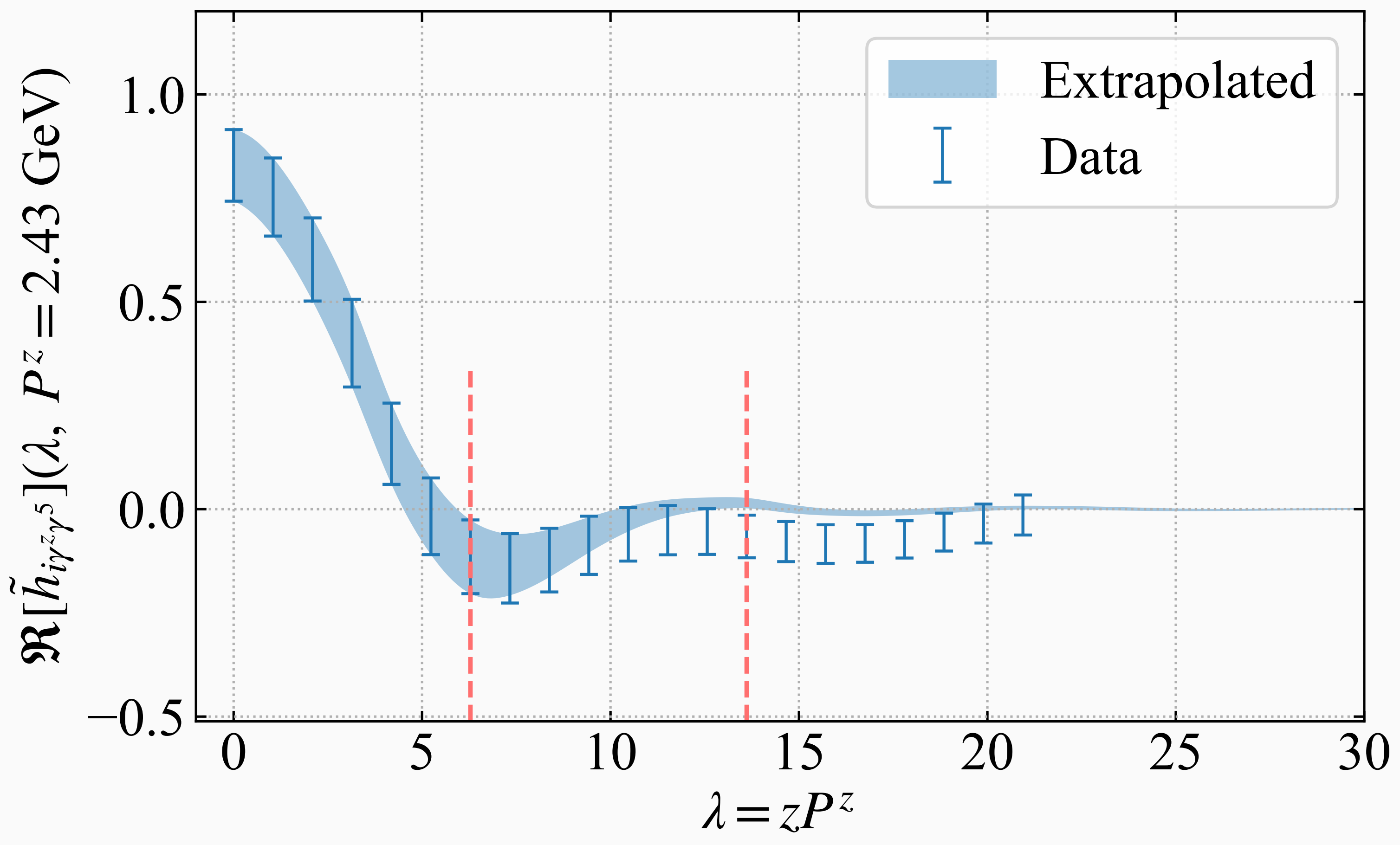
Backup

Normalization at z=0

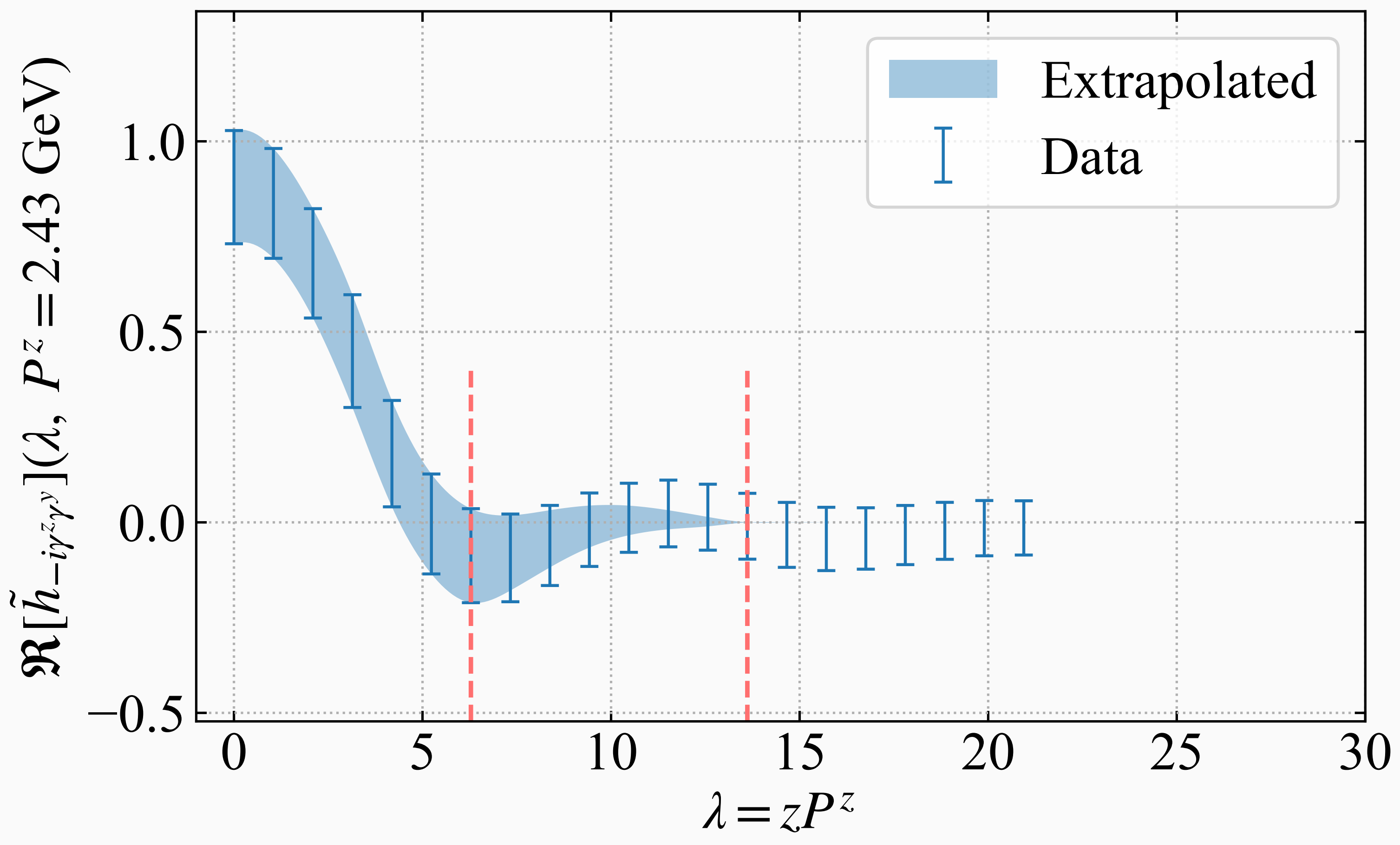
Unpolarized



Helicity



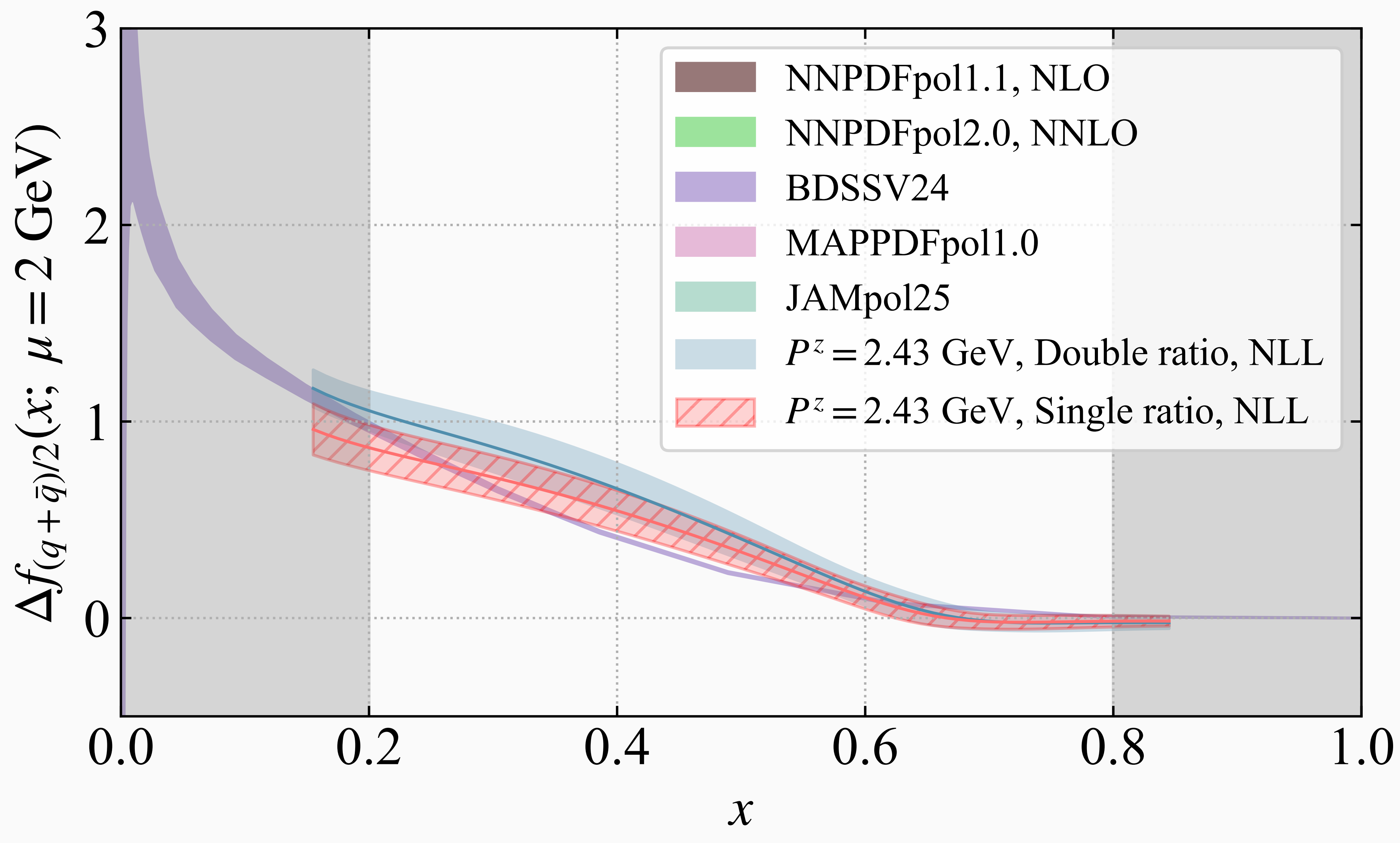
Transversity



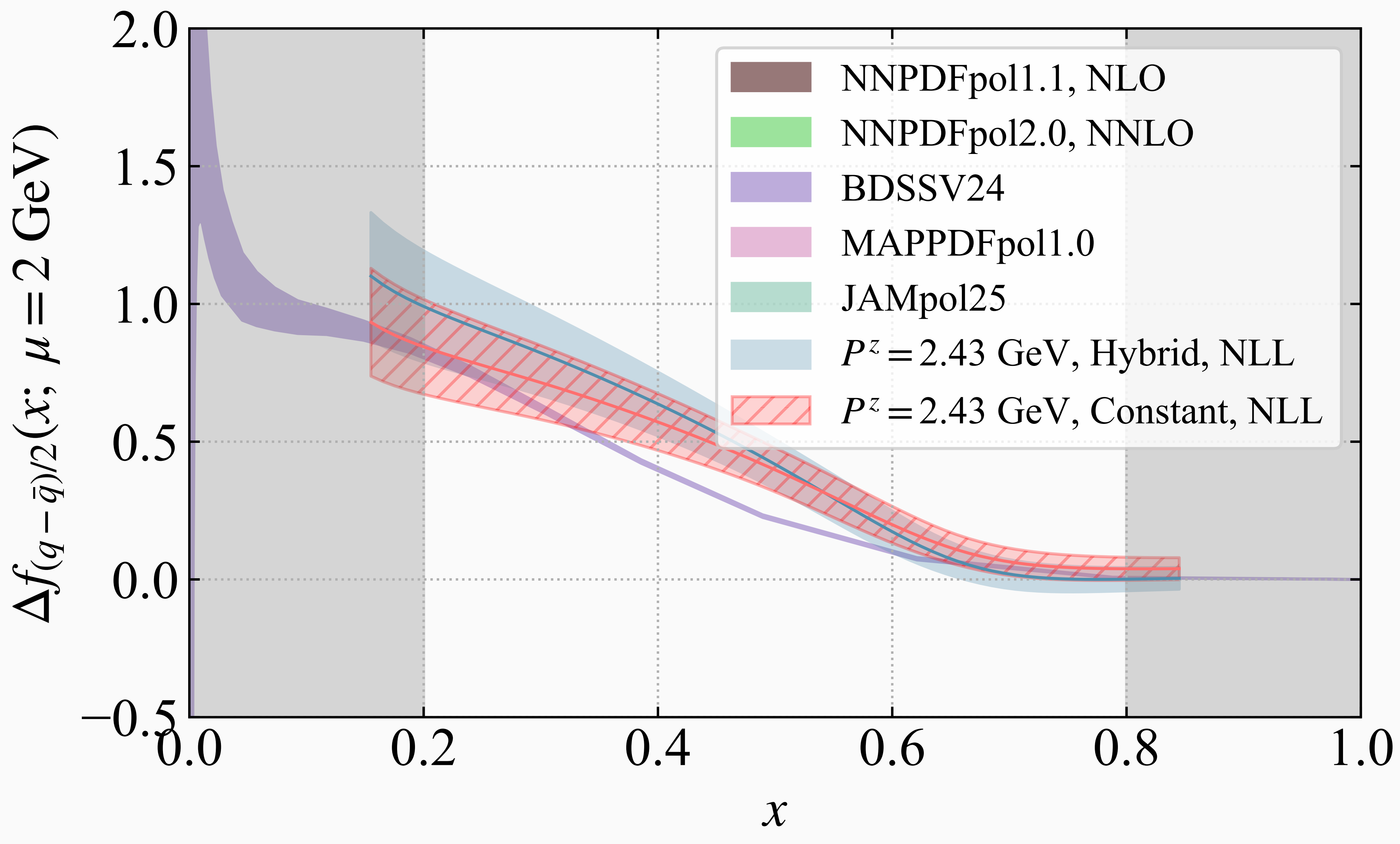
- **It is clear that factor** $\dfrac{\tilde{h}^0_\Gamma(0,0;a)}{\tilde{h}^0_\Gamma(0,P^z;a)}$ **at** $z = 0$ **cannot be used to normalize