Updates on Gluon GPDs and the Hadronic Structure of the Photon from Lattice QCD

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Center for Frontiers in Nuclear Science
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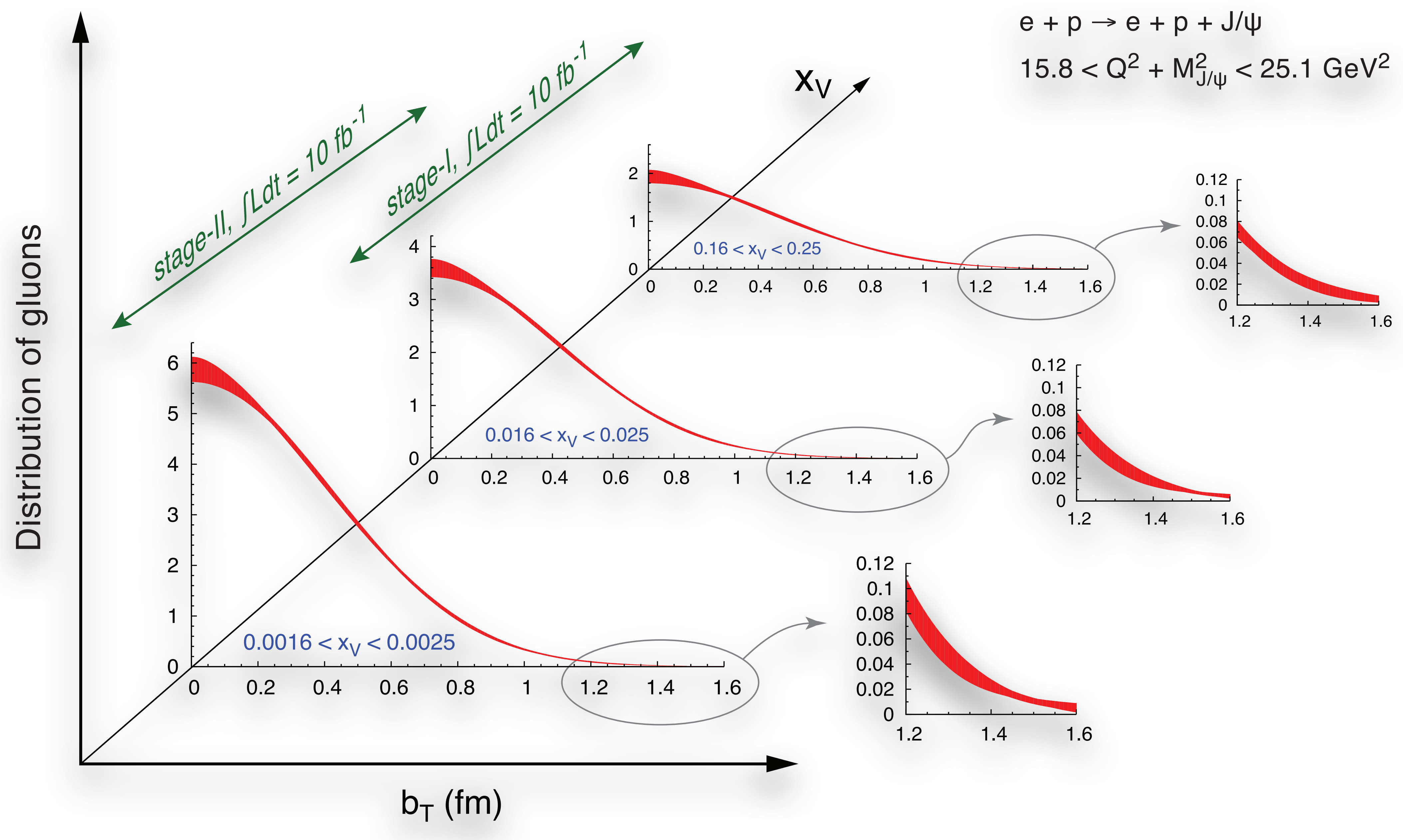
Status of gluon generalized patron distributions (GPDs)

- Gluon GPDs from theory or experiment almost unknown (no LQCD calculation yet)



EIC white paper (projection)

- In experimental processes, gluon GPDs enter indirectly, only through higher-order effects

- Number of independent observables in experiments is typically insufficient to fully isolate and constrain all 8 gluon GPDs

Toward determination of gluon GPDs from LQCD

- Given experience from gluon helicity PDF, is it possible to isolate all 8 gluon GPDs in LQCD calculations?

Schoenleber, RSS, Izubuchi, Yang [PRD 2025]

  - Lorentz-covariant parameterization of the matrix elements in terms of a linearly independent basis of tensor structures
  - Projection of LQCD matrix elements onto light cone distributions

- Are projections of LQCD gluonic matrix elements onto light cone distributions unique?

- Is LQCD matrix element with one set of operator better than the other?

LQCD calculation in progress Jakob Schoenleber**, Michael Engelhardt, Taku Izubuchi, Tanjib Khan, Huey-Wen Lin, Sergey Syritsyn and students/postdocs to sign up for

Questions related to choice of operators for gluon PDFs

- For unpolarized gluon PDF: $M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$

X. Ji [PRL 2013]

- A combination of matrix elements on the lattice:

$$M_{0i;i0} = \langle p|\, G_{0i}(z)\, [z,0]\, G_{i0}(0)\, |p\rangle = 2\, p_0^2 \mathcal{M}_{pp} + 2\mathcal{M}_{gg}$$

$$M_{ji;ij} = -2\mathcal{M}_{gg} \quad i,j \rightarrow x,y$$

$$M_{0i;i0} + M_{ji;ij} = 2p_0^2\, \mathcal{M}_{pp}$$

Balitsky, Morris, Radyushkin, et al [PLB 2020]

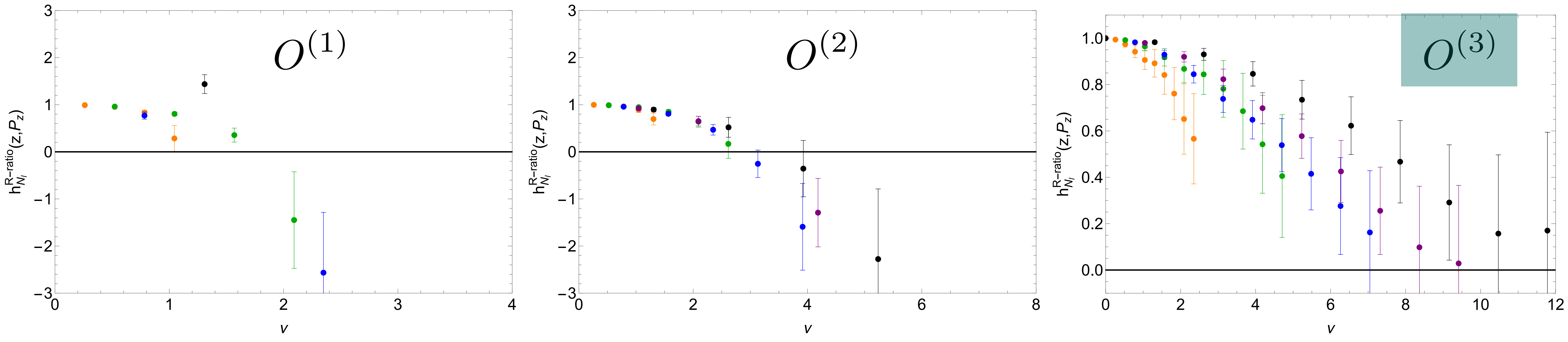
$$\mathcal{M}_{pp}(\omega, z^2=0) = \frac{1}{2}\int_{-1}^{1} dx\; e^{-ix\omega}\; xg(x)$$

- Other choices of multiplicatively renormalizable operators

Zhang, et al [PRL 2019]
Li, Ma, Qiu p[PRL 2019]

$$O^{(1)}(z) = F^{zi}(z)W(z,0)F^{z}_{\;i}(0)$$

$$O^{(2)}(z) = F^{z\mu}(z)W(z,0)F^{z}_{\;\mu}(0)$$



Good, Hasan, Lin [J. Phys. G [2025]

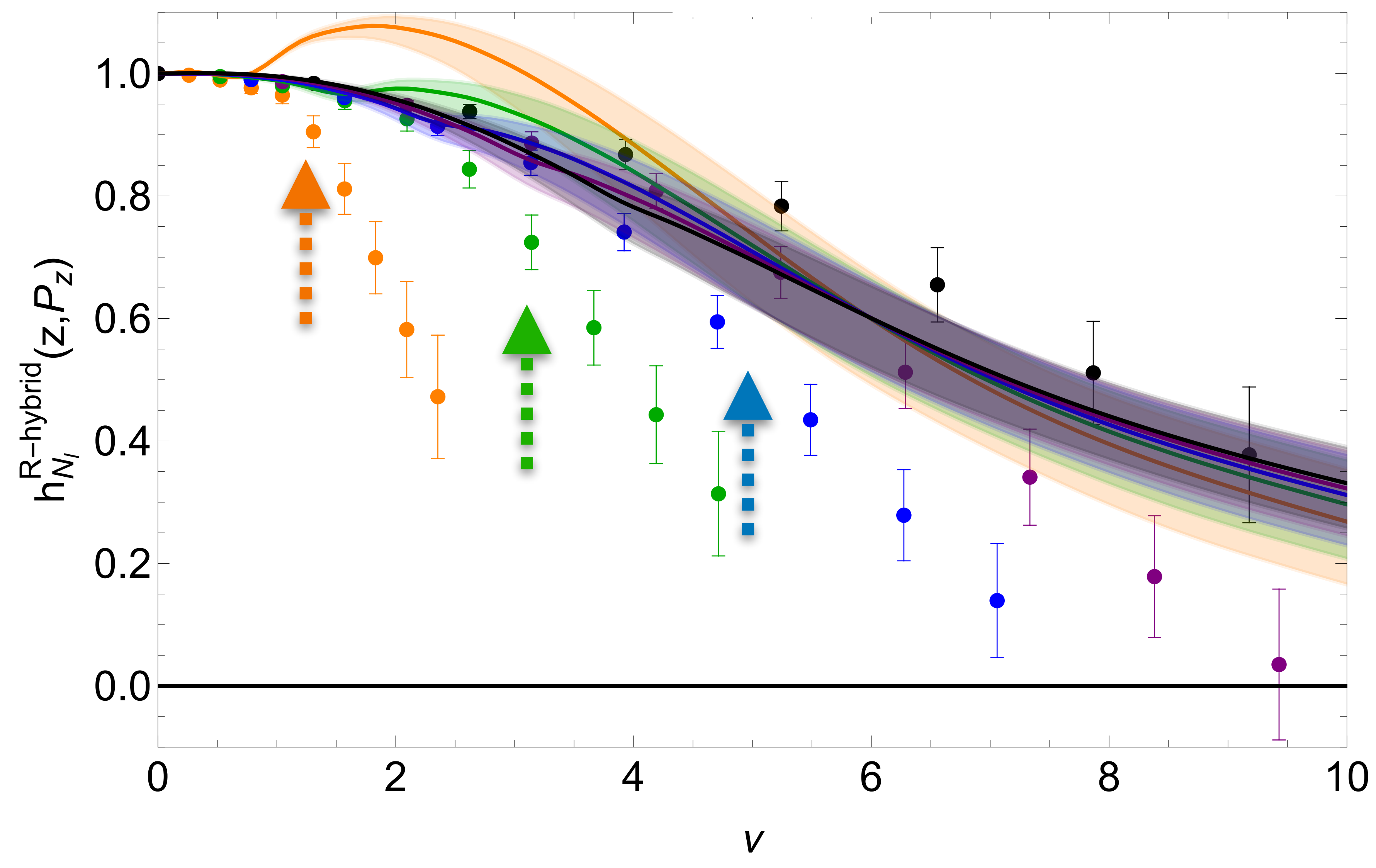
Questions related to choice of operators for unpolarized gluon PDFs

- A new calculation with LaMET matching

Good, Yao, Lin [arXiv: 2505.13321]

- Operator choice as in

Balitsky, et al [PLB 2020]



$$z_{\max} \sim 1.1 \text{ fm}, \;\; p_z \sim 2.15 \text{ GeV}$$

- Large power corrections (for fixed momentum, starts to deviate after $z_{\max} \sim 0.5$ fm)?

