Lattice Parton Collaboration

The **Renormalization** of quasi-DAs: from Mesons to **Baryons**

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LPC, arXiv:2508.08971 (2025)

OUTLINE

1 **Renormalizations**
Why Renormalization; Schemes

2 **Baryon quasi-DAs on Lattice**
Set up; Results; Divergence; Ratio scheme

3 **Self renormalization**
Self parameterization & Two step fitting

4 **Hybrid scheme**
Regions; Hybrid renormalization

5 **Summary**
Quais-DA of Lambda & Proton

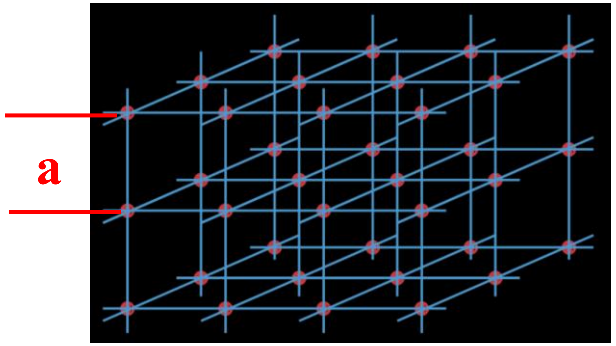
Part 1

Why Renormalization

Why Renormalization

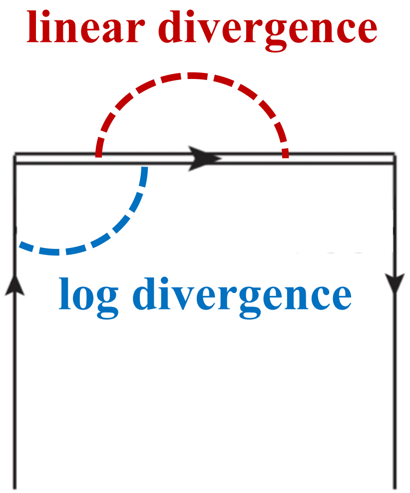
- **Scheme Conversion**

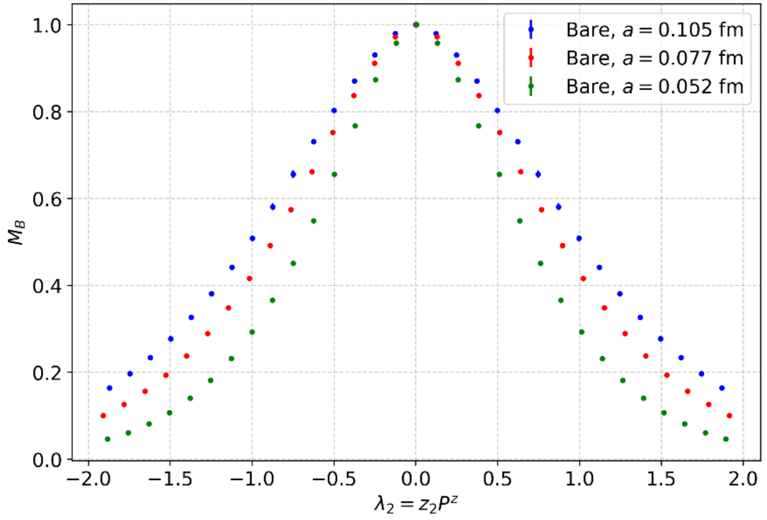
$$\frac{1}{a}\ cut \ \rightarrow \ \text{certain scheme}$$



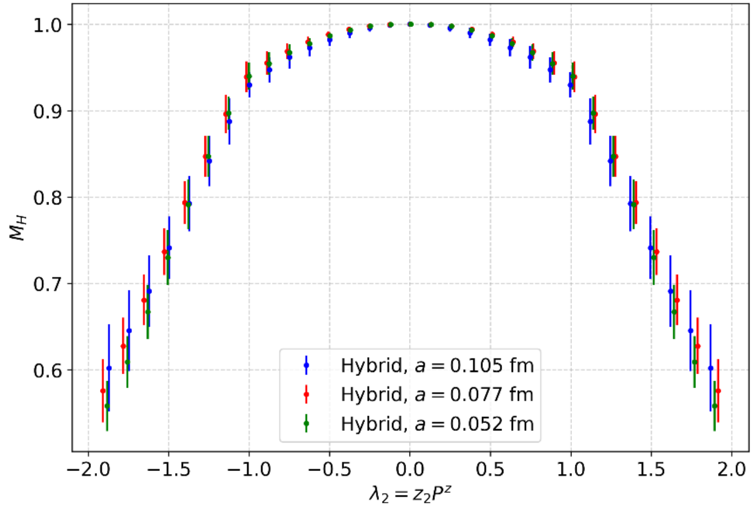
Renorm → $\overline{\text{MS}}$

- **Divergence Subtraction**

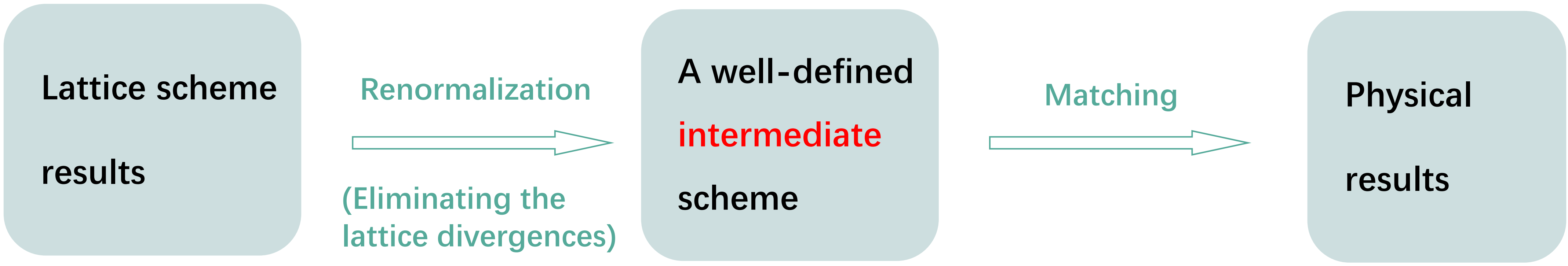




Renorm →



Renormalization: **Bridging** the lattice scheme with the physics



RI-MOM, Ratio,
Self, Hybrid, ···

- 1) Renormalization requires the ability to eliminate the divergence caused by the lattice effect.
- 2) The converted scheme is well-defined for effective matching or running …

Comparison of Different Renormalization

Part ①

Scheme	Pros	Cons
RI/MOM RI/SMOM Scheme *Martinelli et.al, NPB (1994);* *Alexandrou et.al, NPB (2017); Stewart, Zhao, PRD (2018)* *He et.al, PRD (2022); Bi et.al, PRD (2023)*	Strict subtraction Well established Widely applied ···	Not suitable for non-local (e.g. Gauge dependence, Signal problem ···)
Ratio Scheme *Radyushkin et.al, PRD (2017)*	Easy to use Well subtraction at short-distance	Extra IR behavior at large-distance
Self Renormalization and Hybrid Scheme *Ji et.al, NPB (2021); Huo et.al, NPB (2021)*	—— **The best match for the LaMET formalism**	