all** z**, or it will introduce unnecessary discretization effects in all matrix elements;**
- **The discretization effects at** $z = 0$ **can be estimated and subtracted once we have multiple lattice spacings;**
- **The FT is a summation, a single matrix element at** $z = 0$ **has a small contribution in the momentum space;**
- **Both the discretization and excited state contamination at** $z = 0$ **will be studied in the future work to restore the normalization condition.**

Scheme Comparison

Helicity PDF $(q+\bar{q})/2$



Helicity PDF $(q-\bar{q})/2$



Gauge Fixing in Lattice QCD

Continuous Theory

$$F_{\mathrm{CG}}[A,\Omega] \equiv \frac{1}{2}\sum_{\mu=1}^{3}\int d^4x A^a_{\Omega\mu}(x)A^{\mu a}_{\Omega}(x)$$

$$\delta F_{\mathrm{CG}}[A,\Omega] = -\sum_{\mu=1}^{3}\int d^4x (D^{\Omega}_{\mu ab}\theta_b)A^{\mu a}_{\Omega}$$

$$= -\sum_{\mu=1}^{3}\int d^4x (\partial_\mu\theta_a - g f^{cab}A^c_{\Omega\mu}\theta_b)A^{\mu a}_{\Omega}$$

$$= \sum_{\mu=1}^{3}\int d^4x \theta_a(\partial_\mu A^{\mu a}_{\Omega})$$

$$* A_{\Omega\mu}(x) \equiv \Omega^\dagger(x)A_\mu(x)\Omega(x) + \frac{i}{g}\Omega^\dagger(x)\partial_\mu\Omega(x)$$