- Does not show $O^{(3)}$ has small $\mathcal{O}(z^2)$ corrections

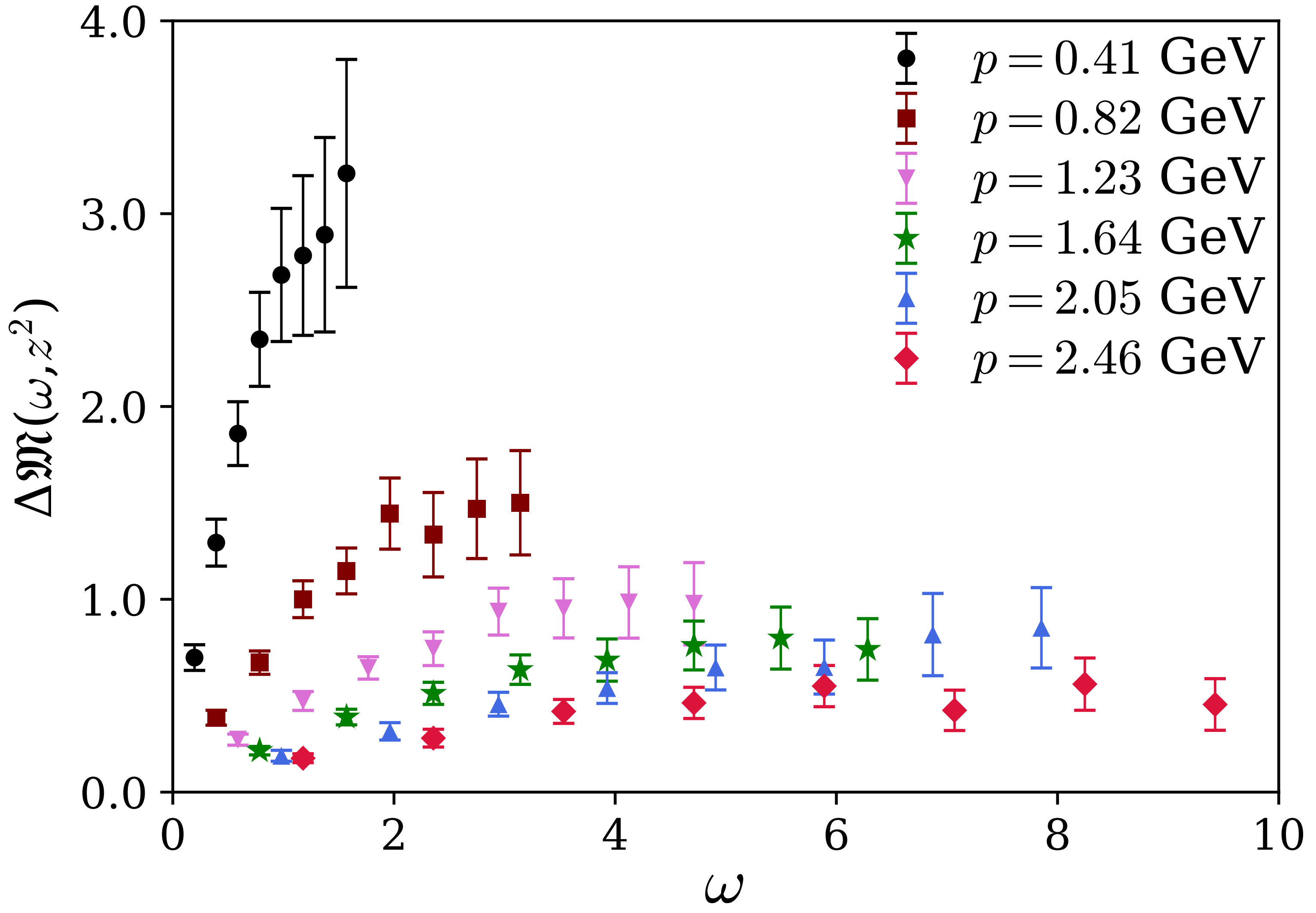
A more complicated problem related to gluon helicity PDF

- For polarized gluon PDF: $\Delta M_{\mu\alpha;\lambda\beta}(z,p,s) = \langle p,s|\, G_{\mu\alpha}(z)\, W[z,0] \widetilde{G}_{\lambda\beta}(0)\, |p,s\rangle$ X. Ji [PRL 2013]

- "Contamination term" present in LQCD matrix element — Balitsky, Morris, Radyushkin [JHEP 2022]

- Renormalized matrix element: $\Delta\mathfrak{M}(\omega,z^2) = [\Delta\mathcal{M}_{sp}^{(+)}(\omega,z^2) - \left(1+\frac{m_p^2}{p_z^2}\right)\omega\Delta\mathcal{M}_{pp}(\omega,z^2)]$

Unknown nonperturbative functions



Can one come up with a better choice of operators?

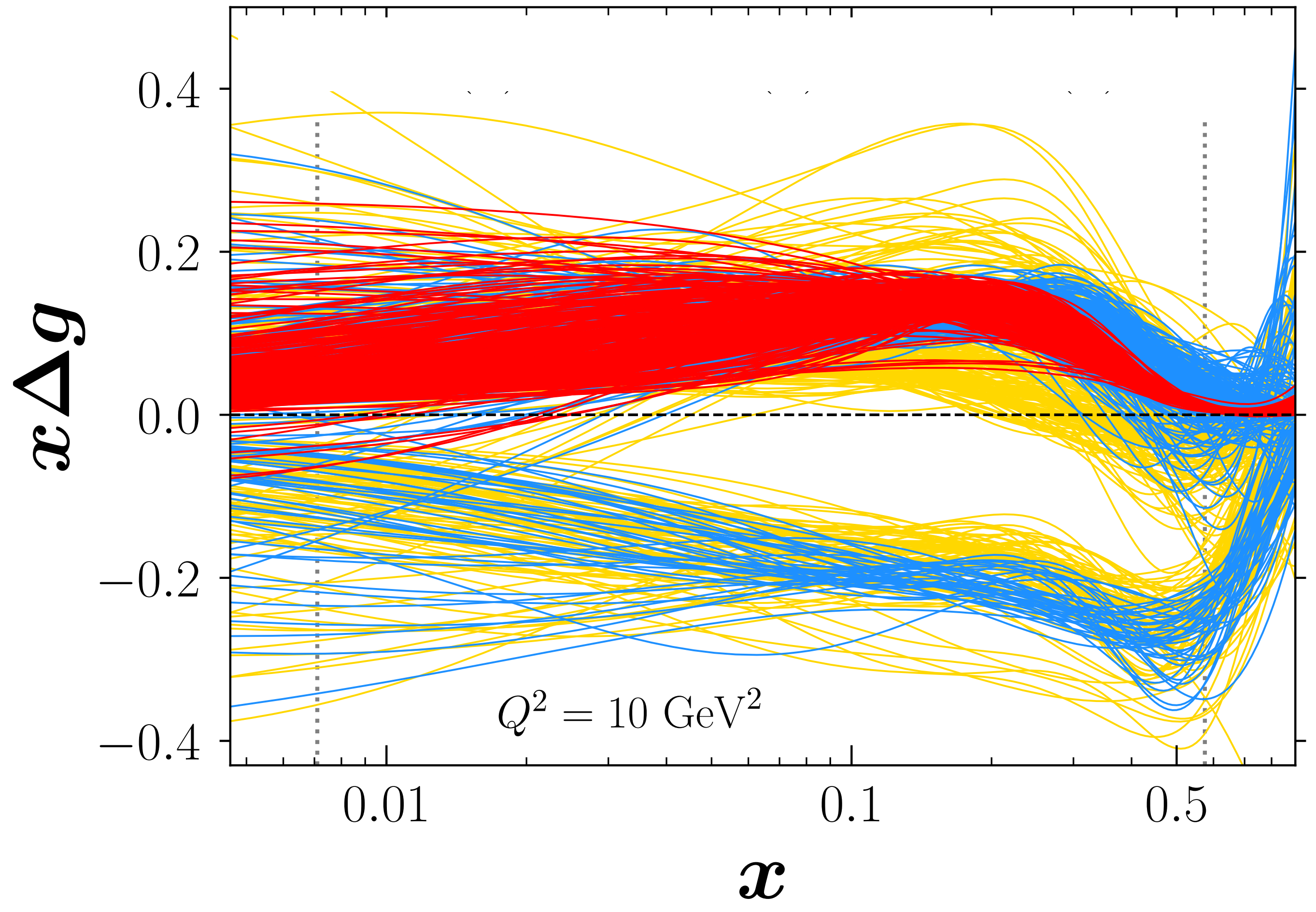
- Two different $p_n = \frac{2\pi n}{La}$ data sets are related by

$$\Delta\mathfrak{M}_g(\omega) \equiv \Delta\mathcal{M}_{sp}^{(+)}(\omega) - \omega\Delta\mathcal{M}_{pp}(\omega) = \frac{r^2\Delta\mathfrak{M}(\omega)\big|_{p_k} - \Delta\mathfrak{M}(\omega)\big|_{p_l}}{r^2-1} \qquad r = k/l$$