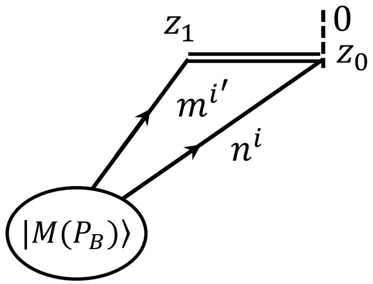
Part 2

Baryon quasi-DAs on Lattice

Meson and Baryon Quasi-DAs

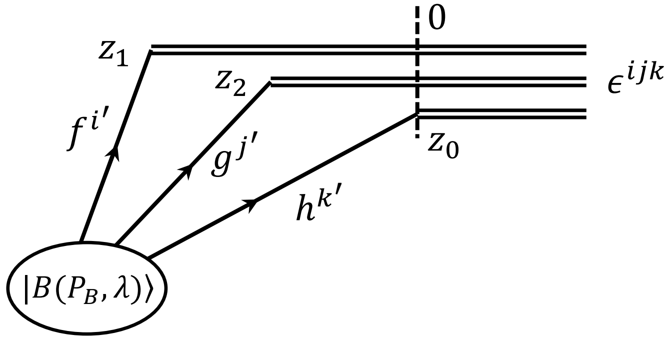
Part ②

- **Definition of Meson Quasi-DAs:**



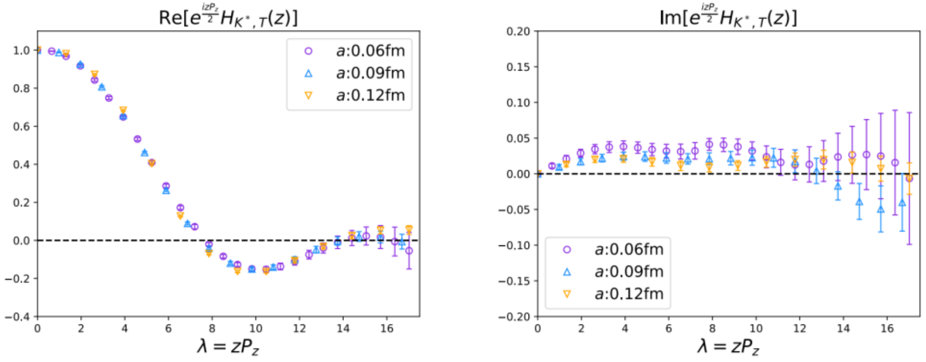
1D → 2D

- **Definition of Baryon Quasi-DAs:**

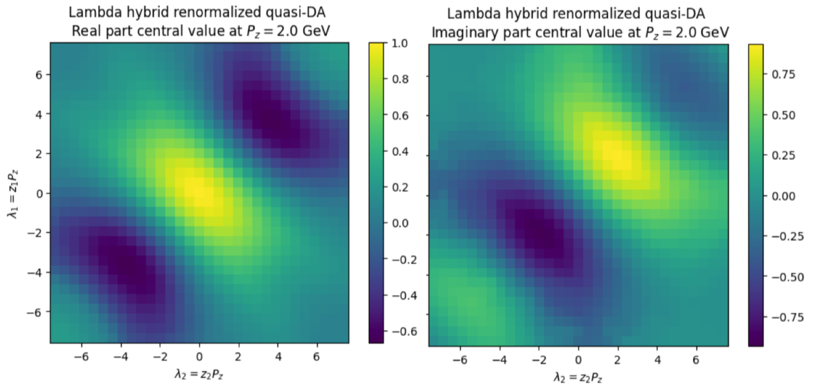


$$\hat{O}(\vec{x},t;z) \quad = m_\alpha^{i'}(\vec{x}+zn_z,t)\tilde{\Gamma}_{\alpha\beta}W^{i'i}(\vec{x}+zn_z,\vec{x},t)n_\beta^i(\vec{x},t)$$

$$\hat{O}_\gamma(\vec{x},t;z_1,z_2) = \epsilon^{ijk}W^{ii'}(\infty,\vec{x}+z_1n_z)f_\alpha^{i'}(\vec{x}+z_1n_z,t)$$
$$\times\tilde{\Gamma}_{\alpha\beta}W^{jj'}(\infty,\vec{x}+z_2n_z,t)g_\beta^{j'}(\vec{x}+z_2n_z)$$
$$\times W^{kk'}(\infty,\vec{x})h_\gamma^{k'}(\vec{x},t)$$



LPC, PRL 127, 062002 (2021)



LPC, arXiv:2508.08971 (2025)

Lattice Setup

Part ②

- **Based on CLQCD Ensembles**

CLQCD, PRD 109, 054507

  - Three different lattice spacing

Ensemble	Volume	Lattice spacing	π mass	measurement	P^z
C24P29	$24^3 \times 72$	0.105 fm	293 MeV	864*4	1.96, 2.45, 2.94, 3.43 GeV
F32P30	$32^3 \times 96$	0.077 fm	303 MeV	777*4	1.99, 2.50, 2.99, 3.49 GeV
H48P32	$48^3 \times 144$	0.052 fm	317 MeV	550*6	1.99, 2.48, 2.98, 3.48GeV

Bare quasi-DA results

Part ②

Normed Bare quasi-DA $\tilde{\psi}$: $\tilde{\psi}(z_1, z_2, P^z) = \psi(z_1, z_2, P^z)/\psi(z_1 = z_2 = 0\,, P^z)$

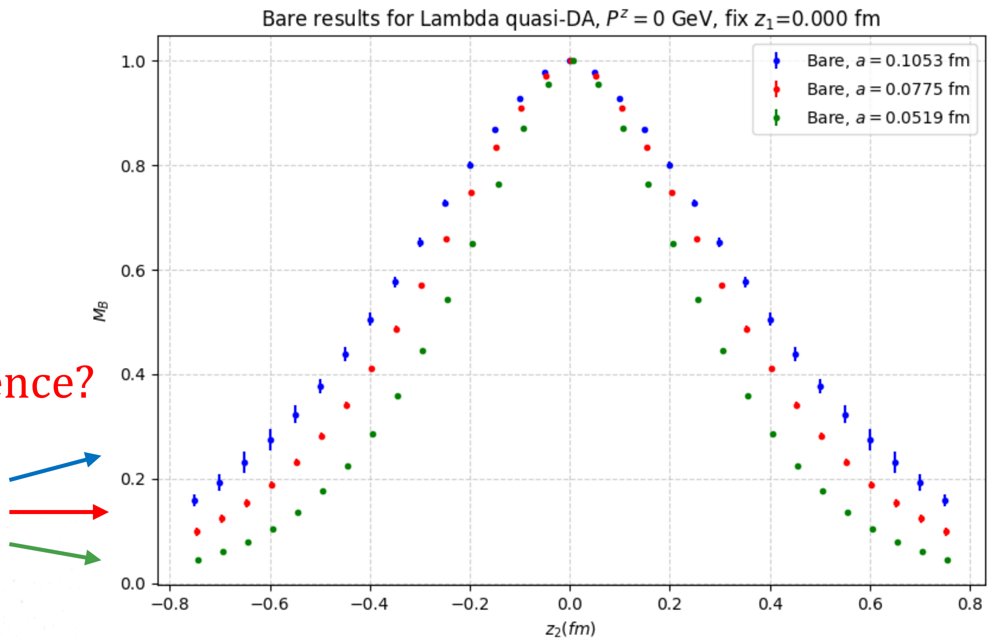
- z-dependence of normed bare $\tilde{\psi}(z_1, z_2, P^z)$ on different lattice spacing

Lambda axial term

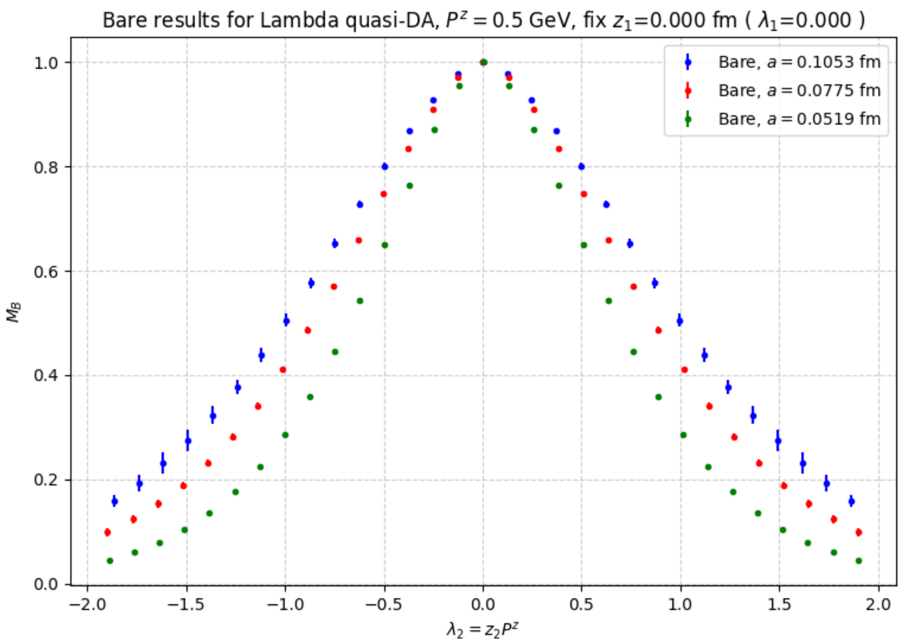
fix z_1=0 fm

P^z=0 GeV

Divergence?