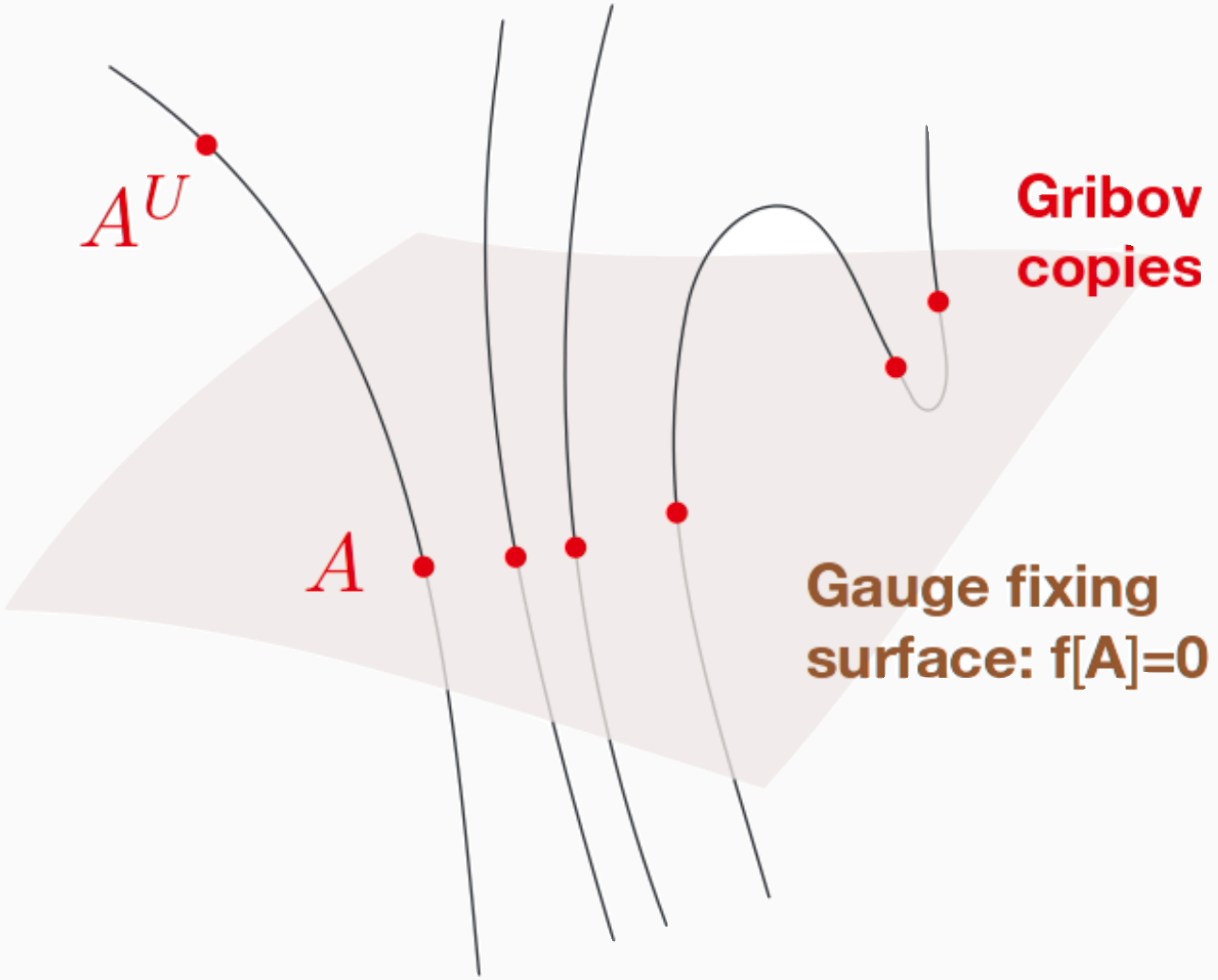
Lattice Theory

$$F_{\mathrm{CG}}[U,\Omega] \equiv -\Re\left[\mathrm{Tr}\sum_{x}\sum_{\mu=1}^{3}\Omega^\dagger(x+\hat{\mu})U_\mu(x)\Omega(x)\right]$$

Find stationary points of the functional value.