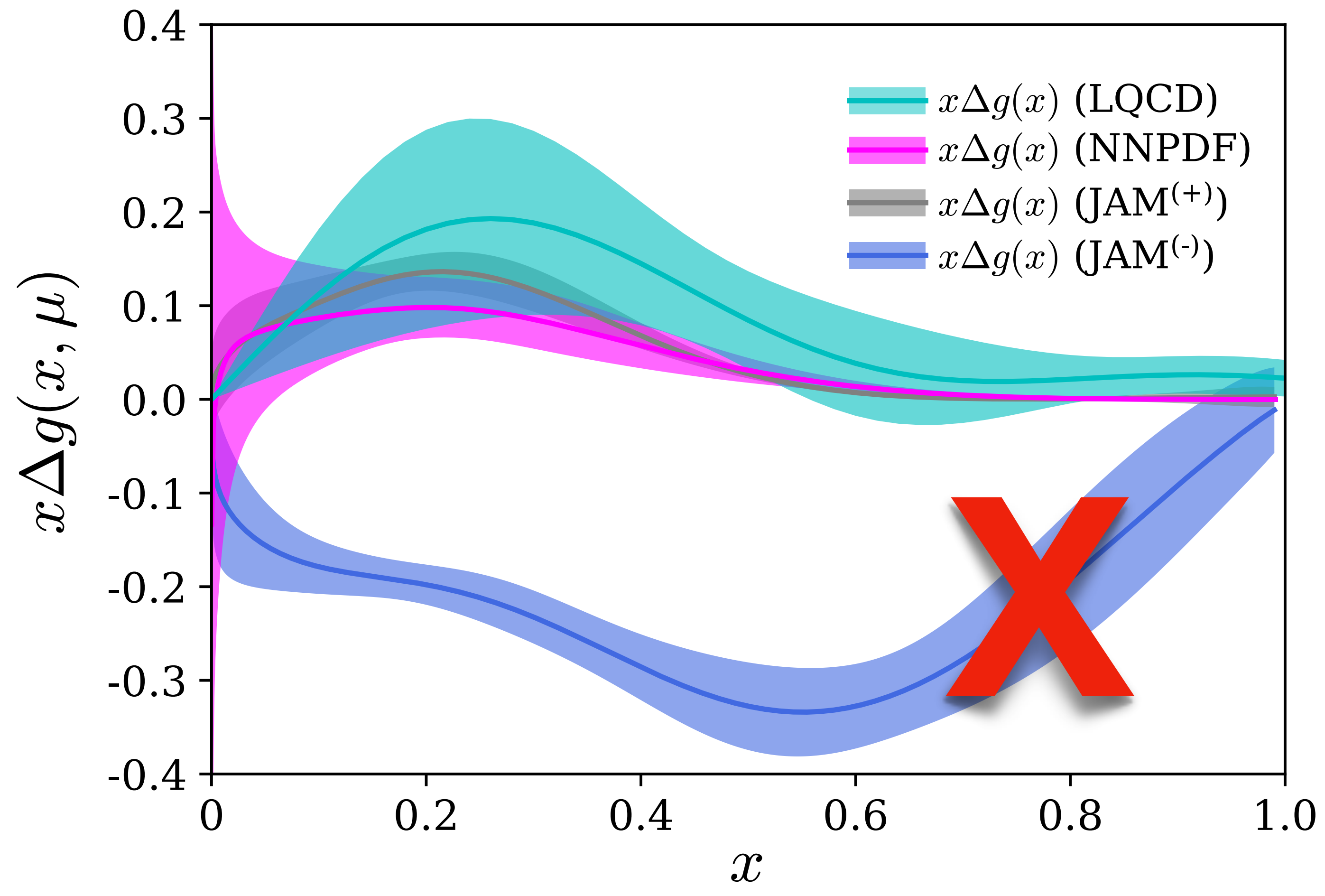
▶ ***Main regulator for the ML algorithms***

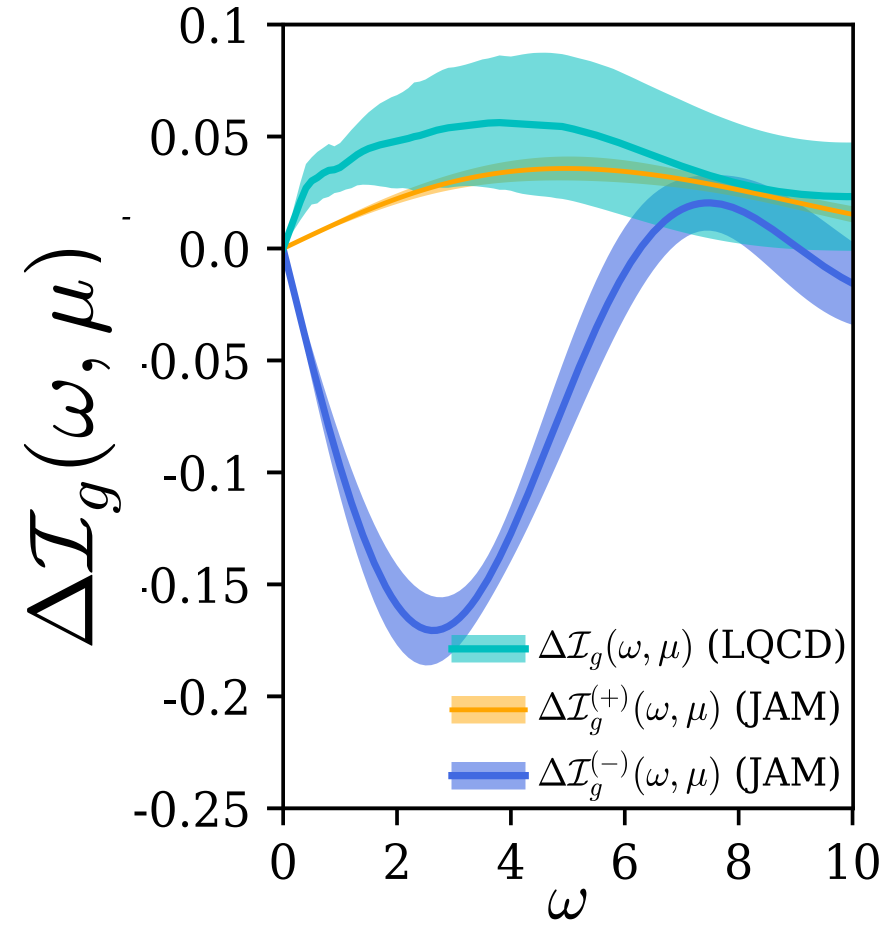
How Lattice QCD & ML can help determining gluon helicity PDF

- **LQCD + ML tends to rule out negative** gluon helicity PDF in the mid-to-moderately large x region



JAM Collaboration global fit (PRD 2022)
positive and negative solutions

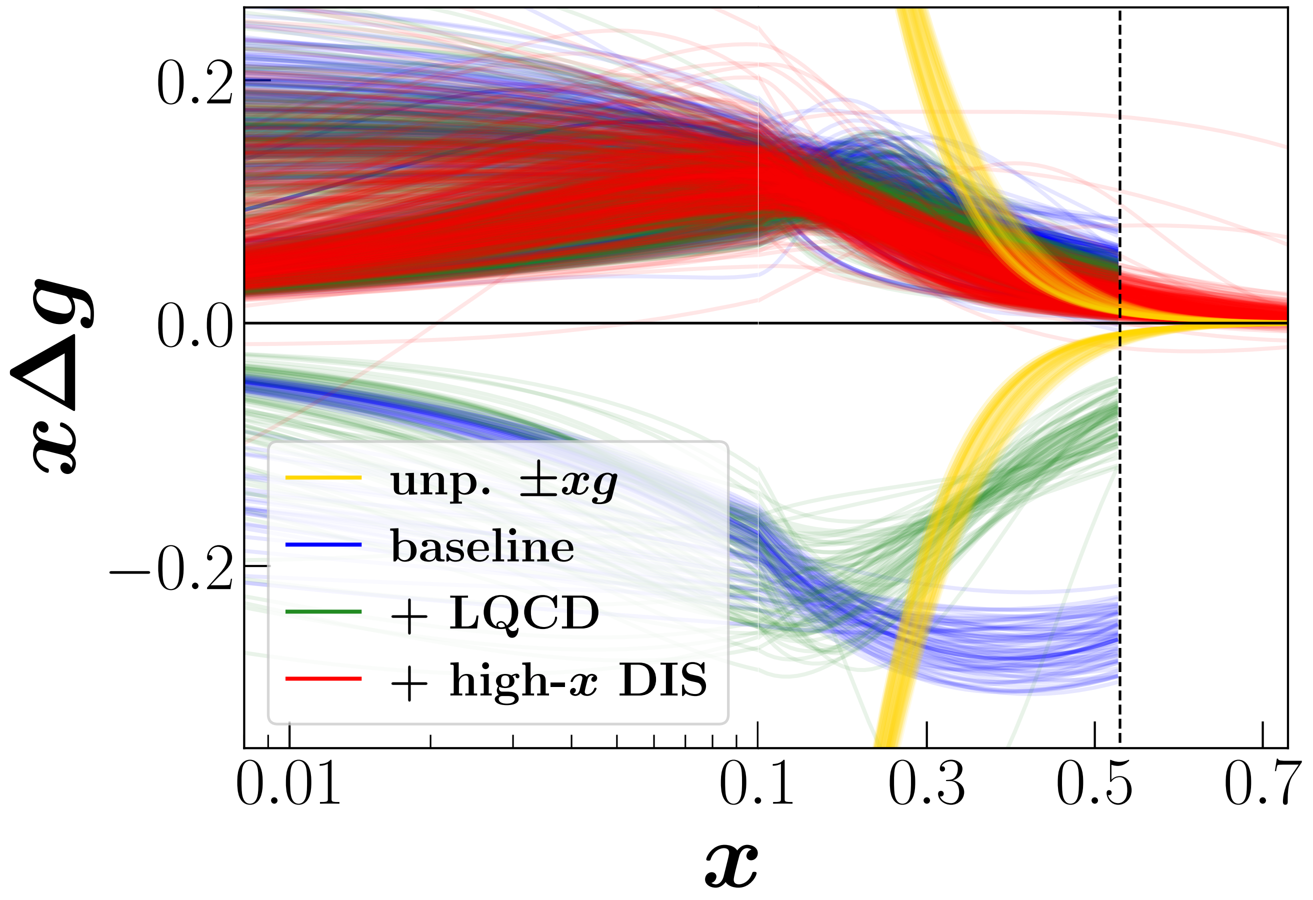




LQCD + Neural Network: only positive gluon helicity PDF
Liu, Khan, RSS [PRD 2023]

- Negative Δg leads to negative cross-sections for Higgs boson production (de Florian, Forte, Vogelsang [PRD 2024])

  - JAM re-analysis (PRL 2024)



LQCD (+ ML) predictions hold

Toward determination of gluon GPDs from LQCD

- General matrix element: $M^{\mu\nu;\alpha\beta}_{s's}(z) = \langle p', s' |\, G^{\mu\nu}(-z/2) W(-z/2, z/2) G^{\alpha\beta}(z/2)\, | p, s \rangle$

  - For spatial vector z, $\quad M^{\mu\nu;\alpha\beta}(\omega, \eta, t, z^2) = \sum_\ell \mathcal{M}_\ell(\omega, \eta, t, z^2)\, \mathcal{T}_\ell^{\mu\nu;\alpha\beta}(\omega, \eta, t, z^2)$

    UV divergences must be renormalized ($\mathcal{M}_\ell$)

    Possible tensor structures ($\mathcal{T}_\ell^{\mu\nu;\alpha\beta}$)