P^z=0.5 GeV



Bare quasi-DA results: Linear divergence   Part ②

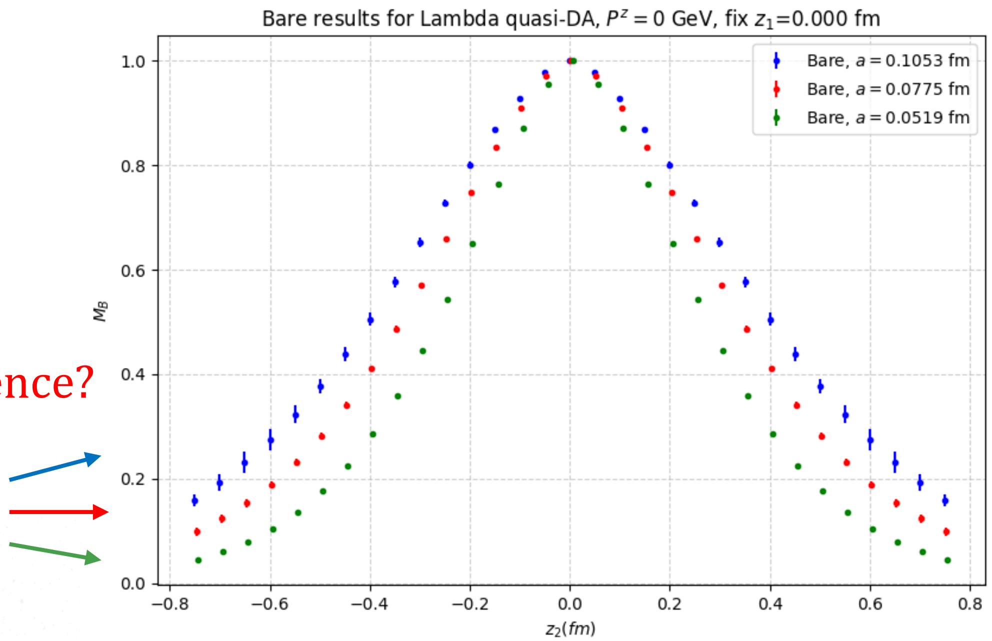
Linear divergence form for Normed Bare quasi-DA $\tilde{\psi}$:

$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

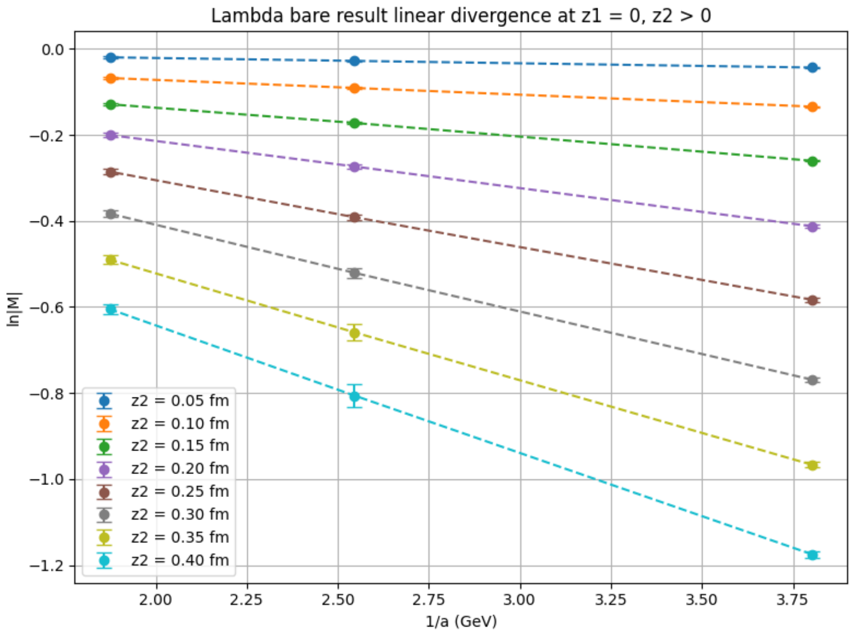
$$M(z_1, z_2; P_z; a) = exp\left[\left(\frac{k}{a\ln(a\Lambda_{\mathrm{QCD}})} + m_0\right)\tilde{z}\right] \times m(z_1, z_2; P_z; a)$$

P^z=0 GeV

Divergence?



Log scale

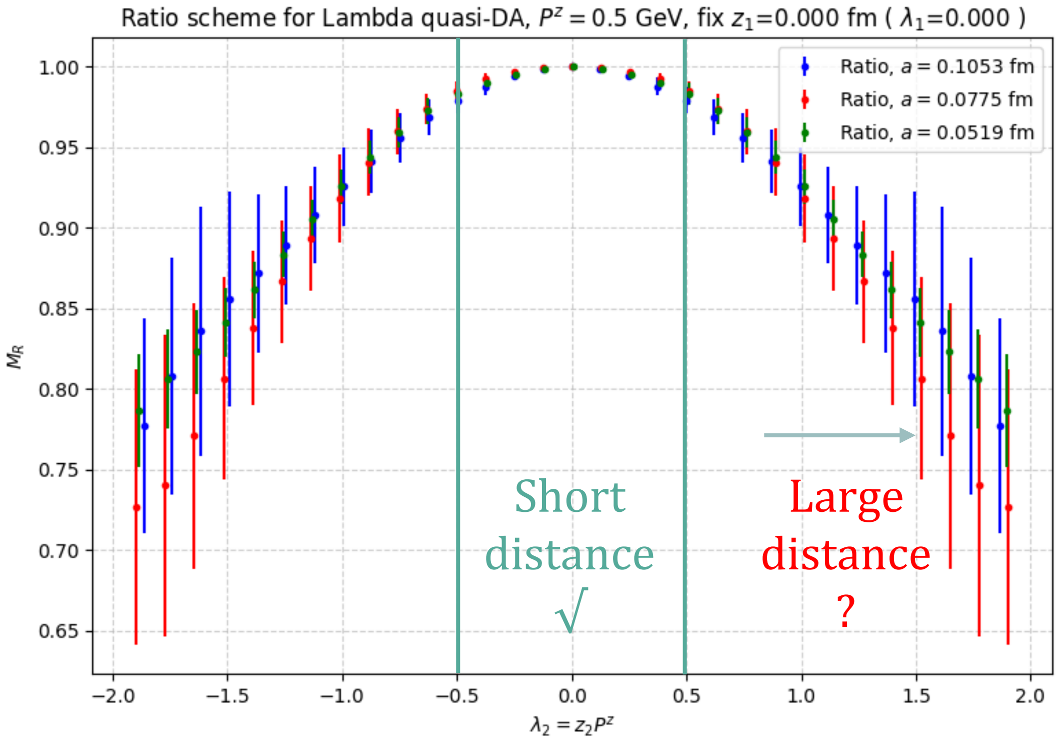


Ratio scheme quasi-DA results Part ②

Ratio scheme quasi-DA $\tilde{\psi}^{\text{ratio}}$ **:** $\tilde{\psi}^{\text{ratio}}(z_1, z_2, P^z) = \tilde{\psi}(z_1, z_2, P^z)/\tilde{\psi}(z_1, z_2, P^z = 0)$

- z-dependence of ratio scheme quasi-DA $\tilde{\psi}^{\text{ratio}}(z_1, z_2, P^z)$ on different lattice spacing

Lambda axial term P^z=0.5 GeV fix z_1=0 fm



$\tilde{\psi}(z_1, z_2, P^z)/\tilde{\psi}(z_1, z_2, P^z = 0)$

gives extra IR divergence at large distance

Part 3

Self renormalization

Self renorm: Use Lattice data to **renorm them·selves**

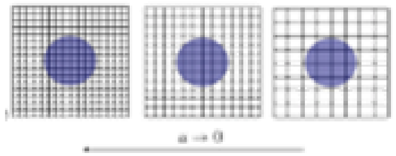
- 1) Lattice perturbation theory gives a parametrization of the Lattice divergences

$$\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu) = \frac{\hat{M}(z_1, z_2, 0, P^z, a)}{Z_R(z_1, z_2, a, \mu)}$$

$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

$$Z_R(z_1, z_2, a, \mu) = \exp\left[\left(\frac{k}{a \ln[a\Lambda_{\mathrm{QCD}}]} - m_0\right)\tilde{z} + \frac{\gamma_0}{b_0}\ln\left[\frac{\ln[1/(a\Lambda_{\mathrm{QCD}})]}{\ln[\mu/\Lambda_{\mathrm{MS}}]}\right] + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right] + f(z_1, z_2)a^2\right]$$

- 2) With data from different lattice ensembles we can fit the terms with lattice spacing dependence



- 3) Remaining terms can be obtained by matching with the perturbative result at the short-distance region.