Gribov Copies

- The gauge fixing condition may have many solutions in Lattice QCD.



Ph. D. Thesis of Diego Fiorentini

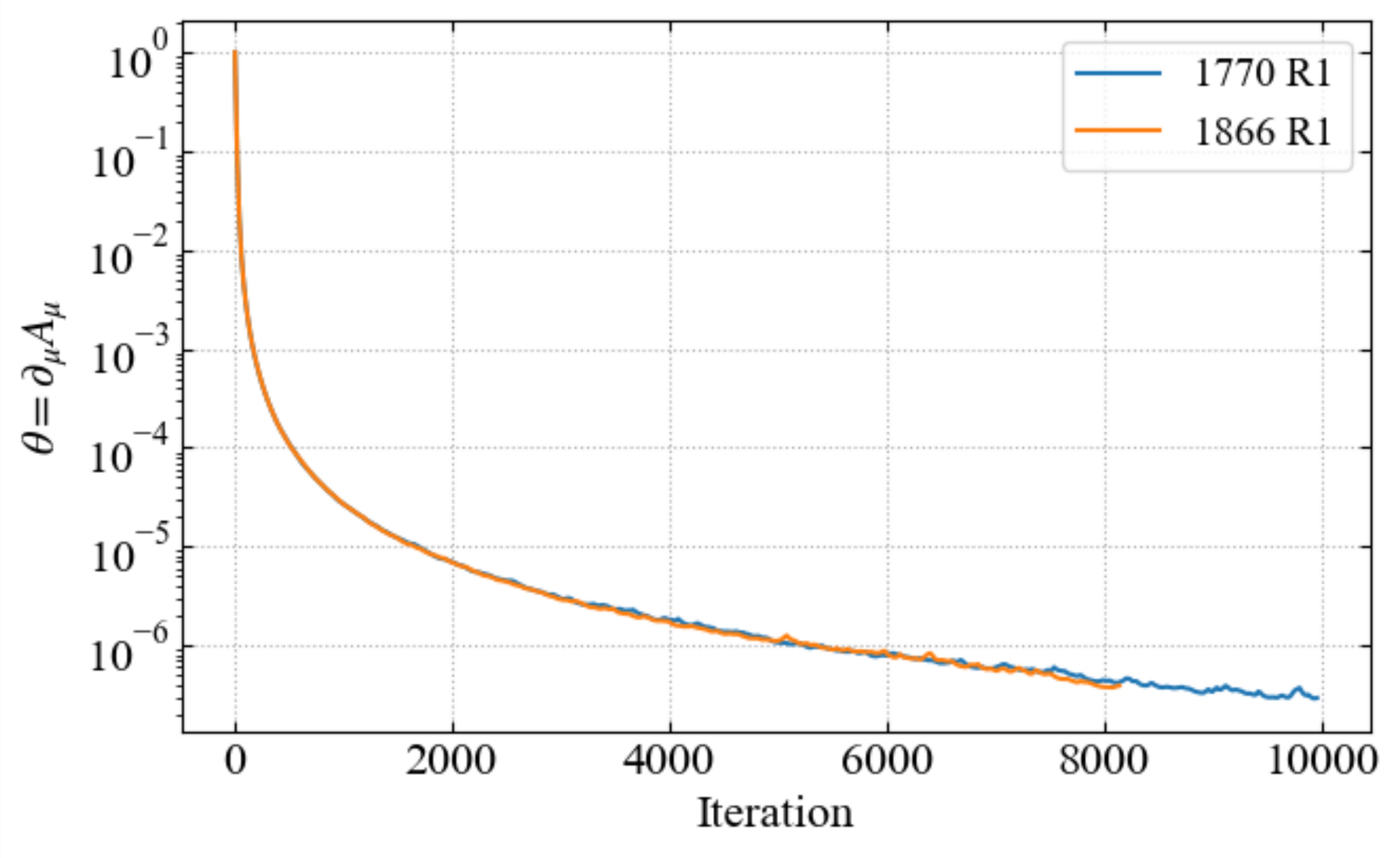
Criteria of Gauge Fixing

- Variation of the functional

$$\delta F / F < 10^{-8}$$

- Residual gradient of the functional