  - Write down possible tensor structures,

$$
\begin{aligned}
\mathcal{T}_{gg} &= g^{\alpha\nu} g^{\beta\mu} - g^{\alpha\mu} g^{\beta\nu}, \\
\mathcal{T}_{PP} &= P^\alpha P^\mu g^{\beta\nu} - P^\beta P^\mu g^{\alpha\nu} - P^\alpha P^\nu g^{\beta\mu} + P^\beta P^\nu g^{\alpha\mu}, \\
\mathcal{T}_{P\Delta} &= P^\alpha \Delta^\mu g^{\beta\nu} - P^\beta \Delta^\mu g^{\alpha\nu} - P^\alpha \Delta^\nu g^{\beta\mu} + P^\beta \Delta^\nu g^{\alpha\mu}, \\
&\vdots \\
\mathcal{T}_{\Delta Pzz} &= -P^\nu \Delta^\alpha z^\beta z^\mu + P^\mu \Delta^\alpha z^\beta z^\nu + P^\nu \Delta^\beta z^\alpha z^\mu - P^\mu \Delta^\beta z^\alpha z^\nu, \\
\mathcal{T}_{\Delta\Delta zz} &= \Delta^\alpha \Delta^\mu z^\beta z^\nu - \Delta^\beta \Delta^\mu z^\alpha z^\nu - \Delta^\alpha \Delta^\nu z^\beta z^\mu + \Delta^\beta \Delta^\nu z^\alpha z^\mu.
\end{aligned}
$$

  - Ensure remaining tensors form a linearly independent set

$$
\mathfrak{F} = \sum_\ell f_\ell\, \mathcal{M}_\ell|_{z^2=0} \quad \longrightarrow \quad H_{g/h}(x, \xi, t) = \int_{-\infty}^{\infty} d\omega\, e^{i\omega x} \mathfrak{F}(\omega, \xi, t)
$$

Toward determination of gluon GPDs from LQCD

- The projection problem: $(\mathcal{P}, M) = \mathcal{P}_{\mu\nu;\alpha\beta} M^{\mu\nu;\alpha\beta} = \sum_\ell f_\ell \mathcal{M}_\ell$
- For the pion (e.g. two possibilities):
  - $$(\mathcal{P}[\mathfrak{F}], M) = \frac{z_3^2 \xi^2}{\eta^2}(M_{0i;0i} + M_{ij;ij}) + \frac{4\omega\xi^2(\xi-\eta)z_3}{\eta^2\Delta_1}(M_{02;12} + M_{12;02})$$
  - $$(\mathcal{P}'[\mathfrak{F}], M) = \frac{z_3^2 \xi^2}{\eta^2} M_{0i;0i} + \frac{\xi^2[16\omega^2\xi(\xi-\eta) + z_3^2(4\xi^2 P^2 - t)]}{2\eta^2\Delta_1^2} M_{ij;ij} + \frac{4\omega\xi^2(\xi-\eta)z_3}{\eta^2\Delta_1}(M_{02;12} + M_{12;02})$$
- Difference between these projections:

$$(\mathcal{P}[\mathfrak{F}] - \mathcal{P}'[\mathfrak{F}], M) = \frac{\xi^2}{\eta^3\Delta_1^2}\left[16\omega^2\xi^2(\xi-\eta) - z_3^2\left(t\eta + 4P^2\xi^2(3\eta - 4\xi)\right)\right]\mathcal{M}_{gg} \quad \xrightarrow{\Delta\to 0} \quad \frac{\omega^2}{p_0^2}\mathcal{M}_{gg}$$

The projection problem does not provide a justification that certain projections “reduce” power corrections. Nevertheless, it is useful to identify, given a choice of basis, linear combinations of invariant amplitudes that gives the light cone distribution, so that their z^2-dependence may be studied explicitly on the lattice.

Gluon GPDs of the nucleon from LQCD

- A lot more complicated case, many tensor structures
    - Basis of 72 linearly independent tensors
    - Projection onto $\mathfrak{E}_g{}^T$:

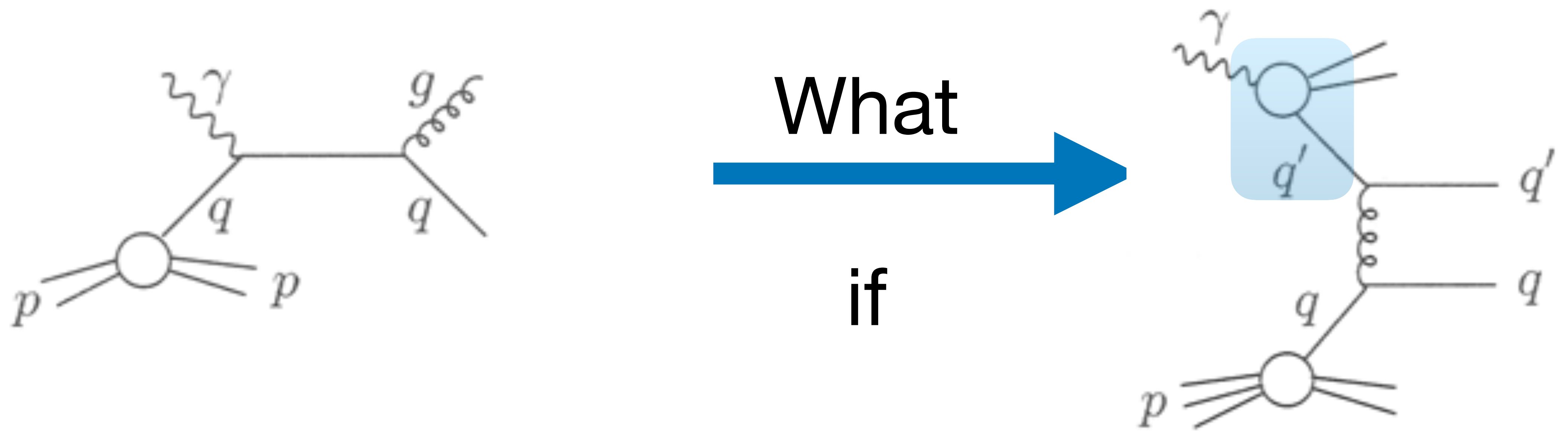
$$(\mathcal{P}[\mathfrak{E}_g{}^T], \mathrm{M}) = \frac{8m\xi^2 z_3^2(t-4m^2)}{3\Delta_1^2\eta^2 t\omega^2}\mathrm{tr}[\mathbf{M}_{01;01} - \mathbf{M}_{02;02} + \mathbf{M}_{12;12}] + \frac{4m^3\xi z_3^2}{\Delta_1^2\eta t\omega^2}\mathrm{tr}[i\gamma_5\mathbf{M}_{01;02} + i\gamma_5\mathbf{M}_{02;01}]$$
$$\times \frac{4m\xi z_3^2(2t-5m^2)}{3\Delta_1^2\eta^2 t\omega^2}\mathrm{tr}[i\sigma_{30}\mathbf{M}_{01;01} - i\sigma_{30}\mathbf{M}_{02;02} + i\sigma_{30}\mathbf{M}_{12;12}].$$

- Perturbative matching in progress with Jakob Schoenleber (LaMET and SDF)
- LQCD calculation in progress [with Engelhardt, Izubuchi, Lin, Schoenleber, Syritsyn]

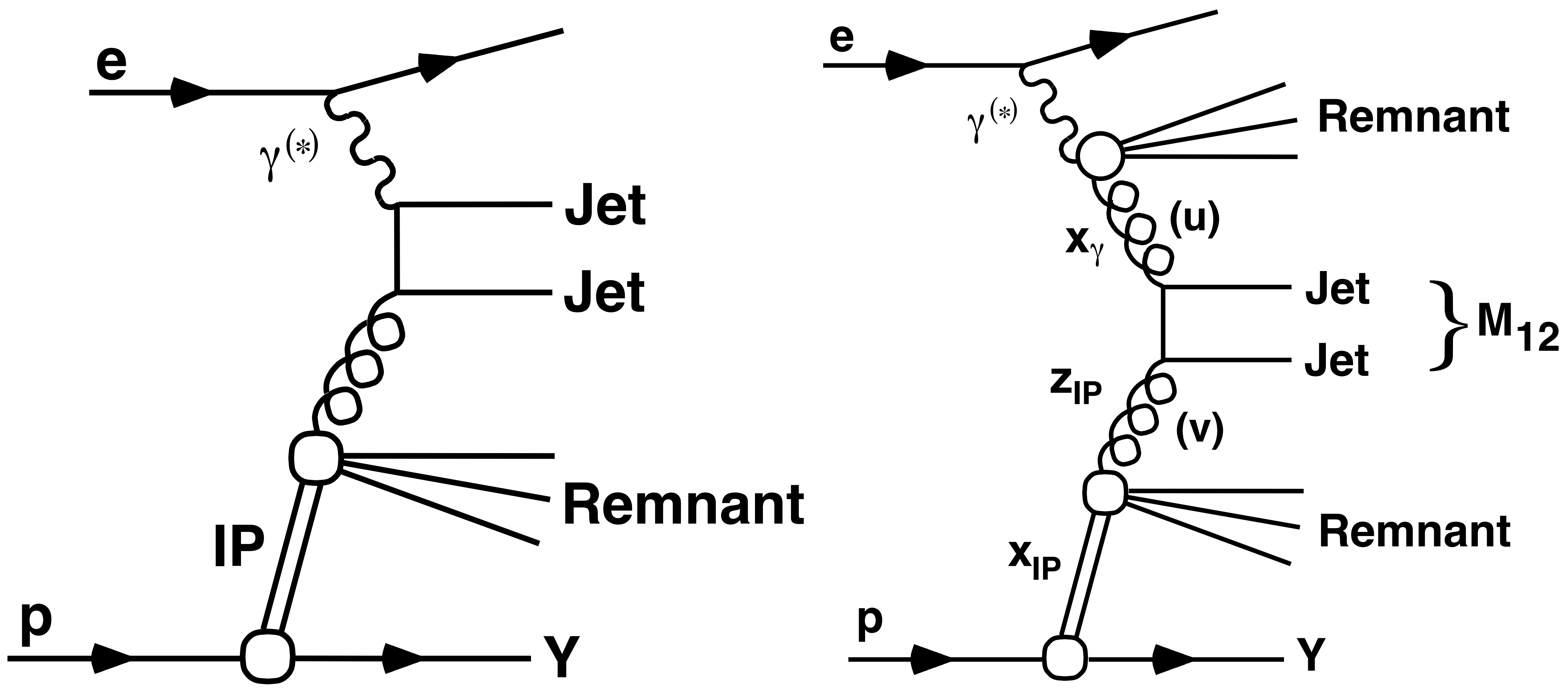
ONLY LQCD data will tell which operators have small power corrections or better than other choices