Ensemble	Volume	Lattice spacing
C24P29	$24^3 \times 72$	0.1052 fm
F32P30	$32^3 \times 96$	0.0775 fm
H48P32	$48^3 \times 144$	0.0520 fm

Self-renormalization: 2-step fitting

$$\tilde{z} = \begin{cases} |z_1 - z_2| & z_1 z_2 < 0 \\ \max(z_1, z_2) & z_1 z_2 \geq 0 \end{cases}$$

Part ③

- **Step-1** (fitting the lattice spacing dependence)

b0

$$\ln M(z_1, z_2; P_z = 0; a) = \frac{k}{a \ln(a\Lambda_{\mathrm{QCD}})}\tilde{z} + g(z_1, z_2) + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]$$

fix $\Lambda_{\mathrm{QCD}} = 0.2$ GeV

- **Step-2** (matching the perturbative result)

$$g(z_1, z_2) - \ln Z_{\overline{\mathrm{MS}}}(z_1, z_2; \mu, \Lambda_{\overline{\mathrm{MS}}}) = m_0\tilde{z} + b_0$$

$$Z_{\overline{\mathrm{MS}}}(z_1, z_2; \mu, \Lambda_{\overline{\mathrm{MS}}}) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{z_1^2\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{z_2^2\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{(z_1 - z_2)^2\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

- Re- matrix element & renorm factor

$$M_R(z_1, z_2; P_z = 0) = \exp\left(g(z_1, z_2) - m_0\tilde{z} - b_0\right)$$

$$Z_R(z_1, z_2; a) = \exp\left[\frac{k}{a \ln(a\Lambda_{\mathrm{QCD}})}\tilde{z} + m_0\tilde{z} + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]\right]$$

Self fitting step-1

Part ③

$$\ln M(z_1, z_2; P_z = 0; a) = \frac{k}{a\ln(a\Lambda_{\mathrm{QCD}})}\tilde{z} + g(z_1, z_2) + f(z_1, z_2)a^2 + \frac{\gamma_0}{b_0}\ln\frac{\ln(1/a\Lambda_{\mathrm{QCD}})}{\ln(\mu/\Lambda_{\overline{\mathrm{MS}}})} + \ln\left[1 + \frac{d}{\ln(a\Lambda_{\mathrm{QCD}})}\right]$$

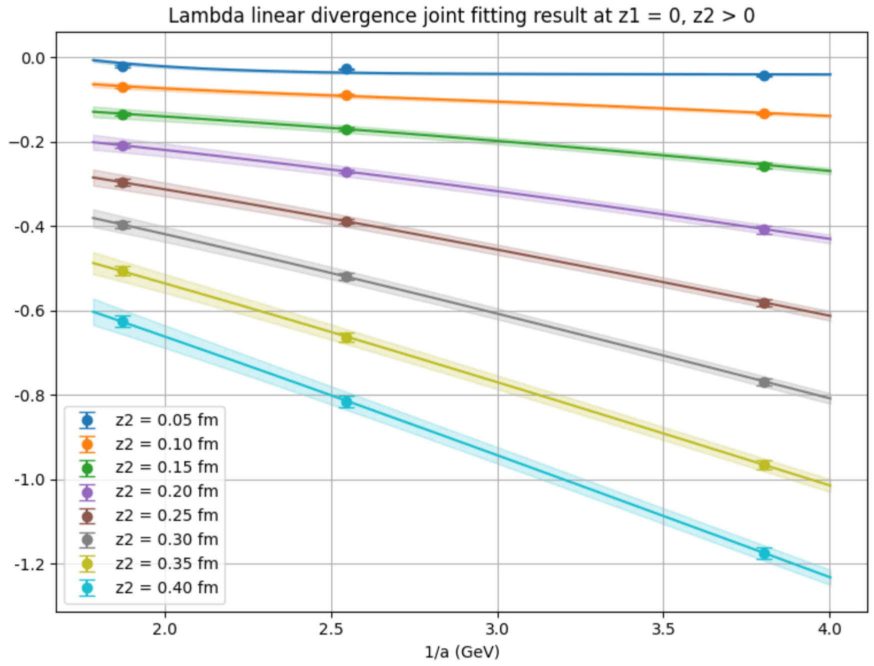
- Step-1 fit result

Interpolate to a0 = 0.05 fm

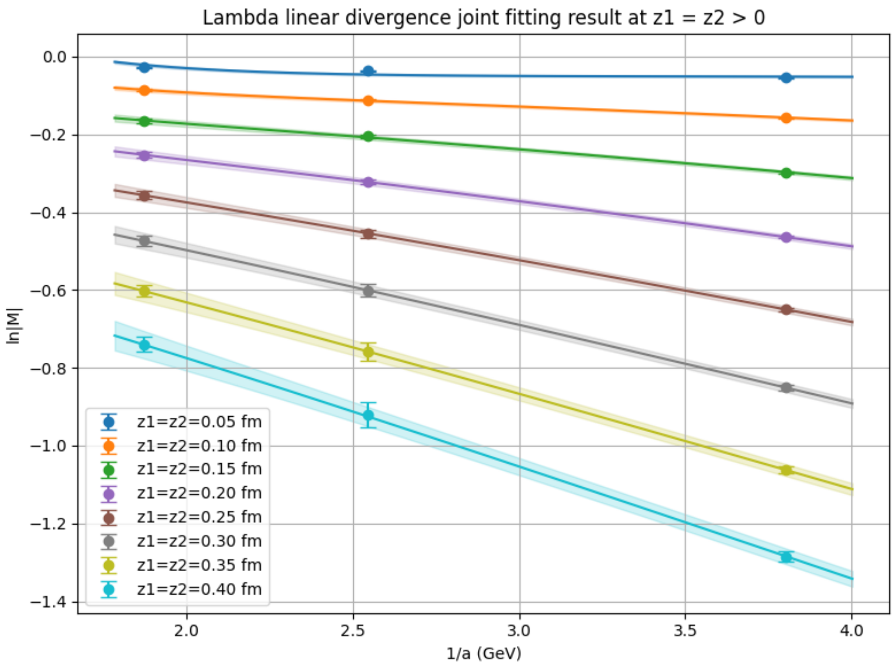
z1, z2: 0.15~ 0.70 fm

```
Parameters:
              k   0.7792 (27)     [     0 ± 10 ]
      g_-14_-10    1.020 (39)     [     0 ± 10 ]
      f_-14_-10     1.2 (5.9)     [     0 ± 10 ]
       g_-14_-9    1.066 (40)     [     0 ± 10 ]
       f_-14_-9     0.4 (5.5)     [     0 ± 10 ]
       g_-14_-8    1.100 (43)     [     0 ± 10 ]
       f_-14_-8    -0.3 (5.4)     [     0 ± 10 ]
       g_-14_-7    1.124 (46)     [     0 ± 10 ]
       f_-14_-7    -0.7 (5.4)     [     0 ± 10 ]
       g_-14_-6    1.135 (47)     [     0 ± 10 ]
       f_-14_-6    -0.6 (5.4)     [     0 ± 10 ]
       g_-14_-5    1.133 (46)     [     0 ± 10 ]
       f_-14_-5    -0.2 (5.3)     [     0 ± 10 ]
       g_-14_-4    1.119 (44)     [     0 ± 10 ]
       f_-14_-4     0.5 (5.1)     [     0 ± 10 ]
        g_-14_4    1.209 (43)     [     0 ± 10 ]
        f_-14_4     6.0 (6.2)     [     0 ± 10 ]
        g_-14_5    1.253 (49)     [     0 ± 10 ]
...
```

Fit result z1=0 (log scale)