$$\theta^G \equiv \frac{1}{V} \sum_x \theta^G(x) \equiv \frac{1}{V} \sum_x \mathrm{Tr}\left[\Delta^G(x) \left(\Delta^G\right)^\dagger(x)\right]$$

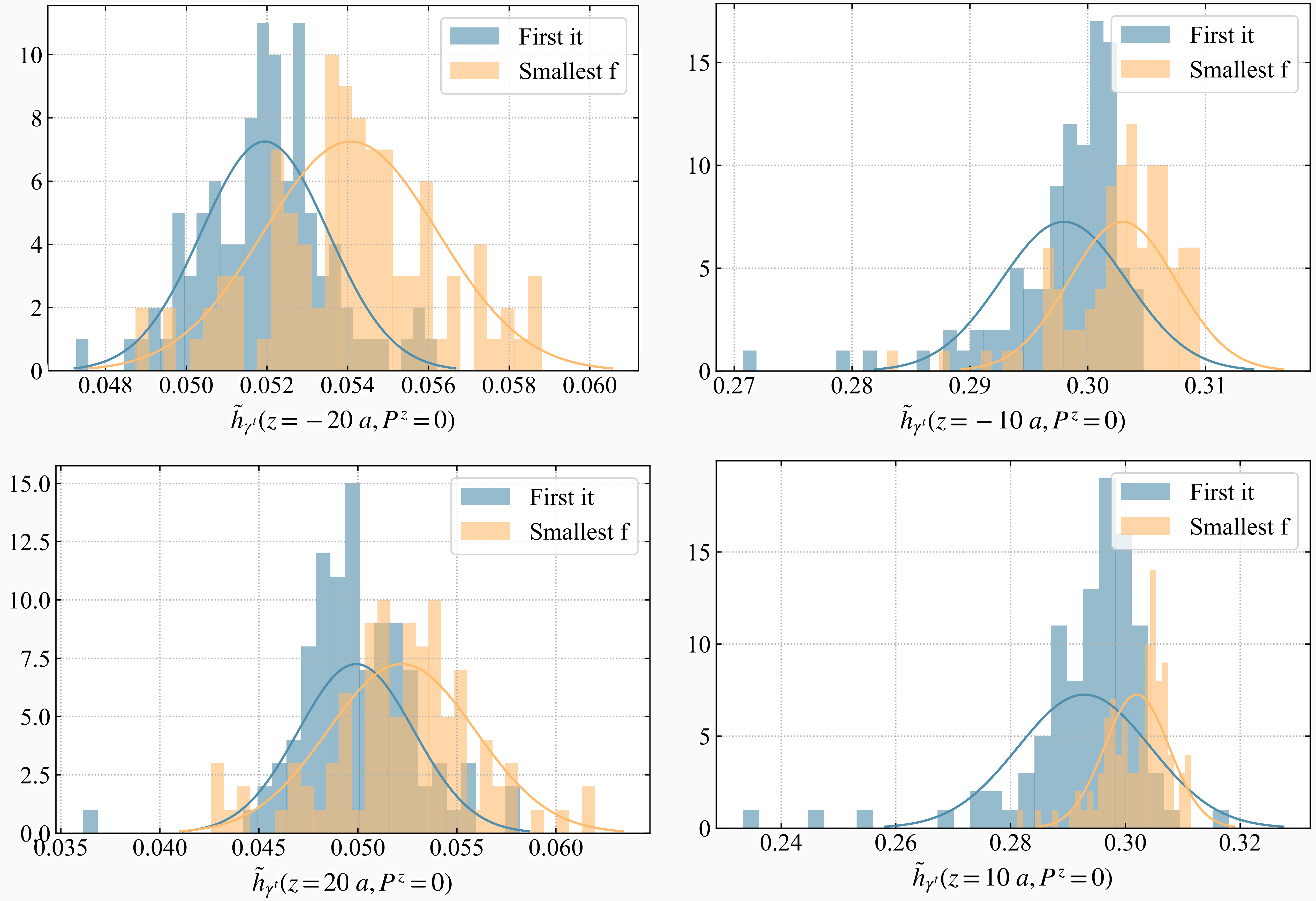


$$* \ \Delta^G(x) \equiv \sum_\mu \left(A_\mu^G(x) - A_\mu^G(x - \hat{\mu})\right)$$