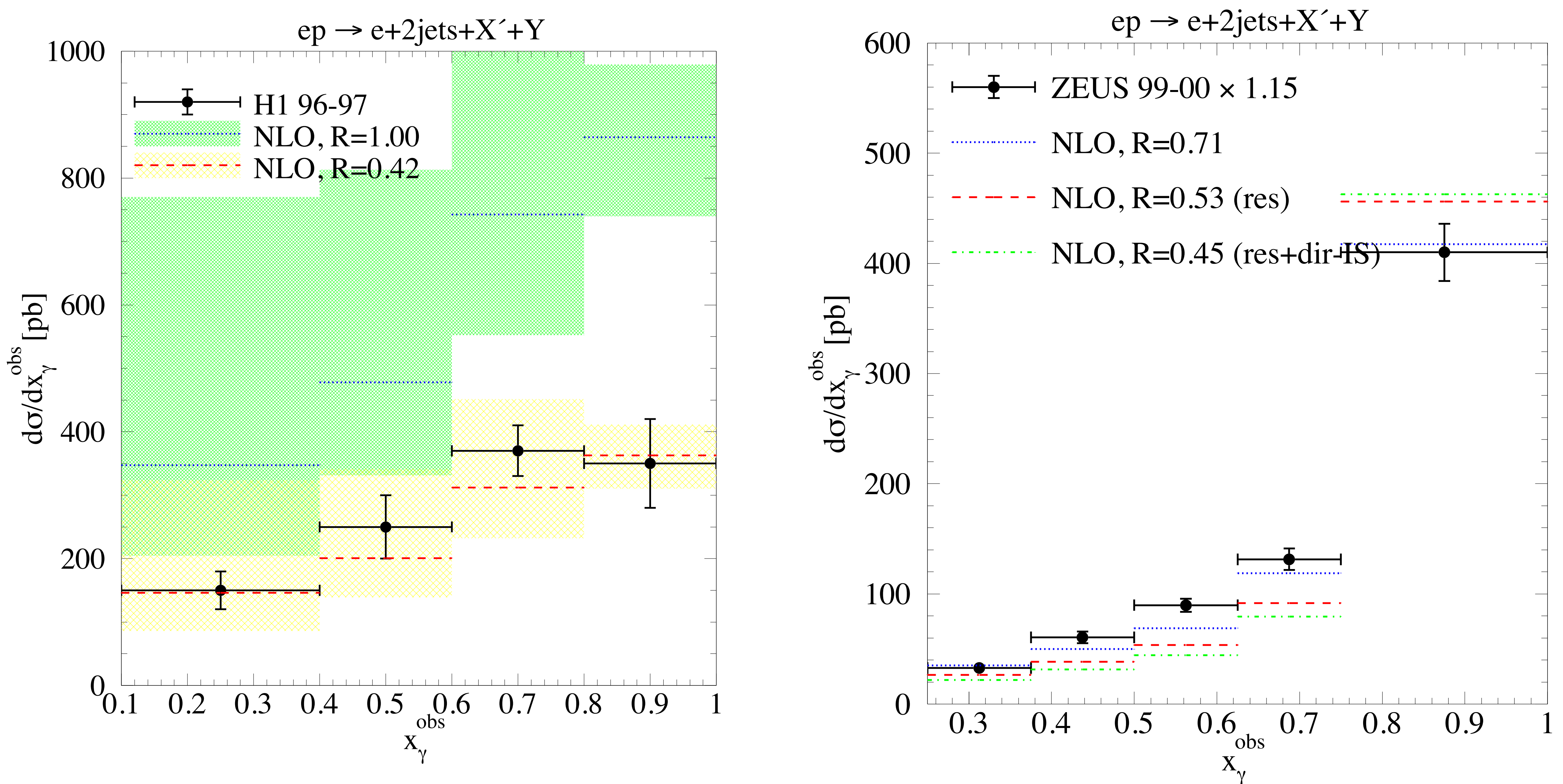
Toward hadronic structure of the “resolved” photon from LQCD

- QCD allows photon to exhibit a hadronic structure due to quantum fluctuations!



- Test of QCD factorization “breaking” (or lack of understanding of resolved photon structure)





Differential cross sections for dijet photo production measured by H1 and ZEUS [Phys. Lett. 2008]

Toward hadronic structure of the "resolved" photon from LQCD

- Critically important for EIC and constraining gluon PDF [Chu, Aschenauer, Lee 2017]

- LQCD calculation of moment and formalism for calculating 4-pt functions

X. Ji, C. Jung [PRL 2001, PRD 2001]

- First LQCD calculation of quasi-PDF in collaboration with Atlantis Moses* (NMSU), Masaaki Tomii**(BNL), Taku Izubuchi, Luchang Jin

  - Evaluate electromagnetic part of the Green's function in the lowest-order perturbation theory and write the matrix element

$$\langle\gamma(\vec{p})|\, O_f(z,0) \exp\left[-ieq_f \int_0^z A_\mu(t)dt_\mu\right] |\gamma(\vec{p})\rangle$$

$$\downarrow$$

$$h_f(z,\vec{p}) = -\frac{1}{2}e^2 \int d^4x \int d^4y \sum_{\lambda=1,2} e^{|\vec{p}|(x_t-y_t)-i\vec{p}\cdot(\vec{x}-\vec{y})} \langle J_\lambda(x) O_f(z,0) J_\lambda(y)^\dagger \rangle$$

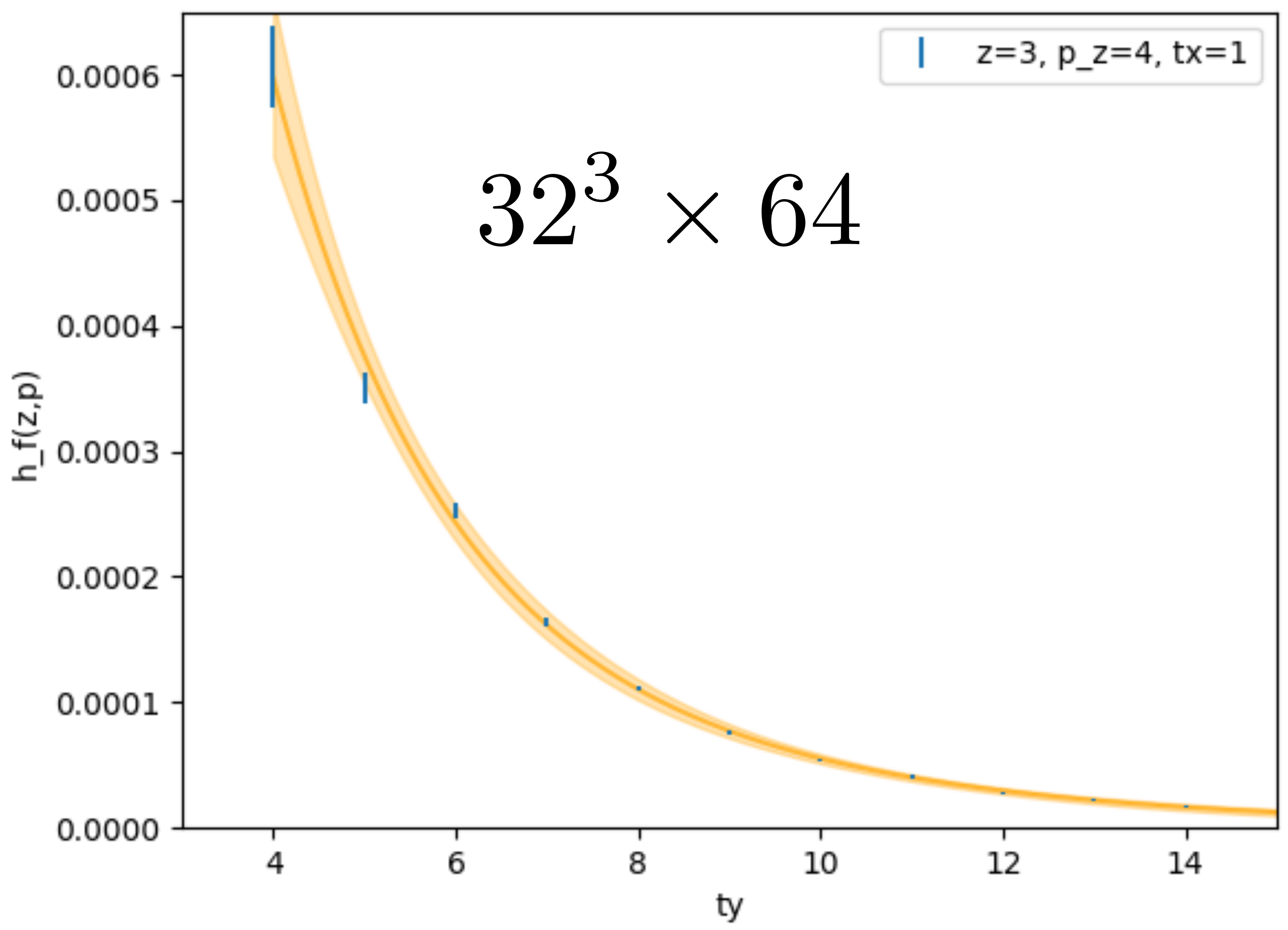
$$\boxed{O_f(z,0) = \bar{q}_f(z)\gamma_4 U(z,0) q_f(0)}$$