Fit result z1=z2 (log scale)



Self fitting step-2

Part ③

$$g(z_1, z_2) - \ln Z_{\overline{\rm MS}}(z_1, z_2; \mu, \Lambda_{\overline{\rm MS}}) = \underline{m_0}\tilde{z} + \underline{b_0}$$

$$Z_{\overline{\rm MS}}(z_1, z_2; \mu, \Lambda_{\overline{\rm MS}}) = 1 + \frac{\alpha_s C_F}{2\pi}\left[\frac{7}{8}\ln\frac{\boxed{z_1^2}\mu^2 e^{2\gamma_E}}{4} + \frac{7}{8}\ln\frac{\boxed{z_2^2}\mu^2 e^{2\gamma_E}}{4} + \frac{3}{4}\ln\frac{\boxed{(z_1 - z_2)^2}\mu^2 e^{2\gamma_E}}{4} + 4\right]$$

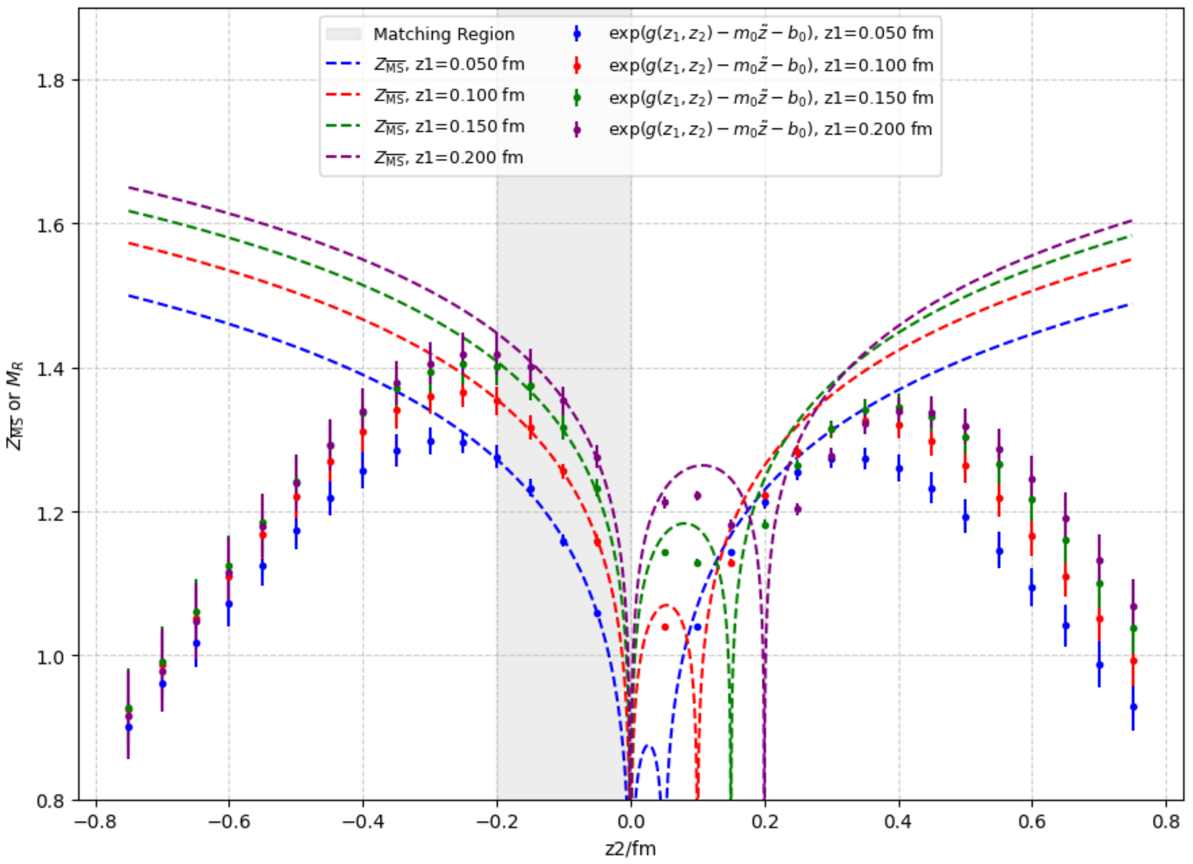
- Step-2 fit result

  z1-, z2-: 0.05 ~ 0.20 fm

```
Parameters:
    m0     1.254 (11)      [     0 ± 10 ]
    b0    0.1093 (23)      [     0 ± 10 ]

Fit:
      key          y[key]       f(p)[key]
-----------------------------------------
    g_-4_1    0.6632 (58)     0.6636 (11)
    g_-4_2    0.7873 (73)     0.7897 (15)
    g_-4_3    0.8843 (88)     0.8905 (20)
    g_-4_4     0.960 (10)     0.9808 (25)  *
    g_-3_1    0.5688 (45)    0.56330 (87)  *
    g_-3_2    0.7001 (60)     0.6943 (11)
    g_-3_3    0.8029 (74)     0.7981 (15)
    g_-3_4    0.8843 (88)     0.8905 (20)
    g_-2_1    0.4438 (39)    0.44705 (95)
    g_-2_2    0.5897 (45)    0.58605 (87)
    g_-2_3    0.7001 (60)     0.6943 (11)
    g_-2_4    0.7873 (73)     0.7897 (15)
    g_-1_1    0.2911 (23)     0.2924 (13)
```

Lambda result of matching the lattice matrix elements to the MSbar scheme perturbative expression



Self scheme quasi-DA $\tilde{\psi}^{\mathrm{self}}$

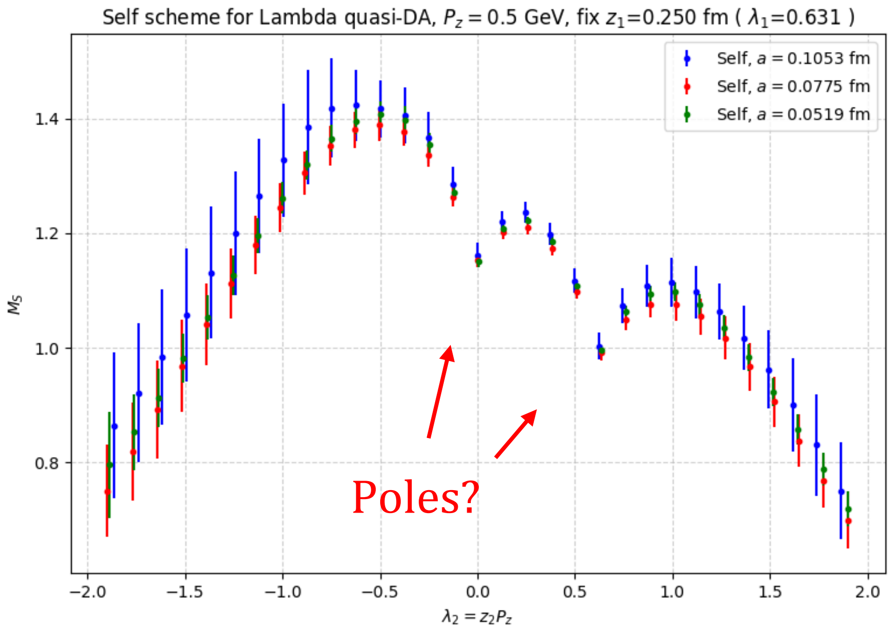
Self scheme quasi-DA $\tilde{\psi}^{\mathrm{self}}$:

$$M_R(z_1, z_2; P_z) = \frac{M(z_1, z_2; P_z; a)}{Z_R(z_1, z_2; a)}$$

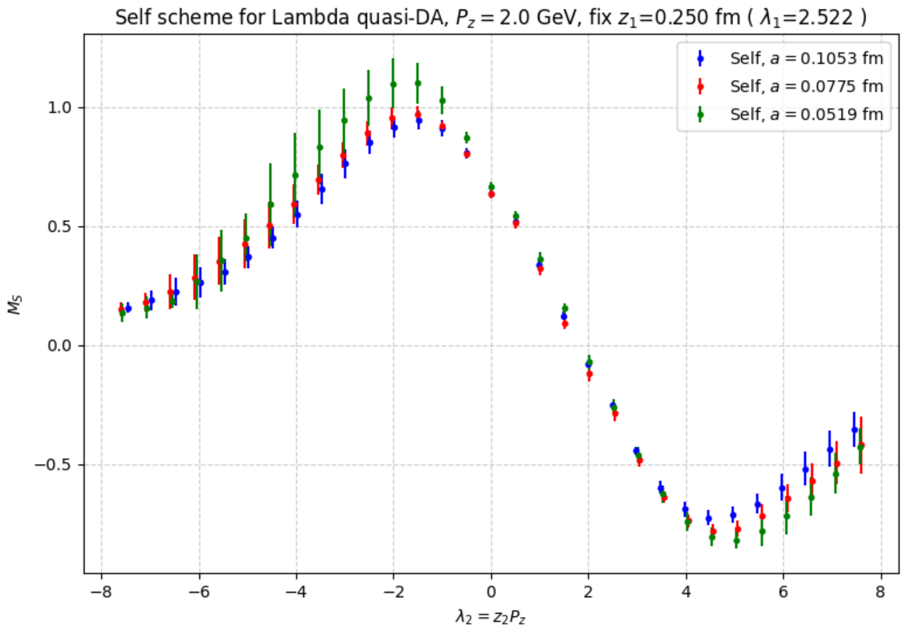
- z-dependence of quasi-DA $\tilde{\psi}^{\mathrm{self}}(z_1, z_2, P^z)$ on different lattice spacing

Lambda Axial term fix z_1=0.250 fm

P^z=0.5 GeV



P^z=2.0 GeV

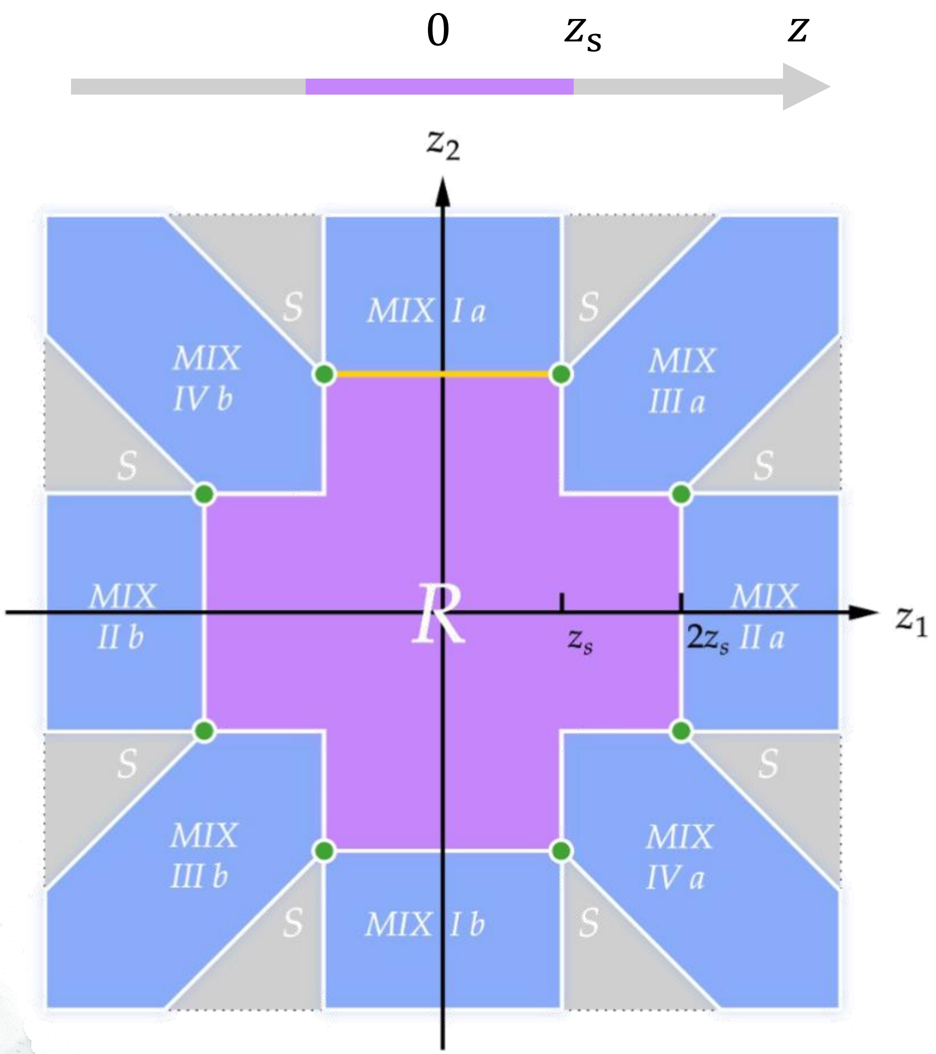


$\overline{MS}$: $\ln(\mu^2 z_1^2)$, $\ln(\mu^2 z_2^2)$, and $\ln(\mu^2 (z_1 - z_2)^2)$

Part 4

Hybrid scheme

Hybrid scheme: Use suitable schemes for different regions



- 1) **Short-distance region**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, 0, \mu)} \cdot S_{\mathrm{short}}(z_1, z_2)$$

- 2) **Long-distance region**

$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(\mathrm{sign}(z_1)z_s, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)} \cdot S_{\mathrm{long}}(z_1, z_2)$$

- 3) **Mixing regions**

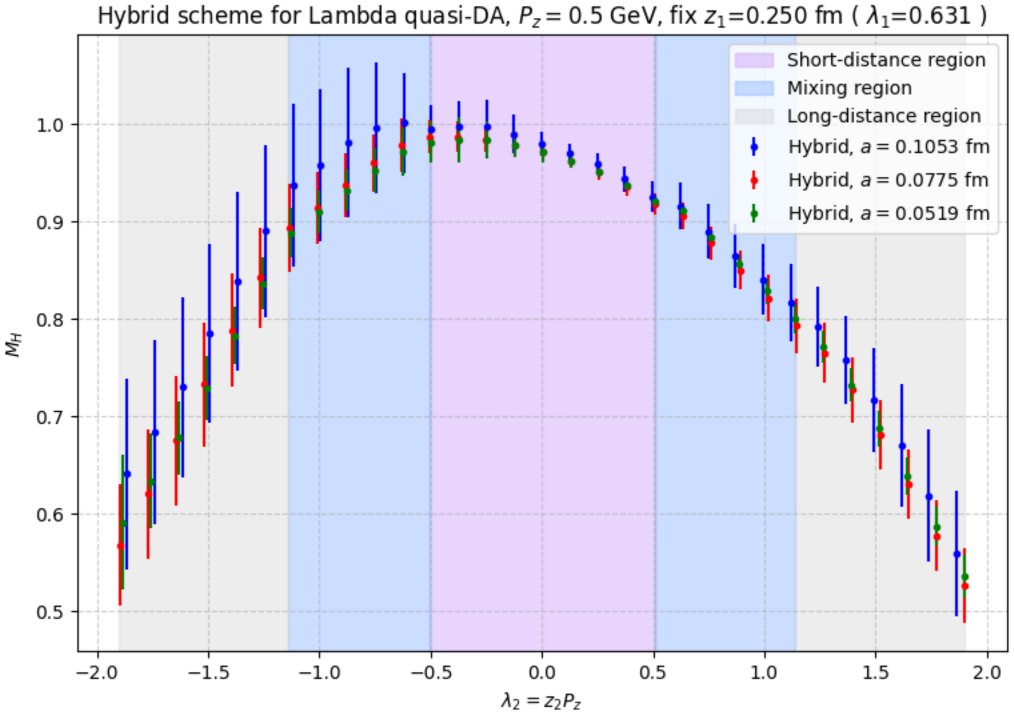
$$\frac{\hat{M}_{\overline{\mathrm{MS}}}(z_1, z_2, 0, P^z, \mu)}{\hat{M}_{\overline{\mathrm{MS}}}(z_1, \mathrm{sign}(z_2)2z_s, 0, 0, \mu)} \cdot \theta(z_s - |z_1|)\theta(|z_2| - 2z_s)$$