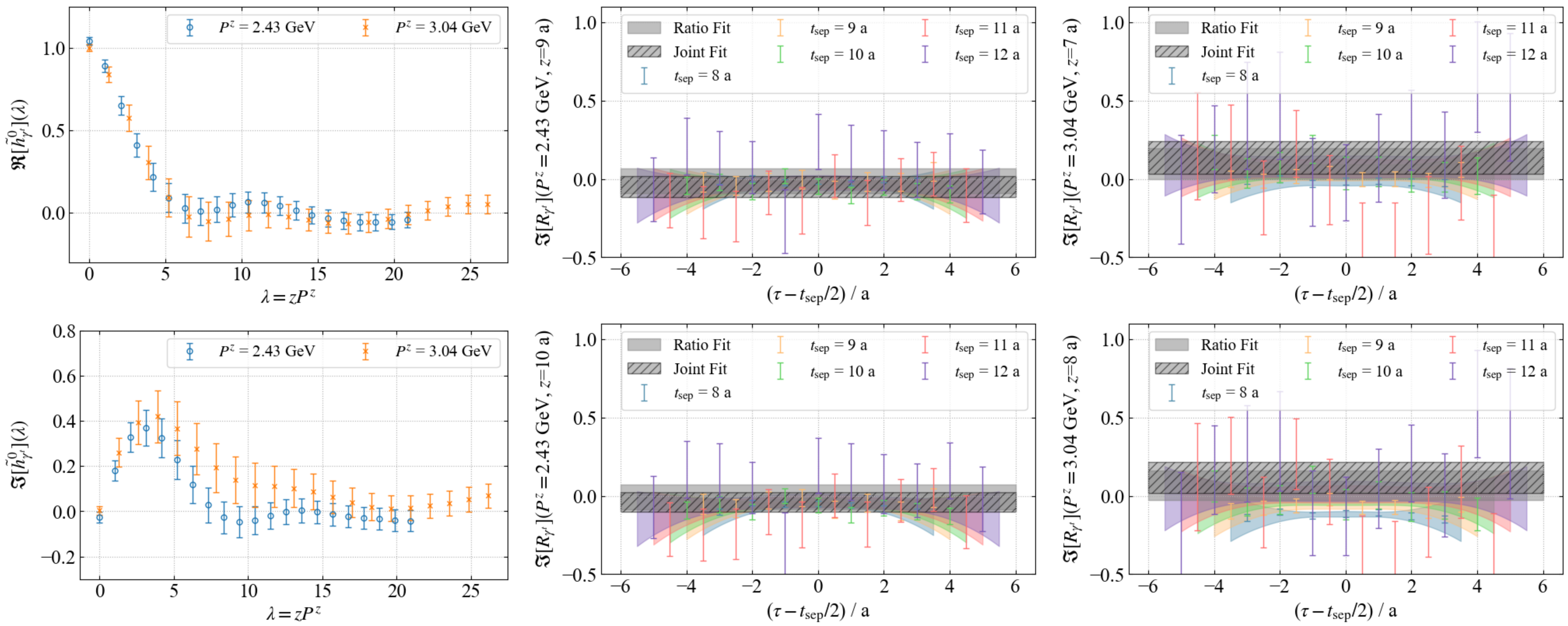
Quasi-distribution under the Coulomb Gauge

- Quasi-distribution of Pion:

$$\tilde{h}_{\gamma^t}(z, P^z, \mu) = \frac{1}{2P^t}\langle \vec{P} = \vec{0} \,|\, \bar{\psi}(z)\gamma^t\psi(0) \,|_{\vec{\nabla}\cdot\vec{A}=0} \,|\, \vec{P} = \vec{0}\rangle$$



Excited-State Contamination



Tsep dependence