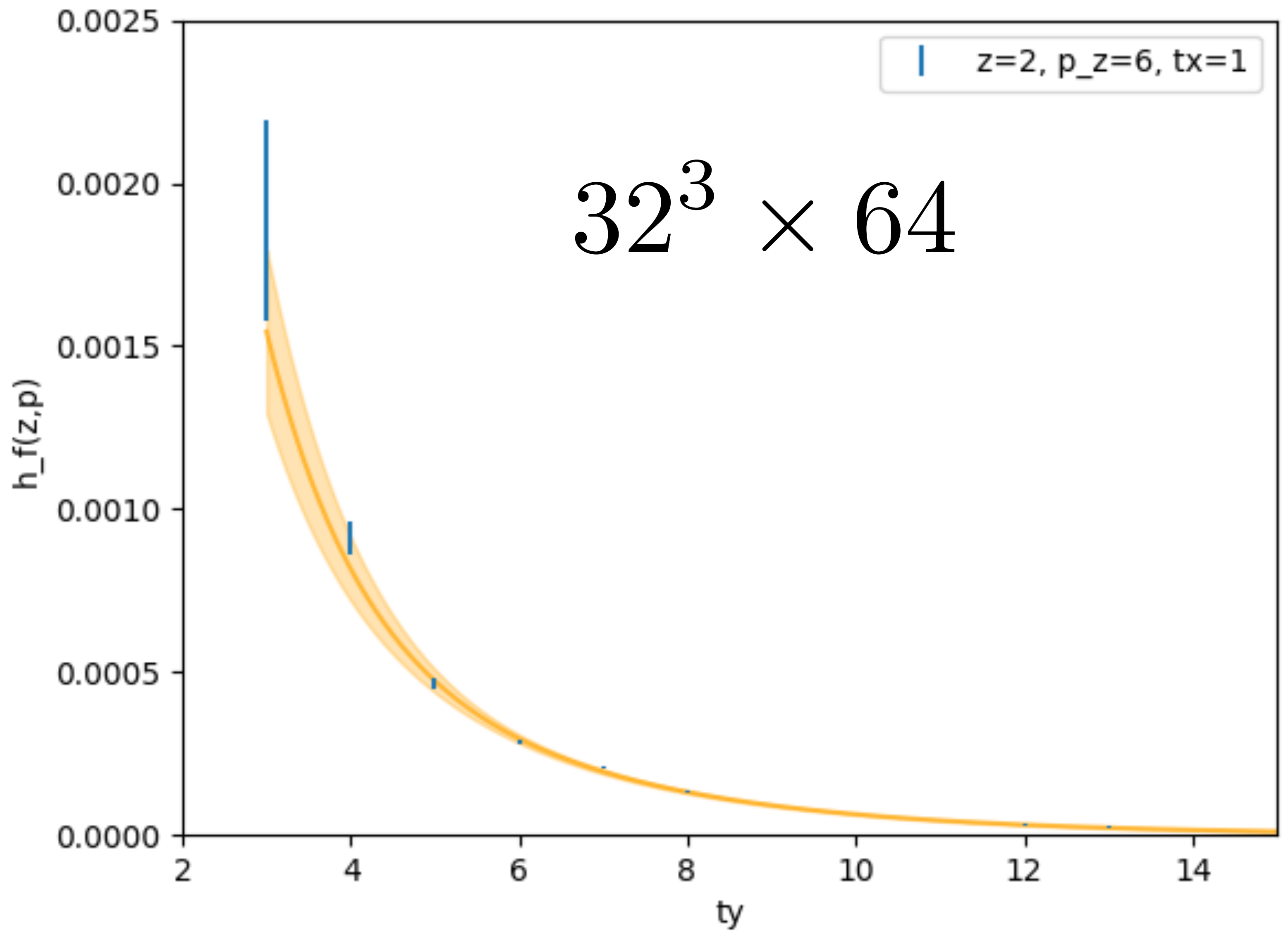
- RBC/UKQCD domain wall fermion

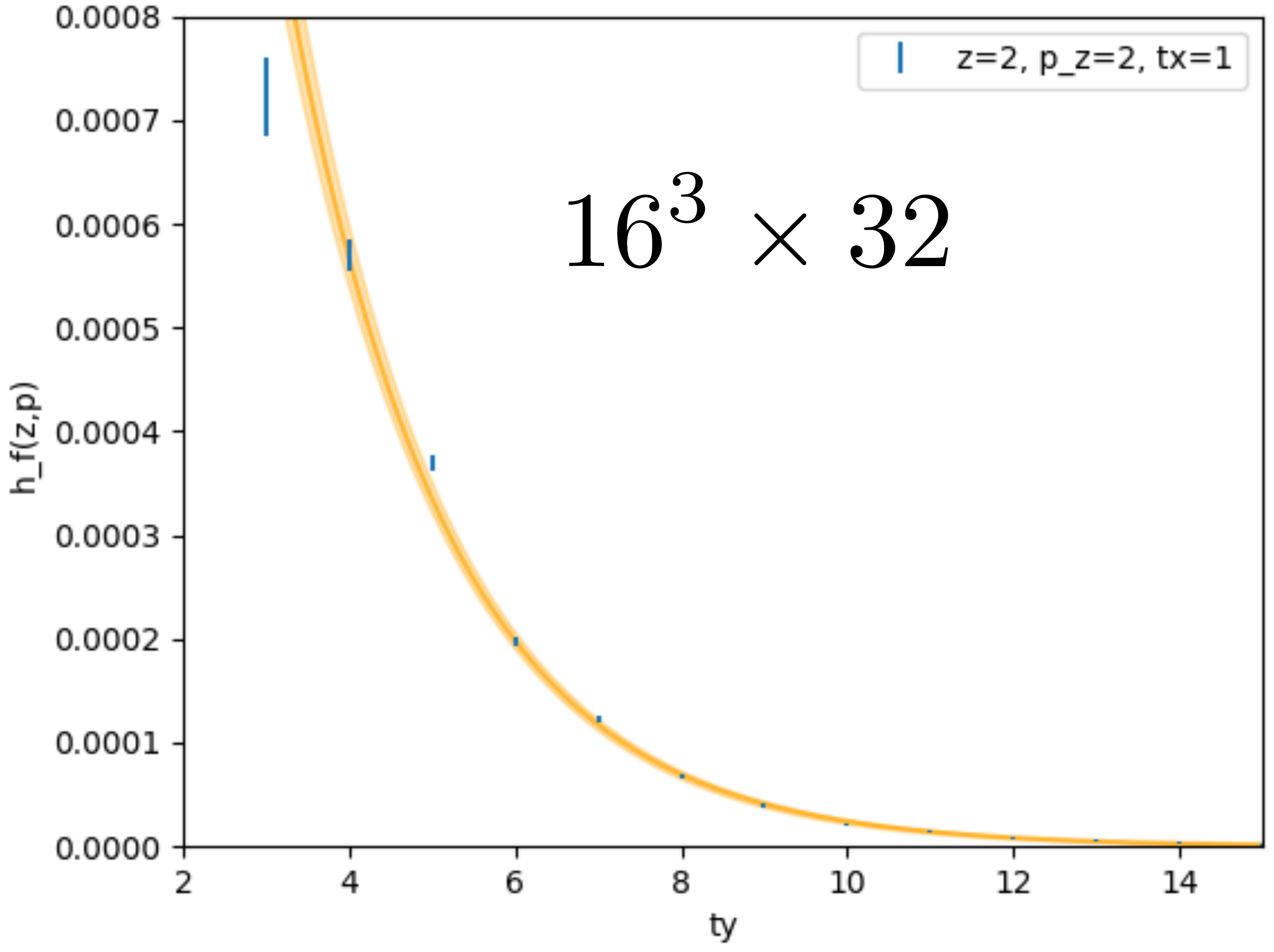
$$\left.\begin{array}{l} 16^3 \times 32\,(a \approx 0.12\,\text{fm}) \\ 32^3 \times 64\,(a \approx 0.063\,\text{fm}) \end{array}\right] \text{Data analysis}$$

▶ 3rd ensemble with a different lattice spacing

- Preliminary matrix elements:







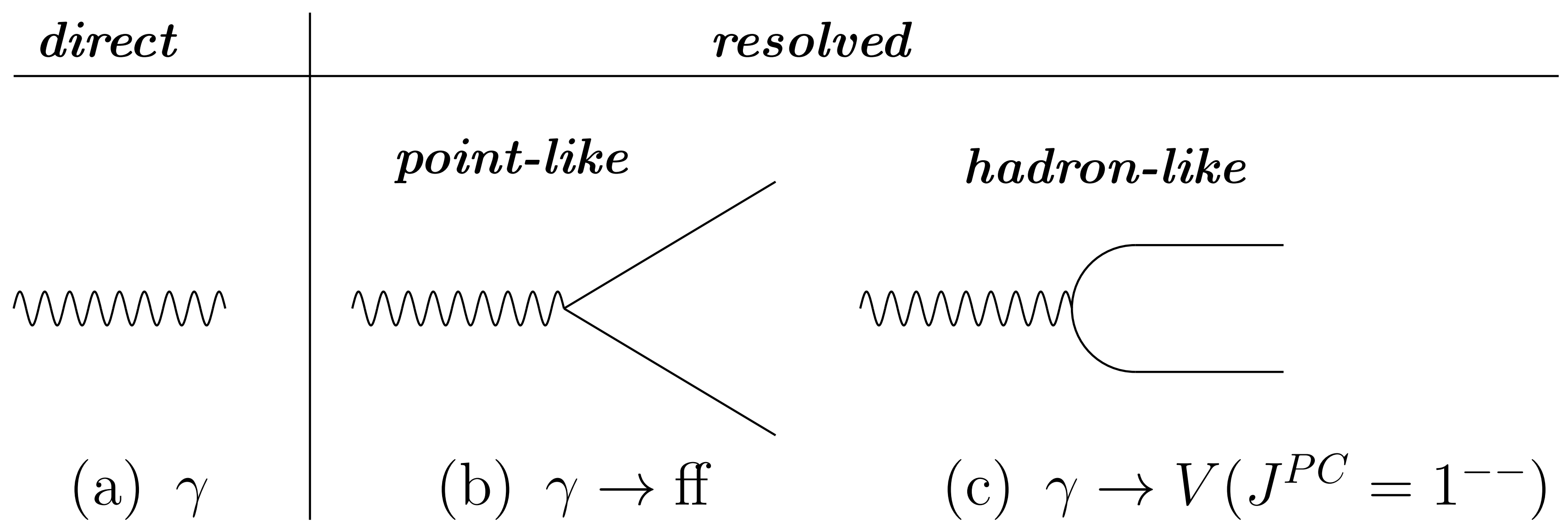
Toward hadronic structure of the “resolved” photon from LQCD

- Renormalization

$$O_\Gamma(z)_R = Z_O^{-1} e^{\delta \bar{m} z} O_\Gamma(z)$$

  - Ratio-renormalization [Radyushkin 2017] might not be suitable

- Possible choices of renormalization
  - Self renormalization [Ji, et. al. NPB 2021]
  - Other ideas