Hybrid scheme quasi-DA $\tilde{\psi}^{hybrid}$

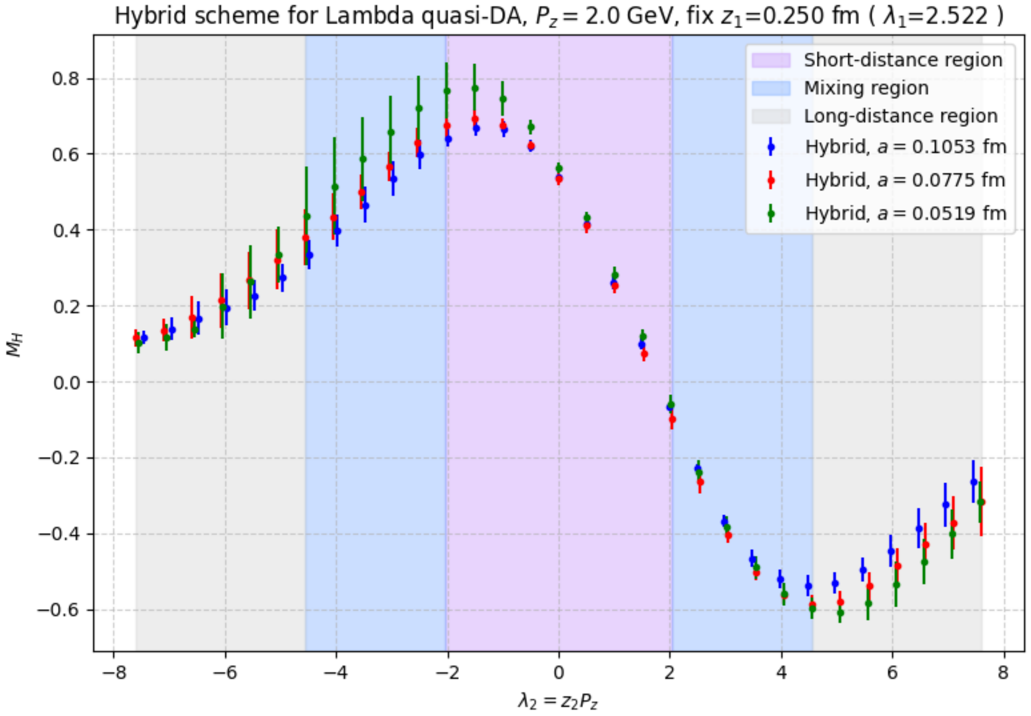
- z-dependence of quasi-DA $\tilde{\psi}^{hybrid}(z_1, z_2, P^z)$ on different lattice spacing

Lambda A term fix z_1=0.250 fm

P^z=0.5 GeV



P^z=2.0 GeV

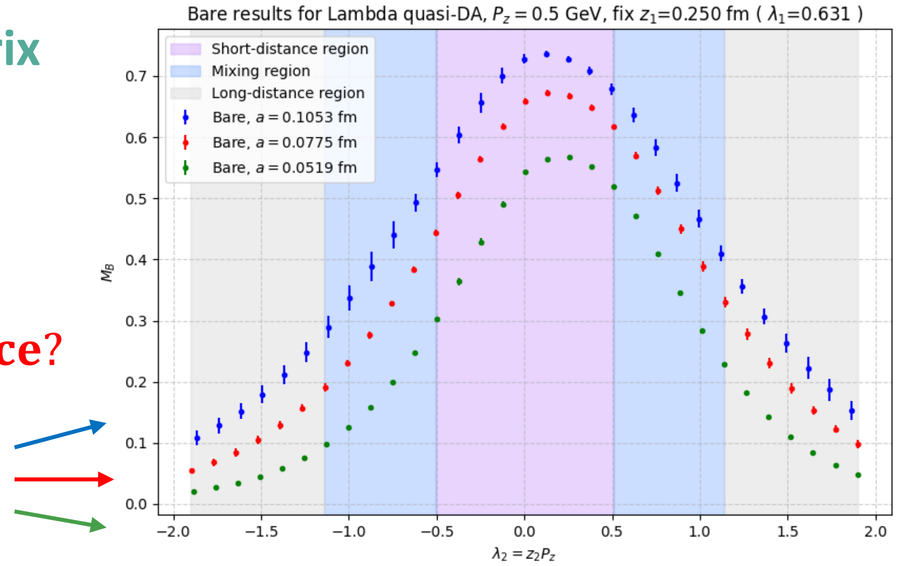


Schemes Comparison (Hybrid)

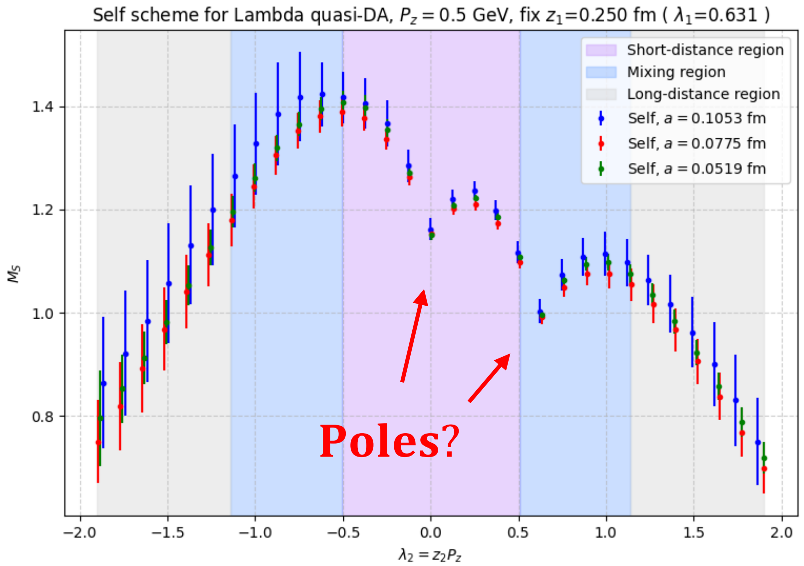
Part ④

Bare Matrix Element

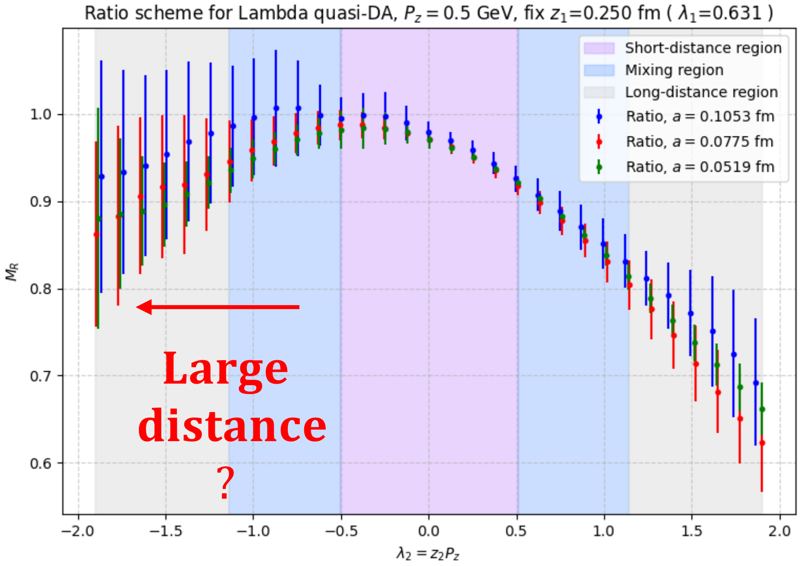
Divergence?