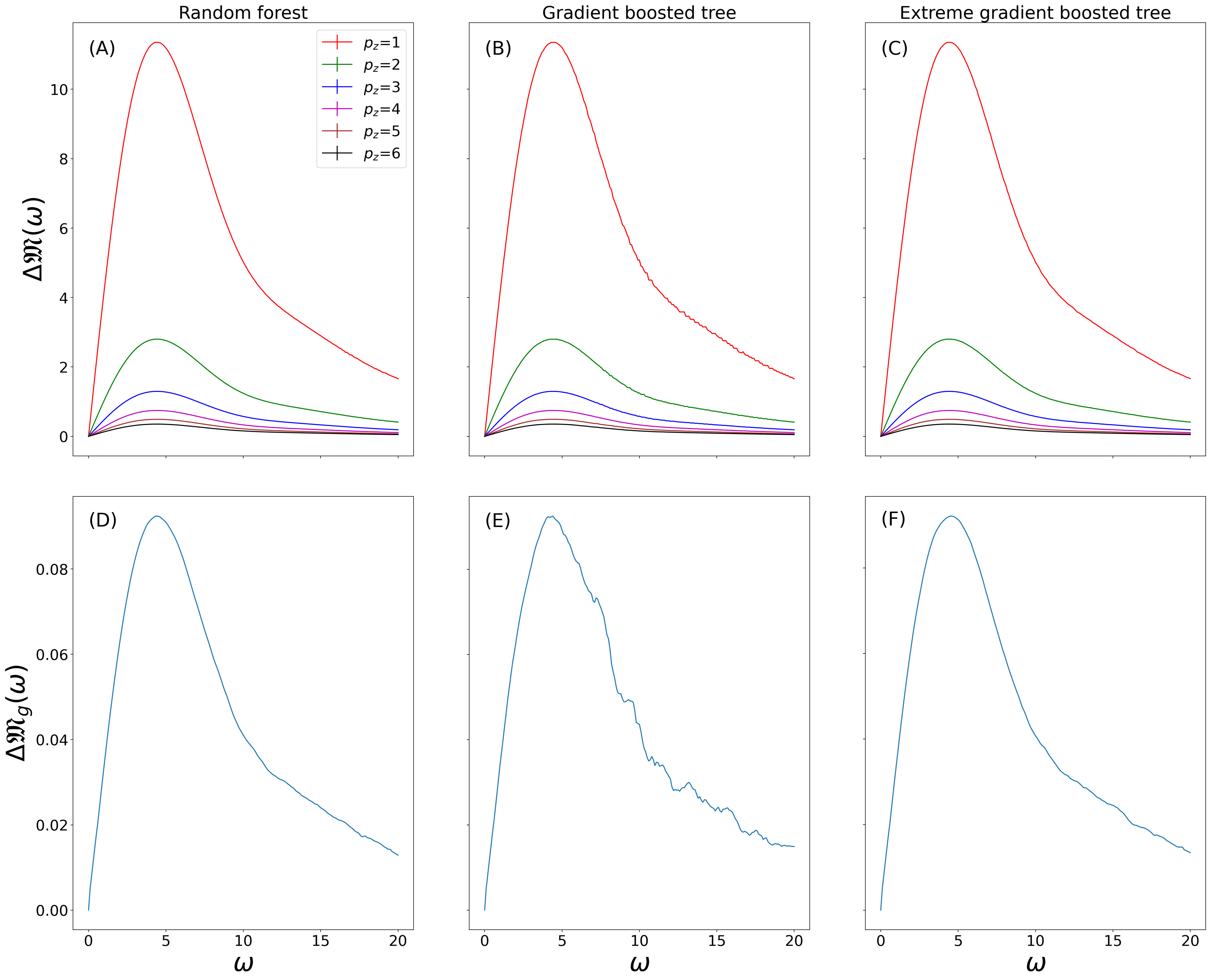


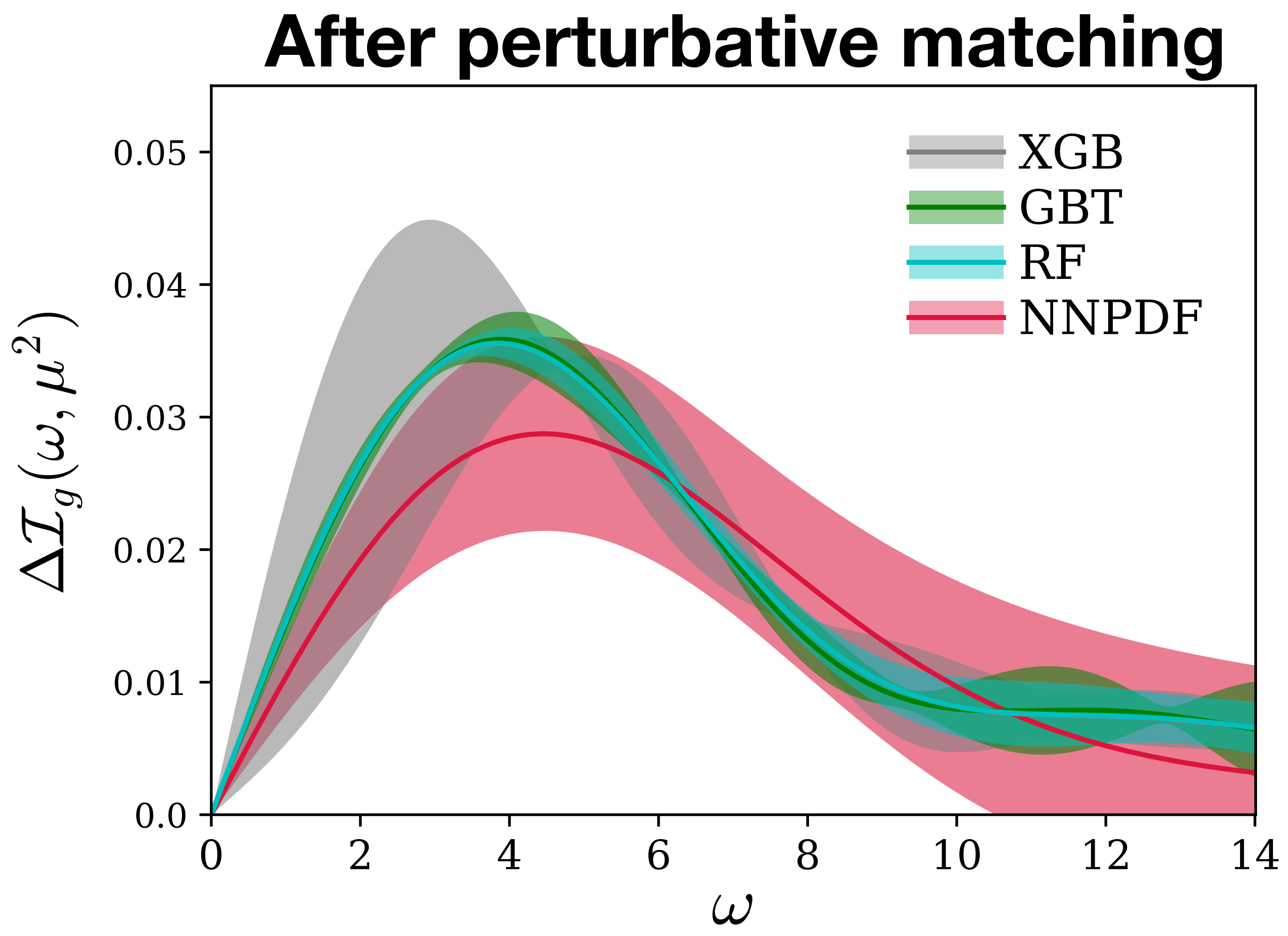
- LQCD calculation of the Hadronic tensor [K.F. Liu, PRL 94]
  - Positive scaling violation of F_2^γ at large x [Christian Zimmermann]

Thank you!

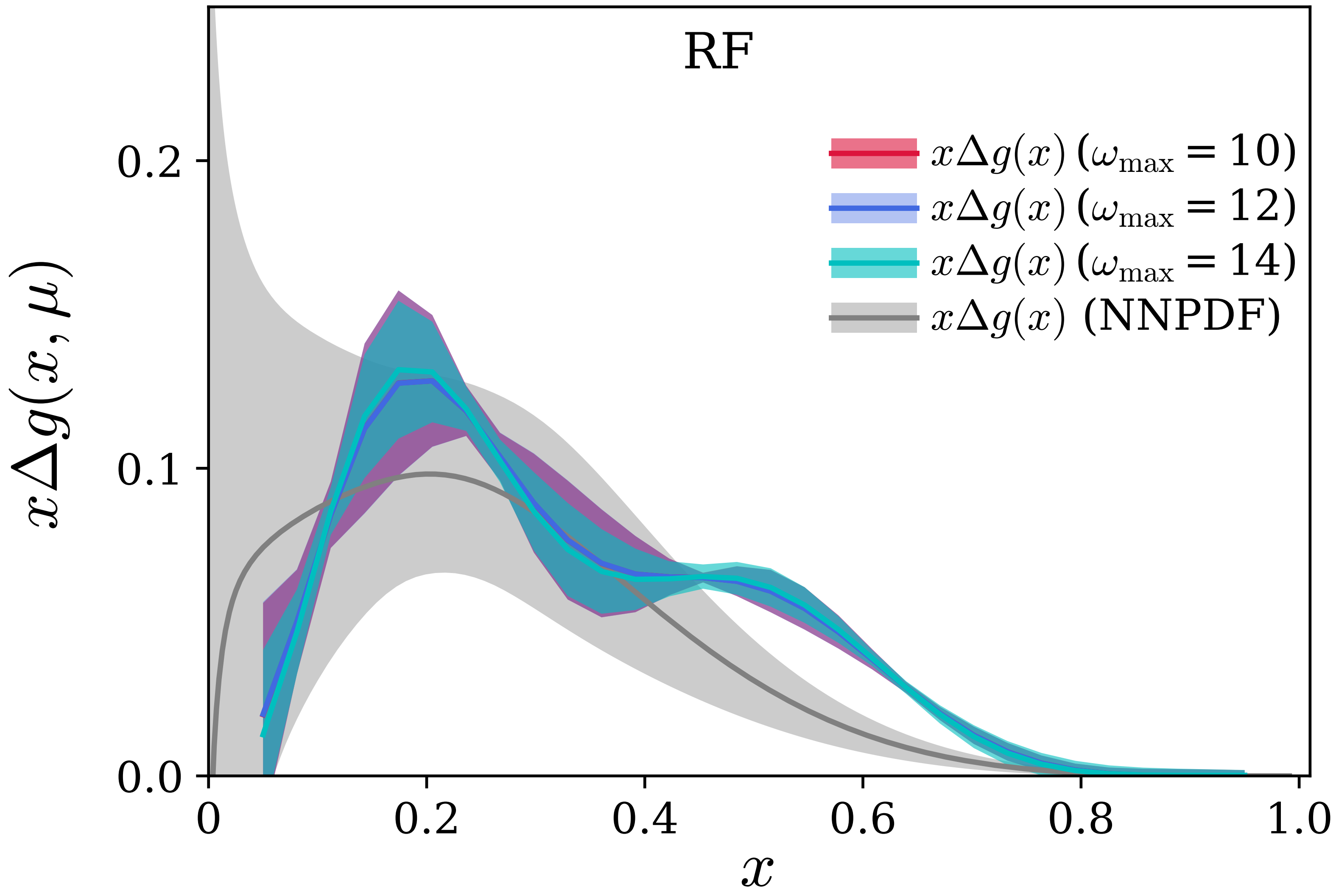
Impact of LQCD calculations on polarized gluon PDFs

- Physics informed machine learning + phenomenological constraint





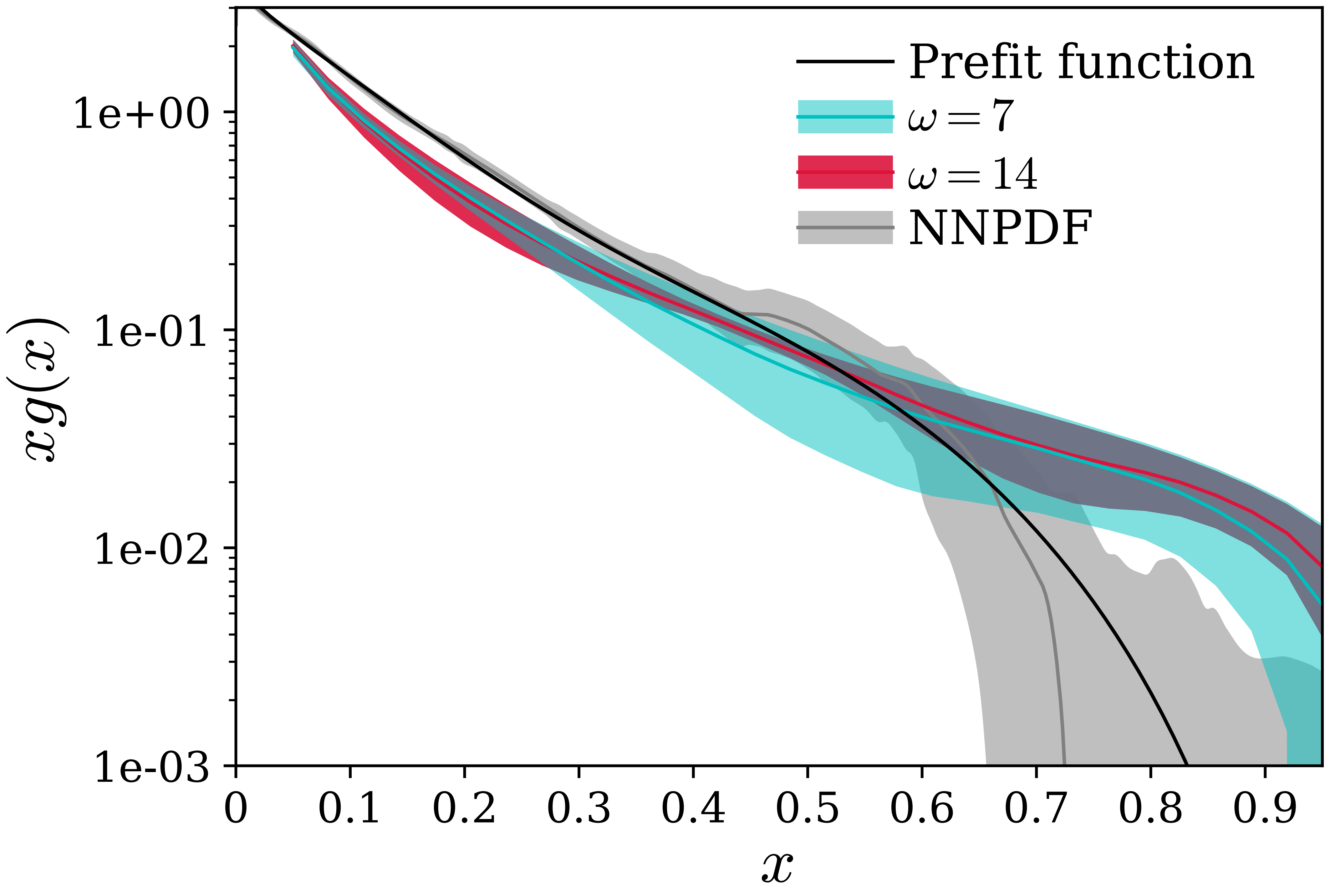
Fourier transform using Neural Network



- LQCD can provide strong constraint on the gluon helicity PDF

ML application to unpolarized gluon PDF

- Insight into unpolarized gluon PDF in the mid-to-moderately large x region

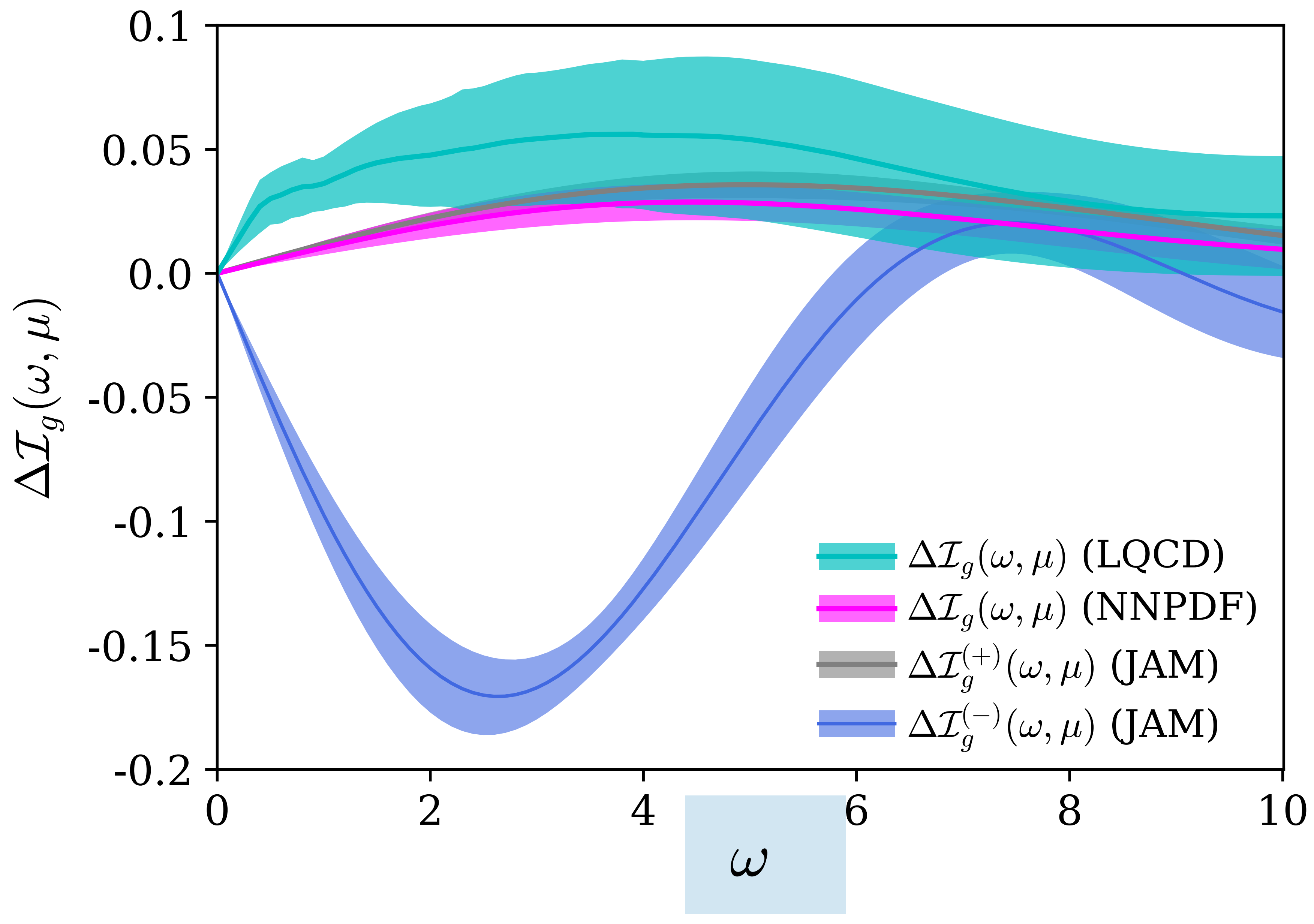


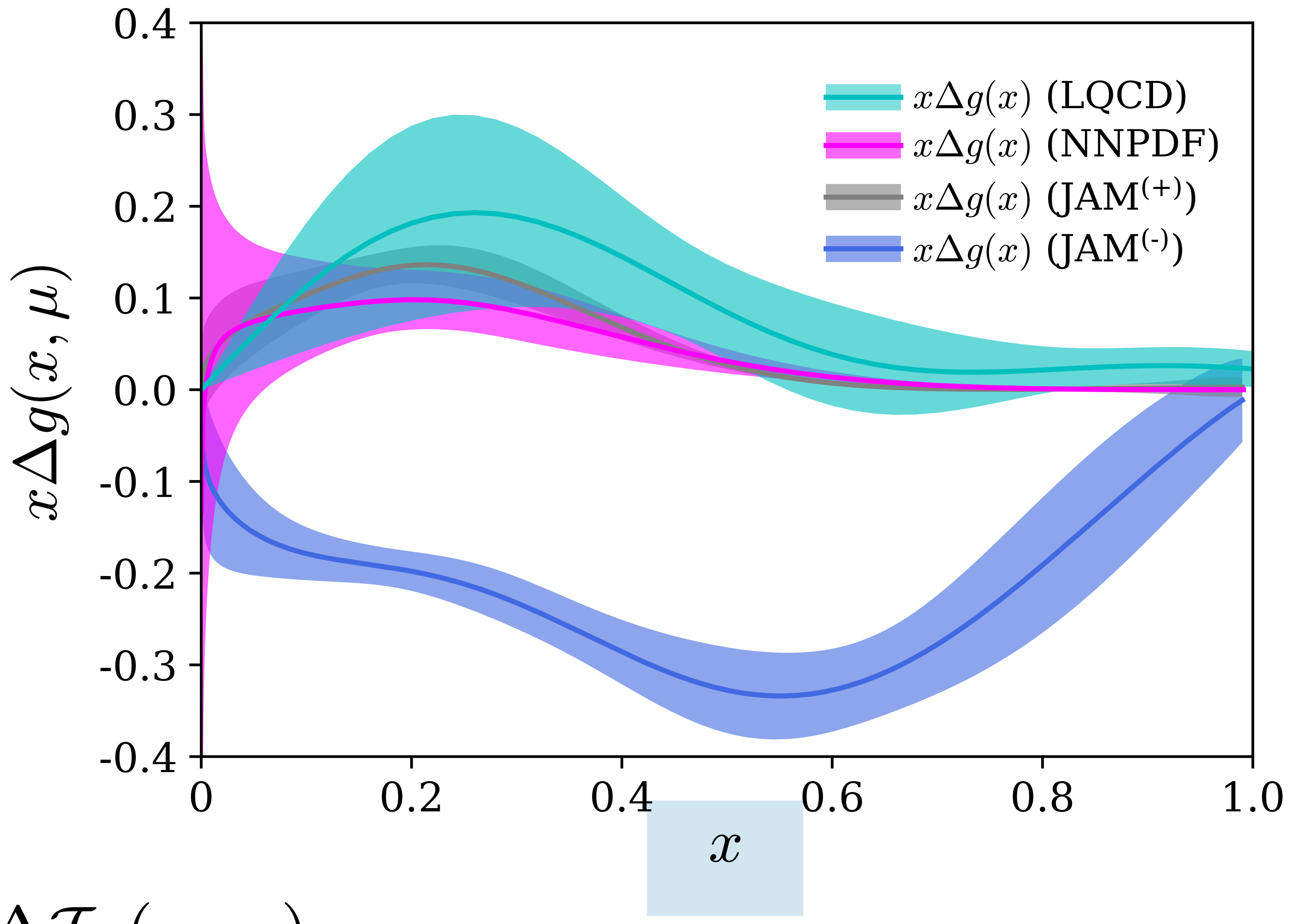
Kamruzzaman (Sandia National Lab), RSS, et al [2409.17234]

- Caution: Physical pion mass, continuum and infinite volume limits to be considered in future calculations

- Only LQCD (with application of **machine learning**) calculation

***RSS*, Liu, Khan [PRD 2023]**





$$x\Delta g(x,\mu) = \frac{2}{\pi}\int_0^\infty d\omega\ \sin(x\omega)\ \Delta\mathcal{I}_g(\omega,\mu)$$

$$\Delta G^L(\mu) \equiv \int_0^{\omega_{\max}} d\omega\ \Delta\mathcal{I}_g(\omega,\mu) = 0.405(196)(081)$$

- Negative gluon polarization is disfavored at the level of matrix elements