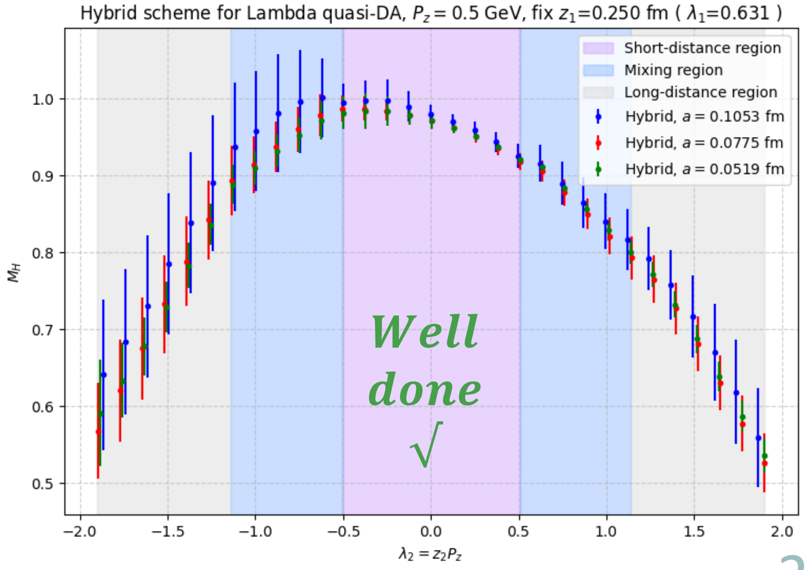
Self scheme



Ratio scheme



Hybrid scheme

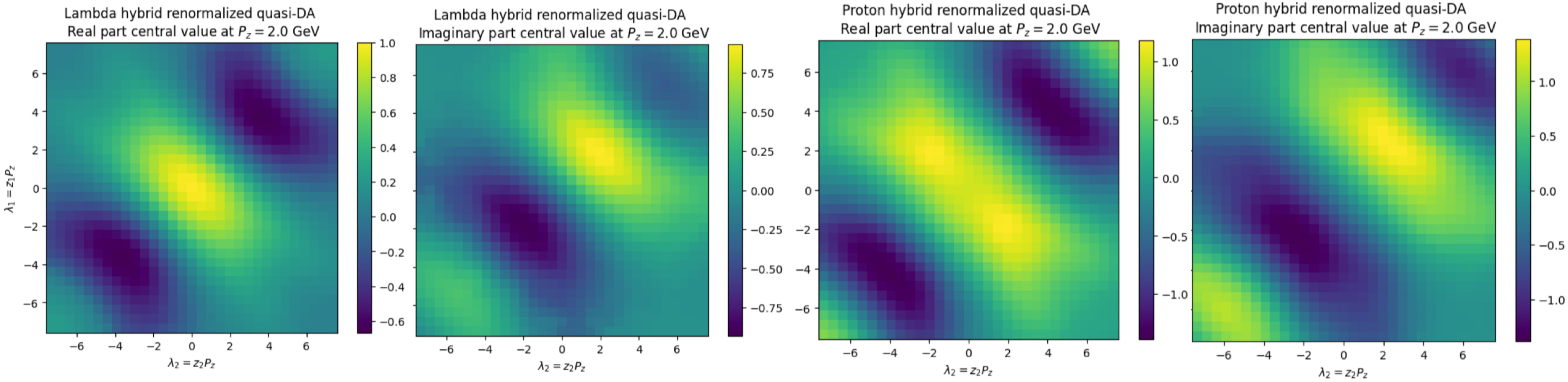


Quasi-DA results under Hybrid renormalization Part ④

- Hybrid renormalized large momentium baryon quasi-DA ($P^z = 2.0\text{GeV}$)

Λ quasi-DA **Proton quasi-DA**

Re Im Re Im



well-defined and **smooth** in all regions

Part 5

Summary

Towards Reliable Light Baryon LCDA on Lattice:

Lattice scheme results → Renormalization → A well-defined scheme → LaMET Matching → Light cone results

(Calculating the LaMET matching kernel under self scheme) Done!

Ensemble	Volume	Lattice spacing
C24P29	$24^3 \times 72$	0.105 fm
F32P30	$32^3 \times 96$	0.077 fm
G36P29	36^3x108	0.068 fm
H48P32	$48^3 \times 144$	0.052 fm

New (C24P29, G36P29, H48P32)

Ratio scheme

Self renormalization New

Hybrid scheme New

LPC, PRD 111, 034510 (2025)

LPC, arXiv:2508.08971 (2025)

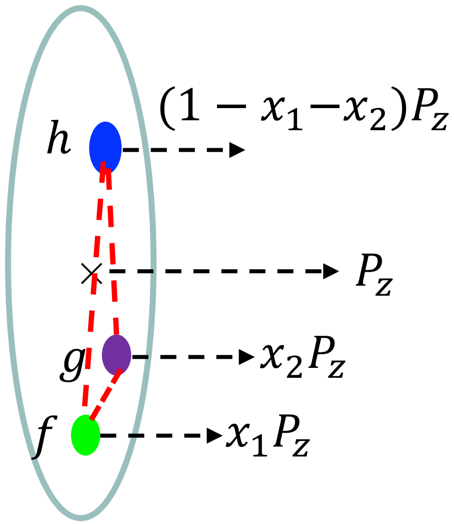
Thanks For Your Attention!

Part 6

Backup

Baryon LCDAs

Part ①



- **Definition of baryon LCDA:**

$$M_L(\xi_1,\xi_2;P)=\langle 0|\epsilon^{ijk} f_\alpha^{i'}(\xi_1 n)W_{i'i}(\xi_1,\xi_0)g_\beta^{j'}(\xi_2 n)W_{j'j}(\xi_2,\xi_0)h_\gamma^k(\xi_0 n)|B(P,\lambda)\rangle$$

$$\Phi(x_1,x_2,\mu)=\int\frac{\mathrm{d}P^+\xi_1}{2\pi}\frac{\mathrm{d}P^+\xi_2}{2\pi}\mathrm{e}^{\mathrm{i}x_1P^+\xi_1+\mathrm{i}x_2P^+\xi_2}\frac{M_L(\xi_1,\xi_2;P,\mu)}{M_L(0,0;P,\mu)}$$

C.Han et.al. JHEP 07019 (2024);
V.L.C & I.R.Z NPB 24652(1984); G.R.Farrar et.al. NPB 311585(1989)

- **Leading twist octet baryon LCDA:**

Octet	n	p	Λ
fgh	d d u	u u d	u d s

$$\begin{aligned}&\langle 0|f_\alpha(z_1n)g_\beta(z_2n)h_\gamma(z_3n)|B(P_B,\lambda)\rangle\\&=\frac{1}{4}f_V\left[(\not{P}_BC)_{\alpha\beta}(\gamma_5u_B)_\gamma V^B(z_in\cdot P_B)+(\not{P}_B\gamma_5C)_{\alpha\beta}(u_B)_\gamma A^B(z_in\cdot P_B)\right]\\&+\frac{1}{4}f_T(i\sigma_{\mu\nu}P_B^\nu C)_{\alpha\beta}(\gamma^\mu\gamma_5u_B)_\gamma T^B(z_in\cdot P_B),\end{aligned}$$

Lattice Setup

Definition of the Operators on Lattice

Matrix element

$$C_2(z_a, z_2; t, \vec{P}) = \int \mathrm{d}^3x e^{-\mathrm{i}\vec{P}\cdot\vec{x}} \langle 0|\mathcal{O}_{\mathrm{Sink}}(\vec{x}, t; z_1, z_2)\bar{\mathcal{O}}_{\mathrm{Src}}(0,0;0,0)T|0\rangle$$

Source-side Operator

$$\bar{O}^P_{\mathrm{mod}} = (u^T C\gamma_5\gamma^t d)u,$$

$$\bar{O}^\Lambda_{\mathrm{mod}} = \frac{2(u^T C\gamma_5\gamma^t d)s + (u^T C\gamma_5\gamma^t s)d + (s^T C\gamma_5\gamma^t d)u}{\sqrt{6}}.$$

Projection Operator

$$T = \gamma_t + \gamma_z$$

Sink-side Operator

$$O^V_P = [u^T(z_1 n_z)(C\gamma^t)u(z_2 n_z)]\gamma_5 d(z_3 n_z)$$

$$O^A_P = [u^T(z_1 n_z)(C\gamma_5\gamma^t)u(z_2 n_z)]d(z_3 n_z)$$

$$O^T_P = [u^T(z_1 n_z)\frac{1}{2}C[\gamma^t, \gamma^\mu]u(z_2 n_z)]\gamma_5\gamma_\mu d(z_3 n_z)$$

$$O^V_\Lambda = [u^T(z_1 n_z)(C\gamma^t)d(z_2 n_z)]\gamma_5 s(z_3 n_z)$$

$$O^A_\Lambda = [u^T(z_1 n_z)(C\gamma_5\gamma^t)d(z_2 n_z)]s(z_3 n_z)$$

$$O^T_\Lambda = [u^T(z_1 n_z)\frac{1}{2}C[\gamma^t, \gamma^\mu]d(z_2 n_z)]\gamma_5\gamma_\mu s(z_3 n_z)$$